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MARINER 1964 PROBABILITY OF SUCCESS MODEL

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I. INTRODUCTION

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The purpose of this report is to describe a mathematical model which evolved out of the effort to determine the probability of obtaining various combinations of the Mariner 1964 mission objectives. The model assumes that the mission and spacecraft can be divided into independent, functional blocks whose series and parallel combination comprise the totality of the mission. Various parameters, which define the independent functional blocks stochastically, are then introduced and the results are presented as functions of these parameters. Specific values for these parameters are chosen and results are derived and discussed.

author

II. ANALYSIS

A. DESCRIPTION OF MATHEMATICAL MODEL AND DESIRED RESULTS

The probability of successfully completing various combinations of the Mariner 1964 mission objectives is determined as a function of the probability of launch, the probability of a successful boost, the probability of the space-craft living through injection, and the probability of the hardware surviving the cruise until the time at which it accomplishes its particular objective. Those results thought to be most representative of the spectrum of probable mission outcomes are the following:

1. Probability of obtaining at least one set of three month cruise data out of two programmed launches (P_{CD}).
2. Probability of obtaining at least one Occultation, Television, OR Fields and Particles success out of two programmed launches (P_{ANY}).
3. Probability of obtaining at least one Occultation, Television, OR Fields and Particles success out of two programmed launches, given that only one spacecraft is functioning properly at injection (P_{IF1}).
4. Probability of obtaining at least one Occultation, Television, OR Fields and Particles success out of two programmed launches, given that both spacecraft are functioning properly at injection (P_{IF2}).
5. Probability of obtaining at least one set of Occultation, Television, AND Fields and Particles successes out of two programmed launches (P_{ALL}).

The remainder of this report will be devoted to deriving the mathematical model and the particular results expressed above.

In creating a mathematical model to represent the Mariner 1964 mission, the following assumptions were made:

1. A launch policy which advocates launching the first spacecraft on a Type II trajectory and the second spacecraft on a Type I trajectory will be pursued. Both spacecraft will be aimed for points at which TV data, Occultation data, and Fields and Particles data are obtainable.

2. The functions which comprise the Mariner mission are assumed to be independent of one another. In other words, the launch vehicle performance does not influence the spacecraft, or the performance of the Central Computer and Sequence (CC&S) subsystem does not affect the performance of the Transmitter subsystem.
3. The a priori probability of launching a spacecraft on a given day is a constant. The calculation of this daily launch probability takes into consideration range holds, equipment malfunction, and weather and includes their respective probable delays. (See Ref. 1).
4. Only launch days in which the booster can carry 3σ propellant reserves will be considered. Thus, it is extremely probable that the spacecraft will be injected onto a correctable trajectory if lift-off occurs and the booster functions normally.
5. The spacecraft will either completely survive or completely fail during the boost phase. A partially operable spacecraft at the end of the boost is not considered by the model.
6. A successful spacecraft is defined as one in which all component parts function their respective required length of time. Parts are assumed to fail randomly with time, and fail completely when they fail. No failures arising from component drift, design deficiency, or statistical error is considered.
7. The spacecraft is tested beyond the infant mortality phase of all of its components prior to launch and no component reaches its wearout point before the termination of the mission.
8. All TV pictures of the planet obtained will be considered of equal value, irrespective of the trajectory flown or the point from which the pictures were taken. The same statement holds for Fields and Particles data.
9. Only occultation obtained in two-way communication lock will be considered. All occultation data obtained from aiming points falling within the occultation zone at encounter will be considered of equal value.

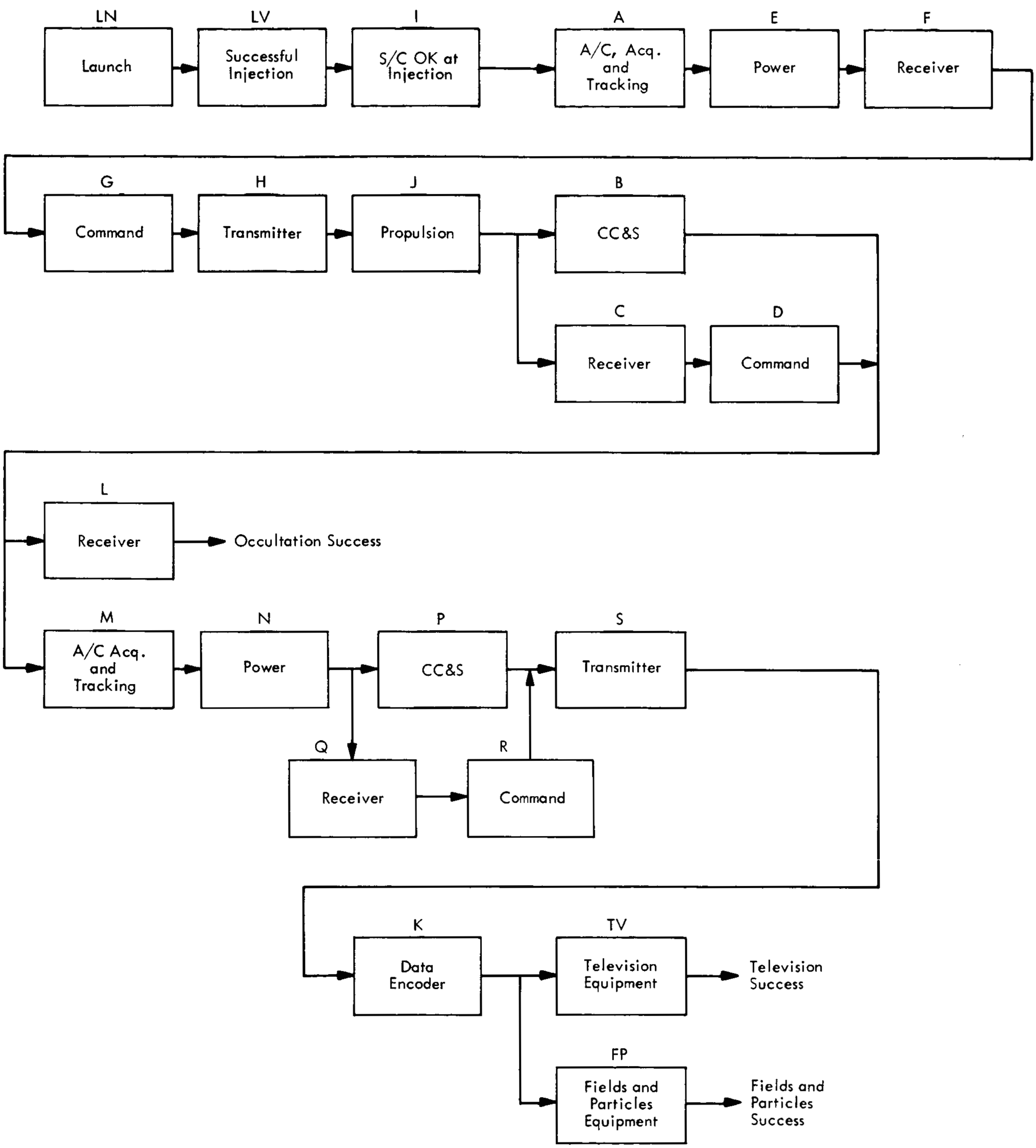
10. Magnetometer data and data from either the Trapped Radiation Detector or the Ion Chamber is required for a successful Fields and Particles experiment.

A functional block diagram of the mission was prepared and the various mission outcomes were represented by serial and parallel combinations of these blocks. Recall that these functional blocks were assumed to be independent of one another and therefore a simplified mathematical treatment is allowable. Each functional block was analyzed and either was assigned an independently derived reliability or was represented by a time-dependent random failure model. In the latter case, a new parameter, β, is introduced; β is a failure rate multiplier which adjusts the failure rates assigned by the PRC analysis (See Ref. 2 and 3) to values deemed more representative. Refer to Section III-B for discussion of this point. If a particular functional block is maintained in the stored condition for some period of time, its reliability is shown also to be a function of α, a failure rate multiplier which accounts for the fact that stored electronic components age at a different rate than those energized. The probability of achieving the various mission outcomes is then presented as a function of the independently derived probabilities, β, and α.

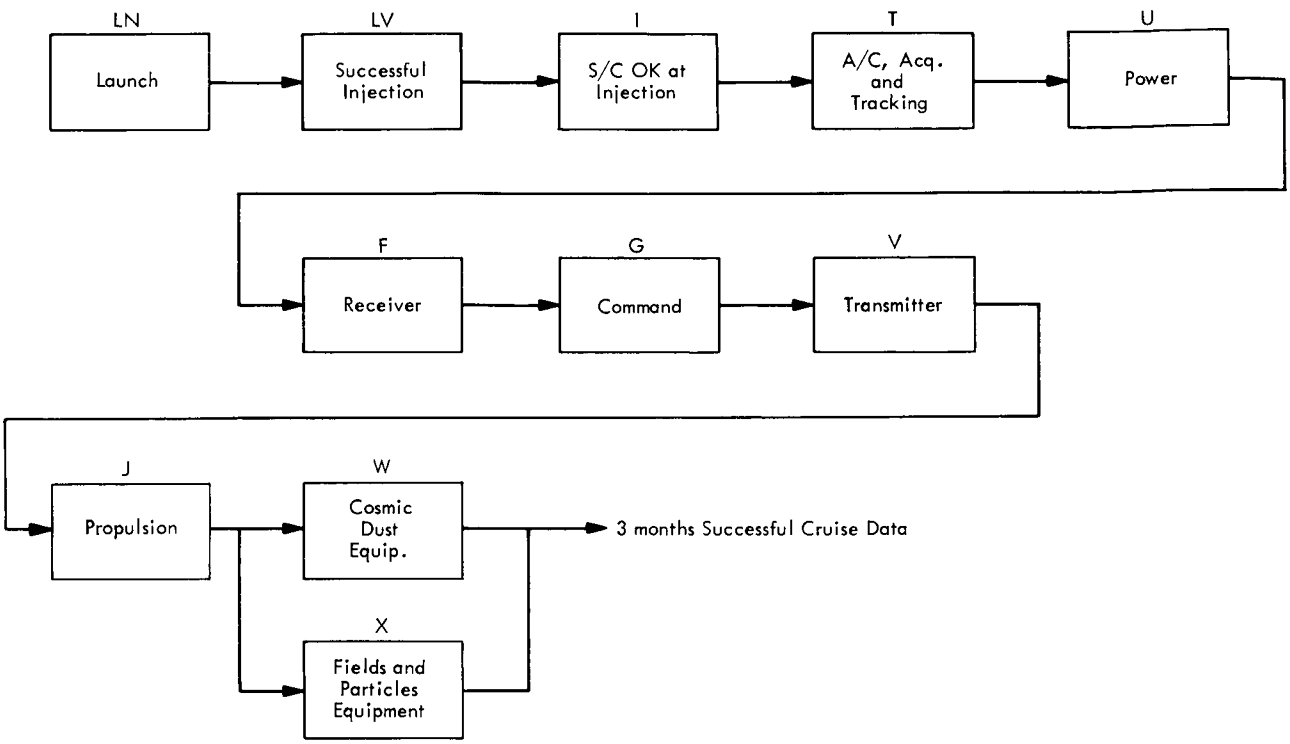
B. FUNCTIONAL BLOCK DIAGRAMS

Illustrated below are two functional block diagrams which represent the Mariner 1964 mission. The planetary objectives are contained in the first diagram whereas the interplanetary objectives are contained in the second.

1. Planetary Objectives



2. Interplanetary Objectives



Supplementary functional block diagram data

Block	Min. operational time required (hr)	Conditions
A. A/C, Acq. and Tracking	6006	
E. Power	6006	
F. Receiver	120	
G. S/C Command	120	
H. Transmitter	6006	
J. Propulsion	120	
B. CC&S	5280	
C. Receiver	5280	Given that it worked at 120 hr
D. S/C Command	5280	Given that it worked at 120 hr
L. Receiver	6006	Given that it worked at 120 hr
M. A/C, Acq., and Tracking	6213	Given that it worked at 6006 hr
N. Power	6213	Given that it worked at 6006 hr
P. CC&S	6213	Given that it worked at 120 hr
Q. Receiver	6213	Given that it worked at 120 hr
R. Command	6213	Given that it worked at 120 hr
S. Transmitter	6213	Given that it worked at 6006 hr
K. Data encoder	6213	
TV. Television equipment	6213	
FP. Fields and Particles equipment	6213	
T. A/C, Acq., and Tracking	2200	
U. Power	2200	
V. Transmitter	2200	
W. Cosmic dust equipment	2200	
X. Fields and Particles equipment	2200	

C. RELIABILITY ANALYSIS

The following section describes the reliability analysis of each independent functional block. In some cases the block's reliability is stated discretely. In other cases, it is stated as a function of α and β. Note that in several instances the model developed is more complex than required to serve the purpose, but this representation is consistent with existing computer programs used to obtain numerical results.

1. Probability of Launch - Block LN

The following analysis was used to determine the probability of launching the two programmed spacecraft during the launch period. It assumes the following:

1. There exists a probability, p, that a spacecraft will be launched on any given day during the launch period. p is assumed to be a function of the magnitude of the daily launch window. $q = 1-p$ represents the probability of not launching on a given day. (See Section III-A for further discussion on p.)
2. Attempts will be made to launch the first spacecraft on successive days until successful, and then the same procedure will be applied to the second spacecraft commencing the following day.
3. Only one launch attempt per day will be made.
4. There exists a total of N_2 days in the launch period of which the first N_1 are Type II launch days and the latter $N_2 - N_1$ are Type I launch days. ($N_1 = 6$ days and $N_2 = 26$ days were taken as nominal for the 1964 opportunity.)
5. A maximum of one spacecraft will be launched during the Type II launch period. In other words, if the first spacecraft is launched during the Type II launch period, the launch crew will wait until the beginning of the Type I period before attempting to launch the second spacecraft.

Under such assumptions, there exist five mutually exclusive and exhaustive outcomes. These appear below with their respective probabilities of occurring.

1. Event of launching the first spacecraft during the Type II launch period and the second during the Type I launch period.

$$P_{II,I} = (1 - q^{N_1})(1 - q^{N_2 - N_1})$$

2. Event of failing to launch the first spacecraft during the Type II period and launching both spacecraft during the Type I launch period.

$$P_{I,I} = q^{N_1} \left\{ 1 - q^{N_2 - N_1 - 1} \left[1 + p\,(N_2 - N_1 - 1) \right] \right\}$$

3. Event of launching the first spacecraft during the Type II period and failing to launch the second during the Type I period.

$$P_{II,0} = (1 - q^{N_1})\, q^{N_2 - N_1}$$

4. Event of failing to launch the first spacecraft during the Type II period and launching only one spacecraft during the Type I period.

$$P_{I,0} = q^{N_1} \left\{ (N_2 - N_1)\, pq^{N_2 - N_1 - 1} \right\}$$

5. Event of failing to launch either spacecraft during the entire N_2 day launch period.

$$P_{0,0} = q^{N_2}$$

The above five probabilistic expressions represent the chances of all possible launch outcomes occurring for a standard launch model where no unusual events occur. If one wishes to determine the probabilities of the five launch outcomes where a delay is introduced immediately following the launch of the first spacecraft in order that any anomalies occurring in the launch, the launch vehicle, or the spacecraft might be resolved prior to the launch of the second spacecraft, the following analysis can be used. The same assumptions as were made for the standard launch model still hold with the addition of an assumed R day delay following the first launch before attempting to launch the remaining spacecraft. Furthermore R was assumed to be always equal to or greater than N_1, the magnitude of the Type II launch period. The probabilities for the five outcomes now become:

$$P_{II,I} = 1 - q^{N_1} - N_1 pq^{N_2 - R - 1} \tag{1}$$

$$P_{I,I} = q^{N_1} - q^{N_2 - R - 1} \left[1 + p\,(N_2 - N_1 - R - 1) \right] \tag{2}$$

$$P_{II,0} = N_1 pq^{N_2 - R - 1} \tag{3}$$

$$P_{I,0} = q^{N_2 - R - 1} \left[1 + p\,(N_2 - N_1 - R - 1) \right] - q^{N_2} \tag{4}$$

$$P_{0,0} = q^{N_2} \tag{5}$$

Obviously, the best representation of the probabilities of the expected launch outcomes would combine the results obtained from the standard model with the results obtained where non-standard delays are introduced. If we use, as subscript notation, O for an outcome corresponding to a standard launch (no delay), and 8, 12, 16 for outcomes corresponding to non-standard launches incurring 8, 12, and 16 days delays, respectively, and introduce weighting factors, W_i, which represent the relative number of times we can expect the various delays, we can arrive at the overall probabilities of the five mutually exclusive and exhaustive launch outcomes occurring:

$$P_{II,I} = W_O (P_{II,I})_O + W_8 (P_{II,I})_8 + W_{12} (P_{II,I})_{12} + W_{16} (P_{II,I})_{16} \quad (1)$$

$$P_{I,I} = W_O)P_{I,I})_O + W_8 (P_{I,I})_8 + W_{12} (P_{I,I})_{12} + W_{16} (P_{I,I})_{16} \quad (2)$$

$$P_{II,O} = W_O (P_{II,O})_O + W_8 (P_{II,O})_8 + W_{12} (P_{II,O})_{12} + W_{16} (P_{II,O})_{16} \quad (3)$$

$$P_{I,O} = W_O (P_{I,O})_O + W_8 (P_{I,O})_8 + W_{12} (P_{I,O})_{12} + W_{16} (P_{I,O})_{16} \quad (4)$$

$$P_{O,O} = W_O (P_{O,O})_O + W_8 (P_{O,O})_8 + W_{12} (P_{O,O})_{12} + W_{16} (P_{O,O})_{16} \quad (5)$$

where $\sum W_i = 1; \quad i = O, 8, 12, \text{ and } 16$

If one lets:

P_{LN2} = Probability of launching both programmed vehicles during launch period.

P_{LN1} = Probability of launching exactly one vehicle during the launch period.

P_{LN0} = Probability of launching neither vehicle during the launch period.

Then:

$$P_{LN2} = P_{II,I} + P_{I,I}$$

$$P_{LN1} = P_{II,0} + P_{I,0}$$

$$P_{LN0} = P_{0,0}$$

2. Successful Injection - Block LV

P_{LV} is obtained from Ref. 4, 5, and 6 where it is derived. Refer to Section III-C for further discussion.

3. Spacecraft Surviving Boost Environment - Block I

P_I is obtained from Ref. 7 where it is derived. See Section III-D for further discussion.

4. Spacecraft

In the remaining cases, the functional block's reliability is determined from the reliabilities of the serial/parallel combinations of the components which comprise it. The component's failure rate is obtained from the PRC report (see Ref. 2, 3, 8, and 9) and the following exponential failure model is assumed.

$$P\text{ (meeting objective)} = \left[e^{-\beta\lambda T_{on}}\right]\left[e^{-\alpha\beta\lambda T_{off}}\right]$$

where:

λ = component's failure rate

T_{on} = required operating time of component

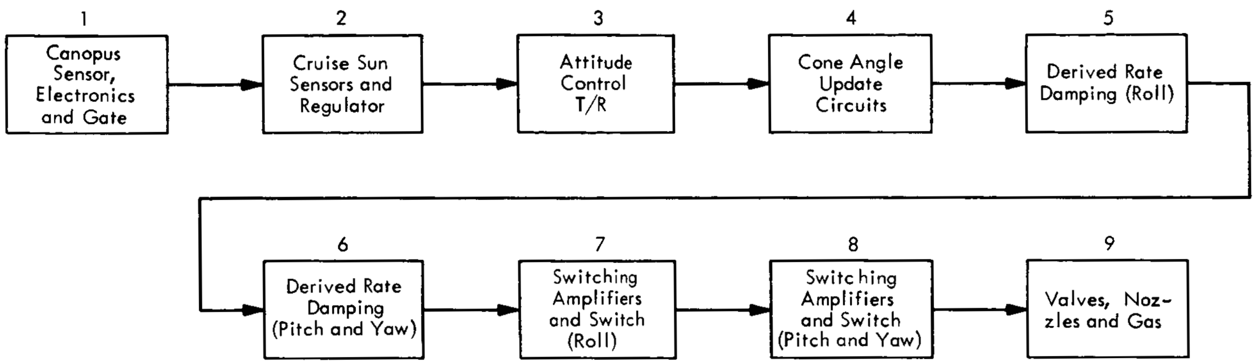
T_{off} = required storage time of component

β = multiplier which adjusts PRC failure rates to correspond to Mariner R flight data (see Ref. 10)

α = multiplier which adjusts operating failure rates to stored failure rates.

Note that in the majority of cases, $T_{off} = 0$.

a. Attitude Control, Acquisition and Tracking



Component	PRC failure rate (failures per 10^6 hrs)
1 Canopus Sensor, Electronics, and Gate	33.53
2 Cruise Sun Sensors and Regulator	3.68
3 Attitude Control Transformer Rectifier	1.94
4 Cone Angle Update Circuits	5.71
5 Derived Rate Damping (Roll)	1.54
6 Derived Rate Damping (Pitch and Yaw)	2.33
7 Switching Amplifiers and Switch (Roll)	10.22

(Cont)

Component	PRC failure rate (failures per 10^6 hr)
8 Switching Amplifiers and Switch (Pitch and Yaw)	32.81
9 Valves, Nozzles, and Gas	0.00

$$P(A/C, \text{Acq.}, \text{Tracking}) = P_1 P_2 P_3 P_4 P_5 P_6 P_7 P_8 P_9$$

Block A (T_{on} = 6006 hr)

$$P_A = (.5759)^{\beta}$$

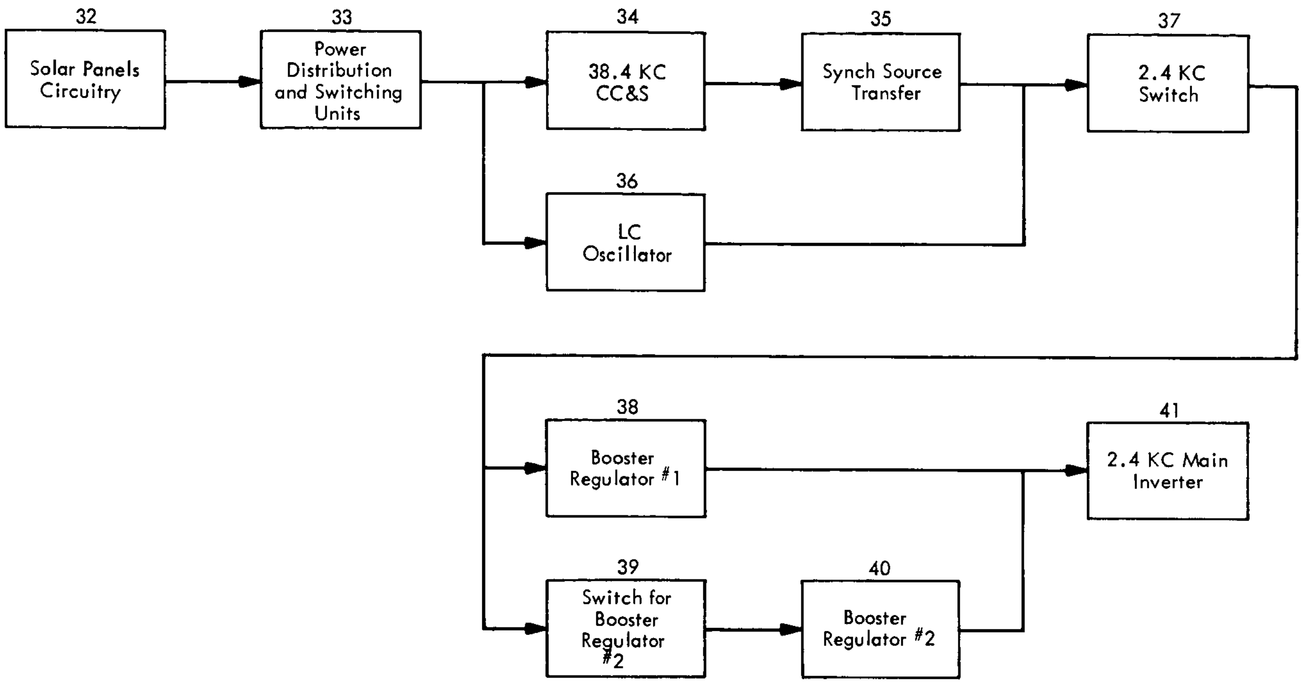
Block M (T_{on} = 6213 hr given that it worked to 6006 hr)

$$P_M = (.9812)^{\beta}$$

Block T (T_{on} = 2200 hr; 3 months)

$$P_T = (.8170)^{\beta}$$

b. Power



Component	PRC failure rate (failures per 10^6hr)
32 Solar Panels Circuitry	29.36
33 Power Distribution and Switching	2.96
34 38.4 KC CC&S	12.42
35 Synch Source Transfer	4.85
36 LC Oscillator	2.42
37 2.4 KC Synch	4.49
38 Booster Regulator #1	16.10
39 Switch for Booster Regulator #2	8.26
40 Booster Regulator #2	16.85
41 2.4 KC Main Inverter	11.44

$$P(\text{Power}) = P_{32}\, P_{33}\, (P_{34}\, P_{35} + P_{36} - P_{34}\, P_{35}\, P_{36})\, P_{37}\, (P_{38} + P_{39}\, P_{40} - P_{38}\, P_{39}\, P_{40})\, P_{41}$$

Block E (T_{on} = 6006 hr)

$$P_E = (.7478)^{\beta} \left[(.9003)^{\beta} + (.9856)^{\beta} - (.8873)^{\beta}\right] \left[(.8604)^{\beta} + (.9080)^{\beta} - (.7813)^{\beta}\right]$$

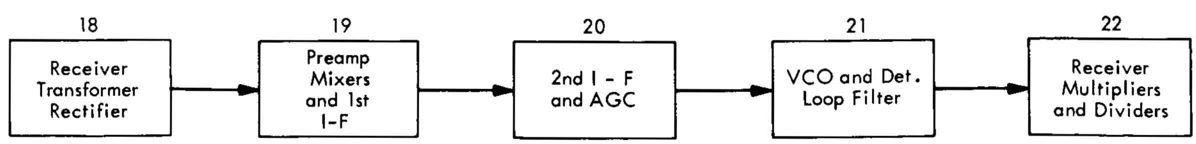
Block N (T_{on} = 6213 hr, given that it worked to 6006 hr)

$$P_N = (.7403)^{\beta} \left[(.8970)^{\beta} + (.9851)^{\beta} - (.8836)^{\beta}\right] \left[(.8560)^{\beta} + (.9050)^{\beta} - (.7747)^{\beta}\right]$$

Block U (T_{on} = 2200 hr; 3 months)

$$P_U = (.8990)^{\beta} \left[(.9622)^{\beta} + (.9947)^{\beta} - (.9571)^{\beta}\right] \left[(.9464)^{\beta} + (.9653)^{\beta} - (.9136)^{\beta}\right]$$

c. <u>Receiver</u>



Component	PRC failure rate (failures per 10^6 hr)
18 Receiver Transformer Rectifier	2.47
19 Preamp Mixers and 1st I - F	7.00
20 2nd I - F and AGC	18.51
21 VCO and Det. Loop Filter	16.54
22 Receiver Multipliers and Dividers	15.68

$$P(\text{Rec}) = P_{18}\, P_{19}\, P_{20}\, P_{21}\, P_{22}$$

Block F (T_{on} = 120 hr)

$$P_F = (.9928)^{\beta}$$

Block C (T_{on} = 5280 hr, given that it worked to 120 hr)

$$P_C = (.7330)^{\beta}$$

Block Q (T_{on} = 6213 hr, given that it worked to 120 hr)

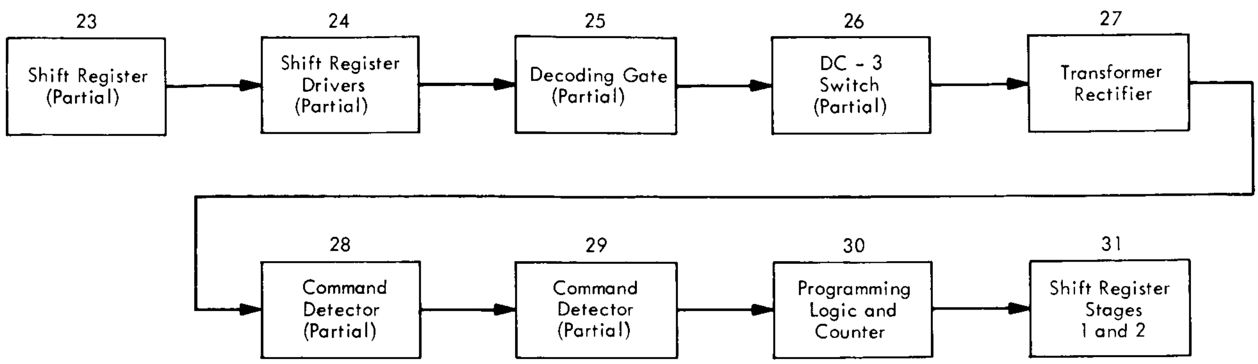
$$P_Q = (.6950)^{\beta}$$

Block L (T_{on} = 6006 hr, given that it worked to 120 hr)

$$P_L = \left(\frac{P_B + P_D - P_B\, P_D}{P_{BCD}}\right) (P_{18}\, P_{19}\, P_{20}\, P_{21}\, P_{22})$$

$$= \left(\frac{.7187^{\beta} + .5896^{\beta} - .4237^{\beta}}{.7187^{\beta} + .4222^{\beta} - .3148^{\beta}}\right) (.7017)^{\beta}$$

d. Command



Component	PRC failure rate (failures per 10^6 hr)
23 Shift Register (Partial)	2.72
24 Shift Register Drivers (Partial)	10.56
25 Decoding Gate (Partial)	0.34
26 DC - 3 Switch (Partial)	0.51
27 Transformer Rectifier	3.44
28 Command Detector (Partial)	62.35
29 Command Detector (Partial)	2.23
30 Programming Logic and Counter	23.84
31 Shift Register Stages 1 and 2	2.12

$$P(\text{Command}) = P_{23}\, P_{24}\, P_{25}\, P_{26}\, P_{27}\, P_{28}\, P_{29}\, P_{30}\, P_{31}$$

Block G (T_{on} = 120 hr)

$$P_G = (.8869)^{\beta}$$

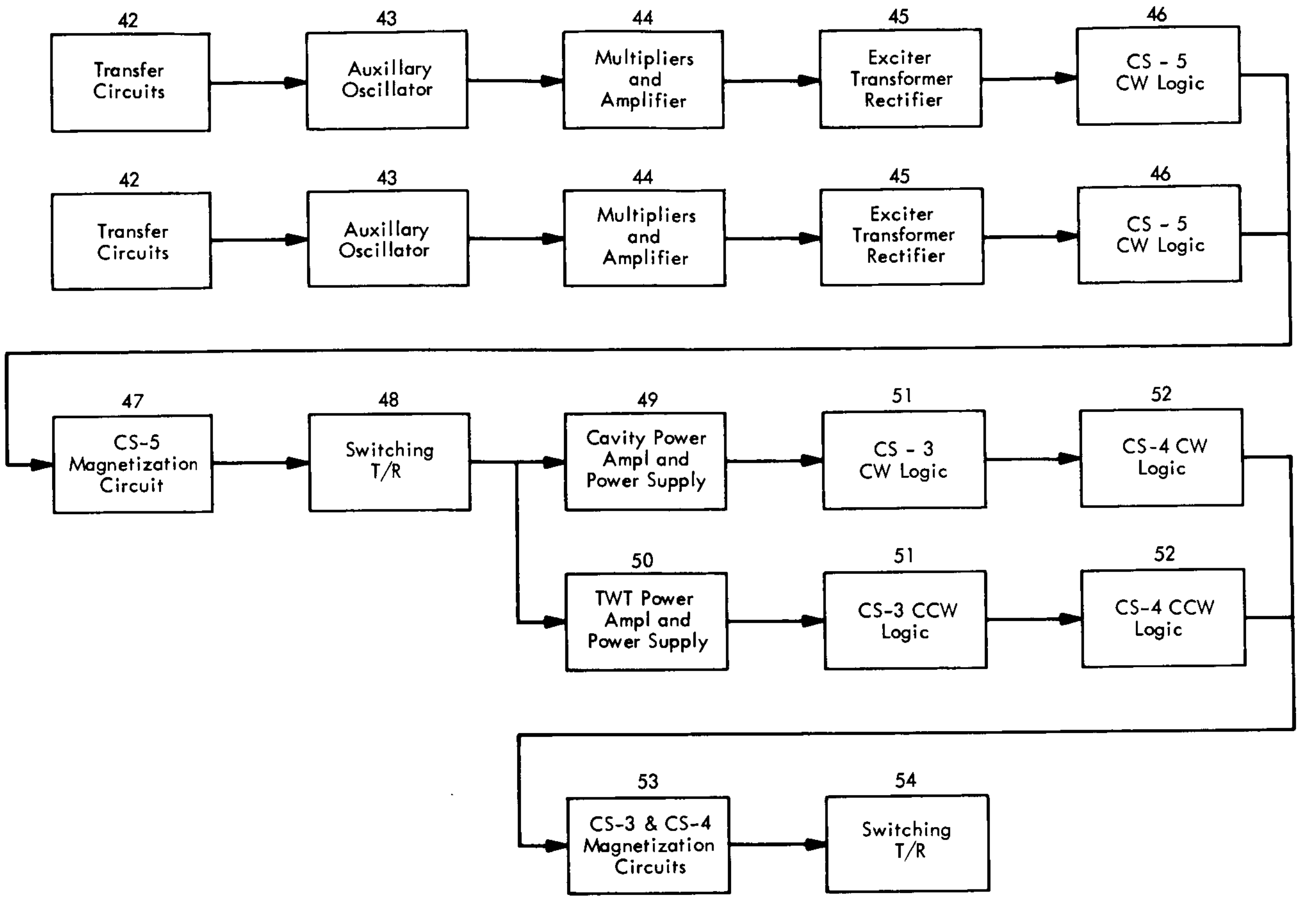
Block D (T_{on} = 5280 hr, given that it worked to 120 hr)

$$P_D = (.5896)^{\beta}$$

Block R (T_{on} = 6213 hr, given that it worked to 120 hr)

$$P_R = (.5170)^{\beta}$$

e. Transmitter



Component	PRC failure rate (failures per 10^6 hr)
42 Transfer circuits	0.31
43 Auxillary Oscillator	2.65
44 Multipliers and amplifiers	14.12
45 Exciter Transformer Rectifier	2.47
46 CS-5 CW Logic	1.44
47 CS-5 Magnetization Circuit	0.05
48 Switching Transformer Rectifier	2.47
49 Cavity Power Ampl. and Power Supply	21.23
50 TWT Power Ampl. and Power Supply	26.30
51 CS-3 CW Logic or CS-3 CCW Logic	1.44
52 CS-4 CW Logic or CS-4 CCW Logic	1.44
53 CS-3 and CS-4 Magnetization Circuits	0.10
54 Switching Transformer Rectifier	2.47

$$P(\text{Transmitter}) = \left[1 - (1 - P_{42} P_{43} P_{44} P_{45} P_{46})^2\right]$$

$$\left[P_{47} P_{48} (P_{49} + P_{50} - P_{49} P_{50} P_{51} P_{52}) \quad P_{51} P_{52} P_{53} P_{54}\right]$$

Block H (T_{on} = 6006 hr)

$$P_H = (.9662)^{\beta} \left\{1 - \left[1 - (.8814)^{\beta}\right]^2\right\} \left\{(.8799)^{\beta} + (.8546)^{\beta} - (.7490)^{\beta}\right\}$$

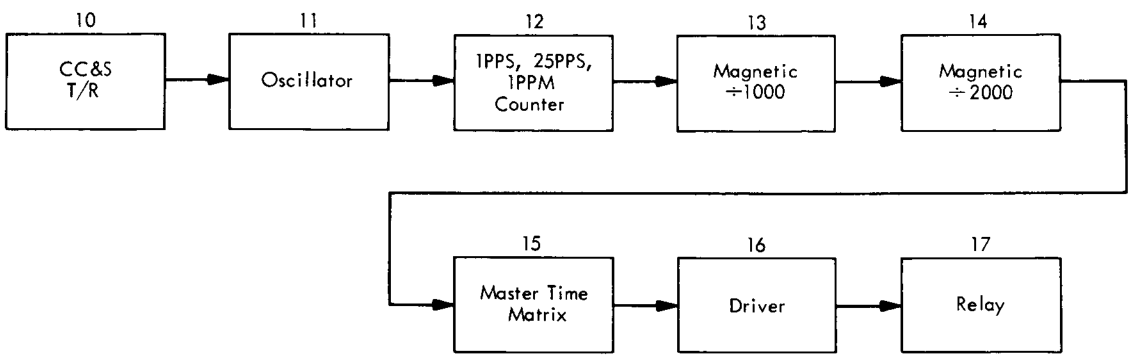
Block S (T_{on} = 6213 hr, given that it worked to 6006 hr

$$P_S = (.9651)^{\beta} \left\{1 - \left[1 - (.8776)^{\beta}\right]^2\right\} \left\{(.8760)^{\beta} + (.8500)^{\beta} - (.7416)^{\beta}\right\} \div P_H$$

Block V (T_{on} = 2200 hr; 3 months)

$$P_V = (.9875)^{\beta} \left\{1 - \left[1 - (.9548)^{\beta}\right]^2\right\} \left\{(.9542)^{\beta} + (.9441)^{\beta} - (.8996)^{\beta}\right\}$$

f. Central Computer and Sequencer



Computer	PRC failure rate (failures per 10^6 hr)
10 CC&S T/R	4.60
11 Oscillator	7.80
12 1PPS, 25PPS, 1PPM Counter	25.44
13 Magnetic ÷ 1000	6.33
14 Magnetic ÷ 2000	8.19
15 Master Time Matrix	10.09

(Cont)

Component	PRC failure rate (failures per 10^6 hr)
16 Driver	1.70
17 Relay	0.75

$$P(CC\&S) = P_{10}\, P_{11}\, P_{12}\, P_{13}\, P_{14}\, P_{15}\, P_{16}\, P_{17}$$

Block B (T_{on} = 5280 hr)

$$P_B = (.7187)^{\beta}$$

Block P (T_{on} = 6213 hr)

$$P_P = (.6780)^{\beta}$$

g. Propulsion

Block J (T_{on} = 120 hr)

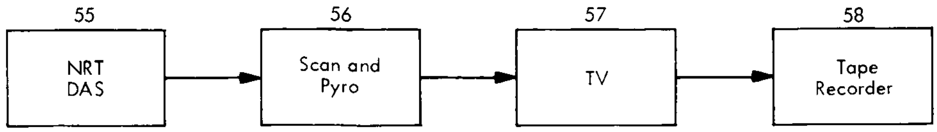
$$P_J = (.9780)^{\beta}$$

h. Data Encoder

Block K (T_{on} = 6213 hr)

$$P_K = (.5380)^{\beta}$$

i. Television



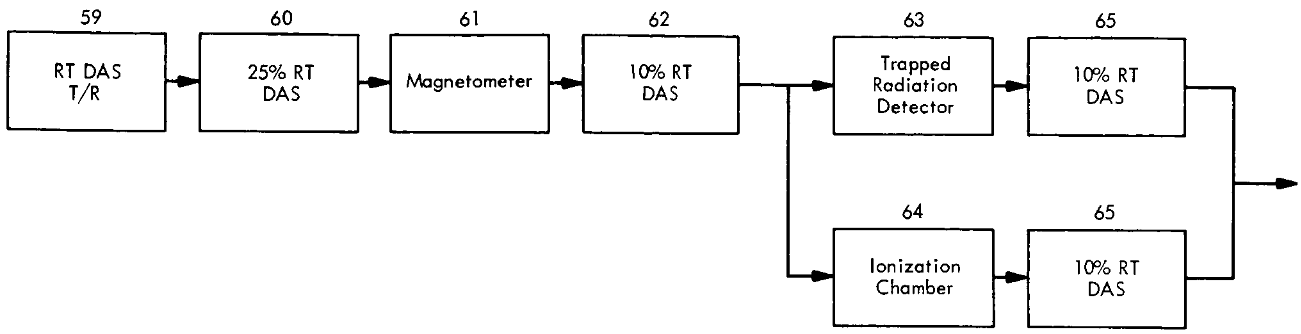
Component	PRC failure rate (failures per 10^6 hr)
55 NRT DAS	376.00
56 Scan and Pyro	-
57 TV	250.00
58 Tape Recorder	157.00

$$P(\text{Television}) = P_{55}\, P_{56}\, P_{57}\, P_{58}$$

Block TV (T_{on} = 8 hrs; T_{off} = 6205 hrs)

$$P_{TV} = (.9580)^{\beta(1 + 775.6\alpha)}$$

j. Fields and Particles



Component	PRC failure rate (failures per 10^6 hr)
59 RT DAS T/R	4.60
60 25% RT DAS	138.75
61 Magnetometer	244.19
62 10% RT DAS	55.23
63 Trapped Radiation Detector	29.80
64 Ionization Chamber	26.33
65 10% RT DAS	55.23

$$P(\text{Fields and Particles}) = P_{59}\, P_{60}\, P_{61}\, P_{62}\, (P_{63} + P_{64} - P_{63}\, P_{64}\, P_{65})\, P_{65}$$

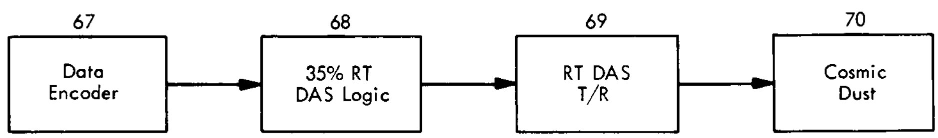
Block FP (T_{on} = 6213 hr)

$$P_{FP} = (.0453)^{\beta} \left[(.8310)^{\beta} + (.8490)^{\beta} - (.5075)^{\beta} \right]$$

Block X (T_{on} = 2200 hr; 3 months)

$$P_X = (.3343)^{\beta} \left[(.9366)^{\beta} + (.9437)^{\beta} - (.7865)^{\beta} \right]$$

k. Cosmic Dust



Component	PRC failure rate (failures per 10^6 hr)
67 Data Encoder	99.70
68 35% RT DAS Logic	193.98
69 RT DAS T/R	4.60
70 Cosmic Dust	77.72

$$P(\text{Cosmic Dust}) = P_{67} P_{68} P_{69} P_{70}$$

Block W (T_{on} = 2200 hr; 3 months)

$$P_W = (.4371)^{\beta}$$

1. Miscellaneous

Block BCD (T_{on} = 5280 hr)

$$P_{BCD} = P_B + P_C P_D - P_B P_C P_D$$

$$= (.7187)^{\beta} + (.4422)^{\beta} - (.3178)^{\beta}$$

Block PQR (T_{on} = 6213 hr)

$$P_{PQR} = \frac{P_P + P_Q P_R - P_P P_Q P_R}{P_{BCD}}$$

$$= \frac{(.6780)^{\beta} + (.3593)^{\beta} - (.2434)^{\beta}}{(.7187)^{\beta} + (.4422)^{\beta} - (.3178)^{\beta}}$$

D. ANALYTIC RESULTS

With the functional blocks described in terms of independently derived reliabilities, α, and β, it is now possible to represent the probabilities of achieving each of the set of mutually exclusive and exhaustive encounter outcomes from which the desired results can be determined.

The subscripts of the terms on the left sides of the equations refer to the number of successes of Occultation, Television, and Fields and Particles, respectively.

If we let:

$$\phi_1 = P_{LV} P_I P_A P_E P_F P_G P_H P_J P_{BCD}$$

$$\phi_2 = P_M P_N P_{PQR} P_S P_K$$

$$\phi_3 = P_{LV} P_I P_T P_U P_F P_G P_V P_J$$

Then:

$$P_{222} = P_{LN2} \phi_1^2 P_L{}^2 \phi_2^2 P_{TV}{}^2 P_{FP}{}^2$$

$$P_{221} = P_{LN2} \phi_1^2 P_L{}^2 \phi_2^2 P_{TV}{}^2 2P_{FP} (1 - P_{FP})$$

$$P_{220} = P_{LN2} \phi_1^2 P_L{}^2 \phi_2^2 P_{TV}{}^2 (1 - P_{FP})^2$$

$$P_{212} = P_{LN2} \phi_1^2 P_L{}^2 \phi_2^2 2P_{TV} (1 - P_{TV}) P_{FP}{}^2$$

$$P_{211} = P_{LN2} \phi_1^2 P_L{}^2 \left[\phi_2^2 2P_{TV} (1 - P_{TV}) 2P_{FP} (1 - P_{FP}) + 2\phi_2 (1-\phi_2) P_{TV} P_{FP}\right]$$

$$P_{210} = P_{LN2} \phi_1^2 P_L{}^2 \left[\phi_2^2 2P_{TV} (1 - P_{TV})(1 - P_{FP})^2 + 2\phi_2 (1-\phi_2) P_{TV} (1-P_{FP})\right]$$

$$P_{202} = P_{LN2} \phi_1^2 P_L{}^2 \phi_2^2 (1 - P_{TV})^2 P_{FP}{}^2$$

$$P_{201} = P_{LN2} \phi_1^2 P_L{}^2 \left[\phi_2^2 (1 - P_{TV})^2 2P_{FP} (1 - P_{FP}) + 2\phi_2 (1 - \phi_2) (1-P_{TV}) P_{FP}\right]$$

$$P_{200} = P_{LN2}\ \phi_1^2\ P_L^{\ 2}\ \Big[\phi_2^2\ (1-P_{TV})^2\ (1-P_{FP})^2 + 2\ \phi_2\ (1-\phi_2)\ (1-P_{TV})$$

$$(1-P_{FP}) + (1-\phi_2)^2$$

$$P_{122} = P_{LN2}\ \phi_1^2\ 2P_L\ (1-P_L)\ \phi_2^2\ P_{TV}^{\ 2}\ P_{FP}^{\ 2}$$

$$P_{121} = P_{LN2}\ \phi_1^2\ 2P_L\ (1-P_L)\ \phi_2^2\ P_{TV}^{\ 2}\ 2P_{FP}\ (1-P_{FP})$$

$$P_{120} = P_{LN2}\ \phi_1^2\ 2P_L\ (1-P_L)\ \phi_2^2\ P_{TV}^{\ 2}\ (1-P_{FP})^2$$

$$P_{112} = P_{LN2}\ \phi_1^2\ 2P_L\ (1-P_L)\ \phi_2^2\ 2P_{TV}\ (1-P_{TV})\ P_{FP}^{\ 2}$$

$$P_{111} = P_{LN2}\ \Big\{\phi_1^2\ 2P_L\ (1-P_L)\ \Big[\phi_2^2\ 2P_{TV}\ (1-P_{TV})\ 2P_{FP}\ (1-P_{FP}) + 2\phi_2\ (1-\phi_2)$$

$$P_{TV}\ P_{FP}\Big] + 2\phi_1\ (1-\phi_1)\ P_L\ \phi_2\ P_{TV}\ P_{FP}\Big\} + P_{LN1}\ \phi_1\ P_L\ \phi_2\ P_{TV}\ P_{FP}$$

$$P_{110} = P_{LN2}\Big\{\phi_1^2\ 2P_L(1-P_L)\qquad 2P_{TV}(1-P_{TV})(1-P_{FP})^2 + 2\phi_2\ (1-\phi_2)\ P_{TV}(1-P_{FP})\Big]$$

$$+ 2\phi_1\ (1-\phi_1)\ P_L\ \phi_2\ P_{TV}\ (1-P_{FP})\Big\} + P_{LN1}\ \phi_1\ P_L\ \phi_2\ P_{TV}\ (1-P_{FP})$$

$$P_{102} = P_{LN2}\ \phi_1^2\ 2P_L\ (1-P_L)\ \phi_2^2\ (1-P_{TV})^2\ P_{FP}^{\ 2}$$

$$P_{101} = P_{LN2}\ \Big\{\phi_1^2\ 2P_L\ (1-P_L)\ \Big[\phi_2^2\ (1-P_{TV})^2\ 2P_{FP}(1-P_{FP}) + 2\phi_2\ (1-\ {}_2)$$

$$(1-P_{TV})\ P_{FP}\Big] + 2\phi_1\ (1-\phi_1)\ P_L\ \phi_2\ (1-P_{TV})\ P_{FP}\Big\} + P_{LN1}\ \phi_1\ P_L\ \phi_2\ (1-P_{TV})\ P_{FP}$$

$$P_{100} = P_{LN2} \Big\{ \phi_1^2\, 2P_L (1-P_L) \Big[\phi_2^2 (1-P_{TV})^2 (1-P_{FP})^2 + 2\phi_2 (1-\phi_2)(1-P_{TV})$$
$$(1-P_{FP}) + (1-\phi_2)^2 \Big] + 2\phi_1 (1-\phi_1) P_L \Big[\phi_2 (1-P_{TV})(1-P_{FP}) + (1-\phi_2) \Big] \Big\} +$$
$$P_{LN1} \phi_1 P_L \Big[\phi_2 (1-P_{TV})(1-P_{FP}) + (1-\phi_2) \Big]$$

$$P_{022} = P_{LN2} \phi_1^2 (1-P_L)^2 \phi_2^2 P_{TV}{}^2 P_{FP}{}^2$$

$$P_{021} = P_{LN2} \phi_1^2 (1-P_L)^2 \phi_2^2 P_{TV}{}^2\, 2P_{FP} (1-P_{FP})$$

$$P_{020} = P_{LN2} \phi_1^2 (1-P_L{}^2 \phi_2^2 P_{TV}{}^2 (1-P_{FP})^2$$

$$P_{012} = P_{LN2} \phi_1^2 (1-P_L)^2 \phi_2^2\, 2P_{TV} (1-P_{TV}) P_{FP}{}^2$$

$$P_{011} = P_{LN2} \Big\{ \phi_1^2 (1-P_L)^2 \Big[\phi_2^2\, 2P_{TV} (1-P_{TV})\, 2P_{FP} (1-P_{FP}) + 2\phi_2 (1-\phi_2)$$
$$P_{TV} P_{FP} \Big] + 2\phi_1 (1-\phi_1)(1-P_L) \phi_2 P_{TV} P_{FP} \Big\} + P_{LN1} \phi_1 (1-P_L) \phi_2 P_{TV} P_{FP}$$

$$P_{010} = P_{LN2} \Big\{ \phi_1^2 (1-P_L)^2 \Big[\phi_2^2\, 2P_{TV} (1-P_{TV})(1-P_{FP})^2 + 2\phi_2 (1-\phi_2) P_{TV}$$
$$(1-P_{FP}) \Big] + 2\phi_1 (1-\phi_1)(1-P_L) \phi_2 P_{TV} (1-P_{FP}) \Big\} + P_{LN1} \phi_1 (1-P_L) \phi_2 P_{TV} (1-P_{FP})$$

$$P_{002} = P_{LN2} \phi_1^2 (1-P_L)^2 \phi_2^2 (1-P_{TV})^2 P_{FP}{}^2$$

$$P_{001} = P_{LN2} \Big\{ \phi_1^2 (1-P_L)^2 \Big[\phi_2^2 (1-P_{TV})^2\, 2P_{FP} (1-P_{FP}) + 2\phi_2 (1-\phi_2)(1-P_{TV}) P_{FP} \Big] +$$
$$2\phi_1 (1-\phi_1)(1-P_L) \phi_2 (1-P_{TV}) P_{FP} \Big\} + P_{LN1} \phi_1 (1-P_L) \phi_2 (1-P_{TV}) P_{FP}$$

$$\begin{aligned}P_{000} = P_{LN2} & \Big\{\phi_1^2\ (1-P_L)^2\ \Big[\phi_2^2\ (1-P_{TV})^2(1-P_{FP})^2 + 2\phi_2\,(1-\phi_2)(1-P_{TV})(1-P_{FP}) + \\ & (1-\phi_2)^2\Big] + 2\phi_1\ (1-\phi_1)(1-P_L)\Big[\phi_2\,(1-P_{TV})(1-P_{FP}) + (1-\phi_2)\Big] + (1-\phi_1)^2\Big\} + \\ & P_{LN1}\ \Big\{\phi_1\,(1-P_L)\ \Big[\phi_2\,(1-P_{TV})(1-P_{FP}) + (1-\phi_2)\Big] + (1-\phi_1)\Big\} + P_{LNO}\end{aligned}$$

where

$$\sum P_{ijk} = 1; \quad i = 0,\ 1,\ 2$$

$$j = 0,\ 1,\ 2$$

$$k = 0,\ 1,\ 2$$

And finally:

$$\begin{aligned}P_{CD} = P_{LN2} & \Big\{\phi_3^2\ \Big[1 - (1-P_W)^2(1-P_X)^2\Big]\quad 2\phi_3\,(1-\phi_3)(P_W + P_X - P_W\,P_X)\Big\} + \\ & P_{LN1}\ \phi_3\,(P_W + P_X - P_W\,P_X)\end{aligned} \tag{1}$$

$$P_{ANY} = 1 - P_{000} \tag{2}$$

$$\begin{aligned}P_{IF1} = (\phi_1/P_{LV}\,P_I) & \Big\{P_L + \phi_2\ \Big[P_{TV} + P_{FP} - P_{TV}\,P_{FP}\Big] - \\ & P_L\,\phi_2\quad P_{TV} + P_{FP} - P_{TV}\,P_{FP}\Big]\Big\}\end{aligned} \tag{3}$$

$$P_{IF2} = 2P_{IF1} - P_{IF1}^2 \tag{4}$$

$$P_{ALL} = P_{222} + P_{221} + P_{212} + P_{211} + P_{122} + P_{121} + P_{112} + P_{111} \tag{5}$$

which correspond to the desired results itemized in Section IIA.

III. INPUT ASSUMPTIONS

A. PROBABILITY OF DAILY LAUNCH, p

The probability of launching various combinations of the two programmed launch vehicles was shown to be a function of p, the daily launch probability. In Ref. 1, p is shown to be a function of the magnitude of the daily launch window. For Mariner 1964, a constant 2 hr daily launch window is available throughout the launch period. This launch window is equivalent to a 0.3 probability of daily launch. Included in this calculation is the average of all delays arising from weather, equipment malfunction, and range holds. Not taken into consideration are factors such as seasonal weather conditions or reliability growth of equipment and procedures which might tend to raise or lower the daily launch probability.

B. SPACECRAFT PARAMETERS, β AND α

The spacecraft reliability was shown to be a function of the parameters, β and α, when an exponential reliability model was assumed for the components comprising the spacecraft. From Ref. 10, "Estimate of the Failure Rate Multiplier for PRC - type Reliability Calculations for the Mariner C Spacecraft," a value of $\beta = 0.19$ is estimated. Here it is also estimated that β lies between 0.07 and 0.42 with a 95% confidence. In addition, a value of $\alpha = 0$ was estimated. This infers that no degradation to components stored aboard the spacecraft occurs during transit.

C. LAUNCH VEHICLE RELIABILITY

In Ref. 4, the probability of attaining a successful injection is determined. It is based on current Atlas/Agena/Shroud performance data. In this reference, the probability of a successful injection is determined to be 0.74 nominally.

D. BOOST ENVIRONMENT

In Ref. 7, the probability of a spacecraft being operational after injection is shown to be 0.98. This number is based on the results of spacecraft shock and vibration testing.

IV. NUMERICAL RESULTS

The equations exhibited in Section IID were solved with the following values taken as nominal for the independent parameters:

p	the probability of launch on a given day, = 0.3
W_0	the probability of no delay between launches, = 0.37
W_8	the probability of an 8 day delay between launches, = 0.31
W_{12}	the probability of a 12 day delay between launches, = 0.21
W_{16}	the probability of a 16 day delay between launches, = 0.11
P_{LV}	the probability of a successful injection, = 0.74
P_I	the probability of the spacecraft being operable at injection, = 0.98
α	the ratio of storage failure rates to operational failure rates, = 0.00
β	the ratio of PRC failure rates to Mariner R failure rates, = 0.19

The probabilities of the desired outcomes, for this particular set of independent parameter values, become:

1. Probability of obtaining at least one set of three month cruise data out of two programmed launches, $P_{CD} = 0.86$.
2. Probability of obtaining at least one Occultation, Television, OR Fields and Particles success out of two programmed launches, $P_{ANY} = 0.81$.
3. Probability of obtaining at least one Occultation, Television, OR Fields and Particles success out of two programmed launches, given that one spacecraft is functioning properly at injection, $P_{IF1} = 0.80$.
4. Probability of obtaining at least one Occultation, Television, OR Fields and Particles success out of two programmed launches, given that both spacecraft are functioning properly at injection, $P_{IF2} = 0.96$.

5. Probability of obtaining at least one set of occultation, Television, AND Fields and Particles successes out of two programmed launches, $P_{ALL} = 0.49$.

If each of the independent parameters is, in turn, varied about its nominal value as chosen in the previous example while the remainder are held fixed, one can observe the variation or the sensitivity of the results to uncertainties in each of the independent parameters of the model. In Fig. 1 through 4, where p, P_{LV}, P_I, and β, respectively are chosen as the variable, the variations in the results can be observed. Note that the desired probabilities are quite insensitive to large variations in p, P_{LV}, and P_I whereas they appear very sensitive to small changes in β over its entire domain. Interpreted, this sensitivity analysis indicates that if one accepts the model derived in this report, he should accept its numerical results only if he is very confident in the selected value of β. The results will not vary significantly, however, if it should occur that p, P_{LV}, or P_I is, in reality, appreciably different from the value selected in this report. Since the probability of mission success is very sensitive to the component failure rates, improvements in this area will most directly enhance the probability of mission success.

V. REFERENCES

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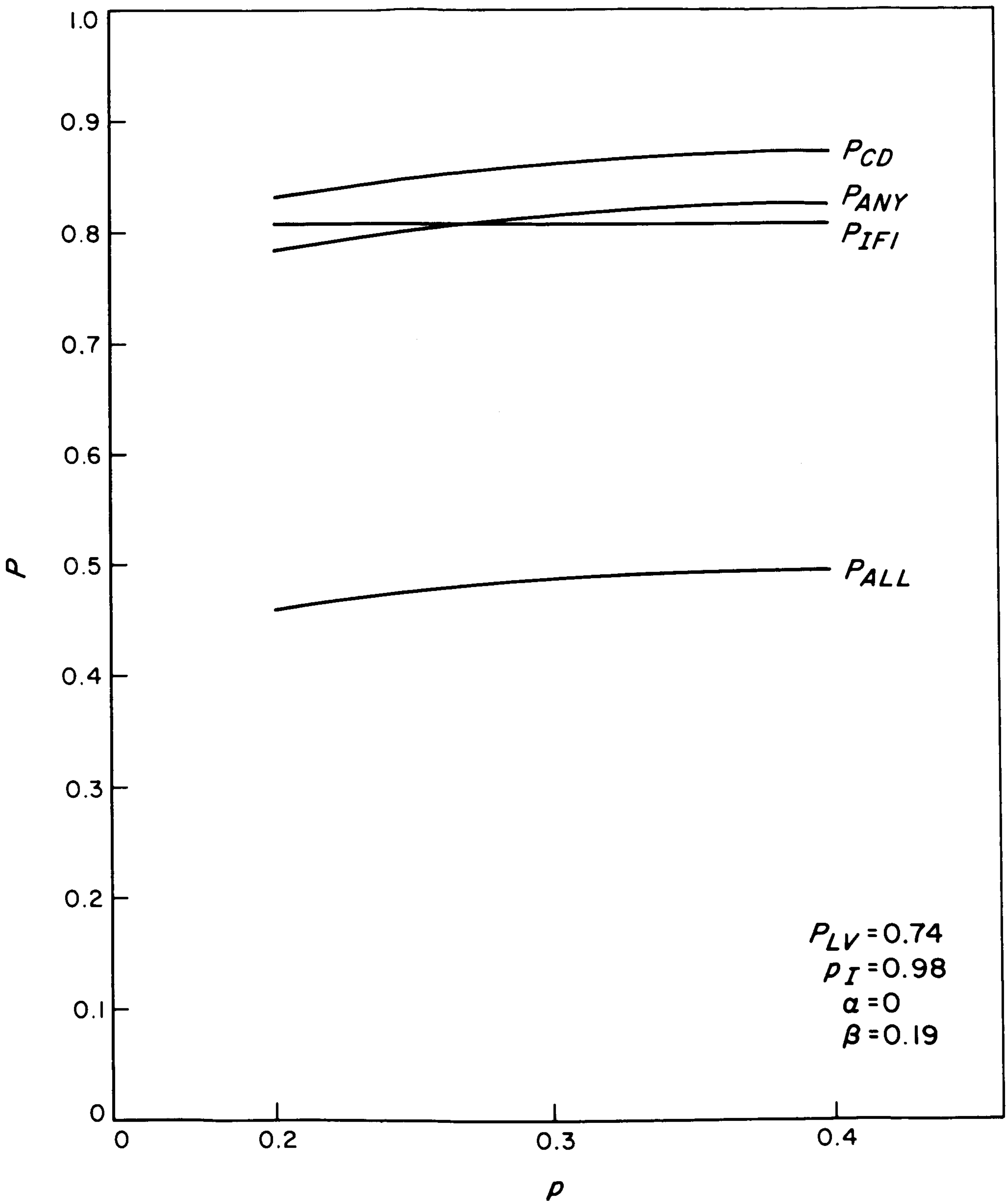

Fig. 1. Probability of mission success versus daily probability of launch

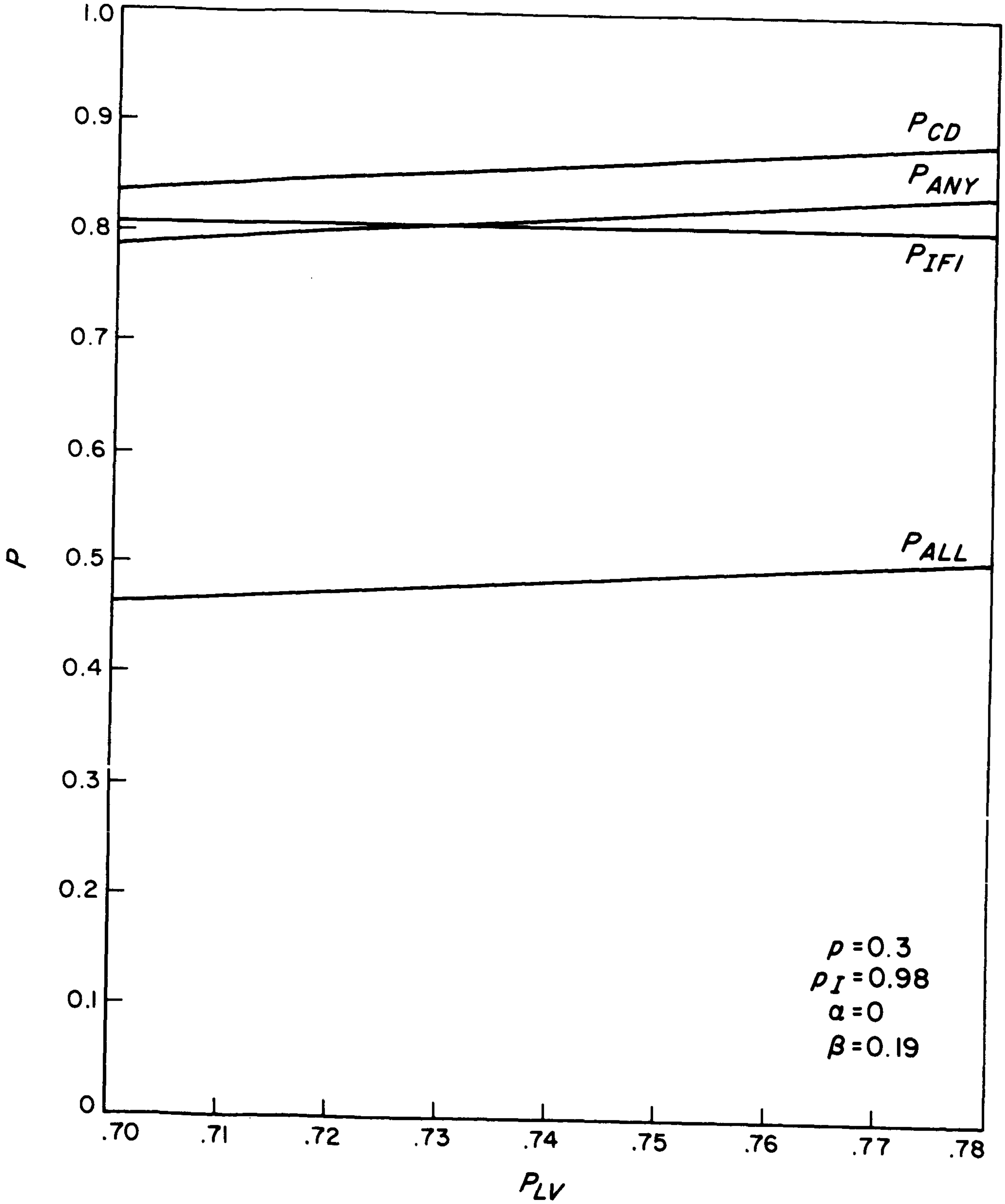

Fig. 2. Probability of mission success versus probability of a successful injection

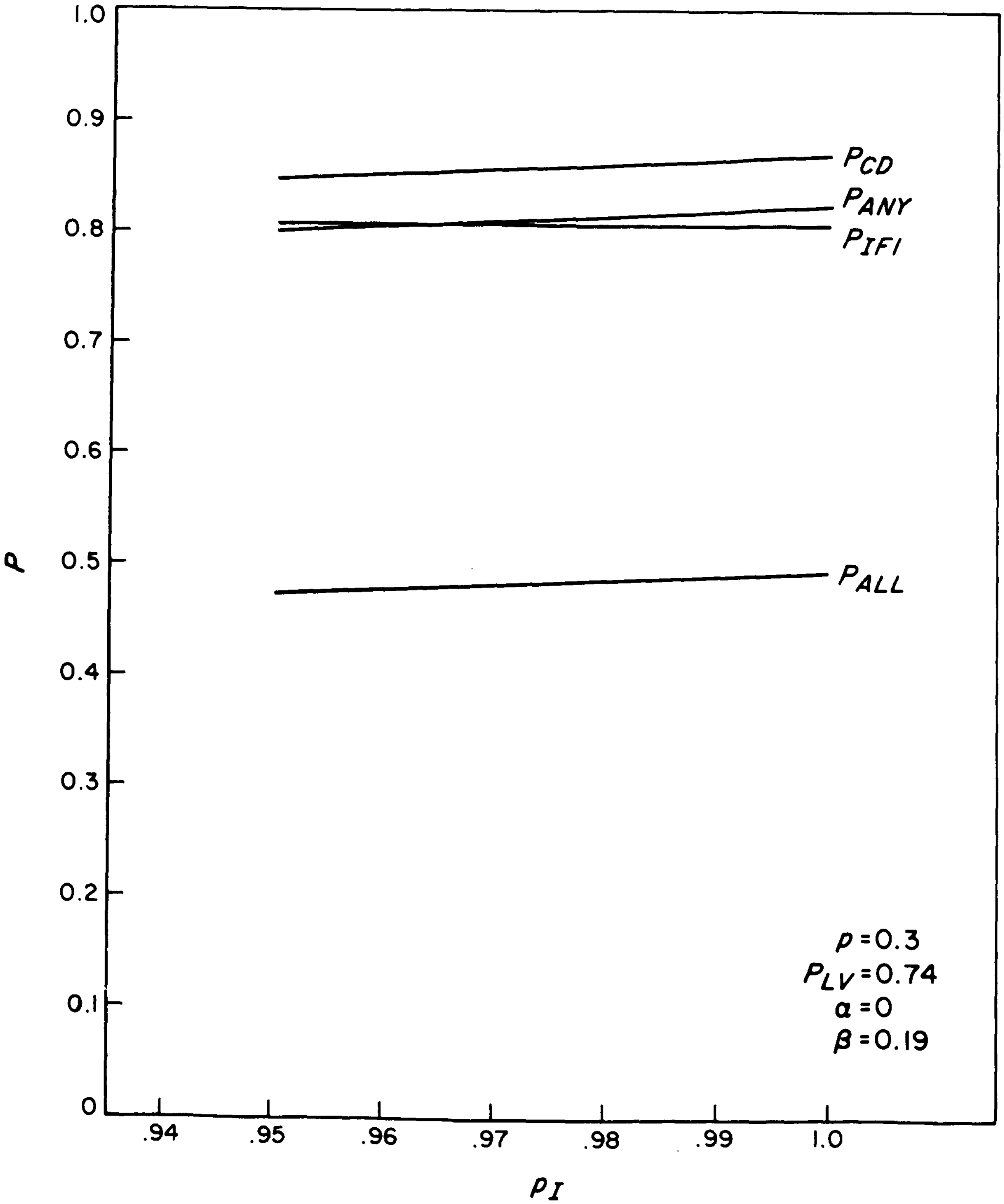

Fig. 3. Probability of mission success versus probability of spacecraft surviving booster environment

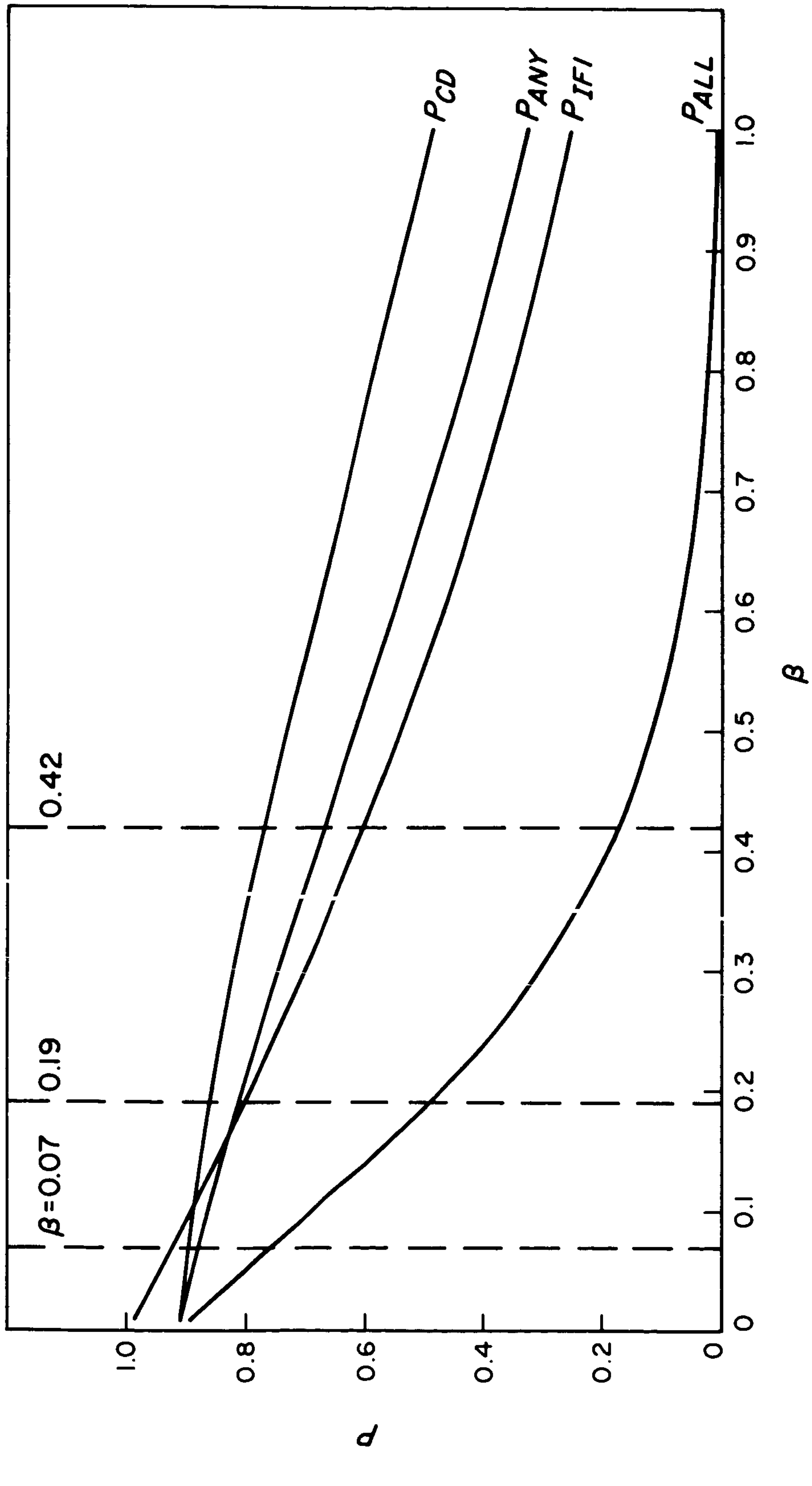

Fig. 4. Probability of mission success versus failure rate multiplier