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CONTROL SYSTEM STUDY
SEPTEMBER PROGRESS REPORT

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FOREWORD

This document contains a description of work performed on Contract NAS8-11496, "Control System Study." This study is being performed by Lockheed Missiles & Space Company, Huntsville Research & Engineering Center for NASA/MSFC, Aero-Astrodynamics Laboratory. The period of performance covered by this report is from 8 September to 7 October 1965.

The principal contributors to the work reported herein were Dr. C. T. Striebel and R. M. Chapman of LMSC/Huntsville Research & Engineering Center.

SUMMARY

A review is made of the overall objectives of this contract. The relationship of the present effort to these objectives is established.

In addition, the results of the present effort to establish the significance of the second moment of the spectral density in characterizing the maximum of the stationary time series is presented.

INTRODUCTION

By way of introduction, the overall objectives of the present study are reviewed.

The engineering problem is control system and structural design of a missile or spacecraft subjected to a flight environment of random atmospheric disturbances. The dynamic analysis problem is the prediction of the vehicle flight dynamic response in a probabilistic sense. In the present study, methods for obtaining approximate mathematical solutions to this problem are being developed.

The primary objective in the present study is to obtain a parametric solution for the probability distribution of the maxima of a particular class of nonstationary processes (those with "separable" covariance). Secondly, it is desired to establish conditions for "stochastic dominance", whereby an already obtained approximate solution can be selected which stochastically dominates the true nonstationary dynamic response as characterized by its mean and covariance matrices.

DISCUSSION

In continuing the study of autoregressive techniques, effort has been concentrated in the area of verifying that the second moment of the spectral density is the significant parameter in characterizing the maximum of the stationary time series. The method selected for this verification is outlined in Reference 2. Using this method the autoregressive coefficients are determined with very little effort and at the same time an adequate sampling interval can be determined.

Reference 2 demonstrated the mechanics of this selection by defining a spectral density form,

$$\Phi(\eta) = \frac{1}{\left| e^{2\pi i \eta} - \zeta \right|^{2n}}, \quad -\tfrac{1}{2} < \eta < \tfrac{1}{2} \qquad (1)$$

and from its expansion establishing the following relationships for the autoregressive coefficients,

$$A = 1$$

$$B_k = \frac{-n! \ (-\zeta)^k}{k! \ (n-k)!}, \quad k = 1, 2, \ldots, n \qquad (2)$$

where $r = n + 1$ is the order of the scheme.

The second moment of the spectral density is related to the variance of the first difference by (Equation 9, Reference 2).

$$\sigma_\Delta^2 = \frac{K\Delta^2}{2}, \qquad (3)$$

where K is the second moment of the spectral density and Δ is the sampling increment.

In terms of the original parameters of Equation (1) i.e., n and ζ the variance of the first difference is (Equation 5, Reference 2),

$$\sigma_{\Delta}^{2} = 1 - \left\{ \frac{\zeta n \left[\sum_{k=0}^{n-1} \frac{(2n-2-k)!}{k!\,[(n-1-k)!]^{2}\,(n-k)} \left(\frac{1}{\zeta^{2}} - 1 \right)^{k} \right]}{\sum_{k=0}^{n-1} \frac{(2n-2-k)!}{k!\,[(n-1-k)!]^{2}} \left(\frac{1}{\zeta^{2}} - 1 \right)^{k}} \right\} \quad (4)$$

This function is shown graphically in Figure 1 for $1 \leq n \leq 10$ and $0 \leq \zeta \leq 1$.

For the present study, two spectral densities are being considered, namely, cases 2 and 3 from Reference (1) with second moments of K=3.24 and K=17.83 respectively. Using these values for the second moments and a given interval size, the variance σ_{Δ}^{2} is determined from Equation (3). With this value for the variance and a chosen value of n, the parameter ζ is found from Figure 1. Substitution of ζ into Equation (2) results in a set of autoregressive coefficients which are tabulated in Table I for the two spectra considered. Also for case 2, a larger sample interval was chosen (Δ = 0.5) for comparison purposes.

The result of using these autoregressive coefficients in determining the distribution of the maxima are compared in Tables II, and III to the results obtained in Reference 1 for the first four statistical moments.

Even though the order chosen to demonstrate the method was n+1 = 6, the results compare favorably with those obtained in Reference 1 where the order of the scheme was nine.

Since the above results are very encouraging, the study will be pursued further in order to improve the comparison by considering a higher order, i.e., a larger n, and by considering more intricate spectral density shapes.

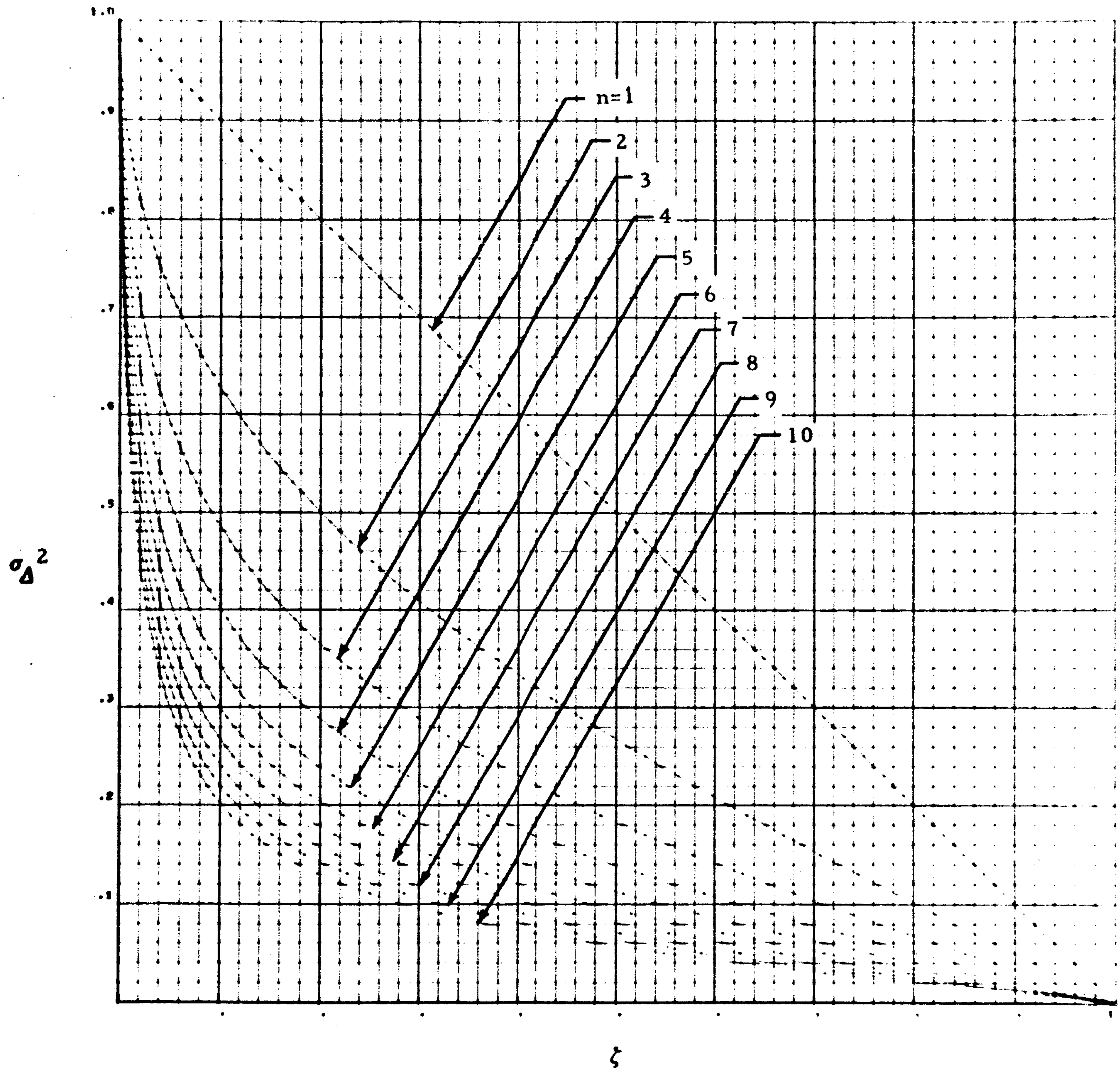

Figure 1 - Graphic Representation for $1 \leq n \leq 10$ and $0 \leq \xi \leq 1$.

FUTURE WORK

Future work will include the effect of varying the order of the scheme chosen (n+1) and to study in some detail the relationship between interval size (Δ) and the order. In addition, other shapes of the spectral density will be considered.

REFERENCES

1. Bieber, R. E., R. M. Chapman, and C. T. Striebel, Study of Guidance and Control Systems, Phase II, Technical Report, LMSC/H-64-11, September 1964.

2. Control System Study July Progress Report, LMSC/HREC A711774, 23 August 1965.

TABLE I

k	B_k k = 3.24 (Δ = .2)	B_k k = 3.24 (Δ = .5)	B_k k = 17.83 (Δ = .257)
1	2.05	.70	.45
2	-1.681	-.196	-.081
3	.6892	.02744	.0073
4	-.1413	$-.1921 \times 10^{-2}$	$-.328 \times 10^{-3}$
5	.01159	$.5378 \times 10^{-4}$	$.590 \times 10^{-5}$

TABLE II

K = 3.24	Ref. 1 (Case 2)	Δ = .2	Δ = .5
μ_1	3.236	3.231	3.332
μ_2	.3780	.3789	.3730
μ_3	.7039	.7137	.7108
μ_4	3.816	3.820	4.039

TABLE III

K = 17.83	Ref. 1 (Case 3)	Δ = .257
μ_1	3.378	3.433
μ_2	.3529	.3417
μ_3	.7666	.6729
μ_4	4.102	3.668