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DIGITAL COMPUTER PROCESSING OF PFM TELEMETRY DATA

by R. M. Ginnings

Prepared under Contract No. NAS 5-2664 *by*
LITTON INDUSTRIES
College Park, Md.
for Goddard Space Flight Center

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ABSTRACT

The problems which arise in processing pulse frequency modulation (PFM) telemetry data by means of a digital computer are increased because of the large number of correlations which must be made to take advantage of the coded nature of the signal. In order to decrease the computation time, the data is severely amplitude limited; the polarity information which remains can be more easily processed by the computer.

This paper estimates the frequency of the pulse-frequency-modulated data and computes the probability of error that is involved in this estimate.

CONTENTS

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by

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INTRODUCTION

Low input signal-to-noise ratios make coded telemetry the natural response to the inherent difficulties of spacecraft data transmission. That advantage, however, is not without its price. The complexity of the data processing imposes numerous hardware requirements and problems.

A number of these problems are eliminated by a technique known as polarity-coincidence correlation: the signal is severely amplitude-limited or "clipped," and what remains is polarity and zero-crossing information, which the computer can process much more easily. This technique obviates the need for conventional analog-to-digital conversion, thus reducing the amount of information the computer must digest and hence the total computation time.

This paper compares the error probability for this polarity-coincidence data processing system with that for the optimum system using matched filters, and presents the theoretical difference in system performance.

DESCRIPTION OF THE TELEMETRY SIGNAL

The information which will be considered is time limited, i.e., such that $D(t) = 0$ when $t < 0$ and when $t > T$, and consists of a sinusoidal signal plus additive Gaussian noise. The signal, $S(t)$, is known to have a positive-going zero crossing at the start of the period T. Thus, $S(t) = S_0 \sin 2\pi f_s t$, $0 < t < T$, where it is also known that f_s will be one of M discrete frequencies $(f_1, f_2, \cdots, f_M)$ within the interval $f_A < f_s < f_B$, and $f_s >> 1/T$.

Recovery of the telemetry data depends upon correctly estimating which one of the M possible signal frequencies is present in the noise. The use of polarity-coincidence correlation requires that the data be severely amplitude-limited so that only the polarity remains to be considered. This polarity indication is in the form of zero-crossing times.

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If the telemetry information were unperturbed by noise, the easiest method of determining the signal frequency would be to count the number of zero crossings in a given period of time. However, in actual telemetry, noiseless data is the exception rather than the rule. Given noisy data, the counting of zero crossings would yield very little information concerning the frequency of the signal.

CORRELATION PROCEDURE

In considering what is known *a priori* about the nature of the signal, the most obvious characteristic is that the signal, regardless of its frequency, always has a zero crossing at the beginning of the data period. Thus, if one were to examine the severely limited data a minimum number of times during each data period, the optimum time to do so would be when the signal was known to be at a relative maximum or minimum.

If the signal frequency, f_s, is assumed to be f_i during a particular data period, then the optimum times at which to examine the signal would occur at

$$t_{ij} = \frac{2j - 1}{4f_i} \qquad j = 1, 2, \cdots, 2f_i T . \tag{1}$$

where j indexes the number of half cycles of $\sin 2\pi f_i t$, i indicates the particular sampling frequency, and T is the length of the data period. A polarity coincidence between the data information and the frequency, f_i, will occur if the information is positive when j is odd (i.e., $j = 1,3,5,\cdots$) or if it is negative, when j is even (i.e., $j = 2,4,6, \cdots$). If we denote the number of polarity coincidences by m_i and divide this by $2f_i T$, then the result $\frac{m_i}{2f_i T}$ is a normalized measure of the correlation between the data and the frequency f_i.

To determine the correlation between the data and the frequency, f_i, the computer generates the numbers $t_{i1}, t_{i2}, t_{i3}, \cdots, t_{i2f_iT}$, and compares these numbers with the data zero crossing times $t_1', t_2', t_3', \cdots$. The two sets of numbers are then ordered and each t_{ij} and t_n' that preceded it are examined. If the j and n subscripts of the particular pair under examination are both even or both odd, then a polarity coincidence will have occurred. If, for example, no noise were present and the sampling frequency was equal to the signal frequency, the ordered sequence would appear as shown in Table 1. It is easily seen from the Table 1 sequence that the two sets of numbers are polarity coincident. If noise were present to perturb the data zero crossing times, then the ordered sequence might appear as shown in Table 2. In this table, a polarity

Table 1

Sampling at the Signal Frequency in the Absence of Noise

t_1'		t_2'		t_3'		t_4'		$t_5'\cdots$	
	t_{i1}		t_{i2}		t_{i3}		t_{i4}		$t_{i5}\cdots$

Table 2

Sampling at the Signal Frequency in the Presence of Noise

t_1'		t_2'		t_3'	t_4'		t_5'		t_6'	t_7'		$t_8'\cdots$	
	t_{i1}		t_{i2}			t_{i3}		t_{i4}			t_{i5}		$t_{i6}\cdots$

coincidence would occur between the following pairs of numbers: (t_1', t_{i1}), (t_2', t_{i2}), (t_7', t_{i5}), (t_8', t_{i6}). The above correlation procedure is performed a total of M times (i.e., $i = 1,2,3,\ldots, M$), and the number of coincidences m_i, that result from each correlation is recorded, and the maximum of all of the $\frac{m_i}{2f_i T}$ is noted. The frequency f_i, that corresponds to this maximum is the estimate of the frequency of the signal.

The set of frequencies f_i, $i = 1,2,3, \cdots, M$, is an orthogonal set over the interval T since the frequencies are chosen such that each f_i will be an integral multiple of $\frac{1}{2T}$. Thus,

$$\int_0^T \sin 2\pi f_i t \sin 2\pi f_j t \, dt = 0 \qquad \text{for } i \neq j \ . \tag{2}$$

This orthogonality is sufficient to guarantee that the frequency f_i, which corresponds to the maximum $\frac{m_i}{2f_i T}$ in the above correlation procedure, will be the actual signal frequency f_s, provided that the signal is unperturbed by noise.

ERROR PROBABILITY

It is now necessary to determine the probability of error that is involved in the estimate of the signal frequency in the presence of noise.

Given the signal frequency, f_s, and given a sampling frequency which is not identical to thê signal frequency (i.e., $i \neq s$), the probability of a polarity coincidence occurring on the j^{th} sample at the i^{th} sampling frequency ($i \neq s$) is the probability that the data had the same polarity as the sampling frequency at the time of that particular trial. This is given by

$$P_{ij} = \frac{1}{\sqrt{2\pi}\sigma} \int_0^{\infty} \exp\left[\frac{-\left\{x - S_0 \sin\left[(2j-1)\left(\frac{f_s}{f_i}\right)\left(\frac{\pi}{2}\right)\right]\right\}^2}{2\sigma^2}\right] dx \qquad \begin{matrix}\text{for } j \text{ odd} \\ i \neq s\end{matrix} \ , \tag{3}$$

and

$$P_{ij} = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{0} \exp\left[\frac{-\left\{x - S_0 \sin\left[(2j-1)\left(\frac{f_s}{f_i}\right)\left(\frac{\pi}{2}\right)\right]\right\}^2}{2\sigma^2}\right] dx \qquad \begin{matrix}\text{for } j \text{ even} \\ i \neq s\end{matrix} \ . \tag{4}$$

This is a result of the j^{th} sample at the i^{th} sampling frequency having a Gaussian amplitude probability density function with a variance equal to σ^2, and a mean equal to the signal amplitude at the time of sampling of

$$S_0 \sin\left[(2j-1)\left(\frac{f_s}{f_i}\right)\left(\frac{\pi}{2}\right)\right] . \tag{5}$$

The P_{ij} in Equation 3 and Equation 4 have been calculated for a typical set of parameters: data period $T = 0.010$ second, frequency range defined by $f_A = 5{,}000$ cps and $f_B = 15{,}000$ cps, number of possible signal frequencies $M = 100$, and for ratios of (S_0/σ) up to 1.0. The calculations were performed for $f_s = f_1$ and for $f_s = f_{100}$. The value $(S_0/\sigma) = 1.0$ corresponds to a value of β signal (energy per bit/noise power density) of 30.1 (see Appendix A).

Each probability, (P_{ij}), was treated as a variable with respect to j. The mean and the standard deviation of (P_{ij}) were calculated for each value of $i (i \neq s)$, β, and for $s = 1$ and $s = 100$. It was found that the mean of (P_{ij}) fell within the range $0.5 \pm 2 \cdot 10^{-6}$ for each value of i, β, and s. Given a fixed value of β, the standard deviation of (P_{ij}) remained the same for any value of i and s. This standard deviation versus the signal energy per bit/noise power density β, is shown in Figure 1. The standard deviation of the (P_{ij}) will approach a value of 1/2 for large values of β since the noise distribution will then be centered with equal probability at either a large positive amplitude or a large negative amplitude and, consequently, the probability of a polarity coincidence approaches 1/2. The standard deviation is seen to be small for small values of β because the signal amplitude will be small compared with the rms noise, and therefore, all values of (P_{ij}) will approach the mean value of 0.5. As a result of this, when the data are being sampled at a frequency other than the actual signal frequency, the amplitude probability density function of any given sample will be approximately Gaussian with a variance equal to σ^2 and a mean equal to *zero*. Thus, the probability of a polarity coincidence occurring on a given sample at any sampling frequency not equal to the signal frequency is given by

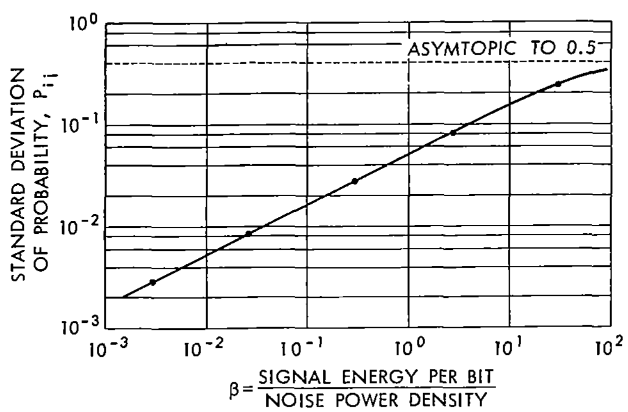

Figure 1—Standard deviations of probability P_{ij} versus the signal energy per bit/noise power density β.

$$P_I = P_{ij} = \int_0^\infty \frac{1}{\sqrt{2\pi}\sigma} \exp\left[\frac{-x^2}{2\sigma^2}\right] dx = 0.5 \quad \text{for all } j,\ i \neq s . \tag{6}$$

The amplitude probability density function of a given sample when the sampling frequency is equal to the signal frequency will be Gaussian with variance equal to σ^2; however, the mean of this probability density function will be equal to the peak value of the signal, S_0, since the data information is being examined when the signal is at its peak value (either a positive or negative peak). The probability of a polarity coincidence occurring on a given sample when the sampling frequency is equal to the signal frequency will be the probability that the signal plus noise had the same polarity as the signal, at the time of that particular sample. This is given by

$$p_2 = P_{ij} = \int_0^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-\left(x - S_0\right)^2}{2\sigma^2}\right] dx \qquad \begin{matrix}\text{for all } j \\ i = s\end{matrix} \quad . \tag{7}$$

If it is now assumed that the noise bandwidth is approximately equal to or greater than $\left(f_B - f_A\right)$, then the probability of a polarity coincidence occurring on any given sample will be *independent* of the probability of a polarity coincidence occurring on any other sample. This independence is clearly inferred from Figure 2 which is a plot of the autocorrelation function of amplitude limited Gaussian noise over the bandwidth of 100 to 20,000 cps. The derivation is given in Appendix B. Hence, we arrive at the conclusion that the probability of x polarity coincidences occurring in k samples is given by

$$\binom{k}{x} p^x (1 - p)^{k-x} \quad , \tag{8}$$

where p is the probability of a polarity coincidence on one sample and $\binom{k}{x}$ is the binominal coefficient.

The probability that m coincidences occur in k samples at the signal frequency is given by

$$\binom{k}{m} p_2^m \left(1 - p_2\right)^{k-m} \quad . \tag{9}$$

The probability that less than m coincidences occur in k samples at *all* of the other $(M - 1)$ sampling frequencies is given by

$$\left[\sum_{x=0}^{m-1} \binom{k}{x} p_1^x \left(1 - p_1\right)^{k-x}\right]^{M-1} \quad . \tag{10}$$

Therefore, the probability of error for the case of k samples at each different sampling frequency will be

$$1 - \sum_{m=1}^{k} \left[\left(\sum_{x=0}^{m-1} \binom{k}{x} p_1^x \left(1 - p_1\right)^{k-x}\right)^{M-1} \binom{k}{m} p_2^m \left(1 - p_2\right)^{k-m}\right] \tag{11}$$

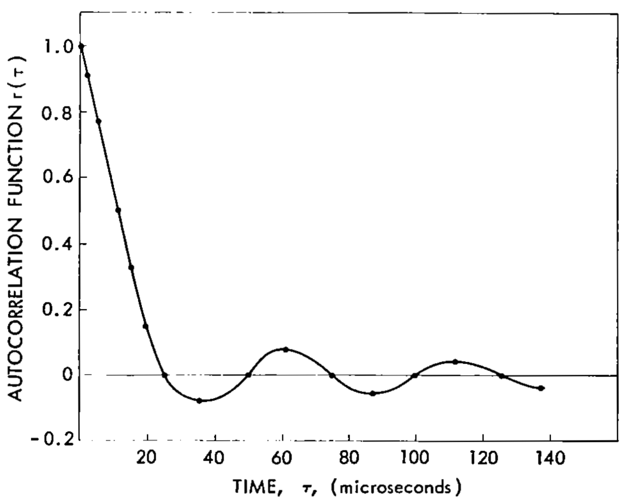

Figure 2—Autocorrelation function of Amplitude Limited Gaussian noise (100 cps—20,000 cps).

The actual case under consideration has a different number of samples at each sampling frequency, so it is necessary to normalize the number of coincidences with respect to the total number of samples. To clarify the result, consider the following example: $T = 0.010$ seconds, $f_A = 5{,}000$ cps, $f_B = 15{,}000$ cps, $M = 100$. The number of samples varies from 101 to 299. If the signal frequency, f_s, is assumed to be f_1, then the probability of error will be given by

$$P_{e_1} = 1 - \sum_{\alpha=\frac{1}{101}}^{\frac{101}{101}} \left[\prod_{L=1}^{99} \left(\sum_{x=0}^{[\alpha(101+2L)-1]} \binom{101+2L}{x} p_1^x \left(1-p_1\right)^{101+2L-x} \right) \cdot \binom{101}{\alpha(101)} p_2^{\alpha(101)} \left(1-p_2\right)^{101-\alpha(101)} \right]. \tag{12}$$

If the signal frequency f_s is assumed to be f_{100}, then the probability of error is given by

$$P_{e_{100}} = 1 - \sum_{\alpha=\frac{1}{299}}^{\frac{299}{299}} \left[\prod_{L=0}^{98} \left(\sum_{x=0}^{[\alpha(101+2L)-1]} \binom{101+2L}{x} p_1^x \left(1-p_1\right)^{101+2L-x} \right) \cdot \binom{299}{\alpha(299)} p_2^{\alpha(299)} \left(1-p_2\right)^{299-\alpha(299)} \right]. \tag{13}$$

The two error probabilities (Equations 12 and 13) represent the upper and lower bounds of the actual error probability since the signal frequency was assumed to be between f_A and f_B.

Equations 12 and 13 were programmed for computation on an IBM 7090 digital computer and the results are plotted in Figure 3. For the purpose of comparison, the word-error probability for the case of matched filters is also plotted.

It is also interesting to determine the effect on the error probability curves of Figure 4,

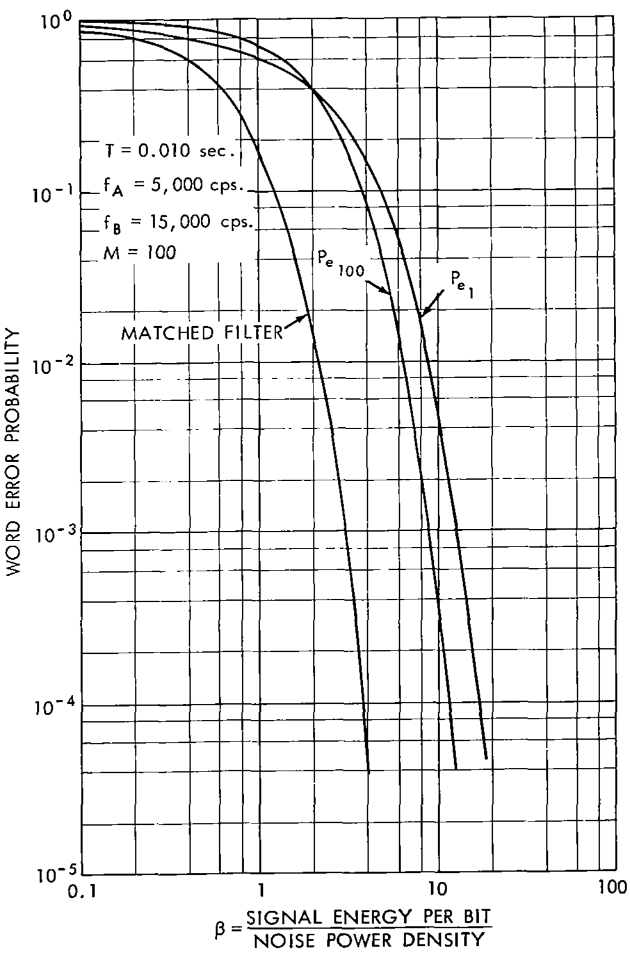

Figure 3—Word error probability versus β.

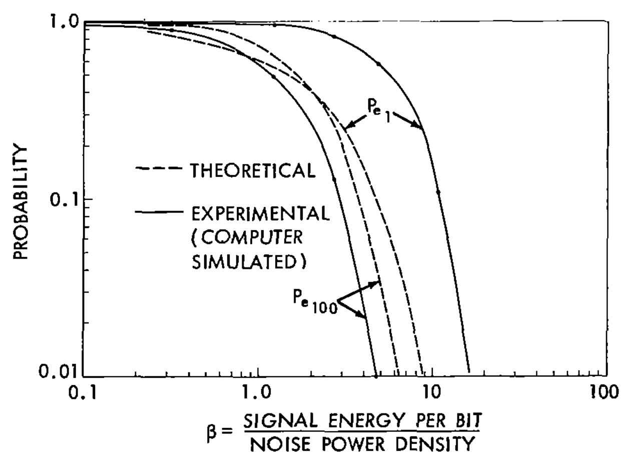

Figure 4—Word error probability, theoretical and experimental.

if, instead of using a sine wave signal, the signal would consist of a square wave. On the assumption that the amplitude of the square wave is the same as the peak value of the sine wave S_0, then the only difference lies in the value of β, which is linearly dependent upon signal power. The signal power for the sine wave is $(S_0^2/2)$, while the signal power for the square wave is (S_0^2). Therefore, the β for the square wave is twice that for the sine wave, or, in other words, the square-wave signal would require twice as much power as the sine-wave signal to achieve the same error probability.

It is easily seen that the preceding statement is correct for the case of equal amplitude sine or square waves since, if the signal plus noise was sampled at the signal frequency, (the samples occur at the maxima and minima of the signal frequency sine wave), then it would be impossible to determine from the result whether or not the signal was a sine wave or a square wave. If the square wave signal plus noise were sampled at a frequency which is different than the signal frequency, the amplitude distribution would again be assumed Gaussian with mean value equal to zero.

COMPUTER SIMULATION OF ERROR PROBABILITY

It was found desirable to perform an experimental check upon the theoretical word-error probability curves. This was done by using the computer to simulate the noisy telemetry data as well as performing the cross-correlation. Simulation of the noisy data was easily done since the zero crossings of the data are the only input to the computer.

Signal frequencies of 5050 cps and 14,950 cps were used in order to calculate the word error probabilities P_{e_1} and $P_{e_{100}}$ respectively. The system bandwidth, and consequently, the noise bandwidth, were previously assumed to be 20,000 cps, which means that the data can be represented by samples taken every 25 μs. The noisy data information is generated by adding a random amplitude, corresponding to the normally distributed noise, to the signal frequency every 25 μs for the entire 10 ms period.

The normally distributed random amplitude is generated using the equation

$$y = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma} e^{-u^2/2\sigma^2} du \quad . \tag{14}$$

The computer generates a random value for y on the interval $0 \leq y \leq 1$ and then solves for x in the above equation. If $|x| > 5\sigma$, then this value of x is discarded and a new value is found to take its place. The random number x has a normal amplitude distribution (truncated) with an rms value equal to σ.

The zero crossings of the signal plus noise are estimated using linear interpolation between the points separated by 25 μs. These zero-crossing times are then cross-correlated with the 100 possible signal frequencies and the signal frequency that gives the highest correlation is then the estimate of the original signal frequency.

The word error probabilities involve a limiting process in a probability sense because if the detection process were performed a very large number of times, then the expected value of the error probability would be that given by Figure 3. The computer time required to simulate the detection process a very large number of times would be prohibitive; therefore, a compromise was made and the number of trials was chosen to be 100. The results are shown in Figure 4 as a function of β for error probabilities from 0.01 to 0.99.

CONCLUSION

The particular data processing technique that was developed was based upon the use of a digital computer because it is felt that the economical advantage of requiring less computer time than conventional methods outweighs the additional error that is introduced as a result of the data being severely limited.

ACKNOWLEDGMENT

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(Manuscript received June 17, 1965)

Appendix A

Representation of Signal-to-Noise Ratio

Signal-to-noise ratios are usually calculated in terms of signal power and noise power over a given bandwidth. However, when several methods of detection are to be compared, it is desirable to represent the signal-to-noise ratio as the ratio of signal energy per bit to the noise power density P_N at the input to the system under consideration. For example, consider the following system:

$T = 0.01$ sec.;

B = noise bandwidth $\approx$ 20 kc;

M = number of discrete frequencies = 100;

$\beta = \dfrac{\text{signal energy per bit}}{\text{noise power density}}$;

n = number of bits = $\log_2 100 = 6.64$

$$\beta = \frac{\dfrac{\left(\dfrac{S_0^2}{2}\right)T}{\log_2 M}}{P_N} .$$

To calculate P_N, we begin with

$$\sigma^2 = \int_{-\infty}^{\infty} P_N\,df = \int_{-B}^{0} P_N\,df + \int_{0}^{B} P_N\,df = BP_N + BP_N = 2BP_N . \tag{15}$$

Then $P_N = \dfrac{\sigma^2}{2B}$,

and as a result:

$$\beta = \frac{\dfrac{\left(\dfrac{S_0^2}{2}\right)T}{\log_2 M}}{\dfrac{\sigma^2}{2B}} = 30.1\left(\frac{S_0}{\sigma}\right)^2 \tag{16}$$

Appendix B

Autocorrelation Function of Amplitude Limited Gaussian Noise

Let $R_1(\tau)$ = autocorrelation function of Gaussian noise over the band from 100 cps to 20,000 cps.

$$R_1(\tau) = 2\pi \int_{10,050-9950}^{10,050+9950} P_N \cos 2\pi f\tau \, df \quad ,$$

$$R_1(\tau) = \frac{2\pi P_N}{\pi\tau} \sin\left[(9950)\, 2\pi\tau\right] \cos\left[(10,050)\, 2\pi\tau\right] \quad ,$$

$$R(\tau) = \text{normalized } R_1(\tau) = \frac{\sin\left[2\pi(9950)\,\tau\right]}{\left[2\pi(9950)\,\tau\right]} \cos\left[2\pi(10,050)\,\tau\right] \quad . \tag{17}$$

Let $r(\tau)$ = autocorrelation function of amplitude limited Gaussian noise over the band from 100 cps to 20,000 cps.

$$r(\tau) = \frac{2}{\pi} \sin^{-1} R(\tau)$$

$$r(\tau) = \frac{2}{\pi} \sin^{-1} \left\{ \frac{\sin\left[2\pi(9950)\,\tau\right]}{\left[2\pi(9950)\,\tau\right]} \cos\left[2\pi(10,050)\,\tau\right] \right\} \quad . \tag{18}$$