10

METHODS FOR CALCULATING NOISE TRANSMITTED TO THE INSIDE OF SPACE VEHICLES FROM RANDOM LOADING ON THE OUTSIDE

Preliminary Ideas and Basic Equations

Joshua E. Greenspon, Dr. Eng.

FACILITY FORM 602
N66-19664 (ACCESSION NUMBER)
(THRU) 1
19 (PAGES)
(CODE)
CR 70722 (NASA CR OR TMX OR AD NUMBER)
23 (CATEGORY)

GPO PRICE $
CFSTI PRICE(S) $
Hard copy (HC) 1.00
Microfiche (MF) .50
ff 653 July 65

National Aeronautics and Space Administration
Goddard Space Flight Center
Contract No. NAS 5-9124
Technical Report No. 4
March, 1965

J G ENGINEERING RESEARCH ASSOCIATES
3831 MENLO DRIVE BALTIMORE 15, MARYLAND

ABSTRACT

19664

This report discusses the basic equations that can be used in computing the noise transmitted to the inside of a shell or shroud from random loading on the outside. The cylindrical shell is treated in some detail and the general relations for an arbitrary structure are presented.

author

TABLE OF CONTENTS

ACKNOWLEDGEMENTS

This work was done under the sponsorship of the Goddard Space Flight Center, NASA. This author would like to acknowledge the guidance and helpful advice of Mr. Thomas G. Butler of Goddard during the frequent discussions which the author had with him during the progress of this work.

LIST OF SYMBOLS

$a_n(S_1, S_2, \omega)$ cross spectral density between normal acceleration at points S_1 and S_2 on the surface of the structure at frequency ω

ω circular frequency

$G(P_1, P_2, \omega)$ cross spectral density of inside pressure at points P_1 and P_2 at frequency ω

P_1, P_2 points in the pressure field

$q_{rn}(S_1)$ component of the rth modal vector which is normal to the surface at point S_1

$q_{kn}(S_2)$ component of the kth modal vector which is normal to the surface at point S_2

$Y_r(\omega) = M_r\sqrt{(\omega^2 - p_r^2)^2 + \omega^2\psi_r^2}$, the frequency response function for the rth mode

p_r natural frequency of the rth mode

$$\psi_r = \frac{K_r}{M_r}\iiint_V(\vec{q}_r \cdot \vec{q}_r)\,dV$$

K_r damping force per unit volume per unit velocity for rth mode of vibration

$M_r = \iiint_V \rho(\vec{q}_r \cdot \vec{q}_r)\,dV$, the generalized mass of the rth mode

$\vec{q}_r$ the modal vector for the rth mode

$$\vec{q}_r = u_r\vec{i} + v_r\vec{j} + w_r\vec{k}$$

u_r, v_r, w_r orthogonal components of the rth modal vector

C_{rk} joint acceptance or correlation integral

θ_r phase angle for rth mode

σ a symbol indicating the surface of the structure

V a symbol indicating the structural volume (includes all parts of the vibrating structure but does not include the volume of the air space)

$g(P_1, S_1, \omega)$	a Green's Function which vanishes over the surface of the structure; P_1, S_1 refer to points in the space with rectangular coordinates x_1, y_1, z_1
$g^*(P_2, S_2, \omega)$	the complex conjugate of $g(P_1, S_1, \omega)$ referred to coordinates in the x_2, y_2, z_2 space
ρ_0	density of the fluid inside the shroud
$w(x, \phi, t)$	radial displacement of a cylinder
x	longitudinal coordinate
ϕ	peripheral coordinate
t	time
$v_{mn}(t), u_{mn}(t)$	functions of time associated with the radial displacement of the cylinder
m	number of axial half waves in the vibration pattern of the cylinder
n	number of full circumferential waves in the vibration pattern
l	length of the cylinder
$\tilde{w}(x_1, \phi_1, x_2, \phi_2, \omega)$	cross spectral density of displacement at points x_1, ϕ_1 and x_2, ϕ_2 on the cylinder
H_w	frequency response function for lateral displacement
$C_{v_{mn}v_{mn}}; C_{u_{mn}v_{mn}}; C_{u_{mn}u_{mn}}$	correlation integrals (or joint acceptances) for the cylinder
$S_f(\xi_1, \eta_1, \xi_2, \eta_2, \omega)$	cross spectral density of pressure exciting the outside of the structure
ξ, η	integration variables for x, ϕ respectively
a	radius of the cylinder
$p_i(r, \phi, x, \omega)$	auto spectral density of inside pressure at point with cylindrical coordinates r, ϕ, x (r is the radial distance from the axis of the cylinder to any point)
H_{p_i}	frequency response function for inside pressure
J_p	Bessel Function of order p
p, q	indices associated with inside Green's Function of a cylinder which has a finite radiating source
C_0	velocity of sound in the air inside the shroud

$\bar{V}$	convection velocity of turbulence (.65 U, where U is the flow velocity
$\alpha, \beta, \gamma, \delta$	empirical constants associated with general relation for jet or boundary layer noise
$\Gamma(x, x', \phi, \phi', \omega)$	general relation for cross spectral density of pressure associated with jet or boundary layer noise
$\tilde{p}(\omega)$	auto spectral density of jet or boundary layer noise

I. Introduction

Space vehicles are contained within shrouds which are subjected to the outside dyanmic environment during launching and flight. There are two main paths by which the space vehicle can receive the loads. One is the air path through the skin of the shroud and the other is the structural path through the attachments of the space vehicle to the shroud and booster. The emphasis in this report will be on noise transmitted through the air inside from random vibration of the casing. To this author's knowledge the first studies of problems of this type were given by Dyer.[1,2] This present study is an extension and generalization of Dyer's work.

II. General equations

A. An arbitrary structure

For a built up elastic structure subjected to random loading the cross spectral density between the normal acceleration at any two points of the structure can be obtained by an extension of Powell's theory[3] given by the present author.[4,5] This cross spectral density, $a_n(S_1, S_2, \omega)$ is

$$a_n(S_1,S_2,\omega) = \omega^4 \sum_r \sum_k \frac{q_{rn}(S_1)\, q_{kn}(S_2)}{|Y_r(\omega)|\,|Y_k(\omega)|} e^{-i(\theta_r-\theta_k)} C_{rk} \qquad [1]$$

$$C_{rk} = \int_\sigma \int_\sigma G(S_1,S_2,\omega)\, q_{rn}(S_1)\, q_{kn}(S_2)\, d\sigma_1\, d\sigma_2$$

where

$G(S_1,S_2,\omega)$ is the cross spectral density of the exciting pressure between points S_1 and S_2 at frequency ω

$q_{rn}(S_1)$ is the normal component of the rth mode evaluated at point S_1 of the surface

$q_{kn}(S_2)$ is the normal component of the kth mode evaluated at point S_2 of the surface

$|Y_r(\omega)| = M_r\sqrt{(\omega^2-p_r^2)^2 + \psi_r^2\omega^2}$, the frequency response function

$\theta_r = \tan^{-1}\frac{\psi_r\omega}{p_r^2-\omega^2}$ the phase angle

p_r is the natural frequency of the rth mode

$\psi_r = \frac{k_r}{M_r}\iiint_V (\vec{q}_r\cdot\vec{q}_r)\,dV$

dV = element of volume of the body

K_r damping force per unit volume per unit velocity

$M_r = \iiint_V \rho (\vec{\phi}_r \cdot \vec{\phi}_r) dV$, the generalized mass

ρ mass density of structural material

$q = \sum_r q_r \phi_r$

where q is the total displacement in some desired direction, ϕ_r is the modal displacement in that direction and q_r is the generalized coordinate for the rth mode

The noise transmitted inside a structure from induced vibrations of the surface can be obtained by employing an extension of Parrent's Theory[6] given by the present author in report referred to above.[5] In that report the cross spectral density of the field pressure in the medium outside a vibrating surface was obtained. By analogous reasoning the statistics of the pressure inside a randomly vibrating surface can be written in terms of pressure cross spectral density, $G(P_1, P_2, \omega)$ as follows:

$$G(P_1, P_2, \omega) = \frac{1}{(4\pi)^2} \int_\sigma \int_\sigma \rho_0^2 a_n(S_1, S_2, \omega)\, \bar{g}(P_1, S_1, \omega)\, \bar{g}^*(P_2, S_2, \omega)\, d\sigma_1 d\sigma_2 \qquad [2]$$

where $G(P_1, P_2, \omega)$ is the cross spectral density of the pressure inside the structure at points P_1 and P_2 at frequency ω

$a_n(S_1, S_2, \omega)$ is the cross spectral density of the normal acceleration at the surface

$\bar{g}(P_1, S_1, \omega)$ is the Green's Function for the medium inside of the structure whose normal derivative vanishes over the surface of the structure (It is the response at inside point P_1 due to unit sinusoidal load of frequency ω at surface point S_1.)

$\bar{g}^*(P_2, S_2, \omega)$ is the complex conjugate of $\bar{g}(P_1, S_1, \omega)$ referred to coordinates in the x_2, y_2, z_2 space

σ denotes the vibrating surface

Equations [1] and [2] constitute the set of equations to determine the noise inside a structure which is exposed to random pressure of known cross spectral density, $G(S_1, S_2, \omega)$ on the surface of the structure. The cross spectral density of the normal acceleration, $a_n(S_1, S_2, \omega)$ in [1] is a function of the mode shapes, ϕ_{rn}, the frequency response function , Yr, and other computable parameters which can be obtained from the characteristics of the structure and the loading. The pressure transmitted to the inside is given by [2] and is dependent on the normal surface acceleration [1] and the inside Green's Function which theoretically can be computed from acoustic theory. Of course there are some practical difficulties in obtaining these Green's Functions in the same way that there are practical difficulties in obtaining mode shapes and frequencies to use in [1]; however for relatively simple shapes and special cases such as low frequency these functions can be readily obtained.

One further simplification in [1] is very helpful and this simplification can be made if there is relatively low damping in the structure. For this case we can employ the logic of Powell[7] and Hurty-Rubenstein[8] and neglect cross product terms thus giving a simpler relation than [1]; this relation is

$$a_n(s_1, s_2, \omega) = \omega^4 \sum_r \frac{q_{rn}(s_1)\, q_{rn}(s_2)}{|Y_r(\omega)|^2} C_{rr} \qquad [3]$$

$$C_{rr} = \int_\sigma \int_\sigma G(s_1, s_2, \omega)\, q_{rn}(s_1)\, q_{rn}(s_2)\, d\sigma_1\, d\sigma_2$$

B. The cylinder

1. The infinitely long cylinder

If we consider an infinitely long stiffened, sandwich, or isotropic cylinder (no longitudinal boundaries) the noise inside of this cylinder due to random loading on the elastic cylindrical surface can be computed readily by use of the methods developed in several previous pieces of work.[9,10,11] We start with the solution for radial displacement, w, of the cylinder under arbitrary loading

$$w(x, \varphi, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \left[V_{mn}(t) \cos n\varphi + U_{mn}(t) \sin n\varphi \right] \sin \frac{m\pi x}{l} \qquad [4]$$

where $2l/m$ is the longitudinal wave length of the structural vibration. For a finite cylinder l can be interpreted as the length and m the number of axial half waves in the vibration pattern. We can consider that we have a finite cylinder of length l from the structural point of view and [4] will be the general solution for the radial displacement if the cylinder has freely supported ends. However, acoustically it is difficult to consider the end effects, so we will continue with the concept of the infinite cylinder as far as the acoustics is concerned.

Following through the solution in a manner similar to that given in the above mentioned reference[10] and neglecting product terms in the manner described in the previous section (Section A), we obtain an expression for the cross spectral density, $\tilde{w}$, of the radial displacement,

$$\tilde{w}(x_1,\phi_1,x_2,\phi_2,\omega) \approx \frac{1}{2\pi}\sum_{m,n} \sin\frac{m\pi x_1}{l}\sin\frac{m\pi x_2}{l}\,|H_w(m,n,\omega)|^2 \times \Big[C_{v_{mn}v_{mn}}\cos n\phi_1\cos n\phi_2 + C_{u_{mn}v_{mn}}\sin n(\phi_1+\phi_2) + C_{u_{mn}u_{mn}}\sin n\phi_1 \sin n\phi_2\Big] \quad [5]$$

where H_w is the complex frequency response function for lateral displacement, i. e. if the loading has the form

$$p(x,\phi,t) = \cos n\phi \sin\frac{m\pi x}{l} e^{i\omega t} \quad [6]$$

the response will be

$$w(x,\phi,t) = H_w(m,n,\omega)\sin\frac{m\pi x}{l}\cos n\phi\, e^{i\omega t} \quad [7]$$

The terms $C_{v_{mn}v_{mn}}$ etc. can be written[8]

$$C_{v_{mn}v_{mn}} = \frac{4}{\pi^2 l^2}\int_0^l\int_0^l\int_0^{2\pi}\int_0^{2\pi} S_f(\xi_1,\eta_1,\xi_2,\eta_2,\omega)\sin\frac{m\pi\xi_1}{l}\sin\frac{m\pi\xi_2}{l}\cos n\eta_1\cos n\eta_2\, d\xi_1 d\eta_1 d\xi_2 d\eta_2 \quad [8]$$

$$C_{u_{mn}v_{mn}} = \frac{4}{\pi^2 l^2}\int_0^l\int_0^l\int_0^{2\pi}\int_0^{2\pi} S_f(\xi_1,\eta_1,\xi_2,\eta_2,\omega)\sin\frac{m\pi\xi_1}{l}\sin\frac{m\pi\xi_2}{l}\sin n\eta_1\cos n\eta_2\, d\xi_1 d\eta_1 d\xi_2 d\eta_2$$

$$C_{u_{mn}u_{mn}} = \frac{4}{\pi^2 l^2}\int_0^l\int_0^l\int_0^{2\pi}\int_0^{2\pi} S_f(\xi_1,\eta_1,\xi_2,\eta_2,\omega)\sin\frac{m\pi\xi_1}{l}\sin\frac{m\pi\xi_2}{l}\sin n\eta_1\sin n\eta_2\, d\xi_1 d\eta_1 d\xi_2 d\eta_2$$

The solution for the internal pressure field follows directly from the analysis given several years ago[9,12] and relation [5] above. We just have to remark that for an infinite cylinder each structural mode gives rise to an acoustic mode inside which has a longitudinal and peripheral pressure distribution the same as the shell radial displacement. The auto spectral density of the pressure inside the cylinder at point r,ϕ,x at frequency ω can then be written

$$p_i(r,\phi,x,\omega) \approx \frac{1}{2\pi}\sum_{m,n}\sin^2\frac{m\pi x}{l}\,|H_{p_i}(r,m,n,\omega)|^2 \Big[C_{v_{mn}v_{mn}}\cos^2 n\phi + 2C_{u_{mn}v_{mn}}\sin n\phi\cos n\phi + C_{u_{mn}u_{mn}}\sin^2 n\phi\Big] \quad [9]$$

where H_{p_i} is the frequency response function for internal pressure, i. e. if the pressure on the surface has the form

$$p_s(x,\varphi,t) = \cos n\varphi \sin\frac{m\pi x}{l}\, e^{i\omega t} \tag{10}$$

then the pressure inside will be determined by the elastic vibration of the wall and the coupling to the air inside and will be given by

$$p_i(r,\varphi,x,t) = H_{p_i}(m,n,r,\omega)\cos n\varphi \sin\frac{m\pi x}{l}\, e^{i\omega t} \tag{11}$$

The frequency response functions H_{p_i} for internal pressure have been programmed for a computer as described in a previous reference.[7] Consider the average internal pressure over cylindrical contours inside the cylinder at various distances r from the center. If we integrate [9] over the area of any of these contours and divide by the area we will obtain

$$[p_i(r,\omega)]_{Ave.} \approx \frac{1}{8\pi}\sum_{m,n} |H_{p_i}(r,m,n,\omega)|^2 [C_{v_{mn}v_{mn}} + C_{u_{mn}u_{mn}}] \tag{12}$$

2. More accurate approximation for the acoustics of finite cylinders

Instead of an infinite cylinder which is radiating sound along its entire length consider an infinitely long tube whose walls are rigid every place except in a segment $0 \leq x \leq l$ where x is the longitudinal coordinate. The tube is assumed to radiate energy to the inside through motion of this segment. The problem is to obtain the sound at any point within the tube.

The Green's Function for this case has been given by Morse and Feshbach[13] and Junger.[14] This function is

$$G(r,\varphi,x;r_0,\varphi_0,x_0;\omega) = \frac{4i}{a^2}\sum_{p,q}\left[\epsilon_p \cos p(\varphi-\varphi_0)\right] \left[\frac{J_p(\pi\alpha_{pq}r/a)\,J_q(\pi\alpha_{pq}r_0/a)}{k_{pq}\left[1-(p/\pi\alpha_{pq})^2\right]J_p^2(\pi\alpha_{pq})}\right] \times \begin{cases} \cos k_{pq}x_0\, e^{ik_{pq}x} & ,\text{if } x > x_0 \\ \cos k_{pq}x\, e^{ik_{pq}x_0} & ,\text{if } x < x_0 \end{cases} \tag{13}$$

α_{pq} is the pth root of $\frac{dJ_p(\pi\alpha)}{d\alpha} = 0$; $k_{pq}^2 = k^2 - \left(\frac{\pi\alpha_{pq}}{a}\right)^2$; $k = \omega/c_0$

Equation [13] expresses the pressure at r, ϕ, x due to a unit harmonic source at r_0, ϕ_0, x_0 . This equation combined with the general relations [1] or [3] and [2] or [5] and [2] ([5] being multiplied by ω^4 to obtain acceleration) can be used to obtain the sound field inside the shell

III. Computation of the correlation integrals (or joint acceptances)

The computation of the frequency response functions and mode shapes to be used in [1], [3], [5], [9], [12] is a problem in structural analysis and computer programs are available for accomplishing these tasks both for cylindrical shells[9] and built up structures by use of matrix procedures such as the programs under development at various aircraft companies (e.g. Program SB028 developed by Martin Company and now available at Goddard). The central issue is to obtain an adequate description of the correlation or cross spectrum of the loading on the outside of the structure due to such sources as boundary layer noise and jet noise. Once this correlation is known then the correlation integrals (or joint acceptances) denoted by C_{rk} in [1], C_{rr} in [3] and $C_{\psi_{mn}\psi_{mn}}$, etc. in [5] can be computed; sometimes, however with considerable difficulty.

Based on examination of the scant evidence available on jet noise[15,16] and the more extensive information on boundary layer noise[11,17,18,19,20] it is believed that an approximation for the cross spectral density of the loading for jet noise and boundary layer noise can be written in the form

$$\Gamma(x,x',\phi,\phi',\omega) = \tilde{p}(\omega)\, e^{-\alpha|x'-x|} e^{-\gamma|\phi'-\phi|} e^{-i\beta|x'-x|} e^{-i\delta|\phi'-\phi|} \qquad [14]$$

where $\tilde{p}(\omega)$ is the auto spectral density of the pressure and $\alpha, \beta, \gamma, \delta$ are empirical constants. Based on the experimental evidence it seems that for <u>boundary layer noise</u>[11]

$$\alpha \approx .11\frac{\omega}{\bar{V}} \quad , \beta \approx \frac{\omega}{\bar{V}} \qquad [15]$$
$$\gamma \approx .60\frac{\omega}{\bar{V}} \quad , \quad \delta = 0$$

where ω is the frequency and $\bar{V}$ the convection velocity (about .65 U , where U is the flow velocity)

For <u>jet noise</u> it seems that $\delta = 0$ [15] but the remaining parameters are questionable. Equation [14] says that the cross spectral density depends only upon the distance between points and not on the exact location of the points themselves. Powell[21] has obtained a general result for such correlation functions for structures whose mode shapes are sinusoidal and for the case where the correlation falls to a small value in a distance which is small compared to the structural wave length. For the particular case of boundary layer noise within cylinders (turbulent pipeflow) Rattaya and Junger[11] have computed the complete correlation integral.

IV. Some approximate relations

A. The infinitely long cylinder

Equation [12] gives the average spectral density of the pressure over a surface located at distance r from the center of the cylinder. Consider the noise transmitted trhough the shell from boundary layer turbulence. Assuming low flow velocities in the sense of Rattaya and Junger[11] and assuming a constant boundary layer thickness on the outside of the shell it can be shown that

$$[p_i(r,\omega)]_{Ave} \approx \tilde{p}(\omega)\,\frac{\bar{C}}{4\pi}\sum_{m,n}\left|H_{p_i}(r,m,n,\omega)\right|^2 \qquad [16]$$

$$\bar{C} = \left(\frac{2\bar{V}}{\omega}\right)^2 \frac{\epsilon_n\,(.11)(1.67)}{\pi \ell a}$$

where $\tilde{p}(\omega)$ is the autospectral density of the outside pressure $\bar{V}$ is .65 U (U being the flow velocity) $\epsilon_n = 1$ for $n=0$; $\epsilon_n = 2, n>0$ ℓ is the length of the shell, and a is the radius of the shell. Equation [16] can be written in the form:

$$\frac{[p_i(r,\omega)]_{Ave}}{\tilde{p}(\omega)} = \frac{\bar{C}}{4\pi}\sum_{m,n}\left|H_{p_i}(r,m,n,\omega)\right|^2 \qquad [17]$$

This relation has a familiar form if we remember the simple input-output relation of a linear black box[22]

$$\bar{\phi}(\omega) \longrightarrow \boxed{H(\omega)} \longrightarrow \bar{\psi}(\omega)$$

$$\bar{\psi}(\omega) = |H(\omega)|^2\,\bar{\phi}(\omega) \qquad [18]$$

where $\phi(\omega)$ is the power spectral density of the input, $\psi(\omega)$ is the power spectral density of the output and $H(\omega)$ is the frequency response function. Equation [17] is more complicated than [18] since it contains many modes, however the right hand side of [17] can be interpreted as the complete transfer function for this problem since it gives the power spectral density of the output $[p_i(r,\omega)]_{Ave}$ as a factor of the power spectral density of the input $\tilde{p}(\omega)$.

As can be seen from Reference 9 $H_{p_i}(r, m, n, \omega)$ involves many factors; it includes the complete coupling between shell and its surroundings so it involves the resonances of the shell, the damping in the shell, and other complicating factors entering the problem such as the resonances in the air column inside, and the air damping outside as effected by speed. The combined shell fluid solution for this infinitely long shell is completly described in some previous references.[9,12] The H_{p_i} functions have been programmed and are available at Goddard.

B. The general case

Equation [1] or [3] and [2] describe the solution of the general case. Fortunately equation [1] is now being programmed so that the response and acceleration cross spectral density can be computed for built up structures. The basic shape of most of the shrouds containing spacecraft is conical or a combination of conical and cylincrical sections. Therefore for purposes of computing the approximate inside field it would not seem to be in great error to form an equivalent cylinder from the shroud and use the Green's Function for the cylinder as given by [13]. Equation [3] would then reduce to [5] with an ω^4 in front of the Σ to take care of acceleration. Equation [13] and [5] can then be substituted into [2] or [3] and letting $P_1 = P_2$ we will obtain the auto spectral density of the sound pressure at P_1 inside the cylinder. The integrations and computations in volved in the above mentioned relations are quite tedious but straightforward and can be done on a computer with little anticipated difficulty.

SUMMARY

Some projected developments have been suggested in this report for computing the sound field inside a structure due to noise exciting the structure at the boundary. A few of the methods use the idea that the structure is an approximate cylinder. One of these cylindrical methods assumes no end effects in the inside acoustics and the result, equation [9] with simplifications resulting in [17], leads to a simple relation which has already been programmed for a computer. Another method (eq. [5], [13], [2]) is more accurate and does consider the finiteness of the vibrating source that produces the field inside but this method is much more involved. If the structure cannot be assumed as a uniform cylinder, but must be considered as a built up structure containing attached masses in which the coupling between motion of masses and shell has to be considered then equation [1] or [3] (which are currently being programmed) coupled with [2] must be used.

REFERENCES

1. I. Dyer, "Sound Radiation into a Closed Space from Boundary Layer Turbulence," Bolt, Beranek and Newman, Inc., Contract Nonr-2321(00), Report No. 602, 12 Dec., 1958.

2. I. Dyer, J. Acoust. Soc. Am., 30, 833-841, 1958.

3. A. Powell, "On the Response of Structures to Random Pressures and to Jet Noise in Particular," Random Vibration, edited by S. Crandall, John Wiley and Sons, 1958, p. 187.

4. J. E. Greenspon, "Modelling of Spacecraft Under Random Loading," NASA CR-132, Nov., 1964.

5. J. E. Greenspon, "Approximation to the Mean Square Farfield Pressure of Randomly Vibrating Structures at Low Frequencies," Contract Nonr-2733(00) (FBM), Tech. Rep. No. 12, Nov., 1964.

6. G. B. Parrent, Jr., Jour, of Optical Soc. Am., 49,8,787 (1959).

7. A. Powell, "On Structural Vibration Excited by Random Pressures with Reference to Structural Fatigue and Boundary Layer Noise," Douglas Aircraft Co., Report No. SM-22795, May 9, 1957.

8. W. C. Hurty and M. F. Rubenstein, "Dynamics of Structures," Prentice Hall, Inc., Englewood Cliffs, New Jersey, 1964, Chap. 11.

9. J. E. Greenspon, "Random Loading and Radiation from Stiffened and Sandwich Type Cylindrical Shell Structures, Part I. Basic Equations and Some Preliminary Results," Contract Nonr-2733(00), Tech. Rep. No. 7, Sept., 1962.

10. J. E. Greenspon, "Random Loading and Radiation from Stiffened and Sandwich Type Cylindrical Shell Structures, Part II. Unsymmetrical Loading with Known Space Distribution," Contract Nonr-2733(00)(FBM) Tech. Rep. No. 10, Oct., 1964.

11. J. V. Rattayya and M. C. Junger, Jour. Acoust. Soc. Am., 36, No. 5, 878-884 (1964).

12. J. E. Greenspon, "Vibrations of Thick and Thin Cylindrical Shells Surrounded by Water," Contract Nonr-2733(00), Tech. Rep. No. 4, Sept., 1960.

13. P. M. Morse and H. Feshbach, "Methods of Mathematical Physics," Part II, McGraw Hill Book Co., Inc., p. 1515.

14. M. C. Junger, "Sound Radiation Within an Infinite Cylinder for Arbitrary Source Distribution on its Surface," Internal Memo., Harvard University Acoustics Research Laboratory, March 20, 1952.

15. I. Dyer, P. A. Franken, and E. E. Ungar, "Noise Environments of Flight Vehicles," Noise Control, Jan/Feb. 1960, p. 31.

16. E. Mollo-Christensen, "Measurements of Near Field Pressure of Subsonic Jets," AGARD Report 449, April, 1963.

17. H. P. Bakewell, Jr., J. Acoust. Soc. Am., 35, 936 (1963).

18. G. M. Corcos, J. Acoust. Soc. Am. 35, 192 (1963).

19. W. W. Willmarth and C. E. Wooldridge, "Measurements of the Fluctuating Pressure at the Wall Beneath a Thick Turbulent Boundary Layer," Univ. Mich. ORA Proj. 02920 (Apr., 1962).

20. D. H. Tack, M. W. Smith, and R. F Lambert, J. Acoust. Soc. Am. 33, 410 (1961).

21. A. Powell, J. Acoust. Soc. Am., 36, No. 4, 783 (1964).

22. R. L. Bisplinghoff, H. Ashley, R. L. Halfman, "Aeroelasticity," Addison-Wesley Publishing Co., Inc. 1955, p. 825.