"Wake of the Moon"

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Abstract

It is shown that the wake of the moon can be described by a closed set of moment equations derived from the Vlasov equations. The problem of the steady wake is solved. By comparison of the results with experiment it is shown that the wake must be unsteady. If the wake is turbulent, it is shown that such a turbulence possesses features not analogous to those which occur in a viscous fluid. By requiring that there be a non-linear cascade from larger scales, it follows that the mean magnetic field is small compared to the fluctuating magnetic field. This result is in agreement with observation.

I. Introduction

Recently Ness[1] has reported magnetic field measurements made on board the Imp I satellite which show a disturbance in the solar wind some 200 moon radii downstream of the moon. At a distance of 200 radii, the wake is about 50 radii wide. The wake was detected by noticing that the satellite abruptly encountered a rapidly varying magnetic field. If the magnetic field were steady (in time), then the length scale over which the magnetic field was observed to vary in space was about one radius. The fluctuations in magnetic field strength were of the same size as the magnetic field in the solar wind before it reaches the earth.

Such fluctuations in magnetic field are also found in the region between the collisionless shock and the magnetosphere. It is commonly thought that they are (1) three dimensional, (2) unsteady, and (3) random. Hence the phrase "turbulent magnetic field" is employed to describe the observations. An important motivation in the present study is to clarify the implications of this idea.

The solar wind, through which the moon moves, has a mean velocity of about 400 km/sec, and a density of 10 protons/cc. The magnetic field associated with the solar wind is about 5 γ, so that the ratio of mean velocity to Alfvén speed is about 10. The Mach number, M, of the protons is about 10, and the Mach number of electrons is about 1/10. The ratio of Debye length to moon radius is 10^{-7}. The ratio of proton Larmor radius, based on mean velocity, to moon radius, is about 5. The mean free path for electron-proton collisions is about 10^5 moon radii.

Previously, Ja. L. Al'pert et al.[2] have analyzed the rarefied gas flow over satellites, and have discussed properties of wakes generated by satellites. However, they discuss problems where the magnetic field is given by its free stream value, which, in view of Ness' results, is a poor assumption. Also they ignore the emission of plasma waves from the satellite, under the assumption that such waves will be strongly damped when their wave length is less than a mean free path. We show here that when the wave length is greater than a proton Larmor radius, the opposite is in fact the case.

In the next section, it is shown that the Vlasov equations are the proper starting point for the discussion. An asymptotic expansion of these equations is given, based upon the size of the moon and properties of the solar wind. This expansion leads to a closed set of moment equations. These equations describe the motion of a compressible, perfectly conducting fluid, under the action of Lorentz forces. This fluid has no pressure in the gas dynamic sense. The relevant expansion of the upstream boundary conditions, so as to include temperature effects, is given.

In the third section, it is shown that if the flow is steady, the resulting wake must be conical flow. In the axisymmetric case, this conical flow is analyzed in detail, and a solution describing it is given. The magnetic field in the wake, described in this way, does not agree with the observations of Ness. Thus the flow must be unsteady if Ness' observations are correct.

The fourth section shows that a three dimensional, unsteady, random flow described by the moment equations derived in section II possesses features having no analogue in the turbulence occurring in a viscous fluid.

II. The Equations of Motion

In the solar wind, the overall Lorentz force acting on the plasma is much greater than the interparticle force. In fact, the ratio is

$$U_\infty B_\infty \left(\frac{4\pi \varepsilon_o 1^2}{a} \right) \sim 10^{+3}$$

Where U_∞ is the mean velocity of the solar wind, B_∞ is the magnetic field in the solar wind, a is electron charge, ε_o is the dielectric constant of free space, and l is the interparticle distance. Collisions can be ignored as the mean free path is much greater than the radius of the moon, L (see Sec. I). Under these assumptions, the plasma equations describe how the Lorentz force is produced by the motion of the plasma, and how the plasma reacts to this force.

Lengths, $\vec{x}$, are nondimensionalized with L, time, t, with L/U_∞, velocity, $\vec{v}$ with U_∞, the magnetic field $\vec{b}$, with B_∞, and the electric field, $\vec{e}$, with $U_\infty B_\infty$. Let f be the proton distribution function, nondimensionalized with U_∞, L, and the proton number density at upstream infinity, n_∞. Then

$$f(t, \vec{x}_o, \vec{v}_o)\, d\vec{v}\, d\vec{x}$$

is the (nondimensional) number of protons in $\vec{x}_o < \vec{x} < \vec{x}_o + d\vec{x}$, $\vec{v}_o < \vec{v} < \vec{v}_o + d\vec{v}$. Likewise, f_e is the electron distribution function.

The motion of the plasma is described by the Vlasov equations (m/m_e is the proton to electron mass ratio $\approx 10^3$):

$$\frac{\partial f_e}{\partial t} + \vec{v}\frac{\partial f_e}{\partial \vec{x}} + \frac{m}{m_e}\,\Gamma\,(\vec{e} + \vec{v} \times \vec{b})\frac{\partial f_e}{\partial \vec{v}} = 0, \qquad (1)$$

$$\frac{\partial f}{\partial t} + \vec{v}\frac{\partial f}{\partial \vec{x}} - \Gamma\,(\vec{e} + \vec{v} \times \vec{b})\frac{\partial f}{\partial v} = 0 \qquad (2)$$

$$\mathrm{curl}\,\vec{b} = \alpha \iiint v(f - f_e)\, d^{(3)} v, \qquad (3)$$

$$\mathrm{div}\,\vec{e} = \frac{\alpha}{\beta} \iiint (f - f_e)\, d^{(3)} v, \qquad (4)$$

$$\frac{\partial \vec{b}}{\partial t} = -\overrightarrow{\mathrm{curl}\, e}, \quad \mathrm{div}\,\vec{b} = 0. \qquad (5)$$

The parameters are (μ_o is the permeability of free space, c the velocity of light)

$$\alpha = \frac{a\, U_\infty\, L\, \mu_o\, n_\infty}{B_\infty} \approx 10^2$$

$$\beta = (U\infty/c)^2 \sim 10^{-5}$$

$$\Gamma = \frac{L\, B_\infty\, a}{U_\infty\, m} \sim 5$$

In equation (3), displacement currents are ignored, bieng of order β.

The upstream boundary conditions, assuming thermodynamic equilibrium, and the mean velocity in the $\vec{1}z$ direction,

$$\left\{ \begin{aligned} f(\infty) &= (\pi\,\gamma\,M^2)^{-3/2} \exp\left[-\frac{\gamma M^2}{2}(\vec{v} - \vec{1}z)^2\right] \\ f_e(\infty) &= \left(\pi\,\gamma\frac{m_e}{m}\,M^2\right)^{-3/2} \exp\left[-\frac{\gamma m_e}{2m}\,M^2\,(\vec{v} - \vec{1}z)^2\right] \end{aligned} \right. \tag{6}$$

For a perfectly absorbing moon, the electrons and ions striking the lunar surface are immediately neutralized. This means that there are no charged particles at the lunar surface.

$$f(\text{surface}) = f_e\,(\text{surface}) = 0 \tag{7}$$

Of course, other, more realistic surface boundary conditions could be specified. But the deductions made below about the properties of the flow are valid for any surface boundary conditions.

Since α is large, and β small, the lowest approximation to the flow has two simple properties. From equation 3, the mean velocities of the protons and electrons are equal. From equation 1, the electron density is equal to the proton density. The electrons are stuck to the protons. The mean velocity and number density describe the mean flow. Thus a single fluid description is appropriate. (This is only true when the length scale is suitably large. On the order of a Debye length, charge separation can occur, and single fluid equations are not appropriate. A similar remark applies to time scales.)

The proton Mach number is large. This imples that an expansion whose lowest term describes a cold plasma is appropriate. The expansion is

$$f \sim f^{(0)} + \frac{1}{\Gamma} f^{(1)} \quad (8)$$

Although equation (8) is of the same form as that given by Rosenbulth and Simon[3], the present results are not equivalent to the finite Larmor radius theory given in [3].

Substituting equation 8 into equation 2, and letting $\Gamma \to \infty$ yields

$$(\vec{e} + \vec{v} \times \vec{b}) \frac{\partial f^{(0)}}{\partial \vec{v}} = 0 \quad (9a)$$

$$\frac{\partial f^{(0)}}{\partial t} + \vec{v} \frac{\partial f^{(0)}}{\partial \vec{x}} - (\vec{e} + \vec{v} + \vec{b}) \frac{\partial f^{(1)}}{\partial \vec{v}} = 0 \quad (9b)$$

The upstream boundary condition can be expanded in a Taylor series about zero temperature. This gives a series of delta functions multiplied by powers of $1/M^2$. The first term in the series is

$$f^{(0)} (\infty) = \delta(\vec{v} - \vec{1}_z) \quad (10)$$

It will be shown that $f^{(0)}$ describes a cold (zero temperature) flow of protons.

The electron Mach number is small. For electrons, a first approximation which gives a cold plasma is incorrect. The first approximation to f_e, $f_e^{(0)}$, must satisfy a more complicated equation than that satisfied by $f^{(0)}$ (eq. 9a). The equation for f_e must contain some convective terms. Further, the upstream boundary condition on f_e is not a solution to the Vlasov equation (1) for $B_\infty \neq 0$), except in an approximate sense. The upstream boundary condition makes sense only when expanded in an asymptotic series of delta functions.

These difficulties are remedied by setting

$$f_e \sim f_e^{(0)} (\vec{x}, \vec{v}^+, t) + \frac{1}{\Gamma} f_e^{(1)} (\vec{x}, \vec{v}^+, t) + \ldots \qquad (11)$$

where $\vec{v}^+ = \frac{m\Gamma}{m_e} \vec{v}$. The equations for $f_e^{(0)}$, $f_e^{(1)}$ are

$$\begin{cases} \vec{v}^+ \cdot \dfrac{\partial f_e^{(0)}}{\partial \vec{x}} + \vec{v}^+ \times \vec{b} \cdot \dfrac{\partial f_e^{(0)}}{\partial \vec{v}^+} = 0 & (12a) \\ \vec{v}^+ \cdot \dfrac{\partial f_e^{(1)}}{\partial \vec{x}} + \vec{v}^+ \times \vec{b} \cdot \dfrac{\partial f_e^{(1)}}{\partial \vec{v}^+} = 0 & (12b) \end{cases}$$

These equations describe how the electrons perform a spiral motion about the field lines. The electron velocity, $O(\frac{L B_\infty a}{m_e})$, is just that required to make the Larmor radius the same order of magnitude as the moon radius.

The upstream boundary conditions may now be expanded in a series of delta function in $\vec{v}^+$, analogous to equation (10).

This gives

$$f_e^{(0)}(\infty) = \delta(\vec{v}^+)$$

$$f_e^{(1)}(\infty) = 0 \qquad (13)$$

We proceed to derive the moment equations associated with equations (1) - (13). First, notice that $f_e^{(1)}$ is zero both on the surface of the moon and at upstream infinity. The equation (12b), for $f_e^{(1)}$ contains no forcing function. Hence

$$f_e^{(1)} = 0$$

From equation (4), as $\alpha/\beta >> 1$, the density of $f^{(1)}$ is the same as that of $f_e^{(1)}$, thus

$$\int\int\int f^{(1)} \, d^{(3)} v = 0 \tag{14}$$

Call the first approximation to mean velocity $\vec{u}$. From (3), as $\alpha >> 1$, the mean velocity of protons is equal to the mean velocity of electrons, to lowest order.

$$\vec{u} = \int\int\int \vec{v} \, f_e^{(0)} \, d^{(3)} v = \int\int\int \vec{v} \, f^{(0)} \, d^{(3)} v.$$

Taking the first two moments of equation (9a) for $f^{(0)}$ gives, using the definition of $\vec{u}$,

$$\vec{e} + \vec{u} \times \vec{b} = 0, \tag{15}$$

$$\int\int\int v_i \, v_j \, f^{(0)} \, d^{(3)} v = n \, (U_i \, U_j), \tag{16}$$

where n is the (non dimensional) number density. Equation (15) is Ohm's law for a perfect conductor. Equation (16) shows that $f^{(0)}$ describes a cold plasma.

Taking moments of equation (9b) gives continuity and momentum equations. The right hand side of the momentum equation has (no pressure) only a $\vec{j} \times \vec{b}$ term, where $\vec{j}$ is given by

$$\vec{j} = \int\int\int v \, f^{(1)} \, d^{(3)} v.$$

Using $f_e^{(1)} = 0$ and equation (3) gives

$$\vec{j} = \frac{\alpha}{\Gamma} \, \overrightarrow{\text{curl b}}. \tag{16}$$

With (15), (16), (17), a closed set of moment equations may be written down. In dimensional form (ρ = proton density, capital letters denote dimensional quantities)

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \rho \vec{U} = 0, \tag{18}$$

$$\rho \frac{\partial \vec{U}}{\partial t} + \rho (\vec{U} \cdot \nabla) \vec{U} = \frac{1}{\mu} \operatorname{curl} \vec{B} \times \vec{B}, \tag{19}$$

$$\frac{\partial \vec{B}}{\partial t} + \overrightarrow{\operatorname{curl}} (\vec{U} \times \vec{B}) = 0; \ \operatorname{Div} \vec{B} = 0 \tag{20}$$

The absence of pressure in (19) is due to the fact that the protons control the motion, and to first approximation, the protons have zero temperature, and also to the fact that there are no close collisions in the plasma.

III. The Steady Wake

Consider the steady wake of a body moving in a fluid described by equations (18), (19), (20). Assume at upstream infinity that B_∞ is parallel to U_∞. The body is axisymmetric. Under these circumstances, the wake is axisymmetric. For the wake of the moon, the flow is hyper alfvénic: the ratio of free stream velocity to Alfvén speed is about 10.

Far downstream, the size of the body is of no importance. In a steady flow, equations 18, 19, 20 have no length scale. Hence the flow far downstream is conical. For the axisymmetric case, let z be in the direction of the free stream, and r the cylinder radius. As the flow is conical, there is a similarity solution for equations (18), (19), (20). The similarity variable is

$$\eta = r/z. \tag{21}$$

The flow properties are constant on rays drawn from the body (located at $r = z = 0$).

Letting $(\ )_\infty$ denote upstream conditions, the flow quantities in the wake behave in the following way:

$$\begin{cases} \rho = \rho_\infty \ \sigma(\eta) \ , \\ \vec{U} = U_\infty \ [\ u(\eta),\ v(\eta)\ ] \ , \\ \vec{B} = B_\infty \ [\ b(\eta),\ c(\eta)\ ] \ . \end{cases} \tag{22}$$

Here u, b are the components of the velocity and magnetic field in the z direction; v, c are the components in the radial direction.

Upon substituting (22) into equations (18)-(20), the continuity (equation 18) and induction (equation 20) may be integrated as

$$\begin{cases} \sigma \ , = \eta\sigma \ v + g_1, \\ b = \eta c + g_2, \\ vb - uc = g_3, \end{cases} \tag{23}$$

where g_i denotes a constant of integration. The momentum equations may be written

$$\begin{cases} \dfrac{dv}{du} = -\dfrac{b}{c} \ , \\ g_1 \ S^2 \ \dfrac{du}{d\eta} + c\ (\eta \ \dfrac{db}{d\eta} + \dfrac{dc}{d\eta}\ ) = 0 \end{cases} \tag{24}$$

where S is the ratio of free stream velocity to the free stream Alfvén speed,

$$S = \frac{U_\infty \sqrt{\mu_0 \rho_\infty}}{B_\infty} \ .$$

The boundary conditions on the axis of symmetry are

$$v(0) = c(0) = 0. \tag{25}$$

The jump conditions across the collisionless shock may be written

$$\begin{cases} [\vec{b}]_n = 0 \\ [\sigma \vec{U}]_n = 0 \\ [\vec{u} \times \vec{b}]_t = 0 \\ [S^2 \sigma \vec{u}\,\vec{u} - (-I\frac{b^2}{2} + \vec{b}\,\vec{b}]_n = 0 \end{cases} \tag{26}$$

where $[\vec{b}]_n$ denotes the jump in normal component of $\vec{b}$ across the shock, t denotes tangential components. In the last equation of (26), I is the unit matrix.

For a shock inclined at an angle $\tan\theta = \eta_o$, the first three equations of (26) can be shown to imply $g_1 = 1$, $g_2 = 1$, and $g_3 = 0$.

The last equation states that, across the shock, the jump in momentum flux is balanced by the jump in magnetic stress. As S is large, an expansion in terms of $S \to \infty$ is appropriate. Now, if the shock were weak, i.e. if S were near one, then, from linearized theory, we expect that θ would be of order $1/S^2 - 1$. In the present case, S is large, and the strong shock has a deflection angle greater than that given by linearized theory.

$$\theta = \eta_o \sim \delta/S \tag{27}$$

Associated with this deflection, the other flow quantities behave as

$$\begin{cases} u \sim 1 + \dfrac{u^+(\eta^+)}{S^4} \quad \ldots, \\ v \sim v^+(\eta^+)/S, \\ b \sim 1 + 0(\dfrac{1}{S^4}), \\ c \sim c^+(\eta^+)/S, \\ \sigma \sim \sigma^+(\eta^+), \text{ where } \eta = \eta^+/S \;. \end{cases} \tag{28}$$

Substitution of this expansion into the last equation of (26) gives

$$\begin{cases} -\delta = -\delta\sigma^{+} + \sigma^{+}v^{+} \\ 0 = -\sigma^{+}v^{+}\delta + \sigma^{+}v^{+2} + 1/2 \end{cases} \qquad (29)$$

The first equation in (29) assures the balance between the magnetic stress and the momentum flux in the z direction; the second, the balance in the r direction. These equations yield

$$\begin{cases} v^{+} = \dfrac{1}{2\delta^2} \\ \sigma^{+} = \dfrac{2\delta^2}{2\delta^2 - 1} \end{cases} \qquad (30)$$

~~The solution may~~ be gotten as follows. Since $g_3 = 0$, the first equation of (24) ~~may be~~ integrated using the last equation of (23), to give

$$u^2 + v^2 = 1 + O\left(\frac{1}{S^4}\right) \qquad (31)$$

Here, the constant of integration is evaluated at the shock, $\eta^{+} = \delta$, using the expansion (28).

Using the expansion (28) for all $0 < \eta^{+} < \delta$ reduces the second equation of (24) to

$$\frac{du^{+}}{d\eta^{+}} = \frac{-c^{+}\, dc^{+}}{d\eta^{+}} ,$$

which gives

$$u^{+} = \frac{-(c^{+})^2}{2} \qquad (32)$$

the constant of integration being evaluated on $\eta^{+} = 0$, (equation 25). Equations (31), (32), and (23) are the solution.

Using (30) and (23), the value of δ is determined to be

$$\delta = \frac{1}{\sqrt{2}} \quad . \tag{33}$$

Using (31), (32) and (23), the variation of c^+, u^+ and v^+ with η^+ is found to be

$$\left\{ \begin{aligned} u^+ &= -2(\eta^+)^2 \\ v^+ &= -2\eta^+ \\ c^+ &= -2\eta^+ \end{aligned} \right. \tag{34}$$

A sketch of this simple solution is shown in Figure 1. The linear variation of the magnetic field with η^+, and the small size of the change in magnetic field across the wake $0(\frac{1}{S})$ both contradict the observations of Ness.

IV. Comparison with Turbulence

In this section, we show that a three dimensional random flow described by equation (18)-(20) possesses features having no analog in the turbulence occurring in a viscous fluid. For purposes of comparison we shall use the hypersonic wake.[4] At the outset, it should be noted that equations (18)-(20) have no dissipation terms. Such terms undoubtedly come into a higher order approximation of the plasma equations. But the next higher approximation to (18)-(20) requires detailed knowledge of $f_e^{(0)}$, and the calculation of $f_e^{(0)}$ appears quite difficult.

In view of this difficulty, we compare equations (18)-(20) to the inviscid (Euler) equations. Consider for comparison an eddy in the hypersonic wake to be a lump of fluid surrounded by thin shear layers. It is known[4] that the density changes in this eddy are negligible. Thus, the flow inside the eddy may be considered incompressible, to first approximation. The dissipation occurs in the shear layers which enclose the eddy. Hence the flow inside is irrotational. There is a velocity potential ϕ satisfying Laplace's equation. The internal flow is specified by giving either the normal or tangential velocity on the boundary of the eddy. The fundamental feature of this description is that any size or shape of eddy is allowed, as the solution ot the boundary value problem posed above always exists and is unique.

In a similar vein, we ask if such latitude is available in equations (18)-(20). As, to some order, there is dissipation associated with rotational magnetic or velocity fields, the question amounts to examining the class of irrotational solutions permitted by (18-20). It is easy to see that the system of equations which are satisfied by the velocity potential ϕ , and the magnetic potential $\psi(\vec{B} = \vec{\nabla}\psi)$, are redundant. In general there are no solutions to an eddy problem posed in the same way as in the hypersonic wake.

The reason is that wave propagation, on the scale of an eddy, must be dominant in flows governed by (18-20). For example, using Ness' data for magnetic fields, and estimating that the fluctuating velocity field is 10% of the mean velocity, we find that the magnetic energy in an eddy is roughly the same as kinetic energy. From this, it follows that Alfvén waves may not be ignored in the dynamics of an eddy (in contrast to the hypersonic case where acoustic waves were unimportant inside an eddy). The reason that no nontrivial irrotational motions exist inside a closed eddy is that the equations are hyperbolic. Solutions exist only for problems posed on an initial line, not a closed surface. These solutions are, in general, rotational.

Thus the concept of vortex stretching in equations (18-20) is also significantly different from that in a viscous fluid. This is most easily seen in the case where the density fluctuations are small. In that case, set $\vec{P} = \vec{U} + \vec{B} / \sqrt{u\rho}$, $\vec{Q} = \vec{U} - \vec{B}/\sqrt{u\rho}$. Then if

$\vec{r} = \text{curl}\,\vec{P}$, $\vec{s} = \overrightarrow{\text{curl}\,Q}$,

an equation for the evolution of r and s can be written as[5]

$$\left[\frac{\partial r}{\partial t} + \text{curl } r \times Q\right] + \left[\frac{\partial s}{\partial t} + \text{curl } s \times P = 0\right] \quad (35)$$

Assuming the r is almost always zero in regions where s is non-zero leads to the idea that r is stretched by Q and s by P.

Consider a coordinate system moving with the mean velocity, U. Assume that the mean value of the magnetic field is non-zero. If, at t = 0, a disturbance creates r and s, according to (35), the r and s filaments will tend to separate with a mean velocity $\vec{\bar{B}} / \sqrt{\overline{\mu \vec{\rho}}}$ (bar denotes mean value). If this happens, the mechanism for stretching s by P becomes linear, for, if r and s are separated by large distance, the irregularities in P caused by variations in r will not be felt by the s filaments, and vice versa. Thus, if the mean magnetic field is non-zero in the wake, the random rotational elements in the flow separate, and, after a long time, are convected with the mean magnetic field. In this circumstance, a cascade can proceed only through stretching by mean field gradients. From section III, we anticipated that mean field gradients in the wave may be small.

There is every reason to believe that a cascade does exist. The above is an argument that the mean magnetic field in the turbulent wake of the moon should be near zero: let $B = \bar{B} + B'$, where B' is the fluctuating part of B. Then $\sqrt{\overline{(B')^2}} / \bar{B} << 1$, so that the Alfvén waves are propagated mainly by the instantaneous magnetic field rather than its mean value. This is in rough agreement with Ness' results.

Because the vortex stretching mechanism is very different from the classical case, and because eddies cannot exist with the same form that they have in the classical case, one should expect the random three dimensional unsteady flow governed by (18-20) possess features not analogous to turbulence as it occurs in a viscous fluid.

Finally, it seems worthwhile to point out that Cohen[6] has shown that the solar wind has fluctuations on the scale of 800 km, which is roughly comparable with the scales of motion seen by Ness. If most stars have solar winds, and if most solar winds are turbulent, and if the speculations of this section apply, about half the matter in the universe is in this sort of turbulent motion.

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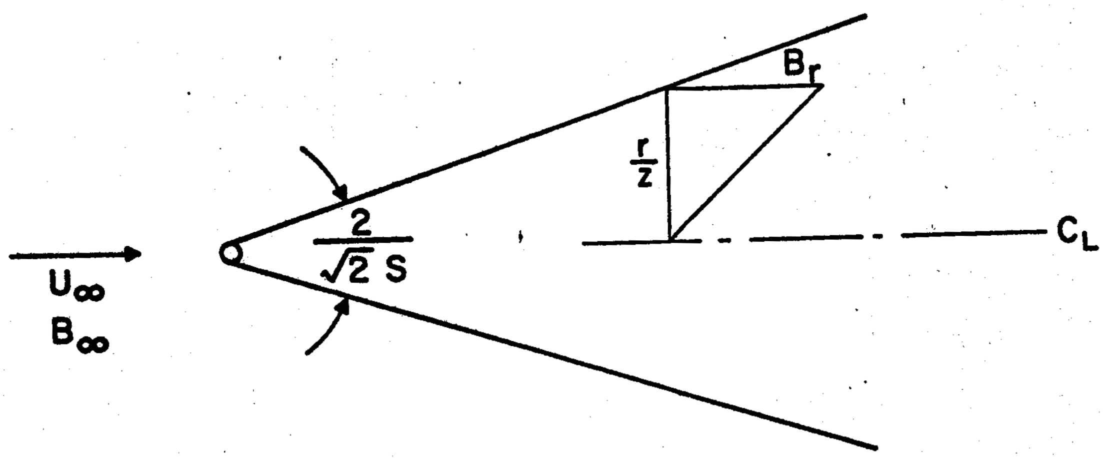

$$B_r = -\frac{2rB_\infty}{z} = \frac{B_\infty c^+(\eta^+)}{S}$$

FOR LARGE z, THE GRADIENT OF THE RADIAL FIELD IS SMALL, $O(\frac{1}{z})$

FIG. I. STEADY WAKE OF MOON (EQ. 34)