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Interim Report
for
Theoretical Investigation: The Scattering of Light by a Planetary Atmosphere

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(22 Sept. 1965 - 22 Dec. 1965)

Contract No. NAS5-9678

Prepared by

TRW Systems
One Space Park
Redondo Beach, Calif.

for

Goddard Space Flight Center
Greenbelt, Maryland

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ABSTRACT

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This Interim Report covers the first half of a second contract to make a theoretical study of the characteristics of radiation that is scattered from a planetary atmosphere. This contract continues a previous contract that was financed by NASA (Contract No. NAS5-3891). The new feature of this research is that the lower boundary of the atmosphere reflects radiation specularly according to the Fresnel law.

The work on this contract is divided into three parts: The accuracy of the computed X- and Y- functions of Chandrasekhar; the albedo of diffuse skylight from a smooth water surface; and the characteristics of the neutral points, which refer to the directions where the degree of polarization of light from the atmosphere vanishes. Apparent inaccuracies in the X- and Y- functions were caused by a mistake in the computational routine. Data on albedo from a smooth water surface has been superseded by the albedo data in Mullamaa's new book[13], about the computed characteristics of reflected light from a wind-roughened sea surface. The neutral point study is yielding new and useful information.

Author

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1. INTRODUCTION

Chandrasekhar[2] solved the problem of radiative transfer in a planetary atmosphere in an exact way, and he included reflection from the ground. He assumed that the radiation was reflected from the ground according to the Lambert law, which states that the reflected radiation is unpolarized and isotropic. Because of its simplicity, the Lambert law, or some variation of it,[11] was used in all subsequent problems of radiative transfer in planetary atmospheres until Malkevich[12] studied the effect of non-Lambert reflection. However, he did not consider the polarization of the radiation. Sekera was the first to use non-Lambert reflection and include polarization effects.[18] He replaced the Lambert law by the Fresnel law of specular reflection from a smooth surface. According to the Fresnel law, in contrast to the Lambert law, the reflected radiation is highly directional and is polarized. The Fresnel law is relatively easy to use because of its mathematical simplicity. Also, the approximate characteristics of the radiation emerging from either the base or the top of the atmosphere in oceanic regions can be computed from the planetary model with Fresnel reflection at the ground.

In this report the radiation parameters will be computed for two models. The properties of each model will be specified now. Both models have the same atmosphere: A plane-parallel Rayleigh atmosphere. Both are illuminated from above by parallel radiation. The distinction between the two models is determined by the law of reflection at the ground, which is at the base of the atmosphere. One model is the Fresnel model where the radiation is reflected from a smooth water surface according to the Fresnel law. The index of refraction of the water is about 1.34. The reflected radiation is isotropic and unpolarized for the other model -- the Lambert model. The ground albedo of the Lambert model is made equal to the albedo of the Fresnel model.

Sekera[18] compared the characteristics of light leaving the bottom of the Fresnel model and of an identical one, except that no light was reflected from the ground of the latter model. He found that Fresnel reflection had only a small effect on the intensity and the degree of polarization of the light leaving the bottom of the atmosphere. However, the neutral point positions were considerably different for the two models. If a comparison is made for the Fresnel and Lambert models*, the same conclusions are reached.

The research that Sekera started was continued on Contract No. NAS5-3891. The purpose of this continuation was to compute the parameters that characterize the radiation leaving the top of the same model that was used by Sekera -- the Fresnel model -- and to compare the parameters with corresponding ones for the Lambert model. While comparing the parameters, one must bear in mind that the albedo at the ground is usually small. The ground albedo is less than ten percent when the solar zenith angle is less than 65°.

A comparison of radiation parameters at the top of the atmosphere for the two models shows the following: The fluxes of radiation differ slightly.

*This comparison was made with data computed on Contract No. NAS5-3891, but has not been published.

If the total normal optical thickness of the atmosphere is greater than 0.5, the intensity, degree of polarization, and neutral points for the one model are about the same as the corresponding parameters for the other model. As the optical thickness decreases below 0.5, these parameters show increasingly greater differences for the two models. For example, the neutral points occur only in the vertical plane of the sun for the Lambert model, but disappear from the sun's vertical plane for certain ranges of optical thickness and of solar zenith angle for the Fresnel model.[9]

Upon completion of this previous research three problems remained. First, a question had arisen concerning the accuracy of the Chandrasekhar X- and Y- functions, which were used in the computations. Second, considerable data on the albedo of the skylight at the ground was unanalyzed. Third, a computational routine was nearly ready to explore the course of the neutral points when they leave the sun's vertical plane. Because of the sensitivity of the neutral point positions to the total normal optical thickness of the atmosphere,[17] to aerosols in the atmosphere,[3,8,17] and to the nature of the ground reflection,[9,17,18,19] the neutral point investigation is the most important of the three areas of study. The present contract was given to study these three problems.

During the first half of this contract the apparent inaccuracies of the X- and Y- functions were discovered to be caused by an error in the computational routine. Next, the computational routine that was developed on the previous contract was modified to compute the Stokes parameters for radiation traveling in directions away from the vertical plane through the sun. Additional albedo calculations were also added to the computational routine. The computations that are required for the present research have been completed. About one-half of the data has been analyzed. The remaining data will be analyzed and the Final Report will be written in the next half of the contract period.

2. X- AND Y- FUNCTIONS

One part of this contract required an investigation into the accuracy of the computed X- and Y- functions, which were introduced by Chandrasekhar.[2] These functions are basic for present methods of finding numerical solutions to problems of radiative transfer in planetary atmospheres. Mulliken[14] had questioned the accuracy of these functions before the previous contract had commenced. These questions were based on the non-uniqueness of solutions for the X- and Y- functions. However, no estimate of the magnitude of such errors seemed to exist. The computations on the previous contract seemed to show that the X- and Y- functions contained errors.

These apparent errors appeared in the computation of the flux balance [Eq. (3.61) of reference 9]. The flux balance equals the net flux of radiation into the atmosphere relative to the incident solar flux. The fluxes that appear in the flux balance equation were computed from the calculated values of the intensity. The flux balance would have been zero for either the Fresnel or the Lambert model if the computations were accurate. The computed flux balance was zero for the Fresnel model. This result indicated that the computational routine was correct. However, when the ground albedo of the diffuse light was increased by 0.6 by increasing the index of refraction of the ground to 10, a one percent error in the flux balance appeared. The cause of this error seemed to be in the accuracy of the X- and Y-functions.

The first question investigated on the present contract was: Does the flux balance for the Lambert model? It was found that the computed flux balanced for the Lambert model. Since the same X- and Y- functions were used in the computations for both the Lambert and Fresnel models, the result of this investigation indicated that these functions were essentially accurate.

Parenthetically, it should be mentioned that instead of the integrating the intensity to obtain the flux, the flux at either the top or the bottom of the atmosphere for the Lambert model can be computed more easily with Chandrasekhar's $\gamma_{\ell}^{(1)}$ and $\gamma_r^{(1)}$ functions [Eqs. (240) and (241), p. 280, reference 2]. If the $\gamma_{\ell}^{(1)}$ and $\gamma_r^{(1)}$ functions are used, the flux balance is identically zero.

When it appeared that the values of the X- and Y- functions were accurate, Dr. J. V. Dave of the National Center for Atmospheric Research at Boulder, Colorado checked the computational routine again and found an error in it, and that the error was caused by taking only the first iteration of the $D_{\ell}^{(0)}$- and $D_r^{(0)}$- functions [Eq. (3.34) of reference 9], instead of the final iteration. The first iteration excludes the integral appearing in

Eq. (3.34) of reference 9. The $D_{\ell}^{(0)}$- and $D_{r}^{(0)}$- functions account for that part of the radiation that has been reflected from the ground and is then scattered by the atmosphere. The first iteration of these functions accounts for the radiation that is reflected from the ground just once. When the albedo of the ground is small, or when the atmospheric optical thickness is small, the first iteration is sufficient. However, when both the ground albedo and the atmospheric optical thickness become moderate, more than one iteration is required to maintain accurate computations of the radiation parameters. For example, when the total normal optical thickness of the nonabsorbing atmosphere is 0.5 and the ground albedo is 0.6, about four iterations are required if the error in the computed flux balance is to be less than 10^{-4}.

After the computational routine was corrected, the error in the flux balance was reduced. The X- and Y- functions that were computed by Dr. J. V. Dave and were used in the present research appear to have the accuracy that was presented in reference 9. As a result, if the incident solar beam contains one unit of flux, the Stokes parameters of the scattered light seem to be computed from Dave's X- and Y- functions with an error less than 10^{-4}, except that the error may be ten times as large for the model with the optical thickness of two, which was the largest used.

The errors in most of the existing computed X- and Y- functions seem to become larger as the optical thickness increases beyond one. These errors depend on the non-uniqueness of the solutions. A method for computing unique X- and Y- functions have been given by Mulliken.[14] Dr. Diran

Deirmendjian of the Rand Corporation has used Mulliken's method to calculate unique X- and Y- functions for large optical thickness. He is using these data to calculate new tables of the radiation parameters for planetary atmospheres.

3. ALBEDO

Until the publication of Mullamaa's recent book[13] on the reflection of light from wind-roughened sea surfaces, albedo data that were computed on the previous contract could have filled in gaps concerning the knowledge of the albedo of the skylight. Before we had seen Mullamaa's book, which was after the present contract had commenced, we were proceeding on the following bases: The reflection of the skylight from wind-roughened surfaces had been computed for only a few models of the skylight intensity, and these models neglected the polarization of the skylight.[1,5,15] Funk[10] used measured values of the polarization and intensity of the skylight to give one representation of these parameters as a function of the solar zenith angle. He used this model to compute the albedo of the skylight at a smooth water surface as a function of the solar zenith angle. Funk found that the polarization of the skylight had only a small effect on the albedo. Our data show a stronger polarization dependence than found by Funk. Also, only our computations showed the dependence of the albedo on radiation wavelength. Additional computations have been made on the present contract to show the relative contributions of the direct sunlight and of the skylight to the albedo.

However, Mullamaa[13] has published most of the albedo data of the type that we have. He made the more difficult computations for a wind-roughened surface, rather than for a smooth surface as we did. Since Mullamaa did not give albedo data for a smooth surface, his data can not be compared directly with our data.

Mullamaa did not include some investigations that could be made with his data. We shall make these investigations with our data for a smooth surface. They include the question of neglecting the polarization of skylight when computing its albedo, the relative contribution of the skylight and of the direct sunlight to the total albedo, and a comparison of the computed and measured albedos.[1,7,15,16]

4. NEUTRAL POINTS

The computed neutral points at both the bottom and top of the atmosphere disappear from the sun's vertical plane for a limited range of optical thickness and of solar zenith angle for the Fresnel model.[9,18] Only one set of observations gives a clue on the locations of the neutral points when they disappear from the sun's vertical plane. Soret[19] observed that the Babinet and Brewster points -- the two neutral points that are seen near the sun from the ground -- disappeared from the sun's vertical plane when the sun was about 20° above a large lake, presumeably smooth, and a dense haze covered the lake. At the same time a single neutral point appeared on each side of the sun in the solar almucantor and about 20° from the sun. Since Soret's observations were made for a non-Rayleigh atmosphere, no evidence existed at the start of this contract about the neutral point positions in a Rayleigh atmosphere after they had disappeared from the sun's vertical plane. The principle task on this contract was to establish the history of the neutral points when they disappear from the sun's vertical plane.

The simplest method to investigate the neutral point characteristics appears to be by numerical methods, rather than analytically. The degree of polarization, as used in this study, is defined by the formula

$$P = (Q^2 + U^2 + V^2)^{\frac{1}{2}}/I \quad ,$$

where the four Stokes parameters, I, Q, U, V are defined on page 29 of reference 2. The parameter V is identically zero for the Fresnel and Lambert models. Since the degree of polarization was expected to vanish for discrete directions, which in general would differ from the discrete directions for which the data were calculated, it was decided to determine the neutral points from graphical intersections of $Q = 0$ and $U = 0$ lines.

The original computational routine that was made on the previous contract had calculated the Stokes parameters only for the sun's vertical plane. Since the Stokes parameters for the neutral point study had to be computed for azimuths away from the sun's vertical plane, the azimuthal dependence was introduced into the original routine by substituting Eqs. (3.7) and (3.8) into (3.3) and (3.4) of reference 9. Mr. W. H. Walker of the National Center for Atmospheric Research constructed the original routine and also made the new modifications. He also made the computations for the present contract. All of these computations were completed in December, 1965, except for the possibility that new and important features may still appear and require additional computations.

The results of a preliminary analysis of the neutral point data computed for the base of the atmosphere of the Fresnel model will be discussed first. The neutral point positions are a function of the sun's zenith angle. The positions can be discussed with respect to either the rising or the setting sun. The rising sun will be the reference in this discussion. When the sun appears at the horizon, the Babinet neutral point exists above it. As the sun rises the Babinet point moves towards the sun, a Brewster point appears below the sun when the sun's elevation is about 10°, and then the two neutral points approach each other and merge slightly below the sun when the sun's elevation is about 15°. At the same time a neutral point appears on each side of the sun near to, but not on the solar almucantor. The further course of the neutral points depends on the optical thickness. If the optical thickness is small with a value less than 0.1, the neutral points continue to remain outside of the sun's vertical plane and return only when the sun reaches the zenith.

The fact that neutral points appear outside of the sun's vertical plane for small solar zenith angles is unexpected, since Soret did not make such an observation. Also, preliminary data analyses indicate that the neutral point distance from the sun's vertical plane decreases as the total normal optical thickness of the atmosphere increases. When the optical thickness becomes large enough -- between 0.5 and one -- the Babinet and Brewster points do not disappear from the vertical plane of the sun.

The third neutral point that is seen from the ground is the Arago point. It also lies in the sun's vertical plane. When the ground reflects radiation according to approximately the Lambert law, the computed and observed Arago point is roughly 20° above the anti-solar horizon when the solar zenith angle is 90°, approaches the horizon as the solar zenith decreases, and disappears when the solar zenith angle is about 70°. If the ground reflects radiation according to the Fresnel law, the computed Arago point disappears at a larger zenith angle than for the Lambert model. When the Arago point disappears for the Fresnel model, no neutral points replace it.

The three neutral points that one would see when above a planetary atmosphere that is specified by the Lambert model are designated on the accompanying figure. These neutral points lie in the sun's vertical plane. However, when the ground reflects radiation according to the Fresnel law, the neutral points disappear from the sun's vertical plane at the top of the atmosphere for a range of optical thickness and of solar zenith angle that is larger than the range for disappearance at the ground. To show what occurs on top of the atmosphere consider the Fresnel model whose atmospheric optical thickness is 0.25. Assume that the sun rises from the horizon and moves towards the zenith. When the sun rises above the horizon, two neutral points appear in the sun's vertical plane; neutral point

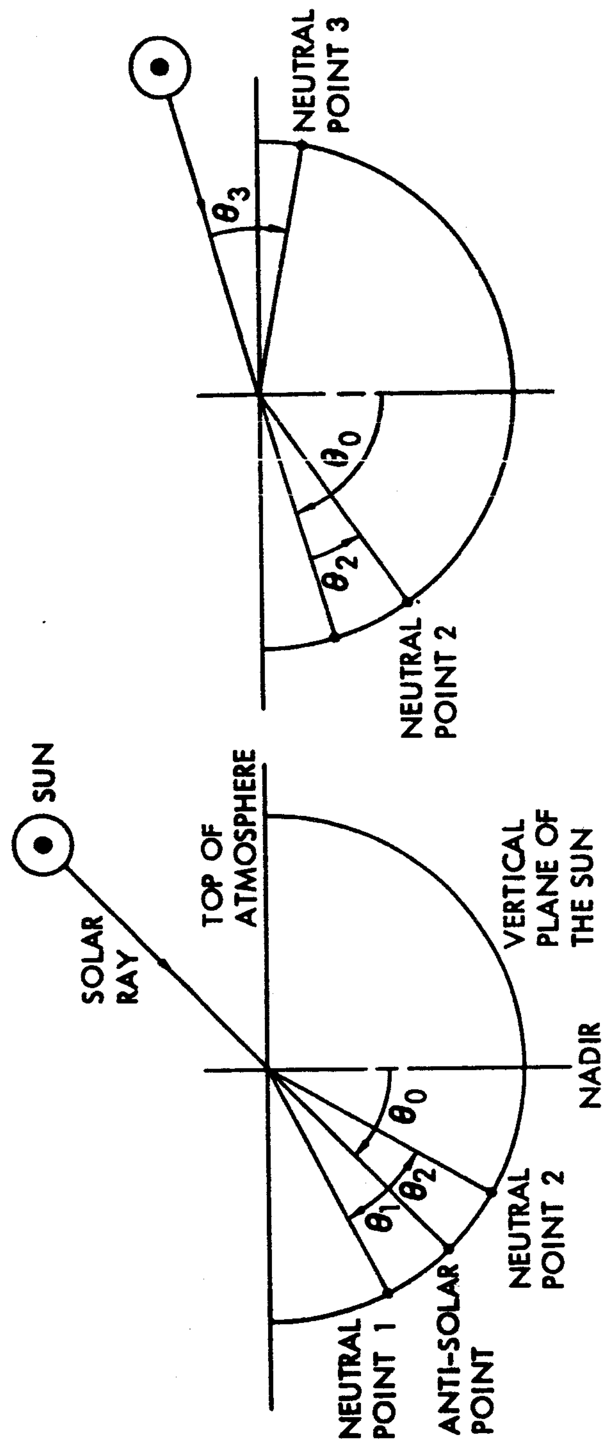

Figure 1. Schematic representation of the neutral points that would appear to an observed above a Lambert model atmosphere. The left drawing shows the two neutral points that would appear when Θ_o is small. The right drawing shows the two neutral points that would appear when Θ_o is large.

three exists about 18^{o} below the sun, and neutral point two exists about 16^{o} below the anti-solar point. As the sun rises, neutral point three approaches the horizon; and neutral point two approaches the anti-solar point. When the sun reaches an elevation of 15^{o}, neutral point three disappears, and neutral point two crosses the sun. Also, neutral point one becomes visible at the horizon above the anti-solar point. As the sun continues to rise neutral points one and two approach each other in the vertical plane of the sun and merge between the anti-solar point and the horizon. At this time the solar elevation is 20^{o}. Next, two neutral points immediately appear -- one on each side of the sun's vertical plane and near the anti-solar point. The azimuth of these two neutral points quickly increases to about 10^{o} from the sun's vertical plane and then changes slowly as the sun continues to rise. The two neutral points move back to the anti-solar point when the sun reaches the zenith. Quantitative data on the neutral point characteristics and related data will be given in the Final Report.

5. ACKNOWLEDGEMENTS

I want to express my gratitude to the National Center for Atmospheric Research (NCAR) for the help that they gave on the computations. Mr. W. H. Walker of NCAR expertly modified the computational routine and made all computations, which were done at NCAR. These services were made without charge to my employer -- TRW Systems.

6. PROGRAM FOR NEXT THREE MONTHS

The computed data will be analyzed for information about the ground albedo and for characteristics of the neutral point positions, when they occur outside of the sun's vertical plane. The Final Report will be written.

7. CONCLUSIONS AND RECOMMENDATIONS

1. Accuracy of the computed Chandrasekhar X- and Y- functions

Dr. Dave of the National Center for Atmospheric Research calculated the X- and Y- functions that were used in the research being reported on here. These functions are sufficiently accurate for all anticipated applications. Accurate X- and Y- functions for large values of optical thickness are now being computed by Dr. Deirmendjian of the Rand Corporation.

2. Albedo

Albedo studies of smooth sea surfaces have little value since publication of Mullamaa's book[13] on the albedo of wind-roughened surfaces. Mullamaa's data on the Stokes parameters of the radiation reflected from the rough sea can be used without unreasonable difficulty in planetary radiation problems instead of the more restricted Stokes parameters of the radiation reflected from smooth water.

3. Neutral points

The present study will give a complete picture of the neutral point behavior for the Fresnel model. These same type of data can be computed for more complex models of ground reflection and of atmospheres. However, such neutral point data for the more complex models should be a by-product of general studies of radiative transfer for the complex models.

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