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LANDING DYNAMICS STUDY FOR LUNAR LANDING RESEARCH VEHICLE

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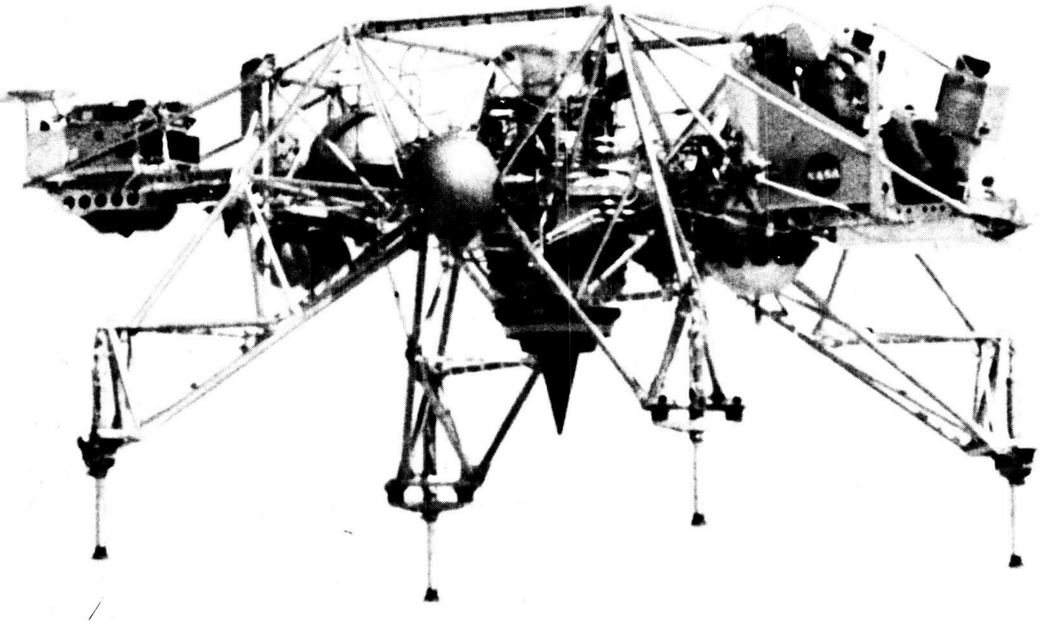


Figure 1. Lunar Landing Research Vehicle

INTRODUCTION

The mission of the Lunar Landing Research Vehicle (Figure 1) is to accomplish basic research on lunar landing and training problems. The basic goals in this vehicle design were to simulate lunar gravity, to provide the "feel" of operation in a vacuum, and to preserve the basic geometry of a true lunar landing vehicle.

The height, base width, and landing gear configuration are generally comparable to an actual two-man lunar landing vehicle. The airframe is of simple welded aluminum tubing truss construction. Although this structure is not aerodynamically clean, the design provides performance capabilities adequate for the initial research tasks to be accomplished. The actual dimensions and weights are given in Table 1, Physical Characteristics.

The jet engine is installed vertically and is able to fly the vehicle to some prescribed attitude then the thrust can be cut back to five-sixths' earth gravity, allowing the vehicle to react as if under the influence of the gravity on the moon.

The unique feature in the design of this vehicle is the method of automatically providing this five-sixths' earth gravity thrust. Mounted within the airframe is a two-axes gimbal ring within which is mounted the jet engine. In order for the jet thrust vector to counteract five-sixths' earth gravity, the thrust vector passes through the vehicle center of gravity and is gyro-stabilized to an earth vertical, regardless of vehicle attitude. Provision is also made in the automatic control to tilt the engine to counteract vehicle drag as well. This is done to give the pilot the "feel" of operating in a vacuum, that is, without any aerodynamic effects.

In this lunar mode of operation, the descent velocity is controlled by two throttleable hydrogen peroxide lift rockets operated by the pilot. There are also rocket motors to provide attitude control for each of the three vehicle axes.

The landing legs are easily removable to facilitate testing of alternate gear designs. The shock absorbers are the conventional air-oil type utilizing a snubber arrangement to minimize rebound effects. They have rubber shock mounts to give them some slight lateral flexibility to absorb lateral shock loads. Also, they are fitted with pads as ground contacting elements; however, incorporation of casters can be accomplished simply. A much more detailed description of the vehicle can be obtained from Reference 1 or 2.

The metering pin design for this strut was based on two different conditions. Condition one was a straight vertical velocity of ten feet per second without any horizontal velocity. Condition two was a vertical impact velocity of six feet per second in combination with a horizontal velocity of three feet per second.

Even though there is a variance in the total energy of these two conditions, there is one strut that does approximately the same amount of work for either condition. Since the impact velocities of the two conditions are significantly different, the metering pin design has to be a compromise between the two conditions. Thus, although the strut does not have a high efficiency for either condition, it will satisfy both conditions.

This report covers a study to investigate the landing dynamics of the Lunar Landing Research Vehicle. This study was made to determine the effects of certain combinations of landing variables on the strut, the leg, or the vehicle landing performance, and to define the limitations of the vehicle under a variety of realistic landing conditions. In addition to horizontal and vertical impact velocities, the other variables which are covered in this report are pitch attitude, pitch rate, engine mode or thrust, ground slope, and ground friction coefficient.

These parameters are varied individually and in combinations that give realistic landing condition. The results have been grouped to show the effect of the initial landing conditions.

The computer program used in this study is for planar motion which means that the motion of the vehicle is restricted to one plane. Because of this, the investigation into leg orientation is restricted to 2-2 or 1-2-1 yaw.

Appendix A contains the tables of the critical values for all the series. It is from these tables that the operational profiles were plotted. The curves represent an interpolation of this data. Thus, if a point is barely within the limits, the curve is drawn close to this point so that the point is also barely within the profile. In a similar manner for a point just outside the limits, the curve would be drawn to barely exclude that point.

The study described by this report was made by the Analytical Mechanics Department of Bendix Products Aerospace Division, The Bendix Corporation, South Bend, Indiana under the supervision of Mr. R. J. Black with Mr. D. C. Irwin as Project Engineer. The computer work was done by Mr. J. Cadoret.

TABLE I

PHYSICAL CHARACTERISTICS

Dimensions, feet (meters) -	
Overall length	22.50 (6.85)
Overall width	15.08 (4.60)
Overall tread	13.35 (4.07)
Height with footpads attached and struts extended (plus 0.62 ft (0.19 m) if casters are attached; minus 1.17 ft (0.36m) if struts are fully retracted):	
Overall	10.00 (3.05)
Center of gravity	6.50 (1.98)
Cockpit floor	5.83 (1.78)
Weights, pounds (kilograms) -	
Primary structure including engine gimbal ring	500 (227)
Landing gear (plus 38 lb (17.2 kg) when casters replace the pads)	140 (64)
Manual controls	45 (20)
Control avionics and wiring	160 (73)
Jet-engine system including hydraulics	800 (363)
Rocket system	370 (168)
Flight instruments, radar sensors, wiring, and console	115 (52)
Electrical system	115 (52)
Ejection-seat parachute and breathing oxygen	135 (61)
Drogue parachute and attachments	20 (9)
Research instrumentation and telemetry system	105 (48)
Communications	5 (2)
Normal empty weight	2510 (1139)
Useful load at takeoff:	
Pilot	185 (84)
JP4 fuel (430 lb (195 kg) less 6 min of idle at 8 lb/min (3.6 kg/min))	382 (173)
Hydrogen peroxide (672 lb (305 kg) less 60 lb (27.2 kg) for preflight checks)	612 (285)
Helium gas	5 (2)
Engine oil	8 (4)
Total useful load	1192 (548)
Takeoff weight	3702 (1687)

COMPUTATIONAL PROCEDURES

A detailed description of the equations of motion is presented in Appendix B. The following brief discussion outlines the steps the computer goes through in order to generate a complete operational profile.

The equations of motion of the LLRV system were programmed on a Bendix G-20 computer. A step-by-step time solution was carried out for various initial conditions using a computational time increment of .002 seconds and a type-out time increment of .02 seconds. In addition to the dynamical solution, all critical variables such as stroke load, horizontal load, stroke, and vehicle stability angle were checked at each time increment. If any exceeded allowable limits, a diagnosis of the particular problem area was automatically printed out by the computer.

In order to obtain the operational boundaries of the system, an array of vertical and horizontal initial impact velocities was set up for computation. The array was set up in a series of rays in the V_V (vertical velocity) and V_H (horizontal velocity) plane. The computer would begin at a specified initial point and compute a complete simulated landing. If the landing was satisfactory, the computer would automatically advance to the next set of velocities on the original ray. This would continue until there was a transition from a successful to an unsuccessful landing. At this point, the computer would switch to the next adjacent ray at the same level at which it was working on the first ray. Depending upon whether or not a successful landing was found at this point, the computer would either proceed to a higher or lower velocity on this new ray. This would continue until a transition was again found. This process was continued throughout the complete array, thus establishing the operational limitations.

The computer program used in this study was previously developed by Bendix Products Aerospace Division for its own preliminary investigation into the alightment dynamics of this vehicle which was reported in Bendix Report No. SD-63-2 (870).

OPERATIONAL PROFILES

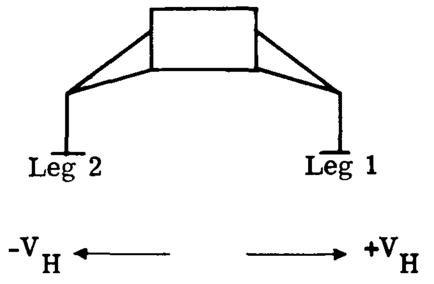


Figure 2. 2-2 Yaw

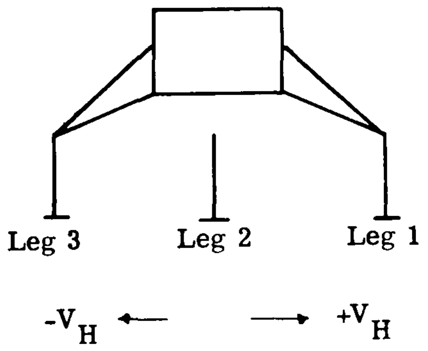


Figure 3. 1-2-1 Yaw

In referring to the legs of the vehicle by number, the following system was used. For 2-2 yaw orientation, (See Figure 2) leg one is the leading leg for position horizontal velocities and leg two is the leading leg for negative horizontal velocities. Therefore, with 2-2 yaw, reference to leg one or leg two really refers to a pair of legs.

The 1-2-1 yaw orientation (See Figure 3) has leg one as the leading leg for positive horizontal velocities and leg three as the leading leg for negative horizontal velocities. The middle legs are referred to as Leg 2. Here reference to Leg 1 or 3 refers to just one leg and reference to leg two would refer to a pair of legs.

Included on the operational profile graphs is the notation for the failure mode that determines the boundary of the profile. This notation is "L" for excessive axial load, "H" for excessive horizontal load, "B" for strut bottoming, and "T" for vehicle toppling. The number with the letter refers to the gear number using the system described above. Thus, if the axial load in Leg 1 is excessive, it is noted as "1L" on the graph.

It was considered that a desired operational boundary should include the points of $V_V = 10$ ft./sec., $V_H = 0$; and $V_V = 6$ ft./sec., $V_H = \pm 3$ ft./sec. These three points will be referred to as the "required" points and used as a measure of the size of the operational profiles.

Standard Series

The "standard" series refers to Series 1, 2, and 7. These series all have a level vehicle without a pitch rate landing on level ground and consider the footpads as nonsliding. Series one and two have 2/3-g thrust with Series 1 having 1-2-1 yaw and Series 2 having 2-2 yaw. Series seven has 5/6-g thrust with a 2-2 yaw orientation.

Series 1 (1-2-1 Yaw, 2/3-g Thrust). - The operational profile for Series 1 is limited in its entirety by the axial load on either leg one or leg three. This profile is symmetrical about zero horizontal velocity with the right side (positive horizontal velocities) being limited by the axial load on leg one and the left side (negative horizontal velocities) being limited by the axial load on leg three. With nonsliding footpads, any horizontal velocity will tend to tilt the vehicle up on the leading leg, hence this leg will do a greater percent of the total work than any of the other legs. The profile is determined by when this energy input becomes too large and causes excessive axial load. (See Figure 4)

The landing velocity range for this series is good and contains the "required" points.

Series 2 (2-2 Yaw, 2/3-g Thrust) - The operational profile for Series 2 is also symmetrical about zero horizontal velocity and is limited by the allowable loads on the vehicle. For vertical velocities over four feet per second, the allowable axial load is exceeded. This is caused by the vehicle tipping up on two legs and these legs doing a large percentage of the work. When the energy input into these legs gets too large the result is excessive axial loads. For vertical velocities less than four feet per second and at approximately horizontal velocities of five feet per second, the allowable horizontal footpad load is exceeded. (See Figure 5)

This profile has an even bigger range than Series 1 and therefore it also contains the "required" points.

Series 7 (2-2 Yaw, 5/6-g Thrust) - This Series 7 profile is very close to the Series 2 profile and the comments on Series 2 apply to Series 7. (See Figure 6)

Landings with Initial Vehicle Pitch

There are six series (5, 6, 10, 11, 15, and 16) to investigate landings for a vehicle with an initial pitch angle. For 2/3-g thrust, Series 10 and 11 have a small (±3°) pitch angle and Series 5 and 6 have a large (±6°) pitch angle. For 5/6-g thrust, Series 15 and 16 have a large pitch angle. These series all have nonsliding footpads, level ground, 2-2 yaw orientation, and no initial pitch velocity.

A symmetrical vehicle with initial back pitch (positive pitch angle) and positive horizontal velocity will react the same as forward pitch (negative pitch angle) and negative horizontal velocity. Similarly, back pitch and minus horizontal velocity will give the same results as forward pitch and positive horizontal velocities. Because of this, Series 5, 10, and 16 are the mirror image of Series 6, 11, and 15, respectively. Therefore, these are comments only on Series 6, 11, and 15, and these comments can be applied (remembering the above) to Series 5, 10, and 16.

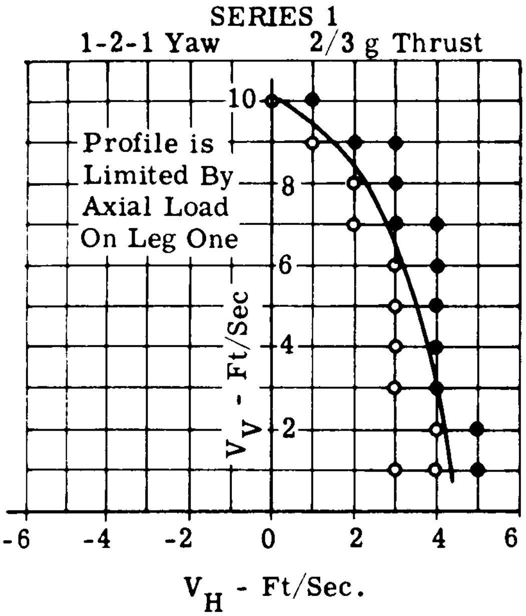


Figure 4. Standard Series With 1-2-1 Yaw

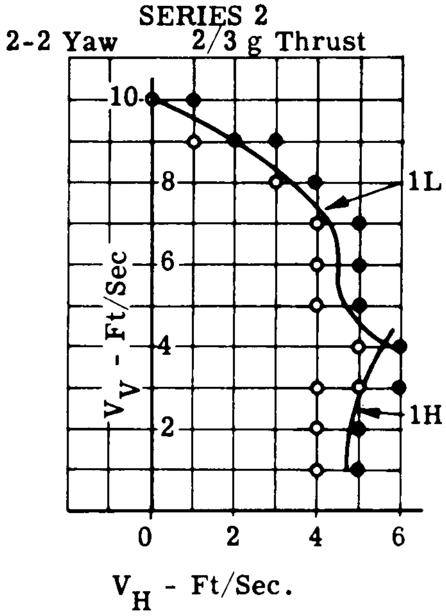
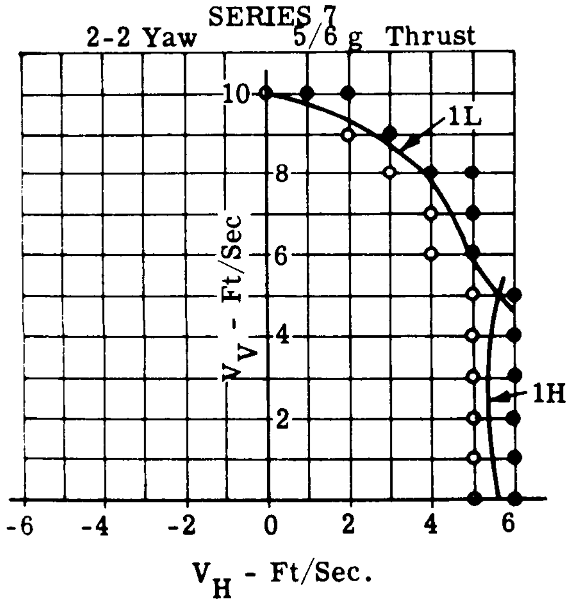


Figure 5. Standard Series With 2-2 Yaw



NOTES: ● Bad Landing
○ Good Landing

These profiles are symmetrical about $V_H = 0$ therefore only the positive V_H side of the profile is shown.

Pitch Angle 0
Pitch Velocity 0
Ground Slope 0
Non-Sliding Footpads

Figure 6. Standard Series With Lunar Mode

Series 6, 11, and 15 have an initial back pitch. With back pitch and positive horizontal velocity, the trailing legs will impact the ground first; and with negative horizontal velocity, the leading legs will impact the ground first.

Series 11 (3° Back Pitch, 2/3-g Thrust) - The operational profile for Series 11 has the general shape of a trapezoid. The sides are limited by the horizontal load on the leading legs due to the horizontal velocity and the pitch motion induced.

The top of the profile is limited by the impact velocity of leg one. (See Figure 7) The impact velocity of this leg consists of two components, the translational and rotational velocity of the vehicle. Roughly speaking, if the impact velocity of any leg exceeds ten feet per second, the allowable axial load in that leg will be exceeded. A three degree pitch angle reduces the operational landing velocity's limits from that of a level vehicle (Series 2) considerably. This reduction is to such an extent that none of the "required" landing velocities are contained in this profile.

Series 6 (6° Back Pitch, 2/3-g Thrust) - The operational profile for Series 6 is altered from that of Series 11 but the limiting factors are the same. That is, the sides of the profile are limited by the horizontal loads on the leading legs and the top by the impact velocity of leg one. (See Figure 8)

As in the case of three degree pitch, none of the "required" landing velocities are inside of this profile for a vehicle with an initial six degree pitch angle.

Series 15 (6° Back Pitch, 5/6-g Thrust) - The only change in the initial condition for Series 15 from Series 6 is the increased thrust from two-thirds to five-sixths g. This makes the operational profile for Series 15 bigger than for Series 6 but with the same shape. Again, none of the required landing velocities are included in this profile. This is indicated in Figure 9.

The operational profiles for Series 10, 5, and 16 appear as Figures 10, 11, and 12 respectively.

Landings on a Small Ground Slope (3°)

Series 12, 13, and 14 cover that phase of the study investigating the effect of a small ground slope of three degrees.

Series 12 (Level Vehicle, 2/3-g Thrust) - The operational profile for Series 12 is very close to the one for Series 11, with the top of the profiles almost identical. The sides of profile twelve are limited to under four feet per second for downslope horizontal velocities and over three feet per second for upslope horizontal velocities.(See Figure 13)

Series 11 and 12 are limited by the same factors, which were horizontal load on the leading leg for the sides of the profile and the impact velocity of leg one for the top of the profile.

One of the required landing velocities is inside of the operational profile. That one is the vertical velocity of six feet per second with a negative horizontal velocity of three feet per second.

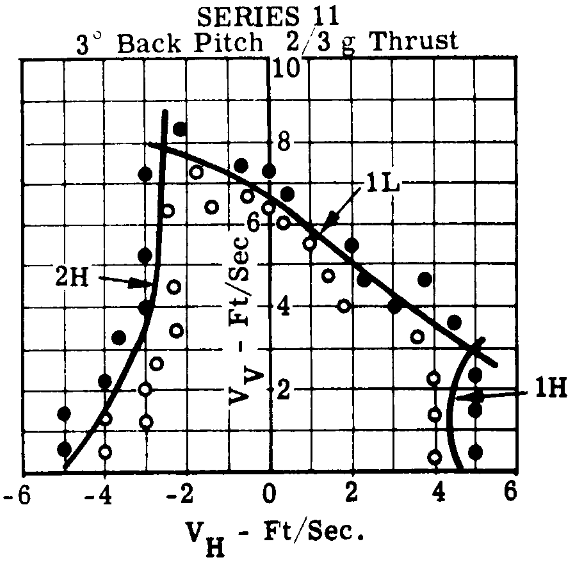


Figure 7. Vehicle Pitch 3°

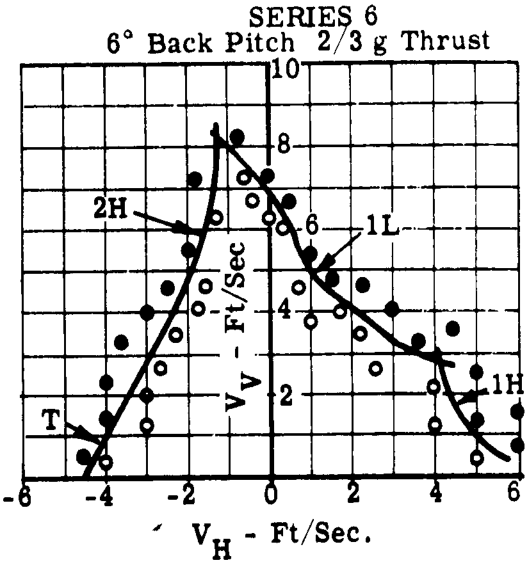
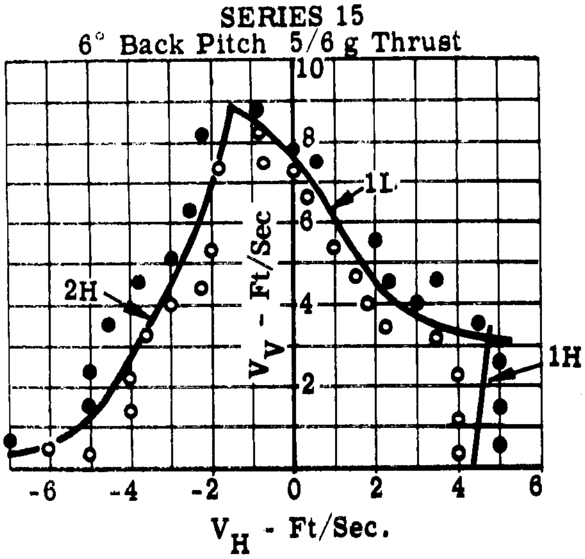


Figure 8. Vehicle Pitch 6°



Pitch Velocity 0
2-2 Yaw
Ground Slope 0
Non-Sliding Footpads

Figure 9. Vehicle Pitch (6°) With Lunar Mode

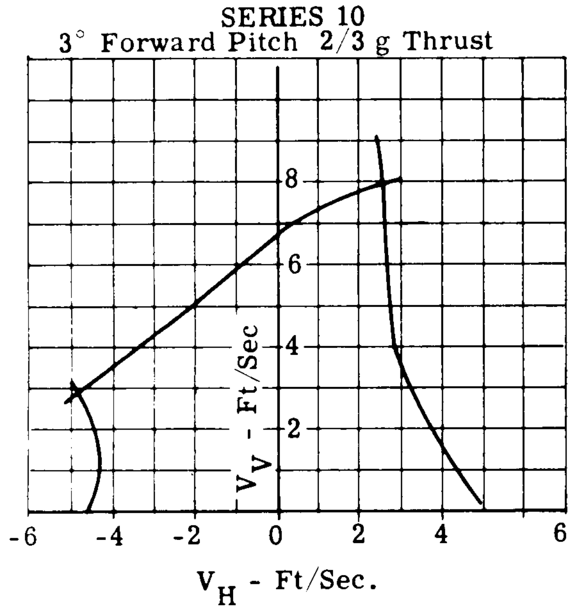


Figure 10. Vehicle Pitch (-3°)

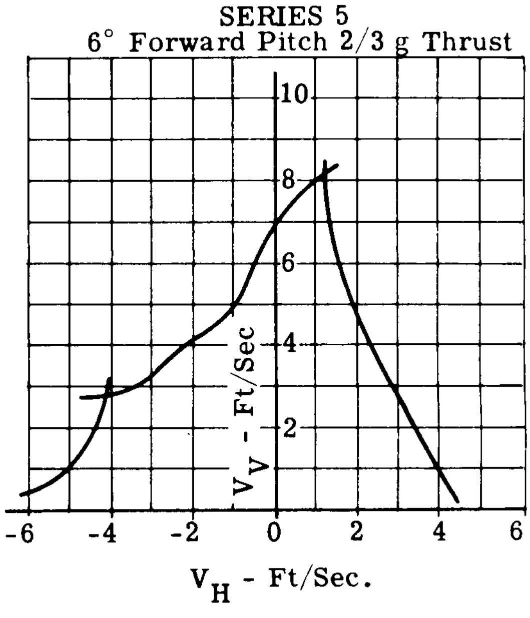
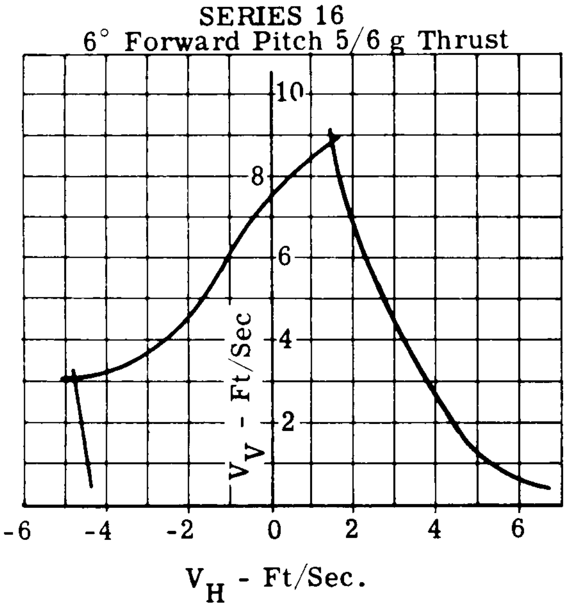


Figure 11. Vehicle Pitch (-6°)



Pitch Velocity 0
2-2 Yaw
Ground Slope 0
Non-Sliding Footpads

Figure 12. Vehicle Pitch With Lunar Mode

Series 13 (Level Vehicle, 5/6-g Thrust) - The only difference in the initial conditions for Series 13 from Series 12 is the increased thrust from two-thirds to five-sixths g. This change has very little effect on the operational profile and the profile for Series 13 is very similar to Series 12 and the comments on Series 12 apply to Series 13.(See Figure 14)

Series 14 (6° Back Pitch on Vehicle, 2/3-g Thrust) - The six-degree back pitch on the vehicle makes a nine-degree included angle between the vehicle and the ground at the time of initial ground contact.

The operational profile for Series 14 is roughly in the shape of a triangle with the vertices of the base at a horizontal velocity of plus and minus five feet per second with zero vertical velocity and the top vertice at a vertical velocity of 8.4 ft./sec., and a negative horizontal velocity of one-half foot per second.This is illustrated in Figure 15.

The up-slope (negative V_H) horizontal velocity side of the "triangle" is limited by the horizontal load on Leg 2. At vertical velocities under 1.5 ft./sec. and horizontal velocities over 4 ft./sec. the vehicle toppled backwards. Since the horizontal load on Leg 2 in this region is near or over its allowable, this portion of the curve blends in with that portion that is limited by horizontal load only.

Most of the down-slope (positive V_H) horizontal velocity side of the "triangle" is limited by excessive axial load caused by the high impact velocity of Leg 1. At vertical velocities less than 3 ft./sec. and horizontal velocities greater than 3.5 ft./sec. the profile is limited by the horizontal load on Leg 1.

The top vertice of this triangle is in a region where three loads are near their limit. These are the horizontal loads on Legs 1 and 2 and the axial load of Strut 1. Again, none of the required landing velocities are included in this profile.

Landings with Pitch Velocity

Series 8 and 9 cover the effect of landing with a pitch velocity of plus or minus three-tenths of a radian per second. Series 8, which has a back pitch velocity, is the mirror image of Series 9, which has a forward pitch velocity. A back pitch velocity with positive horizontal velocity will give the same results as forward pitch velocity with negative horizontal velocity.

Except for this reversal of pitch velocity direction, Series 8 and 9 have identical initial landing conditions of non-sliding footpads, 2-2 yaw orientation, five-sixths g thrust, level ground, and level vehicle. The level vehicle means that even though the vehicle is rotating, all four legs contact the ground simultaneously.

Series 9 (.3 Radians per second forward Pitch Velocity, Five-sixths g Thrust) - The operational profile for series nine is large, having high horizontal velocity capability in the same range as Series 7, which is the comparable Series without pitch velocity. The maximum vertical velocity reached is under nine feet per second compared to ten feet per second for Series 7. (See Figure 16)

At the higher vertical velocities (roughly above 8.5 feet per second) the impact velocity of Leg 1 causes excessive axial load in that leg. This high impact velocity of Leg 1

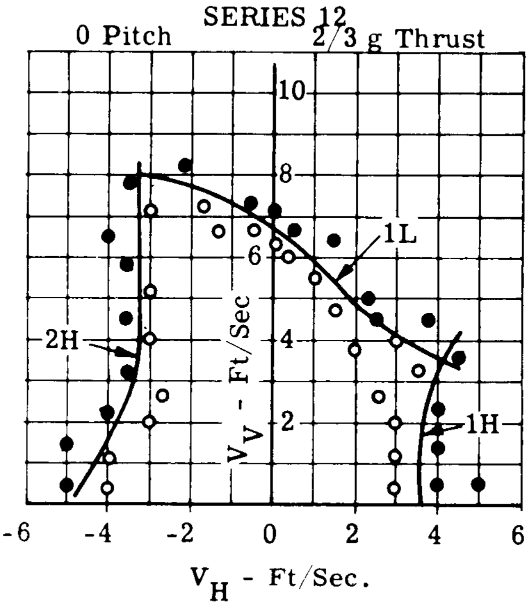


Figure 13. Ground Slope

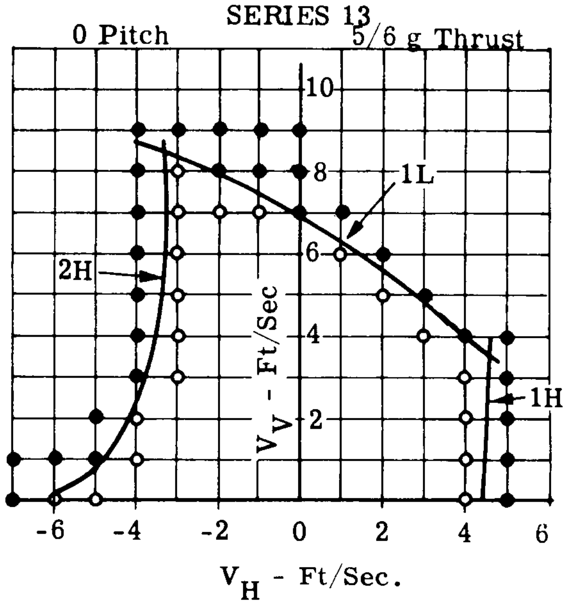
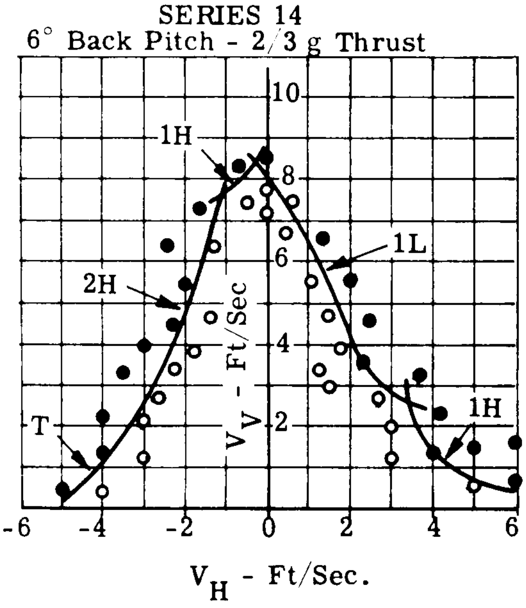


Figure 14. Ground Slope With Lunar Mode



Pitch Velocity 0
2-2 Yaw
Ground Slope 3°
Non-Sliding Footpads

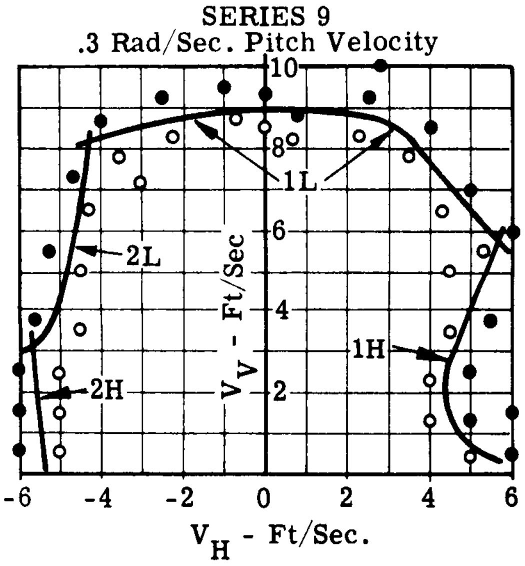
Figure 15. Ground Slope With Vehicle Pitch (+6°)

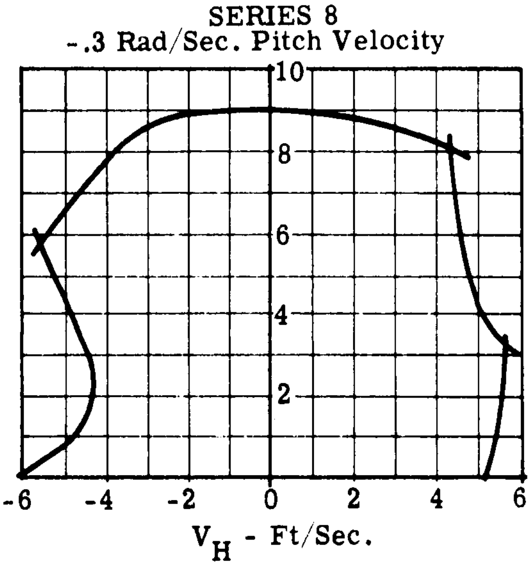
comes from the vertical velocity plus the additional velocity input from the pitching of the vehicle. There is another part of the profile limited by the axial load on leg one when the high axial load is due to an excessive energy input into leg one. (See Figure 13). The vehicle tips up on leg one since it is moving and pitching in that direction, so that almost all the energy goes into this leg. The right side of the profile is limited by the horizontal load on leg one caused by the horizontal velocity and the pitching motion. Part of the left side of the profile is limited by the axial load on leg two. This high axial load is caused by high energy inputs into leg two.

In this portion of the profile, the pitching direction is towards leg one with the horizontal velocity in the direction of leg two. This results in a relatively high axial load on leg one which lasts only a short time. The end result being a small amount of work done by leg one which means that leg two has to do a large share of the total work.

As has been indicated, the required landing velocities of V_V = 6 ft./sec., V_H = ± 3 ft./sec. have been met but the V_V = 10 ft./sec., V_H = 0 condition has not been satisfied.

Series 8 (.3 Radians per second Back Pitch Velocity, Five-sixths g Thrust) - This operational profile is the mirror image of Series nine, so the comments on Series nine can be applied to Series eight.(See Figure 17)





Pitch Angle 0 Ground Slope 0
2-2 Yaw Thrust 5/6 g
Non-Sliding Footpads

Figure 16. Pitch Velocity, Plus

Figure 17. Pitch Velocity, Minus

Sliding Footpad Landings

These runs change the nonsliding footpads to footpads that may slide, depending on the loading and the coefficient of friction μ between the footpads and the ground.

An investigation into the values of μ to use was conducted, using the initial landing conditions of Series 3 and 4 and contact velocities of $V_V = 4$ ft./sec. and $V_H = 6$ ft./sec. Nine different values of μ were used.

The peak axial strut load occurs in the forward struts and is influenced to a large extent by the pitching motion of the vehicle. This pitching is caused primarily by the horizontal loads, so for increasing μ the pitching moment increases, making higher axial loads in the forward struts.

The strut is bent by the horizontal ground loads. In the following discussion, the term "spring-back" refers to that time when the strut after being bent to a maximum begins to "spring-back" to its original configuration.

The axial load is increased when the leg springs back. The strut axial load due to spring-back is not as great as that due to the pitching moment. For μ between 0.5 and 1.0, the leg springs back at about the same time as the peak load load is occurring due to pitching of the vehicle. This increases the peak axial load for this range of μ but it is not as great as the peak axial load for nonsliding footpads caused by increased pitching of the vehicle. (See Figures 18 through 21)

From Table 2 it can be seen that the horizontal load becomes a maximum at $\mu = 1.0$, actually greater than for nonsliding footpads. F_H is almost a direct measure of bending of the strut. For both $\mu = 1.0$, and $\mu = 10$, the footpad could be considered as not sliding. Since the bottom of the strut does not move in either case, a measure of

Table 2. Loads for Various Coefficients of Friction
Series 3 Condition; $V_H = 6$ ft./sec.; $V_L = 4$ ft./sec.

	Max. for Leg 1		Max. for Leg 2
μ	F_S	F_H	F_H
.15	888	150	106
.40	1246	546	238
.50	1462	810	294
.75	2189	1684	393
1.00	1774	2396	520
1.25	1793	2361	654
1.50	1810	2337	705
2.0	1842	2315	829
10.0	2362	2040	1216
Nonsliding	2300	2050	1131

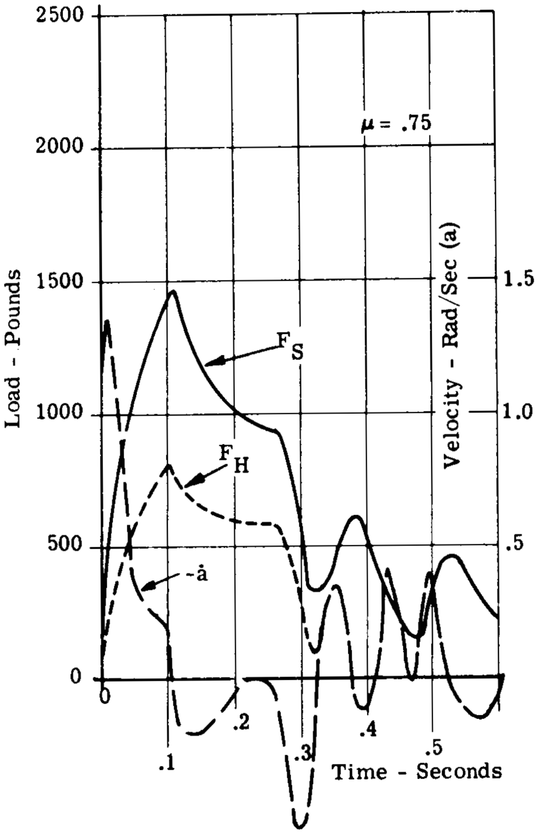


Figure 18. Sliding Footpad Investigation
μ = .50

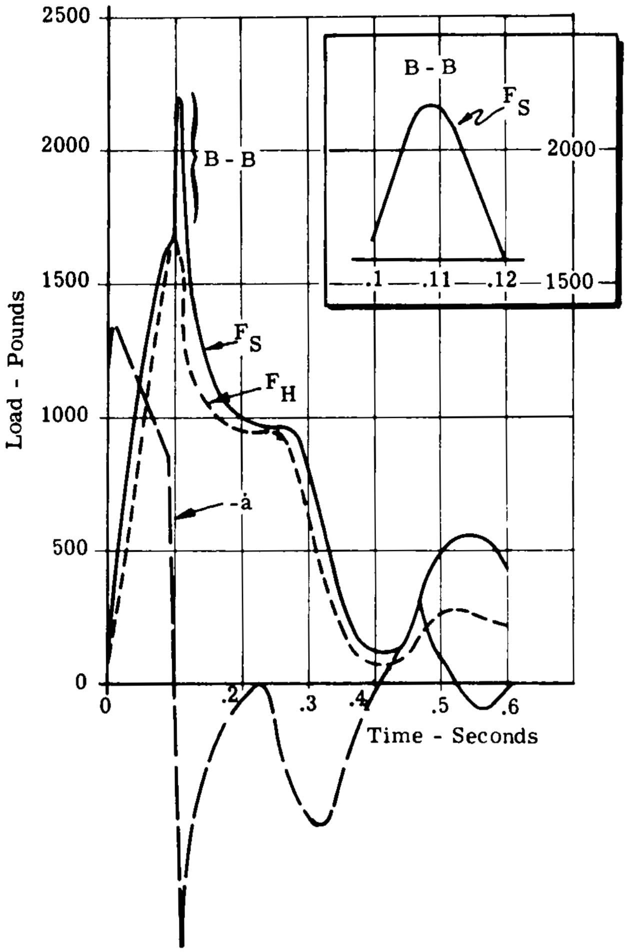


Figure 19. Sliding Footpad Investigation
μ = .75

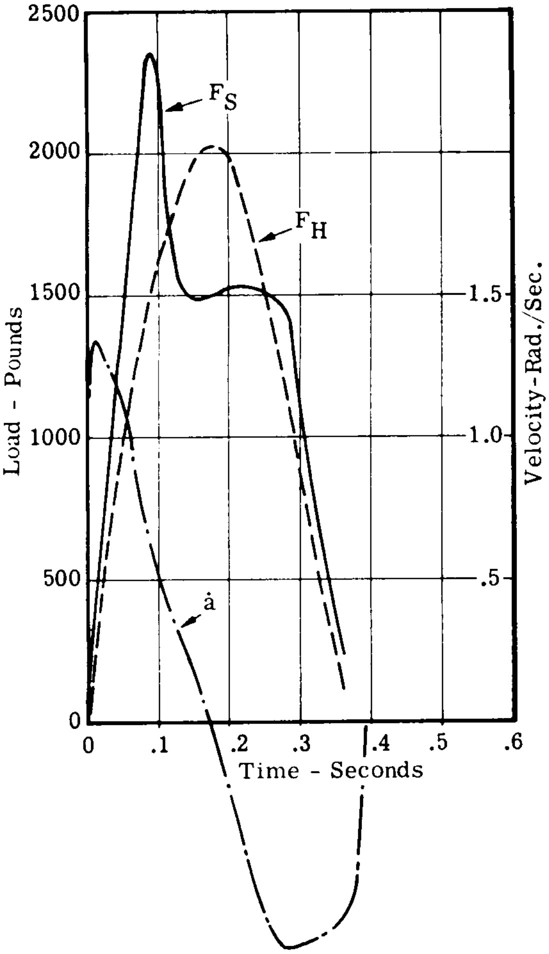


Figure 20. Sliding Footpad Investigation

μ = 1.0

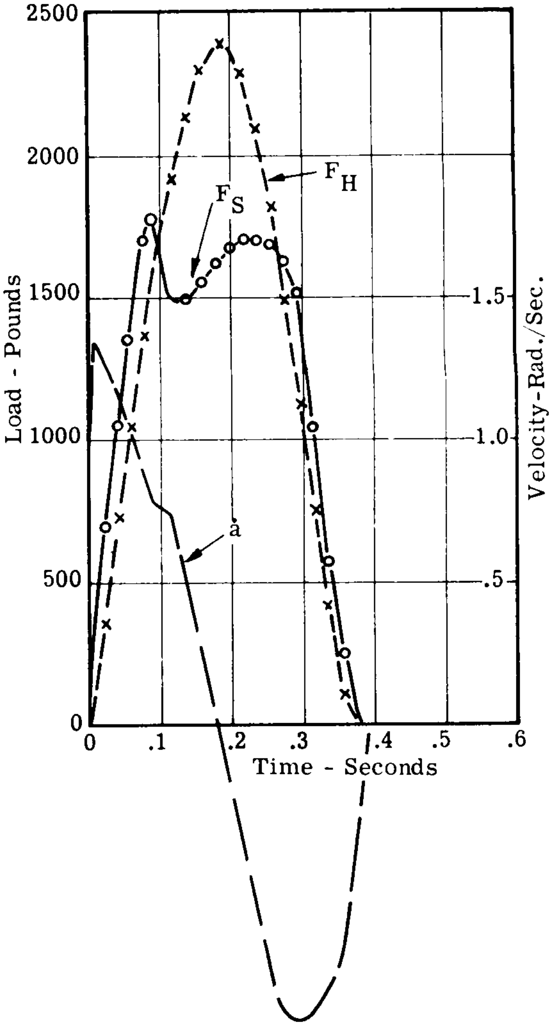


Figure 21. Sliding Footpad Investigation

μ = 10.0

bending is the movement of the top of the strut which is a combination of translation and rotation. In both cases, the amount of rotation at the peak, F_H is about the same, so the difference in the two conditions is the translation of the strut. Because of the higher horizontal loads on the rear legs for higher μ the translational movement is more for $\mu = 1.0$, causing a greater bending of the front leg and hence higher F_H in the front leg.

From this data it was decided ground friction coefficients of 0.75 and 1.0 would give the best information on the effect of sliding. It is anticipated that a coefficient under 0.5 would result in sliding out of the vehicle without exceeding the horizontal loads. That is, the profile would be independent of the horizontal velocity. Higher horizontal velocities would merely increase the slide-out distance. Also, for coefficients exceeding 1.0, the operational profile would be close to the nonsliding case.

In Figures 22 and 23, the positive horizontal velocity side of the profiles three and four are shown and the negative velocity side would be symmetrical to this.

The initial conditions for Series 3 and 4 vary from those of Series 2 only in that the footpads are allowed to slide for Series 3 and 4, while for Series 2, the footpads were considered as nonsliding.

Series 3 ($\mu = .75$). - The allowable loads on the vehicle determine the operational limits for this profile. For vertical velocities above seven feet per second there is very little change from the nonsliding case (Series 2). For positive horizontal velocities, this portion of the profile is limited by the axial load on leg one. For vertical velocities from four to seven feet per second, the profile is limited by the horizontal load on Leg 1. This excessive load is due to the horizontal velocity and the pitching motion induced. For vertical velocities under four feet per second, the vehicle slides out to a stop without exceeding the loads or toppling. As indicated in Figure 22, page 34, this was checked up to a horizontal velocity of nine feet per second.

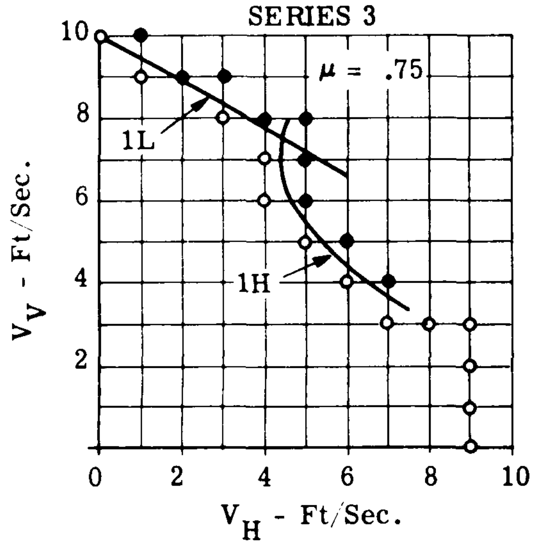
Series 4 ($\mu = 1.00$). - The operational profile for Series 4 has very little change from the nonsliding case (Series 2) for vertical velocities above seven feet per second. For vertical velocities from seven feet per second down to two feet per second, the profile is limited by the horizontal load on Leg 1. The limit is about four and one-half feet per second, which is less than for the nonsliding case. This was indicated in our preliminary analysis of coefficients or friction; however, the change is small and still allows horizontal velocities up to 4.5 feet per second.(See Figure 23)

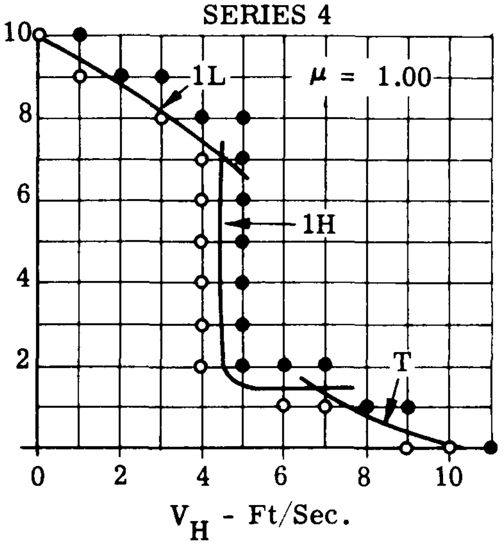
Both Series 3 and 4 meet all three of the required landing velocities.

Ultimate Load Series

Since the landing velocity range of this vehicle was limited mostly by the allowable loads on it, it was decided to investigate the range of landing velocities using the ultimate load as a limiting factor instead of the limit load. Ultimate load is 150% of limit load. This was done for a vehicle with an initial back pitch of six degrees landing on level ground (Series A) and landing on a three degree ground slope (Series B).

Series A and B Increase Load to Ultimate (150% Limit). - Series A and B have the





Pitch Angle 0
Pitch Velocity 0
2-2 Impact
Ground Slope 0
Thrust 2/3 g

Figure 22. Sliding Footpad, μ= .75

Figure 23. Sliding Footpad, μ= 1.00

indentical conditions of Series 6 and 14, respectively. The reasons for reaching the ultimate load in Series A and B are the same as the reasons for reaching the limit load in Series 6 and 14. The increase in load extends the operational limits of the profile but retains the original shape; however, the "requirements" of landing capabilities at V_V = 10 ft./sec., V_H = 0 ft./sec., and V_V = 6 ft./sec., V_H = 3 ft./sec. are not met even using ultimate loads.(See Figures 24 and 25)

There was one region where an increase in loads did not increase the profiles. That was for low (under two feet per second) vertical velocities and high (over four feet per second) horizontal velocities in the direction of the striking leg. In this region the profiles are limited by vehicle toppling and a change in loads has no effect on this part of the profile.

On the comparison plots of A and B with 6 and 14, respectively, the profile for yield load (135% of limit) load has been drawn. (See Figures 26 and 27)

Pitch Angle 6° Thrust 2.3 g
Pitch Velocity 0 2-2 Yaw

Non-Sliding Footpads

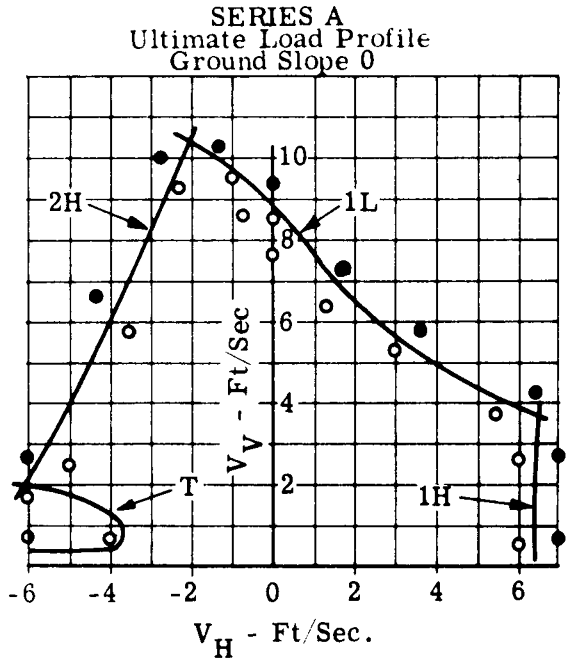


Figure 24. Ultimate Load With Pitched Vehicle (6°)

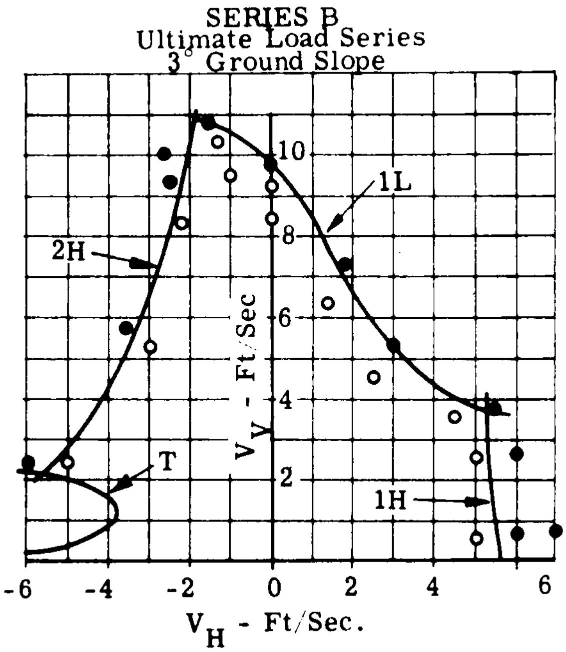


Figure 25. Ultimate Load With Pitched Vehicle (6°) and Small Ground Slope

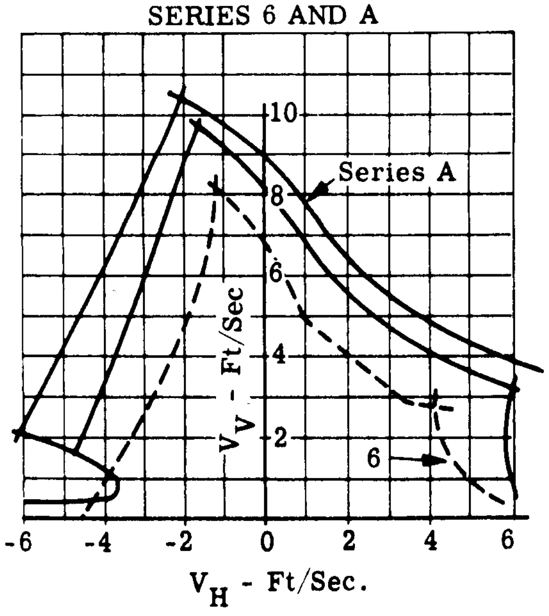


Figure 26. Effect of Increased Load -Pitched Vehicle

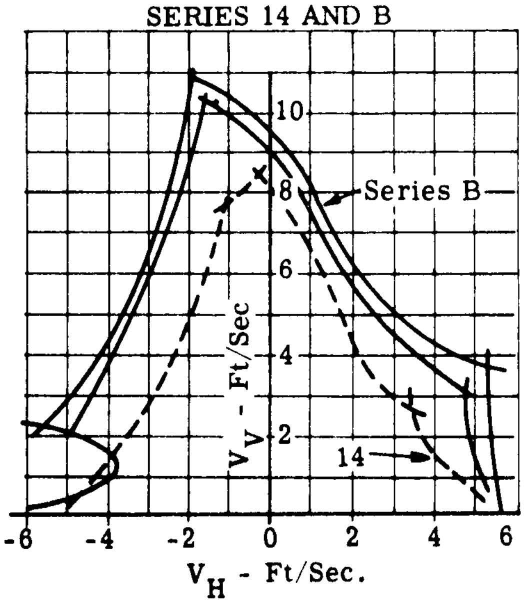


Figure 27. Effect of Increased Load Pitched Vehicle with Small Ground Slope

Engine Failure Series

All other runs were made with either 2/3 g or 5/6 g lift from the main engine. An investigation was conducted to see how severe an engine failure ('0' g lift) would be on the landing profile. Since this would be an emergency condition, the investigation was made using ultimate loads. The two series covering this phenomena are for a level vehicle landing on level ground. Series C has a 2-2 leg orientation and Series D has a 1-2-1 leg orientation.

These profiles are symmetrical about $V_H = 0$ and only half of the profile is drawn. Both profiles indicate a capability of eleven feet per second vertical velocity without a horizontal velocity. This eleven feet per second compares to a vehicle hovering two feet off the ground with an instantaneous loss of power or lift. In such an event, the vehicle would impact the ground in approximately one-third of a second at about eleven feet per second velocity.

Series C (2-2 Yaw). - As can be seen in Figure 28, this profile was limited by the horizontal load on leg one vertical velocities up to four feet per second. For vertical velocities from four feet per second to around eleven feet per second, strut bottoming was the limiting factor. At the very top, excessive axial load determined the profile.

This profile is fairly large and contains all the required points. It should be remembered that this profile is for ultimate loads.

Series D (1-2-1 Yaw). - For this configuration, a small horizontal velocity tips the vehicle up enough so that one strut is bottomed out by excessive energy input. This occurs at about two feet per second horizontal velocity for vertical velocities up to eleven feet per second. The top of the profile is limited by excessive axial load.(See Figure 29)

1-2-1 Yaw Investigation

In general, the 1-2-1 yaw landing would be more stable but throws more of the load and energy into one leg than a 2-2 landing. Since the operational limits have been defined by loads, the 1-2-1 yaw landings have a smaller range than the 2-2 landings. Series E and F are for a vehicle with a pitch angle of six degrees landing on level ground (E) and landing on a ground slope of three degrees (F). Series J and K are for a vehicle with a pitch angle of three degrees landing on level ground (J) and landing on a ground slope of three degrees (K).

Series E (6° Pitch, 0° Ground Slope). - Series E indicates limits of roughly three feet per second vertical velocity and horizontal velocities of three feet per second in the direction of the striking leg and four feet per second in the other direction. The profile is limited by axial load with most of the profile limited by the impact velocity of Leg 1. (See Figure 30)

Series F (6° Pitch, 3° Ground Slope). - Series F never exceeds two feet per second vertical velocity. The horizontal velocity limit is three feet per second in either direction. Most of this profile is limited by the impact velocity of Leg 1. For zero horizontal velocity the vehicle is limited to less than one foot per second.(See Figure 31)

Pitch Angle 0 Ground Slope 0
Pitch Velocity 0 Thrust 0
Non-Sliding Footpads

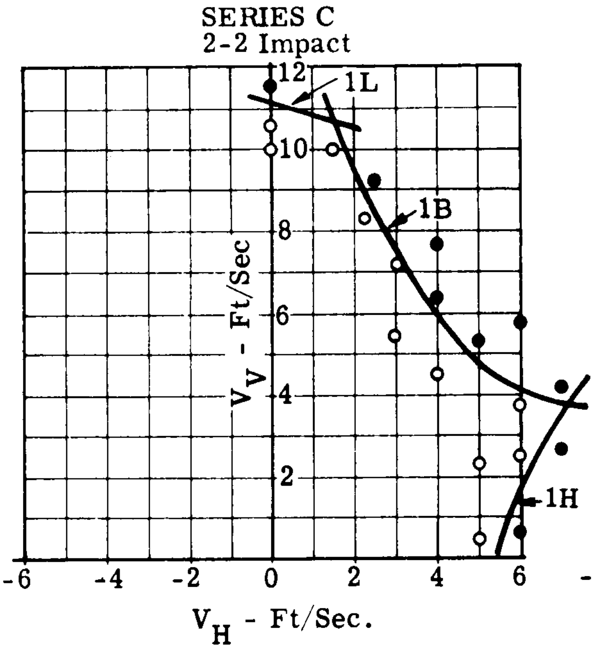

SERIES C
2-2 Impact

Figure 28. Engine Failure, 2-2 Yaw

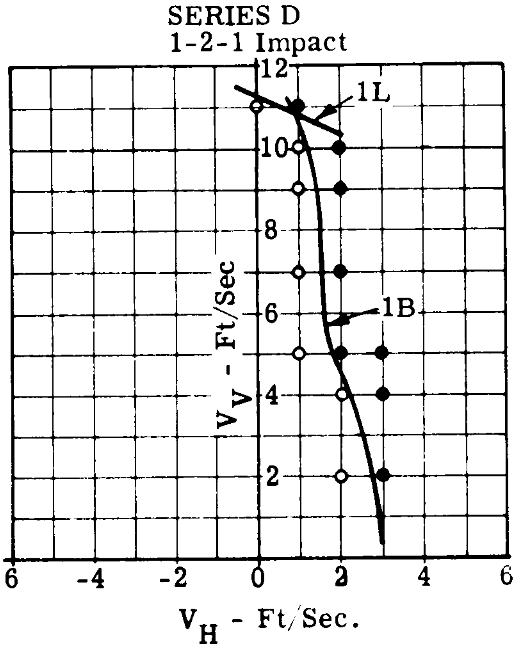

SERIES D
1-2-1 Impact

Figure 29. Engine Failure, 1-2-1 Yaw

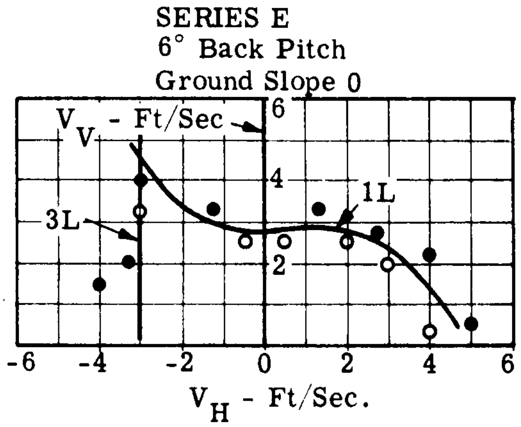

SERIES E
6° Back Pitch
Ground Slope 0

Figure 30. 1-2-1 Yaw With 6° Pitch

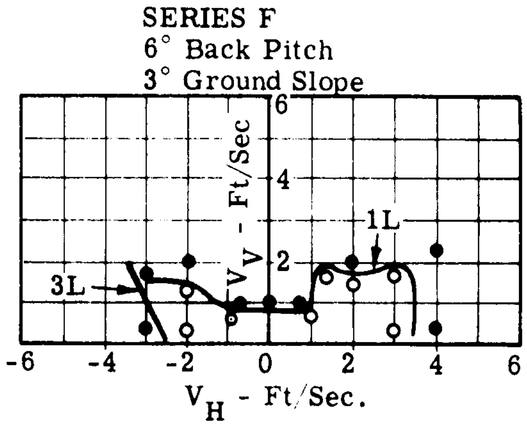

SERIES F
6° Back Pitch
3° Ground Slope

Figure 31. 1-2-1 Yaw With 6° Pitch and 3° Ground Slope

Both of these profiles indicate very narrow velocity regions where the vehicle can land safely in a 1-2-1 leg configuration with most of the limitations due to the impact velocity of leg one. The reason for this can be seen by an examination of the case when the vehicle is allowed to free fall pivoting about the number three leg. Upon contact of Leg 1 for Series E, the center of gravity drop is .99 feet and the loss of potential energy is

$$M\ g\ h = 105.5 \times 10.67 \times .99 = 1110 \text{ foot-pounds.}$$

This potential energy is converted to kinetic energy in the form of angular rotation about the number three leg.

The moment of inertia about leg three is

$$I = I_0 + m\ell^2 = 1720 + 105.5\,(11.75)^2 = 16{,}300 \text{ slug-feet}^2$$

Equating the kinetic energy of the rotational motion to the loss in potential energy gives

$$1/2\, I\ \omega^2 = M\ g\ h \text{ or } \omega^2 = \frac{2\ M\ g\ h}{I}$$

$$\omega^2 = \frac{2(1110)}{16{,}300} = .136 \text{ or } \omega = .37$$

The velocity of the number one leg is .37 x 18.9 = 7 feet per second.

In a similar manner for Series F the velocity of leg one is 8.47 feet per second.

This is the overriding factor influencing the impact velocity of Leg 1. For the actual profiles, adding to this are the translational velocity of the vehicle and the additional angular motion due to the forces on Leg 3.

Series J (3° Pitch, 0° Ground Slope, 2/3-g Thrust). - The allowable axial load, in either Leg 1 or Leg 3,limited the entire profile. The sides are limited to between three and four feet per second horizontal velocity due to excessive energy input into the forward leg in the direction of the horizontal velocity. The top of the profile is limited to slightly over six feet per second vertical velocity due to the impact velocity of Leg 1. (See Figure 32)

Series K (1-2-1 Yaw, 3° Pitch, 3° Ground Slope, 2/3-g Thrust). - As with Series J, the limit axial load in either Leg 1 or Leg 3 determined the entire profile. The horizontal velocity limits are about three feet per second downslope and three-and-one-half feet per second upslope due to the excessive energy input into the leading leg. The allowable vertical velocity is limited to less than three feet per second for horizontal velocities between plus and minus two feet per second due to the high impact velocity of Leg 1. (See Figure 33)

SERIES J
3° Back Pitch
0 Ground Slope

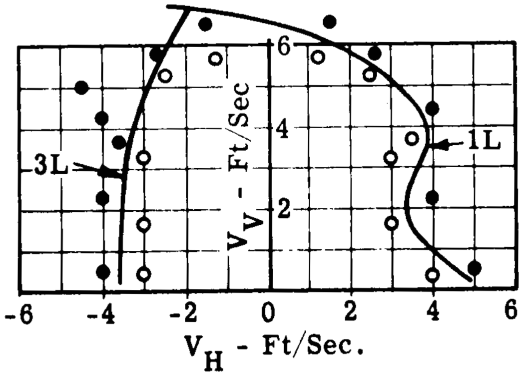


Figure 32. 1-2-1 Yaw With 3° Pitch

SERIES K
3° Back Pitch
3° Ground Slope

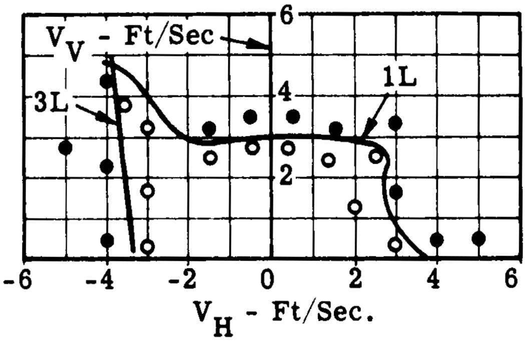


Figure 33. 1-2-1 Yaw With 3° Pitch and 3° Ground Slope

Landing on a 15° Ground Slope

Series G (0° Pitch, 15° Ground Slope, 5/6-g Thrust). - The investigation into the landing velocity range on a fifteen degree ground slope was limited to a 2-2 yaw orientation. For this configuration the conversion of potential energy into kinetic energy gives an impact velocity of about six feet per second. This is with the vehicle rotating in the 2-2 plane with two legs on the ground and impacting the other two legs after rotating fifteen degrees.

In the actual profile with the top limited by the impact velocity of these legs the initial translational velocity is about eight feet per second and only about four feet per second when leg one impacts. This means that the legs that initially contact the ground absorb a significant amount of energy before the other set of legs impact.

The sides of the profile are limited by the horizontal loads on the leading legs.

This profile indicates a good range for a fifteen-degree ground slope; however, none of the required points are contained in this profile.(See Figure 34)

Series H (2-2 Yaw, 15° Forward Pitch, 15° Ground Slope, 5/6-g Thrust). - This profile is limited by the allowable loads on the vehicle. For downslope horizontal velocity it is limited at approximately four feet per second and for upslope velocities, it is limited at over five feet per second. The vertical velocity is limited to slightly over nine feet per second from zero horizontal velocity to almost four feet per second upslope velocity. This profile has a wide range of landing velocities and contains the "required points" of V_V = 6 ft./sec., V_H = ± 3 ft./sec.

If the V_V, V_H axis is rotated counterclockwise 15°, this profile closely resembles Series 2, zero-degree ground slope, and zero pitch angle. (See Figure 35)

Obstacle Impact Runs

Six runs were made where a pair of legs struck an obstacle six inches high. Three runs were made with the leading legs striking the obstacle and three with the trailing legs hitting the obstacle. The critical values for these runs are shown in Table 3.

These runs indicated very little difference from the comparable nonsliding footpad case (Series 2). In either case (with or without the obstacle) the footpads could not move. The only difference occurs when the footpads climb over the six-inch obstacle. During this time period there is a force tending to retard the movement of the footpad over the obstacle; however, this force is relatively small (defined as the coefficient of friction times the force normal to this ledge) and lasts for a short time interval so the overall effect on the vehicle is small.

Table 3. Summary Table of Critical Values
for
Obstacle Impact Runs

V_V	V_H	Max. Stroke	Min. Beta	Max. Vertical	Max. Horizontal
The leg number for the value given is in parentheses.					
		Forward Legs Strike Obstacle			
1	-5	.80 (2)	.39 (1)	1264 (2)	-1635 (2)
2	-5	.86	.44	1223	-1648
3	-5	.89	.52	1578	-1615
		Rear Legs Strike Obstacle			
1	+5	.81 (1)	.40 (2)	1581 (1)	1571 (1)
2	+5	.87	.43	1416	1665
3	+5	.89	.49	1578	1605

Pitch Velocity 0
2-2 Yaw

Non-Sliding Footpads
Thrust 5/6 g

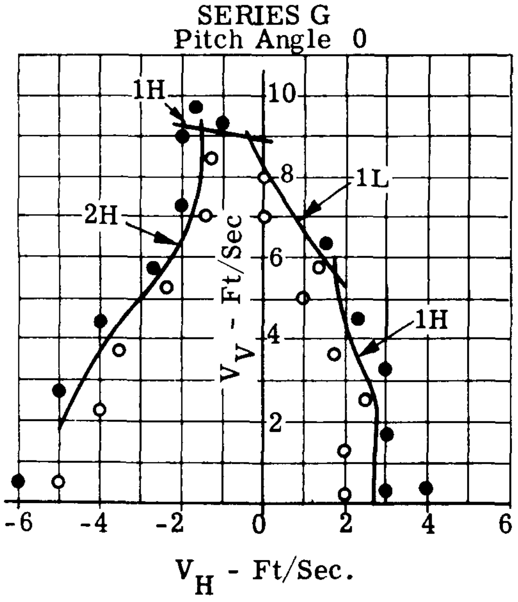


Figure 34. 15° Ground Slope

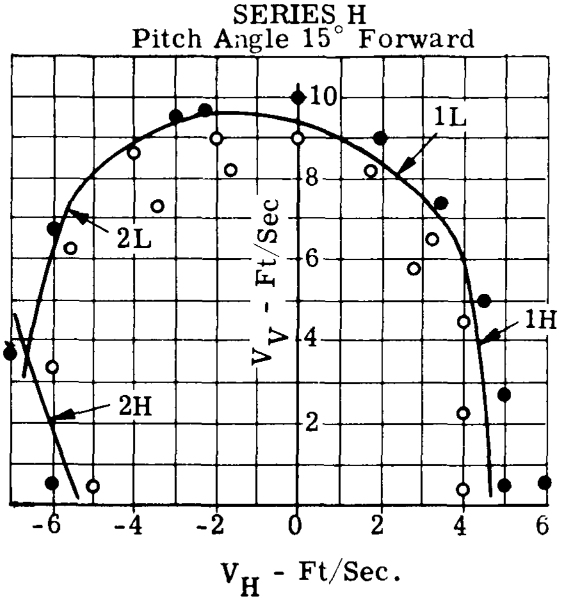


Figure 35. 15° Ground Slope With -15° Pitch

Since the obstacle impact runs duplicated the nonsliding footpad case, the obstacle impact runs were discontinued.

COMPARISON DATA

The Effect of Yaw

In almost every case the limitations on the vehicle have resulted from excessive loads, either axial or horizontal load on the legs. There is a considerable reduction in the operational range of the vehicle for any condition that places most of the energy or loads onto one or two legs. This reduction is especially large if most of the energy or loads transmitted to one leg versus being transmitted to two legs.

This means that the effect of yaw is significant in all cases but most significant when combined with pitch and/or ground slope. This is illustrated in Figures 36, 37, and 38.

In the comparison of Series 1 and 2, while the profile for Series 1 is reduced substantially from that of Series 2, both profiles contain all the required points so they are considered to have a large landing velocity range. (See Figure 39.)

Effect of Thrust

As can be seen in Figures 40 thru 43, the difference in the operational limitations for a 2/3 or 5/6 g thrust is small. This is true for a pitched vehicle or for a vehicle landing on a small ground slope.

Effect of Pitch

2-2 Yaw - The first comparison is for level ground, nonsliding footpads, and a 2-2 yaw orientation. This is a comparison of Series 2, 11, and 6 as shown in Figure 44. A three-degree pitch angle (Series 11) considerably reduces the operational limitations of the zero pitch (Series 2) profile. The step from three degrees to six degrees does not produce as pronounced a change as the initial three degree step but does shrink the operational range of the horizontal velocities even further. Figure 45 is a similar comparison except that the engine thrust is 5/6 g in Figure 45 (Series 7 and 15) and 2/3 g in Figure 44.

Without any vehicle pitch all of the required points are within the limits of the profile, but with a three- or six-degree pitch, none of the required points were met. In examining these profiles it should be pointed out that for zero pitch angle the excessive strut axial load is caused by excessive energy input while for a pitched vehicle, the excessive axial load results from a high impact velocity.

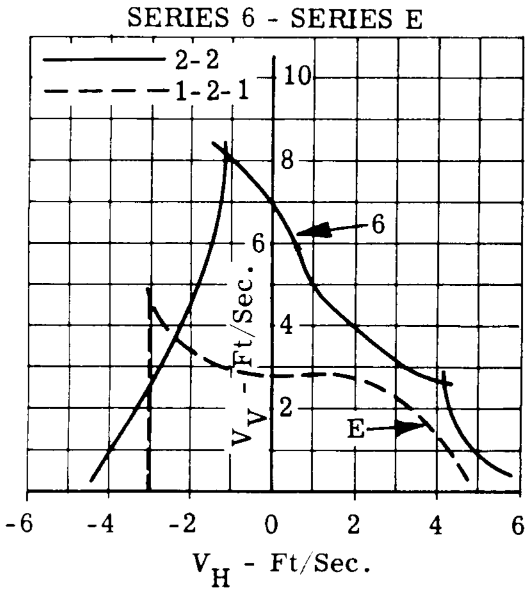


Figure 36. Effect of Yaw
Pitched Vehicle (6°)

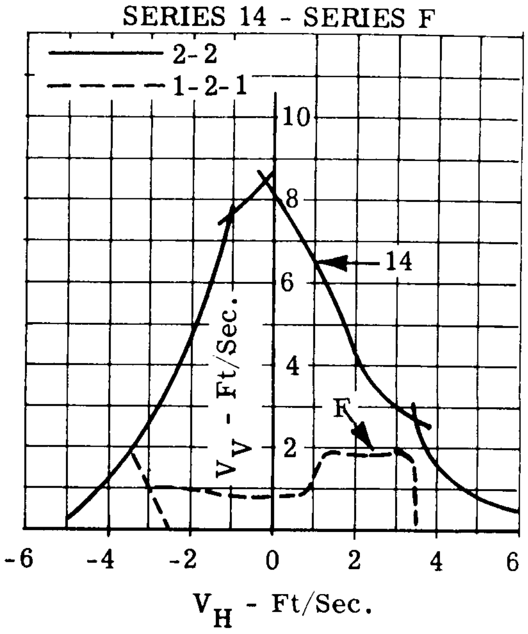


Figure 37. Effect of Yaw
Pitched Vehicle (6°) With Ground Slope

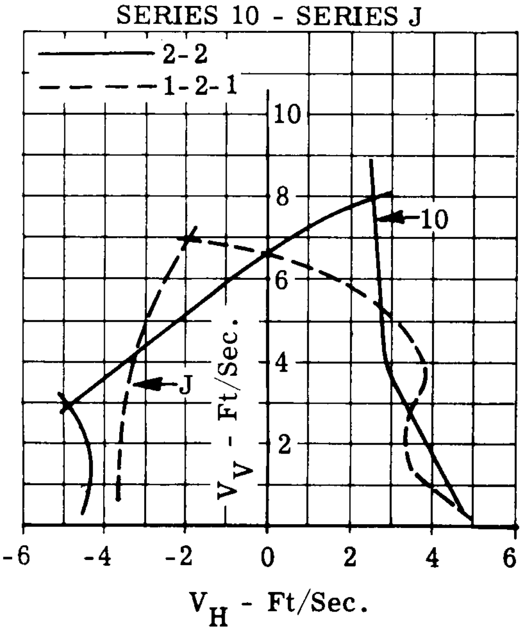


Figure 38. Effect of Yaw
Pitched Vehicle (3°)

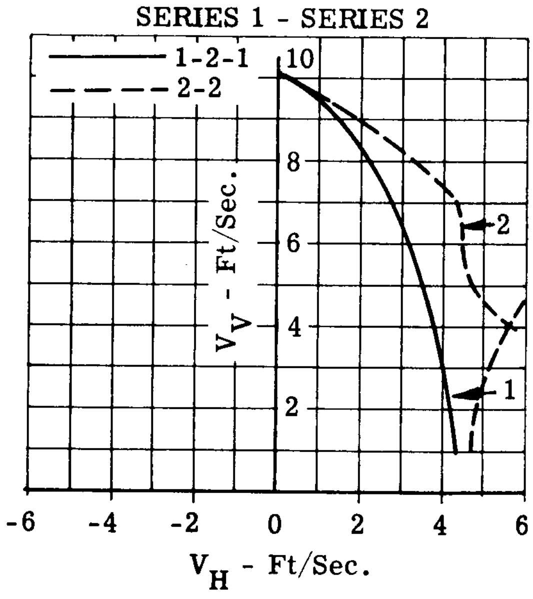


Figure 39. Effect of Yaw
Standard Landing

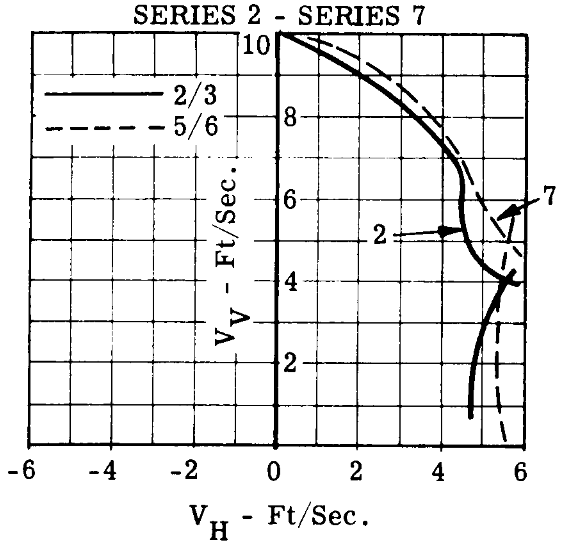


Figure 40. Effect of Thrust
Standard Landing

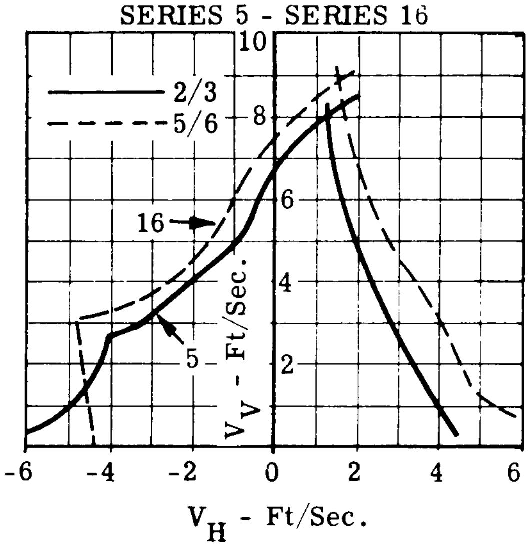


Figure 41. Effect of Thrust
Pitched Vehicle (-6°)

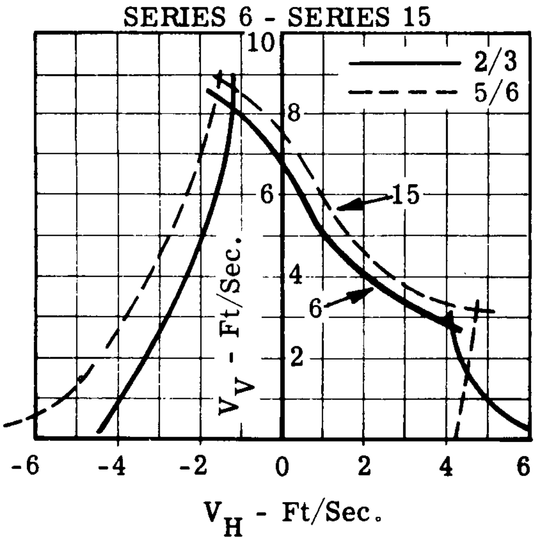


Figure 42. Effect of Thrust
Pitched Vehicle (6°)

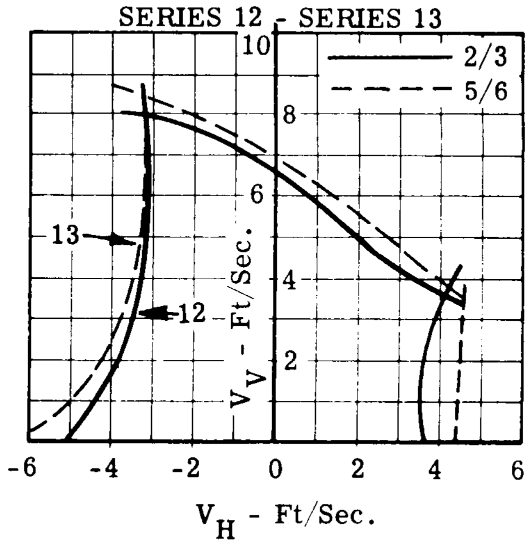


Figure 43. Effect of Thrust
Ground Slope 3°

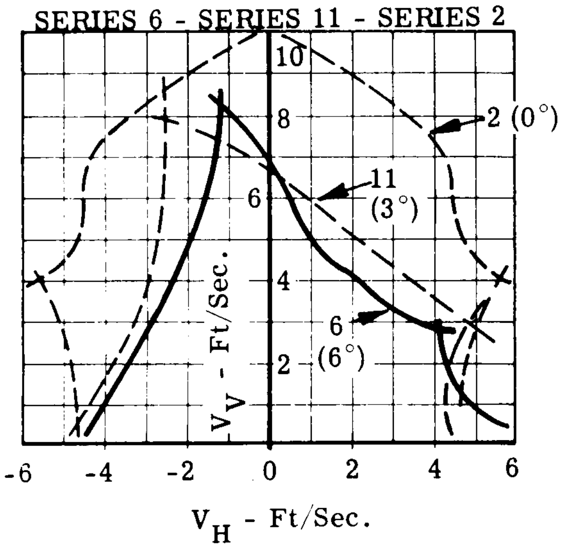


Figure 44. Effect of Pitch, 2-2 Yaw

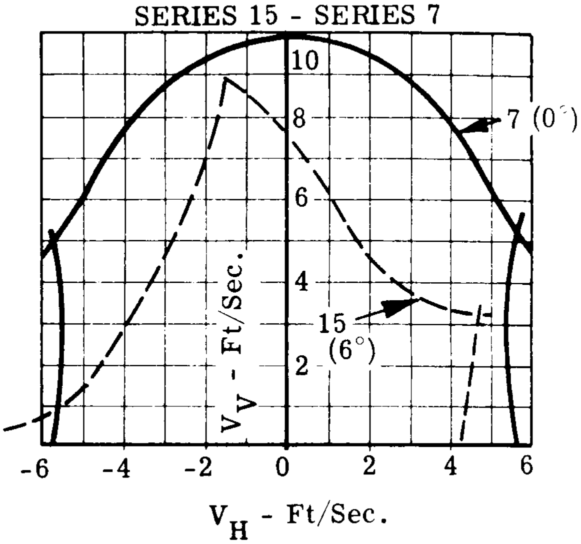


Figure 45. Effect of Pitch, 2-2 Yaw, Lunar Mode

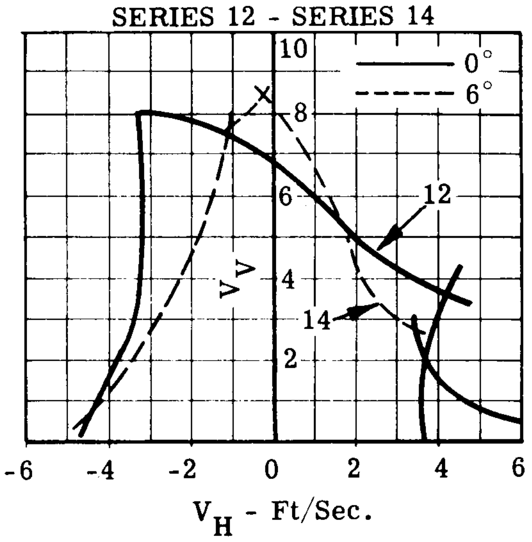


Figure 46. Effect of Pitch, 2-2 Yaw, 3° Ground Slope

With a small ground slope, the effect of vehicle pitch on the operational profile is to alter the shape from an approximate trapezoid (zero pitch angle) to an approximate triangle (6° pitch angle). Thus the horizontal velocity range is reduced by vehicle pitch but the maximum vertical velocity is about the same. (See Figure 46.)

1-2-1 Yaw - Any pitching of the vehicle with this leg orientation reduces the landing velocity limitations. The limiting factor with these conditions is either the leg off of the ground at the time of initial vehicle contact will impact the ground at a high velocity or the leg making the initial ground contact will have to absorb a high percentage of the total vehicle energy.

As can be seen in Figure 47, a 1-2-1 leg orientation combined with a pitched vehicle will greatly curtail the landing velocity capability of the vehicle.

This reduction in landing velocity range is increased if the landing conditions are compounded by landing on a small ground slope. (See Figure 48.)

Effect of Ground Slope (3°)

Although the ground slope of three-degrees is termed small, it reduces the limits of the operation profiles considerably. Comparing Series 2 with 12 (Figure 49) or Series 7 with 13 (Figure 50) illustrates this reduction. In the upper portion of the profiles the reduction is due to the high impact velocity of Leg 1. The level ground series is limited in this region from an excessive energy input while the series with ground slope is limited from an excessive impact velocity.

The limits on the "sides" of the profile, due to horizontal load, are also reduced. This is caused primarily from the vehicle being tipped up on two legs for a longer period of time for the ground slope series than for the level ground series.

For a vehicle with large back pitch (6°) the effect of ground slope is negligible. This is illustrated by the comparison of Series 6 (level ground) to Series 14 (3° ground slope) in Figure 51. These profiles are very close. For horizontal velocities in the direction of the leading legs, the vehicle is tipped up on these legs in either case so that the horizontal load inputs are very close. The portion of the profiles limited by the impact velocity of leg one is altered by the three-degree ground slope but the change is not great.

Effect of Pitch Velocity

If the vehicle lands on all four legs the fact that it is pitching does not have a big influence on limit landing velocities. The profile for a pitching vehicle flattens at high vertical velocities. The rotating velocity adds to the translational velocity of the vehicle giving an impact velocity of ten feet per second for one set of legs for translational velocities less than ten feet per second. The changes in the horizontal velocity limitations are insignificant. (See Figure 52.)

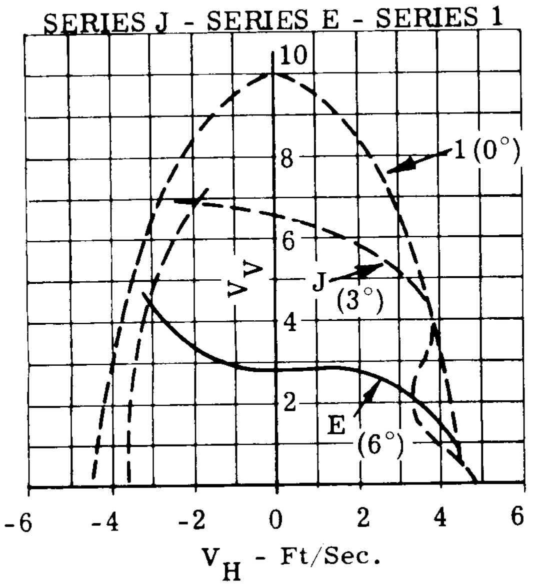


Figure 47. Effect of Pitch, 1-2-1 Yaw

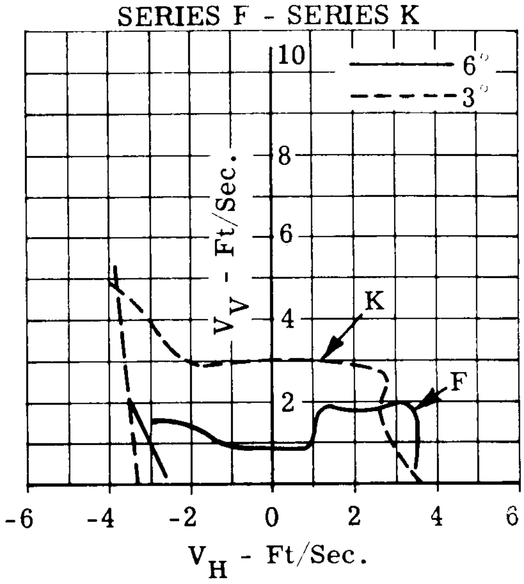


Figure 48. Effect of Pitch, 1-2-1 Yaw With 3° Ground Slope

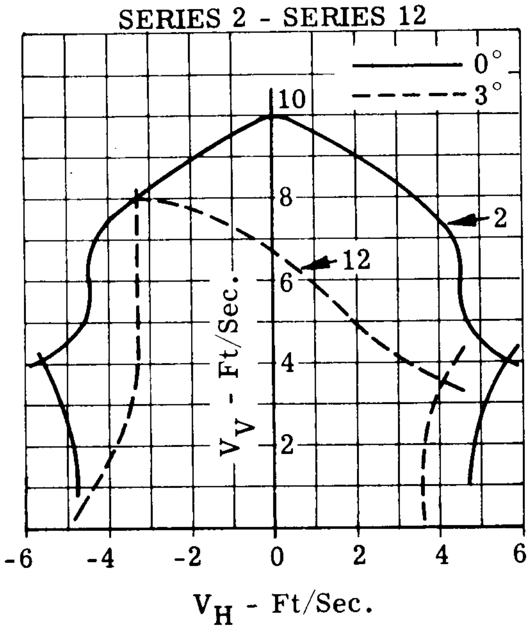


Figure 49. Effect of Ground Slope

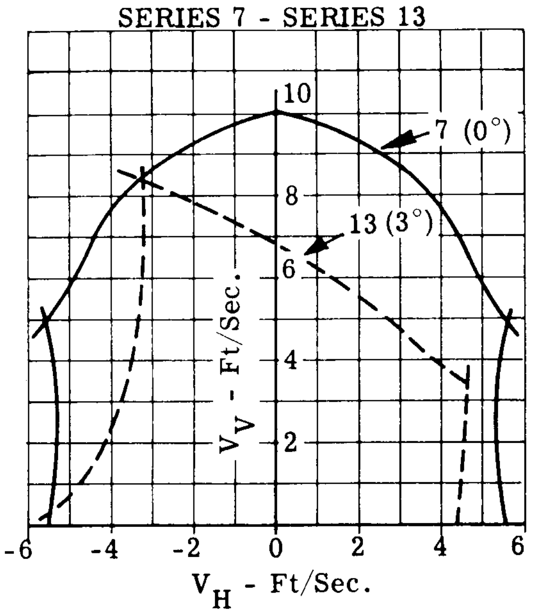


Figure 50. Effect of Ground Slope Lunar Mode

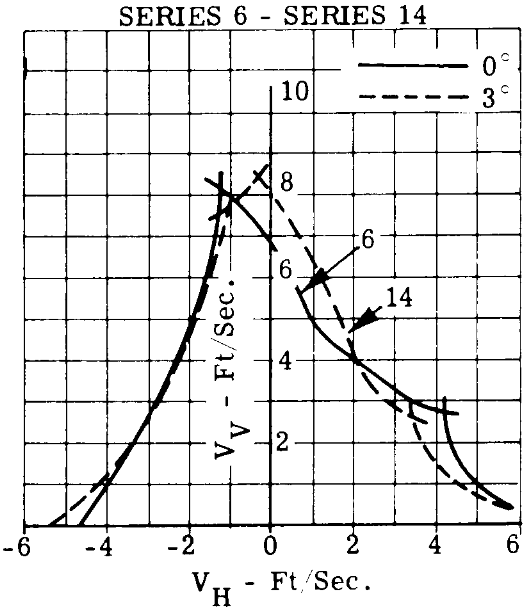


Figure 51. Effect of Ground Slope Pitched Vehicle (6°)

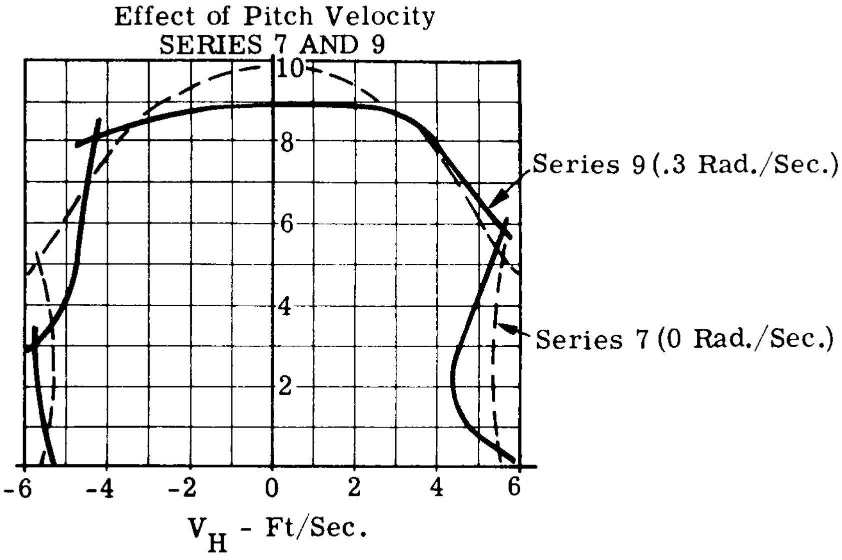


Figure 52. Effect of Pitch Velocity

Effect of Sliding Footpads

The sliding footpads do not affect the operational limits for vertical velocities above seven feet per second. In this region, the limit axial strut load defines the profiles and the effect of sliding footpads is negligible. For vertical velocities under seven feet per second, the profiles are changed considerably when the effect of sliding footpads allows the vehicle to slide out and not exceed the horizontal footpad load. This is true for Series 3 ($\mu = .75$), and would be amplified even more for lower coefficients-of-friction.

For some coefficients-of-friction the profile could be reduced as shown by Series 4, $\mu = 1.00$. However, this is small and the range of horizontal velocities is still large, about 4.5 feet per second. (See Figure 53.)

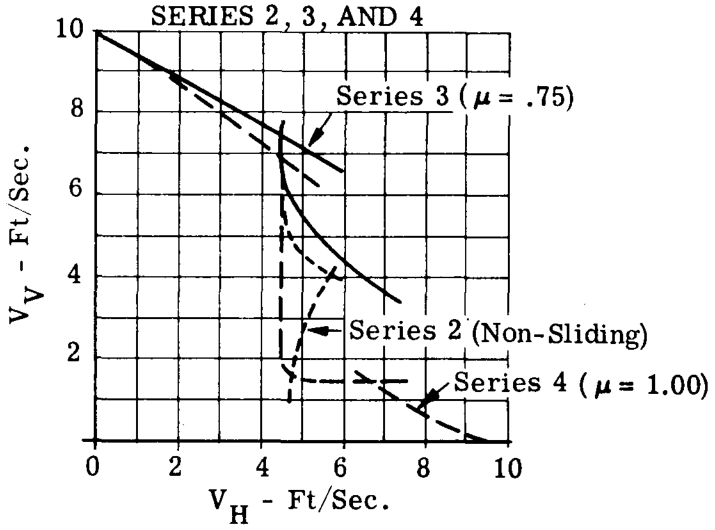


Figure 53. Effect of Sliding Footpads

CONCLUSIONS

For normal landing conditions of a level, nonrotating vehicle landing on level ground the landing velocity range is adequate. This is basically true any time all four legs contact the ground initially. Thus landing on a ground slope is not too severe if the vehicle is pitched to contact all legs upon initial ground contact. This is also true of a rotating vehicle landing on level ground.

When only one or two legs of the vehicle strike the ground initially, the landing velocity range of the vehicle is considerably reduced. This would occur with a pitched vehicle landing on level ground or level vehicle (with the gravitational field) landing on a ground slope. These conditions place a high percentage of the work into one or two legs thus reducing the landing operational range.

Throughout this study, including the various parameter changes, the limitation of the vehicle, almost exclusively, has been the loads on it. While there could be changes in the strut to reduce the axial load on the legs, this would not affect the horizontal footpad load. Thus, increasing the strut length to reduce the axial load on the leg would not change the horizontal footpad load appreciably. This indicates that to increase the capability of the vehicle would require an increase in the allowable loads.

The series run using ultimate loads instead of limit loads indicated that this increase the velocity range approximately twenty per cent. While this increase did not include all the required points for all conditions, it should be considered a major increase in the capability of the vehicle.

Of course, if any major change was undertaken to increase the allowable loads of the structural part of the vehicle, then the strut configuration should be investigated. This would cover not only increasing the capability of the struts but also investigating changing the geometry of the legs to improve the landing performance of the vehicle.

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APPENDIX A

SUPPORTING DATA FOR OPERATIONAL PROFILES

This section contains data taken from the computer data generated in this study. These data are the maximum axial load in the strut, the maximum horizontal load, the maximum stroke, and the minimum stability angle for each of the runs made. A table of these data is presented for each of the conditions studied. The operational profiles were plotted from these tables.

Table A-1. Summary Table of Critical Values for Series 1

V_V	V_H	Max. Vert.	Max. Horiz.	Max. Stroke
1	3	(1) 1478	(1) 829	(1) .860
1	4	1893	1186	.943
1	5	2324	1568	.991
2	5	2609	1601	1.154
2	4	1889	1201	1.076
3	4	2278	1213	1.100
3	3	1865	837	.961
4	3	1979	880	1.014
4	4	2378	1229	1.124
5	4	2586	1283	1.152
5	3	2091	924	1.069
6	3	2202	964	1.111
6	4	2960	1334	Bottomed
7	4	3285	1380	Bottomed
7	3	2307	1000	1.137
7	2	1800	644	1.037
8	2	1990	664	1.068
8	3	2460	1030	1.156
9	3	2647	1057	Bottomed
9	2	2323	673	1.093
9	1	2101	329	.967
10	1	2383	337	.994
10	0	2261	0	.830

All values are for leg No. 1 as indicated by the (1) next to the first row of numbers.

Table A-2. Summary Table of Critical Values for Series 2

V_V	V_H	Max. Stroke	Min. Beta	Max. Vertical	Max. Horizontal
The leg number for the value given is in parentheses.					
1	4	.788 (1)		1110 (1)	1344 (1)
1	5	.889		1323	1730
2	5	.927		1329	1740
2	4	.827		1090	1327
3	4	.834		1647	1230
3	5	.944		1627	1675
3	6	1.025		1753	2137
4	6	1.032		2300	2050
4	5	.947		2046	1568
5	5	.980		2269 (@ .084)	1513
5	4	.946		1971	1180
6	4	1.039	.753 (1)	2081	1245
6	5	1.058	.735	2325	1592
7	5	1.114	.744	2446	1655
7	4	1.079	.758 (2)	2221	1295
8	4	1.106	.742	2334	1338
8	3	1.049	.752 (1)	2154	980
9	3	1.078	.752	2478	1008
9	2	1.014	.755	2308	658
9	1	.923	.754 (2)	2136	323
10	1	.956	.756	2426	332
10	0	.84	.773	2337	0

Table A-3. Summary Table of Critical Values for Series 3

V_V	V_H	Max. Stroke	Min. Beta	Max. Vertical	Max. Horizontal
The leg number for the value given is in parentheses.					
0	8	.320 (1)	.7505 (1)	544 (1)	460 (1)
0	9	.320	.7502	544	460
1	9	.472	.7422	685	555
2	9	.582	.7409	945	731
3	9	.663	.7376	1401	1072
3	8	.670	.7368	1427	1120
3	7	.678	.7350	1465	1173
4	7	.794	.7289	2642	1769
4	6	.810	.7325	2189	1684
5	6	.899	.7371	2337	1841
5	5	.913	.7378	1878	1663
6	5	.976	.7449 (1)	2020	1799
6	4	.978	.7420 (2)	1935	1446
7	4	1.017	.7430	2178	1461
7	5	1.027	.7438	2253	1905
8	5	1.064	.7408	2545	1965
8	4	1.051	.7475 (2)	2404	1478
8	3	1.032	.7550 (1)	2216	1024
9	3	1.052	.7550 (2)	2479	1048
9	2	1.014	.7532 (1)	2316	664
9	1	.924	.7545 (2)	2137	326
10	1	.956	.7550 (2)	2486	334
10	0	.842	.7637 (2)	2406	-

Table A-4. Summary Table of Critical Values for Series 4

V_V	V_H	Max. Stroke	Min. Beta	Max. Vertical	Max. Horizontal
The leg number for the value given is in parentheses.					
0	11	.892(1)	Toppled	1137 (1)	1527 (1)
0	10	.870	.5366 (1)	1535	1440
0	8	.822	.6323 (1)	1021	1282
0	9	.840	.6096 (1)	1041	1334
1	9	.855	Toppled	1366	1748
1	8	.943	Toppled	1282	1677
1	7	.924	.524 (1)	1220	1596
2	7	.997	Toppled	1459	1909
2	6	1.009	.5481 (1)	1564	2192
2	5	.942	.6313	1380	1834
2	4	.844	.6929	1136	1404
3	4	.872	.7126	1341	1389
3	5	.976	.6637	1415	1882
4	5	.998	.6897	1750	1883
4	4	.894	.7284	1695	1360
5	4	.929	.7421	1869	1355
5	5	1.018	.7124	1989	1889
6	5	1.036	.7318 (1)	2134	1904
6	4	.975	.7453 (2)	2037	1371
7	4	1.017	.7488 (2)	2227	1391
7	5	1.055	.7211 (2)	2365	1916
8	5	1.075	.7152 (2)	2579	1927
8	4	1.053	.7526 (2)	2404	1416
8	3	1.026	.7489 (1)	2216	995
9	3	1.057	.7526 (1)	2479	1020
9	2	1.014	.7531 (1)	2316	664
9	1	.924	.7545 (2)	2137	326
10	1	.956	.7551 (2)	2486	334
10	0	.842	.7637 (2)	2406	

Table A-5. Summary Table of Critical Values for Series 6

The leg number for the value given is in parentheses.

V_V	V_H	Max. Vertical	Max. Horizontal	Max. Stroke	Min. Beta
.50	5.00	1807.9 (1)	1587.2 (1)	.948 (1)	.6585 (2)
.55	6.00	1731.8	2436.0	1.068	.6118
1.50	6.00	1759.7	2512.2	1.077	.6010
1.40	5.00	1552.8	2128.7	1.020	.6586
1.30	4.00	1507.7	1591.4	.931	.6584
2.20	4.00	1651.2	1595.7	.939	.6585
2.40	5.00	1918.6	2073.0	1.020	.6587
3.55	4.50	2705.6	1599.7	.979	.6588
3.20	3.60	2427.7	1219.3	.922	.6586
2.85	2.70	1952.3	855.0	.863	.6584
3.50	2.25	2121.1	572.7	.847	.6584
4.00	3.00	2779.3	787.6	.914	.6586
4.60	2.40	2728.4	588.3	.886	.6586
3.95	1.80	2157.5	624.3	.829	.6584
4.70	1.50	2349.0	670.7	.833	.6584
3.90	1.00	1625.1	721.7	.770	.6582
4.60	.70	1656.7	758.0	.772	.6583
5.50	1.05	2278.6	714.4	.823	.6584
6.70	.45	2359.4	831.2	.817	.6585
6.00	.30	1849.4	842.9	.786	.6584
6.40	0.	1846.8	-957.5 (2)	.808 (2)	.6583
7.10	0.	2342.4	-997.9	.852	.6584
6.70	-.45	1806.3	-1172.6	.874	.6582
7.40	-.60	2079.6	-1280.1	.927	.6582
8.10	-.75	2386.4	-1388.3	.973	.6583
7.30	-1.75	1959.0 (2)	-1843.8	1.011	.6579
6.40	-1.40	1706.6	-1597.7	.941	.6579
5.50	-2.00	1474.7	-1756.3	.943	.6576
4.70	-1.50	1214.6	-1416.2	.855	.6576
3.95	-1.80	1141.1	-1407.6	.820	.6575
4.60	-2.40	1349.8 (2)	-1762.7	.926	.6573
4.00	-3.00	1433.2 (1)	-1885.9	.949	.6571
3.50	-2.25	1197.5 (2)	-1496.3	.845	.6573
2.85	-2.70	1230.1	-1534.7	.848	.6568
3.20	-3.60	1447.9	-1943.0	.950	.6351
2.20	-4.00	1453.9	-1854.9	.908	.5816
2.00	-3.00	1262.8	-1465.5	.809	.6243
1.20	-3.00	1206.0	-1310.8	.733	.5979
1.30	-4.00	1423.2	-1648.1	.832	.0014
.45	-4.00	2051.4	-1025.2	.809	.6538
.50	-5.00	2090.6	-2119.2	.967	.6529

Table A-6. Summary Table of Critical Values for Series 7

V_V	V_H	Max. Stroke	Min. Beta	Max. Vertical	Max. Horizontal
The leg number for the value given is in parentheses.					
0	4	.013 (1)	.755 (1)	322 (1)	388 (1)
0	5	.016	.751	480	390
0	6	.774	.202 (2)	1474	1800
1	6	.887	.011	1436	2016
1	5	.803	.372	1242	1636
2	5	.858	.437	1220	1648
2	6	.941	.118	1435	2050
3	6	.971	.271	1523	2051
3	5	.886	.514	1487	1617
4	5	.902	.579	1980	1518
4	6	.987	.399	2277 @ .090	1972
5	6	1.002	.482	2321	1892
5	5	.939	.636	2082	1476
6	5	1.007	.654	2267	1542
6	4	.981	.732	2004	1205
7	4	1.044	.724	2064	1258
7	5	1.069	.676	2341	1607
8	5	1.117	.652	2458	1665
8	4	1.076	.679	2264 @ .064	1302
8	3	1.001	.688	2040	952
9	3	1.034	.686	2352	982
9	2	.965	.729	2183	645
10	2	.995	.728	2433	662
10	1	.908	.758 (1)	2337	325
10	0	.788	.773	2297	0

Table A-7. Summary Table of Critical Values for Series 9

V_V	V_H	Max. Vertical	Max. Horizontal	Max. Stroke	Min. Beta
The leg number for the value given is in parentheses.					
.50	5.00	1522.2 (1)	1691.8 (1)	.802 (1)	.3173 (2)
.55	6.00	1483.0	2063.9	.886	.0015
1.50	6.00	1546.5	2260.5	.953	.0005
1.40	5.00	1430.2	1856.6	.872	.3439
1.30	4.00	1145.4	1467.3	.767	.5446
2.20	4.00	1147.4	1524.1	.822	.5693
2.40	5.00	1374.3	1926.0	.925	.4020
3.55	4.50	1587.0	1549.2	.888	.5808
3.90	5.40	1877.7	1917.5	.975	.4637
5.00	4.50	1930.7	1342.9	.919	.6521
5.50	5.25	2202.4	1640.6	.988	.5903
6.00	6.00	2513.8	1946.6	1.047	.5059
7.20	4.80	2297.9	1579.3	1.085	.6788
6.55	4.20	2057.7	1332.6	1.035	.7175
7.90	3.50	2104.3	1151.9	1.049	.6836
8.70	4.00	2368.9	1359.9	1.104	.6596
10.00	2.80	2720.9	964.8	1.062	.6792
9.10	2.45	2378.7	822.4	1.014	.6879
8.20	2.10	2044.9	685.8	.959	.7056
8.80	.90	2262.3	299.4	.869	.7471
8.10	.75	2001.3	247.3	.824	.7486
8.50	0.	2122.2	-34.7	.742	.7633 (1)
9.20	0.	2399.7	-32.6	.772	.7634
8.80	-.90	2248.3	-271.3 (2)	.839 (2)	.7422 (1)
9.50	-1.05	2501.5	-325.7	.883	.7562 (2)
9.10	-2.45	2339.6	-783.6	.988	.7106 (1)
8.20	-2.10	2011.1 (2)	-647.3	.928	.7348
7.10	-3.00	1993.0	-905.2	.947	.6966
7.90	-3.50	2246.5	-1101.9	1.019	.6900
8.70	-4.00	2513.7	-1307.7	1.082	.6632
7.20	-4.80	2421.2	-1518.6	1.071	.6935
6.55	-4.20	2226.2	-1274.8	1.012	.7318
5.50	-5.25	2428.1	-1558.9	.968	.6273
5.00	-4.50	2166.2	-1279.3	.902	.6858
3.90	-5.40	2299.0	-1584.1	.906	.5426
3.55	-4.50	1985.8	-1217.0	.807	.6416
2.40	-5.00	1494.9	-1440.7	.819	.5218
2.60	-6.00	1662.0	-1851.3	.919	.3194
1.50	-6.00	1656.0	-1725.9	.873	.2963
1.40	-5.00	1049.7	-1345.0	.773	.5035
.50	-5.00	1104.5	-1423.7	.745	.4567
.55	-6.00	1686.7	-1770.0	.840	.2295

Table A-8. Summary Table of Critical Values for Series 11

V_V	V_H	Max. Vertical	Max. Horizontal	Max. Stroke	Min. Beta
The leg number for the value given is in parentheses.					
.50	5.00	1406.8 (1)	1869.0 (1)	.969 (1)	.6783 (1)
.45	4.00	1153.8	1438.9	.874	.7106 (2)
1.30	4.00	1285.8	1660.5	.909	.7107 (2)
1.40	5.00	1503.0	2052.7	.996	.6840 (1)
2.40	5.00	1566.4	2007.7	1.002	.7035 (1)
2.20	4.00	1389.0	1498.8	.905	.7109 (2)
3.20	3.60	2038.0	993.5	.844	.7109
3.55	4.50	2452.8	1338.1	.918	.7111
4.50	3.75	2548.1	915.1 (2)	.830	.7111
4.00	3.00	2223.2	700.2	.806	.7109
4.60	2.40	2288.7	577.9	.859	.7109
3.95	1.80	1864.4	403.9	.782	.7107
4.70	1.50	2050.7	343.1 (1)	.797	.7108
5.50	2.00	2477.8	496.0 (2)	.871	.7110
6.40	1.40	2501.1	349.2 (1)	.854	.7110
5.50	1.05	2118.2	326.8	.789	.7108
6.70	.45	2449.7	331.7	.777	.7109
6.00	.30	2059.2	-361.1 (2)	.734	.7108
6.40	0.	2113.2	-463.9	.761 (2)	.7108
7.10	0.	2413.7	-472.2	.802	.7109
6.70	-.45	2130.6	-634.9	.829	.7108
7.40	-.60	2403.2	-704.3	.882	.7108
6.40	-1.40	1769.4	-1034.4	.912	.7106
7.30	-1.75	2089.3	-1212.1	.987	.7106
8.20	-2.10	2461.6	-1389.8	1.049	.7107
7.10	-3.00	2093.1 (2)	-1811.0	1.060	.7104
6.30	-2.50	1867.0	-1545.4	.997	.7104
5.25	-3.00	1822.2	-1762.2	.973	.7102
4.60	-2.40	1578.3	-1453.0	.894	.7102
4.00	-3.00	1548.5	-1712.0	.921	.7100
3.50	-2.25	1330.9	-1329.9	.802	.7101
2.85	-2.70	1175.9	-1428.7	.812	.7099
3.20	-3.60	1403.4	-1850.1	.929	.7060
2.20	-4.00	1414.0	-1814.0	.903	.6631
2.00	-3.00	1216.8	-1398.3	.786	.6954
1.20	-3.00	1173.8	-1267.7	.718	.6725
1.30	-4.00	1401.7	-1635.1	.834	.6300
1.40	-5.00	1594.6	-2022.7	.922	.5763
.50	-5.00	1656.2	-1961.5	.911	.6682
.45	-4.00	1701.9	-934.8	.733	.7026

Table A-9. Summary Table of Critical Values for Series 12

V_V	V_H	Max. Stroke	Min. Beta	Max. Vertical	Max. Horizontal
The leg number for the value given is in parentheses.					
.5	5	1.032 (1)	.591 (1)	1606 (1)	2250 (1)
.45	4	.949	.631	1341	1753
.4	3	.834	.665	1253	1262
1.2	3	.847	.669	1334	1336
1.3	4	.964	.647	1424	1904
2.2	4	.968	.658	1399	1861
2	3	.857	.674	1309	1304
2.85	2.7	.817	.689	1330	948
3.2	3.6	.899	.673	1753	1316
3.55	4.5	.985	.651	2270 @ .108	1727
4.5	3.75	.796	.689	2583	1123
4	3	.760	.693	1943	878 (1)
4.6	2.4	.886	.706	2308 @ .102	696 (2)
3.95	1.8	.826	.705	1788	519
4.7	1.5	.847	.706	1776	480
5.5	2	.914	.704	2472 @ .092	644
6.4	1.4	.909	.704	2395 @ .084	495 (2)
5.5	1.05	.850	.703	2081	414 (1)
6.7	.45	.847	.700	2551 @ .084	405
6	.3	.808	.701	1891	401
6.4	0	.796	.700	1816	410
7.1	0	.820 (1)	.700	2463	406
6.7	-.45	.765 (2)	.700	1751	465
7.14	-.6	.819	.700	2515 (1)	514
6.4	-1.4	.854	.699	1621 (2)	828
7.3	-1.75	.928	.699	2116 (1)	975
8.2	-2.1	.998	.700	2454 @ .072 (1)	1128
7.1	-3	1.029	.680	2005 (2)	1555
7.9	-3.5	1.084	.648	2234	1793
6.55	-4.2	1.073	.609	2205	2140
5.9	-3.6	1.013	.652	1964	1839
5.25	-3	.945	.684	1729	1540
4.5	-3.75	.965	.643	1739	1899
4	-3	.868	.685	1506	1524
3.2	-3.6	.892	.672	1329	1716
2.85	-2.7	.775	.693	1100	1300
2	-3	.746	.697	1155	1296
2.2	-4	.875	.660	1340	1716
1.3	-4	.813	.670	1315	1560
1.4	-5	.911	.616	1483	1960
.5	-5	.841	.697	1494	1793
.45	-4	.743	.704	1293	1434

Table A-10. Summary of Critical Values for Series 13

V_V	V_H	Max. Stroke	Min. Beta	Max. Vertical	Max. Horizontal
The leg number for the value given is in parentheses.					
0	5	.897 (1)	.414 (2)	1344 (1)	1882 (1)
0	4	.804	.613	1153	1525
1	4	.797	.622	1157	1521
1	5	.884	.404	1351	1876
2	5	.906	.452 (2)	1329	1846
2	4	.818	.614 (1)	1125	1500
3	4	.845	.646 (1)	1393	1523
3	5	.935	.533 (2)	1525	1880
4	5	.954	.594 (2)	2346 @ .104	1857
4	4	.837	.655 (1)	2304 @ .106	1291
4	3	.700	.681	1836	828 (1)
5	3	.757	.699	2352 @ .098	773 (2)
5	2	.769	.707	2200	524
6	2	.828	.704	2451 @ .090	585 (2)
6	1	.747	.703	1921	344 (1)
7	1	.801	.703	2626 @ .082	367
7	0	.694	.701	2297 @ .084	335
8	0	.744	.703	2665 @ .074	338
9	0	.787 (1)	.704	2616	340 (1)
9	-1	.885 (2)	.703	2574 @ .066	661 (2)
8	-1	.842	.704	2431	668
7	-1	.789	.689	1570	670
7	-2	.897	.665	1649 (2)	1078
8	-2	.943	.668	2339 (1) @ .074	1085
0	-5	.766	.481	1077 (2)	1294
0	-6	.854	.307	1211	1635
0	-7	.947	Toppled	1440	2072
1	-7	.935	Toppled	1770	2500
1	-6	.873	Toppled	1618	2139
1	-5	.797	.303 (1)	1424	1780
1	-4	.697	.482	1220	1435
2	-4	.784	.518	1265	1584
2	-5	.877	.351	1445	1956
3	-5	.943	.439	1510	2143
3	-4	.856	.548	1300	1742
3	-3	.739	.620	1096	1357
4	-3	.813	.581	1308	1489
4	-4	.914	.417	1390	1912
5	-4	.964	.396	1722	2001
5	-3	.859	.624	1555	1521
6	-3	.921	.637	1713	1534
6	-4	.988	.415	1926	2017
7	-4	1.033	.595	2103	2019
7	-3	.975	.639	1858	1541
8	-3	1.018	.638	2056 (1)	1538
8	-4	1.071	.592	2215 (2)	2031
9	-4	1.103	.593	2359 (2)	2030
9	-3	1.054	.637	2605 (1) @ .068	1538
9	-2	.982	.679	2307 (1)	1079

Table A-11. Summary Table of Critical Values for Series 14

V_V	V_H	FS(Max.)	FH(Max.)	S(Max.)	Beta(Min.)
The leg number for the value given is in parentheses.					
.50	5.00	1953.4 (1)	1623.8 (1)	.974 (1)	.6530 (1)
.55	6.00	2215.5	2370.3	1.086	.6096
1.50	6.00	2014.7	2873.2	1.119	.5711
1.40	5.00	1777.1	2502.0	1.082	.6405 (1)
1.30	4.00	1716.4	1892.3	1.008	.6584 (2)
1.20	3.00	1605.1	1196.7	.933	.6582
2.00	3.00	1660.3	1417.9	.943	.6583
2.20	4.00	1882.6	2033.7	1.021	.6585
3.20	3.60	2520.5	1673.2	.992	.6586
2.85	2.70	2153.3	1209.1	.942	.6584
3.50	2.25	2332.8	885.9	.932	.6584
3.00	1.50	1879.0	653.0	.871	.6582
3.30	1.20	1615.2	645.5	.869	.6582
3.95	1.80	2187.7	736.3	.919	.6584
4.60	2.40	2712.5	832.6	.966	.6586
5.50	2.00	2824.0	1054.8	.969	.6586
4.70	1.50	2150.6	1092.2	.930	.6584
5.50	1.05	2103.2	1177.2	.929	.6584
6.40	1.40	2916.1	1173.4	.964	.6586
7.40	.60	2573.2	1345.8	.957	.6586
6.70	.45	2033.2	1322.6	.936	.6585
7.10	0.	1873.2	1461.1	.939	.6584
7.80	0.	2195.7	1521.3	.953	.6585
8.50	0.	2635.4 (1)	1570.6	.963 (1)	.6585
8.10	-.75	1906.5 (2)	1760.3	.979 (2)	.6583
7.40	-.60	1694.4 (2)	1645.8	.942 (1)	.6582
6.40	-1.40	1453.9 (1)	-1669.0 (2)	.936 (2)	.6579 (2)
7.30	-1.75	1607.9 (2)	-1970.0	1.012	.6468 (1)
6.30	-2.50	1635.3 (1)	-2137.6	1.012	.6091 (1)
5.50	-2.00	1456.2	-1773.6	.931	.6576 (2)
4.70	-1.50	1327.0	-1432.7	.831	.6576
3.95	-1.80	1319.6	-1414.6	.822	.6575
4.60	-2.40	1736.4	-1766.8	.926	.6574 (2)
4.00	-3.00	1572.3	-1874.9	.946	.6483 (1)
3.50	-2.25	1587.4 (1)	-1493.3	.844	.6573 (2)
2.85	-2.70	1223.6 (2)	-1519.3	.842	.6568
3.20	-3.60	1431.8 (2)	-1914.1	.942	.6354
2.20	-4.00	1594.7 (1)	-1811.2	.895	.5827
2.00	-3.00	1248.9 (2)	-1440.0	.799 (2)	.6246
1.20	-3.00	1427.0 (1)	-1285.5	.775 (1)	.5985
1.30	-4.00	1400.1 (2)	-1602.7	.814 (2)	.0067
.45	-4.00	1877.4	-922.6	.810	.6529
.50	-5.00	1551.3	-1802.9	.838	.0074

Table A-12. Summary Table of Critical Values for Series 15

V_V	V_H	FS(Max.)	FH(Max.)	S(Max.)	Beta(Min.)
The leg number for the value given is in parentheses.					
.50	5.00	1396.3 (1)	1956.6 (1)	.917 (1)	.4363 (2)
.45	4.00	1211.6	1616.7	.836	.6240
1.30	4.00	1137.5	1511.8	.820	.5907
1.40	5.00	1326.0	1843.8	.908	.4293
2.40	5.00	1485.7	1838.6	.926	.4879
2.20	4.00	1293.8	1498.9	.832	.6197
3.20	3.60	1873.4	1388.2	.804	.6586
3.55	4.50	2422.7	1683.9	.902	.6316
4.50	3.75	3000.0	1176.9	.852	.6588
4.00	3.00	2304.3	953.4	.783	.6586
3.50	2.25	1807.0	739.0	.708	.6584
3.95	1.80	1785.8	504.1	.712	.6584
4.60	2.40	2340.0	581.6	.782	.6586
5.50	2.00	2508.2	597.7	.791	.6586
4.70	1.50	1927.1	613.7	.715	.6584
5.50	1.05	2029.3	690.6	.717	.6584
6.40	1.40	2597.5	673.0	.785	.6586
7.40	.60	2472.5	791.7	.761	.6586
6.70	.45	2072.0	799.3	.723 (1)	.6585
7.10	0.	1965.0	906.3 (1)	.747 (2)	.6584
7.80	0.	2316.0	-939.5 (2)	.800	.6585
7.40	-.60	1778.8	-1168.0	.833	.6582
8.10	-.75	2141.7	-1281.6	.890	.6583
8.80	-.90	2505.3 (1)	-1391.0	.941	.6500
8.20	-2.10	1951.0 (2)	-1939.0	1.001	.4791
7.30	-1.75	1677.4	-1685.2	.926	.5630
6.30	-2.50	1363.7	-1876.4	.936	.5179
5.50	-2.00	1157.8	-1526.0	.825	.6481
4.60	-2.40	1160.5 (2)	-1520.6	.814	.6573 (2)
5.25	-3.00	1420.7 (1)	-1883.0	.918	.6031 (1)
4.50	-3.75	1444.0 (2)	-2024.0	.938	.4347
4.00	-3.00	1229.2	-1631.3	.837	.6071
3.20	-3.60	1288.5	-1685.0	.833	.5646
3.55	-4.50	1477.5	-2090.2	.931	.4464
2.40	-5.00	1519.6	-1993.2	.875	.3253
2.20	-4.00	1326.2	-1614.1	.780	.5085 (1)
1.30	-4.00	1243.5	-1446.5	.689	.5385 (2)
1.40	-5.00	1473.2	-1771.9	.782	.3949 (1)
.50	-5.00	1530.1	-1200.8	.763	.5769
.55	-6.00	1638.6	-1659.9	.890	.3899
.60	-7.00	1664.0	-2092.9	.967	.0012

Table A-13. Summary Table of Critical Values for Series A

V_V	V_H	Max. Stroke	Min. Beta	Max. Vertical	Max. Horizontal
The leg number for the value given is in parentheses.					
0.6	7	1.120 (1)	Toppled	2076 (1)	3249 (1)
.55	6	1.068	.61 (2)	1743	2432
2.6	6	1.081	.61 (2)	2090	2469
2.8	7	1.122	Toppled	2300	2857
4.25	6.3	1.099	.60 (2)	3824	2496
3.9	5.4	1.044	.66	3268	2031 (1)
5.25	3	.94	.66	3292	630 (2)
5.9	3.6	.992	.66	3972	814 (2)
7.3	1.75	.922	.66	3584	684 (1)
6.4	1.4	.873	.66	2838	687 (1)
7.8	0	.892	.66	2718	-1033 (2)
8.5	0	.928	.66	3129	-1062
9.2	0	.960	.66	3572	-1092
8.8	-9	1.014	.66	2763	-1499
10.2	-1.2	1.081	.66	3677	-1717
9.5	-1.05	1.051	.66	3136 (1)	-1606
9.1	-2.45	1.115 (2)	.66	2456 (2)	-2349
10	-2.8	1.151	.62 (2)	2753 (1)	-2605
6.55	-4.2	1.122	.53 (1)	2070 (2)	-3082
5.9	-3.6	1.075	.65 (1)	1770	-2544
2.4	-5	.984	.52 (2)	1618	-2257
2.6	-6	1.042	.47 (2)	3487	-2664

Table A-14. Summary Table of Critical Values for Series B

V_V	V_H	Max. Stroke	Min. Beta	Max. Vertical	Max. Horizontal
The leg number for the value given is in parentheses.					
.6	7	1.156 (1)	Toppled	2561 (1)	4174 (1)
.55	6	1.104	.61 (1)	2126	2600
.5	5	.976	.65	1946	1646
2.4	5	1.077	.63	2088	2414
2.6	6	1.116	.57 (1)	2290	2770
3.6	4.5	1.052	.66 (2)	3092	2173
3.9	5.4	1.099	.64 (1)	3482	2538
4.6	2.4	.968	.66 (2)	2824	836
5.25	3.0	1.011	.66 (2)	3551	951
7.3	1.8	.998	.66	3767	1169
6.4	1.4	.963	.66	2918	1172
8.5	0	.964	.66	2705	1569
9.2	0	.972 (1)	.66	3148	1610
9.9	0	.996 (2)	.66	3621	1647
9.5	-1.05	1.056	.66 (2)	2586	1952
10.2	-1.2	1.085	.66 (1)	2976	2038 (1)
10.9	-1.35	1.110	.64 (1)	3432 (1)	-2162 (2)
10	-2.8	1.143	.54 (2)	2559 (2)	-2912
9.1	-2.45	1.110	.60 (1)	2259	-2608
8.2	-2.1	1.069	.62	1948 (2)	-2290
5.25	-3	1.011	.60	2125 (1)	-2152
5.9	-3	1.011	Toppled	2216 (1)	-2551
2.6	-6	1.030	.47	1761 (2)	-2595
2.4	-5	.972	.51	1667 (2)	-2206

Table A-15. Summary Table of Critical Values for Series C

V_V	V_H	Max. Stroke	Min. Beta	Max. Vertical	Max. Horizontal
The leg number for the value given is in parentheses.					
.55	6	1.141 (1)	.69 (1)	2171 (1)	2663 (1)
.5	5	1.102	.71	1878	2222
2.4	5	1.104	.73	2371	1861
2.6	6	1.143	.72	2624	2433
2.8	7	Bottomed	.70	2740	2967
4.2	7	1.164	.71	3141	2569
3.9	6	1.122	.73	2830	2041
5.9	6	Bottomed	.74	3593	2144
5.25	5	Bottomed	.75	2932	1713
4.6	4	1.155	.76	2457	1311
6.4	4	Bottomed	.76	2875	1404
5.5	3	1.135	.76	2262	1006
7.1	3	1.164	.76	2529	1056
7.8	4	Bottomed	.76	3139	1459
8.2	2.1	1.144	.76	2610	746
9.1	2.45	Bottomed	.76	2918	895
10	1.6	1.148	.76	3104 (1)	585
10	0	1.064 (2)	.76 (2)	2870 (2)	0
10.8	0	1.076	.76	3188	0
11.6	0	1.087	.76	3541 (2)	0

Table A-16. Summary Table of Critical Values for Series D

V_V	V_H	Max. Stroke	Min. Beta	Max. Vertical	Max. Horizontal
2	3	Bottomed	.93	2590	1090
2	2	1.107	.93	1865	696
4	2	1.154	.93	2204	745
4	3	Bottomed	.93	3047	1204
5	3	Bottomed	.93	3224	1247
5	2	Bottomed	.93	2354	807
5	1	1.082	.93	1691	387
7	1	1.119	.93	2098	409
7	2	Bottomed	.93	2612	847
9	2	Bottomed	.93	2842	884
9	1	1.147	.93	2668	427
10	1	1.159	.93	3032	435
10	2	Bottomed	.93	3165	900
11	1	Bottomed	.93	3417	443
11	0	1.079	.93	3278	0
All values for Leg 1.					

Table A-17. Summary Table of Critical Values for Series E

V_V	V_H	Max. Stroke	Min. Beta	Max. Vertical	Max. Horizontal
The leg number for the value given is in parentheses.					
.5	5.0	Bottomed (1)	.83 (3)	3011 (1)	1734 (1)
.45	4.0	1.151	.83	2177	1278
2.2	4.0	Bottomed	.83	2427	1445
2	3	1.675	.83	1972	1017
2.85	2.7	1.034	.83	2386	828
2.5	1.8	.907	.83	2212	540
3.3	1.2	.792	.83	2451	426
2.65	0.6	.642	.83	2237	305 (1)
2.8	0	.578	.83	2272	-347 (2)
2.65	-0.6	.677	.83	2216	-491
3.3	-1.2	.826	.83	2307	-766
3.2	-3	1.063 (3)	.83	2230 (3)	-1355
3.2	-2	.927 (3)	.83	2231 (1)	-1016
4	-3	1.115 (3)	.83	2268 (3)	-1499
2	-3.3	1.019 (3)	.83	2514 (3)	-1178 (2)
1.4	-4	1.088 (3)	.83	2779 (3)	-1361 (1)

Table A-18. Summary Table of Critical Values for Series F

V_V	V_H	Max. Stroke	Min. Beta	Max. Vertical	Max. Horizontal
The leg number for the value given is in parentheses.					
.4	4	Bottomed (1)	.83 (3)	2782 (1)	1610 (1)
.3	3	1.130	.83	2086	1143
1.7	3	1.159	.83	2242	1334
2.2	4	1.161	.83	3108	1903
2.0	2	1.065	.83	2351	909
1.6	2	1.054	.83	2115	869
.8	1	.889	.83	2149	449
1.6	1.1	.934	.83	2133	514
1.0	.7	.832	.83	2361	416
1.0	0	.669 (1)	.83	2310	299
1.0	-.7	.643 (3)	.83	2294	-343 (1)
2.0	-2	.843	.83	2563	-698
.7	-1	.722	.83	2022	-417
1.2	-2	.875	.83	1973	-680
1.7	-3	.964	.83	2413 (1)	-981
.3	-3	.992	.83	2291 (3)	-999
.2	-2	.855 (3)	.83	2123 (1)	-729

Table A-19. Summary Table of Critical Values for Series G

V_V	V_H	Max. Stroke	Min. Beta	Max. Vertical	Max. Horizontal
The leg number for the value given is in parentheses.					
.4	4	.974 (1)	.32 (1)	1473 (1)	2119 (1)
.3	3	.928	.42	1352	1876
.2	2	.806	.46	1345	1227
1.2	2	.821	.47	1390	1235
1.7	3	.914	.39	1308	1806
3.2	3	.917	.28	1874	1762
2.6	2.5	.872	.41	1493	1609
3.8	1.9	.867	.46	2013	1565
4.5	2.2	.915	.46	2413	1718
5	1	.889	.45	1716	1421
5.8	1.2	.915	.44	2035	1414
6.4	1.35	.918	.44	2351	1380
7	0	.933	.41	1936	1270
8	0	1.025	.43	2223	1508
9.4	-1	1.077	.40	2074	1806
9.8	-1.7	1.069	.34	2166	1892
9	-2	.981	.36	2059	-1838 (2)
8.6	-1.3	.999	.38	1966	-1480
7	-1.4	.805	.29	1912	-1345
7.3	-2.2	.881 (2)	.32	2364	-1715
5.9	-2.8	.888	.24	2080	-1755
5.2	-2.5	.817	.26	2042 (1)	-1525
3.8	-3.5	.834	.17	1230 (2)	-1644
4.4	-4	.917	Toppled	1395	-1943
2.2	-4	.751	.29	1242	-1510
2.7	-5	.875	.11	1425	-1929
.5	-5	.717	.45	1284	-1527
.6	-6	.749	.48	1518	-1813

Table A-20. Summary Table of Critical Values for Series H

V_V	V_H	Max. Stroke	Min. Beta	Max. Vertical	Max. Horizontal
The leg number for the value given is in parentheses.					
.6	6	.930 (1)	.22 (2)	1567 (1)	2258 (1)
.5	5	.944	.26 (1)	1400	1940
.4	4	.731	.32	1591	1432
2.2	4	.773	.33	1462	1478
2.7	5	.959	.29	1404	1990
4.4	4	.906	.38	1774	1656
5.0	4.5	.969	.37	2141	1909
5.9	2.8	.908	.46	2014	1260
6.6	3.1	.972	.45	2185	1454
7.3	3.4	1.026	.44	2328	1655
8.2	1.8	1.063	.49	2167	1275
9	2	1.108	.49	2375	1450
10	0	1.037	.48	2492	901
9	0	.989	.48	2158	797
8.2	-1.8	.793	.48	1796	449
9	-2	.828	.48	2073	417
9.8	-2.2	.860	.48	2386 (1)	417
9.6	-3	.827 (2)	.49	2392 (2)	455
8.7	-4	.935	.47	2252	642
7.3	-3.4	.856	.48	1754	470
6.8	-6	1.070	.30	2322	-1282 (2)
6.2	-5.5	1.023	.37	2114	-1138
3.7	-7	.993	.15	2386	-1746
3.2	-6	.907	.28	2090	-1410
.6	-6	.901	.03	1729 (1)	-1841
.5	-5	.795	.21	1354 (1)	-1460

Table A-21. Summary Table of Critical Values for Series J

V_V	V_H	Max. Vertical	Max. Horizontal	Max. Stroke	Min. Beta
The leg number for the value given is in parentheses.					
.50	5.00	3142.4 (1)	1856.4 (1)	1.166 (1)	.8821 (3)
.40	4.00	2243.0	1377.6	1.150	.8820
2.20	4.00	2336.9	1263.8	1.166	.8823
1.70	3.00	1624.1	1022.1	1.054	.8821
3.20	3.00	1771.5	829.8 (2)	1.055	.8823
3.80	3.50	2003.9	1014.5	1.121	.8824
4.40	4.00	2389.7	1209.3	1.165	.8825
4.50	2.20	1954.5	600.0	.957	.8824
5.20	2.50	2183.1	716.9	1.010	.8825
5.90	2.80	2447.9	840.7 (2)	1.062	.8826
6.60	1.40	2498.2	393.2 (1)	.891	.8826
5.80	1.20	2233.6	351.1 (1)	.841	.8824
5.80	-1.20	1978.0	-766.2 (2)	.980 (3)	.8820
6.60	-1.40	2262.6	-866.1	1.041	.8821
5.90	-2.80	2335.0 (3)	-1372.7	1.150	.8817
5.20	-2.50	2015.6	-1227.1	1.098	.8817
5.00	-4.50	3677.1	-1927.9	1.163	.8812
4.40	-4.00	2954.7	-1720.2	1.164	.8813
3.80	-3.50	2267.6	-1493.9	1.130	.8813
3.20	-3.00	1967.5	-1273.3	1.044	.8813
1.70	-3.00	2064.2	-1018.9	.896	.8811
2.20	-4.00	2377.6	-1445.8	1.066	.8810
.40	-4.00	2444.9	-1222.0	1.034	.8808
.30	-3.00	2011.0	-940.8	.903	.8809

Table A-22. Summary Table of Critical Values for Series K

V_V	V_H	Max. Vertical	Max. Horizontal	Max. Stroke	Min. Beta
The leg number for the value given is in parentheses.					
.50	5.00	3763.0 (1)	2161.6 (1)	1.163 (1)	.8560 (1)
.40	4.00	2950.5	1704.4	1.162	.8653
.30	3.00	2112.4	1251.4	1.144	.8645
1.70	3.00	2303.9	1418.8	1.164	.8623
1.20	2.00	1549.9	888.3	1.037	.8776
2.60	2.50	2251.6	1105.1	1.118	.8668
3.20	3.00	2439.3	1244.1	1.163	.8635
3.10	1.60	2381.4	751.4	.997	.8799
2.40	1.30	2175.0	667.4	.961	.8812
2.60	.40	2244.8	458.2	.807	.8750
3.40	.60	2441.6	534.6	.839 (1)	.8752
3.40	-.60	2406.1	476.2	.609 (3)	.8751
2.60	-.40	2205.9	371.6	.610 (1)	.8756
2.40	-1.30	2166.7	-470.8 (2)	.644 (3)	.8763
3.10	-1.60	2288.6	-661.3	.760	.8720
3.20	-3.00	2225.3	-1141.1	.971	.8510
3.80	-3.50	2253.2	-1419.8	1.085	.8384
4.40	-4.00	2641.2 (3)	-1688.2	1.164	.8217
2.70	-5.00	2777.4	-1736.8	1.147	.8398
2.20	-4.00	2460.9	-1260.9 (2)	1.021	.8489
1.70	-3.00	2049.9	-880.7 (1)	.887	.8666
.30	-3.00	2009.2	-853.8 (2)	.904	.8631
.40	-4.00	2396.1	-1151.2 (2)	1.026	.8526

APPENDIX B

EQUATIONS OF MOTION

X - Y Coordinate system with the ground as reference (Y normal to ground)

R - Vehicle radius from centerline to footpads

i - Subscript used to denote ith leg

θ_i - Angle of leg i from the X-axis

ϕ - Angular orientation of vehicle with respect to X-Y axis

α_i - Angular deflection of leg i.

ζ - Ground slope

H - Vertical distance from c.g. to hardpoint of strut

L - Length of shock absorber fully extended

s_i - Amount of stroke (Compression) of the ith strut

AH - Cross-sectional area of inner cylinder (I.D.)

A_o - Net orifice area

g - Gravity

T - Thrust of engine

FA - Force due to air spring of shock absorber

FF - Friction of internal parts of shock absorber

FS - Total shock absorber axial force

FH - Force due to bending of the shock absorber

M_i - Moment about hardpoint of leg i due to FH(i)

m - Mass of vehicle

ρ - Mass density of fluid in shock absorber

μ - Coefficient of friction between footpad and surface

ν - coefficient of friction between footpad and obstacle

c - Discharge coefficient of orifice

FGV - Force normal to ground surface

FGH - Force along ground surface

FLG - Force along ledge (obstacle)

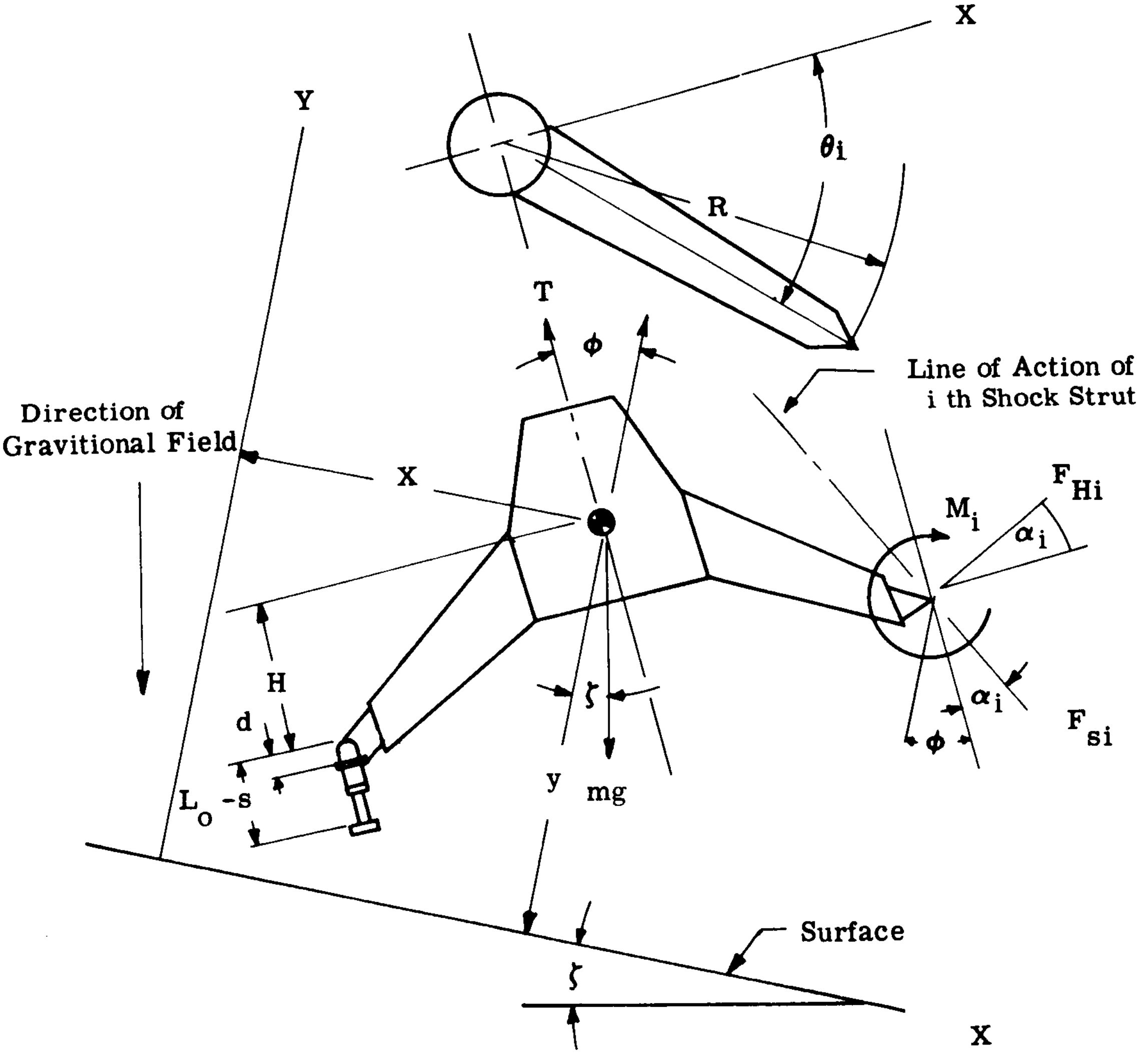

Figure B-1. Schematic Diagram of Vehicle

A diagram of the vehicle with appropriate symbols is shown in Figure B-1. The directions indicated in this diagram are positive. As is indicated the X-Y coordinate system chosen is parallel (X) and normal (Y) to the ground surface.

The vehicle motion is assumed confined to translation and rotation in the plane of its approach path which is the X-Y plane.

The only force acting on the vehicle during landing are gravity, thrust of the engine, the shock absorber forces, and the force due to bending of the shock absorber.

The axial load in the strut consists of three parts: 1) hydraulic load, 2) air load, and 3) friction of the internal parts. The formula for the total axial load in the shock absorber is: $FS = \rho (AH)^3 \dot{s}^2/2(c A_0)^2 + FA + FF$

The first expression is the formula for hydraulic load. The air load, FA, at any stroke, s, is dependent on the extended air pressure and the stroke s. For this study the extended air pressure was not changed so that the air load becomes a direct function of s and was put into the computer in the form of a table. The friction load FF was assumed equal to zero. The horizontal load (FH) due to bending of the strut was calculated from the graph shown in Figure B-2. The formula for this graph is given by:

For $-AB \leq \alpha \leq AB$ $\quad FH = - KMT/(L - s)$

and for $\alpha > AB$ $\quad FH = [- KMT * AB - KBT (\alpha - AB)]/(L - s)$

and for $\alpha < - AB$ $\quad FH = [KMT * AB - KBT (\alpha + AB)] /(L - s)$

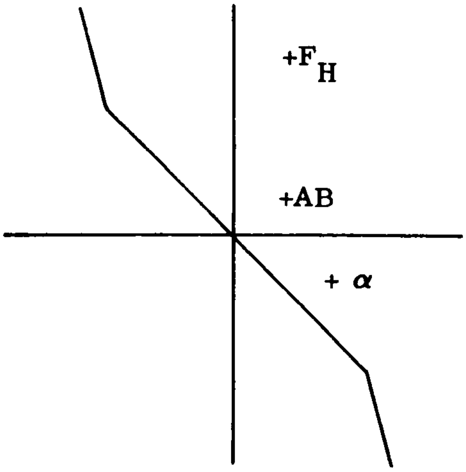

Figure B-2. Shock Mount Loading

The sum of the strut forces resolved into the X-Y direction and resulting moments about the center of gravity are given by

$$FX = \sum_{i=1}^{4} \quad FS\ (i)\ \sin\ (\alpha(i)\ + \phi) + FH\ (i)\ \cos\ (\alpha(i)\ + \phi)$$

$$FY = \sum_{i=1}^{4} \quad FH\ (i)\ \sin\ (\alpha(i)\ + \phi) - FS\ (i)\ \cos\ (\alpha(i)\ + \phi)$$

$$\begin{aligned} Mo = \sum_{i=1}^{4} \quad & M\ (i) + \ (FH\ (i)\ \cos\alpha(i) + FS\ (i)\ \sin\alpha(i)\ )\ H \\ & + (FH\ (i)\ \sin\alpha(i)\ - \ FS\ (i)\ \cos\alpha(i)\ )\ R \cos\ \theta\ (i) \end{aligned}$$

where $M\ (i) = FH\ (L\ -\ s\ (i))$

Now adding in the gravity and thrust forces resolved in the X-Y direction gives

$$FXX = mg\ \sin \zeta - \ T\ \sin\phi\ -\ FX$$

$$FYY = -FY + T\ \cos\phi\ -\ mg\ \cos\zeta$$

Gravity and thrust do not add to the moment about the center of gravity since they act through the center of gravity.

The accelerations are:

$$\begin{aligned} \ddot{X} &= FXX/m \\ \ddot{Y} &= FYY/m \\ \ddot{\phi} &= -Mo/MI \end{aligned}$$

These equations were programmed on a CDC G-20 digital computer.

The process of solving these equations can best be illustrated by outlining the steps involved in the computer program.

1. Read in initial data.

2. Calculate position of each footpad in the X-Y coordinate system.

3. Determine which footpads are on the surface and which are off the surface.

4. Calculate s, $\dot{s}$, and α for each strut whose footpad is on the surface. (These terms are zero if the footpad is off the surface.)

5. Calculate FS, FH individually and sum them (torque can be calculated from known geometry).

6. Sum all forces and moments acting.

7. Using these forces and moments, the center of gravity motion and angular

motion about the center of gravity can be determined at the end of the time interval Δt. Rectangular integration is used for the vehicle motion.

8. Set all parameters for next time interval.
9. Check for vehicle stopped (within epsilon quantities). If not, return to step 2 for another iteration.

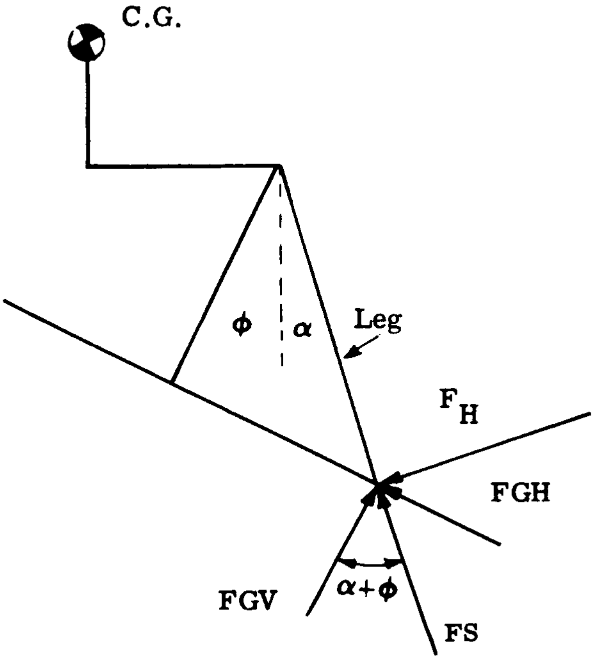

Figure B-3. Footpad Force Diagram

The time interval of t = .002 seconds proved to be sufficiently small to adequately describe the vehicle motion for all cases.

The method of calculating α and FH were changed to handle those runs where the footpads were allowed to slide. In order to check for sliding, the forces normal to the ground, FGJ, and along the ground, FGH, had to be calculated. The condition of sliding is FGH = μ FGV. For FGH less than μFGV the footpads would not slide.

The computations were made on the assumption that the footpads did not slide and FGH was compared to μFGV. If FGH was less than μFGV, the assumption was correct and the calculations for that time interval completed and the program continued for the next time interval.

If FGH was greater than μFGV, then sliding would have occurred, which means that the position of the footpad has to be found. This can be done by finding the correct angle that the strut would be bent to fulfill the condition of sliding. The measure of bending of the strut, α, had been calculated on the assumption that the footpad did not slide. Now α is calculated so that the condition of sliding is fulfilled.

The average of these two values of α is used for the recalculations made for this time interval. Because of the interdependency of α, FGH, and FGV, it was considered that just using α based on the condition of sliding might be an over-correction of strut bending. The values recalculated for this time interval are $\dot{\alpha}$, $\dot{s}$, FS, FH, FGV, and FGH. These new values of FGV and FGH are then tested for the condition of sliding and the process of revaluing α repeated until FGH $= \alpha$ FGV within an epsilon value. When this happens the program continues with the other computations and the last values of FS and FH are used in the equations of motion.

There were a few runs made to study the dynamic effect on the vehicle of the footpads striking an obstacle. This obstacle was defined as a six-inch ledge as shown in Figure B-4. For these runs there was an additional force, FLG, along the ledge when the footpad was moving along and in contact with the ledge. This force would be opposite the direction of movement and equal to the coefficient of friction, ν , times the force normal to the ledge, FGH. When this force, FLG, was acting it was added into the equations of motion

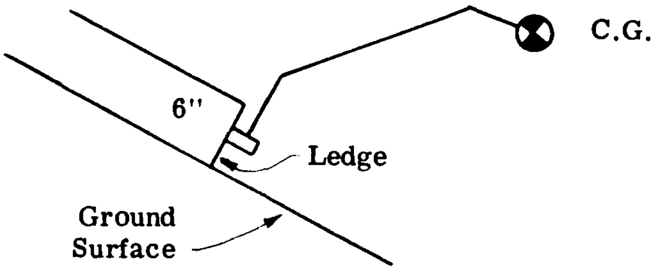

Figure B-4. Obstacle Impact

NASA-Langley, 1966 CR-428