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FACILITY FORM 602

N66 21651 (ACCESSION NUMBER)

31 (PAGES)

CR-71405 (NASA CR OR TMX OR AD NUMBER)

(THRU)

1 (CODE)

07 (CATEGORY)

GPO PRICE $

CFSTI PRICE(S) $

Hard copy (HC) 2.00

Microfiche (MF) .50

ff 653 July 65

CRES

THE UNIVERSITY OF KANSAS • CENTER FOR RESEARCH INC
ENGINEERING SCIENCE DIVISION • LAWRENCE, KANSAS

Technical Report
61-1

Fresnel Zone Processing of Synthetic Aperture Radar Data

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January 3, 1966

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Theory of Synthetic Aperture

The simplest method of describing a synthetic aperture radar is by comparing it with a conventional radar. In a conventional system, all the elements of an array transmit in phase. The signals received at each element will differ in phase, this difference depending mainly on the angle of the target with respect to the array. If the signals from each element are added as they are received, their sum will go through maximums and minimums as the angle to the target changes. This is the familiar pattern of major and minor lobes of conventional antennas.

There is no reason, other than simplicity, why the signals received at the different array elements must be summed as they reach the array. If the signals received by each element could be stored individually, they could be recombined later. The proper phase corrections could then be made to recombine the signals as if they were all received in phase. The amplitude of the phase-corrected signal would be much more nearly the signal desired. Also, there is no good reason why all the elements of the array must transmit simultaneously.

In the synthetic system, signals are transmitted and received in sequence. The physical antenna used becomes a new element of the synthetic array with each transmission. The signals received are then stored in such a manner that their phases and amplitudes are preserved. It becomes obvious that using only these techniques, a very long synthetic antenna can be realized with a short physical antenna. The result is a narrow beam which is nearly equivalent to that obtainable with a large, linear array.

This narrow beam, although a vast improvement over conventional techniques, still spreads with range, and therefore the minimum azimuthal resolution at long range with such a system would be substantially larger than the resolution obtainable at short range. However, consider the geometry in Figure 1.

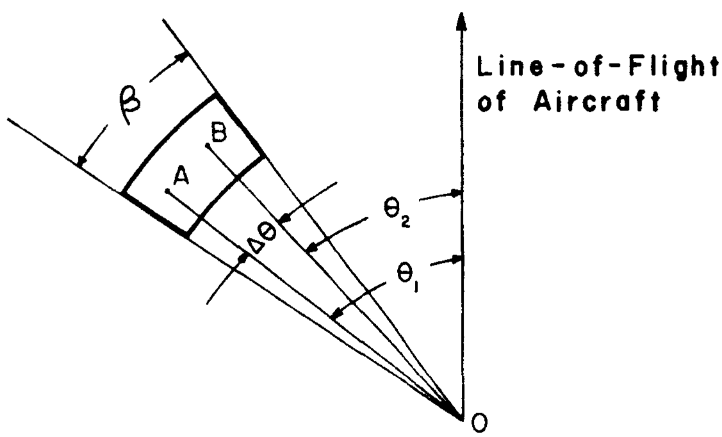


Figure 1

The illuminated area shown is a function of the range R, the beam width β, and the pulse length. If the two points A and B in this area reflected the transmitted signal equally well, it would be impossible to distinguish between these points using either a conventional radar system or the synthetic system as previously described. However, the signal reflected from point A will experience a doppler shift f_d given by the relationship

$$f_d = \left(f_t \frac{c+v}{c-v} - f_t\right) \cos \theta_1$$

where f_t = transmitted frequency, c = the speed of propagation, v = velocity of the plane, and Θ_1 is the angle the plane motion makes with the radial line to the target. Since the plane velocity is much less than c, this relationship becomes

$$f_d = \left(\frac{2v}{c}\right) f_t \cos \theta_1$$

The signal from the point B will also have a doppler shift, but the doppler frequency of this signal will be determined by the angle Θ_2. The two doppler frequencies will differ by

$$\Delta f_d = \left(\frac{2v}{c}\right) f_t \left(\cos \theta_2 - \cos \theta_1\right) \simeq \left(\frac{2v}{c}\right) f_t \sin \theta \, \Delta\theta$$

This difference in doppler frequency at a given range allows the two points A and B to be distinguished. It is interesting to note that if the beam is directed at an angle $\Theta = 90^\circ$ with respect to the plane, $\sin\Theta$ will be unity and a signal received from the center of the beam will be at zero doppler. By utilizing both doppler information and phase considerations, it is possible to design a system capable of greatly improved resolution. As it turns out, the resolution attainable with such a system is dependent only on the size of the physical antenna, and is independent of both range and wavelength. For the focused synthetic aperture the minimum resolvable area is given by

$$\text{Resolution} = \frac{D}{2}$$

where D is the length of the physical aperture.

Method of Obtaining and Storing Data

Consider the following figure. A plane with side-looking radar with beam width β is flying along the line shown at a perpendicular range R to the target T. If a pulse is transmitted and received at each of the points A through G, information about the target T will only appear in the signals received at the points B, C, D, E, and F, times when the target is in the beam width of the radar. If for each pulse, the amplitude information at the carrier frequency is removed from the received signal, the resulting signal will be one form of the return information as a funotion of range. Signals returned from the particular range R, as used in Figure 2, will contain information about the target T if the plane is at either B, C, D, E, or F. However, the target T will be at a different angular position with respect to the plane at each of these points. The doppler information in the received signal from this target allows the target to be distinguished

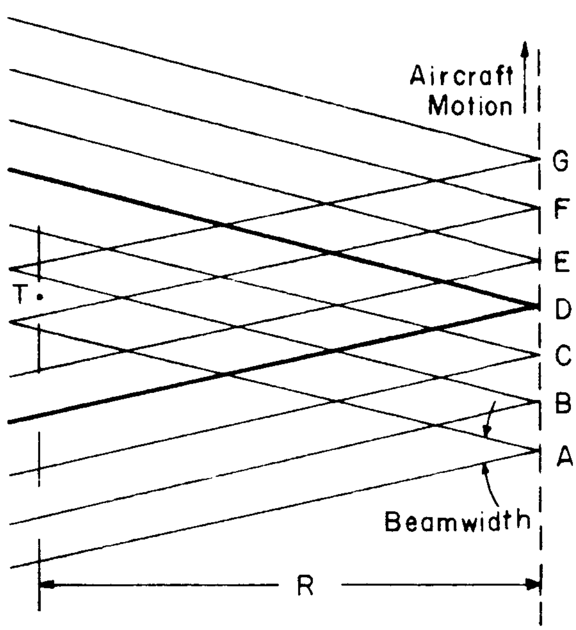
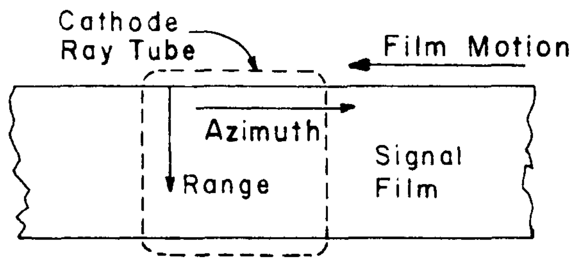


Figure 2

from adjacent targets, which enter and leave the beam at different times although at the same range.

It becomes obvious that a great deal of information must be stored and processed before a radar map can be constructed. One method of information storage presently used is signal film. Figure 3 shows the basic idea of data storage on signal film.



The film is swept past a cathode ray tube in the direction shown. For each received pulse, the return signal, modified as previously stated, is used as intensity modulation on the cathode ray tube. The vertical deflection on the tube corresponds to range to the target. Since the signal used for intensity modulation can be either positive or negative, a gray level is chosen to represent a zero signal. As was stated, signal film is simply a method of storing the received data, so no phase corrections are made on the signal film. It is the purpose of this paper to outline a method of converting the data stored on signal film to a radar map. To fully understand the problem, we need to determine: (1) just how information about a point on the ground is converted to information on signal film, and (2) how this information is removed from the signal film and made to represent a point on a radar map. Consider the following figures.

Ground — Signal Film — Radar Map

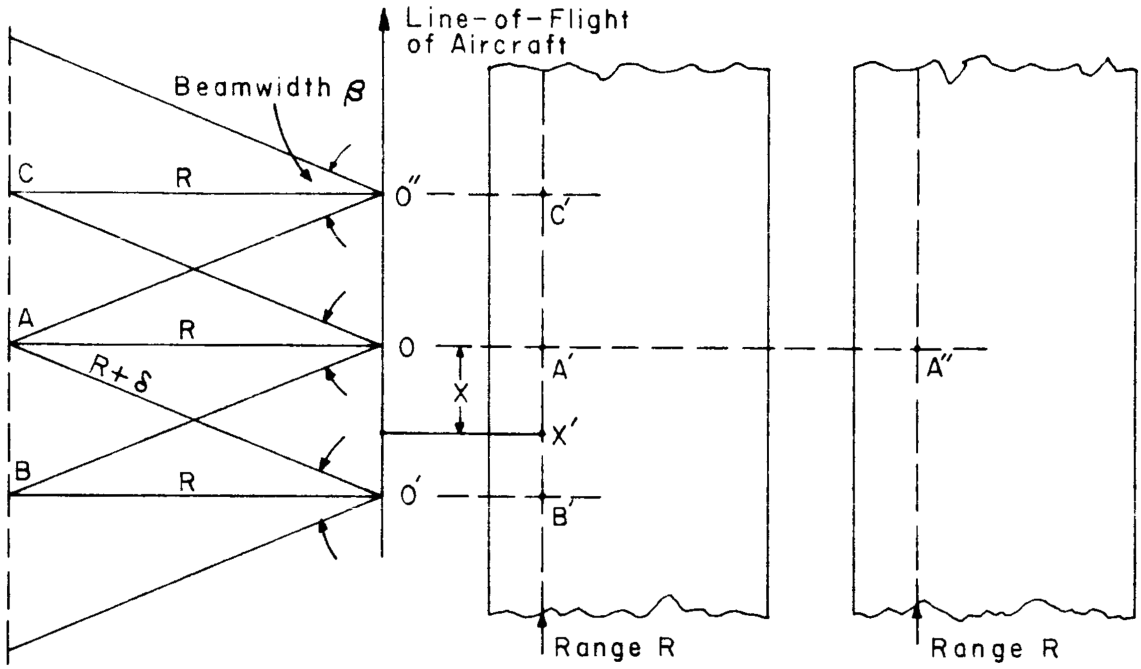


Figure 4 — Figure 5 — Figure 6

Figure 4 shows the plane and the targets C, A, and B on the ground. The plane first transmits and receives at O'. As was stated, the demodulated return signal is used as intensity modulation on the signal film along the horizontal line through B' (on the film), as shown in Figure 5. As the plane moves to O and O" and again transmits and receives, the horizontal lines through A' and C' on the signal film are recorded. The target A is seen to be in the beamwidth of the radar at each of the points O', O, and O" and hence has contributed to the returned signal at each of these points. Therefore, in order to use all the information available concerning the point A on the ground in forming the point A" on a radar map (Figure 6) one must analyze the information at the points C', A', and B' on the signal film. Information is therefore read off the signal film in a direction perpendicular to the direction in which information is read onto the film. Although the points A, B, and C are all the same perpendicular distance R from the line of flight of the plane, the distances $\overline{OA}$ and $\overline{O'A}$ are not equal. In other words, the signal reflected from target A when the plane is at O' will necessarily travel a distance 2δ farther than the signal reflected from A when the plane is at O. Since we are trying to form the point A'' on the radar map, we are primarily interested in correcting the phase of those parts of the signals at B' and C' which were contributed by target A. From the geometry of Figure 4 we have:

$$\overline{OA}^2 + \overline{OO'}^2 = \overline{O'A}^2$$

or,

$$R^2 + \overline{OO'}^2 = (R+\delta)^2 = R^2 + 2R\delta + \delta^2$$

If the distance $\overline{OO'}$ is much less than R, we can write

$$\delta = \frac{\overline{OO'}^2}{2R}$$

A's contribution to the signal received at O' must therefore travel a distance of $\frac{\overline{OO'}^2}{R}$ farther than A's contribution to the signal received at O. In terms of phase, this distance corresponds to

$$\gamma = \frac{2\pi}{\lambda}(2\delta) = \frac{2\pi\overline{OO'}^2}{R\lambda} \text{ radians.}$$

In actuality, many pulses would be transmitted as the plane traveled from O' to O and to O". If we denote by X the location of the plane at transmission with respect to O, the amount of phase change necessary to make A's contribution to the signal returned at X in phase with A's contribution to the signal returned at O would be

$$\gamma = \frac{2 \pi X^2}{R \lambda} \text{ radians.}$$

The distance between the points O' and O" corresponds to the length of the synthetic aperture at the range R. The necessary length is simply the width of the beam at this range, or

$$L_{eff} = R \beta$$

The beamwidth β for a physical array is given by

$$\beta = \frac{\lambda}{D}$$

where λ is the transmitted wavelength, and D is the length of the physical aperture. Therefore, the effective length of a synthetic aperture is given by

$$L_{eff} = \frac{R \lambda}{D}$$

Since a target can contribute to a return any time it is in the beamwidth, the maximum amount of phase change necessary will be

$$\gamma_{max} = \frac{2\pi}{R\lambda}\left(\frac{L_{eff}}{2}\right)^2 = \frac{2\pi}{R\lambda}\left(\frac{R^2\lambda^2}{4D^2}\right) \text{ radians}$$

or,

$$\gamma_{max} = \frac{\pi R\lambda}{2D^2} \text{ radians,}$$

which may be many multiples of $\frac{\pi}{2}$.

The ideal signal film processor would scan the line C'B' in Figure 5, shifting the phase of each signal appropriately, and would sum the phase-shifted signals to give the point A" on the radar map. This would bring all the returned signals resulting from target A into phase, while signals resulting from all other targets would undergo phase cancellation.

Although processors of this nature are available, the purpose of this paper is to describe a processor which would be both small and inexpensive. This processor, however, would not be capable of utilizing the information of the signal film as effectively as the ideal processor, and azimuthal resolution, as would be expected, is somewhat degraded.

<u>Theory of Processing Unit</u>

In terms of signal film, the unfocused synthetic aperture system would simply sum the signals read from the line $\overline{C'B'}$ in Figure 5 to form the point A" on the radar map. The unfocused and the focused system are two extremes of the method of processing.

In the unfocused system, the number of intervals is made very small (namely one), and an average is taken over that interval. It is the premise of this paper that one can choose a discrete number of intervals, sum the signals in each interval, shift the phase of these summed signals by an amount appropriate for the particular interval, and sum these signals to form a point on a radar map. If signals along the line $\overline{C'B'}$ were divided into intervals of a length which corresponded to a phase shift of one degree, and these intervals were processed, the resolution obtainable on a radar map would hardly differ at all from that obtainable with the continuous phase shift available with an ideal processor. As the number of intervals is decreased, however, the minimum azimuthal resolution must necessarily become larger. It is convenient to think of each point on the signal film as the sum of a number of vectors, each vector representing the signal returned from a target during a particular pulse at a particular range. In forming a point on a radar map we are only interested in one of these vectors for each point on the signal film. The question is, just how much do we degrade azimuthal resolution by adding these vectors when they are not in phase with each other? The addition of two vectors more than 90° out of phase can result in an amplitude less than the largest vector. In this paper, we shall divide the vectors into two groups. The first froup will contain those vectors that differ in phase from the vector at A' by no more than $\pi/2 \pm 2n\pi$ radians. The other group, naturally, will contain all other vectors.

As has been shown, the phase of the signal from some point X', a distance X from the point A' in Figure 5, differs from the phase of the signal at A' by an amount

$$\gamma = \frac{2\pi X^2}{R\lambda} \quad \text{radians}$$

The boundaries of the intervals desired will be determined by

$$\frac{(2N+1)}{2}\left(\pi\right) = \frac{2\pi X^2}{R\lambda} \qquad N = 0,\ 1,\ 2,\ \ldots.$$

or,

$$X = \pm\sqrt{\frac{(2N+1)\,R\lambda}{4}} \qquad N = 0,\ 1,\ 2,\ \ldots.$$

at a particular range R, and wavelength λ, the boundaries of the intervals are located at a distance

$$X = \pm\, C\sqrt{(2N+1)} \qquad C = \text{constant}$$

from the central point, A'. The number of the interval is N. However, since the interval boundaries are a function of range, these boundaries will not be horizontal lines on the signal film shown in Figure 5, but will diverge with range. These zones are shown on a section of signal film in Figure 7. These zones can be likened to Fresnel zones, where the phases of signals reflected from targets represented by the zone boundaries differ by π radians. To form a point on a radar map at range R1, as shown in Figure 7, it would be necessary to scan the line $\overline{DE}$, shifting the phase of signals from points lying in the odd-numbered zones by π radians and adding these signals to the signals from points lying in the

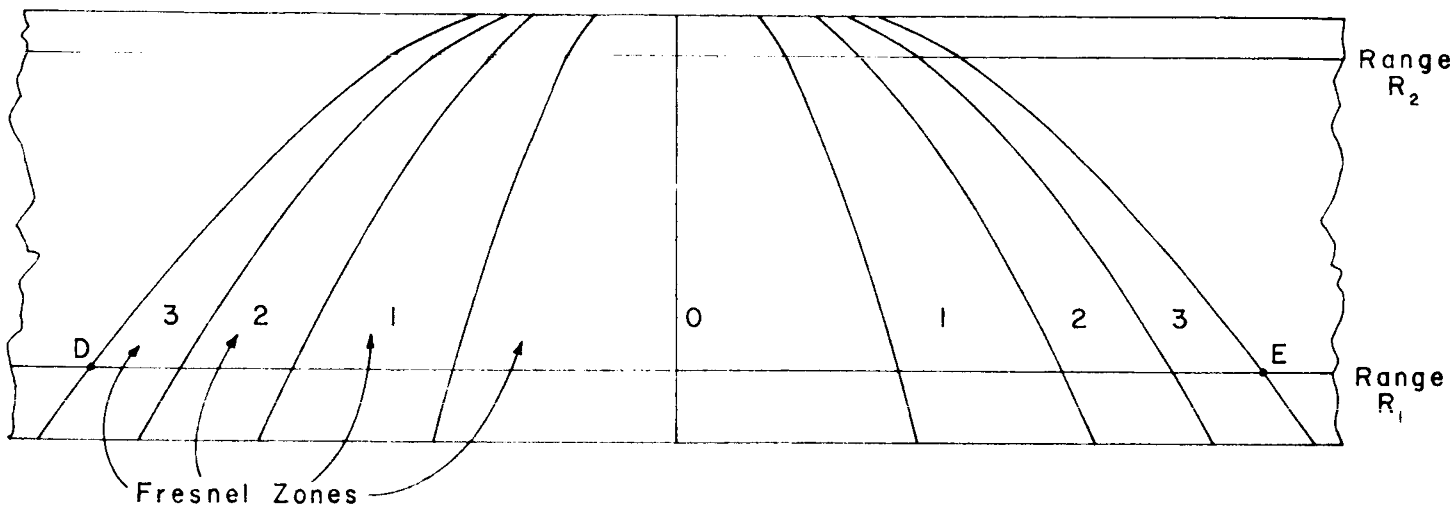


Figure 7

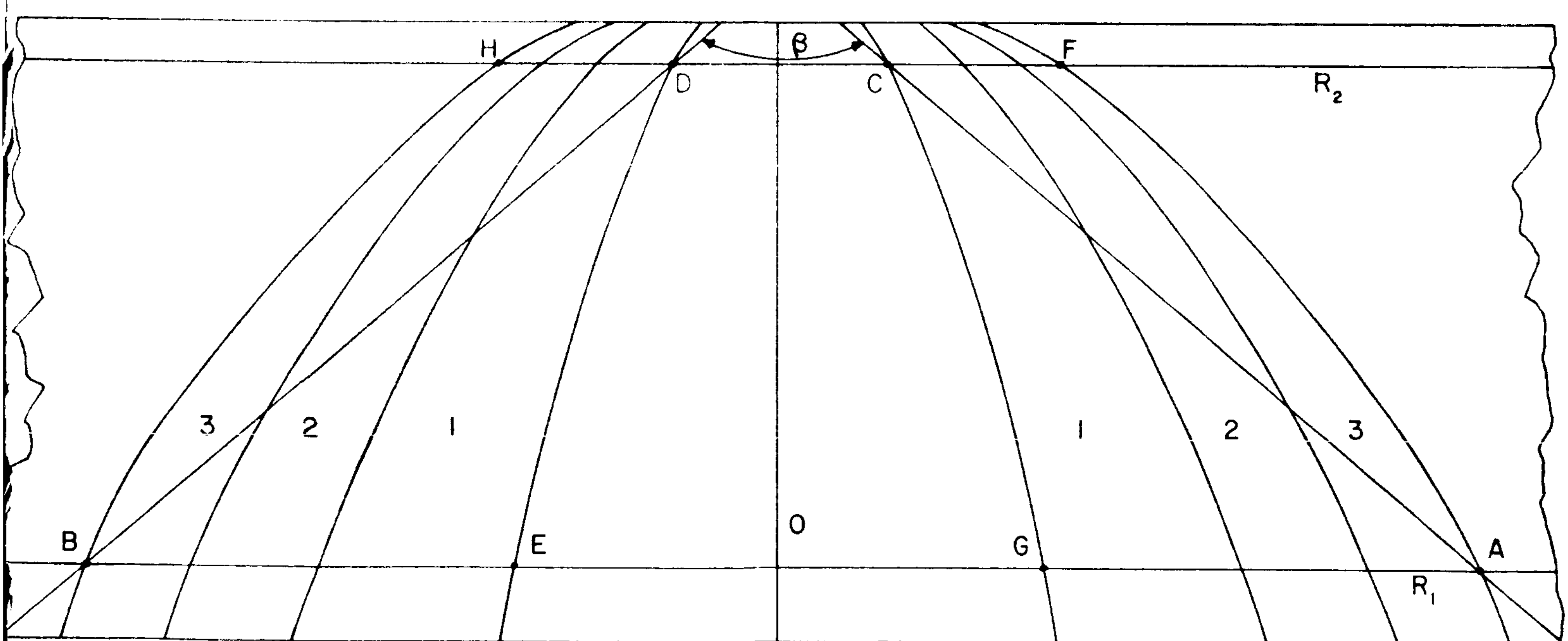


Figure 8

even numbered zones. The length of the line $\overline{DE}$, of course, is the signal film equivalent of the beamwidth at range R_1, or the effective length of the synthetic aperture at this range. Although the zones look as shown in Figure 7, with the boundaries varying as the square root of range, the effective length that must be scanned decreases more rapidly than the zone boundaries as range is decreased. This is shown in Figure 8. The lines $\overline{BD}$ and $\overline{AC}$ represent the signal film equivalent of the beamwidth, and the lines $\overline{AB}$ and $\overline{DC}$ represent the lengths of film that must be scanned to form points on a radar map.

Although there appear to be a number of ways to scan the film and introduce the proper phase-shifts at the proper times, a simple solution would be to make constants (independent of range) of the times when phase is shifted. This can be done if the maximum deflection voltage on the flying-spot scanner that is used to read the film varies as the zone boundaries vary with range. In Figure 8, if we consider R_1 the maximum range and R_2 the minimum range, the maximum deflection voltage would be that voltage necessary to produce a deflection that followed the boundary of the Fresnel zone passing through the points A and B. If the film is scanned from top to bottom in time as shown in Figure 8, the absolute value of the maximum horizontal deflection voltage would appear as shown in Figure 9. This voltage increases as the square root of (time + constant) over one period, and then falls back to its initial value and begins again. Each time interval, t, corresponds to the time required for the scanner to sweep vertically from range R_2 to range R_1.

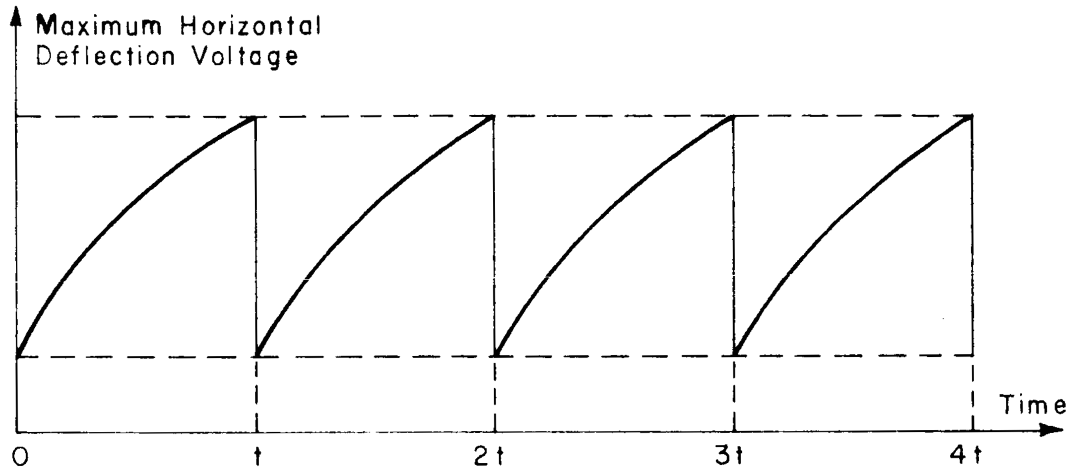


Figure 9

The actual horizontal deflection voltage waveform would appear as shown in Figure 10.

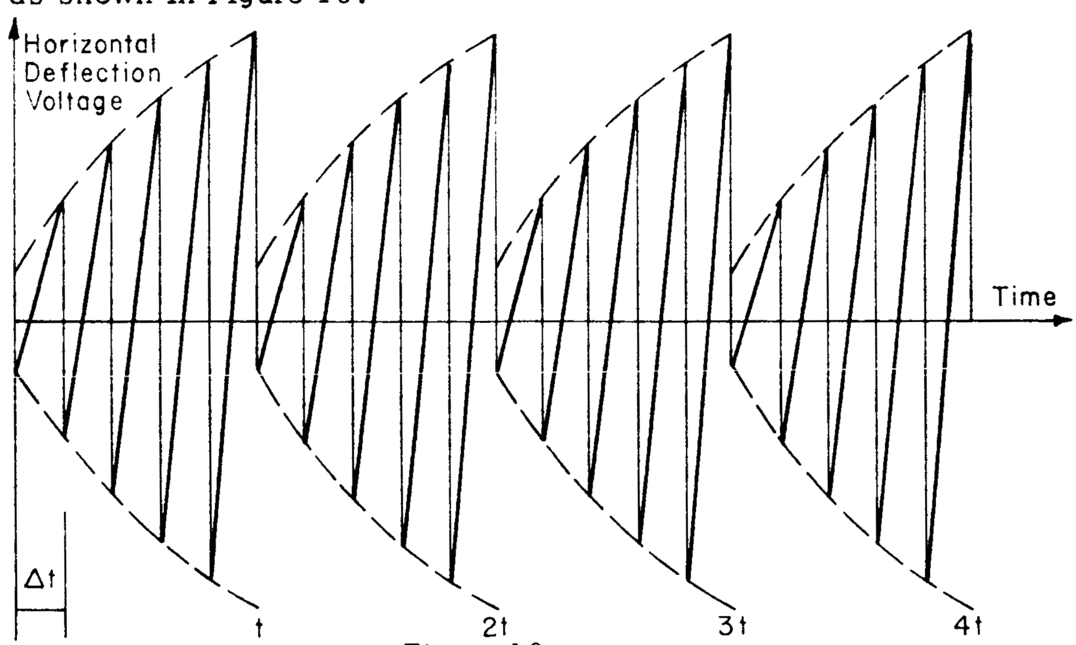


Figure 10

The number of voltage "oscillations" in each envelope would be simply the number of horizontal lines scanned on the film. In the example of Figure 8, there are three zones on each side of the central zone "0".

Using this example, phase-switching would occur in time as is shown in Figure 11.

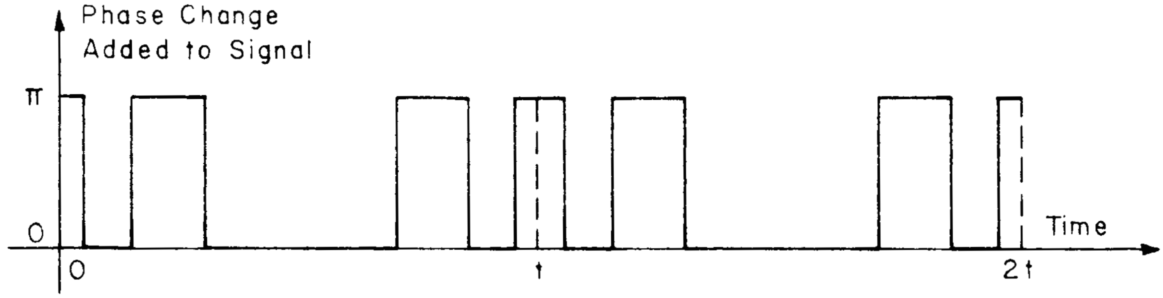


Figure 11

Since the maximum deflection follows a zone boundary with range, the points of phase-switching will also fall at zone boundaries. In a geometric sense, this is explained by the relationship

$$\frac{\overline{BA}}{\overline{EG}} = \frac{\overline{HF}}{\overline{DC}} \qquad \text{(refer to Figure 8)}$$

Since the area to be scanned does not follow a zone boundary, it will be necessary to mask the film to exclude areas that do not fall in the signal film equivalent of the beamwidth. The horizontal deflection voltage will follow the boundary of the largest zone that appears in the sweep, the zone in which point B in Figure 8 falls. At shorter ranges, the first and last parts of each sweep line will fall behind the mask. The mask, chosen to give zero signal output, will not affect the results. The associated circuitry necessary to build a processor of this type could be built in several ways. One possibility is shown in Figure 12.

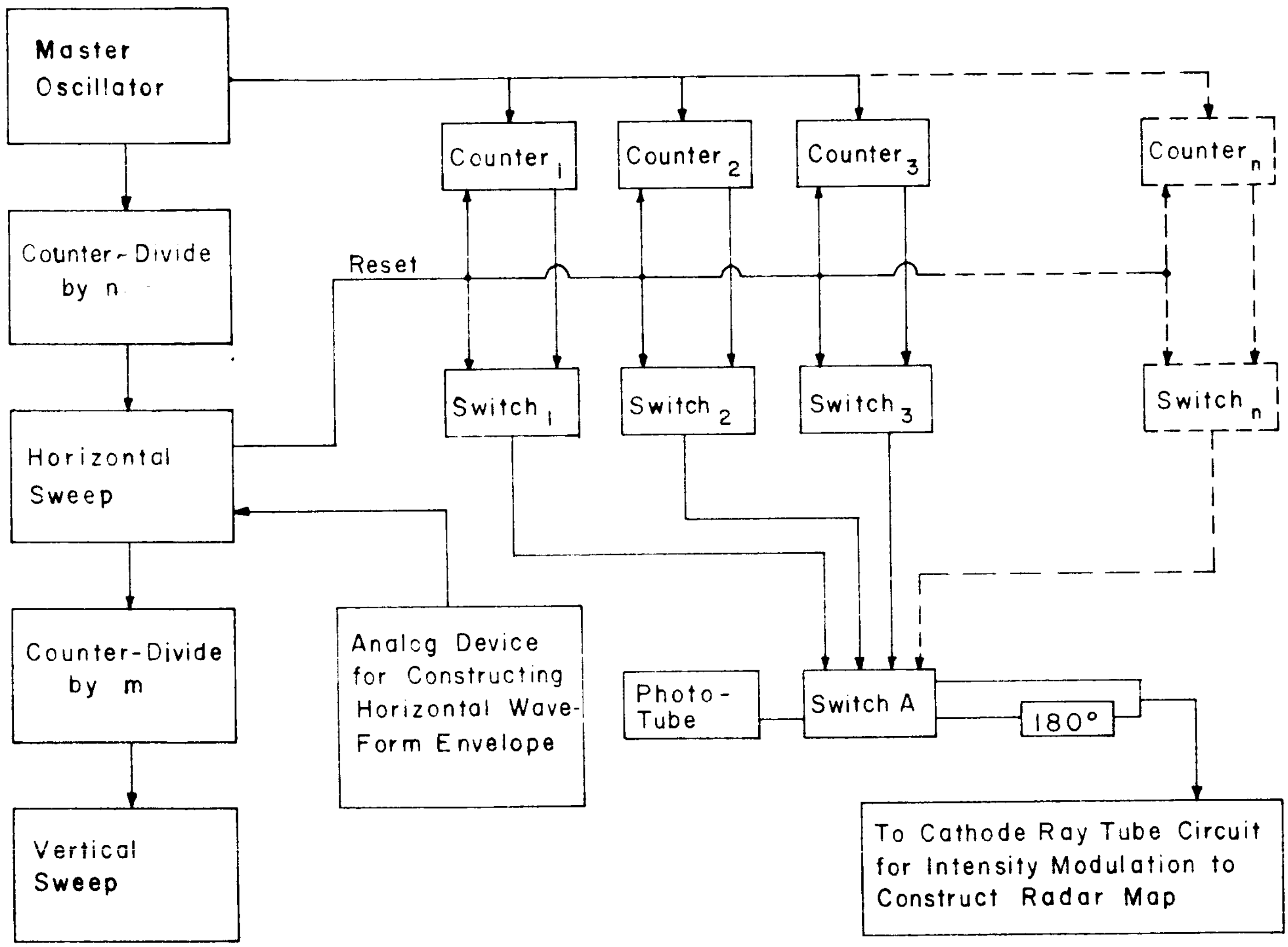

Master Oscillator
Counter-Divide by n
Horizontal Sweep
Counter-Divide by m
Vertical Sweep
Analog Device for Constructing Horizontal Wave-Form Envelope
Reset
Counter 1
Counter 2
Counter 3
Counter n
Switch 1
Switch 2
Switch 3
Switch n
Photo-Tube
Switch A
180°
To Cathode Ray Tube Circuit for Intensity Modulation to Construct Radar Map

Figure 12

Special Case #1

It was shown that the horizontal deflection voltage envelope increases as the square root of (time + constant) over one period. In terms of the radar return signal, the constant corresponds to the distance from the aircraft to the minimum range interval, and time corresponds to the distance from the minimum range to the maximum range. This is shown in Figure 13.

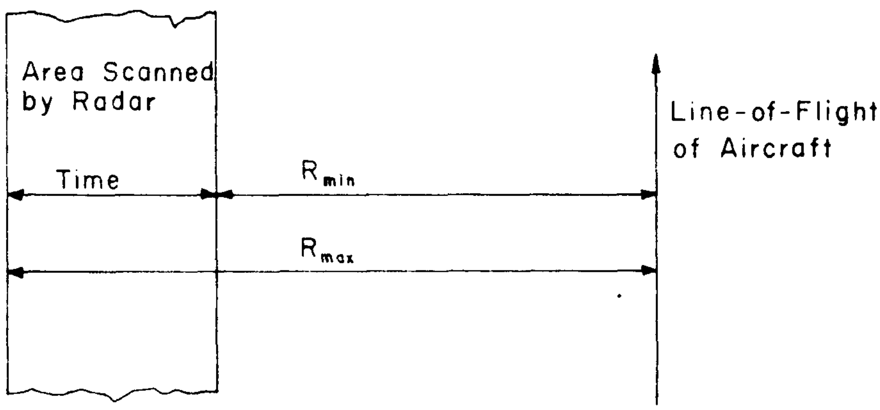


Figure 13

If the minimum range R_{min} is large compared to $(R_{max} - R_{min})$, the constant term will dominate the equation for the waveform envelope. This fact may allow the use of a sawtooth-type waveform envelope. If the distance from the center of the central Fresnel zone at range R_{min} to the boundary of the zone which the deflection voltage envelope must follow is

$$d_1 = k \sqrt{c}$$

where k is some constant and c corresponds to the range R_{min}, then the distance to this same zone at range R_{max} is simply

$$d_2 = k\sqrt{c+t}$$

where t corresponds to $(R_{max} - R_{min})$. This is shown graphically in Figure 14.

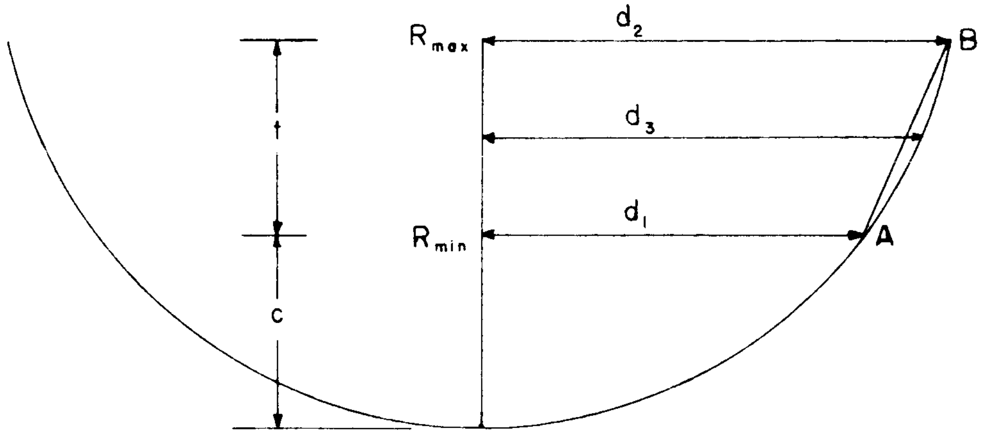


Figure 14

The linearity of the line $\overline{AB}$ in Figure 14 can be checked by comparing the average of d_2 and d_1 with the value of d at a point centrally located between d_2 and d_1. Or, compare

$$\frac{\sqrt{c+t} + \sqrt{c}}{2} \quad \text{with} \quad \sqrt{c + \frac{t}{2}}$$

If the difference of these quantities is divided by the average value of d_2 and d_1 and the result is denoted δ, this quantity will represent the deviation of the curve from a straight line.

$$\delta = -1 + \frac{d_3}{\left(\frac{d_1 + d_2}{2}\right)} = -1 + \frac{2\sqrt{c + \frac{t}{2}}}{\sqrt{c+t} + \sqrt{c}}$$

$$\delta = -1 + \frac{2\sqrt{1 + \frac{t}{2c}}}{1 + \sqrt{1 + \frac{t}{c}}}$$

If the straight line approximating the curve is centered between the curve and the average value at the midpoint of the interval, δ will be $\pm$ 1/2 as large as is calculated above. Or,

$$\delta_{\%} = \pm 100 \left(-.5 + \frac{\sqrt{1+\frac{t}{2c}}}{1+\sqrt{1+\frac{t}{c}}}\right) \quad \text{percent.}$$

As an example, if the minimum range is 1 and the maximum range is 2, $t = c$ and $\delta\% = 0.8\%$. δ can be expressed in terms of range as

$$\delta_{\%} = \pm 100 \left[-.5 + \frac{\sqrt{.5 + \frac{R_{max}}{2R_{min}}}}{1+\sqrt{\frac{R_{max}}{R_{min}}}}\right] \quad \text{percent.}$$

Resolution of System

The resolution capability of this system is not easily calculated. The minimum azimuthal distance capable of being resolved should fall between the distances resolvable with the unfocused and the perfectly focused systems, or, between $1/2\sqrt{R\lambda}$ and $D/2$ respectively, where R = range to target, λ = transmitted wavelength, and D = length of physical aperture. The resolution of the proposed system shall be determined by comparing the amplitude of the resultant signal from a perfect reflector at the point of focus with the amplitude of the resultant signal from another perfect reflector located an azimuthal distance d from focus. Calculation of these amplitudes involves summing the integrals of the real parts of each vector on the signal film over all the Fresnel zones. As was stated on page 8, the amount of phase change in the returned signal from some point out of focus a distance x is

$$\gamma = \frac{2\pi x^2}{R\lambda} \text{ radians}$$

The necessary calculation will be

$$A = \sum_{\substack{\text{All Fresnel} \\ \text{Zones}}} \int_{\substack{\text{Zone} \\ \text{Boundaries}}} \cos \frac{2\pi x^2}{R\lambda} dx,$$

a form of the Fresnel integral. To convert to the actual Fresnel integral form, let

$$v^2 = \frac{4x^2}{R\lambda}, \qquad dv = \frac{2dx}{\sqrt{R\lambda}}$$

then

$$A = \sum_{\substack{\text{All Fresnel} \\ \text{Zones}}} \frac{\sqrt{R\lambda}}{2} \int_{\substack{\text{Zone} \\ \text{Boundaries}}} \cos \frac{\pi v^2}{2} dv$$

The boundaries of the zones will be determined by solving

$$\gamma = \frac{2\pi x^2}{R\lambda} = \frac{(2N-1)\pi}{2}$$

or,

$$x = \pm \frac{\sqrt{(2N-1) R\lambda}}{2} \qquad N = 1, 2, 3, \cdots.$$

These limits can be put in Fresnel form as

$$\text{limit} = \left(\frac{\sqrt{(2N-1)R\lambda}}{2}\right)\left(\frac{2}{\sqrt{R\lambda}}\right) = \sqrt{2N-1} \qquad N = 1, 2, 3, \cdots.$$

The total number of Fresnel zones in the beamwidth at some particular range can be determined by solving

$$\gamma_{max} = \frac{\pi R\lambda}{2D^2} = \frac{(2m-1)\pi}{2}$$

or,

$$m = \frac{R\lambda}{2D^2} + \frac{1}{2} = \text{total number of zones}$$

If the phase of signals from odd-numbered zones is shifted by 180°, the amplitude of the signal returned from a perfectly reflecting target at the point of focus will therefore be

$$A = 2\left(\frac{\sqrt{R\lambda}}{2}\right)\int_0^1 \cos\frac{\pi}{2}v^2 dv + \sum_{N=1}^{m} 2\left(\frac{\sqrt{R\lambda}}{2}\right)(-1)^N \int_{\sqrt{2N-1}}^{\sqrt{2N+1}} \cos\frac{\pi}{2}v^2 dv$$

These same calculations can be carried out for the return signal from a perfectly reflecting target at a distance d from the focus point. The result of these calculations is

$$B = \frac{\sqrt{R\lambda}}{2}\int_0^{1+\frac{2d}{\sqrt{R\lambda}}} \cos\frac{\pi}{2}v^2 dv + \frac{\sqrt{R\lambda}}{2}\int_0^{1-\frac{2d}{\sqrt{R\lambda}}} \cos\frac{\pi}{2}v^2 dv$$

$$+ \sum_{N=1}^{m}(-1)^N \frac{\sqrt{R\lambda}}{2}\left[\int_{\sqrt{2N-1}+\frac{2d}{\sqrt{R\lambda}}}^{\sqrt{2N+1}+\frac{2d}{\sqrt{R\lambda}}} \cos\frac{\pi}{2}v^2 dv + \int_{\sqrt{2N-1}-\frac{2d}{\sqrt{R\lambda}}}^{\sqrt{2N+1}-\frac{2d}{\sqrt{R\lambda}}} \cos\frac{\pi}{2}v^2 dv\right]$$

The resolution capability of the system can now be defined as that distance d which causes the ratio B/A to be .707. This choice of the 3db. point is highly arbitrary, however. It may be that with this definition of resolution, distances slightly greater or slightly smaller than d can be resolved on a radar map.

A computer program was constructed to calculate the relationship between minimum resolution, range, and transmitted wavelength. The ratio B/A was calculated for various values of c, the distance from focus (in a form comparable with the Fresnel integrals), and for various values of the number of zones averaged. Values of c and the necessary number of zones were found such that B/A = .707, .500, and .400. These values were plotted on log-log paper and the results are shown on the following page. Analysis of these graphs yields the following results:

$$\#\text{zones necessary} = a\,c^{-b} + h$$

Depending on the definition of resolution, the constants a, b, and h are as follows:

B/A = .707

a = .140

b = 1.89

h = .18

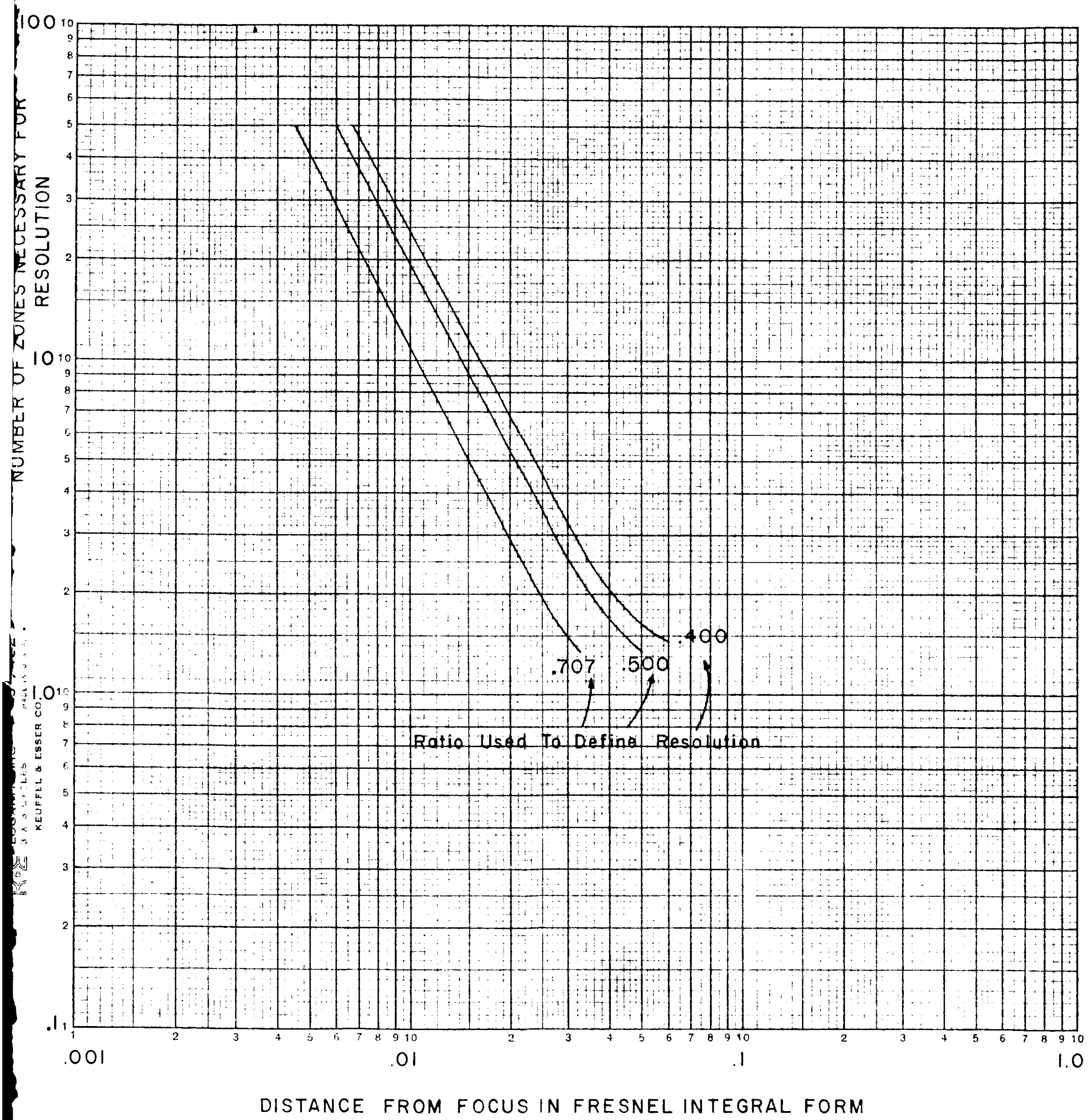
B/A = .500

a = .275

b = 1.85

h = .35

PLOT OF THE NUMBER OF FRESNEL ZONES NECESSARY FOR RESOLUTION VS. THE DISTANCE FROM FOCUS IN FRESNEL INTEGRAL FORM



B/A = .400

a = .360

b = 1.84

h = .50

The actual azimuthal distance capable of being resolved is

$$d = \frac{c\sqrt{R\lambda}}{2}$$

or,

$$c = \frac{2d}{\sqrt{R\lambda}}$$

The number of zones available for processing (the number in the beamwidth) was found to be

$$m = \frac{R\lambda}{2D^2} + .5$$

where D = length of physical antenna. Equating the number of zones necessary to the number of zones available yields

$$\frac{R\lambda}{2D^2} + .5 = a\left(\frac{2d}{\sqrt{R\lambda}}\right)^{-b} + h$$

Solving this expression for d,

$$d = \frac{(R\lambda)^{\frac{1}{2}}}{2}\left(\frac{a}{\frac{R\lambda}{2D^2} + .5 - h}\right)^{\frac{1}{b}}$$

If, $\frac{R\lambda}{2D^2}$ >> (.5 - h), this reduces to

$$d = (2)^{\frac{1-b}{b}} (a)^{\frac{1}{b}} (D)^{\frac{2}{b}} (R\lambda)^{\frac{b-2}{2b}}$$

For the three definitions of resolution chosen, the results are:

B/A = .707

$$d = .255\, D^{1.06}\, (R\lambda)^{-.029}$$

B/A = .500

$$d = .362\, D^{1.08}\, (R\lambda)^{-.041}$$

B/A = .400

$$d = .418\, D^{1.09}\, (R\lambda)^{-.043}$$

This result at first seems unusual in that the resolution capability of the system improves as range increases. However, the exponent on the quantity $(R\lambda)$ lies between -.043 and -.029, so for all reasonable values of $(R\lambda)$, $(R\lambda)$ raised to these powers will not differ greatly from 1.

Therefore, it appears that the definition of resolution used does not greatly effect the range and wavelength dependence of the system. The definition of resolution used does determine, however, the constant term in the value of d.