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Abstract.--Measured velocities in pairs, groups, and clusters of galaxies have been used to estimate average masses. In pairs, these estimates depend strongly on the morphological type of the galaxies involved. In clusters, the measured motions imply much larger average masses, or the existence of intergalactic matter, or instability, and present a serious difficulty in identifying the members of clusters. Other measurable characteristics of galaxies in pairs -- their orientations, dimensions, types, and luminosities -- are also correlated, suggesting a common origin; but the effects of selection, as pointed out by Neyman, are shown to affect the results. These studies are all related to the problem of evolution of galaxies summarized in a brief resumé.

Introduction

The masses of galaxies are important in several areas of astronomy and physics. In cosmology the mean mass is used to derive the average density of matter in the universe, a quantity which is related to the curvature of space in the cosmological models of general relativity. In any theory of the origin and evolution of galaxies, the masses are important in the dynamical aspects. Also, the wide range in mass estimates must be explained by a statistical theory of the origin of galaxies.

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[1] Revision of a chapter prepared early in 1965 for a Festschrift honoring Jerzy Neyman, Emeritus Professor of Statistics, University of California, Berkeley, California.

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Statistics have been involved in practically all phases of these studies, and one of the basic problems concerns observational selection (Neyman and Scott, 1964). The luminosities of nearby galaxies range from 10^8 to 10^{12} suns, and it is clear that only the most luminous ones are observed at large distances. Moreover, they have a wide variety of forms, and there is further selection due to confusing distant galaxies of circular projection with foreground stars on photographs.

Projection introduces a second statistical problem, since most galaxies appear to have an axis of symmetry similar to that of a disk or oblate spheroid. Each is viewed in one projection at an unknown angle to the axis. Masses are determined from motions perpendicular to the plane of projection (radial velocities), generally on such simplifying assumptions as these: (1) the average internal motions in a galaxy are circular and in the equatorial plane; (2) the velocities of individual galaxies in a cluster are directed at random; (3) the orbits of double galaxies are circular, randomly oriented, and equally likely to be viewed at any angle to the line of centers; (4) the only forces involved are gravitational; and (with a few exceptions) (5) the observed forms, groupings, and distributions are relatively stable over long periods of time.

Distributions of luminosities, sizes, distances, and derived masses of galaxies are not only confused by the effects of selection noted above, and by possible systematic errors introduced as a result of the assumptions listed, but also by fairly large observational errors, by small sample sizes, and by interdependent errors. The distance of a galaxy, for example, is often inferred from its apparent brightness compared with its assumed luminosity. Its dimensions and mass are also derived from this distance, so that correlations between mass, dimensions, and luminosity are subject to bias.

A summary of mass determinations was collected at a special conference organized by Neyman, Scott, and myself (1961, p. 619). Corrections and additions have since been made (Holmberg, 1964) as shown in Table 1.

The first column of Table 1 gives the NGC catalog number of the galaxy (or Messier number or Vorontsov-Velyaminov number). In the second column, m_{pg} is the total photographic magnitude (in general, larger m_{pg} implies lower accuracy). The morphological types indicate forms from Ir (irregular) through Sc, Sb, Sa (spirals), SBc, SBb, SBa (barred spirals), and S0 (smooth lenticulars) to E (ellipticals) of projected ellipticity 0.7 (E7) to 0 (circular E0). The corrected redshift radial velocity, V, is relative to the Milky Way nucleus, and is used as a distance indicator; D=V/100 in Mpc except when V < 300 km/sec. Under "Method," "Ls" stands for optical spectra taken with a long slit extending across the galaxy image to determine rotation; "H II" stands for separate optical spectra giving orbital velocities of ionized-hydrogen gas clouds about the center of a galaxy; "21 cm" stands for radio doppler shifts used to determine rotation; "Circular orbits" refers to the double-galaxy analysis presented in the next section; and "Stat" refers to the statistical studies of stellar radial velocities showing the rotation of our Milky Way galaxy. References refer to the list at the end of this paper with the abbreviations "de V" for de Vaucouleurs, "BBP" for Burbidge, Burbidge, and Prendergast, "Z H" for Zwicky and Humason, "v d B" for van den Bergh, "Min" for Minkowski, and "D-A" for Duflot-Augard. In the cases of NGC 598, "1942,59,62" stands for Wyse and Mayall (1942), Volders (1959), and Dieter (1962), in the case of NGC 224, "1942,57" stands for Wyse and Mayall (1942) and Schmidt (1957), and in the case of NGC 3115, "1959,61" stands for Münch (1959) and Poveda (1961). The last two columns give the estimated mass, M, and mass-luminosity ratio, M/L, both in solar units. Both of these estimates are subject to r.m.s. errors of 50% or more; the least accurate values are enclosed in parentheses. In the previous listings by the authors cited, by Holmberg (1964) and Page (1961), distances were based on Hubble's Law, D = V/H, with H = 75 km/sec Mpc (BBP), 80 km/sec Mpc (Holmberg), and 100 km/sec Mpc (Page). All the mass estimates are proportional to the inverse of H used, and the M/L estimates are proportional to the value of H used, except in a few cases (such as M31 and LMC) where other distance indicators have been used. In Table 1 they have all been converted to H = 100 km/sec Mpc (corresponding to cosmological age 10^{10} years), and very rough averages have been listed for the various types. These are plotted on Figure 1.

Table 1.--Masses of galaxies.

(in solar units, distances based on the Hubble Law with H = 100 km/sec Mpc)

Galaxy	m_{pg}	Type	V/100	Method	Reference	$M/10^{10}$	M/L
NGC							
55	7.9	IrSc	1.0	H II	de V 1961	3.	2.
1613*	10.0	Ir I	(0.)	21 cm	Volders 1961	0.03	4.
3034 (M82)	9.6	Ir II	3.2	Ls	Mayall 1960	1.	9.
3556	10.6	ScIr	7.6	Ls	BBP 1960	1.	1.
6822	9.7	Ir I	0.7	21 cm	Volders 1961	( )	
-- LMC	0.5	Ir S	(0.)	21cm, H II	de V 1963	1.1	5.
-- VV254		Ir	45.9	Ls	BBCRP 1963	9.8	
(Mean Ir)			0.7 to 45.9			2	5
NGC							
157	11.2	Sc	18.4	Ls	BBP 1961	4.4	1.5
253	6.9	Sc	1.0	Ls	BBP 1962	20.	2.
598 (M33)	6.2	Sc	(0.)	21cm, H II	1942, 59, 62	1.	3.
613	11.0	SBc	14.9	Ls	BBRP 1964	10.	10.
1084	11.1	Sc	14.5	Ls	BBP 1963	0.8	1.
1365	10.5	SBc	15.1	Ls	BBP 1960	2.5	1.5
2146	11.3	Sc	9.9	Ls	BBP 1959	1.3	1.5
2903	9.5	Sc	5.1	Ls	BBP 1960	4.0	2.1
3646	11.8	Sc	42.0		BBP 1951	20.	4.
4631	9.7	Sc	6.5	Ls	de V 1963	2.4	1.8
5144 (M51)	8.6	Sc	(4.)	Ls	BB 1964	4.3	11.
5248	11.0	Sc	11.4	Ls	BBP 1962	4.	1.5
5457 (M101)	8.5	Sc	4.2	21 cm	Volders 1959	(1.0)	(17.)
6503	10.7	Sc(dwf)	3.5	Ls	BBCRP 1965	0.13	0.8
7320	13.	Sc	10.7	Ls	BB 1961	(4.4)	(8.)
(Mean Sc)			0.5 to 42.0			6	2
(Mean Ir Sc)			0.5 to 45.9			5	3
16 Double systems	10.to 13.	2 Ir 32 S	6. to 76.	Circular orbits	Page 1962 and Table 2	4.0	3.2
NGC							
224 (M31)	4.3	Sb	(0.8)	21 cm, H II	1942, 57, 59	34.	8.4
1068 (M77)	10.	Sb(em)	12.0	Ls (em)	BBP 1959	2.0	2.7
1097	10.4	SBb	12.1	Ls (em)	BB 1960	0.6	0.5
3031 (M81)	8.1	Sb	(0.8)	H II	Münch 1959	12.	6.
3504	11.6	SBb	14.7	Ls (em)	BBP 1960	0.8	1.

Table 1.--(cont'd).

Galaxy	m_{pg}	Type	V/100	Method	Reference	$M/10^{10}$	M/L
3521	9.6	Sb	6.4	Ls (em)	BBCRP 1963	8.	5.
4258	8.9	Sb	5.3	Ls	BBP 1963	10.	2.4
5005	10.5	Sb	10.8	Ls (em)	BBP 1961	10.	2.5
5055 (M63)	9.3	Sb	6.0	Ls (em)	BBP 1960	4.5	2.
5383	12.4	Sb	23.7	Ls	BBP 1962	4.	7.
7479	11.6	Sb	26.6	Ls	BBP 1960	(>0.8)	0.5
Milky Way	--	Sb	--	21cm,Stat	Schmidt	18.	5.6
(Mean Sb)			0.8 to 22.6			10	4
NGC							
2782	12.5	Sa	25.1	Ls	D-A 1960	11.	7.5
3623 (M65)	10.2	Sa	6.4	Ls	BBP 1961	20.	7.2
7469	12.7	Sa (em)	50.2	Ls	BBP 1963	0.8	0.5
(Mean Sa)			6.4 to 50.2			20	7
(Mean Sb Sa)			0.8 to 50.2			11	4.5
(Mean Ir S)			0.5 to 50.2			7	3.5
NGC							
221 (M32)	9.7	E2	(0.8)		Fish 1964	0.3	11.
3115	10.1	E7	4.2		1959, 61	15.	46.
3379	10.5	E1	7.5		Fish 1964	13.	20.
4111	11.6	ES0	8.4		Poveda 1961	4.	14.
4278	11.2	E	6.2		Poveda 1961	5.	14.
4406 (M86)	10.3	E3	(0.)		Fish 1964	96.	39.
4472 (M49)	10.	E1	8.6		Fish 1964	110.	19.
4486 (M87)	9.6	E0	11.9		Fish 1964	260.	60.
5128	8.	Epec	4.0	Ls (em)	BB 1959	15.	13.
(Mean E)			0.8 to 11.9			70	30
28 Double systems	9. to 14.	33E 13S0	7. to 48.	Circular orbits	Page 1962 and Table 2	60.0	90.
Mean groups		4S,4E	3. to 91.	Virial theorem	Table 4	250.	280.
Mean clusters		100	7. to 67.	Virial theorem	Table 4	130.	600.

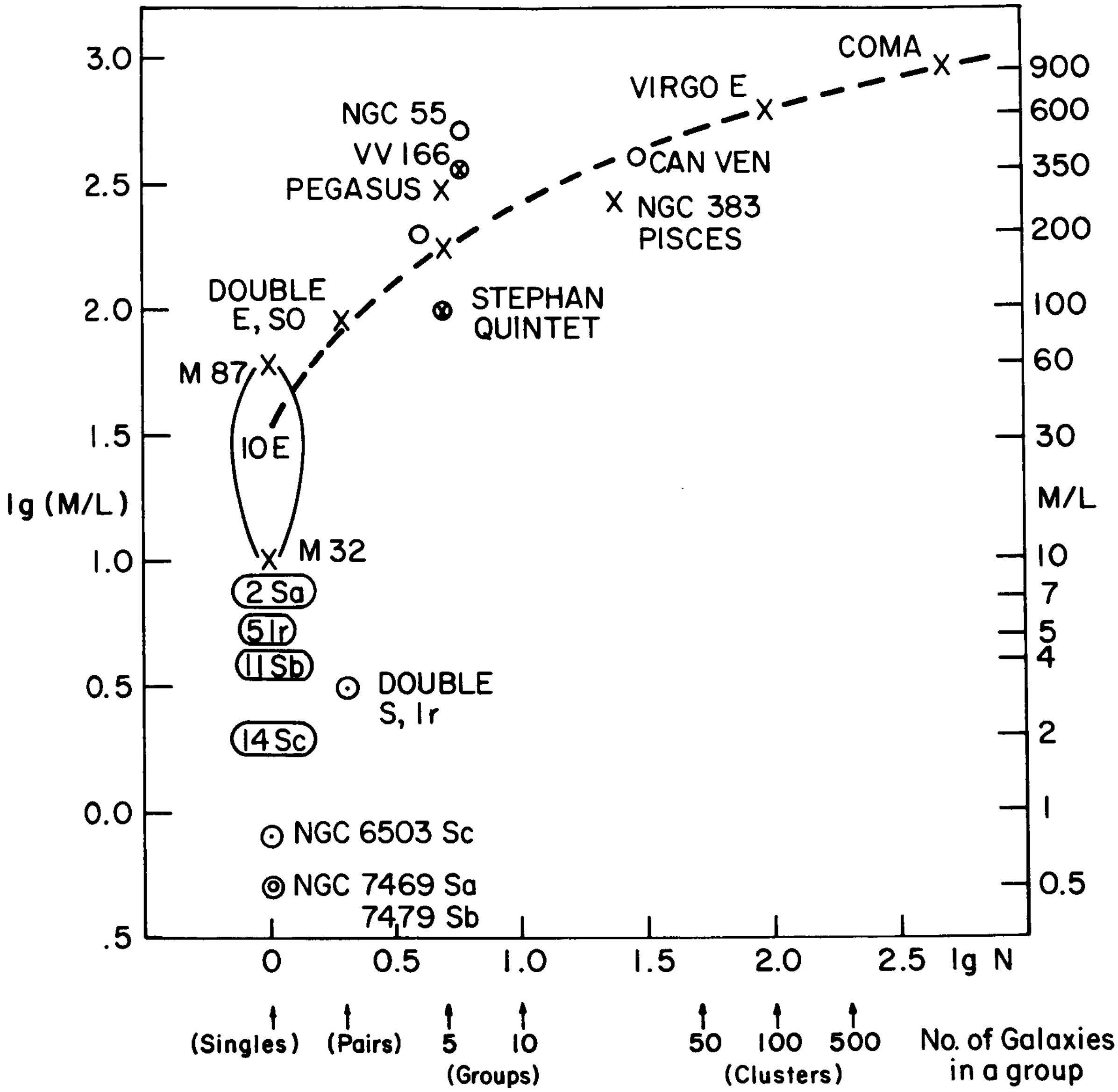

Figure 1.--Average values of M/L for galaxies.

Mean masses in pairs of galaxies

The masses of individual galaxies listed in Table 1 are subject to systematic errors on the low side because the circular (rotation) velocities of stars or luminous (H II) gas clouds cannot be measured near the outer edge of a galaxy. In effect, this ignores the mass in an outer rim of a spiral, where the luminosity is too low for optical velocity measurements to be made. However, most authors correct for this by extrapolating the mass distribution, assuming that the density drops off smoothly to zero, thus adding an amount roughly equal to 0.3 M. Errors of measurement are estimated to be of the order 0.5 M (r.m.s.).

Masses of pairs of galaxies must be derived statistically on assumption (3) in the Introduction, and most of the observational data were combined in an analysis by Page (1962) which can be briefly summarized as follows: The observations consist of separate radial velocities for each galaxy in a pair or pair-like group. The mean of these two velocities for one pair, V, is used as the distance indicator. From Hubble's law of redshifts,

$$V = h \times 10^{-4} D , \qquad (1)$$

where V is in km/sec, and $h \approx 1$ from all the recent studies of distances of galaxies. A detailed study of measurement errors in V showed a standard deviation of 90 km/sec, with weights of single observations ranging from 0.05 to 20. There is a further dispersion in equation (1) due to errors in D, and values of h have been used ranging from 0.75 to over 1.5 in the literature.

The difference between the two radial velocities in a pair, ΔV, is assumed to be the projection of a circular orbital velocity, v, and the observed angular separation, S (in minutes of arc), is the projection of the line of centers (of length, r) divided by the distance, D, and by the number of minutes in a radian. By Kepler's harmonic law of two-body gravitational orbits, rv^2 is proportional to the sum of the masses; hence

$$M_1 + M_2 = 675 \frac{SV(\Delta V)^2}{h} \cos^3 \varphi \cos^2 \psi \geq 675 \frac{SV(\Delta V)^2}{h} , \qquad (2)$$

where φ is the projection angle of r, having an unknown value between 0 and $\pi/2$, v is assumed to be perpendicular to r (circular orbit), and ψ is another angle involved in the projection of v, having some value between 0 and 2π. When $\varphi = \pi/2$ one galaxy is behind the other, and the pair would not be recognized as a double. At the other extreme, very wide pairs (large S) were not selected for observation. Holmberg (1954) had found from an analysis of the projected separations of many pairs that the distribution of r is

$$p(r) = K \left[1 + \left(\frac{r}{r_m}\right)^3\right] \qquad (3)$$

for $0.03\, r_m < r \leq r_m$, where K is a normalizing constant, and r_m is determined from approximate distance estimates to be about $(2.3/h) \times 10^5$ psc, apparently the largest possible distance between two galaxies in stable orbit around each other. Equation (3) also applies to double stars with a much smaller value of r_m; it probably represents a statistical result of the condensation of stars (and galaxies) from self-gravitating gas clouds, and the later perturbations of a pair by encounters with single stars (or galaxies).

On the assumptions that φ, ψ, M, and r are independent of each other in a sample of many double galaxies, and that the errors in ΔV are normally distributed (so that the mean square of measured ΔV must be reduced by the variance, σ^2/W)

$$(\Delta V)^2 - \frac{\sigma^2}{W} = 5.92 \times 10^{-8}\, h\bar{M}\left(0.19 + \frac{10^4}{SV}\right) , \qquad (4)$$

where σ is the standard deviation and W the weight of measurements of ΔV, $\bar{M}$ is the mean mass of a single galaxy in all the pairs, and the relative errors in S and V are negligible compared with $\sigma/\Delta V$. Equation (4) is a regression between observed $(\Delta V)^2$ and observed $(0.19 + 10^4/SV)$, and a least-squares solution for $h\bar{M}$ was made from the observations of 33 pairs of galaxies, yielding a value $h\bar{M} = 2.6 \times 10^{11} \pm 1.4 \times 10^{11}$ solar masses (1 solar mass $= 2 \times 10^{33}$ gm). In another 19 cases, observations referred to groups of N galaxies approximating a pair. The simplest of these ($N = 3$) consisted of a close pair of galaxies with a more distant satellite galaxy; the most complex ($N = 5$) consisted of a close group of four with a satellite. These were included with the factor $N/2$ on the right of equation (4) yielding $h\bar{M} = 3.1 \times 10^{11} \pm 1.1 \times 10^{11}$.

Least-squares solutions of equation (4) were also made for subsets of the data, as shown in Table 2, from which it is clear that the mean mass of an elliptical $\bar{M}_E = 30\ \bar{M}_S$, where $\bar{M}_S$ is the mean mass of spirals in these pairs and groups. The mixed systems confirm this fact, which is of importance in the theory of evolution of galaxies. It is also indicated in the individual mass determinations of Table 1, although these vary widely in the case of elliptical (E) galaxies.

The total luminosity of a large group of stars was at first expected to be proportional to the total mass, even though any one star may be 10,000 times more luminous or 1000 times less luminous than the Sun. However, all theories of stellar evolution show that massive stars of very high luminosity are short-lived, so that an old population of stars should have lower luminosity for a given total mass. The ratio M/L in solar units is as small as 10^{-3} for young giant stars and as large as 1000 for long-lived dwarf stars. The luminosity of a galaxy is defined in these solar units as

$$\begin{aligned} L &= D^2\ 10^{0.104-0.4\ m} \\ &= \left(\frac{V}{h}\right)^2 10^{8.104-0.4\ m} \quad , \end{aligned} \tag{5}$$

where <u>m</u> is the measured apparent photographic magnitude of the galaxy. Introducing the sum of <u>N</u> luminosities into equation (4), we get another regression involving the same left-hand side, the desired mean M/hL, and the observables V/S, V^2, and the sum $\Sigma\ 10^{8.104-0.4m}$ on the right. Least-squares solutions for $\overline{M/hL}$ yield the values given in Table 2 and show that the mean M/L for massive E galaxies is 30 to 60 times the value for spirals (S), somewhat more than would be expected if the E galaxies consist simply of older stars. This may indicate an admixture of nonluminous matter in E galaxies,

Table 2.--Average mass and M/L, double galaxies.

No. of systems n	No. of galaxies, ΣN_i (by type) Irr	 S	 SO	 E	Mean mass $h\overline{M}/10^{10}$ (suns)	Mean M/L $\overline{M}/hL$ (solar units)	Notes
52	2	52	17	43	31.2 ± 10.6	38.0 ± 19.9	All systems
33	1	29	10	26	26.0 ± 13.9	31.2 ± 26.0	Pure pairs only
41	1	44	13	33	28.7 ± 9.0	43.8 ± 15.2	High-weight obs. only
16	2	32	0	0	4.0 ± 4.2	3.2 ± 4.2	S and Irr only
10	1	19	0	0	1.6 ± 2.3	1.4 ± 1.8	Pure pairs only
13	1	27	0	0	1.5 ± 1.7	1.3 ± 1.5	High-weight obs. only
18	0	0	11	26	66.2 ± 29.	98. ± 68.	E and SO only
13	0	0	8	18	63.6 ± 38.	92. ± 92.	Pure pairs only
13	0	0	8	19	59.4 ± 15.	90. ± 37.	High-weight obs. only
18	0	20	6	17	31.4 ± 17.	46. ± 23.	Mixed systems
10	0	10	2	8	27.7 ± 23.	41. ± 34.	Pure pairs only
15	0	17	5	14	31.4 ± 18.	46. ± 26.	High-weight obs. only
					$h\overline{M}_E/10^{10}$		Assuming $\overline{M}_E = 30\overline{M}_S$
15	0	17	5	14	60.7 ± 36.		Mixed only
13	0	0	8	19	59.4 ± 15.		E and SO only
28	0	17	13	33	60.0 ± 19.		E, SO, and mixed
13	1	27	0	0	43.4 ± 53.		S and Irr only
41	1	44	13	33	59.6 ± 16.		All high-weight obs.

Notes

Each system includes N_i galaxies in two groups treated as mass-points.

For "pure pairs," $N_i = 2$, and no other galaxy is nearby.

"High-weight observations" include only those systems for which observed relative velocities have weight greater than 0.5.

h is the Hubble constant in units of 10^{-4} km $\sec^{-1}$ Pc^{-1} ($h \simeq 1$).

$\overline{M}$ is the mean mass of one galaxy, in suns.

L is the total photographic luminosity of a galaxy, in suns.

$\overline{M}_E$ is the mean mass of E and SO galaxies.

$\overline{M}_S$ is the mean mass of S, SB, and Irr galaxies.

Each value of $h\overline{M}$ and $\overline{M/hL}$ results from a least-squares solution from which r.m.s. errors of the mean were also determined.

The values of $\overline{M/hL}$ for S and Irr galaxies were incorrectly listed in the first publication (Page, 1962).

although optical evidence of obscuring dust clouds and radio evidence of non-luminous hydrogen are limited to spirals. It is possible that other forms of matter are involved, such as collapsed masses or very low-temperature stars.

The validity of these results has been discussed (Page, 1962) and it is shown that the assumption of circular orbits and the tidal effects neglected in equation (2) are not likely to have affected the results significantly. If $\underline{M}$ is positively correlated with r, so that more massive pairs are systematically of wider separation than less massive ones (a possible result of the mechanics of galaxy formation or of later perturbations by intruders), then the values of $h\overline{M}$ in Table 2 are underestimated. If the observed pairs are all embedded in an intergalactic medium of uniform density ρ, the mass involved in equations (2) and (4) would be $2M + 4\pi\rho r^3/3$ and this dependence on r or SV again results in an underestimate. Motions in clusters of galaxies and cosmological models fitted to the Hubble Law of redshifts imply values of ρ as high as 10^{-28} gm/cm^3. The resulting increase in $\overline{M}$ is approximately $5 \times 10^{35}\ \rho/h^3$ or about 10^7 solar masses, which is insignificant (only one part in 10^3 or 10^4).

Although the selection in $\underline{S}$ has been accounted for, other effects of selection might influence the means in Table 2. Selection of the higher luminosity pairs is to be expected, although small-diameter galaxies and ones of low surface brightness are apt to be overlooked on photographs; high surface brightness is selected for velocity measurements. Because the more luminous galaxies in a class are expected to be the more massive ones, the estimated average masses, $\overline{M}_E$ and $\overline{M}_S$ are undoubtedly biased toward higher values. However, the large ratio $\overline{M}_E/\overline{M}_S$ cannot be explained as a result of this selection, and for three reasons: (1) the E galaxies included in the set of pairs (Page, 1962, pp. 293-94) are somewhat fainter than the S galaxies included; (2) in the mixed pairs, E galaxies are as often brighter than S galaxies as they are fainter; and (3) the results for mixed pairs confirm $\overline{M}_E/\overline{M}_S = 30$. Note, also, that for the pairs selected, the mean luminosity $\overline{L}_E \simeq 0.67\ \overline{L}_S$ if the spread is not extreme.

It has been suggested that galaxies in pairs differ systematically from single galaxies, but this is not supported by the mass estimates for single and double spirals in Table 1. Moreover, the morphological types E0 to E7, S0, Sa, Sb, Sc, SBa, SBb, SBc, and Irr I all appear normal in pairs, although the rare dwarf elliptical and dwarf irregular types are not represented in this sample. These dwarfs probably are much less massive.

The set of observations may include "optical pairs" -- chance lineups of two galaxies, one far beyond the other. The number of such chance pairs, as distinguished from dynamical pairs with $r < r_m$, clearly depends on the number of galaxy images per square degree and on the maximum separation, S_m, which is accepted. Polya (1919) derived the probability that, if n points are distributed at random on a sphere, none of them will fall within angle S from an (n + 1)th point:

$$p(S,n) = (\cos S/2)^{2n} \simeq \frac{e^{-nS^2}}{4.78 \times 10^7} \quad , \tag{6}$$

and this was used by Holmberg (1937) to estimate N_2, the number of chance pairs in a square degree of the sky where N_1 single galaxies are randomly distributed:

$$N_2 \simeq \frac{\pi N_1^2 S^2}{7200} \quad . \tag{7}$$

For separations S less than 6 minutes of arc and $N_1 \simeq 1.3$ galaxies per square degree brighter than $m \simeq 15$, equation (7) yields $N_2 = 0.027$ per square degree, or less than 6% of the pairs counted by Holmberg in photographs covering 15000 square degrees. Thus it is argued that few or none of the 33 pairs studied (Page, 1962) are chance lineups. However, this argument is based on gross oversimplifications: (1) the density of single galaxies brighter than 15^m is not uniform over the sky; (2) some of the nearer pairs have much larger separations; (3) the roughly equal brightnesses of galaxies in a pair have not been taken into account, along with the rapid increase in N_1 as fainter galaxies are counted.

Page, Dahn, and Morrison (1961) refined this statistical treatment and applied it to counts of pairs and single galaxies on the Palomar Atlas photographs in two clusters (Coma and Virgo) and one area outside of known clusters. In these areas totaling 270 square degrees they counted 254 pairs and 8500 singles, classed in six ranges of brightness or magnitude as shown in Table 3. The faintest galaxies counted were about $18^{m}.5$, or about one-twenty-fifth as bright as the 15^{m} galaxies considered above. Pairs were counted only if the separations,

$$S \leq 3(a_1 + a_2) \ , \tag{8}$$

where a_1 and a_2 are the angular diameters of the two galaxies in the pair. It was found empirically that

$$\log a = 2.7 - 0.2\ m \pm 0.1 \ , \tag{9}$$

$$\log N_1(m_1) = \log N_1(m_2) + 0.5(m_1 - m_2) \ , \tag{10}$$

where $N_1(m_1)$ is the number of single galaxies per square degree with brightnesses in the range $m_1 - 0.75 < m < m_1 + 0.75$. The counts of all pairs, $9.5 < m < 18.5$, fitted the expected regression of equation (7) over the range $0.25 < N_1 < 150$:

$$\text{observed } N_2 = (0.80 \pm 0.06)\ N_1^2 \ . \tag{11}$$

Counts were actually made in 2-cm squares on the photographs corresponding to $22'.4 \times 22'.4$, but pairs were often formed with galaxies in the next square, so that the effective areas for the counts of pairs in each brightness class were the A_{ii} given in Table 3, and the expected number of chance pairs is

$$\begin{aligned} N_2 &= \left(\frac{\pi}{2}\right) \sum_{i,j} S_{ij}^2 b_i b_j \left(1 - \frac{1}{A_{ij} D}\right) \qquad (12) \\ &= 0.63\ N_1^2 \ . \end{aligned}$$

Table 3.--Classes of galaxies counted
(Page, Dahn, and Morrison, 1961).

Class, i	Magnitude, m_i	Major diameter, a_i	$N_1(m_i)/N_1 = b_i$	Maximum separation, S_{ii}	Effective area, A_{ii}
1	$18\overset{m}{.}5$-17.0	$0\overset{\prime}{.}11$-$0\overset{\prime}{.}22$	0.823	$0\overset{\prime}{.}95$	590 $(')^2$
2	17.0-15.5	0.22-0.45	0.146	1.90	680
3	15.5-14.0	0.45-0.90	0.0260	3.76	880
4	14.0-12.5	0.90-1.79	0.0040	7.52	1350
5	12.5-11.0	1.79-3.58	0.00082	15.1	2560
6	11.0- 9.5	3.58-7.17	0.00014	30.2	6050

The difference between the observed numbers of pairs and the expected number of chance pairs calculated from equation (12) is (0.17 ± 0.06) N_1^2, which shows, first, that about 0.17/0.80, or 21% of pairs defined by equation (8) and with $m < 18.5$ are physical pairs rather than optical chance lineups, and, second, that physical pairs occur more frequently in clusters (where N_1 is large) than in other regions. Because the sample size is small, this latter conclusion is uncertain; it might be expected as a result of the manner in which pairs of galaxies are formed. Since the number of close passages is proportional to the square of the number of galaxies per unit volume, a higher frequency in clusters might be due to the greater probability of fission there, or to the greater number of captures (although the dynamical capture of one galaxy by another is unlikely).

Masses and stability of clusters of galaxies

The average masses of galaxies were first estimated by Zwicky (1933) and Smith (1936) from velocity dispersions in clusters of galaxies. Deviations from the mean of all measured radial velocities of galaxies in a cluster are interpreted as projections of randomly oriented individual velocities with respect to the center of mass. On the assumption that the measured velocities are a fair sample of all the velocities of member galaxies, and that the cluster is stable, the virial theorem can be applied (Zwicky, 1933), or the largest relative velocities can be equated to the velocity of escape (Smith, 1936). If the distance is known and if symmetry can be assumed so that a distribution of galaxies around the center of mass can be inferred, the mass of the cluster can be determined by either method. This total mass, divided by the number of galaxy images counted on photographs of the cluster, gives an average galaxy mass which is generally 10 to 50 times larger than masses of individual galaxies determined from rotations or orbital motions in pairs, as shown in Tables 1 and 4. Table 4 is taken primarily from the papers discussed in the 1961 Conference, with values of $\overline{M}$ and $\overline{M/L}$ converted to a Hubble constant H = 100 km/sec Mpc where necessary.

Table 4.--Masses of groups and clusters of galaxies
(from de Vaucouleurs, 1961, Burbidge, 1961; H = 100 km/sec Mpc).

Group or cluster	Angular diameter	Total m_{pg}	$\bar{V}/100$	R (Mpc)	N_b	$N_b\bar{M}/10^{10}$	$\bar{M}/10^{10}$	$\overline{M/L}$
VV115 (Seyfert)	1!9		44.	0.01	5	24.	5.	
VV116		12.7	64.		2E,3S	100.	20.	
VV150	1!2		73	0.02	S			
VV166 (NGC 67-72)			67.9		3E,3S			350.
VV288 (Stephan)	3!7	11.8	67.	0.04	5(E,S)	500.	100.	100.
NGC55	500!	8.7	5.5	0.4	6S	600.	100.	500.
NGC383 (Pisces)					25E	12500.	500.	260.
NGC3031-77 (M81)		6.	2.		> 4S		120.	200.
NGC6027 (Serpens)	1!6	14.	45.	0.01	3E,3S			
NGC7619 (Pegasus)	120!	11.	40.	0.3	5E	2500.	500.	300.
Local Group					2I,2S,2E		400.	
Sculptor	950!		3.	0.37	6	1700.	280.	
NGC3561			87.					
NGC6166	2!5	13.0	9.1	0.03	5E	1400.	280.	175.
Abell 2199	12!		90.	0.15	> 19			
(Mean group)				0.15	8	1000	150	280
Can Ven Cluster	19°	6.6	6.8	1.1	30S	4500.	150.	400.
Fornax	5.7		15.	0.75	30	4700.	157.	
Pegasus	2.0		39.	0.67	50	4200.	84.	
U Ma	10.		20.	1.8	50	2800.	56.	
Hercules	1.4		108.	1.3	50S, 30E	5600.	70.	
Virgo E	11.5	6.3	11.	1.1	100E	24000.	240.	600.
Virgo S	11.0		19.	1.8	100S	< 45000.	< 450.	
Coma	9.0	9.4	67.	5.2	500E	75000.	150.	900.
NGC541					500S	5000.	10.	
Mean cluster				1.7	100	10000	130	600

Some of the groups and clusters are identified in the first column by numbers in the catalog of Vorontsov-Velyaminov (1960), some by the NGC number of bright galaxies in them and some by the constellation where they appear. Angular diameters are given in minutes of arc for the smaller groups and in degrees for larger clusters. The total photographic magnitude of the whole group or cluster of galaxies and mean radial velocity in km/sec are given as before, $\overline{V}/100$ being equal to the distance in Mpc. The radius, R of the cluster is in Mpc. The number of bright galaxies, N_b (no fainter than one-fifth to one-tenth of the brightest), in the cluster, is used to obtain the mean mass of a galaxy, $\overline{M}$, from the total mass estimate, $N_b\overline{M}$. (There is a large uncertainty in N_b due to foreground and background galaxies.) The total mass (listed under $N_b\overline{M}$) is obtained from the virial theorem applied to the deviations $V_i-\overline{V}$, assuming that each group or cluster is stable. The ratio $\overline{M/L} = N_b\overline{M}/\Sigma\, L_b$ and is less affected by the uncertainty in N_b but may still be wrong by a factor of two (van den Bergh, 1961). The upward trend of M/L with N_b shown in Figure 1 is as yet unexplained.

Three reasons have been proposed for these excessive cluster masses: (1) the galaxies in large, compact clusters differ systematically from others (in fact, it has been claimed that such cluster members are predominantly or entirely E galaxies); (2) there are other forms of mass in clusters, generally called intergalactic matter; and (3) the clusters are <u>not</u> stable, so that the cluster mass estimate is unfounded. The conference organized by Neyman, Page, and Scott (1961) met primarily to consider this third possibility and the hypothesis proposed by Ambartsumian (1956, 1961). In effect, Ambartsumian assumed sudden release of vast amounts of energy to account for the large dispersion in observed radial velocities of galaxies in some groups and clusters. Discussion revealed two further difficulties in any statistical analysis of motions in a cluster of galaxies: the unwitting inclusion of foreground or background galaxies as cluster members (uncertainty in N_b), and peculiar patterns of motion (contraction and subclustering) that invalidate the conventional application of the virial theorem.

It appeared from this discussion that there are at least four categories of systems with different degrees of stability:

(a) Close pairs of galaxies are probably stable.

(b) Small groups like Stephan's Quintet are most likely to be unstable, often explosive.

(c) Loose irregular clusters such as the Virgo Cluster are suspected to be unstable, but not violently so.

(d) Compact regular clusters such as the Coma Cluster are probably stable.

Six stages of instability-stability were recognized:

(a) Explosive expansion, as assumed by Ambartsumian.

(b) Mild expansion.

(c) Contraction.

(d) Dynamical stability to which the virial theorem applies.

(e) Stability of form involving a continuous exchange of galaxies between a cluster and the field, to which the virial theorem does not apply.

(f) Subclustering, or clusters of clusters, for which the virial theorem must be modified.

The most serious observational difficulty was recognized to be the identification of the members of a cluster or group, excluding foreground and background galaxies, yet including faint members. One of the major theoretical difficulties is that the calculated time for unstable groups and clusters to disperse is generally 10^8 years or so -- much less than the estimated ages of individual member galaxies, and inconsistent with the idea that member galaxies were all formed in the cluster where they now appear. So short a cluster life raises the question of cluster formation and is probably inconsistent with the observed velocity dispersion among field galaxies.

The conference report ends with four more questions:

"What is the evidence that members of a cluster had a common origin?

"Are nongravitational forces involved in the dynamics of small groups of galaxies?

"In what way are the extragalactic radio sources associated with individual galaxies or with clusters?

"What is the mechanism by which the galaxies were formed, and how does it account for clustering?"

In the three years since this was written, direct evidence (both radio and optical) has been obtained of explosive energy release in galaxies. At the same time astronomers have developed greater acceptance of an intergalactic medium and a greater interest in the mechanism of the formation and evolution of galaxies. Lynds and Sandage (1963) discovered clouds of ionized gas apparently "splashed" out of the center of the nearby spiral, M82, about 1.5×10^6 years ago, and Schmidt (1964) discovered the super-luminous Quasi-Stellar Objects (QSO's or "quasars"). Their strong radio emission led to this discovery, and other means of identifying them are now under study. Theoretical studies by several authors have been discussed at special symposia (Robinson, 1964, and Page, 1964), generally starting from a protogalaxy gas cloud assumed to have a density much higher than the present mean density of galaxy matter (product of the number of galaxies per unit volume and the average mass of a galaxy, about 3×10^{-30} gm/cm^3). In fact, Sciama (1964) assumes an intergalactic density of 10^{-28} gm/cm^3 in the form of ionized hydrogen at 100,000°K which would be unobservable in both optical and radio frequencies, and would have thermal instabilities leading to condensing masses of about 10^{11} suns.

The enormous energy output of the QSO's may be due to gravitational collapse (Robinson, 1964) in the few cases where initial conditions were right (zero angular momentum), and other conditions may have led to condensation of pairs, groups, or clusters of galaxies. Pairs of galaxies are the simplest groups, and for this reason I spent a good deal of time with Neyman and his group at Berkeley looking for statistical evidence of a common origin of the two galaxies in a pair. The observable features that are expected to reveal common origin are the orientations and morphological types. In addition, the study of relative sizes of galaxies in pairs leads to a correlation of size with morphological type.

Relative orientations in pairs

The statistical analysis is applied to the following measured quantities:

T_1 = morphological type,

a_1 b_1 = major and minor (angular) dimensions,

θ_1 = position angle of the major axis for one galaxy in a pair (subscript 2 for the second galaxy of the pair),

θ = position angle of line of centers,

s = (angular) separation of centers.

It is found that:

(a) The distribution of θ_1, θ_2 is uniform; therefore, individual galaxies are randomly oriented to the line of sight.

(b) The ratio, $b/a = f(i,e)$ where i is the inclination of the galaxy axis to the line of sight, e is the true ratio of axes of a spheroid matching some standard isophotal surface, and f is the simple projection of an ellipsoid corrected for a systematic error in measurement identified by Holmberg (1945) and for variations in surface brightness with i, allowing for internal absorption.

(c) The true axis ratio, $e(T)$, is a function of type and is roughly known from descriptive studies such as the Hubble Atlas. That is, rough means are:

$\bar{e} = 0.1$ for Sb, Sc; $\bar{e} = 0.2$ for Sa, S0; $\bar{e} = 0.3$ to 0.4 for E.

(d) From (a), the distribution of i should be $f(i) = \cos i$ for a fair sample of individual galaxies. Hence, the distribution of b/a for any type, $n_T(f)$, should determine $\bar{e}(T)$ or the distribution of e for type T. This determination depends primarily on $n_T(f)$ near the value $f = \bar{e}(T)$.

(e) Selection may also affect $n(f)$, but it is likely to be slowly changing with $f = b/a$ over the full range $f = e(T)$ to $f = 1.0$. Hence, the fitting of observed $n(f)$ over the full range in f can determine both the selection factor and $\bar{e}(T)$ or the distribution of $e(T)$.

(f) With $e(T)$ known, i can be determined for each galaxy image. The angle φ between the axes of two galaxies in a physical pair is then calculated for a trigonometric function, $\varphi(i_1, i_2, \theta_1, \theta_2, \theta)$. After allowing for selection in i, the distribution of φ is obtained.

Preliminary results for a list of 120 doubles were inconclusive (Page, 1963). If the derived distribution of φ shows a tendency toward small φ, this will be evidence of the common origin of physical pairs from a rotating mass, both fragments sharing the original angular momentum. A different result may indicate some more complex coupling of the angular momenta of galaxies in close pairs.

It should be noted that the random orientation of the axes of individual galaxies was inferred above from observed position angles, θ_1 and θ_2 in pairs widely separated in the sky. Evidence of parallel axes of individual galaxies in one region of the sky -- that is, in roughly the same direction from us, and possibly in a large cluster -- has been reported by Wyatt and Brown (1955) whose observations were not complete enough for application of the following analysis. If the angular dimensions, _a_ and _b_, can be measured without systematic error, or corrected as suggested by Holmberg (1945), and if $e(T)$ can be estimated, then the inclination _i_ and the position angle θ can be listed for each galaxy together with its direction from us relative to the plane of the Milky Way in terms of galactic coordinates, B and L.

There are three hypotheses to be tested, first for galaxies in all directions and then for galaxies in nearly the same direction for which $B = B_n \pm \Delta B$, $L = L_n + \Delta L$:

Hypothesis 1, axes completely random (as for individuals in pairs).

Then, for n galaxies, and no selection effects, we expect

$$n_{Ia}(\theta)\, d\theta = \frac{n}{180}\, d\theta \; ,$$

$$n_{Ia}(i)\, di = \frac{2\pi n}{180} \cos i \, di \; , \qquad (13)$$

where dθ and di are measured in degrees of arc. In terms of the corrected ratio, $f_c = b/a$, equation (13) becomes

$$n_{Ia}(f_c) = \frac{f_c t}{(1 - e_T^2)^{\frac{1}{2}} (f_c - e_T)^{\frac{1}{2}}}$$
$$= 0 \quad \text{for} \quad f_c < e_T \; , \qquad (14)$$

where t is a constant representing the numbers of galaxies of type T in the standard interval of sin i. A rough preliminary analysis of several hundred measures shows that for elliptical galaxies, $e_T = 0.3$, and for type Sa, $e_T = 0.2$, both with little spread. For over 300 Sb and Sc galaxies, e_T has a spread from 0.08 to over 0.3, with a mean of 0.19. This spread implies errors in the types, T. No account has been taken of selection based on f or T. The determination of $n(e_T)$ may be possible from data of this type and Hypothesis 1, if it can be assumed that the frequency of various morphological types in space is a smooth function of e. With several hundred sets of measures of a and b for galaxies in one cluster, Hypothesis 1 could be tested in this manner for that cluster's members.

Hypothesis 2(a), preferred direction of the axes of galaxies in widely differing directions from us. If this is assumed for all galaxies, it implies a nonisotropic cosmological model. (Another possible hypothesis -- a preferred direction of axes relative to our line of sight -- violates the Cosmological Principle, since it implies that our Milky Way galaxy is in a preferred position.) The data allow calculation of two angles, p and q, defining the orientation of the axis of an external galaxy relative to the axis of the Milky Way, as shown by Holmberg (1946):

$$\cos p = \cos B \cos i \cos \theta' - \sin B \sin i \,, \tag{15}$$

$$\sin (L+q) = - \frac{\cos i \sin \theta'}{\sin p} \,, \tag{16}$$

where θ' is the position angle of the galaxy relative to the pole of the Milky Way and can be computed from the measured θ:

$$\sin(\theta - \theta') = \frac{\sin 27^{\circ}.4 - \sin \delta \sin B}{\cos \delta \cos B} \,. \tag{17}$$

There is a fourfold ambiguity in this determination of the axis direction, allowing two values each of p and q. Hypothesis 2(a) implies that p is small, therefore that i is large for B near 90° and that i is small (edge-on view) for B near 0°. Unfortunately, the obscuration of galaxies by interstellar material in our Milky Way prevents observations for $B < 20^{\circ}$. The most sensitive test may be for differences in

$$\begin{aligned} \sin^2 i &= (\cos p \sin B + \sin p \cos B \cos (L+q))^2 \\ &= \frac{f_c^2 + e_T^2}{1 - e_T^2} \end{aligned} \tag{18}$$

between galaxies with B near 30° and those near 90°.

Hypothesis 2(b), preferred direction of axes in a cluster of galaxies; that is, $p - p_0$ and $q - q_0$ both small, where p_0 and q_0 define the preferred direction. It seems unlikely that selection could affect observed values of a, b, and θ to produce this result. Wyatt and Brown (1955) found a preference for $\theta = 130^{\circ}$ among 800 galaxies in the constellation Cetus ($B = -65^{\circ} \pm 15^{\circ}$, $L = 150^{\circ} \pm 50^{\circ}$).

Further work on this important question of preferred orientations is being continued by Neyman, Scott, Zonn, and others.

Table 5.--Types of galaxies in pairs.

$n_2(T_1,T_2)$ = number of pairs of types T_1,T_2

Type of larger, T_1	Type of smaller galaxy in the pair, T_2							
	E	SO,SBO	Sa	Sb,Sc	(S)	SB	Ir	Total
E	13	14	6	6	(12)	0	1	40
SO,SBO	6	74	11	10	(21)	1	4	106
Sa	10	24	46	28	--	1	3	112
Sb,Sc	10	31	24	44	--	2	12	123
(S)	(20)	(55)	--	--	(142)	(3)	(15)	(235)
SB	0	7	2	2	(4)	2	2	15
Ir	0	14	3	10	(13)	0	4	31
Total	39	164	92	100	(192)	6	26	427
n, galaxies in sample	79 E	270 SO,SBO	204 Sa	223 Sb,Sc	(427) S	21 SB	57 Ir	854 Total

S = Sa, Sb, or Sc. Ir is mainly Ir I.

Counts and types by W. Zonn from Palomar Atlas Prints.

Correlations of types, sizes, and luminosities in pairs of galaxies

If pairs of galaxies were formed at random, the expected numbers $n_2(T_1,T_2)$ involving types T_1 and T_2 would be proportional to the product $n(T_1)n(T_2)$, where $n(T)$ is the number of galaxies of type T in the sample, and $n = 2n_2$ is the total number. Preliminary analysis of 427 pairs typed by Zonn showed that

$$\frac{n_2(T_1,T_2)}{n_2} = \frac{Q(T_1,T_2)n(T_1)n(T_2)}{n^2} , \tag{19}$$

where $Q(T_1,T_2)$ is a "pairing factor" generally different from 1. The actual numbers $n_2(T_1,T_2)$ are shown in Table 5 and the pairing factors in Table 6. There is a clear preference for $Q(T_i,T_i) > Q(T_i,T_j)$, for the few barred spirals, SB (SBa, SBb, and SBc taken as one group). Combining Sa with Sb and Sc in one group, S, yields $Q(S,S) = 1.33$ for 142 spiral pairs.

The pairing factor $Q(T_i,T_j)$ also contains evidence of relative sizes, since T_i refers to the larger galaxy in the pair, and T_j to the smaller one. Thus the ratio $n_2(T_i,T_j)/[n_2(T_i,T_j) + n_2(T_j,T_i)]$ represents the proportion of $T_i - T_j$ pairs in which the one of type T_i is the larger, as shown in Table 7. These seem to indicate a spread of sizes from SO types generally smallest through E, Ir, SbSc, Sa to SB generally largest. Neyman and Scott (1964) have applied this analysis to a more accurate set of data, showing that Sc galaxies in mixed pairs average about three times larger than EO galaxies, although when the Sc is the brighter it can be 10 to 15 times larger than the EO. Unfortunately, these results can be seriously influenced by systematic errors in the types, the tendency being to classify a small image as E type because no structure can be seen.

The best available data on pairs can be found in a catalog of galaxy redshift measurements by Humason, Mayall, and Sandage (1956), where the effects of selection are expected to be extreme because of the difficulty of photographing spectra (in addition to the selection of the galaxies from

Table 6.--Pairing factor, $Q(T_1,T_2)$ in 427 pairs.

Type of larger, T_1	Type of smaller galaxy in the pair, T_2							
	E	SO,SBO	Sa	Sb,Sc	(S)	SB	Ir	Total
E	3.56	1.12	0.63	0.58	0.61	0.0	0.4	1.01
SO,SBO	0.48	1.73	0.34	0.28	0.31	0.3	0.44	0.79
Sa	1.06	0.75	1.88	1.05	--	0.4	0.44	1.10
Sb,Sc	0.97	0.88	0.90	1.51	--	0.7	1.61	1.10
(S)	1.04	0.82	--	--	1.33	0.6	1.05	1.10
SB	0.0	2.11	0.8	0.7	0.76	7.4	2.9	1.43
Ir	0.0	1.56	0.44	1.34	0.91	0.0	2.11	1.09
Total	0.99	1.21	0.90	0.90	0.90	0.57	0.91	1.00

Table 7.-- Relative sizes of galaxies in pairs (portion of mixed pairs in which one type is larger).

Type more often larger	Type more often smaller					
	E	SO,SBO	Sa	Sb,Sc	SB	Ir
E	--	0.7	--	--	--	(1.0)
SO,SBO	--	--	--	--	--	--
Sa	0.63	0.69	--	0.54	--	(0.5)
Sb,Sc	0.63	0.76	--	--	(0.5)	0.55
SB	--	0.9	(0.7)	(0.5)	--	(1.0)
Ir	--	0.78	(0.5)	--	--	--

photographs). Of 920 galaxies listed by HMS, 188 satisfy the pair requirement, $s \leq 3(a_1 + a_2)$ in 94 separate pairs of which 26 were selected by the observers because they were pairs. Galaxies in the other 68 pairs were observed singly. Twenty-six are single, isolated pairs, 19 of them included in the mean mass determinations by Page (1962). These 26 again show a slight tendency toward pairs of the same types -- 5 pairs of E, 3 of Sb or Sc, and 1 of SO -- a total of 9 instead of the 7 expected from random pairing.

More significant, the linear dimensions, A, of all these and many other galaxies in the HMS (1956) catalog can be calculated from the angular diameters in minutes of arc, a, given by de Vaucouleurs (1964) and the redshift velocities, V, using Hubble's Law, equation (1), with h = 1:

$$A = 0.00292 \text{ a V kpc.} \qquad (20)$$

The mean values of A in Table 8, and the distribution of magnitudes shown in Fig. 2, show that galaxies in pairs differ only slightly from single ones of the same type, and that the spiral types have a larger spread in dimensions than ellipticals. (Measurement errors in V are relatively small; $\sigma_V \simeq 100$. Some of the deviations in A may be due to errors in measuring a.) Although the sample by no means represents all the data (de Vaucouleurs, 1964), and although the dispersions are large, the mean absolute dimensions in Table 8 imply that elliptical and lenticular galaxies (E, SO, and SBO) are less than two-thirds of the size of spirals (Sa, Sb, Sc, SBa, SBb, and SBc). Moreover, galaxies of different types in a tight pair $(s < 3a_1 + 3a_2)$ are smaller yet. The average masses of elliptical galaxies in a similar sample of tight pairs (Table 2) is 30 times the average for spirals; hence the density of matter in the former must be over 100 times larger than the density in spirals.

The evolution of galaxies

It is now virtually certain that galaxies slowly change in appearance over periods of billions of years, due primarily to the formation of stars from interstellar gas and the aging of the stars (a process first studied

Table 8.--Mean dimensions of galaxies by types (diameter, A, in kpc ± mean deviation).

	128 single field galaxies		98 galaxies in groups		51 galaxies in isolated tight pairs					
					all pairs		similar types		mixed types	
Type	n	A	n	A	n	A	n	A	n	A
E0 to E7	29	10.8 ± 4.	41	10.1 ± 4.	21	8.5 ± 4.				
S0,SB0	20	11.3 ± 4.	27	15.6 ± 4.	9	7.0 ± 3.				
E,S0,SB0	49	11.0 ± 4.	68	12.3 ± 4.	30	8.1 ± 4.	20	8.8 ± 4.	9	6.7 ± 3.
Sa	12	17.6 ± 7.	8	12.1 ± 3.	2	12.6 ± 5.				
SBa	8	14.5 ± 7.	1	17.8	0					
Sa,SBa	20	16.4 ± 7.	9	12.7 ± 3.	2	12.6 ± 5.				
Sb	13	18.9 ± 5.	4	25.7 ±12.	5	15.9 ± 7.				
SBb	6	24.6 ± 7.	7	27.0 ± 4.	1	7.1				
Sb,SBb	19	20.7 ± 6.	11	26.6 ± 7.	6	14.5 ± 7.				
Sc	18	16.6 ± 4.	4	22.0 ± 7.	10	16.0 ± 6.				
SBc	18	16.4 ± 5.	0		2	13.0 ± 4.				
Sc,SBc	36	16.5 ± 5.	4	22.0 ± 7.	12	14.4 ± 6.				
Sa,Sb,Sc	43	17.6 ± 5.	16	18.0 ± 7.	17	14.8 ± 6.				
SBa,SBb,SBc	32	17.5 ± 6.	8	25.9 ± 4.	3	11.1 ± 4.				
S,SB	75	17.6 ± 6.	24	20.6 ± 6.	20	14.3 ± 6.	9	18.4 ± 7.	9	13.2 ±4.
Ir	4	5.2 ±	6	12.1 ± 7.	1	12.3				
All types	128	14.6	98	14.4	51	10.8				

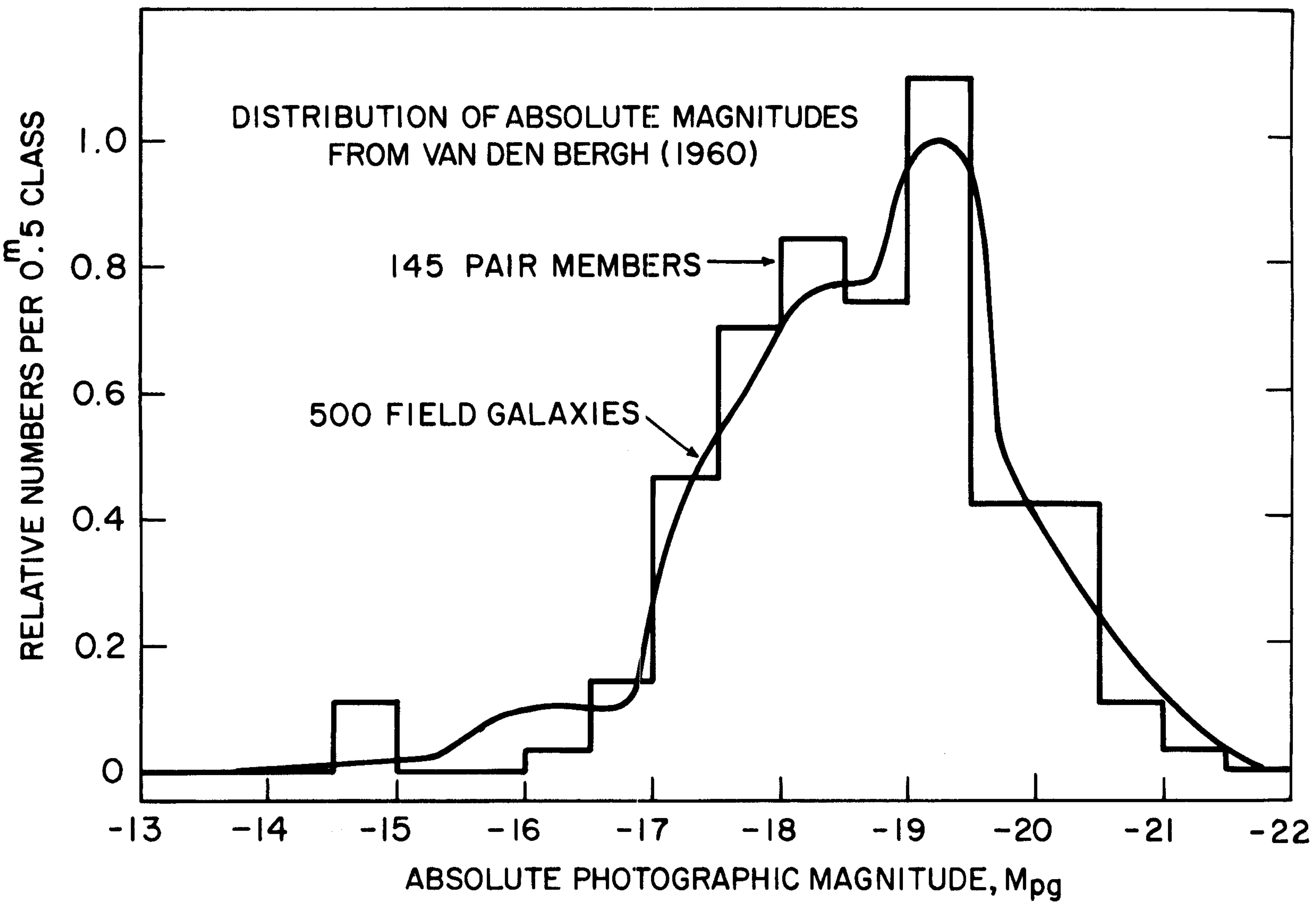

Figure 2.-- Distribution of absolute magnitude from van den Bergh (1960).

statistically 30 years ago, and now the subject of detailed calculations based on nuclear reactions in individual stars). The generally accepted concept is that stars condensed from primordial gas clouds or regions of higher density in a universal gaseous medium. As they age, the stars become redder and less luminous, although their masses remain nearly constant. Since E and S0 galaxies have low luminosity for their large masses, it was at first natural to assume that evolution carried a blue spiral galaxy into the redder E type. However, it is difficult to account in this way for the larger mass of the E galaxies, and for tight pairs consisting of one E and one spiral galaxy.

The evolutionary development of stars in the Milky Way has been worked out by Schmidt (1957) and others on the assumption that the rate of star formation depends on the density of the gas from which they form. Holmberg (1964) then collected mass estimates like those in Tables 1 and 2, and size estimates like those in Table 8, and showed that the resulting average densities of galaxies are correlated with color and morphological type in the sense that high density implies red, E-type galaxies. He argues that the small scatter on a plot of density versus color of galaxies proves that (1) galaxies are all of about the same age and (2) the initial density of each primordial gas cloud determines the morphological type of the galaxy evolved. Dense gas clouds formed stars quickly; these stars aged, reddened, and now have the low luminosity (high M/L) of an E galaxy. In gas clouds of lower density, stars formed later and have not yet aged; hence, we see them as blue, highly luminous spirals of low M/L.

These ideas were discussed at the Congress of the International Astronomical Union (Page, 1964) and it was noted that the initial sizes of the primordial gas clouds, their angular momenta, and possibly their turbulence and chemical content may also affect the morphological types of the galaxies that evolved. In addition to the average densities, colors, and morphological types of galaxies that have been studied so far, it is possible to derive for a large sample of galaxies:

(a) Density distribution (from accurate doppler shifts in many spectra of each galaxy -- as reported by the Burbidges in papers cited, 1960-1965, primarily for spirals).

(b) Approximate central density, from inclinations of lines in individual spectra, now measured for over 100 galaxies by Mayall (1961), Lindblad and Page (as yet unpublished).

(c) Total angular momentum (from the above measures).

(d) Color and luminosity distribution, including central-region colors (as measured by Holmberg (1964) and others).

(e) Mean M/L and the differences between M/L near the center and in outer regions (from the above measures).

(f) Gas content (from the hydrogen 21-cm radio-emission flux).

(g) Distribution of interstellar gas and stars of various types (from the intensities of lines in spectra).

Preliminary results indicate the expected correlation between central densities from (a) and (b) and central colors from (d), and between angular momenta (c), gas content (f), and morphological types. The most serious discrepancy remains in the large values of mean M/L for galaxies (particularly E and SO types), which are not consistent with means of M/L for individual stars with a distribution of masses similar to stars near the sun. It seems likely (Page, 1964) that this may be explained either by large numbers of very small, faint stars in E galaxies, or by large collapsed masses with low or zero luminosity. The formation of small stars and the lower cutoff in frequency distribution of stellar masses probably depend on the turbulence in the primordial gas cloud from which a galaxy condenses. The formation of large nonluminous masses by collapse is possibly a later stage in the evolution of some contracting galaxies with low angular momentum that for a brief period are highly luminous quasi-stellar objects (Robinson, 1964).

Zwicky (1964) finds evidence of a sequence of "compact galaxies" that may be earlier stages in the collapse; he estimates that there are two of these, on the average, in every square degree of the sky as photographed by the large telescopes at Mount Wilson and Palomar. As reliable methods are developed for identifying these small images on photographs (probably by their blue color), and after their distances have been reliably estimated, it will be possible to calculate the relative numbers in a volume of space and provide a statistical basis for theories of evolution of galaxies.

Discrete sizes of elliptical galaxies

A recent statistical result of observations by A. Wilson (1964) and the theoretical work of Edelen (1963) involves measures of diameters more accurate than those given in Table 8. Wilson measured values of a accurate to 0".01 for 130 E-type galaxies in 8 clusters, in each of which the galaxies are presumably all at the same distance from us. These values of a show a clumping at discrete sizes given by the formula

$$\lg a + \lg \overline{V} = K + \frac{1}{2} \lg n(n + 1) \ , \qquad (21)$$

where n is an integer, and K is an empirical constant. A careful review (Page, 1964a) uncovered no other explanation than that the E galaxies were formed in these sizes, a result expected from the field equations of general relativity with one further assumption of stability (Edelen, 1963).

Wilson has further evidence that the distances of clusters of galaxies and the redshifts, $Z = V/c$, related to them by Hubble's Law, equation (1), are also clumped about discrete values given by the formula

$$\frac{Z}{1 + Z} = \frac{M(M + 1)N(N + 1)}{137^2} \ , \qquad (22)$$

where M and N are integers. This set of discrete redshifts implies that the 28 clusters with measured Z are arranged in a pattern rather than distributed at random. It has yet to be confirmed by an analysis of the angular sizes and directions of clusters, and the sample size should be increased.

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