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SECONDARY STRESSES IN TRUSSES

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REVISIONS

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1.0 INTRODUCTION

A widely used method for the analysis of two-dimensional fixed joint trusses is presented in References 1-4, and is commonly called the method of secondary stresses. In this approach, the truss is first analyzed under the assumption that all members meeting at each joint are connected by frictionless pins. Next, it is assumed that the joints of the structure are "locked" i.e., that all members meeting at a joint rotate through the same angle. Further assumptions are then made: (a) that the axial (or, "primary") stresses previously computed have negligible effect upon member stresses from bending, and (b), that the bending (or, "secondary") stresses have negligible effect in modifying the primary stresses. Thus, from a computation of joint displacements followed by a moment distribution,[1] the secondary stresses may be found. This calculation of secondary stresses may be iterated, and then it is seen that successive iterations converge in an alternating manner to some true solution. The method of secondary stresses obviates the need for solving a large set of simultaneous equations; but this is offset by the labor of the computation of joint deflections and moment distribution. Moreover, it seems impractical to apply this technique to a three dimensional structure.

The availability of large computers has made it feasible to analyze a truss (two or three dimensional) by direct application of the energy theorems of structural analysis. For illustration, the following sample problem (from Ref. 2) is solved by the method of secondary stresses and by a direct method, which has been programmed and is available to all Stress Groups at Astronautics (Program No. 2785, Report No. ERR-AN-206).

[1] For an unsymmetrical loading condition, the moment distribution must be modified to account for sidesway. See Ref. 2, page 461 for details.

2.0 SECONDARY STRESS CONCEPT

For an adequate understanding of the method of secondary stresses it is important that the mechanism resulting from the assumptions be explained.

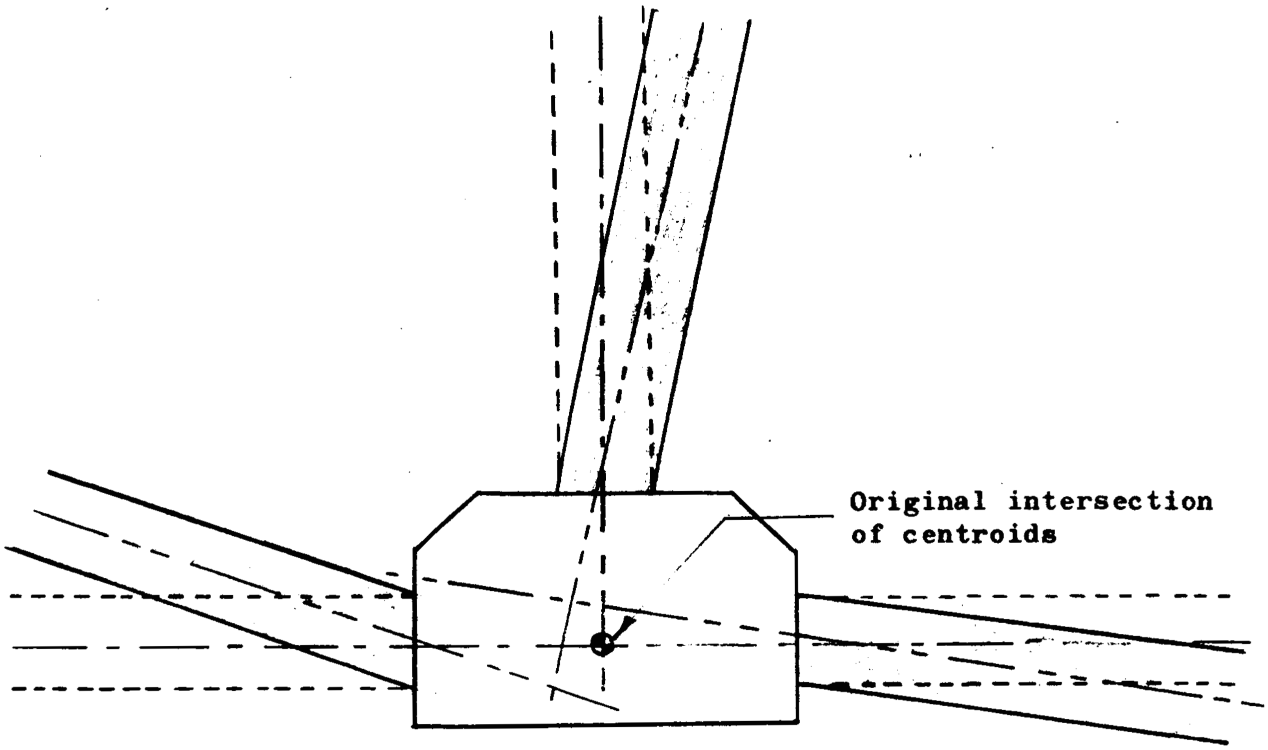


Figure 1 - Fixed Truss - Joint Relaxed

The joint in Figure 1 (1) is taken as a typical fixed joint in a truss. Initially the centroids of all the members framing into the unloaded joint of the structure are assumed to intersect. Allowing for pin joint action at the joint, the members will deflect to the shaded position and their centroids to the doubly dashed center line position. If the joint is truly fixed the relative angles between each of the members can not

(1) In figures 1 and 2 the rotation of the gusset is not shown.

change. Only the entire joint can rotate to an equilibrium position. Hence, after the axial loads in the members have been calculated from the pin joint analysis, the continuity at the "fixed" joint must be restored. To accomplish this, the relative end deflection or rotation of each member must be computed. From these displacements, fixed end moments are calculated and the members are allowed to rotate elastically until the joint continuity is restored. The initial and final deflected positions of the fixed joint are shown in Figure 2.

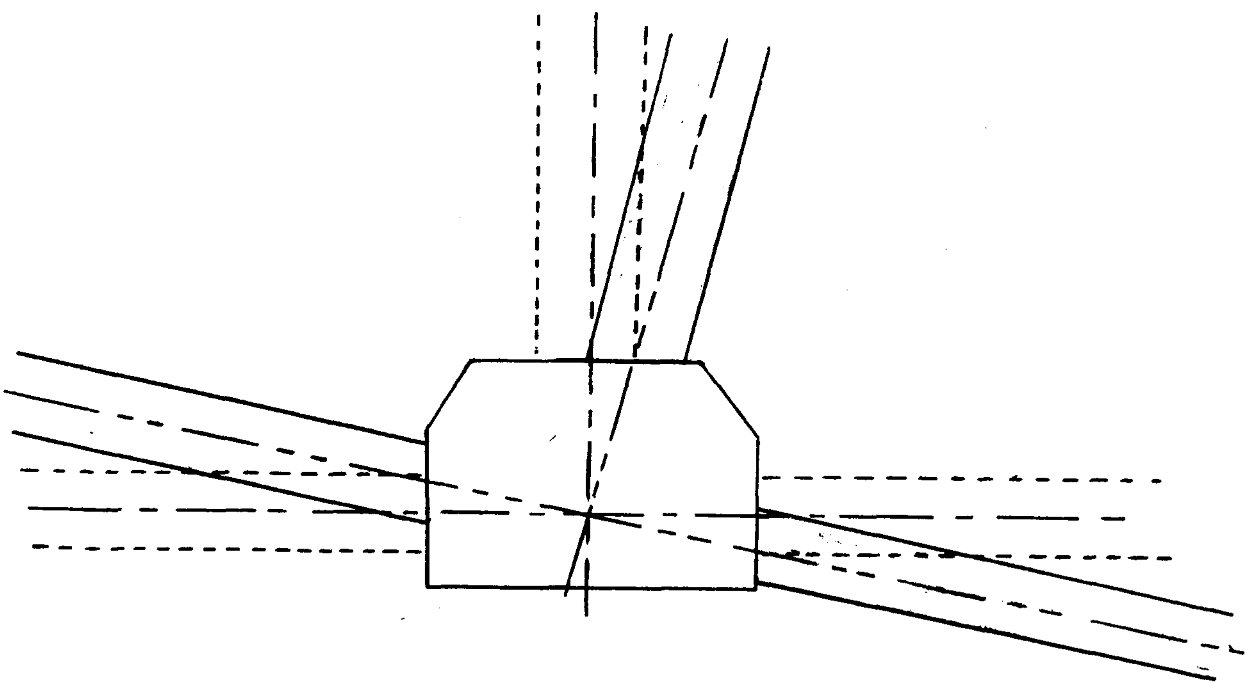


Figure 2 - Fixed Truss - Joint Unloaded and Loaded

3.0 FIXED JOINT TRUSS EXAMPLE

The example solved by the "secondary stress" method and by Astronautics Program No. 2785 is shown in Figure 3. All joints are rigid and all external load is applied at the joints. The structure is assumed to be geometrically and physically symmetric about the vertical member C c. With this condition realized, one-half of the truss may be analyzed after properly constraining the structure at C, c, and a.

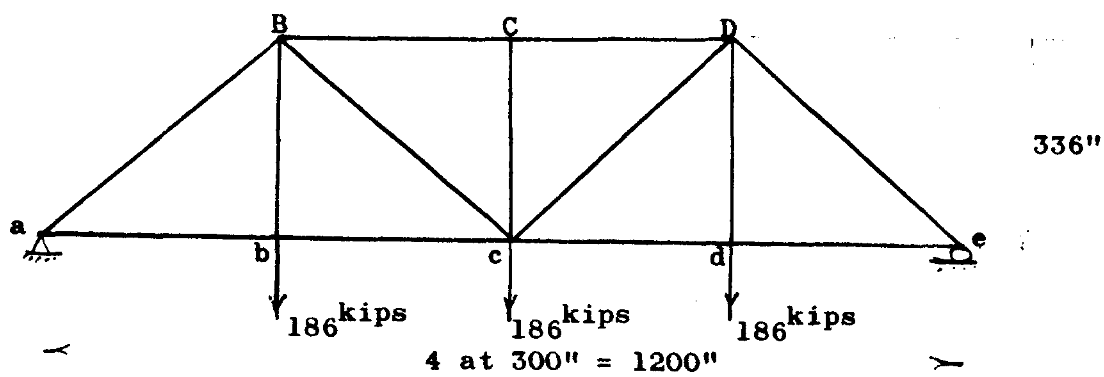


Figure 3 - Example - Fixed Joint Truss

3.1 Manual Solution

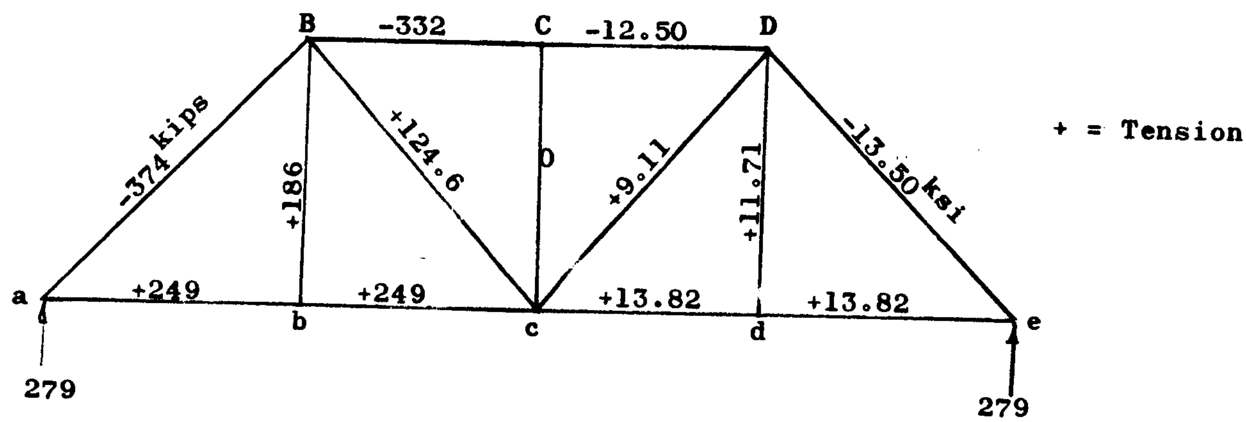


Figure 4 - Bar Forces and Stress Intensities

TABLE I

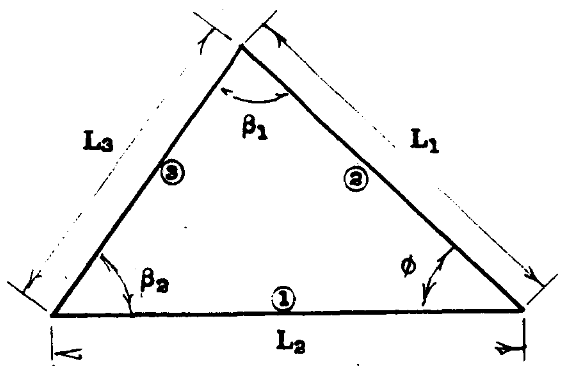
Geometric Properties

BAR	L (in) Length	A(in²) Area	I (in⁴) Inertia	K (in³) Stiffness (Rel.)
ab	300	18.0	174.9	0.583
bc	300	18.0	174.9	0.583
Bb	336	15.88	153.4	0.456
Bc	450.4	13.68	131.4	0.292
Cc	336	11.44	78.9	0.235
aB	450.4	27.68	960.9	2.134
BC	300	26.55	922.7	3.076

The primary bar forces (pin joint analysis) are shown in the line diagram in Figure 4; truss element geometric (and section) properties are presented in Table I. The next step is to compute the Ψ angles for obtaining the fixed end moments (FEM's). These Ψ angles will be calculated by the bar chain method shown in Reference 2.

$$E \Delta \phi = (\sigma_3 - \sigma_1) \cot\beta_1 + (\sigma_3 - \sigma_2) \cot\beta_2$$

where:



β_1 = angle opposite side ①

β_2 = angle opposite side ②

ϕ = angle opposite side ③

The Ψ angles must be computed in order to calculate the fixed end moments for the moment distribution. The angle Ψ is the rotation of the chord joining the ends of the elastic curve referred to the original direction of the member. Ψ is positive when the chord of the elastic curve has rotated clockwise from its original direction. Since the member Cc is assumed to remain vertical after loading, the final position of all the joints may be determined by algebraically summing the $E\Delta\phi$'s which gives the Ψ for each member. The $E\Delta\phi$'s are calculated as in Table II. The Ψ's are calculated as shown in Table II and are then used to calculate the initial end moment (see Table III). Figure 5, following Table III, presents the moment distribution iterations. A comparison of the final iteration and the computer program (No. 2785) results is presented in figures 6 and 7.

TABLE II

Computation of Angle Changes

Angle	$\sigma_3 - \sigma_1$	cot β_1	$\sigma_3 - \sigma_2$	cot β_2	1st Term	2nd Term	$E\Delta\phi$
B-a-b	11.71 + 13.50 = +25.21	1.12		0	28.25	0	+28.25
b-B-a	13.82 + 13.50 = 27.32	0.893		0	24.40	0	+24.40
a-b-B	-13.50 - 13.82 = -27.32	0.893	-13.50 - 11.71 = -25.21	1.12	-24.40	-28.25	-52.65
c-B-b	+13.82 - 9.11 = 4.71	0.893		0	+4.21	0	+4.21
B-b-c	+9.11 - 11.71 = -2.60	1.12	9.11 - 13.22 = -4.11	0.893	-2.91	-4.21	-7.12
b-c-B		0	11.71 - 9.11 = 2.60	1.12	0	+2.91	+2.91
C-B-c		0	0 -9.11 = -9.11	1.12	0	-10.20	-10.20
c-C-B	+9.11 - 0 = +9.11	1.12	9.11 + 12.50 = 21.61	0.893	+10.20	+19.30	+29.50
B-c-C		0	-12.50 - 9.11 = -21.61	0.893	0	-19.30	-19.30

Then

$E\Psi_{Cc}$	=	0	$E\Psi_{aB}$	=	+47.91
$E\Delta\phi_{BcC}$	=	-19.30	$E\Delta\phi_{aBb}$	=	+24.40
$E\Psi_{Bc}$	=	+19.30	$E\Psi_{Bb}$	=	23.51
$E\Delta\phi_{Bcb}$	=	2.91	$E\Delta\phi_{bBc}$	=	4.21
$E\Psi_{bc}$	=	+16.39			
$E\Delta\phi_{Bbc}$	=	-7.12	$E\Psi_{Bc}$	=	+19.30
$E\Psi_{Bb}$	=	+23.51	$E\Delta\phi_{cBC}$	=	-10.20
$E\Delta\phi_{abB}$	=	-52.65			
$E\Psi_{ab}$	=	+76.16	$E\Psi_{BC}$	=	+29.50
$E\Delta\phi_{Bab}$	=	28.25	$E\Delta\phi_{BCc}$	=	+29.50
$E\Psi_{aB}$	=	+47.91			0

TABLE III

Computation of Initial End Moments

BAR	K	EΨ	-6EKΨ
ab	0.583	+76.16	-266.0
bc	0.583	+16.39	-57.3
aB	2.134	+47.91	-613.5
BC	3.076	+29.50	-544.0
Bb	0.456	+23.51	-64.3
Cc	0.235	0	0
Bc	0.292	+19.30	-33.8

End moments are positive when they act clockwise on the ends of the member (for orientation, look at the joint).

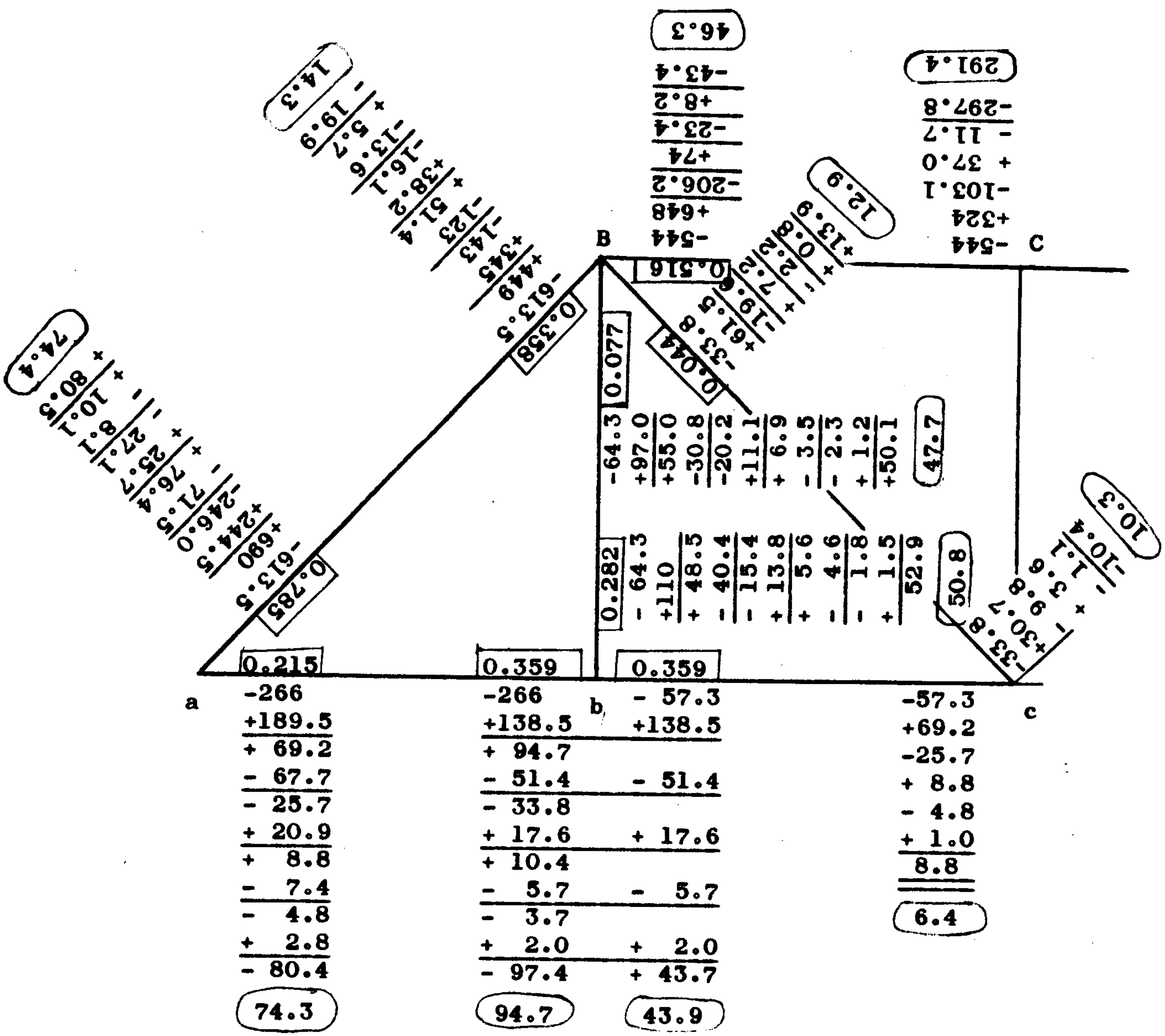


Figure 5 - Moment Distribution on Example Truss

NOTE: Circled final moments are from computer program (No. 2785) results

Circled values are Program 2785 moments at the joints. Uncircled moments are those obtained after one iteration of moment distribution. See Fig. 5 for sign convention.

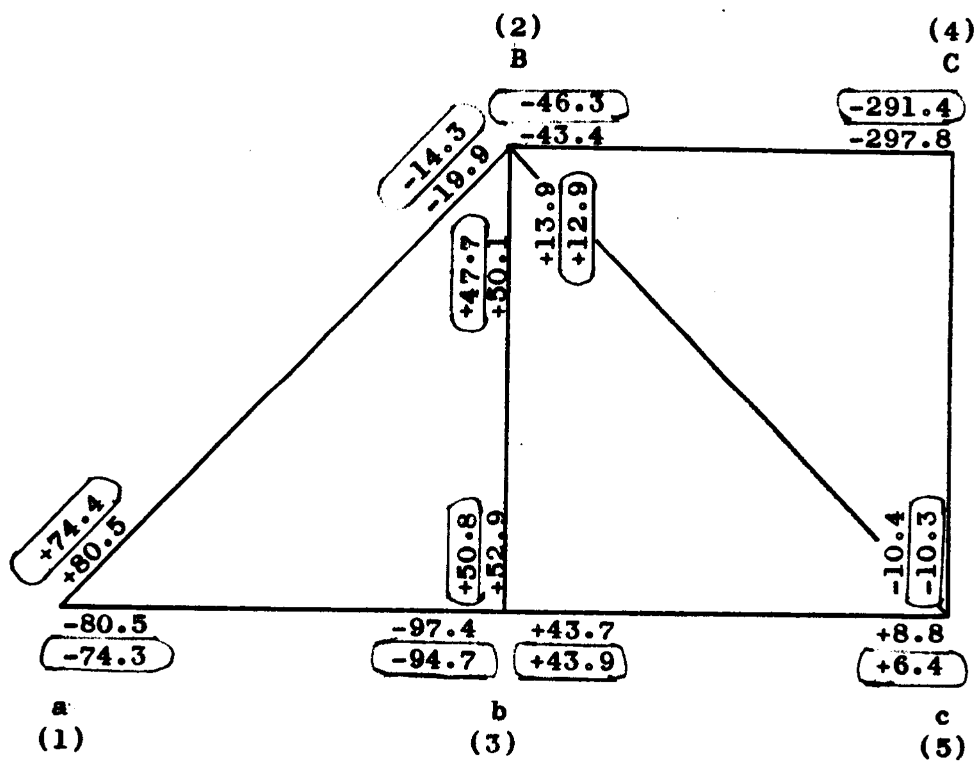


Figure 6 - Final Joint Moments (Kip-in)

Circled values are Program Axial Load results. Others are Axial Loads corrected after the moment distribution. (Positive values denote tension.)

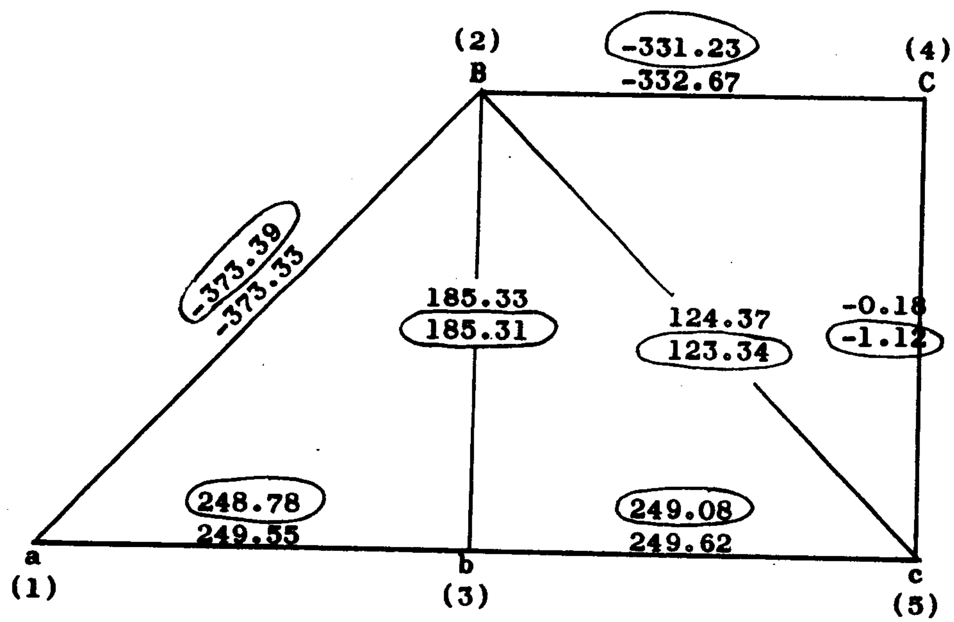
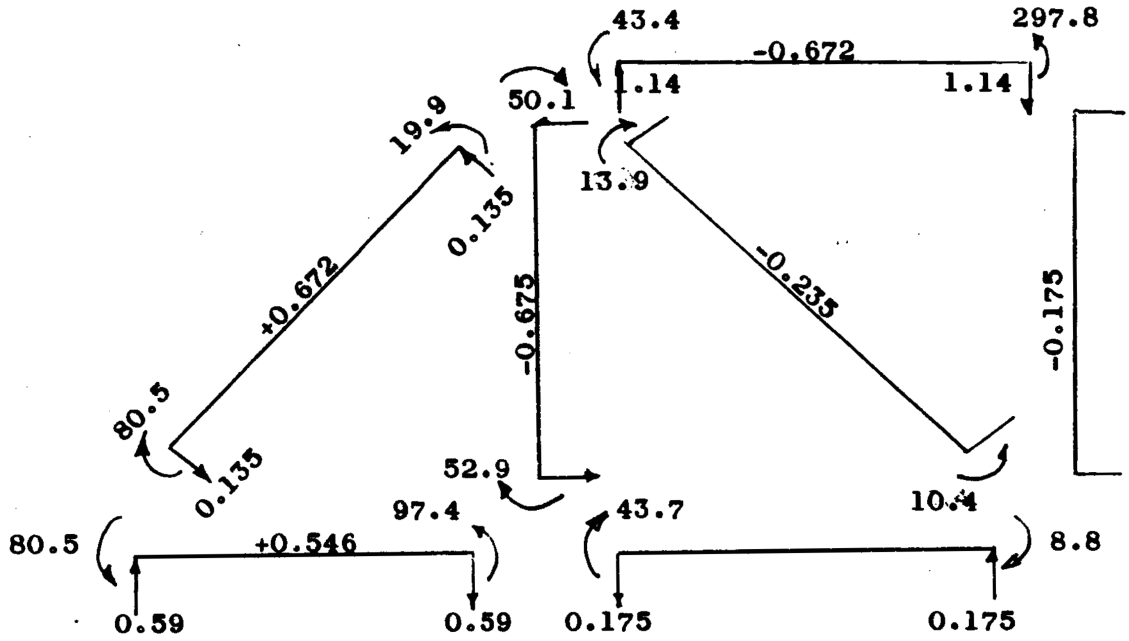


Figure 7 - Final Axial Loads



The calculation performed above shows the shears induced by the fixed end moments and the resultant secondary axial loads induced by the fixed condition. These axial loads are sufficiently small so that no correction of the pin joint axial loads is necessary.

The approximate moments obtained by the first trial moment distribution are somewhat higher than those obtained from the direct approach of the program. As shown in Reference 3 and elsewhere, each cycle of moment distribution will give some alternating answer about the true solution. If several cycles are carried out it would be apparent that the approximate approach converges to the answer obtained by direct application of the Energy Theorems.

3.2 7090/4 Program Solution

The following two pages present print out of the problem input data and a summary of the essential output.

GENERAL TRUSS INPUT INFORMATION - BAY 1

NO. POINTS	NO. BOUNDARY PTS.	NO. GROUND PTS.	NO. SEGMENTS	NO. CONSTRAINTS
5	-0	-0	7	9

POINT	X	Y	Z
1	0.	0.	0.
2	0.	3.00000+02	3.36000+02
3	0.	3.00000+02	0.
4	0.	6.00000+02	3.36000+02
5	0.	6.00000+02	0.

PROPERTIES OF MEMBERS

MEMBER		AREA	X-AREA	Z-AREA	J-FACTOR	I(X)	I(Z)	L	M	N	G	E
1	2	2.768+01	-0.	-0.	10.000+13	9.609+02	10.000+13	0.	90.00	90.00	1.200+07	3.000+07
1	3	1.800+01	-0.	-0.	10.000+13	1.749+02	10.000+13	0.	90.00	90.00	1.200+07	3.000+07
2	3	1.588+01	-0.	-0.	10.000+13	1.534+02	10.000+13	0.	90.00	90.00	1.200+07	3.000+07
2	4	2.655+01	-0.	-0.	10.000+13	9.227+02	10.000+13	0.	90.00	90.00	1.200+07	3.000+07
2	5	1.368+01	-0.	-0.	10.000+13	1.314+02	10.000+13	0.	90.00	90.00	1.200+07	3.000+07
3	5	1.800+01	-0.	-0.	10.000+13	1.749+02	10.000+13	0.	90.00	90.00	1.200+07	3.000+07
4	5	5.720+00	-0.	-0.	10.000+13	3.945+01	10.000+13	0.	90.00	90.00	1.200+07	3.000+07

APPLIED LOADS FOR LOADING CONDITION 1

POINT	LOAD	POINT	LOAD	POINT	LOAD	POINT	LOAD
33	-1.86000+05	53	-9.30000+04				

CONSTRAINTS

POINT	CONSTRAINT	POINT	CONSTRAINT	POINT	CONSTRAINT	POINT	CONSTRAINT
42	10.0000+13	44	10.0000+13	46	10.0000+13	52	10.0000+13
54	10.0000+13	56	10.0000+13	11	10.0000+13	13	10.0000+13
15	10.0000+13						

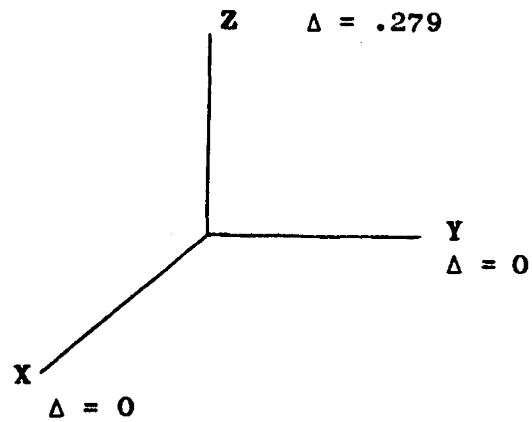
GENERAL TRUSS OUTPUT INFORMATION - BAY 1

LOADING CONDITION 1

JOINT	DISPLACEMENT (X)	DISPLACEMENT (Y)	DISPLACEMENT (Z)	ROTATION (X)	ROTATION (Y)	ROTATION (Z)
1	-0.	-2.76588-01	-2.79000-09	-2.02058-03	-0.	0.
2	-0.	1.24756-01	-6.29867-01	-1.32770-03	-0.	0.
3	-0.	-1.38376-01	-7.60565-01	-1.43848-03	-0.	0.
4	-0.	3.31226-09	-9.20531-01	-2.91375-09	-0.	0.
5	-0.	-3.31226-09	-9.22735-01	-1.67564-10	-0.	0.

MEMBER	LOAD (X)	LOAD (Y)	LOAD (Z)	MOMENT (X)	MOMENT (Y)	MOMENT (Z)
1	-0.	3.73389+05	-1.33316+02	-7.43679+04	-0.	0.
2	0.	-3.73389+05	1.33316+02	1.43171+04	0.	0.
1	-0.	-2.48783+05	5.63660+02	7.43679+04	0.	0.
3	0.	2.48783+05	-5.63660+02	9.47300+04	0.	0.
2	-0.	-1.85311+05	-2.93476+02	-4.77868+04	0.	0.
3	0.	1.85311+05	2.93476+02	-5.08211+04	-0.	0.
2	-0.	3.31226+05	1.12579+03	4.63610+04	0.	0.
4	0.	-3.31226+05	-1.12579+03	2.91375+05	0.	0.
2	-0.	-1.23339+05	-5.64739+00	-1.28912+04	0.	0.
5	0.	1.23339+05	5.64739+00	1.03474+04	-0.	0.
3	-0.	-2.49075+05	-1.24999+02	-4.39088+04	0.	0.
5	0.	2.49075+05	1.24999+02	6.40903+03	0.	0.
4	-0.	-1.12579+03	-1.91330-04	-4.18164-02	0.	0.
5	0.	1.12579+03	1.91330-04	-2.24705-02	-0.	0.

3.3 STATIC CHECK OF PROGRAM OUTPUT



Vertical Reaction at Node 1 (Z Direction)

$R = $ (Z Direction)

$= (-2.790 \times 10^{-9}) \, (10^{14})$

$R = 2.79 \times 10^{5}$ lbs.

Static Reaction (from Fig. 3)

= 279 Kips

Conclusions:

1. Deflections satisfy the boundary conditions.
2. Reactions and external loads satisfy the equations of statics.
3. Solution is correct.

4.0 LIST OF REFERENCES AND BIBLIOGRAPHY

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