NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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Technical Memorandum No. 33-232

The Ephemerides of the Earth–Moon Barycenter, Venus, Mars, and Mercury Considering the Earth and Moon as Separate Bodies

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CONTENTS

ABSTRACT

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The differential equations for the motion of the Earth–Moon barycenter considering the Earth and Moon as separate bodies are discussed. The ephemerides of the Earth–Moon barycenter, Venus, Mars, and Mercury constructed with these differential equations are compared with the ephemerides of the same planets constructed with differential equations in which the Earth and Moon are considered as a point mass located at the barycenter of the Earth–Moon system. Position residuals and angle residuals between the two systems are plotted.

Author

I. INTRODUCTION

Currently, the JPL ephemeris tapes E9510, E9511, and E9512 contain JPL planetary orbit determination program PLOD II-generated position–velocity ephemerides of the planets which were constructed by considering the Earth–Moon system as a point mass located at the barycenter of the Earth–Moon system (Refs. 1–3); i.e., the following second-order nonlinear differential equation was solved by the numerical integration method of PLOD II:

$$\ddot{X}^i = -k^2(1+M)\frac{X^i}{r^3} + \sum_j k^2 m_j \left(\frac{X_j^i - X^i}{(P_j)^3} - \frac{X_j^i}{(r_j)^3}\right) \tag{1}$$

where

$$i = 1, 2, 3$$

$$j = 1, 2, \cdots, 8$$

$$(r)^2 = \sum_{i=1}^{3} (X^i)^2$$

$$(r_j)^2 = \sum_{i=1}^{3} (X_j^i)^2$$

$$(P_j)^2 = \sum_{i=1}^{3} (X_j^i - X^i)^2$$

M is the mass of the planet integrated, m_j are the masses of the other perturbing planets, and X_j^i are the positions of the perturbing planets. In the integration of the Earth–Moon barycenter, M would equal the mass of the Earth plus that of the Moon; for any other planet, M would equal the mass of that planet, and m_3 would equal the mass of the Earth plus that of the Moon, i.e., the planet would be perturbed by the barycenter approximation.

A more precise model of the motion of the Earth–Moon barycenter is obtained by considering the Earth and Moon as separate bodies. The derivation of differential equations to represent this model is reported in Section IV of this Memorandum. This model was considered, but not adopted, in the production of the JPL ephemeris. More recent interest in the possibility of planetary radar experiments as tests of general relativity and the need for more accurate ephemerides for spacecraft data analysis have renewed interest in the refinement of this model. This Memorandum gives the results of a study of this separation effect on the ephemerides of the Earth–Moon barycenter, Venus, Mars and Mercury for the period from December 30, 1949, to August 11, 1968 (Julian dates 2 433 280.5 to 2 440 048.5).

II. METHOD

Given a source ephemeris of the body in question, a unique solution of a differential equation describing the orbit of the body is obtained by fitting to the source ephemeris, in a least-squares sense, a numerical solution of the differential equation. If the differential equations of two least-squares fits to the same source ephemeris differ, the effect of this difference on the generated ephemerides may be seen by plotting the difference between the two fitted ephemerides. An alternative method would be to fit one model of the differential equation to an ephemeris generated by a different model. Both of these methods were used in the present study and were found to give the same results to the accuracy appropriate to the study. Here, a PLOD A fit is defined as a least-squares fit to a source ephemeris using Eq. (1) as the differential equation model, and a PLOD B fit is defined as a least-squares fit to a source, or PLOD, ephemeris using the differential equation model of Section IV.

III. RESULTS

A. Earth–Moon Barycenter

Using PLOD II, a PLOD A fit was made to the current source ephemeris of the Earth–Moon barycenter. As an indication of the goodness of fit, $\sigma = 2.85 \times 10^{-7}$ was obtained, where σ is described by the following equation (Ref. 1):

$$\sigma^2 = \frac{\sum_{j=0}^{N}\left[(\cos\beta_j)\,\delta\lambda_j\right]^2 + (\delta\beta_j)^2 + \left(\frac{\delta\rho_j}{\rho_j}\right)^2}{N} \tag{2}$$

Subsequently a PLOD B fit was made to the same source ephemeris, obtaining $\sigma = 2.79 \times 10^{-7}$. The maximum and minimum values of the residuals of these two runs may be seen in Appendix A and plots of the residuals are presented in Appendixes B and C: Figs. B-1–B-3 and C-1–C-3. Quantities plotted were δx, δy, δz, the residuals in position in the source-minus-computed sense; $\delta\dot{x}$, $\delta\dot{y}$, $\delta\dot{z}$, the residuals in velocity; $\delta\rho/\rho$, the relative residual in the radius vector; $(\cos\beta)\,\delta\lambda$, the cosine of the latitude times the residual in longitude; and $\delta\beta$, the latitude residual.

The PLOD A fitted ephemeris minus the PLOD B fitted ephemeris was then plotted to indicate the effect of the separation of the Earth and Moon (Appendixes A and D: Figs. D-1–D-2). In order to conserve machine time in computation and to obtain more information, it was decided to use the alternative method of obtaining the separation effect as described above; i.e., after a PLOD A fit has been made to a source ephemeris, a PLOD B fit is made to the PLOD A fit and the residuals are plotted. In the case of the Earth–Moon barycenter, this gave essentially the same results, as can be seen in Appendix A and by comparing Figs. D-1–D-2 with Figs. E-1–E-3 in Appendix E. The significant fact to be noted in Fig. E-2 is that a mean of the residuals $\delta\rho/\rho$ gives a value of approximately -2×10^{-8}, as was predicted by Hill in 1878 (Ref. 4) in indicating the principal deviation of the Sun's orbit from a Keplerian ellipse. This value should also be compared with the results reported in Ref. 5.

Also of interest is the fact that the residuals between a PLOD A or a PLOD B run and the source ephemeris are larger than the residuals attributed to the different models of the differential equation used (i.e., PLOD A minus PLOD B). Hence, it appears that the errors in the source ephemeris are larger than the difference in the models being studied here.

B. Venus

Again using PLOD II, a PLOD A fit was made to a current Venus source ephemeris, obtaining $\sigma = 3.00 \times 10^{-7}$, maximum and minimum residuals given in Appendix A, and plots of the residuals in Appendix F: Figs. F-1–F-3. Then, a PLOD B fit was made to the PLOD A fit and the residuals were again plotted (Appendixes A and G: Figs. G-1–G-3), obtaining a σ of 2.81×10^{-11}, indicating a goodness of fit well above the accuracy currently required by ephemeris users. Therefore, the effect of the separation of the Earth and Moon on Venus is negligible when considering the present requirements of the ephemeris system.

C. Mars

A PLOD B fit was made to a previous PLOD A fit to a Mars source ephemeris, and a σ of 1.76×10^{-11} was obtained with the maximum and minimum values of the residuals approximately the same order of magnitude or even smaller than those of Venus (Appendixes A and H: Figs. H-1–H-3). Hence, the separation effect of the Earth and Moon on Mars is very small and thus also negligible when considering the present requirements of the ephemeris system.

D. Mercury

In the PLOD B fit to a previous PLOD A fit to a source ephemeris of Mercury, a σ of 3.64×10^{-12} was obtained. The maximum and minimum values of the residuals are smaller than any of the other planets studied in this Memorandum (Appendixes A and J: Figs. J-1–J-3). This result is to be expected. The data for Mercury are included mainly for completeness of information about the effect of the separation of the Earth–Moon barycenter on all the inner planets.

IV. A DISCUSSION OF THE DIFFERENTIAL EQUATIONS FOR THE MOTION OF THE EARTH–MOON BARYCENTER, CONSIDERING THE EARTH AND MOON AS SEPARATE BODIES

The derivation of the differential equation for the motion of the Earth–Moon barycenter and the method for computing the acceleration in PLOD II (Ref. 1) are presented herein. The subscripts used and the quantities they denote are as follows:

- 0 center of inertial reference frame
- 1 center of Sun
- 2 center of Earth
- 3 center of Moon
- B center of mass of Earth–Moon system, i.e., Earth–Moon barycenter
- j center of Mercury, Venus, Mars, or one of the five outer planets

Consider the n-body problem in an inertial reference frame. The equation of motion of the ith body, according to Newton's law of gravitation, is as follows:

$$\ddot{\mathbf{r}}_{0i} = G \sum_{\substack{k=1 \\ k \neq i}}^{(n)} m_k \frac{\mathbf{r}_{ik}}{r_{ik}^3} \tag{1}$$

where $\mathbf{r}_{ik}$ is the vector from the ith body or point to the kth body or point, r_{ik} is the magnitude of $\mathbf{r}_{ik}$, G is the universal constant of gravitation, and m_k is the mass of the kth body. Using this equation, the case of the solar system can be considered, and the inertial-frame equations of motion of the Sun, Moon, and Earth can be written; however, care must be taken with the algebraic signs.

For the Sun (mass S), Earth (mass E), and Moon (mass M), respectively,

$$\ddot{\mathbf{r}}_{01} = GE \frac{\mathbf{r}_{12}}{r_{12}^3} + GM \frac{\mathbf{r}_{13}}{r_{13}^3} + G \sum_j P_j \frac{\mathbf{r}_{1j}}{r_{1j}^3} \tag{2}$$

$$\ddot{\mathbf{r}}_{02} = -\, GS \frac{\mathbf{r}_{12}}{r_{12}^3} + GM \frac{\mathbf{r}_{23}}{r_{23}^3} + G \sum_j P_j \frac{\mathbf{P}_{2j}}{P_{2j}^3} \tag{3}$$

$$\ddot{\mathbf{r}}_{03} = -\, GS \frac{\mathbf{r}_{13}}{r_{13}^3} - GE \frac{\mathbf{r}_{23}}{r_{23}^3} + G \sum_j P_j \frac{\mathbf{r}_{3j}}{r_{3j}^3} \tag{4}$$

where P_j represents the mass of the planet (either Mercury, Venus, Mars, or one of the five outer planets) at j. The vector $\mathbf{r}_{23}$ will be used throughout this discussion since it is available directly from the lunar theory; $\mathbf{r}_{32}$ will not be used.

For PLOD, we need the equations of motion of the Earth and Moon in a heliocentric frame. The equation for translating to the heliocentric frame for the kth body is as follows:

$$\mathbf{r}_{1k} = \mathbf{r}_{0k} - \mathbf{r}_{01} \tag{5}$$

Differentiating Eq. (5) twice yields the following:

$$\ddot{\mathbf{r}}_{1k} = \ddot{\mathbf{r}}_{0k} - \ddot{\mathbf{r}}_{01} \tag{6}$$

Letting $k = 2$ for the Earth and 3 for the Moon, and using Eqs. (6), (2), (3), and (4), we obtain the desired equations. The heliocentric differential equations for the Earth and Moon are, respectively,

$$\ddot{\mathbf{r}}_{12} = \ddot{\mathbf{r}}_{02} - \ddot{\mathbf{r}}_{01} = -G(S+E)\frac{\mathbf{r}_{12}}{r_{12}^3} + GM\left(\frac{\mathbf{r}_{23}}{r_{23}^3} - \frac{\mathbf{r}_{13}}{r_{13}^3}\right) + G\sum_j P_j\left(\frac{\mathbf{r}_{2j}}{r_{2j}^3} - \frac{\mathbf{r}_{1j}}{r_{1j}^3}\right) \tag{7}$$

$$\ddot{\mathbf{r}}_{13} = \ddot{\mathbf{r}}_{03} - \ddot{\mathbf{r}}_{01} = -G(S+M)\frac{\mathbf{r}_{13}}{r_{13}^3} - GE\left(\frac{\mathbf{r}_{23}}{r_{23}^3} + \frac{\mathbf{r}_{12}}{r_{12}^3}\right) + G\sum_j P_j\left(\frac{\mathbf{r}_{3j}}{r_{3j}^3} - \frac{\mathbf{r}_{1j}}{r_{1j}^3}\right) \tag{8}$$

The forms of Eqs. (7) and (8) can be recognized as simply those of the heliocentric equation for the Earth, perturbed by the j planets and by an additional "planet," the Moon, and the equation for the Moon, perturbed by the j planets and the Earth, respectively. It is undesirable to integrate Eqs. (7) and (8) directly since both contain the term $\mathbf{r}_{23}/r_{23}^3$. This lunar theory term has too short a period and too large an amplitude (about half as large as the solar term in the case of the Moon) to be considered a perturbation. Therefore, Eqs. (7) and (8) should be combined to eliminate these terms. This can be done by investigating the motion of the Earth–Moon barycenter, the heliocentric coordinate of which is given below:

$$\mathbf{r}_{1B} = \frac{E}{E+M}\mathbf{r}_{12} + \frac{M}{E+M}\mathbf{r}_{13} \tag{9}$$

Differentiating Eq. (9) twice, we obtain the acceleration of the barycenter:

$$\ddot{\mathbf{r}}_{1B} = \frac{E}{E+M}\ddot{\mathbf{r}}_{12} + \frac{M}{E+M}\ddot{\mathbf{r}}_{13} \tag{10}$$

If the expressions for $\mathbf{r}_{12}$ and $\mathbf{r}_{13}$ in Eqs. (7) and (8) are substituted into Eq. (10), we obtain

$$\begin{aligned}\ddot{\mathbf{r}}_{1B} = &-\frac{GE(S+E)}{E+M}\frac{\mathbf{r}_{12}}{r_{12}^3} + \frac{GEM}{E+M}\frac{\mathbf{r}_{23}}{r_{23}^3} - \frac{GEM}{E+M}\frac{\mathbf{r}_{13}}{r_{13}^3} \\ &+ \frac{GE}{E+M}\sum_j P_j\left(\frac{\mathbf{r}_{2j}}{r_{2j}^3} - \frac{\mathbf{r}_{1j}}{r_{1j}^3}\right) - \frac{GM(S+M)}{E+M}\frac{\mathbf{r}_{13}}{r_{13}^3} \\ &- \frac{GEM}{E+M}\frac{\mathbf{r}_{23}}{r_{23}^3} - \frac{GEM}{E+M}\frac{\mathbf{r}_{12}}{r_{12}^3} + \frac{GM}{E+M}\sum_j P_j\left(\frac{\mathbf{r}_{3j}}{r_{3j}^3} - \frac{\mathbf{r}_{1j}}{r_{1j}^3}\right)\end{aligned} \tag{11}$$

where the troublesome lunar theory terms $[GEM/(E+M)]\,(\mathbf{r}_{23}/r_{23}^3)$ cancel. After some manipulation, Eq. (11) can be reduced to the following form:

$$\ddot{\mathbf{r}}_{1B} = -G(S+E+M)\left(\frac{E}{E+M}\frac{\mathbf{r}_{12}}{r_{12}^3} + \frac{M}{E+M}\frac{\mathbf{r}_{13}}{r_{13}^3}\right) + G\sum_j P_j\left(\frac{E}{E+M}\frac{\mathbf{r}_{2j}}{r_{2j}^3} + \frac{M}{E+M}\frac{\mathbf{r}_{3j}}{r_{3j}^3} - \frac{\mathbf{r}_{1j}}{r_{1j}^3}\right) \tag{12}$$

Equation (12) is now used by PLOD. It is important that, although Eq. (12) is an equation for the motion of the barycenter, the masses of the Moon and Earth are *not* considered to be at that point (as in the old version of PLOD). To the contrary, Eq. (12) rigorously takes into account the effects of the Earth and Moon separately in their true positions.

In PLOD, the motion of $\ddot{\mathbf{r}}_{1B}$ is integrated. But, the expressions on the right-hand side of Eq. (12) require the heliocentric coordinates of the Earth ($\mathbf{r}_{12}$) and the Moon ($\mathbf{r}_{13}$). These are obtained as follows:

$$\mathbf{r}_{12} = \mathbf{r}_{1B} - \frac{M}{E+M}\mathbf{r}_{23} \tag{13}$$

$$\mathbf{r}_{13} = \mathbf{r}_{1B} + \frac{E}{E+M}\mathbf{r}_{23} \tag{14}$$

where $\mathbf{r}_{1B}$ is taken from the integration, and $\mathbf{r}_{23}$ is obtained from the lunar theory (coordinates of the Moon with respect to Earth).

The effects of the Earth and Moon are separated when integrating the positions of all the other planets (Mercury, Venus, Mars, and the outer planets) by treating the Earth and Moon as two separate perturbing planets. Therefore, Eq. (1) of Section I is used where the summation is over nine "planets": the Earth, the Moon, and seven others (excluding the perturbed planet whose position is being integrated from the group of Mercury, Venus, Mars, and the five outer planets). The heliocentric coordinates of the Earth and Moon are obtained directly from a subroutine which computes them using Eqs. (13) and (14), with both $\mathbf{r}_{1B}$ and $\mathbf{r}_{23}$ being obtained from JPL ephemeris tapes E9510, E9511, and E9512.

APPENDIX A
Maximum, Minimum Residual Values for PLOD Fits

PLOD Fit	Multiplication factor for all values	σ	δx, AU	δy, AU	δz, AU	$\delta \dot{x}$, AU	$\delta \dot{y}$, AU	$\delta \dot{z}$, AU	$\delta \rho / \rho$	$(\cos\beta)\delta\lambda$, rad	$\delta\beta$, rad
PLOD A to Earth–Moon barycenter source ephemeris (Figs. B-1–B-3)	10^{-7}	2.85	4.5,−5.2	5.0,−5.4	3.3,−3.0	0.12,−0.13	0.16,−0.14	0.070,−0.083	1.9,−4.3	4.9,−5.9	2.5,−2.0
PLOD B to Earth–Moon barycenter source ephemeris (Figs. C-1–C-3)	10^{-7}	2.81	4.1,−4.9	5.1,−5.4	3.4,−2.8	0.13,−0.12	0.16,−0.14	0.086,−0.091	1.4,−4.5	4.5,−6.0	2.4,−2.2
PLOD A to Earth–Moon barycenter source ephemeris minus PLOD B to Earth–Moon barycenter source ephemeris (Figs. D-1–D-2)	10^{-8}		9.8,−10.0	7.8,−8.3	9.4,−9.5	0.12,−0.12	0.18,−0.17	0.17,−0.18	—	—	—
PLOD B to PLOD A for Earth–Moon barycenter (Figs. E-1–E-3)	10^{-8}	8.19	9.8,−10.0	7.8,−8.1	9.5,−9.4	0.11,−0.12	0.17,−0.17	0.17,−0.18	2.9,−6.7	9.4,−9.8	9.3,−9.3
PLOD A to Venus source ephemeris (Figs. F-1–F-3)	10^{-7}	3.00	4.5,−6.0	3.7,−3.8	2.9,−2.4	0.17,−0.15	0.15,−0.14	0.077,−0.091	2.8,−3.3	6.5,−8.6	3.8,−3.2
PLOD B to PLOD A for Venus (Figs. G-1–G-3)	10^{-11}	2.81	3.9,−3.8	3.1,−4.0	1.7,−2.3	0.110,−0.072	0.085,−0.085	0.05,−0.06	2.0,−1.8	6.0,−6.1	2.8,−2.9
PLOD B to PLOD A for Mars (Figs. H-1–H-3)	10^{-11}	1.76	5.1,−3.6	4.8,−4.2	2.2,−2.1	0.034,−0.055	0.040,−0.042	0.017,−0.017	0.92,−1.30	4.2,−3.4	0.40,−0.41
PLOD B to PLOD A for Mercury (Figs. J-1–J-3)	10^{-12}	3.64	2.1,−3.3	2.3,−1.4	1.2,−1.1	0.17,−0.21	0.14,−0.23	0.067,−0.11	2.9,−2.9	11.0,−7.4	2.1,−1.8

APPENDIX B

PLOD A Fit to the Earth–Moon Barycenter Source Ephemeris

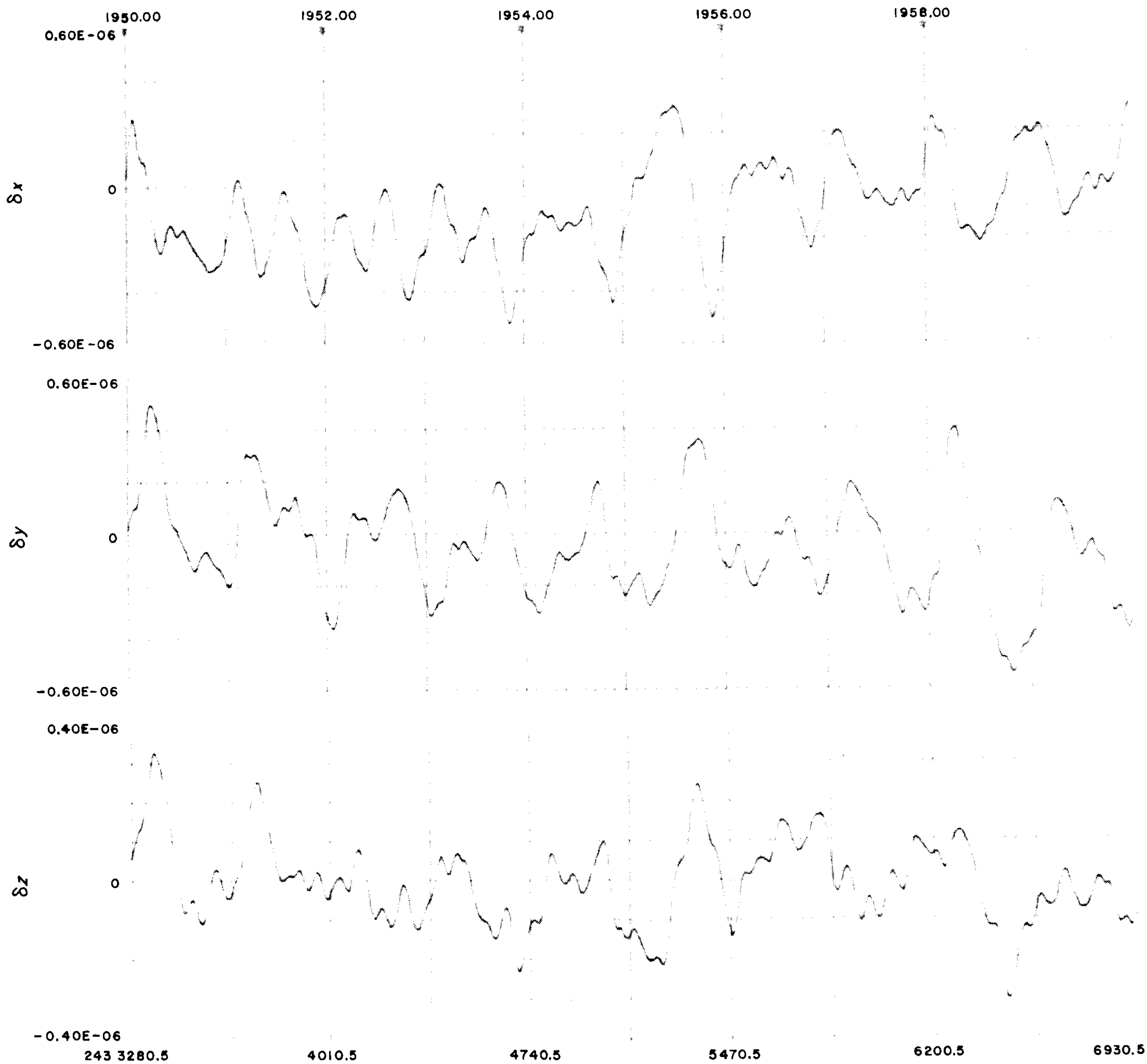


Fig. B-1. PLOD A fit to Earth–Moon barycenter source ephemeris; δx, δy, δz

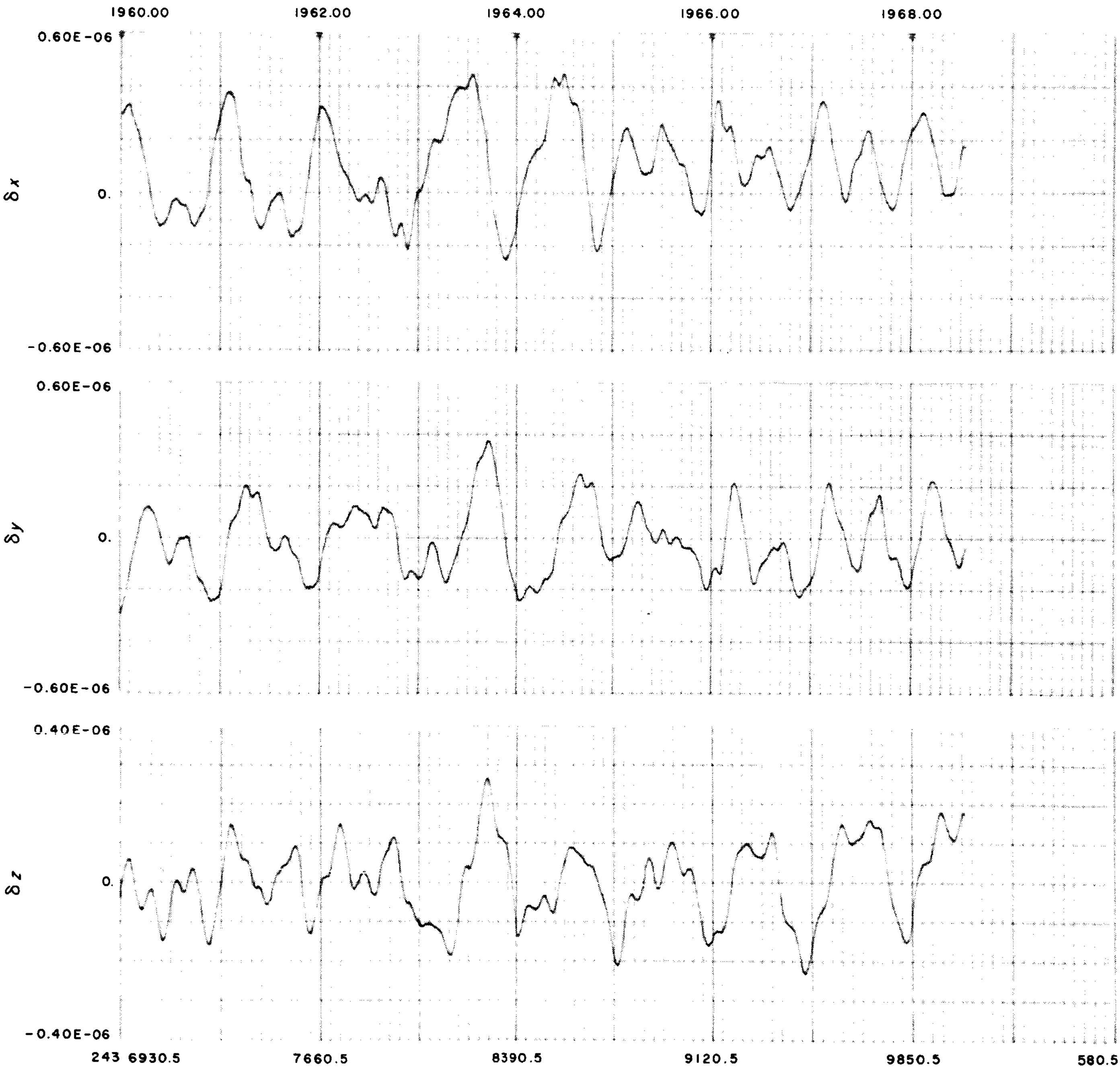

1960.00
1962.00
1964.00
1966.00
1968.00
0.60E-06
δx
0.
-0.60E-06
0.60E-06
δy
0.
-0.60E-06
0.40E-06
δz
0.
-0.40E-06
243 6930.5
7660.5
8390.5
9120.5
9850.5
580.5

Fig. B-1 (cont'd)

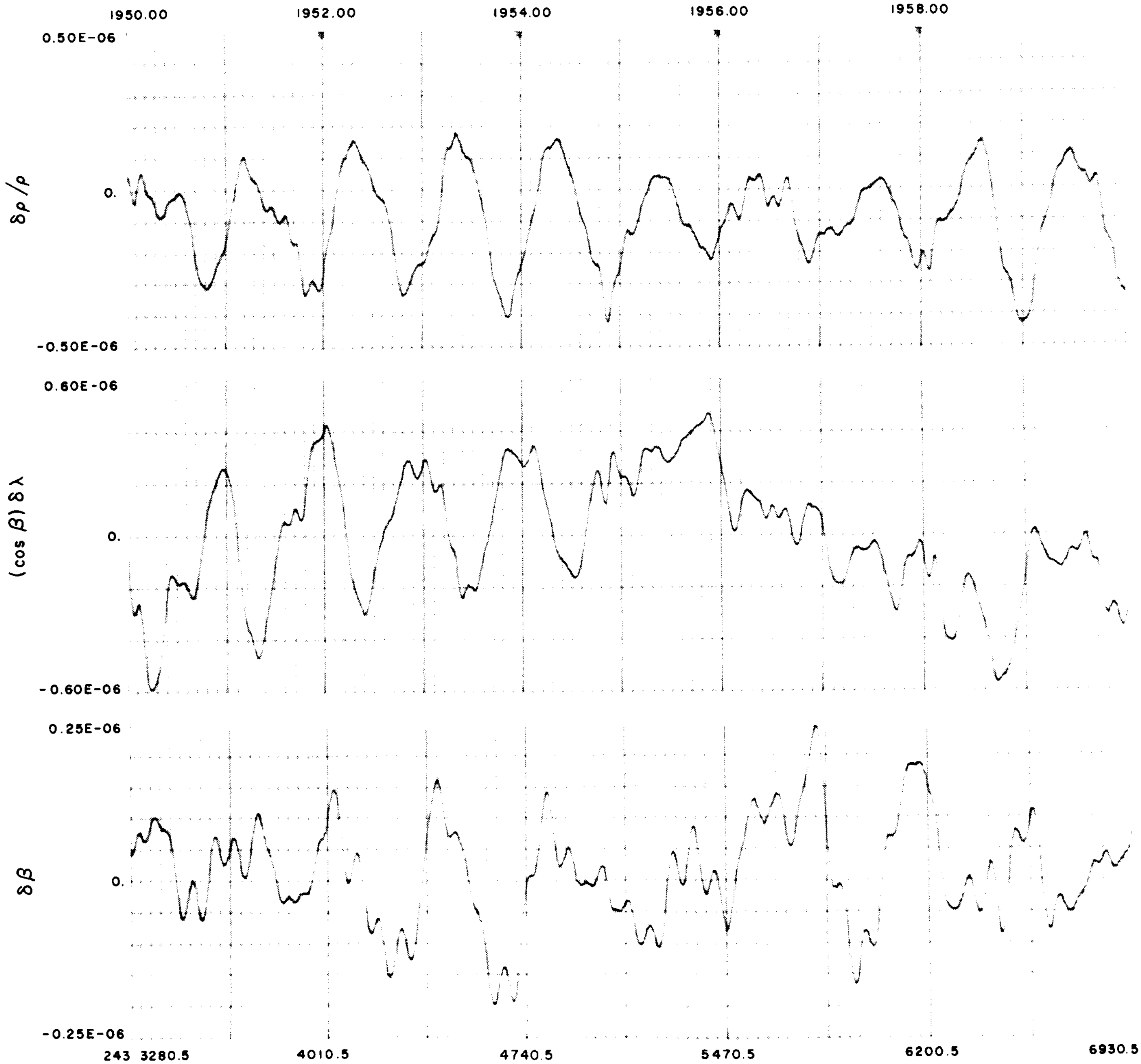


Fig. B-2. PLOD A fit to Earth–Moon barycenter source ephemeris; $\delta\rho/\rho$, $(\cos\beta)\,\delta\lambda$, $\delta\beta$

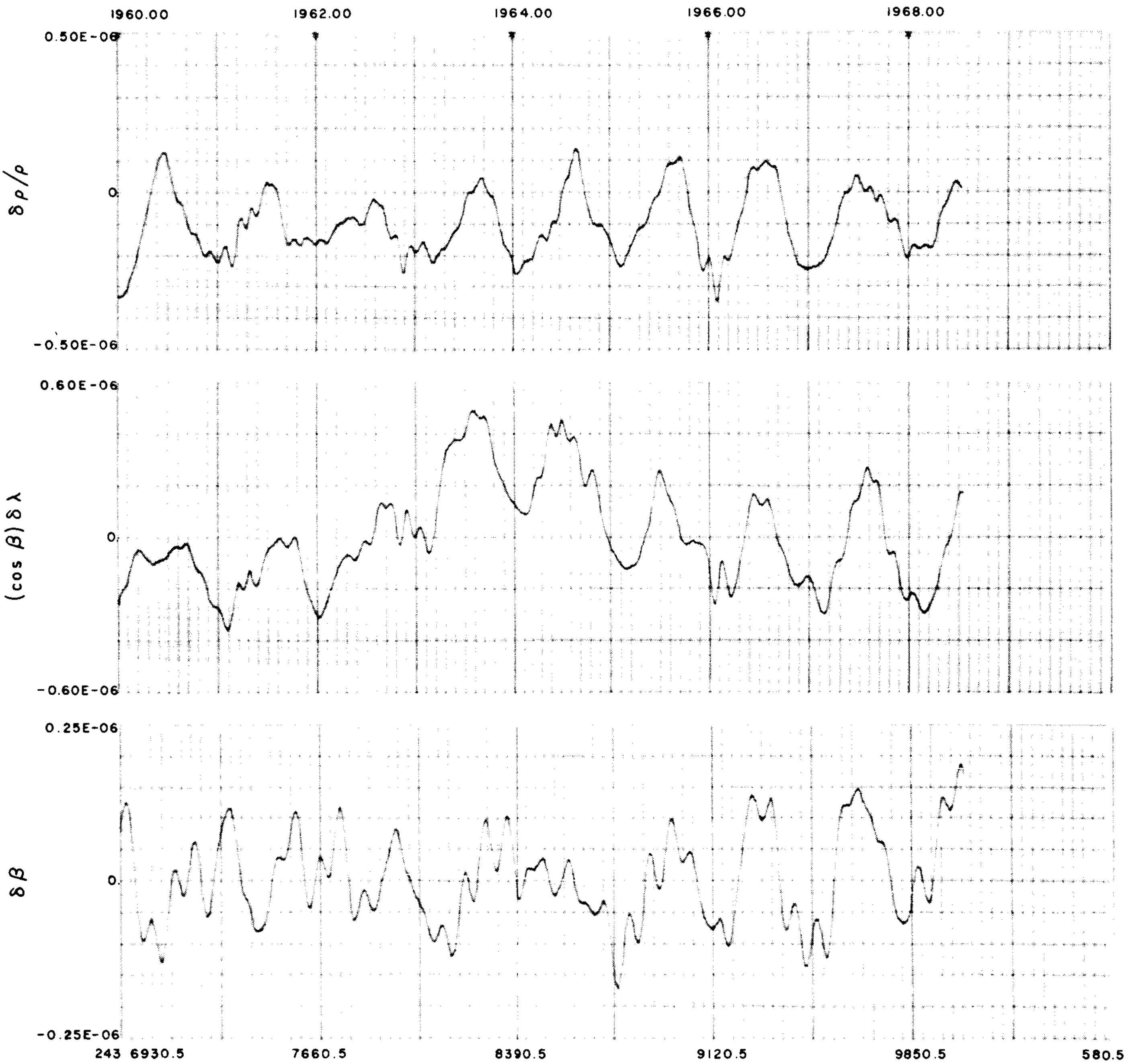

1960.00
1962.00
1964.00
1966.00
1968.00
0.50E-06
0
-0.50E-06
$\delta\rho/\rho$
0.60E-06
0
-0.60E-06
$(\cos\beta)\,\delta\lambda$
0.25E-06
0
-0.25E-06
$\delta\beta$
243 6930.5
7660.5
8390.5
9120.5
9850.5
580.5

Fig. B-2 (cont'd)

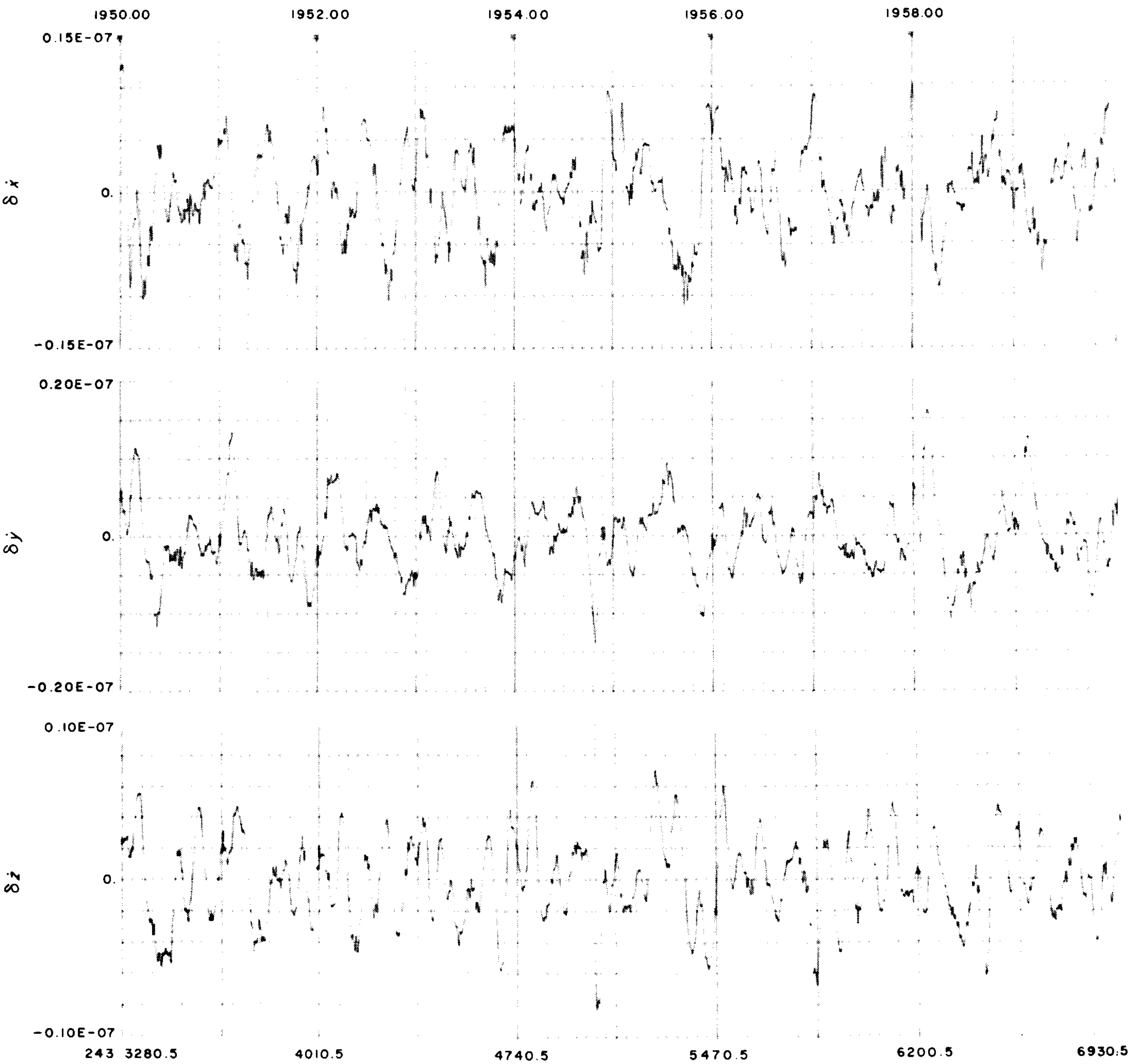


Fig. B-3. PLOD A fit to Earth–Moon barycenter source ephemeris; $\delta\dot{x}$, $\delta\dot{y}$, $\delta\dot{z}$

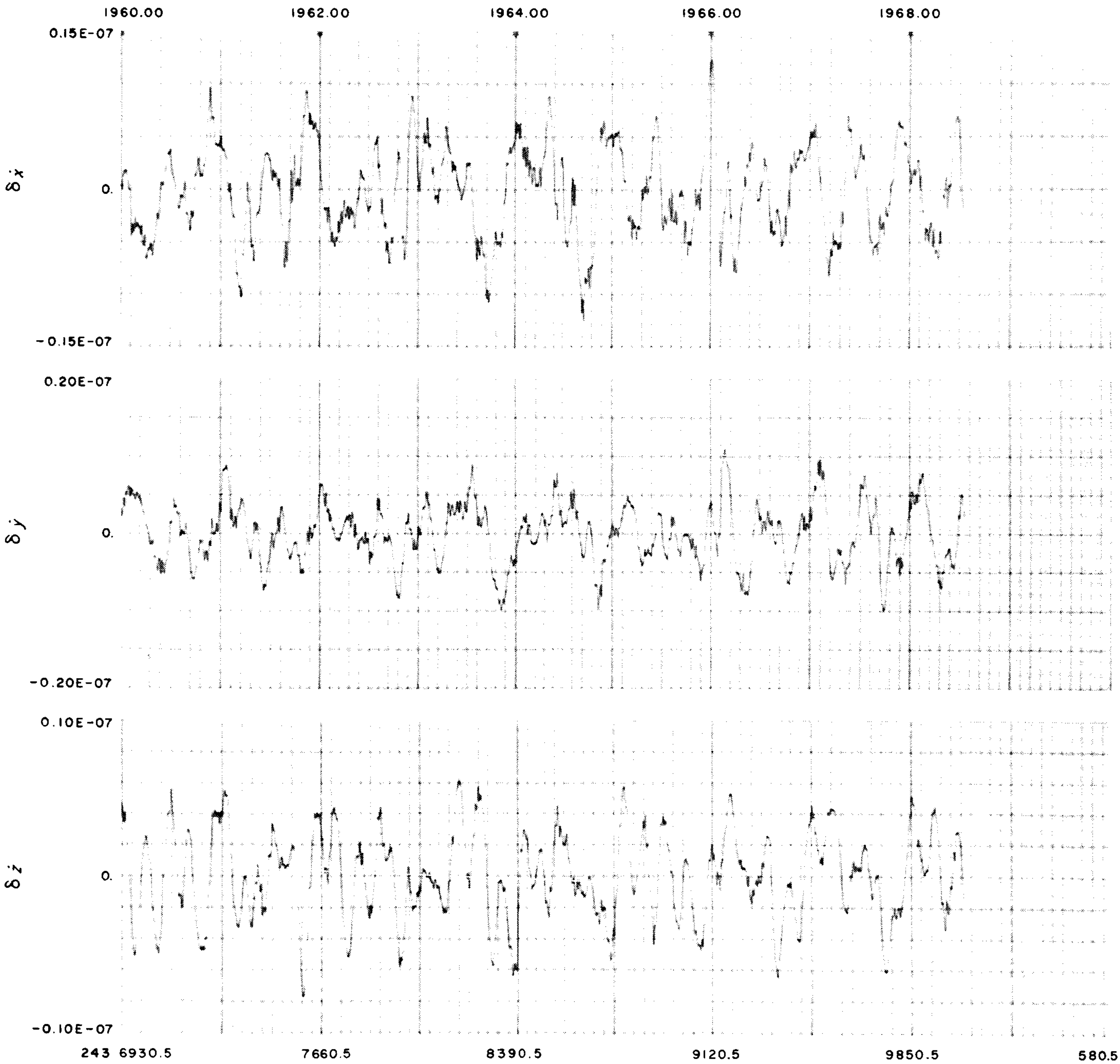

1960.00
1962.00
1964.00
1966.00
1968.00
0.15E-07
$\delta\dot{x}$
0.
-0.15E-07
0.20E-07
$\delta\dot{y}$
0.
-0.20E-07
0.10E-07
$\delta\dot{z}$
0.
-0.10E-07
243 6930.5
7660.5
8390.5
9120.5
9850.5
580.5

Fig. B-3 (cont'd)

APPENDIX C

PLOD B Fit to the Earth–Moon Barycenter Source Ephemeris

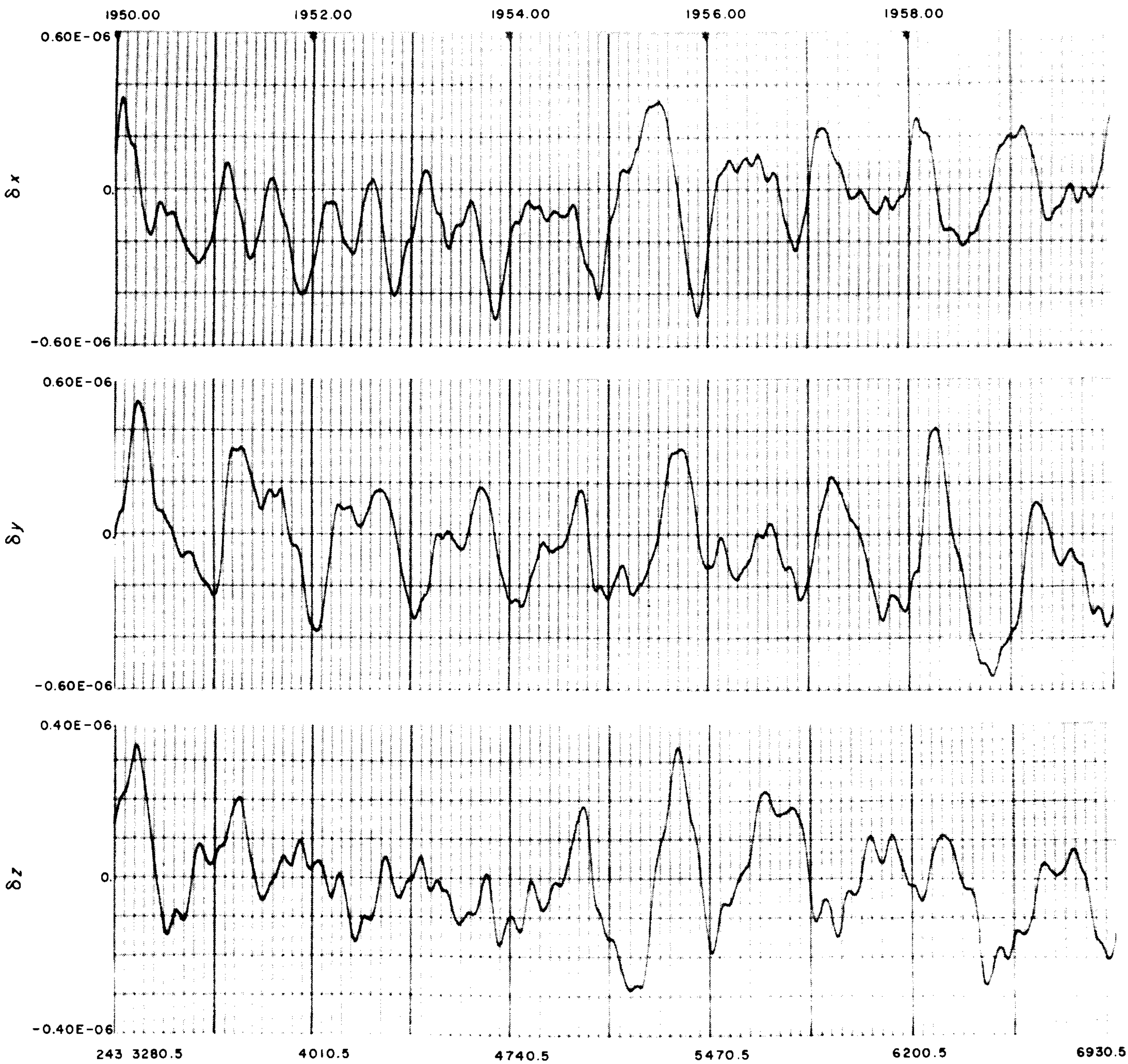


Fig. C-1. PLOD B fit to Earth–Moon barycenter source ephemeris; δx, δy, δz

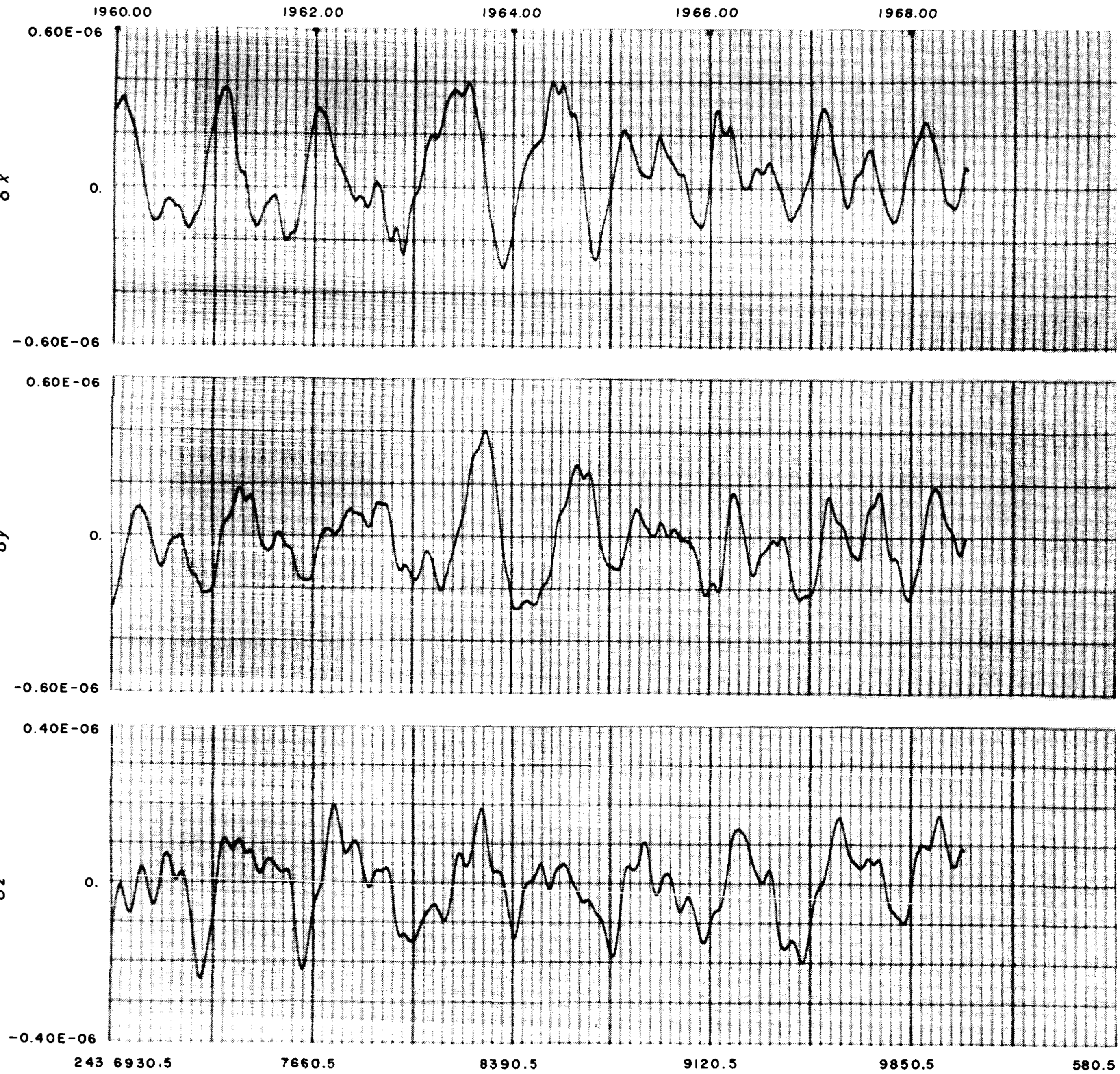

1960.00
1962.00
1964.00
1966.00
1968.00
0.60E-06
δx
0.
-0.60E-06
0.60E-06
δy
0.
-0.60E-06
0.40E-06
δz
0.
-0.40E-06
243 6930.5
7660.5
8390.5
9120.5
9850.5
580.5

Fig. C-1 (cont'd)

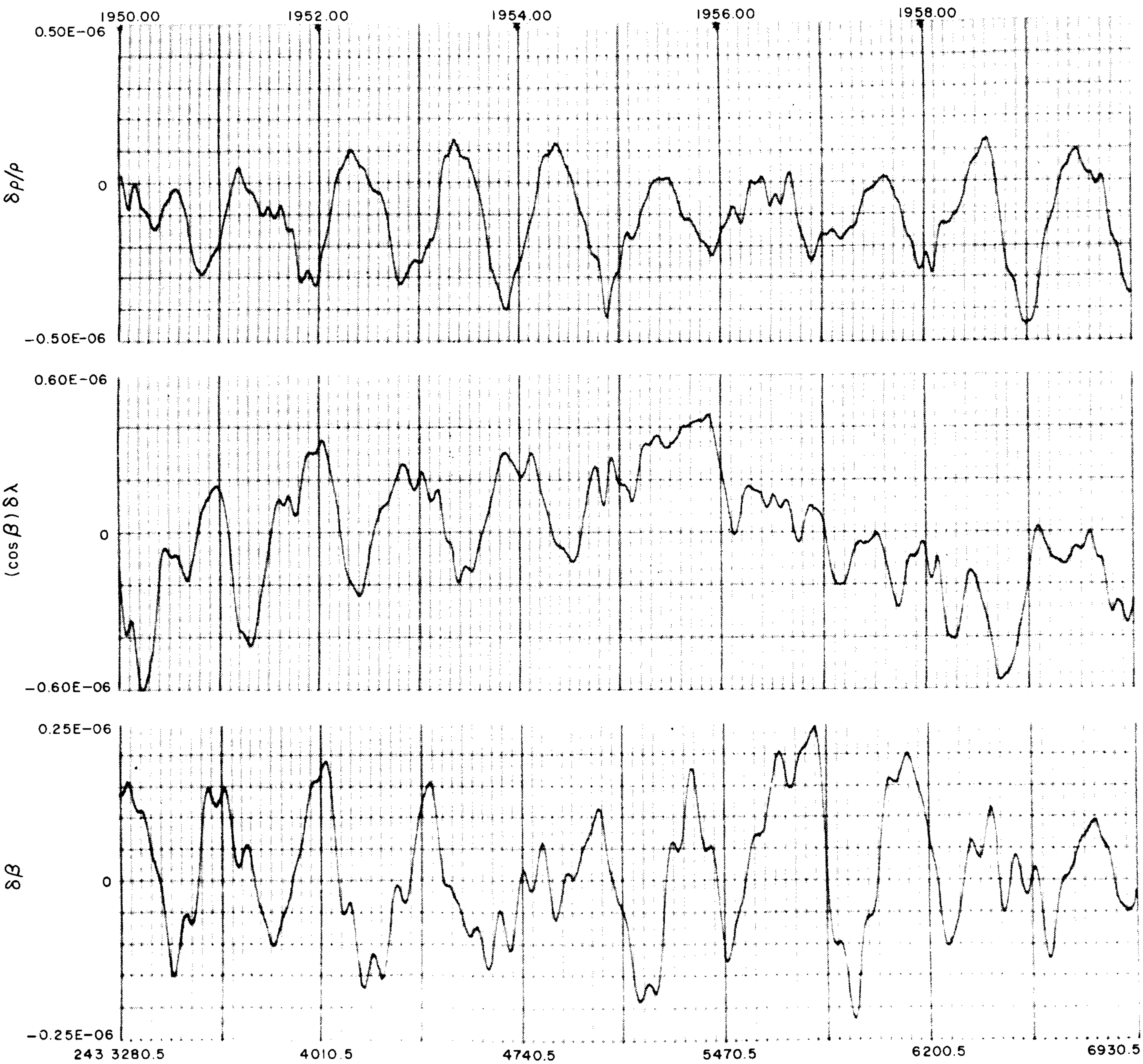


Fig. C-2. PLOD B fit to Earth–Moon barycenter source ephemeris: $\delta\rho/\rho$, $(\cos\beta)\,\delta\lambda$, $\delta\beta$

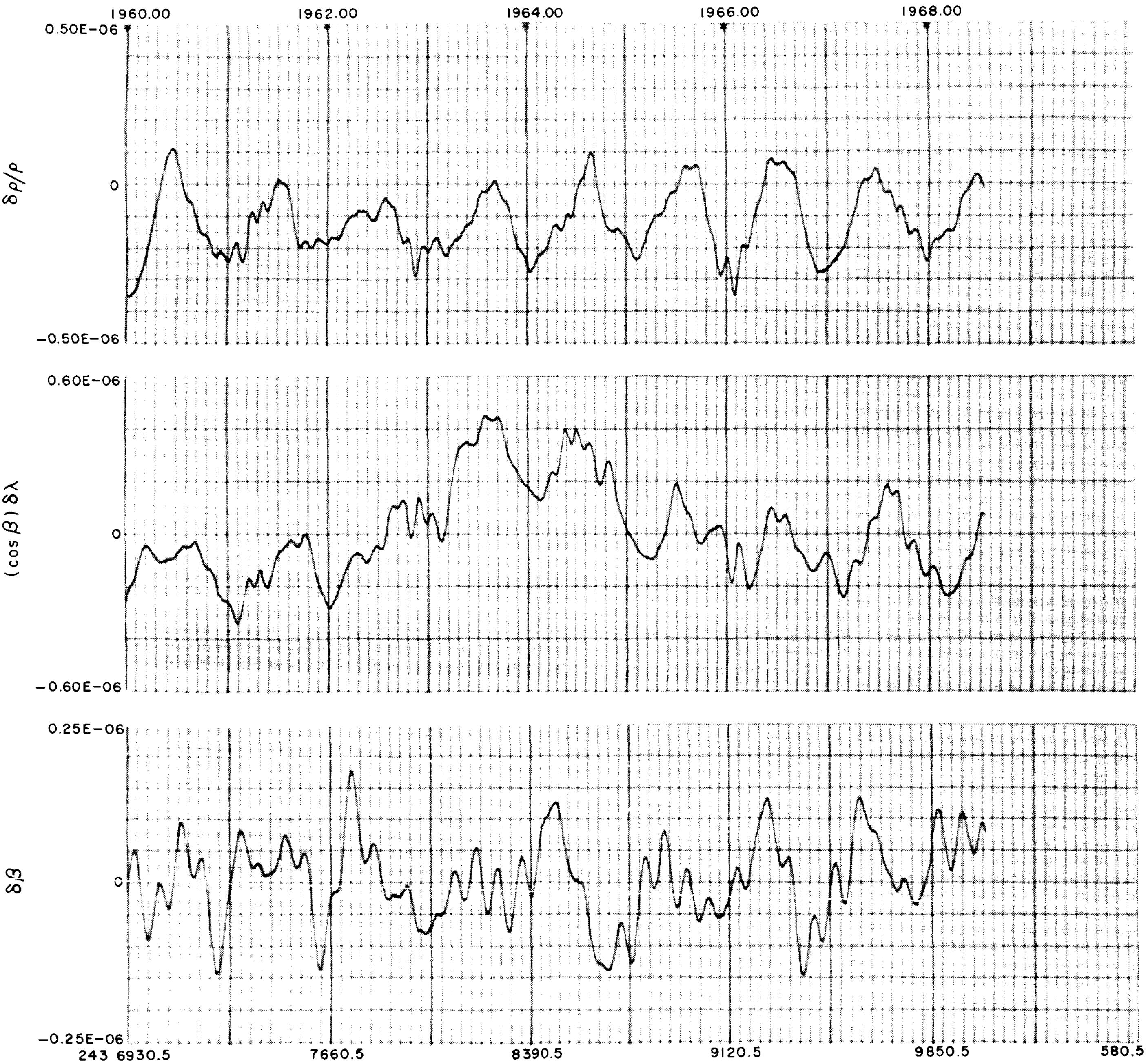

1960.00
1962.00
1964.00
1966.00
1968.00
0.50E−06
0
−0.50E−06
δρ/ρ
0.60E−06
0
−0.60E−06
(cos β) δλ
0.25E−06
0
−0.25E−06
δβ
243 6930.5
7660.5
8390.5
9120.5
9850.5
580.5

Fig. C-2 (cont'd)

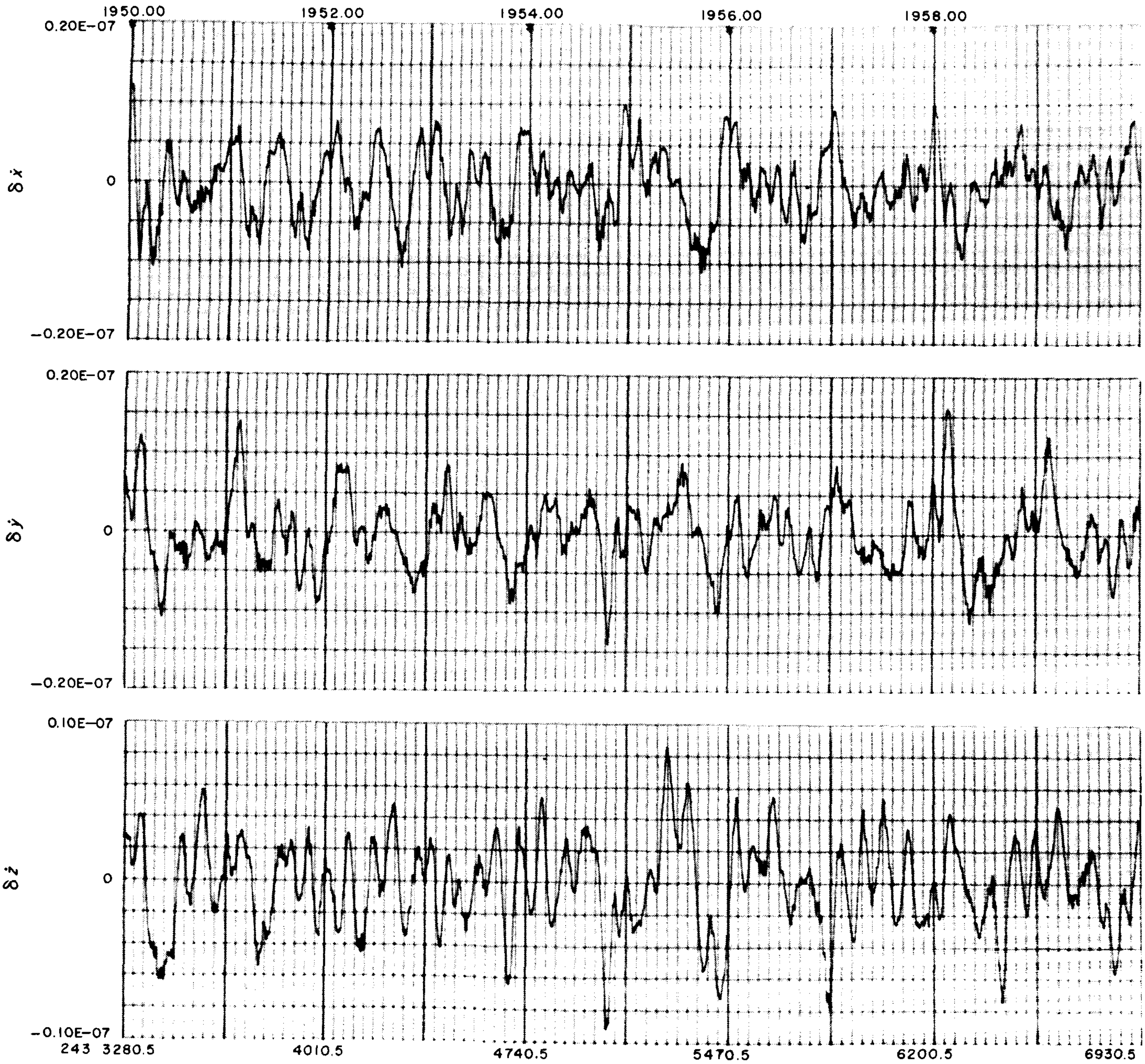


Fig. C-3. PLOD B to fit to Earth–Moon barycenter source ephemeris; $\delta\dot{x}$, $\delta\dot{y}$, $\delta\dot{z}$

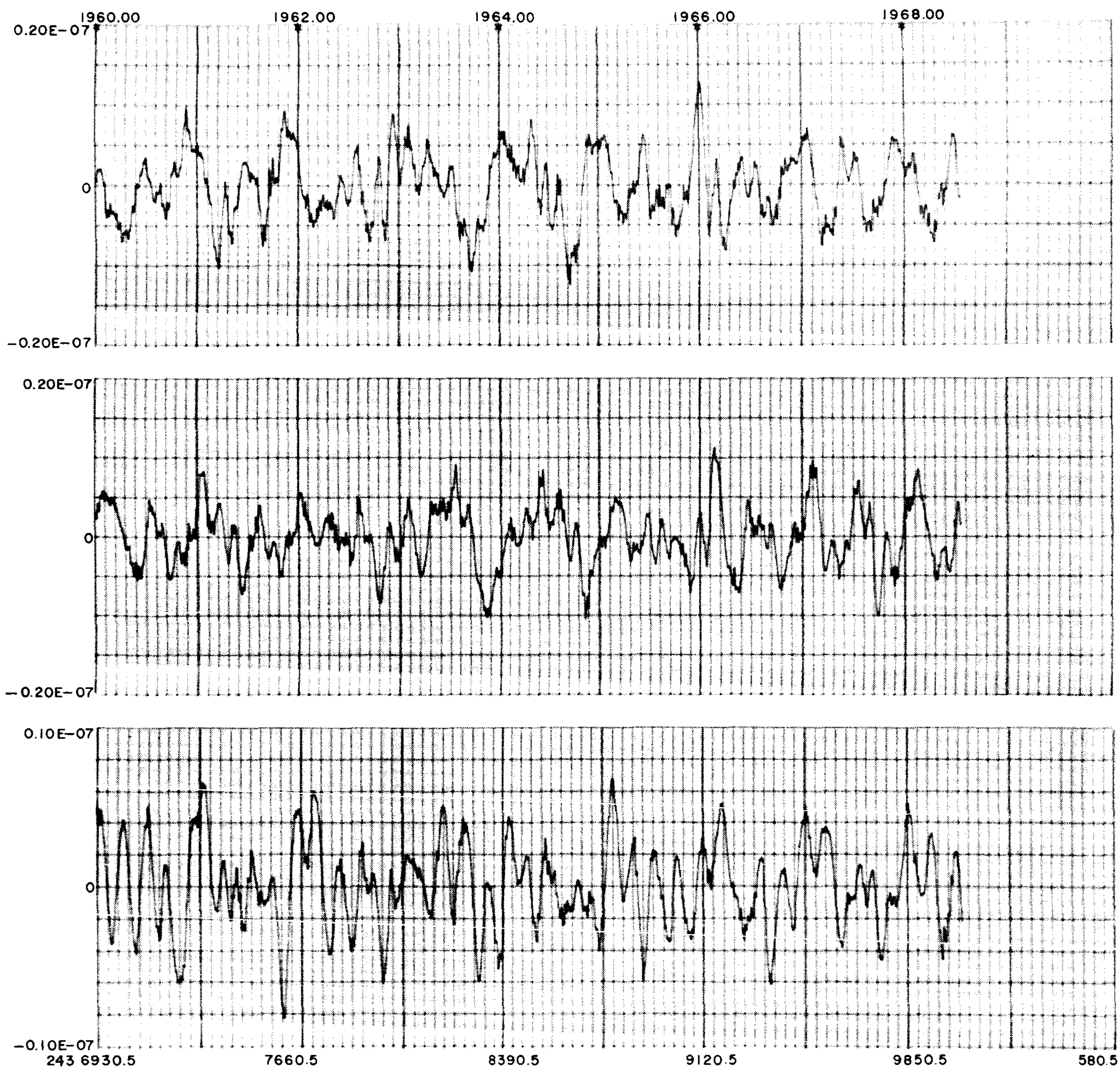

1960.00
1962.00
1964.00
1966.00
1968.00
0.20E−07
$\delta\dot{x}$
0
−0.20E−07
0.20E−07
$\delta\dot{y}$
0
−0.20E−07
0.10E−07
$\delta\dot{z}$
0
−0.10E−07
243 6930.5
7660.5
8390.5
9120.5
9850.5
580.5

Fig. C-3 (cont'd)

APPENDIX D

PLOD A Fit to the Earth–Moon Barycenter Source Ephemeris Minus PLOD B Fit to the Earth–Moon Barycenter Source Ephemeris

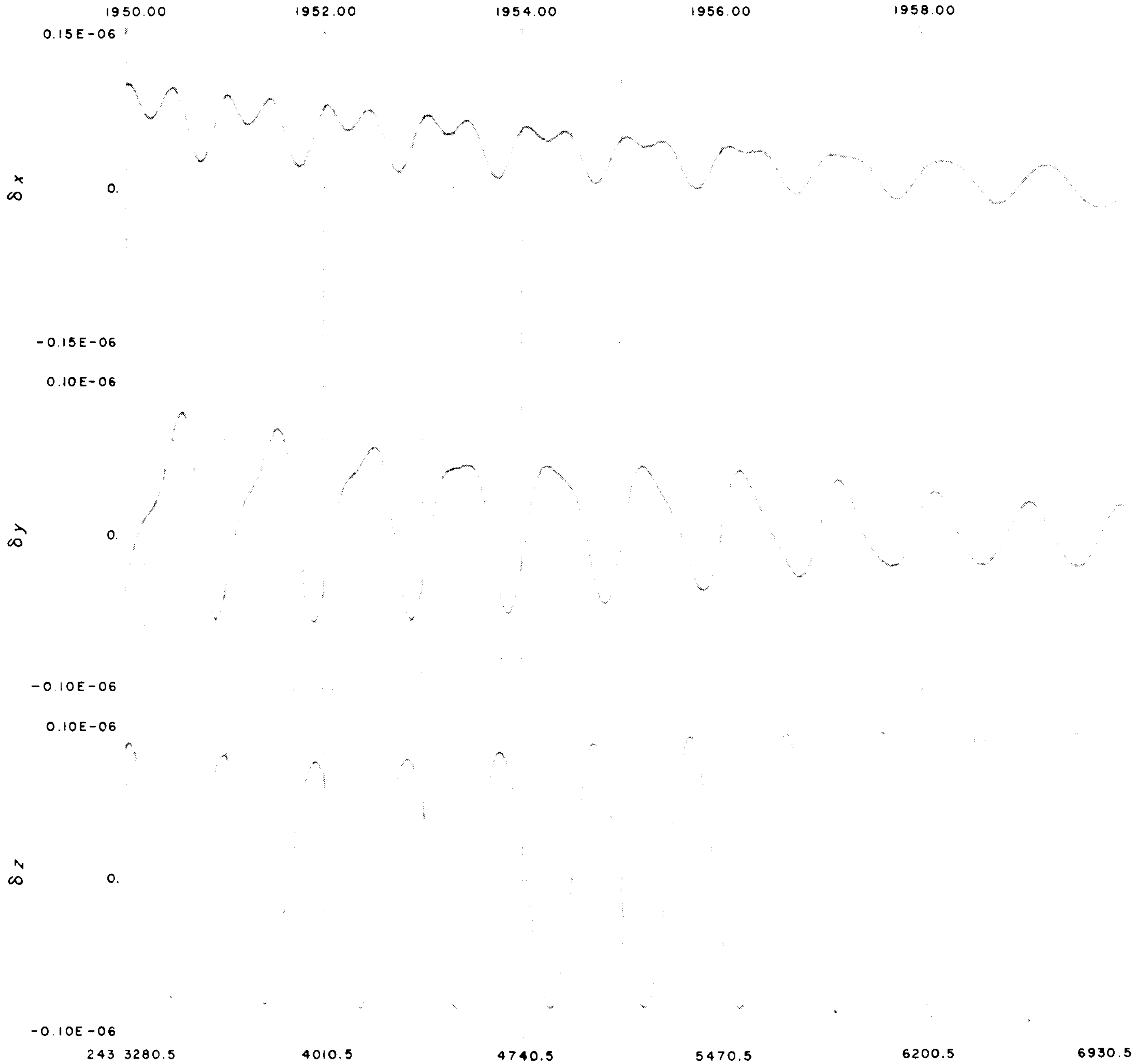


Fig. D-1. PLOD A fit to Earth–Moon barycenter source ephemeris minus PLOD B fit to Earth–Moon barycenter source ephemeris; δx, δy, δz

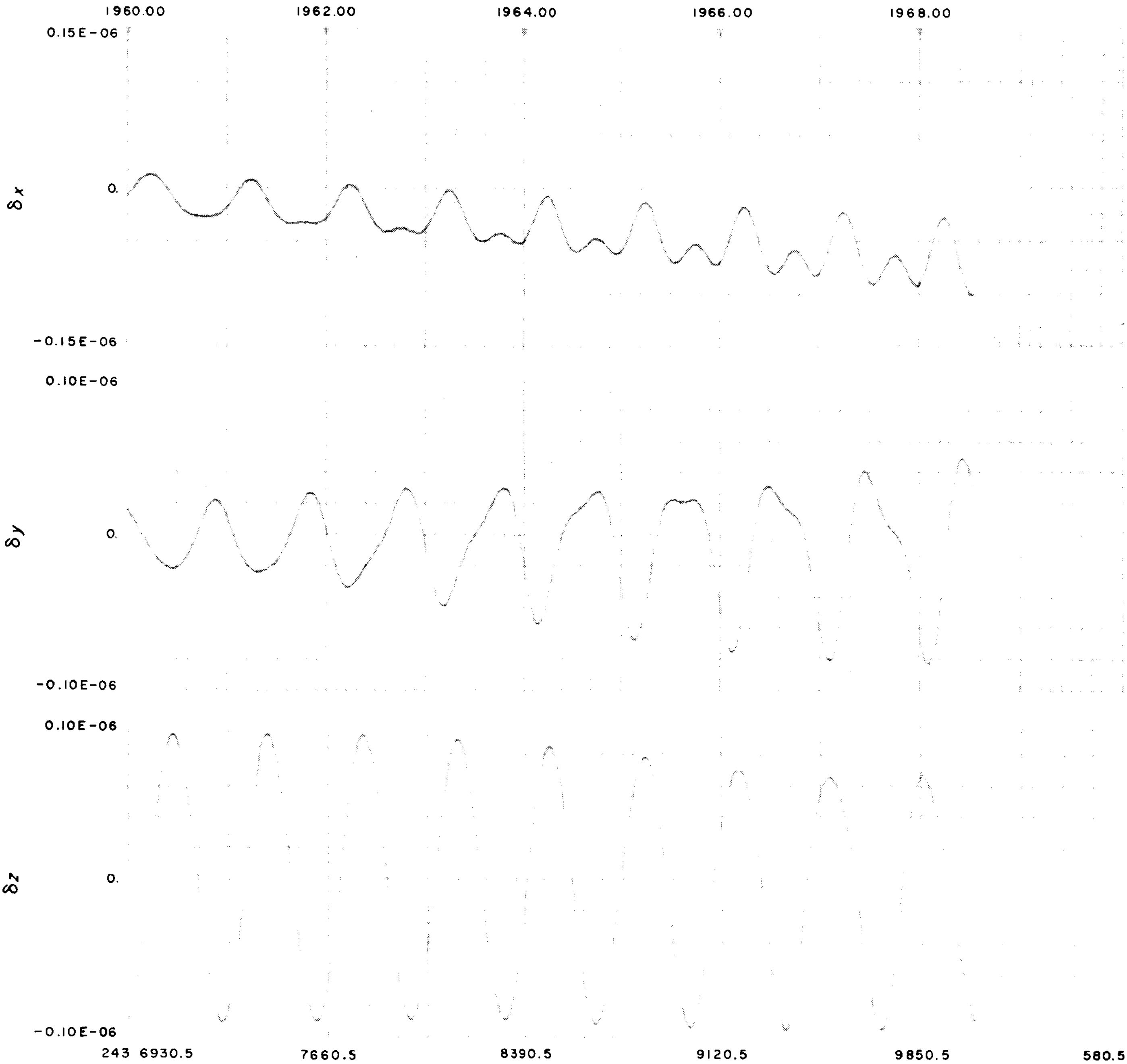

1960.00
1962.00
1964.00
1966.00
1968.00
0.15E-06
δx
0.
-0.15E-06
0.10E-06
δy
0.
-0.10E-06
0.10E-06
δz
0.
-0.10E-06
243 6930.5
7660.5
8390.5
9120.5
9850.5
580.5

Fig. D-1 (cont'd)

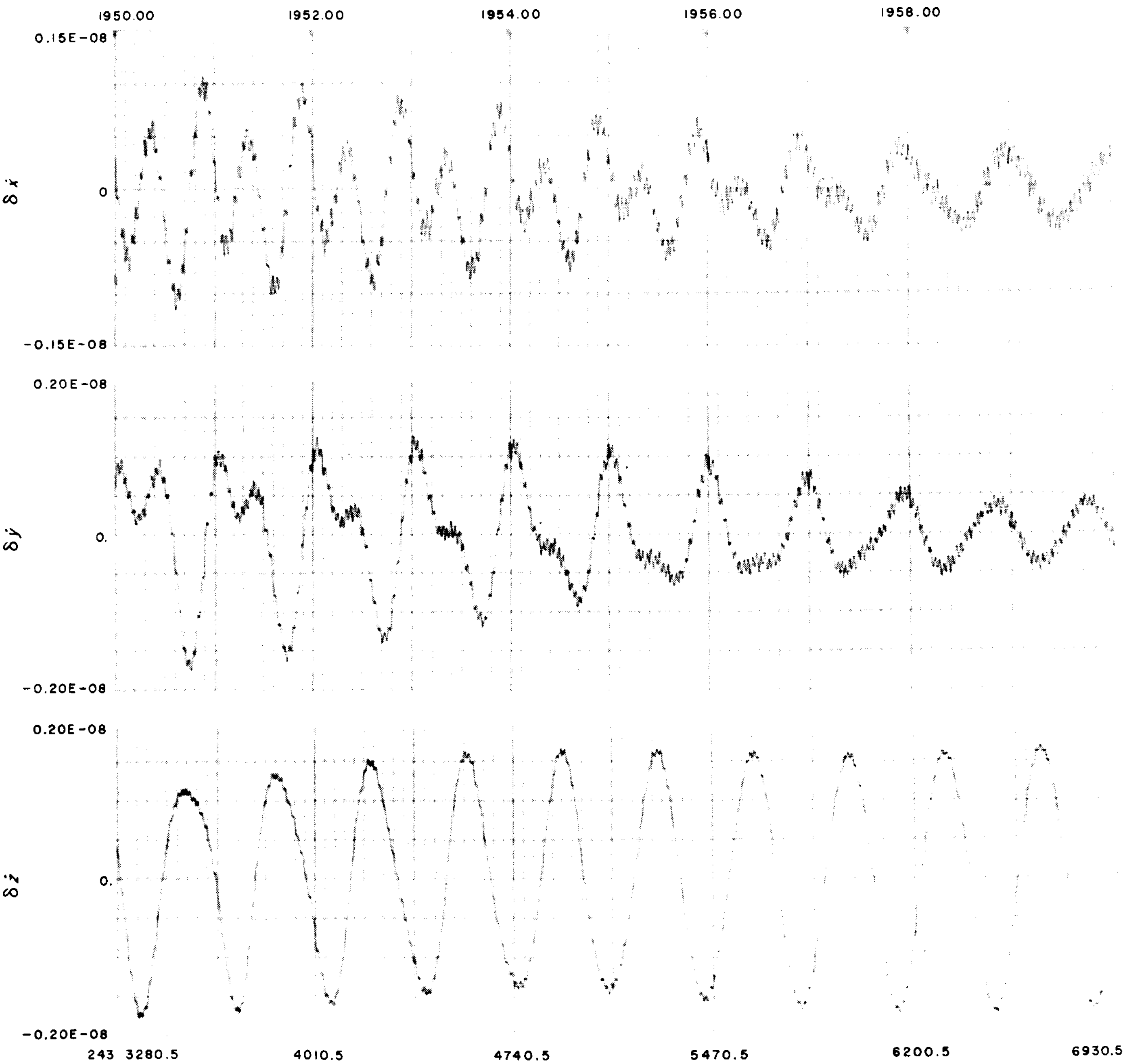
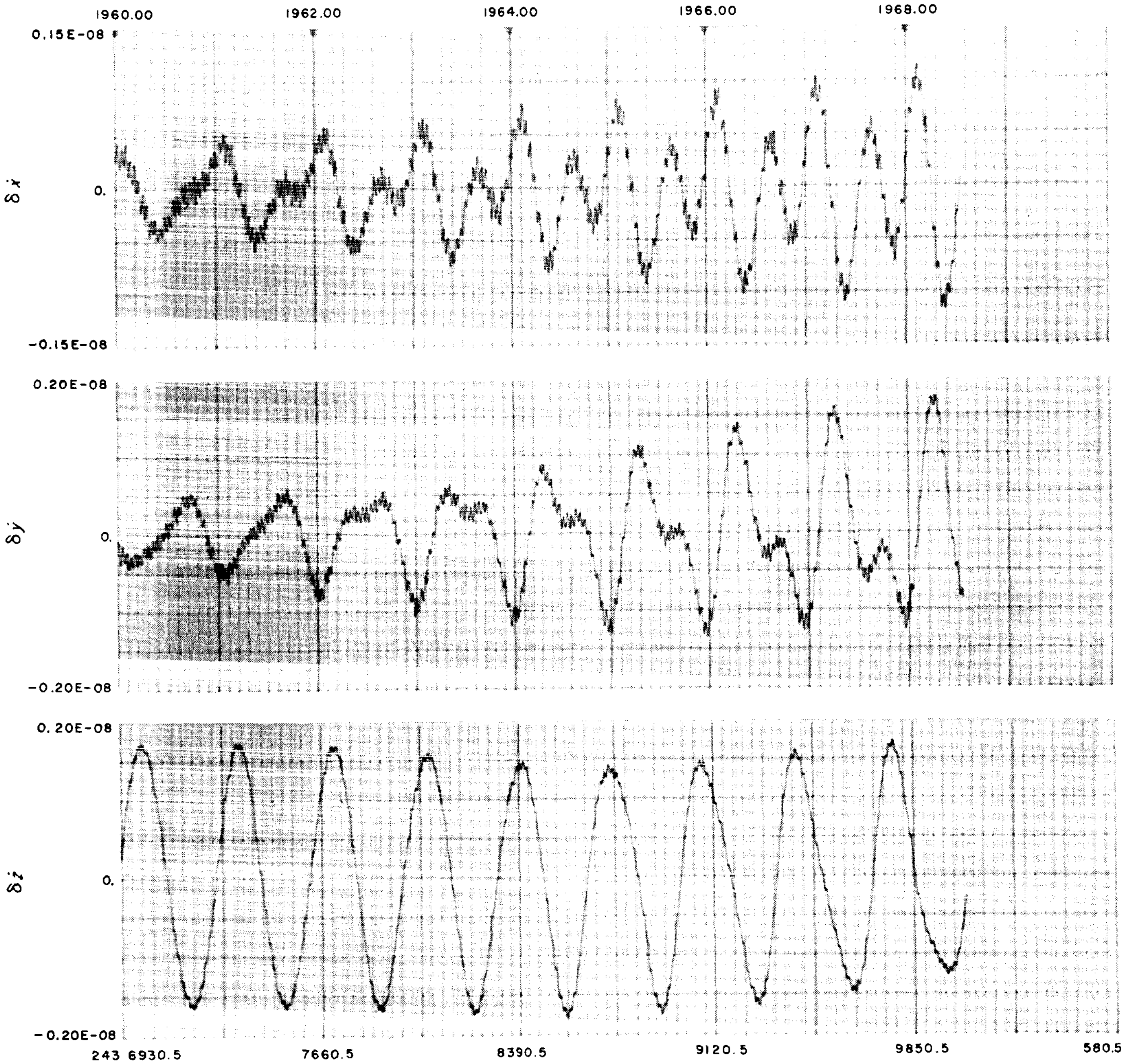


Fig. D-2. PLOD A fit to Earth–Moon barycenter source ephemeris minus PLOD B fit to Earth–Moon barycenter source ephemeris; $\delta\dot{x}$, $\delta\dot{y}$, $\delta\dot{z}$



1960.00
1962.00
1964.00
1966.00
1968.00
0.15E-08
$\delta\dot{x}$
0.
-0.15E-08
0.20E-08
$\delta\dot{y}$
0.
-0.20E-08
0.20E-08
$\delta\dot{z}$
0.
-0.20E-08
243 6930.5
7660.5
8390.5
9120.5
9850.5
580.5

Fig. D-2 (cont'd)

APPENDIX E

PLOD B Fit to PLOD A Fit for the Earth–Moon Barycenter

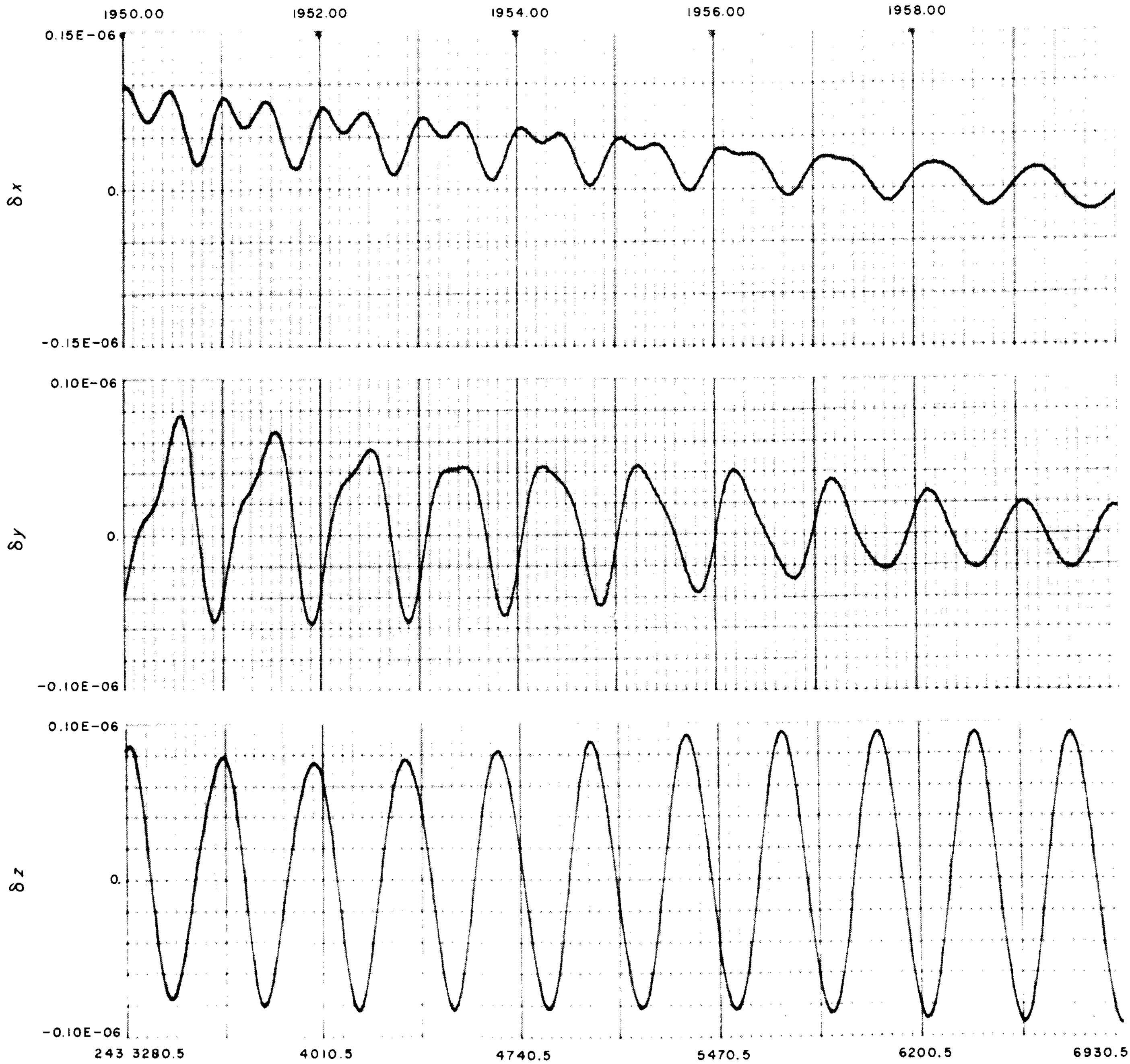


Fig. E-1. PLOD B fit to PLOD A fit for Earth–Moon barycenter; δx, δy, δz

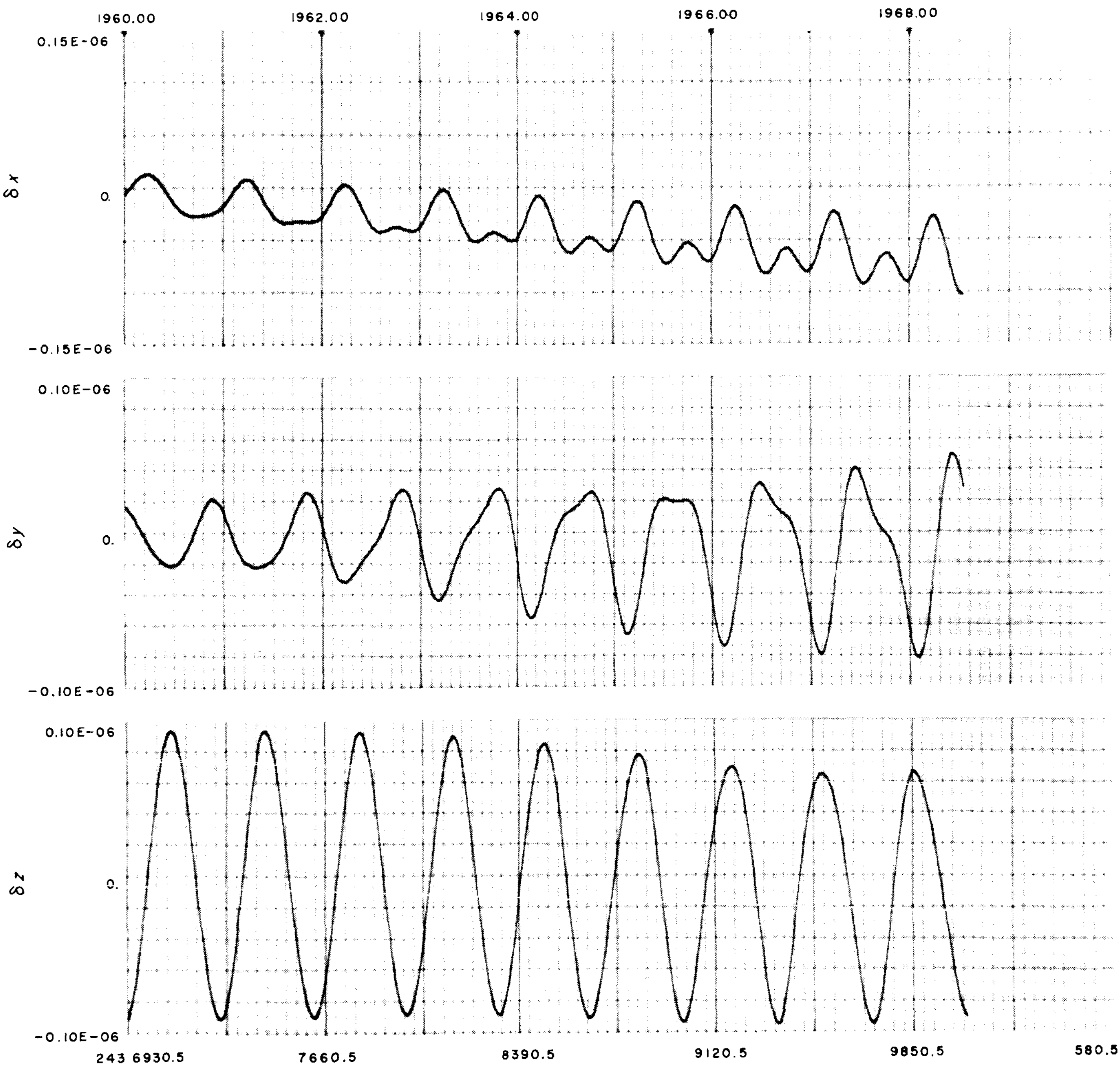

1960.00
1962.00
1964.00
1966.00
1968.00
0.15E-06
δx
0.
-0.15E-06
0.10E-06
δy
0.
-0.10E-06
0.10E-06
δz
0.
-0.10E-06
243 6930.5
7660.5
8390.5
9120.5
9850.5
580.5

Fig. E-1 (cont'd)

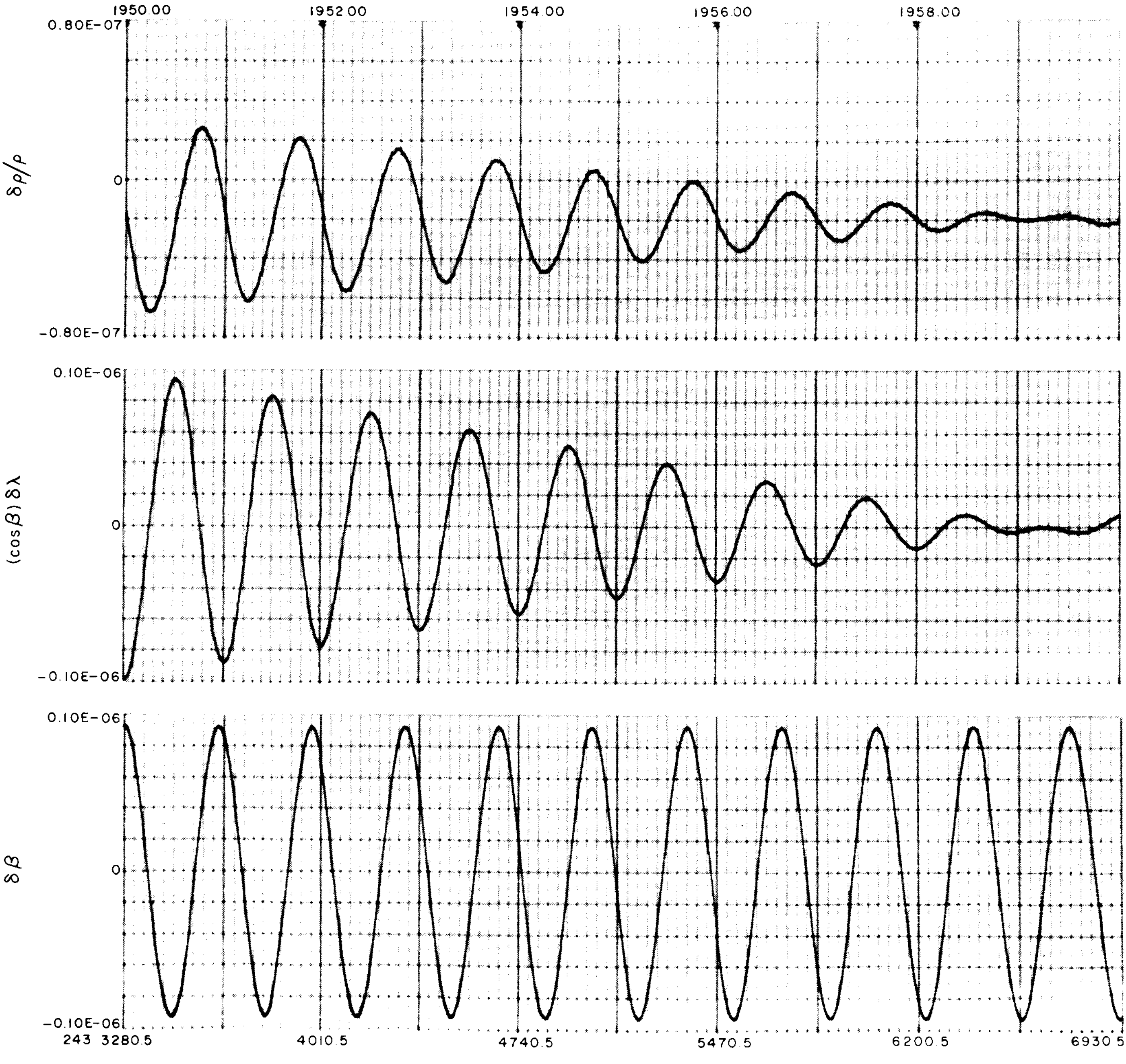


Fig. E-2. PLOD B fit to PLOD A fit for Earth–Moon barycenter; $\delta\rho/\rho$, $(\cos\beta)\,\delta\lambda$, $\delta\beta$

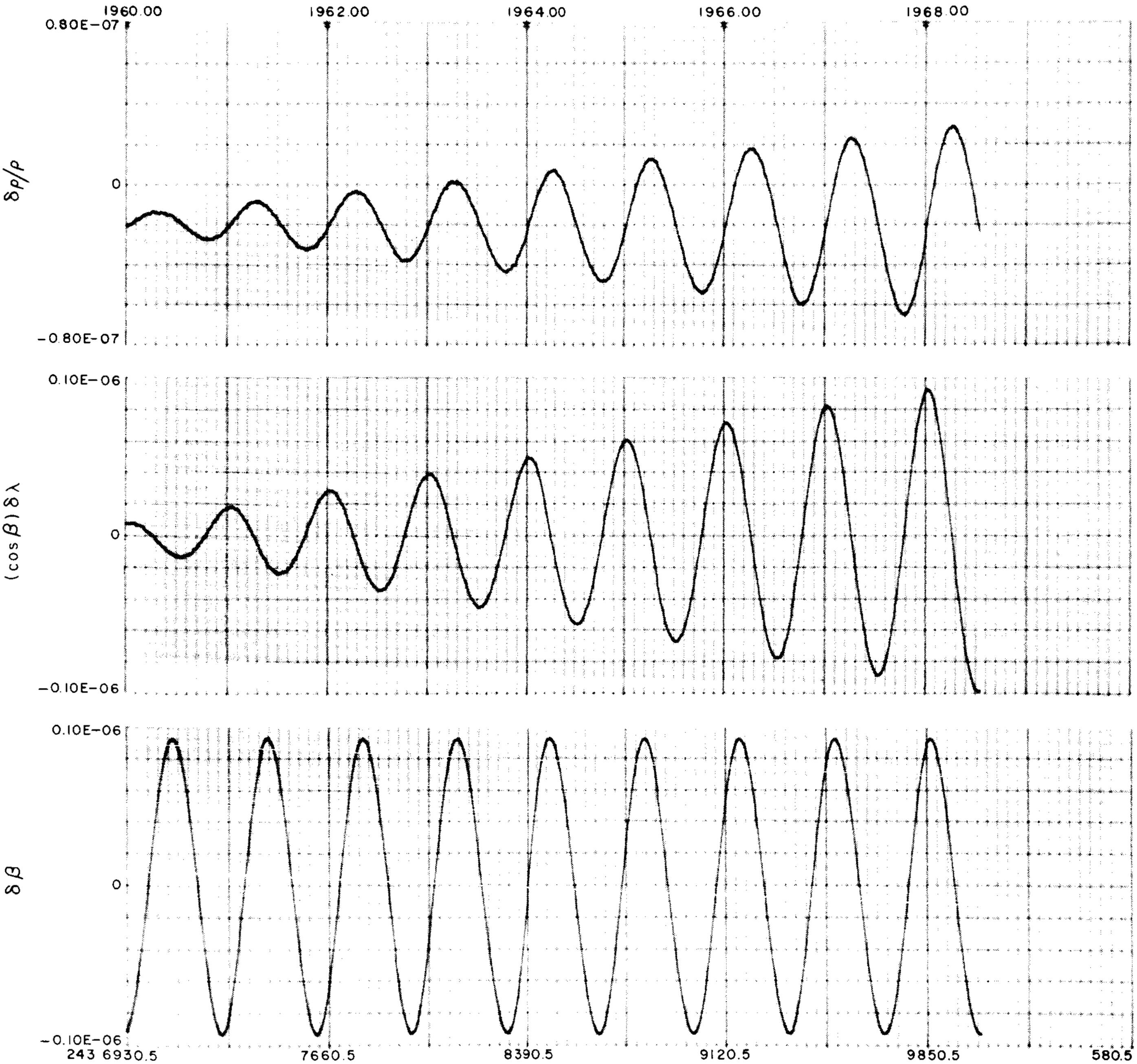

1960.00
1962.00
1964.00
1966.00
1968.00
0.80E−07
0
−0.80E−07
$\delta\rho/\rho$
0.10E−06
0
−0.10E−06
$(\cos\beta)\,\delta\lambda$
0.10E−06
0
−0.10E−06
$\delta\beta$
243 6930.5
7660.5
8390.5
9120.5
9850.5
580.5

Fig. E-2 (cont'd)

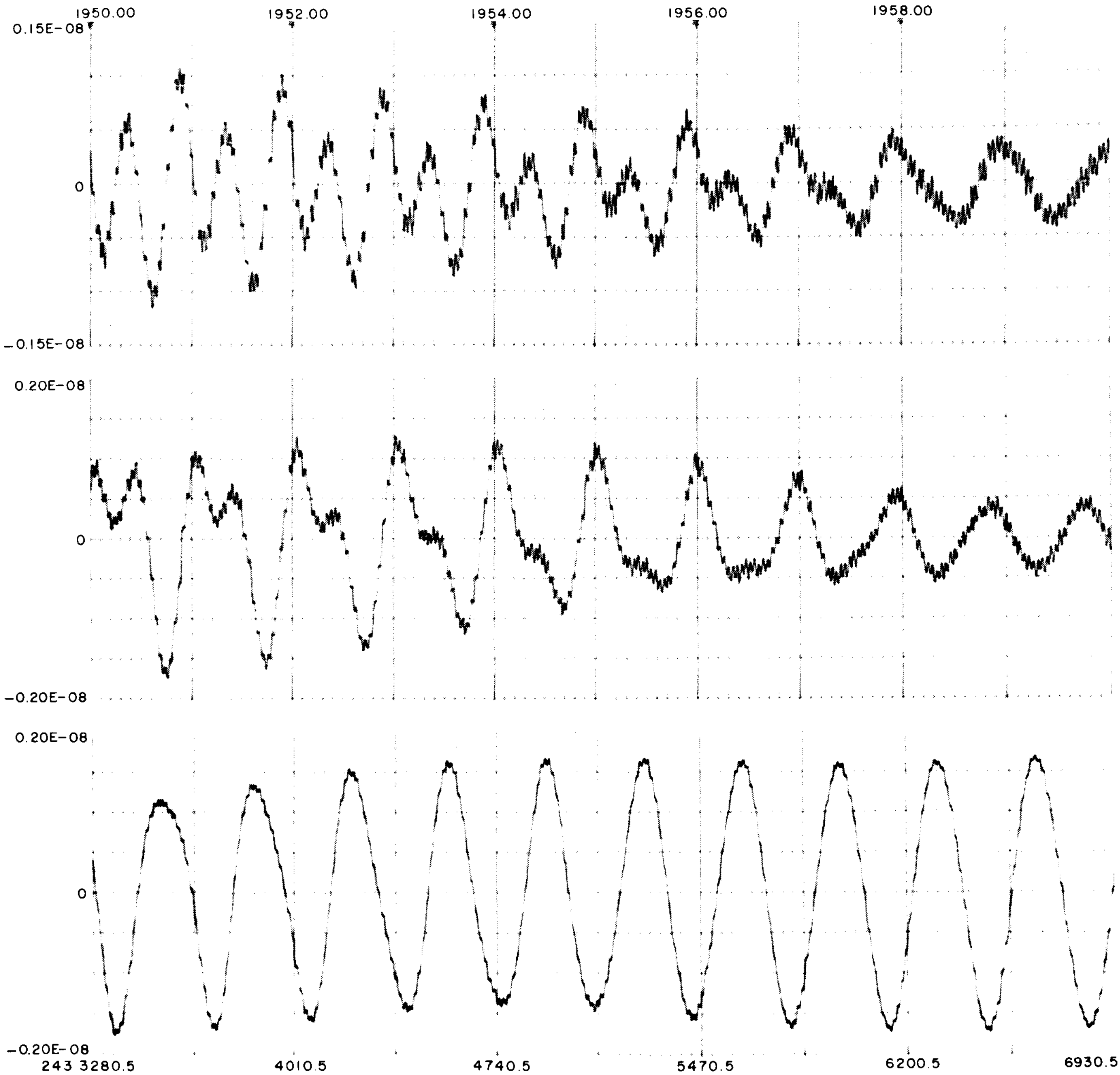


Fig. E-3. PLOD B fit to PLOD A fit for Earth–Moon barycenter; $\delta\dot{x}$, $\delta\dot{y}$, $\delta\dot{z}$

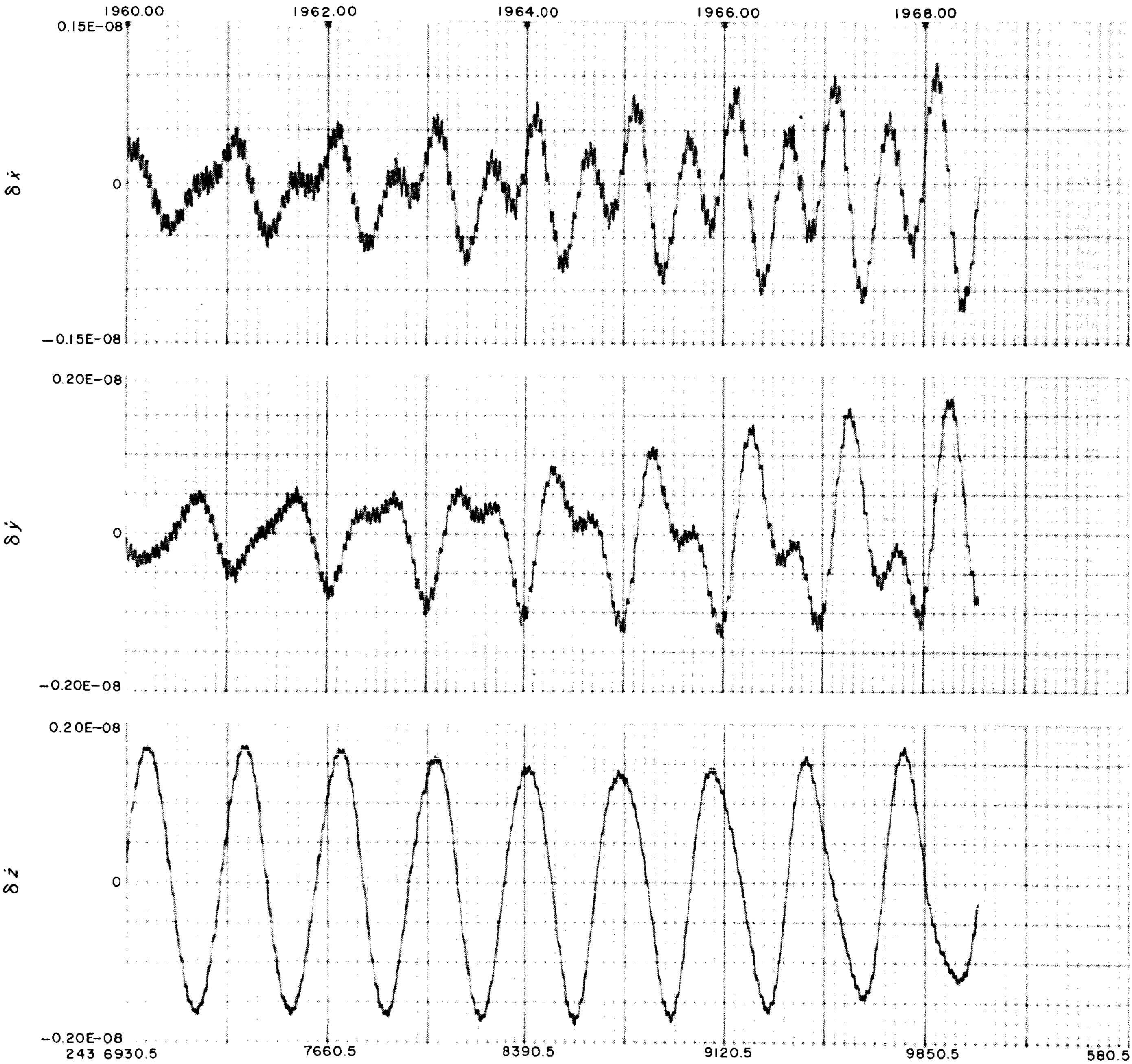

1960.00
1962.00
1964.00
1966.00
1968.00
0.15E-08
0
-0.15E-08
$\delta\dot{x}$
0.20E-08
0
-0.20E-08
$\delta\dot{y}$
0.20E-08
0
-0.20E-08
$\delta\dot{z}$
243 6930.5
7660.5
8390.5
9120.5
9850.5
580.5

Fig. E-3 (cont'd)

APPENDIX F

PLOD A Fit to Venus Source Ephemeris

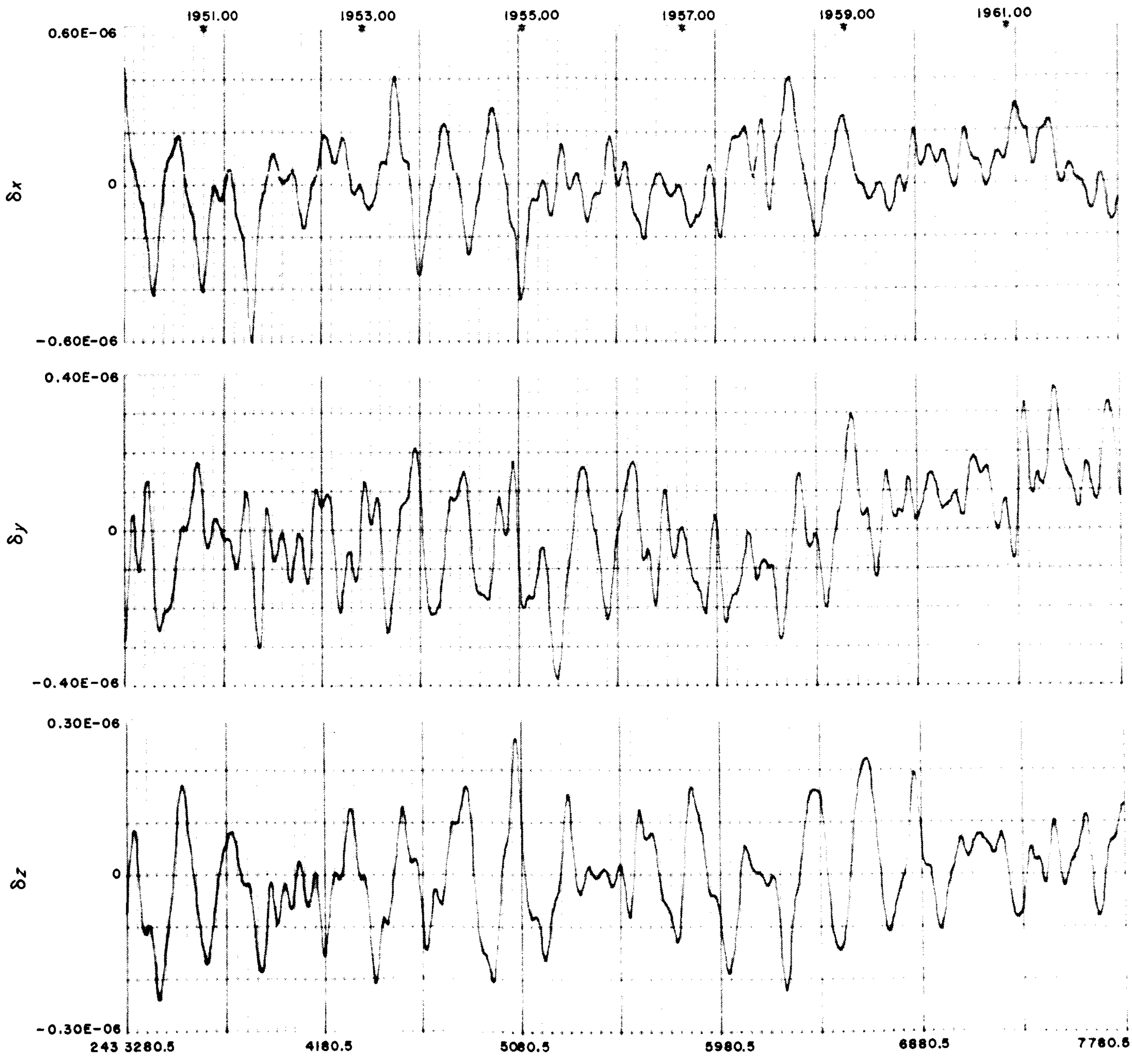


Fig. F-1. PLOD A fit to Venus source ephemeris; δx, δy, δz

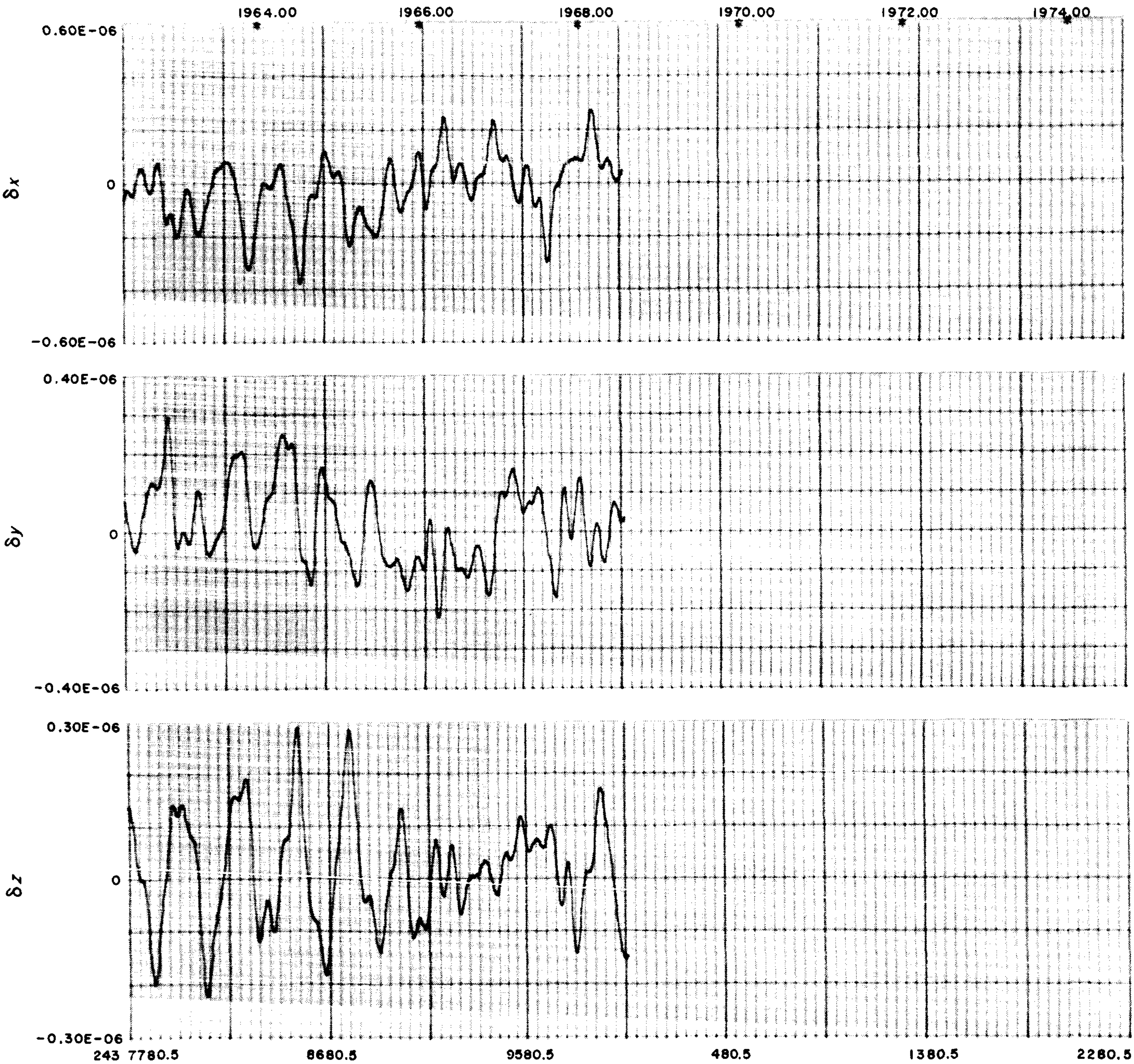

1964.00
1966.00
1968.00
1970.00
1972.00
1974.00
0.60E-06
0
-0.60E-06
δx
0.40E-06
0
-0.40E-06
δy
0.30E-06
0
-0.30E-06
δz
243 7780.5
8680.5
9580.5
480.5
1380.5
2280.5

Fig. F-1 (cont'd)

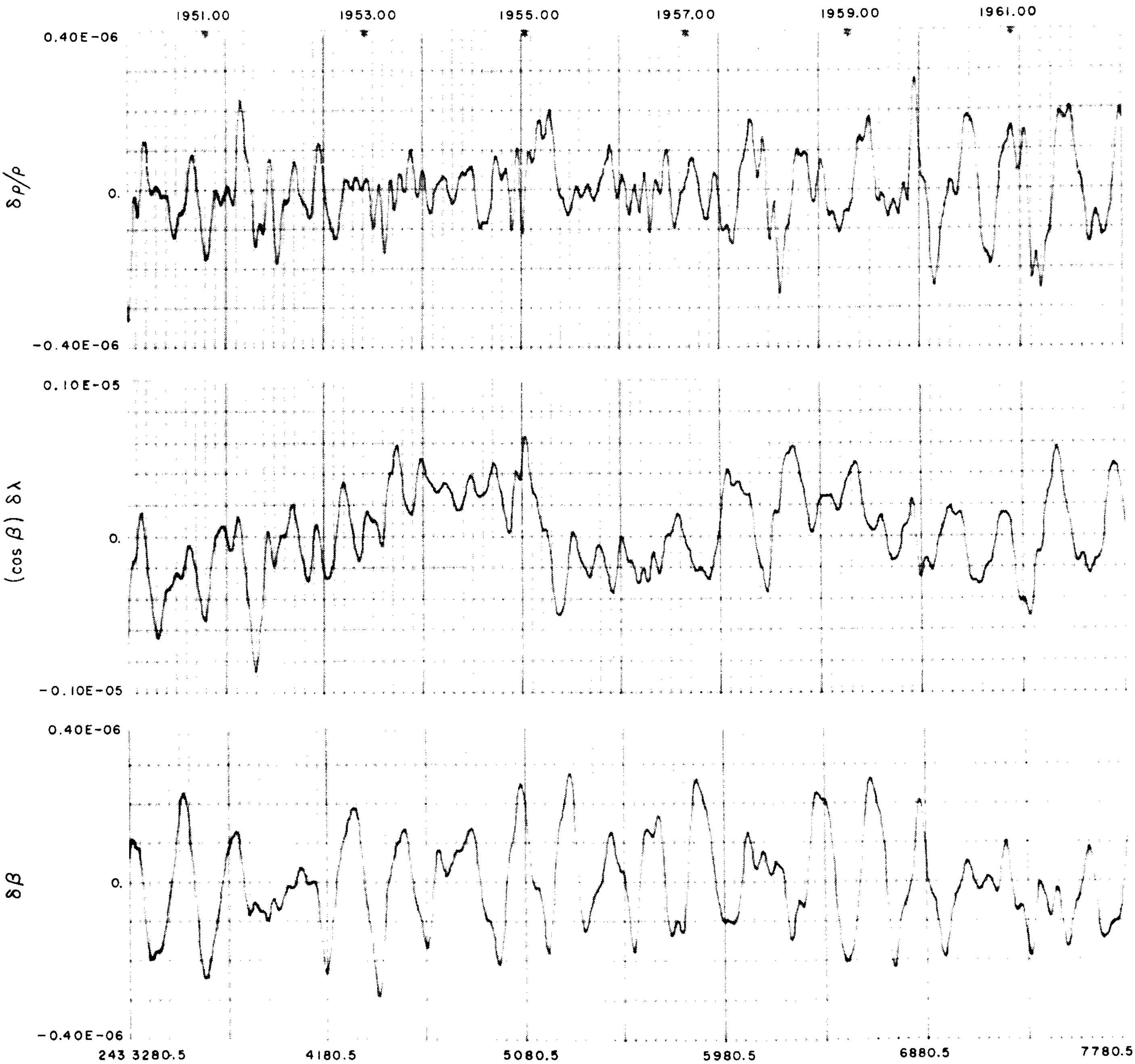


Fig. F-2. PLOD A fit to Venus source ephemeris; $\delta\rho/\rho$, $(\cos\beta)\ \delta\lambda$, $\delta\beta$

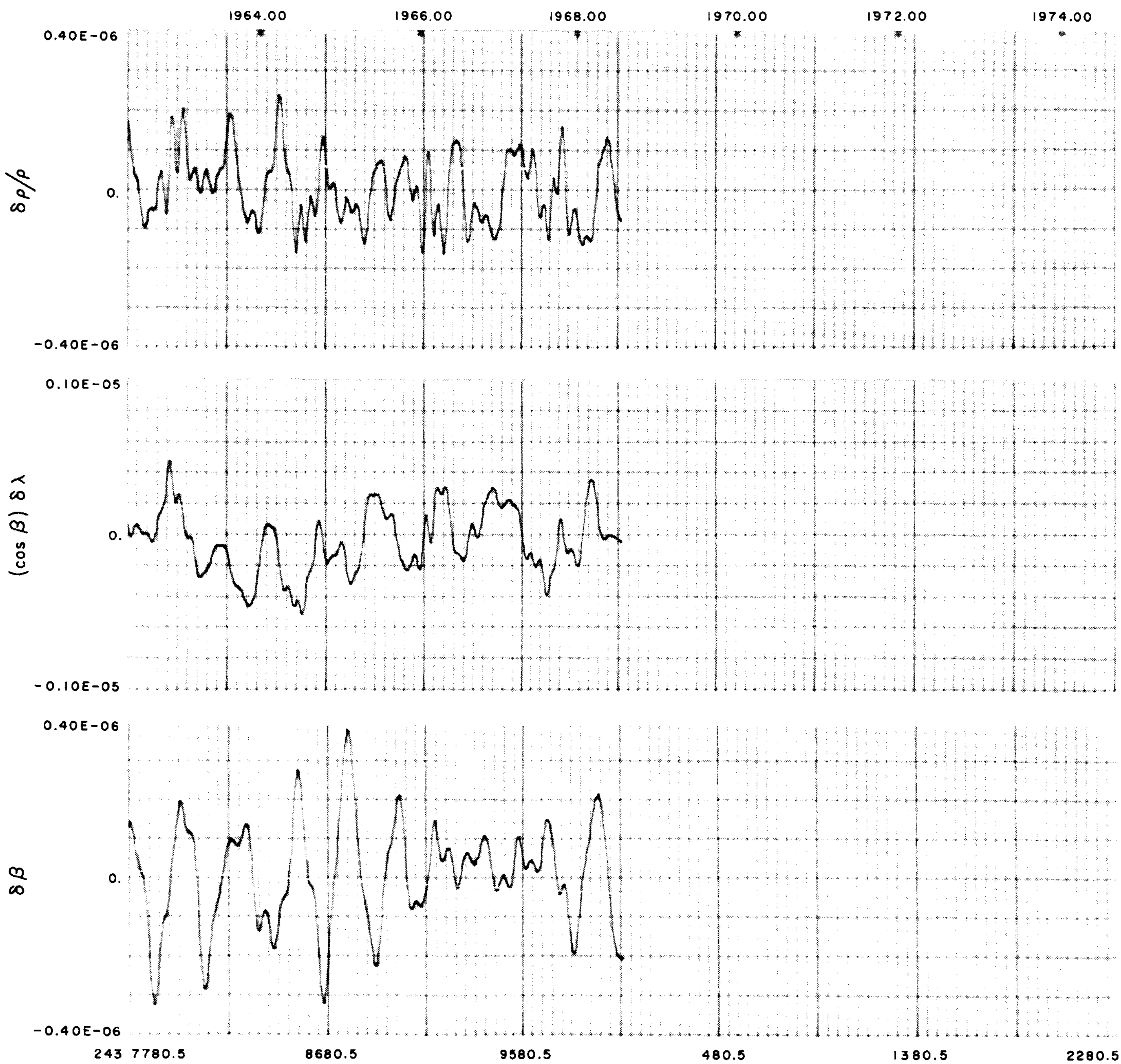

1964.00
1966.00
1968.00
1970.00
1972.00
1974.00
0.40E-06
0.
-0.40E-06
δρ/ρ
0.10E-05
0.
-0.10E-05
(cos β) δλ
0.40E-06
0.
-0.40E-06
δβ
243 7780.5
8680.5
9580.5
480.5
1380.5
2280.5

Fig. F-2 (cont'd)

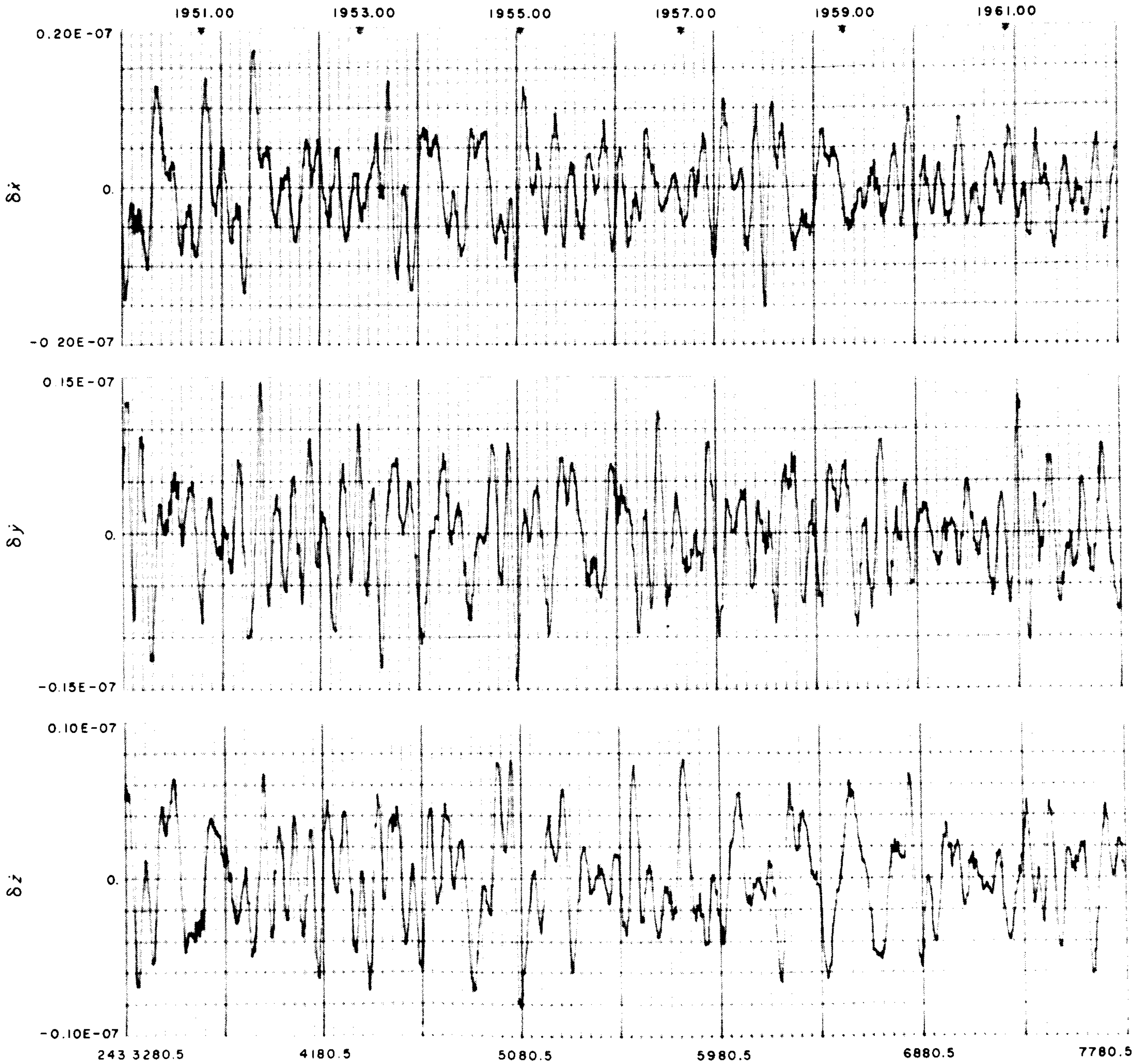


Fig. F-3. PLOD A fit to Venus source ephemeris; $\delta\dot{x}$, $\delta\dot{y}$, $\delta\dot{z}$

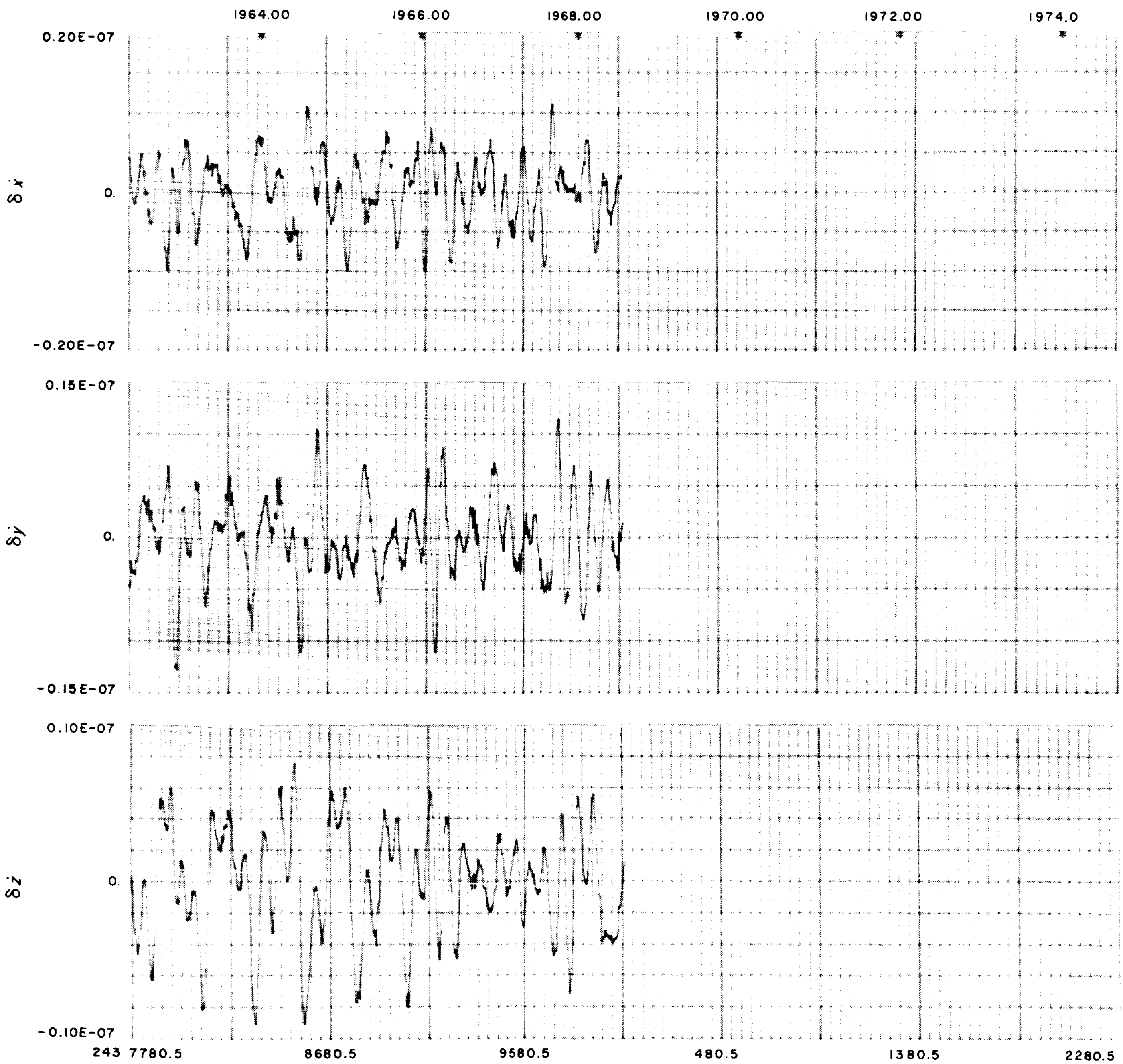

1964.00
1966.00
1968.00
1970.00
1972.00
1974.0
0.20E-07
$\delta \dot{x}$
0.
-0.20E-07
0.15E-07
$\delta \dot{y}$
0.
-0.15E-07
0.10E-07
$\delta \dot{z}$
0.
-0.10E-07
243 7780.5
8680.5
9580.5
480.5
1380.5
2280.5

Fig. F-3 (cont'd)

APPENDIX G

PLOD B Fit to PLOD A Fit for Venus

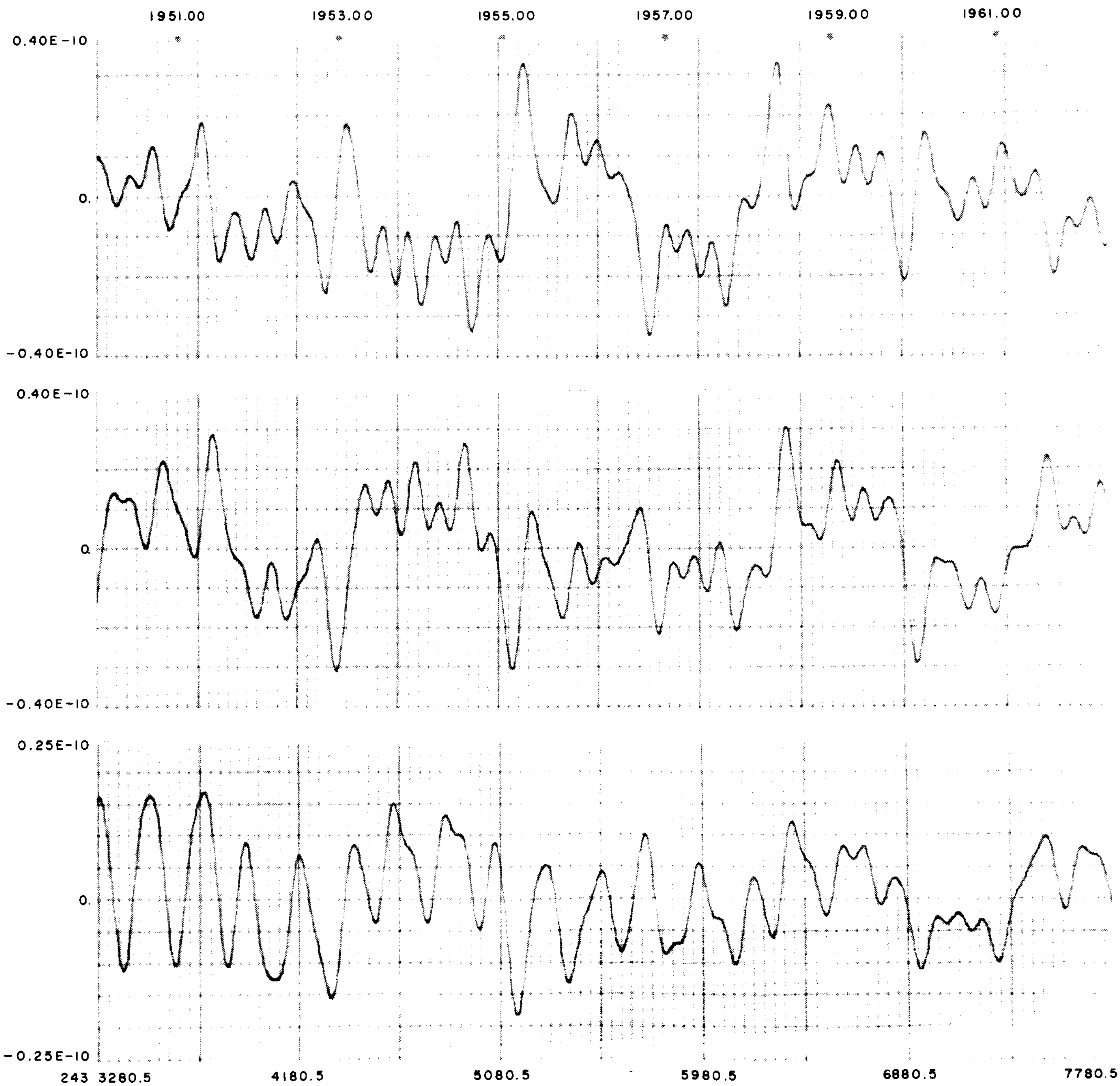


Fig. G-1. PLOD B fit to PLOD A fit for Venus; δx, δy, δz

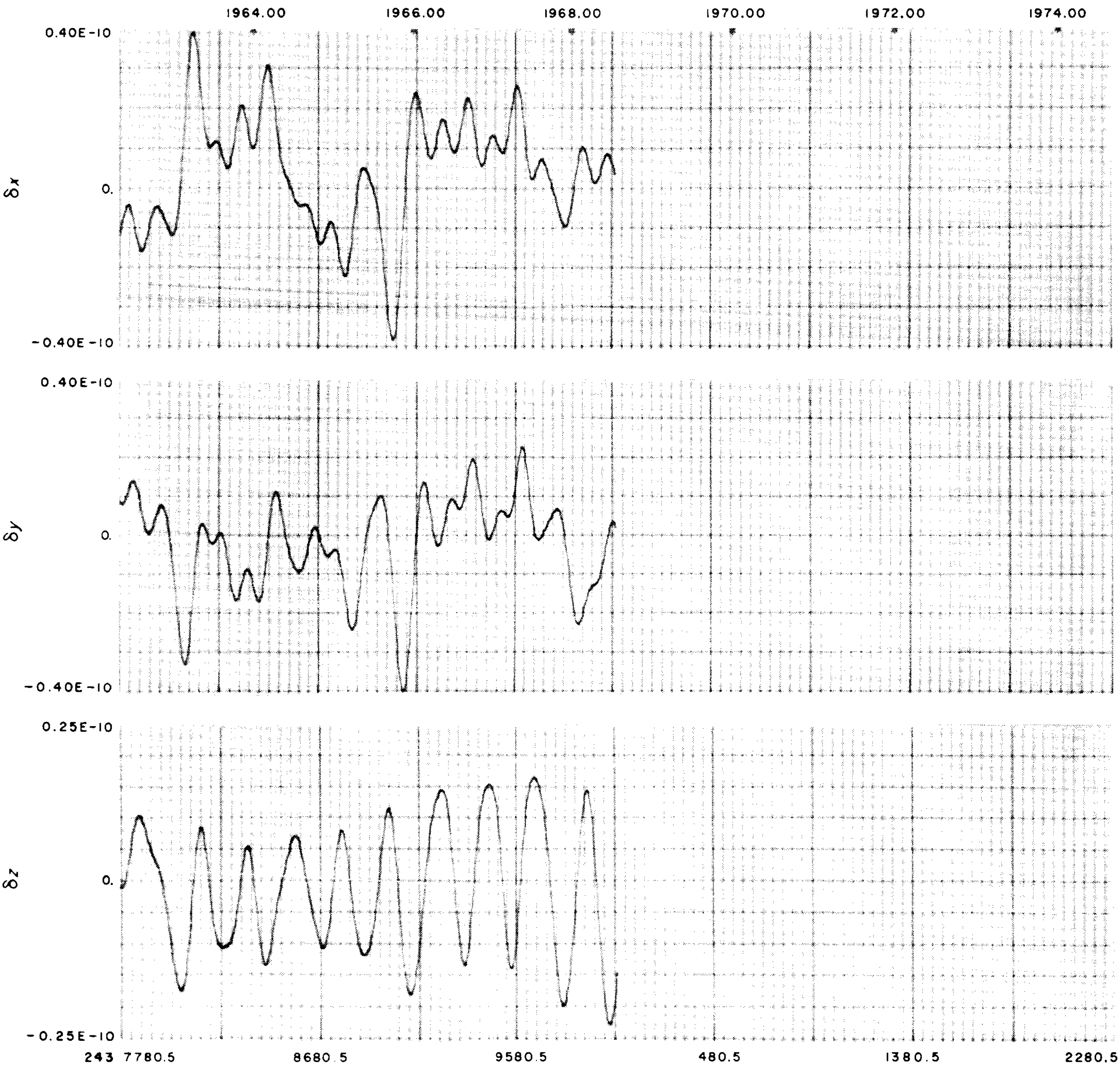

1964.00
1966.00
1968.00
1970.00
1972.00
1974.00
0.40E-10
δx
0.
-0.40E-10
0.40E-10
δy
0.
-0.40E-10
0.25E-10
δz
0.
-0.25E-10
243 7780.5
8680.5
9580.5
480.5
1380.5
2280.5

Fig. G-1 (cont'd)

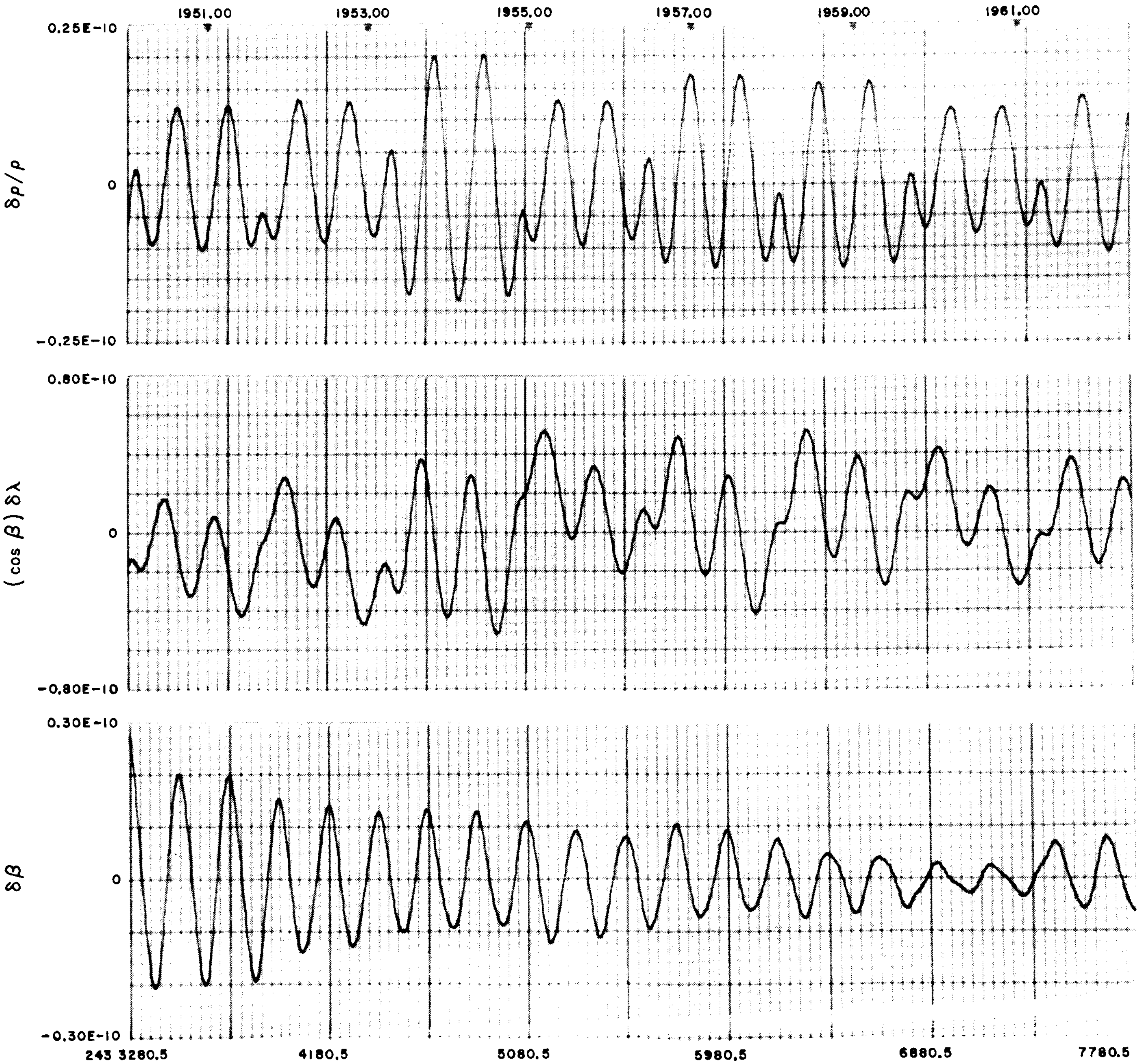


Fig. G-2. PLOD B fit to PLOD A fit for Venus; $\delta\rho/\rho$, $(\cos\beta)\,\delta\lambda$, $\delta\beta$

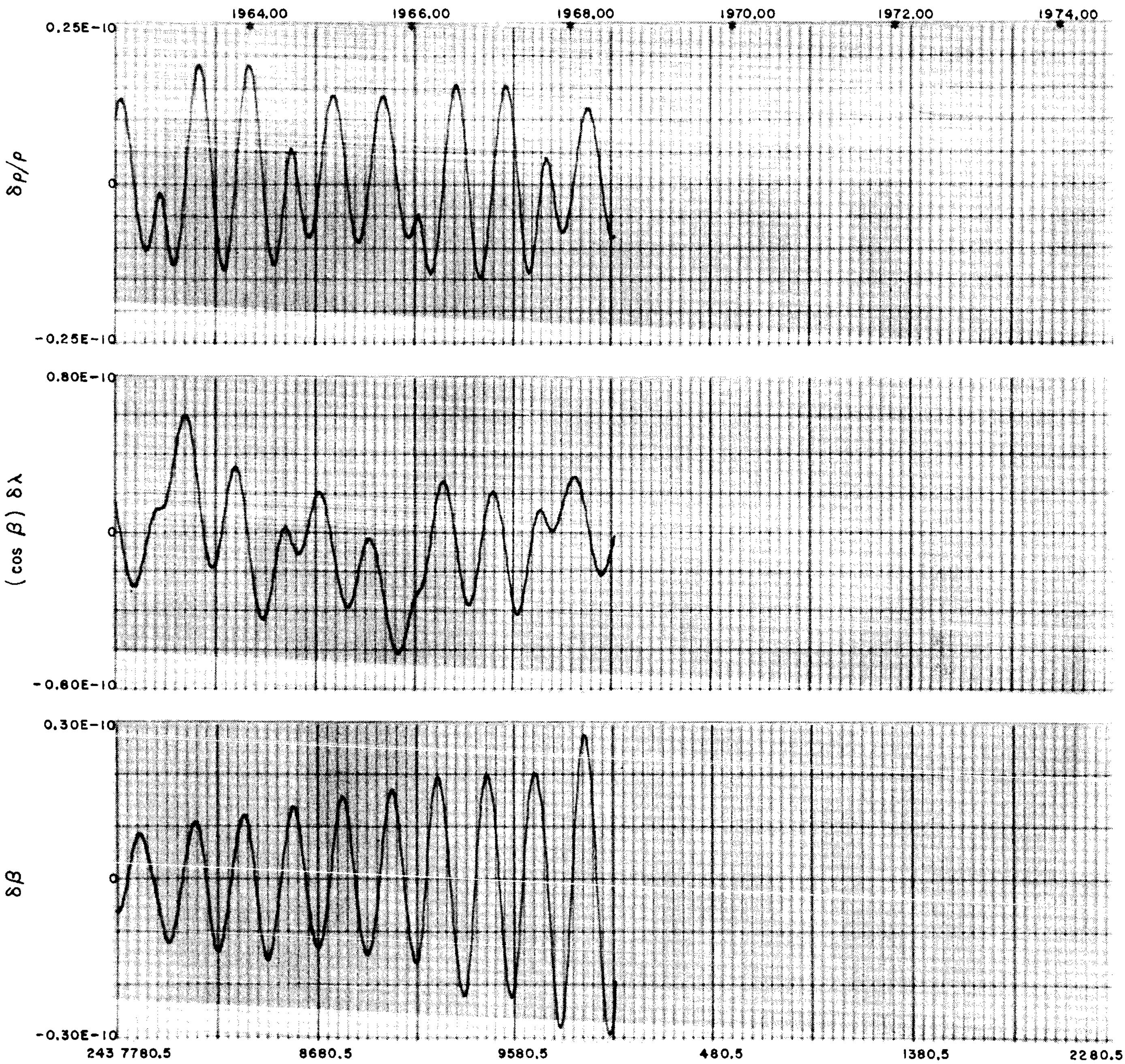

1964.00
1966.00
1968.00
1970.00
1972.00
1974.00
0.25E-10
0
-0.25E-10
δρ/ρ
0.80E-10
0
-0.80E-10
(cos β) δλ
0.30E-10
0
-0.30E-10
δβ
243 7780.5
8680.5
9580.5
480.5
1380.5
2280.5

Fig. G-2 (cont'd)

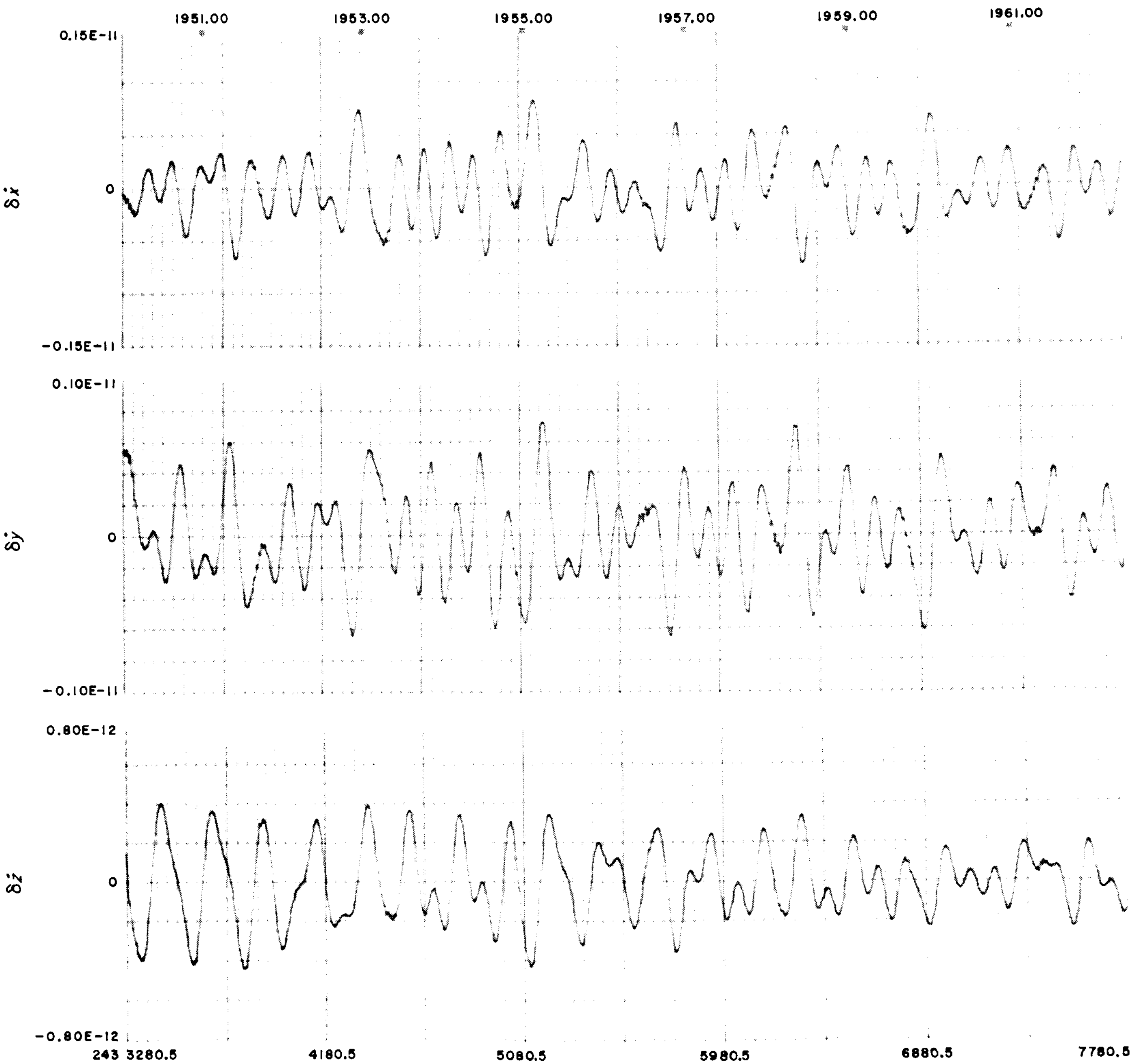


Fig. G-3. PLOD B fit to PLOD A fit for Venus; $\delta\dot{x}$, $\delta\dot{y}$, $\delta\dot{z}$

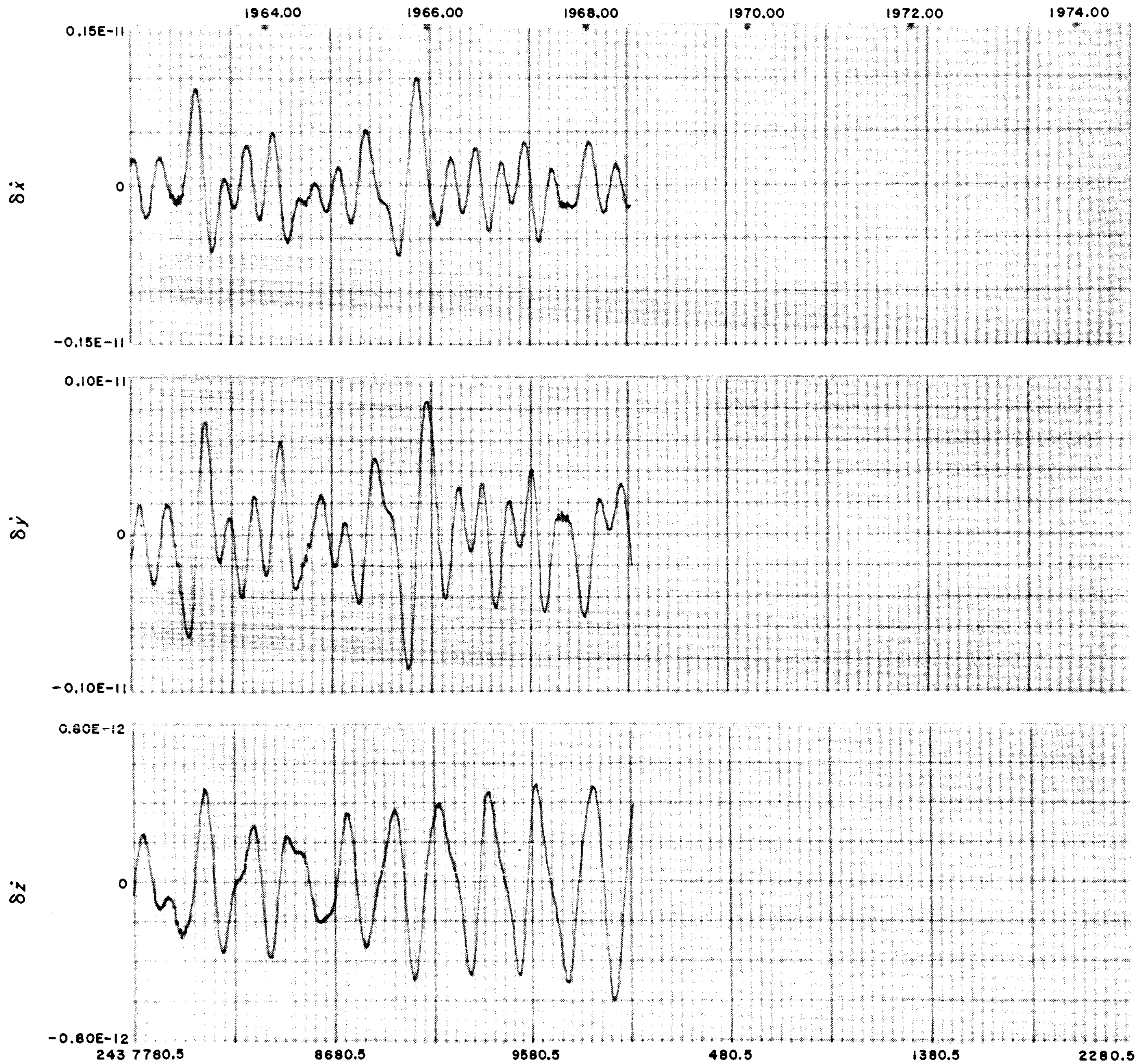

1964.00
1966.00
1968.00
1970.00
1972.00
1974.00
0.15E-11
0
-0.15E-11
$\delta\dot{x}$
0.10E-11
0
-0.10E-11
$\delta\dot{y}$
0.60E-12
0
-0.80E-12
$\delta\dot{z}$
243 7780.5
8680.5
9580.5
480.5
1380.5
2280.5

Fig. G-3 (cont'd)

APPENDIX H

PLOD B Fit to PLOD A Fit for Mars

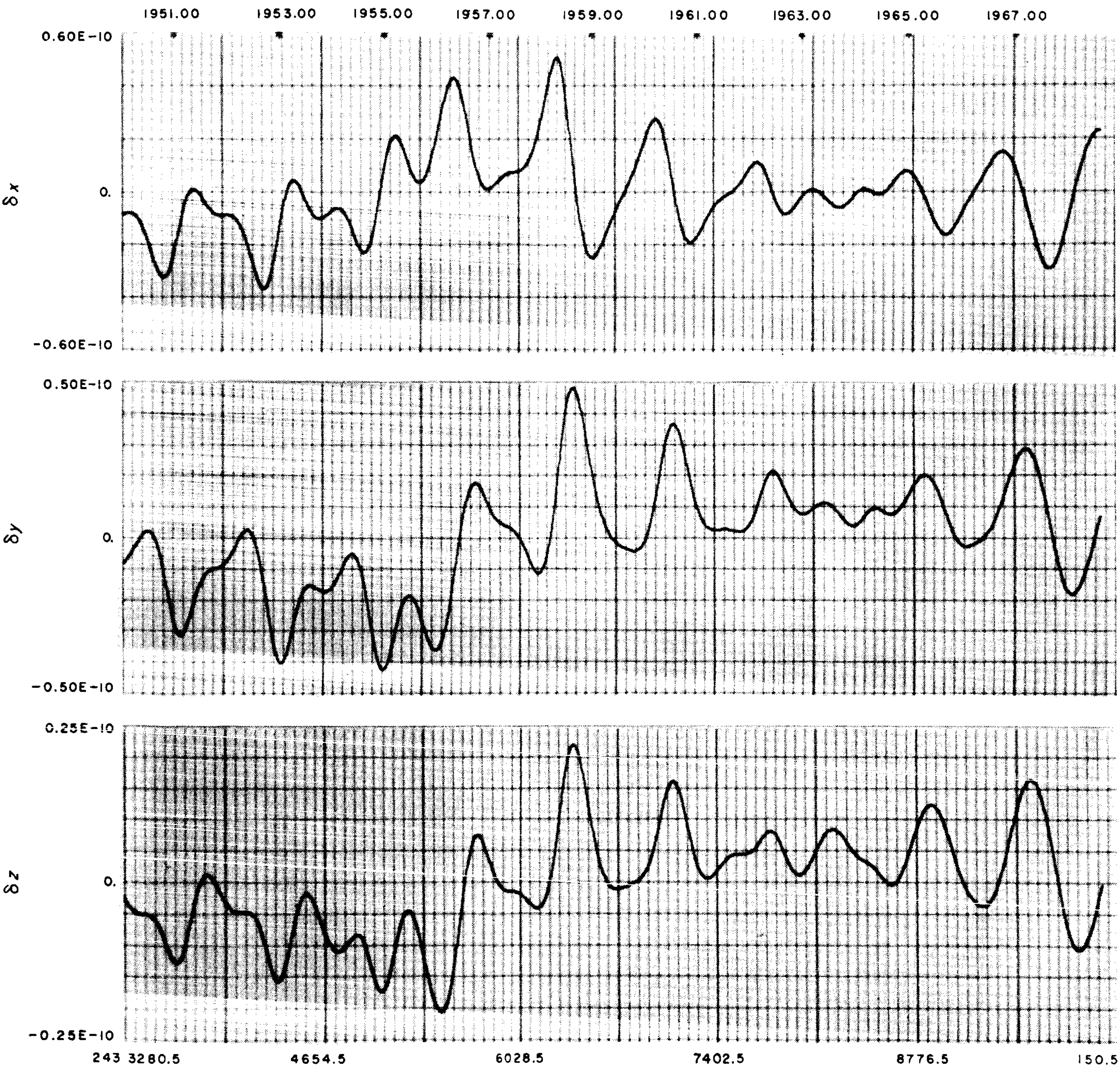


Fig. H-1. PLOD B fit to PLOD A fit for Mars; δx, δy, δz

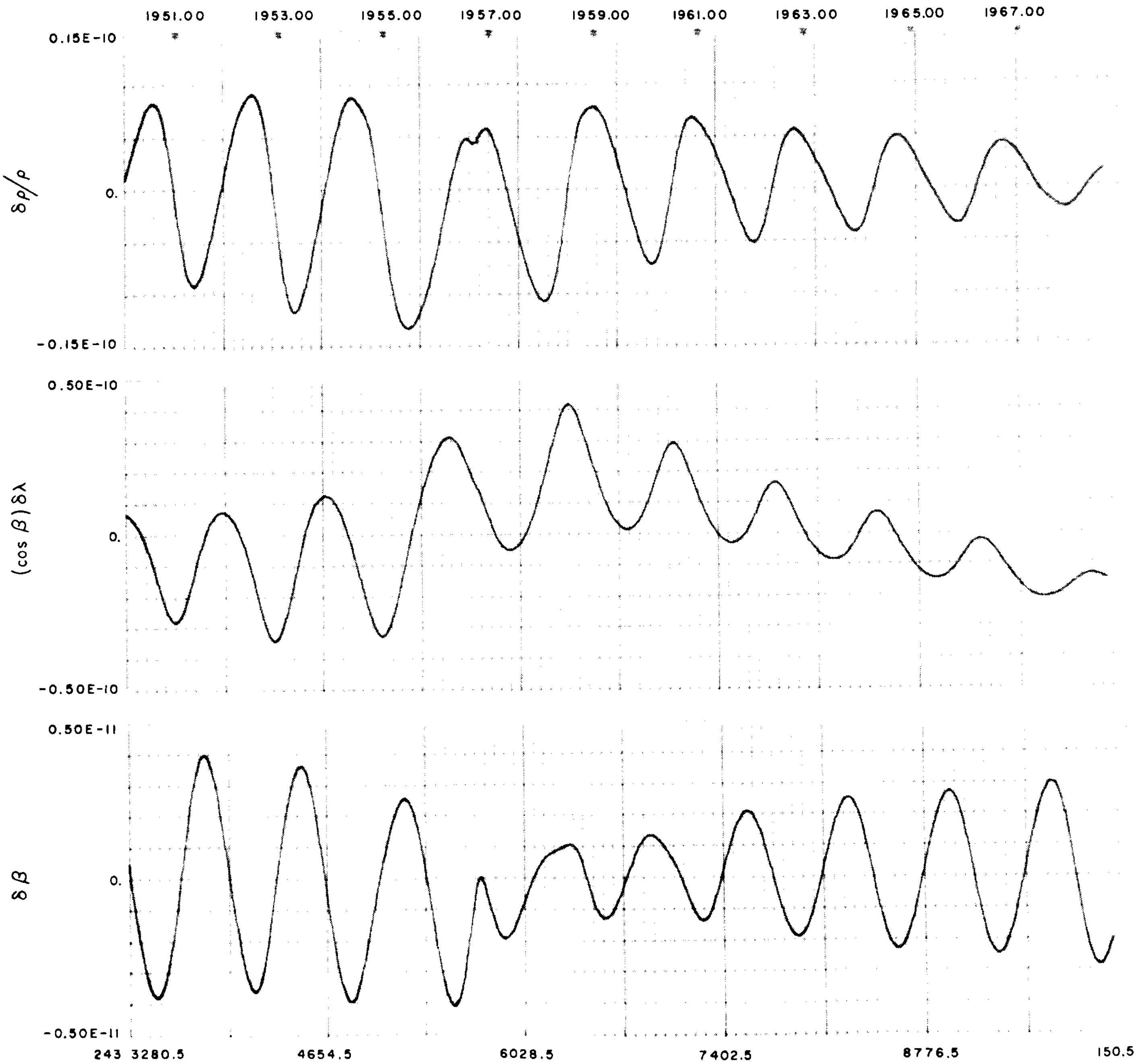


Fig. H-2. PLOD B fit to PLOD A fit for Mars; $\delta\rho/\rho$, $(\cos\beta)\,\delta\lambda$, $\delta\beta$

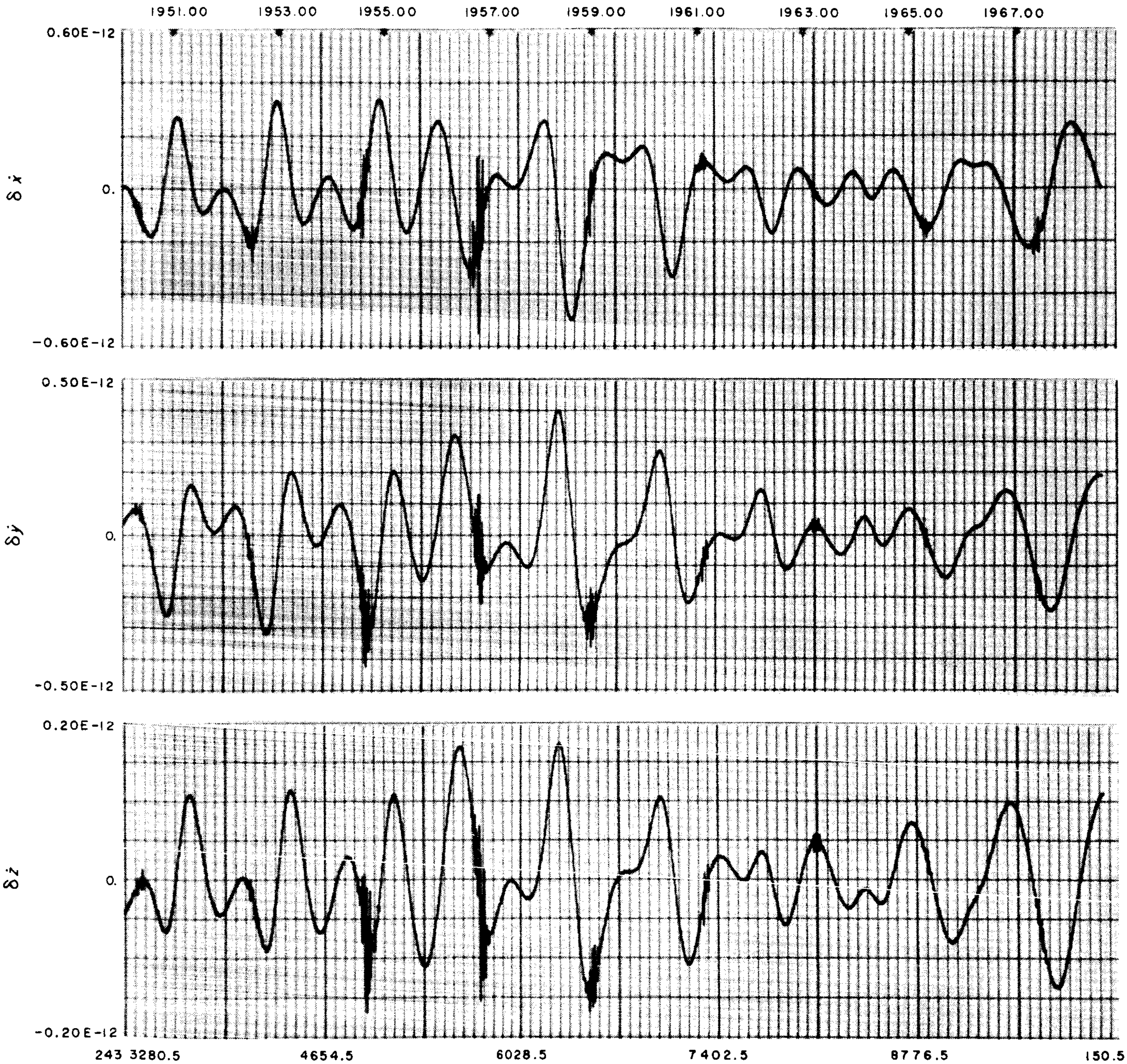


Fig. H-3. PLOD B fit to PLOD A fit for Mars; $\delta\dot{x}$, $\delta\dot{y}$, $\delta\dot{z}$

APPENDIX J

PLOD B Fit to PLOD A Fit for Mercury

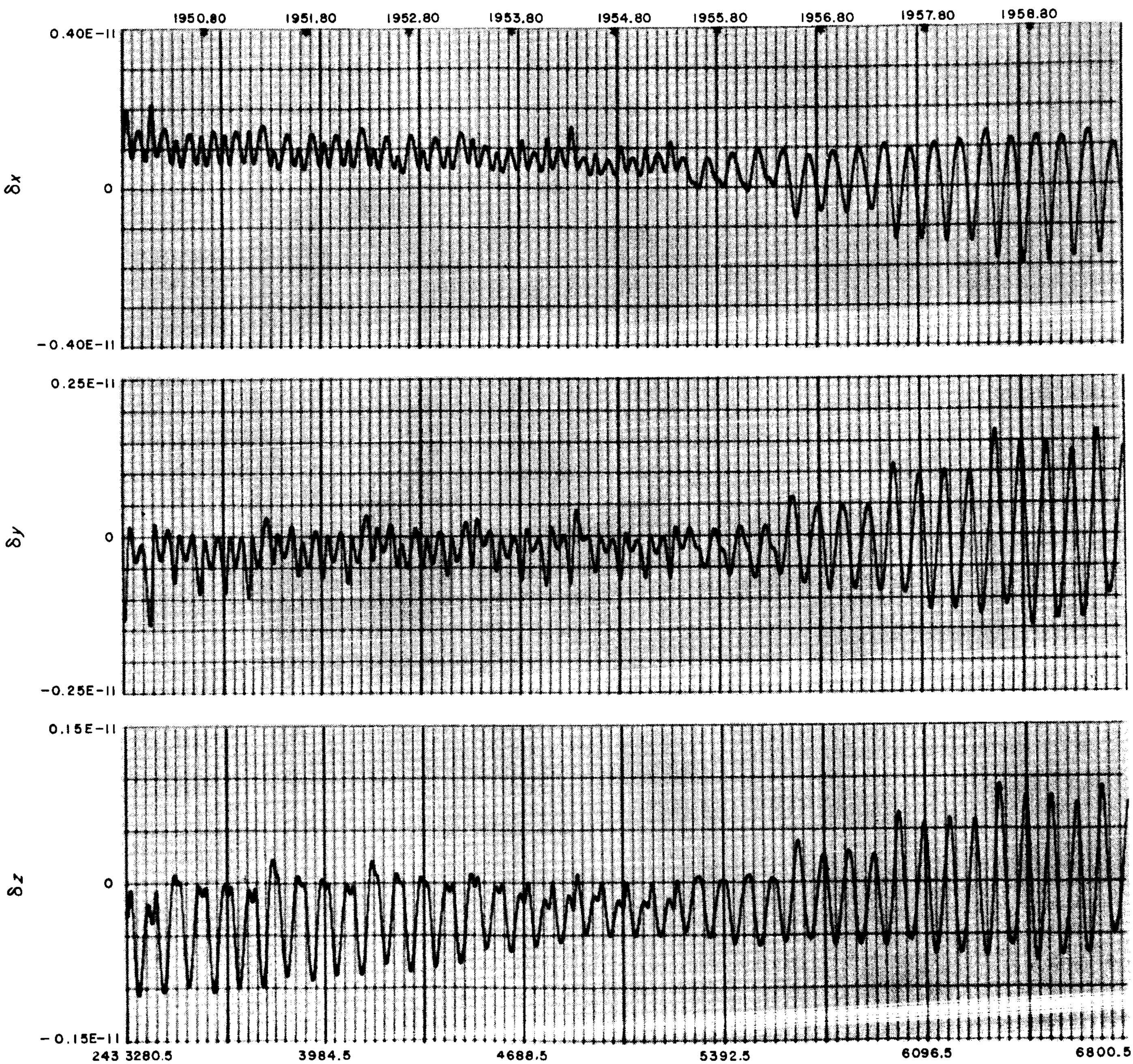


Fig. J-1. PLOD B fit to PLOD A fit for Mercury; δx, δy, δz

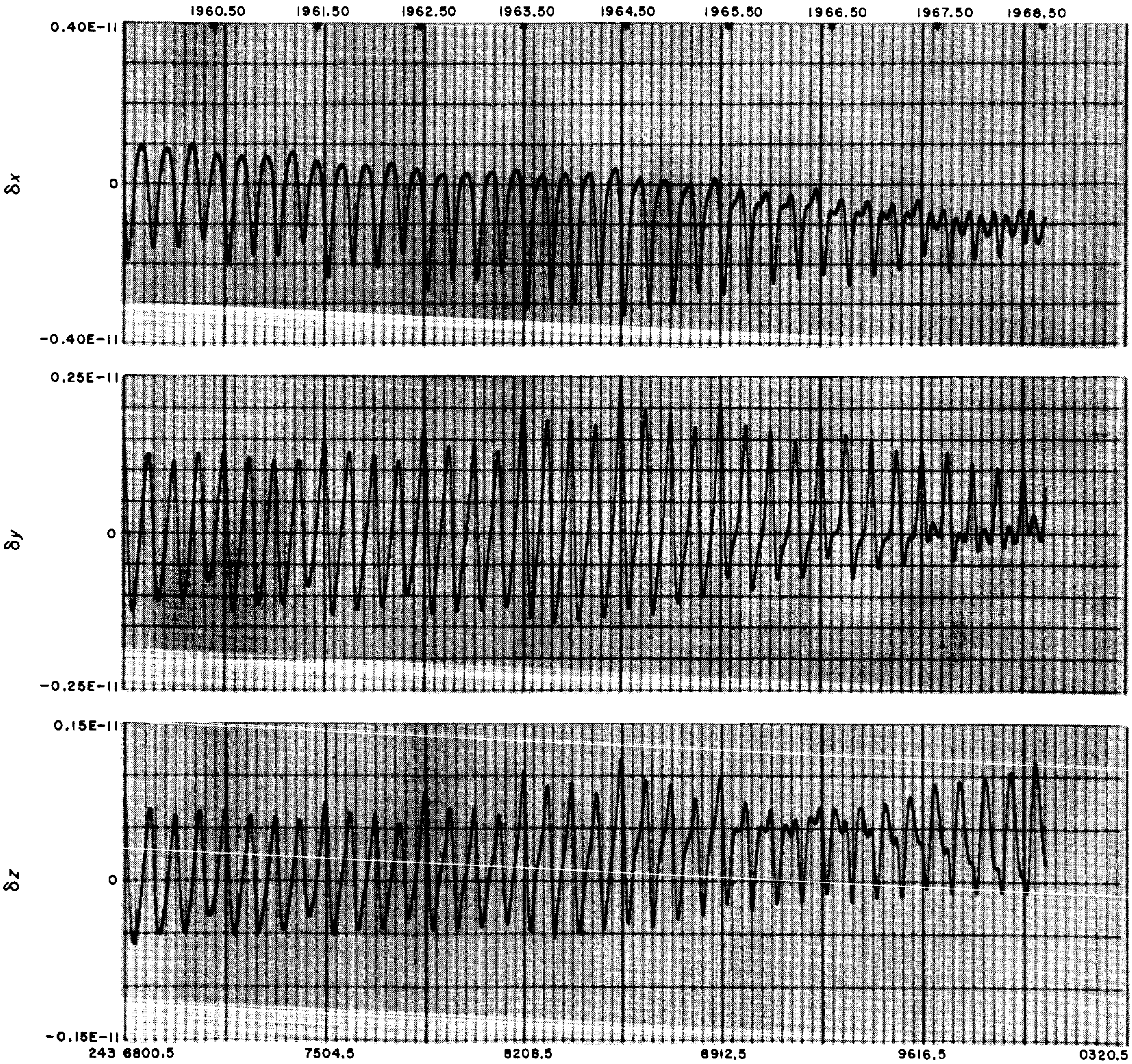

1960.50
1961.50
1962.50
1963.50
1964.50
1965.50
1966.50
1967.50
1968.50
0.40E-11
δx
0
-0.40E-11
0.25E-11
δy
0
-0.25E-11
0.15E-11
δz
0
-0.15E-11
243 6800.5
7504.5
8208.5
8912.5
9616.5
0320.5

Fig. J-1 (cont'd)

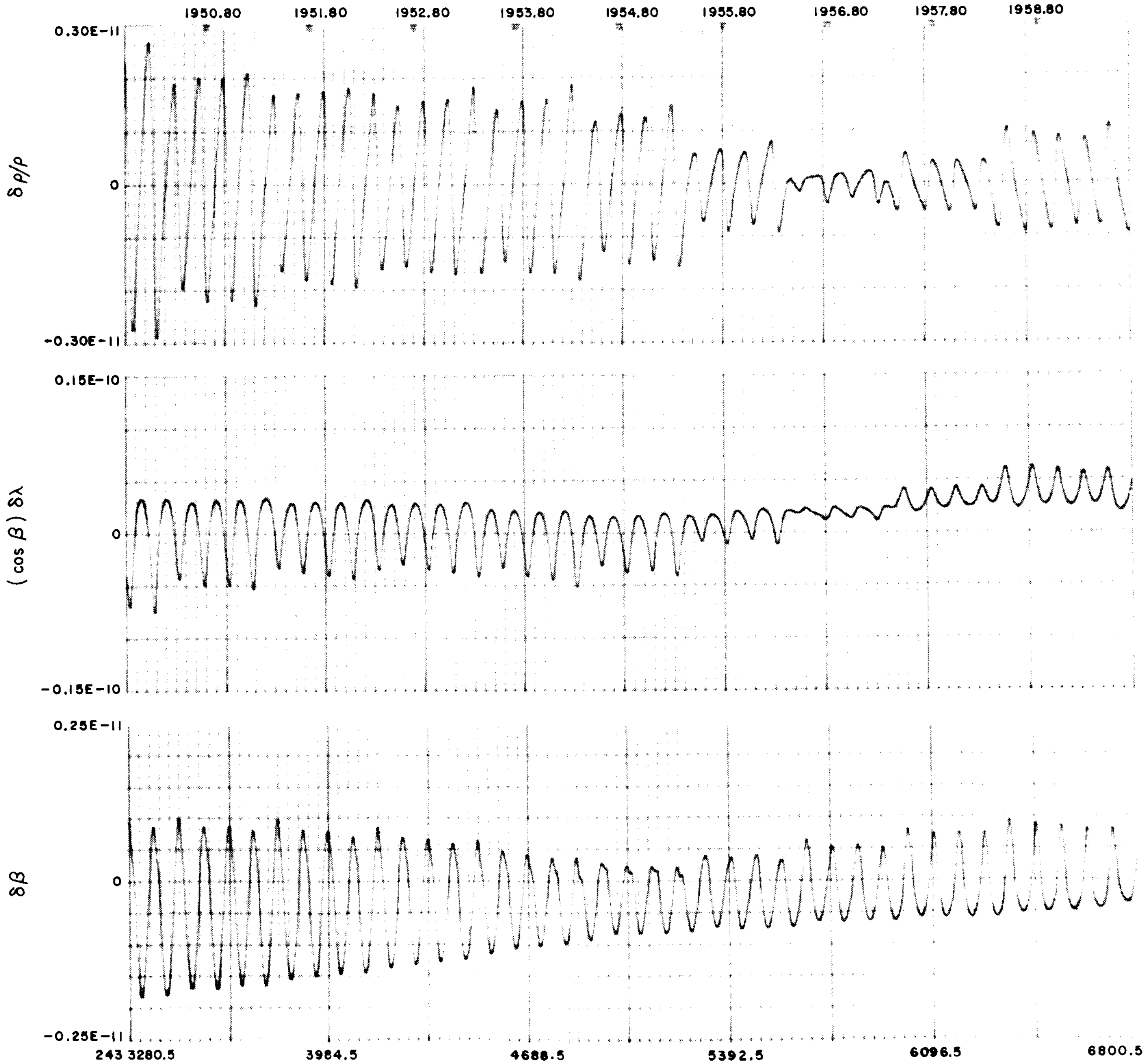


Fig. J-2. PLOD B fit to PLOD A fit for Mercury; $\delta\rho/\rho$, $(\cos\beta)\,\delta\lambda$, $\delta\beta$

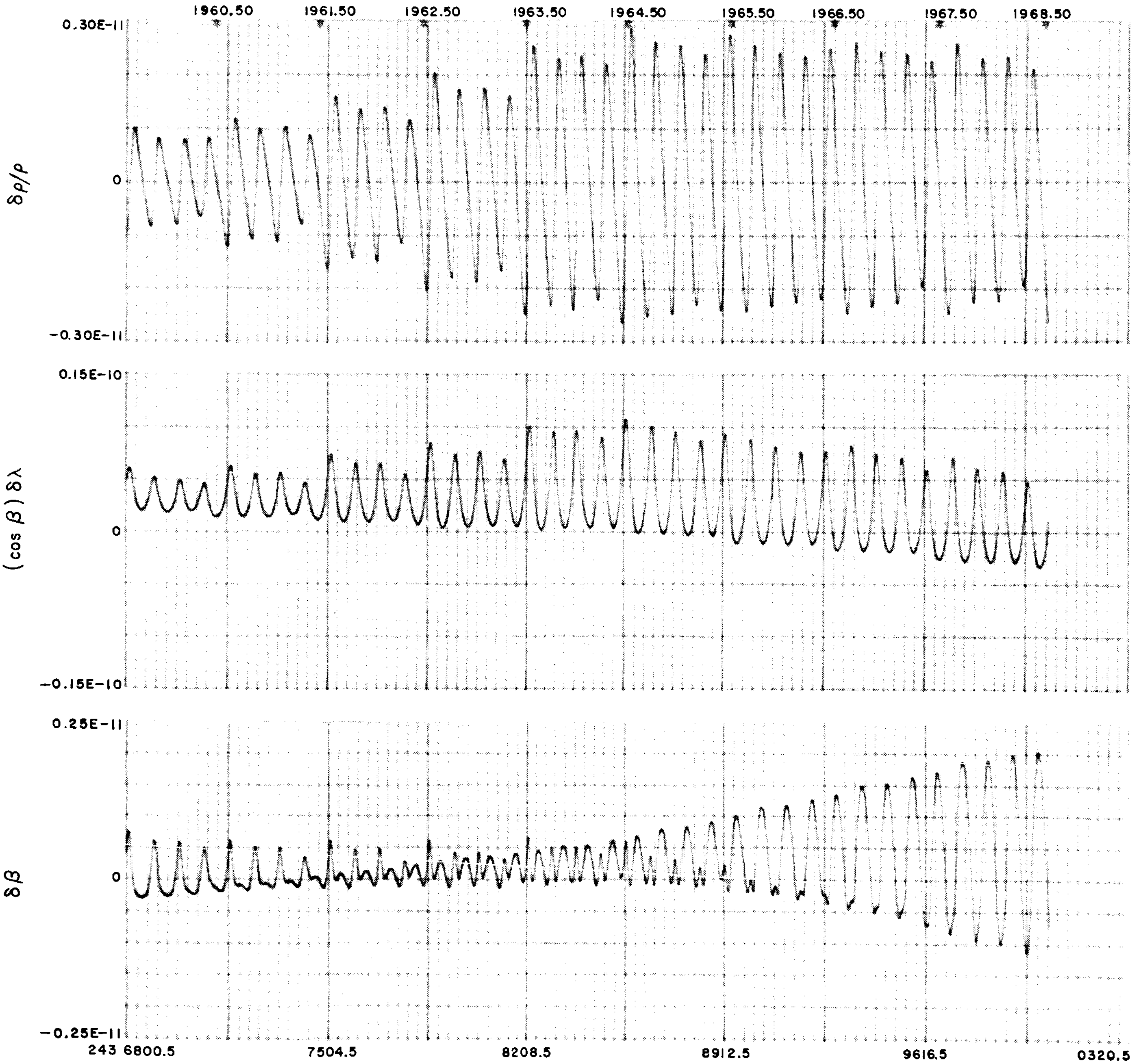

1960.50
1961.50
1962.50
1963.50
1964.50
1965.50
1966.50
1967.50
1968.50
0.30E-11
0
-0.30E-11
$\delta\rho/\rho$
0.15E-10
0
-0.15E-10
$(\cos\beta)\,\delta\lambda$
0.25E-11
0
-0.25E-11
$\delta\beta$
243 6800.5
7504.5
8208.5
8912.5
9616.5
0320.5

Fig. J-2 (cont'd)

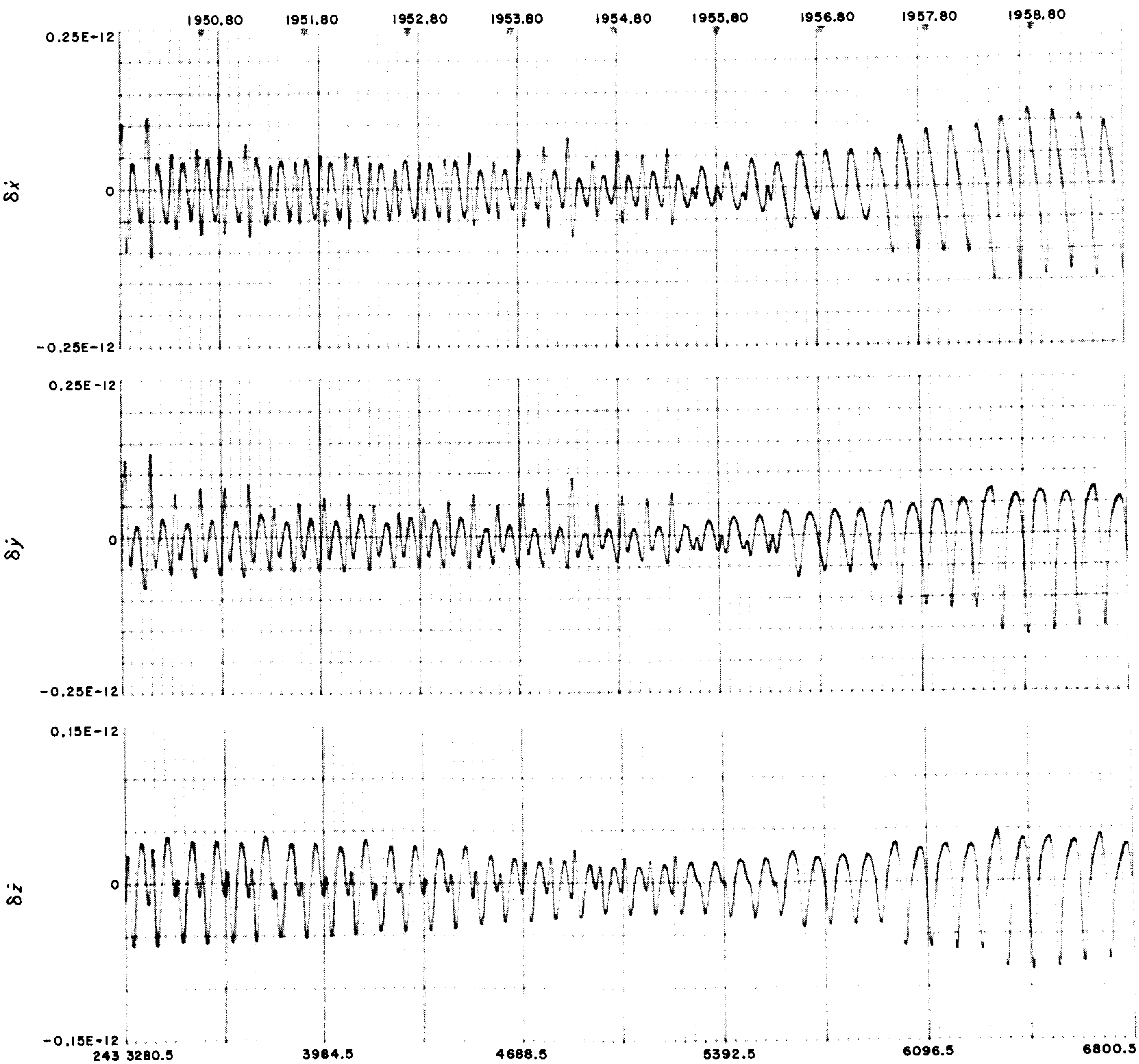


Fig. J-3. PLOD B fit to PLOD A fit for Mercury; $\delta\dot{x}$, $\delta\dot{y}$, $\delta\dot{z}$

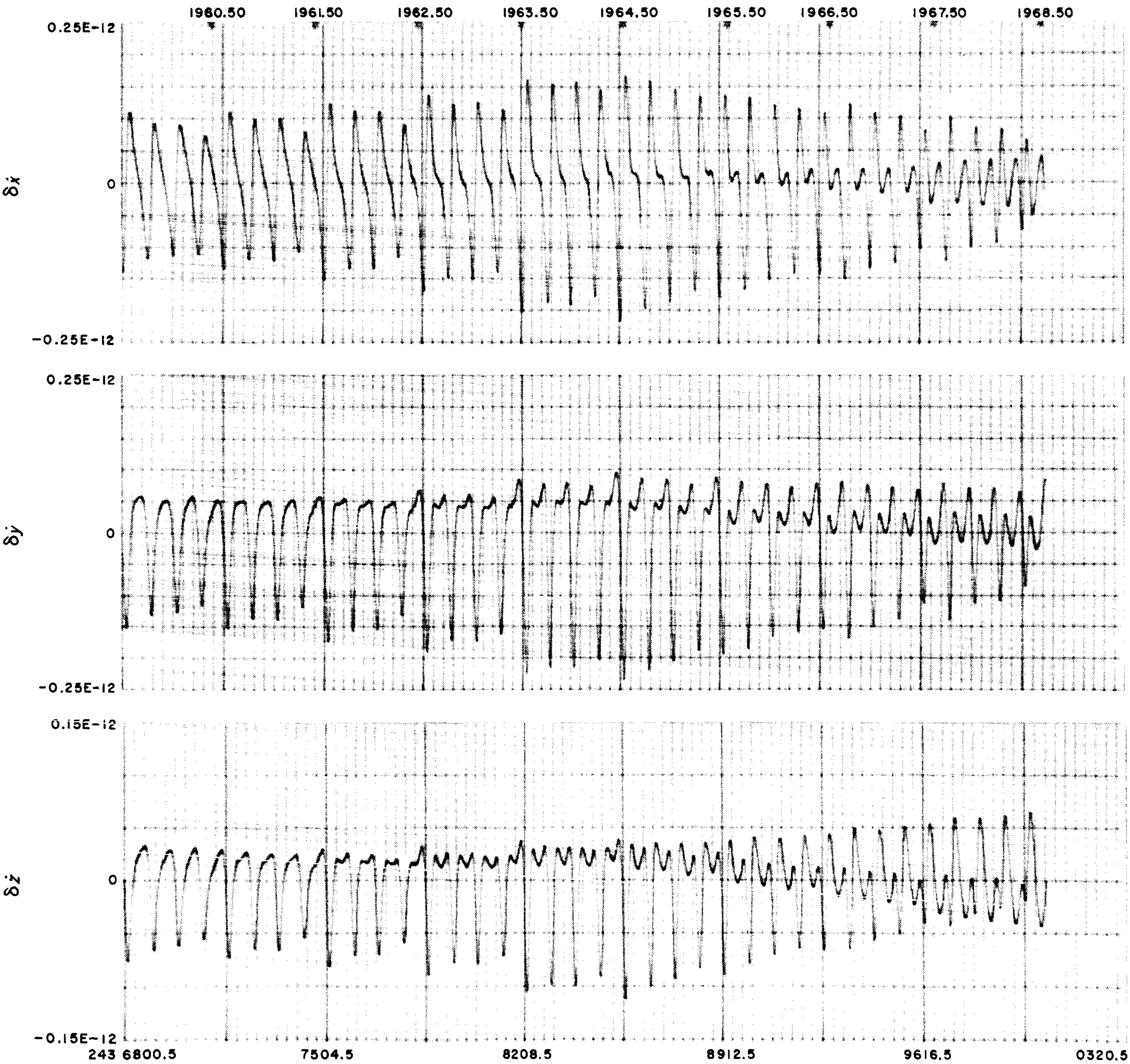

1960.50
1961.50
1962.50
1963.50
1964.50
1965.50
1966.50
1967.50
1968.50
0.25E-12
0
-0.25E-12
$\delta\dot{x}$
0.25E-12
0
-0.25E-12
$\delta\dot{y}$
0.15E-12
0
-0.15E-12
$\delta\dot{z}$
243 6800.5
7504.5
8208.5
8912.5
9616.5
0320.5

Fig. J-3 (cont'd)

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3. Peabody, P. R., Scott, J. F., and Orozco, E. G., *Users' Description of JPL Ephemeris Tapes,* Jet Propulsion Laboratory, Technical Report No. 32-580, Pasadena, California, March 2, 1964.

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