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To disseminate knowledge is to disseminate prosperity — I mean general prosperity and not individual riches — and with prosperity disappears the greater part of the evil which is our heritage from darker times.

—Alfred Nobel

PROGRESS REPORT

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THE ANALYSIS OF STRUCTURALLY ORTHOTROPIC SHELLS BY MEANS OF THE COMPLIANCE METHOD

I. Introduction

Most thin elastic shells when used as structural components possess some type of reinforcement. The reinforcement may take the guise of machined or attached ribs, corrugations of the surface, filament winding or sandwich type construction. Whatever the type of reinforcement, its ultimate purpose is to enable the shell structure to sustain larger loads at a minimal increase in weight.

The usual effect of the stiffness is to give the shell structure directional properties in its strength characteristics. These strength characteristics are measured by the ability of the shell to resist bending and torsional moments and also membrane forces. However directionally dependent strength characteristics may also be observed in an unstiffened shell if the material of the shell is anisotropic, that is, possess directional dependence in its elastic constants. Because of this fact, the stiffened shell is termed as structurally anisotropic while the unstiffened shell composed of an anisotropic material as intrinsically anisotropic.

The stiffeners in a structurally anisotropic shell usually are placed in preferred directions, that is, directions coinciding with the principal directions of the shell reference surface. Then the strength characteristics of the stiffened shell will achieve maximum and minimum values in these directions. The same effect can be achieved in the intrinsically anisotropic shell by using a material orthotropic in its elastic properties and such that the axis of elastic symmetry coincide with the principal directions of the shell surface. In order to differentiate the type of shell being discussed from the general stiffened or anisotropic shell, the former will be termed structurally orthotropic while the latter intrinsically orthotropic. It is the orthotropic shell that is being considered in the research project.

Since there is a similarity in the behavior of the structurally and intrinsically orthotropic shells, the analysis of the latter can be used as the basis of an approximate behavior of the former. To do so requires the postulating of an intrinsically orthotropic shell of the same shell geometry as the stiffened member but whose material constants are unspecified. The equations of the unstiffened shell may then be derived in terms of the unspecified orthotropic elastic constants. Hence the problem of approximating the behavior of the structurally orthotropic shell becomes one of finding the correlation, or equivalently, the compliance, between the elastic constants of the unstiffened shell and the effects of the stiffeners of the structurally orthotropic shell. It is in fact this problem which is the primary purpose of the research grant.

II. Discussion of research in progress and research completed

The stress strain relations for an intrinsically orthotropic material are given as;

$$
\begin{aligned}
e_{\alpha\alpha} &= a_{11}\sigma_{\alpha\alpha} + a_{12}\sigma_{\beta\beta} + a_{13}\sigma_{\gamma\gamma} \\
e_{\beta\beta} &= a_{21}\sigma_{\alpha\alpha} + a_{22}\sigma_{\beta\beta} + a_{23}\sigma_{\gamma\gamma} \\
e_{\gamma\gamma} &= a_{31}\sigma_{\alpha\alpha} + a_{32}\sigma_{\beta\beta} + a_{33}\sigma_{\gamma\gamma} \\
e_{\beta\gamma} &= a_{44}\sigma_{\beta\gamma} \\
e_{\alpha\gamma} &= a_{55}\sigma_{\alpha\gamma} \\
e_{\alpha\beta} &= a_{66}\sigma_{\alpha\beta}
\end{aligned}
$$

Diagonal symmetry in the elastic constants requires that $a_{ij} = a_{ji}$ $(i \neq j)$. In applying the above equations to a shell, the α and β directions correspond to the principal directions of a surface while γ is measured normal to the surface.

As a first attempt in finding a compliance between the effect of stiffners and the orthotropic elastic constants, the first order differential equations for an orthotropic thin elastic shell have been derived. Since such an analysis is based on the Kirchoff hypothesis, then the number of constants appearing in the resulting differential equations are only four in number, namely the constants $a_{11}, a_{12}, a_{22}, a_{66}$. The expressions for the bending and twisting moment stress resultants and the membrane stress resultants are given as;

$$T_{\alpha\alpha} = \frac{\delta}{(a_{11}a_{22}-a_{12}^2)}(a_{22}e_{\alpha\alpha}-a_{12}e_{\beta\beta})$$

$$T_{\beta\beta} = \frac{\delta}{(a_{11}a_{22}-a_{12}^2)}(a_{11}e_{\beta\beta}-a_{12}e_{\alpha\alpha})$$

$$T_{\alpha\beta} = \frac{2\delta}{a_{66}}\,e_{\alpha\beta}$$

$$M_{\alpha\alpha} = \frac{\delta^3}{12(a_{11}a_{22}-a_{12}^2)}(a_{22}K_\alpha - a_{12}K_\beta)$$

$$M_{\beta\beta} = \frac{\delta^3}{12(a_{11}a_{22}-a_{12}^2)}(a_{11}K_\beta - a_{12}K_\alpha)$$

$$M_{\alpha\beta} = \frac{\delta^3}{6a_{66}}\,\tau$$

in these expressions, δ is the shell thickness, K_α and K_β the middle surface, curvature changes in the α and β directions respectively and τ is the twist of the middle surface.

Since there are only four elastic constants but six stress resultants, then only four of the six stress resultants can be made to comply with the stress resultants of the stiffened shell. However the choice of the four to be used is not completely arbitrary. Note that the constant a_{66} appears only in the membrane shear force and the twisting moment. Thus only two of these stress resultants can be made to comply with those of the corresponding stiffened shell.

If the shell reinforcement is of an attached type, then a compliance based on first order theory may yield satisfactory results. However the degree of satisfaction will depend greatly on the inefficiency of the attaching medium in making the reinforcement an integral part of the shell and the type of loading to which the shell is subjected. As an example, consider a rib stiffened shell where the ribs are attached to the shell by means of rivets or some other similar device and assume that the critical loading on the shell structure is from an edge condition which primarily induces bending and twisting moments. If the rib attaching device is such as to allow some relative motion between the rib and the shell in a direction tangent to the shell surface, then the effects of the membrance forces on the shell stress condition will be small and the shell stress condition will be almost completely determined from the bending and twisting moments. Under these circumstances, a compliance may be found by equating the bending and twisting moment stress resultants of the intrinsically and structurally orthotropic shells. Obviously the membrane stress resultants for the two types of shells will not be equal to each other but as previously mentioned, the membrane effects are assumed to be small and therefore of no great concern in determining critical strength characteristics.

Generally, the stiffeners are an integral part of a shell and hence the assumptions inherent in the example of the previous paragraph cannot be made. Thus in order to find a compliance between a structurally and intrinsically orthotropic shell, both the membrane and moment stress resultants of each of the two shells must be equal to each other. First order linear shell theory does not provide a sufficient number of elastic constants for such an equality because first order theory discards the direct effects of transverse shear and hence the elastic constants a_{44} and a_{55}. It also neglects the contraction of the normal and the constants a_{31} and a_{32}, and finally, it suppresses the sixth equation of equilibrium. Thus first order linear shell theory eliminates four elastic constants from consideration in the expressions for the stress resultants.

A plane stress condition is usually regarded as an excellent approximation to the true state of stress in a thin shell. There are then a total of eight arbitrary elastic constants for the orthotropic shell. If now all of the stress resultants were to be expressed in terms of shell deformations, there would also be eight stress resultants, namely, $T_{\alpha\alpha}$, $T_{\beta\beta}$, $T_{\alpha\beta}$, $T_{\alpha\gamma}$, $T_{\beta\gamma}$, $M_{\alpha\alpha}$, $M_{\beta\beta}$, $M_{\alpha\beta}$. Hence there appears to be a sufficient number of independent elastic constants to account for all the stress resultants. If now all of the eight elastic constants could be utilized in the eight stress resultant deformation relations, an equality in stress resultants could be made between a structurally and intrinsically orthotropic shell.

In order to utilize all of the elastic constants in the stress resultant expressions, the effect of the contraction of the normal and the explicit relation of transverse shear stress to deformation must be stated. It is then obvious that first order shell theory is unsatisfactory in predicting the behavior of the intrinsically orthotropic shell. Recourse must be made to second order thin shell theory where in the effects specifically neglected in first order theory are taken into account.

Work is presently progressing in the development of the second order orthotropic shell equations. However prior to attempting the derivation, a separate project as part of a doctoral study had been inaugurated in order to study the second order effects for isotropic shells. To determine directional effects and yet not introduce the complexities of orthotropy, a tracer parameter was introduced into the analysis. This work has recently been completed and it is now being abstracted for publication in the AIAA Journal. The principal conclusions are as follows;

i) For isotropic thin elastic shells, a first order theory is completely justified in that those effects which are discarded in the first order approximation yield small values even in the vicinity of high shear loads.

ii) The effect of the transverse shear on the resulting equations is strongly dependent upon the difference of the elastic properties in the principal curvilinear directions. The greater the difference, the larger the shear effects.

The second conclusion is most important since it guarantees that a second order approximation for an orthotropic shell will yield results significantly different from those of the first order analysis. Hence, hopefully, the second order analysis will incorporate all of the orthotropic elastic constants in the stress resultant deformation equations.

The problem of a stiffened plate has been encountered in engineering practice earlier than the stiffened shell. Further, a plate is a much easier structural member to analyze than a shell. As a consequence there exist far more techniques for dealing with stiffened plates than for stiffened shells. In order to determine whether any of the techniques of plate analysis could be directly applied to shells, a separate research project as part of a Master of Science degree program was initiated in order to study stiffened plates. This program has now been completed and the results are being put in final form.

The important conclusions of the plate study may be summarized as follows;

i) There exists a technique for handling almost any type of plate reinforcement

ii) Equivalent bending and torsional plate stiffnesses may always be found after a detailed study of the external stiffeners.

iii) There does not exist a compliance technique where in the elastic constants of an intrinsically orthotropic plate are related to the stiffener effects.

To some extent, the results of the project were disappointing in that it had been hoped that some compliance technique existed for plates. Thus the compliance technique for shells, when developed, could have been compared to that of the plate and as a check, shown that it degenerated to that of a plate. However the project did provide some useful information in that it discussed and evaluated techniques used in evaluating stiffener effects which could be directly applied to stiffened shells.

Once a compliance is found between the elastic constants of an intrinsically orthotropic shell and the stiffening effects of a structurally orthotropic shell, there will exist the problem of comparing results found by such a method to results found by other more standard techniques. In anticipation of this problem, a project has been initiated as part of a doctoral study to program the intrinsically orthotropic shell equations for a shell of revolution. However, rather than deal with a set of differential equations in finite difference form, a matrix displacement method is being utilized for the programming. This method has the advantage of rapid and accurate convergence and also the capability of dealing with unsymmetric loads and or stiffness properties. Since the compliance technique is not inherently bound with differential equations, then the technique can be programmed as a sub routine to evaluate the orthotropic elastic constants occurring in the matrix displacement program.

Summary

I. Research Completed

1. First order intrinsically orthotropic shell equations derived
2. Second order isotropic shell equations studied. Tracer parameter used to evaluate shear effects
3. Reinforced plate study completed

II. Research In Progress

1. Orthotropic second order shell equations being derived
2. Matrix displacement method being developed for intrinsically orthotropic materials