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THE SOLUTION OF SPECIAL SQUEEZE FILM GAS BEARING PROBLEMS BY AN IMPROVED NUMERICAL TECHNIQUE

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TECHNICAL REPORT

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ABSTRACT

A computer program has been written in order to compile data for special squeeze film gas bearings. In particular, three problems are solved, (1) the spherical squeeze film gas bearing with large axial and large radial displacements and finite axial excursion, (2) the cylindrical squeeze film gas bearing of finite L/D with no restriction on radial displacement or angular misalignment, and (3) the conical squeeze film gas bearing with no restriction on radial displacement or angular misalignment. Some new design data is presented for the bearings mentioned above and comparisons are made with previously published results.

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INTRODUCTION

Until very recently the compressible squeeze-film and squeeze-film bearing has received little attention. The works of Salbu (Ref. 1)* and Malanoski-Pan (Ref. 2) constituted first official record in literature on this subject and were concerned with the flat thrust squeeze-film bearing. Within the past year the pace has quickened and many advances have been made on the squeeze film bearing theory. Of particular interest are Ref. 3, a feasibility study on applying the squeeze film technique and technology to a gimbal axis gyro bearing, Ref. 4, the formulation of an asymptotic analysis for gaseous squeeze film bearings in general and in particular its application to the conical squeeze film bearing, Ref. 5, theory and experiments applicable to the cylindrical squeeze film bearing, and Ref. 6, the basic theory of a spherical squeeze film hybrid bearing.

The present report describes a computer program capable of calculating the performance of the spherical, cylindrical and conical bearings under rather general conditions. The data of Ref. 6 is extended by solving the large radial displacement problem without rotation. The data of Ref. 5 is extended by solving the finite L/D problem for both small and large radial and angular displacements. The data of Ref. 4 is extended to obtain results, also for large radial and angular displacements without rotation.

In the past the numerical solution of large displacement problems usually encountered trouble in obtaining accurate results. However, an advancement in obtaining numerical solutions to finite difference equations has been developed (Ref. 7) and has been incorporated in the present computer program. Because of this recently developed technique accurate design data can be generated relatively fast. A summary of the numerical technique used herein is given in Appendix I of this report.

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*See similarly numbered References in Reference Section.

ANALYSIS

This particular section is sub-divided into three parts. Each part is devoted to a particular type of squeeze film bearing. For each problem the asymptotic formulation ($\sigma \to \infty$) is used (Ref. 4).

Spherical Squeeze Film Bearing without Rotation

The asymptotic normalized lubrication equation for this problem (see Fig. 1) is (Refs. 4, 6):

$$\sin\Phi \frac{\partial}{\partial\Phi}\left[\sin\Phi \frac{\partial}{\partial\Phi}(H_o\Psi_\infty^2) - 3\Psi_\infty^2 \sin\Phi \frac{\partial H_o}{\partial\Phi}\right] + \frac{\partial}{\partial\theta}\left[\frac{\partial}{\partial\theta}(H_o\Psi_\infty^2) - 3\Psi_\infty^2 \frac{\partial H_o}{\partial\theta}\right] = 0, \quad \ldots \quad (1)$$

$$\text{where } \Psi_\infty = PH,$$
$$H = H_o + \varepsilon\cos\Phi \cos\tau, \quad \ldots \quad (2)$$
$$H_o = 1 + \eta_z \cos\Phi + \eta_R \sin\Phi \cos(\theta-\alpha):$$

The boundary conditions for this problem, corresponding to Eq. (1) are:

$$\Psi_\infty^2(\Phi,\theta) = \Psi_\infty^2(\Phi,\theta+2\pi)$$
$$\left.\frac{\partial\Psi_\infty^2}{\partial\theta}\right|_{\Phi,\theta} = \left.\frac{\partial\Psi_\infty^2}{\partial\theta}\right|_{\Phi,\theta+2\pi} \quad \ldots \quad (3)$$

and $\Psi_\infty^2(\Phi_i,\theta) = H_o^2(\Phi_i,\theta) + \frac{3}{2}\varepsilon^2\cos^2\Phi_i$; $i=1,2,$

where Φ_i designates the coordinate of either edge.

According to the numerical technique described in Appendix I, Eq. (1) must be expanded to the general form of,

$$f_1(\Phi,\theta)\frac{\partial^2\Psi_\infty^2}{\partial\Phi^2} + f_2(\Phi,\theta)\frac{\partial^2\Psi_\infty^2}{\partial\theta^2} + f_3(\Phi,\theta)\frac{\partial\Psi_\infty^2}{\partial\Phi} + f_4(\Phi,\theta)\frac{\partial\Psi_\infty^2}{\partial\theta} + f_5(\Phi,\theta)\Psi_\infty^2 = f_6(\Phi,\theta) \quad \ldots \quad (4)$$

$$f_1(\Phi,\theta) = \sin^2\Phi \frac{H_o}{2}$$

$$f_2(\Phi,\theta) = \frac{H_o}{2}$$

$$f_3(\Phi,\theta) = \frac{1}{2}\sin\Phi\cos\Phi H_o - \frac{1}{2}\sin^2\Phi \frac{\partial H_o}{\partial \Phi} = \frac{1}{2}\sin\Phi(\cos\Phi+\eta_z) = f_3(\Phi) \quad . (5)$$

$$f_4(\Phi,\theta) = -\frac{1}{2}\frac{\partial H_o}{\partial \theta} = \frac{1}{2}\eta_R \sin\Phi\sin(\theta-\alpha)$$

$$f_5(\Phi,\theta) = -\sin\Phi\cos\Phi\frac{\partial H_o}{\partial\Phi} - \sin^2\Phi\frac{\partial^2 H_o}{\partial\Phi^2} - \frac{\partial^2 H_o}{\partial\theta^2} = 2\sin^2\Phi[2f_2(\Phi,\theta) - 1]$$

and $f_6(\Phi,\theta) = 0.$

Thus, the solution of Eq. (1) with boundary conditions, (Eq. 3), is found numerically, i.e. the distribution, $\Psi_\infty^2(\Phi,\theta)$ is obtained by solving Eqs. (4) and (5) with boundary conditions, Eq. (3), by applying numerical technique summarized in Appendix I.

Static Performance Parameters of Spherical Bearing

Upon obtaining the distribution of Ψ_∞^2 the following steady state parameters are calculated by Simpson's quadrature formula.

Mean Axial Force,

$$f_z = \frac{F_z}{p_a \pi R^2} = \frac{1}{\pi}\int_{\Phi_1}^{\Phi_2}\int_0^{\pi}\frac{\Psi_\infty \sin 2\Phi}{\sqrt{H_o^2-\varepsilon^2\cos^2\Phi}}\, d\theta d\Phi + \frac{1}{2}[\cos 2\Phi_2 - \cos 2\Phi_1] \quad . . . \quad (6)$$

Mean Radial Force,

$$f_R = \frac{F_R}{p_a \pi R^2} = -\frac{2}{\pi}\int_{\Phi_1}^{\Phi_2}\int_0^{\pi}\frac{\Psi_\infty \sin^2\Phi\cos(\theta-\alpha)}{\sqrt{H_o^2-\varepsilon^2\cos^2\Phi}}\, d\theta d\Phi \quad . \quad . . . . \quad (7)$$

CYLINDRICAL SQUEEZE FILM BEARING WITHOUT ROTATION

The asymptotic normalized lubrication equation for this problem (see Fig. 2) is (Refs. 4,5):

$$\frac{\partial}{\partial\theta}\left[\frac{H_o}{2}\frac{\partial}{\partial\theta}\Psi_\infty{}^2 - \Psi_\infty{}^2\frac{\partial H_o}{\partial\theta}\right] + \left(\frac{D}{L}\right)^2\frac{\partial}{\partial\zeta}\left[\frac{H_o}{2}\frac{\partial}{\partial\zeta}\Psi_\infty{}^2 - \Psi_\infty{}^2\frac{\partial}{\partial\zeta}H_o\right] = 0 \quad \ldots\ldots (8)$$

where

$$\Psi_\infty = PH,$$
$$H = H_o + \varepsilon(\zeta)\cos\tau, \qquad \varepsilon(\zeta) = \varepsilon_o\,[1+a\zeta^2]$$
$$\text{and } H_o = 1 + \eta_R\cos(\theta-\theta_R) + \zeta\eta_\Gamma\cos(\theta-\theta_\Gamma). \quad \ldots\ldots (9)$$

The boundary conditions for this problem, corresponding to Eq. (8) are:

$$\Psi_\infty{}^2(\zeta,\theta) = \Psi_\infty{}^2(\zeta,\theta+2\pi)$$
$$\left.\frac{\partial\Psi_\infty{}^2}{\partial\theta}\right|_{\zeta,\theta} = \left.\frac{\partial\Psi_\infty{}^2}{\partial\theta}\right|_{\zeta,\theta+2\pi} \quad \ldots\ldots (10)$$

and

$$\Psi_\infty{}^2(\zeta_i,\theta) = H_o{}^2(\zeta_i,\theta) + \frac{3}{2}\varepsilon_o{}^2\,[1+a]^2 \quad , \quad i = 1,2$$

where

$$\zeta_{\binom{1}{2}} = (\pm)\,1.$$

According to the numerical technique described in Appendix I, Eq. (8) must be expanded to the general form of:

$$f_1(\zeta,\theta)\frac{\partial^2\Psi_\infty{}^2}{\partial\theta^2} + f_2(\zeta,\theta)\frac{\partial^2\Psi_\infty{}^2}{\partial\zeta^2} + f_3(\zeta,\theta)\frac{\partial\Psi_\infty{}^2}{\partial\theta} + f_4(\zeta,\theta)\frac{\partial\Psi_\infty{}^2}{\partial\zeta}$$
$$+ f_5(\zeta,\theta)\Psi_\infty{}^2 = f_6(\zeta,\theta) \quad \ldots\ldots (11)$$

Thus,

$$f_1(\zeta,\theta) = \frac{H_o}{2}$$
$$f_2(\zeta,\theta) = \left(\frac{D}{L}\right)^2\frac{H_o}{2}$$
$$f_3(\zeta,\theta) = -\frac{1}{2}\frac{\partial H_o}{\partial\theta} = \frac{1}{2}\{\eta_R\sin(\theta-\theta_R) + \zeta\eta_\Gamma\sin(\theta-\theta_\Gamma)\}$$

$$f_4(\zeta,\theta) = -\frac{1}{2}\left(\frac{D}{L}\right)^2 \frac{\partial H_o}{\partial \zeta} = -\frac{1}{2}\left(\frac{D}{L}\right)^2 \eta_\Gamma \cos(\theta-\theta_\Gamma) = f_4(\theta)$$

$$f_5(\zeta,\theta) = -\left\{\frac{\partial^2 H_o}{\partial \theta^2} + \left(\frac{D}{L}\right)^2 \frac{\partial^2 H_o}{\partial \zeta^2}\right\} = 2 f_1(\zeta,\theta) - 1$$

and $$f_6(\zeta,\theta) = 0 \quad \ldots\ldots \quad (12)$$

Thus, as in the previous problem, the distribution of $\Psi_\infty^2(\zeta,\theta)$ is obtained by solving Eqs. (11) and (12) with boundary conditions, Eqs. (10), by applying the numerical technique summarized in Appendix I.

Static Performance Parameters for Cylindrical Bearing

Upon obtaining the distribution of Ψ_∞^2 the following steady state parameters are calculated by Simpson's quadrature formula.

Mean Radial and Tangential Forces

$$\begin{Bmatrix} f_R \\ f_T \end{Bmatrix} = \frac{\begin{Bmatrix} F_R \\ F_T \end{Bmatrix}}{p_a\ LD} = \frac{1}{4}\int_o^{2\pi}\int_{-1}^{+1} \frac{\Psi_\infty \begin{Bmatrix} -\cos\beta \\ \sin\beta \end{Bmatrix} d\zeta d\beta}{\sqrt{H_o^2 - \varepsilon^2(\zeta)}} \quad \ldots\ldots \quad (13)$$

where $\beta = \theta - \theta_R$.

Mean Radial and Tangential Moments

$$\begin{Bmatrix} m_{\Gamma R} \\ m_{\Gamma T} \end{Bmatrix} = \frac{\begin{Bmatrix} M_R \\ M_T \end{Bmatrix}}{p_a L^2\ D} = \frac{1}{8}\int_o^{2\pi}\int_{-1}^{+1} \frac{\Psi_\infty\ \zeta \begin{Bmatrix} -\cos(\beta+\Theta) \\ \sin(\beta+\Theta) \end{Bmatrix} d\zeta d\beta}{\sqrt{H_o^2 - \varepsilon^2(\zeta)}} \quad \ldots\ldots \quad (14)$$

where $\Theta = \theta_R - \theta_\Gamma$.

Perturbation Analysis for the Cylindrical Squeeze Film Bearing

In addition to solving the cylindrical squeeze film bearing problem by the numerical procedure described above a perturbation technique is also used to solve Eqs. (8), (9) and (10) and is reported below.

Consider that one may express $\Psi_\infty^2\ (\zeta,\theta)$ by the equation

$$\Psi_\infty^2\ (\zeta,\theta) = Q_o + \eta_R\ Q_R(\zeta)\ \cos(\theta-\theta_R) + \eta_\Gamma Q_\Gamma(\zeta)\ \cos(\theta-\theta_\Gamma), \quad \ldots\ldots \quad (15)$$

where

$$\eta_R,\ \eta_\Gamma << 1.$$

Substituting Eq. (15) into Eq. (8), one obtains the following three equations:

$$\left(\frac{D}{L}\right)^2 \frac{\partial}{\partial\zeta}\left[\frac{1}{2}\frac{\partial}{\partial\zeta} Q_o\right] = 0 \quad \ldots\ldots \quad (16)$$

$$Q_o - \frac{1}{2} Q_R(\zeta) + \left(\frac{D}{L}\right)^2 \frac{1}{2} \frac{\partial^2 Q_R(\zeta)}{\partial\zeta^2} = 0 \quad \ldots\ldots \quad (17)$$

and

$$\zeta\ Q_o - \frac{1}{2} Q_\Gamma(\zeta) + \left(\frac{D}{L}\right)^2 \frac{1}{2} \frac{\partial^2 Q_\Gamma(\zeta)}{\partial\zeta^2} - \left(\frac{D}{L}\right)^2 \frac{\partial Q_o}{\partial\zeta} = 0 \quad \ldots\ldots \quad (18)$$

The first two boundary conditions of Eq. (10) are already implied by Eq. (15), while the third boundary condition of Eq. (10) is simplified by the perturbation technique to

$$\Psi_\infty^2\ (\zeta_i,\theta) = 1 + \frac{3}{2}\ \varepsilon_o^2[1+a]^2 + 2\eta_R\ \cos(\theta-\theta_R) + 2\zeta\eta_\Gamma\ \cos(\theta-\theta_\Gamma). \quad \ldots \quad (19)$$

Comparing Eq. (15) with Eq. (19) while looking for a solution to Eqs. (16), (17) and (18) one obtains the following:

$$Q_o = 1 + \frac{3}{2}\ \varepsilon_o^2[1+a]^2 \quad \ldots\ldots \quad (20)$$

$$\left.\begin{aligned} &\left(\frac{D}{L}\right)^2 \frac{\partial^2 Q_R}{\partial\zeta^2} - Q_R = -\ 2Q_o \\ &\text{Thus,} \\ &Q_R = 2Q_o + 2(1-Q_o)\ \frac{\cosh\frac{L}{D}\zeta}{\cosh\frac{L}{D}}\ . \end{aligned}\right\} \quad \ldots\ldots \quad (21)$$

$$\left.\begin{aligned} &\left(\frac{D}{L}\right)^2 \frac{\partial^2 Q_\Gamma}{\partial\zeta^2} - Q_\Gamma = -\ 2Q_o\zeta \\ &\text{Thus,} \\ &Q_\Gamma = 2Q_o\zeta - 2\ (Q_o-1)\ \frac{\sinh\frac{L}{D}\zeta}{\sinh\frac{L}{D}} \end{aligned}\right\} \quad \ldots\ldots \quad (22)$$

Perturbed Static Performance Parameters of Cylindrical Bearing

Because of symmetry the sinβ components of force and moment are exactly zero.

Upon substituting the results for Q_o, Q_R and Q_Γ from Eqs. (20), (21) and (22) into Eq. (15) and then substituting Eq. (15) into Eqs. (13) and (14) one obtains the results for the translatory and angular stiffnesses. Thus,

$$f_R = \frac{F_R}{\eta_R p_a LD} = \frac{\pi}{2}\sqrt{1+\frac{3}{2}\varepsilon_o^{\,2}(1+a)^2}\int_o^1\left\{\frac{\varepsilon^2(\zeta)}{[1-\varepsilon^2(\zeta)]^{3/2}}+\frac{\frac{3}{2}\varepsilon_o^{\,2}(1+a)^2}{1+\frac{3}{2}\varepsilon_o^{\,2}(1+a)^2}\left[\frac{\cosh\left(\frac{L}{D}\zeta\right)}{\cosh\frac{L}{D}}\right][1-\varepsilon^2(\zeta)]^{-1/2}\right\}d\zeta\ , \quad (23)$$

and

$$m_{\Gamma_R} = \frac{M_R}{\eta_\Gamma p_a L^2 D} = \frac{\pi}{4}\sqrt{1+\frac{3}{2}\varepsilon_o^{\,2}(1+a)^2}\int_o^1\left\{\frac{\zeta^2\varepsilon^2(\zeta)}{[1-\varepsilon^2(\zeta)]^{3/2}}+\frac{\frac{3}{2}\varepsilon_o^{\,2}(1+a)^2}{1+\frac{3}{2}\varepsilon_o^{\,2}(1+a)^2}\left[\frac{\sinh\left(\frac{L}{D}\zeta\right)}{\sinh\frac{L}{D}}\right](\zeta)\ [1-\varepsilon^2(\zeta)]^{-1/2}\right\}d\zeta \quad (24)$$

Conical Squeeze Film Bearing Without Rotation

The asymptotic normalized lubrication equation for this problem (see Fig. 3) is (Ref. 4):

$$\sin^2\Gamma\ \zeta\frac{\partial}{\partial\zeta}\left[\zeta\left(\frac{H_o}{2}\frac{\partial\Psi_\infty^{\,2}}{\partial\zeta}-\Psi_\infty^{\,2}\frac{\partial H_o}{\partial\zeta}\right)\right]+\frac{\partial}{\partial\theta}\left[\frac{H_o}{2}\frac{\partial\Psi_\infty^{\,2}}{\partial\theta}-\Psi_\infty^{\,2}\frac{\partial H_o}{\partial\theta}\right]=0, \quad (25)$$

where, $\Psi_\infty = PH$

$$H = H_o + \varepsilon_\zeta(\zeta)\sin\Gamma\cos\tau \quad (26)$$

$$H_o = 1 + \eta_z\sin\Gamma + \eta_R\cos\Gamma\cos(\theta-\theta_R) + \eta_\Gamma\frac{\zeta}{\zeta_2}\cos(\theta-\theta_\Gamma)$$

and

$$\varepsilon_\zeta(\zeta) = \varepsilon_{\zeta_1}\left[1 + a\,(\zeta-\zeta_1)^n\right] \quad (27)$$

In Eq. (27), n can be either 1/2, 1 or 2 depending upon the shape of the excursion desired. The values of n previously given correspond respectively to a drooping, linear, or bulging shape.

The boundary conditions for this problem, corresponding to Eq. (25) are:

$$\Psi_\infty^2\ (\zeta,\theta) = \Psi_\infty^2\ (\zeta,\theta+2\pi),$$

$$\left.\frac{\partial\Psi_\infty^2}{\partial\theta}\right|_{\zeta,\theta} = \left.\frac{\partial\Psi_\infty^2}{\partial\theta}\right|_{\zeta,\theta+2\pi}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (28)$$

and

$$\Psi_\infty^2\ (\zeta_i,\theta) = H_o^2(\zeta_i,\theta) + \frac{3}{2}\,\varepsilon_\zeta^2(\zeta_i)\ \sin^2\Gamma, \qquad i = 1,2\ .$$

Following the same procedure as in the previous two cases (spherical and cylindrical bearings), Eq. (25) is expanded to the form of,

$$f_1\ (\zeta,\theta)\ \frac{\partial^2\Psi_\infty^2}{\partial\theta^2} + f_2\ (\zeta,\theta)\ \frac{\partial^2\Psi_\infty^2}{\partial\zeta^2} + f_3(\zeta,\theta)\ \frac{\partial\Psi_\infty^2}{\partial\theta} + f_4\ (\zeta,\theta)\ \frac{\partial\Psi_\infty^2}{\partial\zeta}$$
$$+ f_5(\zeta,\theta)\Psi_\infty^2 = f_6\ (\zeta,\theta)\ \ldots\ldots\ldots \quad (29)$$

The coefficients $f_n\ (\zeta,\theta)$ are:

$$f_1\ (\zeta,\theta) = \frac{H_o}{2}$$
$$f_2\ (\zeta,\theta) = \zeta^2\ \sin^2\Gamma\ \frac{H_o}{2}$$
$$f_3\ (\zeta,\theta) = -\frac{1}{2}\,\frac{\partial H_o}{\partial\theta} = \frac{1}{2}\,[\eta_R\cos\Gamma\ \sin\ (\theta-\theta_R) + \eta_\Gamma\ \frac{\zeta}{\zeta_2}\ \sin(\theta-\theta_\Gamma)] \quad (30)$$
$$f_4\ (\zeta,\theta) = \zeta\sin^2\Gamma\ [\frac{H_o}{2} - \zeta\frac{1}{2}\,\frac{\partial H_o}{\partial\zeta}] = \zeta\sin^2\Gamma\ [f_1\ (\zeta,\theta) - \frac{1}{2}\,\eta_\Gamma\,\frac{\zeta}{\zeta_2}\ \cos(\theta-\theta_\Gamma)]$$
$$f_5\ (\zeta,\theta) = -\zeta\sin^2\Gamma\ \frac{\partial H_o}{\partial\zeta} - \zeta^2\sin^2\Gamma\ \frac{\partial^2 H_o}{\partial\zeta^2} - \frac{\partial^2 H_o}{\partial\theta^2} = 2f_1\ (\zeta,\theta) - (1+\eta_z\sin\Gamma)$$
$$- \eta_\Gamma\,\frac{\zeta}{\zeta_2}\ \sin^2\Gamma\ \cos(\theta-\theta_\Gamma)$$

$$f_6\ (\zeta,\theta) = 0\ .$$

The distribution of $\Psi_\infty^2\ (\zeta,\theta)$ is obtained by solving Eqs. (29) and (30) subject to boundary conditions, Eqs. (28) by applying the numerical technique summarized in Appendix I.

<u>Static Performance Parameters of Conical Bearing</u>

Upon obtaining the distribution of Ψ_∞^2 the following steady state parameters are calculated by applying Simpson's quadrature formula.

Mean Radial and Tangential Forces due to Radial Displacement

$$\begin{Bmatrix} f_R \\ f_T \end{Bmatrix} = \frac{\begin{Bmatrix} F_R \\ F_T \end{Bmatrix}}{p_a\, A_R \cos\Gamma} = \left\{ \frac{\sin^2\Gamma}{\cos\Gamma\left[1-\left(\frac{R_1}{R}\right)^2\right]} \right\} \int_0^{2\pi} \int_{\zeta_1}^{\zeta_2} \zeta \left(\frac{\Psi_\infty}{\sqrt{H_o{}^2-\varepsilon^2(\zeta)\sin^2\Gamma}} \right) \begin{Bmatrix} -\cos\,(\theta-\theta_R) \\ \sin\,(\theta-\theta_R) \end{Bmatrix} d\zeta d\theta \quad \ldots\ldots \quad (31)$$

Center of Pressure Associated with η_R

$$\begin{Bmatrix} \zeta_R \\ \zeta_T \end{Bmatrix}_R = \frac{\left\{ \frac{\sin^2\Gamma}{\cos\Gamma\left[1-\left(\frac{R_1}{R}\right)^2\right]} \right\} \int_0^{2\pi} \int_{\zeta_1}^{\zeta_2} \zeta^2 \left(\frac{\Psi_\infty}{\sqrt{H_o{}^2-\varepsilon^2(\zeta)\sin^2\Gamma}} \right) \begin{Bmatrix} -\cos(\theta-\theta_R) \\ \sin(\theta-\theta_R) \end{Bmatrix} d\zeta d\theta.}{\begin{Bmatrix} f_R \\ f_T \end{Bmatrix}} \quad .(32)$$

Mean Radial and Tangential Forces due to Angular Displacement

$$\begin{Bmatrix} g_R \\ g_R \end{Bmatrix} = \frac{\begin{Bmatrix} G_R \\ G_T \end{Bmatrix}}{p_a\, A_R} = \left\{ \frac{\sin^2\Gamma}{\left[1-\left(\frac{R_1}{R}\right)^2\right]} \right\} \int_0^{2\pi} \int_{\zeta_1}^{\zeta_2} \zeta \left(\frac{\Psi_\infty}{\sqrt{H_o{}^2-\varepsilon^2(\zeta)\sin^2\Gamma}} \right) \begin{Bmatrix} -\cos(\theta-\theta_\Gamma) \\ \sin(\theta-\theta_\Gamma) \end{Bmatrix} d\zeta d\theta \quad (33)$$

Center of Pressure Associated with η_Γ

$$\begin{Bmatrix} \zeta_R \\ \zeta_T \end{Bmatrix}_\Gamma = \frac{\left\{ \frac{\sin^2\Gamma}{\left[1-\left(\frac{R_1}{R}\right)^2\right]} \right\} \int_0^{2\pi} \int_{\zeta_1}^{\zeta_2} \zeta^2 \left(\frac{\Psi_\infty}{\sqrt{H_o{}^2-\varepsilon^2(\zeta)\sin^2\Gamma}} \right) \begin{Bmatrix} -\cos(\theta-\theta_\Gamma) \\ \sin(\theta-\theta_\Gamma) \end{Bmatrix} d\zeta d\theta}{\begin{Bmatrix} g_R \\ g_T \end{Bmatrix}} \quad (34)$$

The location of the center of pressures can be described by its axial distance (measured away from the apex) from the mid-point divided by the height of the cone. This can be expressed as:

$$\delta_P = \left[\zeta_x \cos\Gamma - \frac{1+\frac{R_\perp}{R}}{2}\cot\Gamma\right]\frac{R}{\ell}, \quad \ldots\ldots\ldots\ldots \quad (35)$$

where ζ_x corresponds to $\begin{pmatrix}\zeta_R \\ \zeta_T\end{pmatrix}_{R,\Gamma}$.

In eqs. (31) and (33) $A_R = \left[1 - \left(\frac{R_\perp}{R}\right)^2\right] R^2 \cot\Gamma \quad \ldots\ldots\ldots\ldots \quad (36)$

RESULTS

The results of the perturbation analysis for the cylindrical squeeze film bearing are given by the eqs, (23) and (24). These eqs. have been integrated numerically using Simpson's quadrature formula. The results have been plotted in Figs. 4 through 8.

The discussion which follows refers, at all times, to the normalized variables. In Figs. 4 through 7 it is apparent that the translatory stiffness (radial force) and the angular stiffness (radial moment) increase monotonically with increasing excursion ratio. Also, the more uniform the excursion is ($a \rightarrow 0$) the higher the radial force and moment. One can also conclude that the smaller the slenderness ratio, the more effect the excursion non-uniformity has on the bearing performance, especially at small excursion ratio. For example, consider that for L/D = 0.5, the radial force is reduced by a factor of 4 when 'a' is changed from 0 to -0.8. However, for L/D = 10.0 the radial force is reduced by only a factor of 2 when 'a' is changed from 0 to -0.8, at $\varepsilon_o = 0.1$. The moment for L/D = 0.5 is reduced by a factor of 7 when 'a' is changed from 0 to -0.8. With L/D = 10.0 the moment is reduced by a factor 4 when 'a' is changed from 0 to -0.8, at $\varepsilon_o = 0.1$. When a = -0.8 (large degree of non-uniformity) there is only a slight change in the normalized variables when L/D is varied from 0.5 to 10.0 for all excursion ratios.

In Fig. 8 the effect of L/D is shown on the normalized radial force at various mid-plane excursion ratios when the radial excursion is perfectly uniform ($a \rightarrow 0$). Notice that L/D has little effect at large excursion ratios, $\varepsilon_o \geq .8$. However, the normalized radial force is lower by a factor of approximately 2.5 as L/D is increased from 0.5 to ∞ at $\varepsilon_o = .1$.

For a quick estimate of performance one can use the following formulae for determining the translatory and angular stiffnesses of a cylindrical squeeze bearing: ($\varepsilon_o \sim 0.4$)

$$K_R = 0.3 \frac{p_a LD}{C} \text{ , (lb/in)}$$

$$K_T = 0.02 \frac{p_a L^3 D}{C}, \text{ (in.lb/rad)}$$

Table I, II and III contain results for the Squeeze Film Bearing obtained with the aid of the computer.

Table I contains the only extensive results for the large radial displacement problem. These results are for the spherical bearing described in Ref. 6. The geometry is set with $\Phi_1 = 41.5^o$ and $\Phi_2 = 68.0^o$. The axial eccentricity ratio is zero and the axial uniform excursion ratio is set at 0.5. A computer program check is made by comparing the finite difference results at small η_R with the perturbation analysis results of Ref. 6, (given as a footnote on Table I).

The interesting points brought out by the tabulated data of Table I are.

a) There is a stiffening effect in both the axial as well as the radial direction as η_R is increased.
b) As η_R is increased by a factor of six from 0.1 to 0.6 the axial load capacity is increased by a factor of 1.5, the radial load capacity is increased by a factor of 9, and the radial stiffness is increased by a factor of $\sim$ 3.5.

Table II and Table III are given essentially to show, as a computer program check, the comparison of the finite difference results with the perturbation analysis results for the cylindrical and conical bearing with either small angular or small radial displacement.

Notice that all comparisons made in Tables I, II, and III, are extremely good comparisons and checks between finite difference results and perturbation technique results.

DISCUSSIONS AND CONCLUSIONS

A computer program has been written and "debugged" and now provides a useful tool for generating design data for the non-rotating spherical, cylindrical and conical squeeze film bearings. Large radial, axial and angular displacement data can be obtained. A complete discussion of the program and numerical technique is given in the Appendices.

Some results are provided for a spherical bearing of a particular geometry to show the effect of radial displacement on the bearing performance. All three parameters, axial load, radial load and radial stiffness increase with increasing radial displacement.

Excellent comparisons and computer program checks are given between the finite difference results and the results from the perturbation analyses for all three bearing configurations.

A perturbation analysis (small radial displacement) for the cylindrical squeeze film bearing is given. Design data is provided in chart form in Figs. 4 through 8.

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NOMENCLATURE

Symbol	Meaning
A_R	Radially projected bearing area (see Eq. 36), in^2.
a	Degree of axial non-uniformity of excursion (a=0 is a perfectly uniform excursion).
e	Excursion, in.
C	Mean bearing gap, in.
C.P.	Symbolization for center of pressure.
D	Journal bearing diameter, in.
F	Force, or load capacity (Temporal average), lb.
$f = \overline{F}$	Normalized Force.
$f_{n}, n=1,2,3..,$	Coefficients of partial differential equations.
G	Force, or load capacity due to angular displacement (Temporal average), lb.
H	Normalized bearing gap, h/C.
H_o	Temporal average of H over one period of squeeze motion.
h	Local bearing gap, in.
K	Stiffness, Temporal average (lb/in or in.lb/rad).
L	Journal bearing length, in.
ℓ	Height of frustum of cone, in.
M	Moment, Temporal average (in. lb).
$m = \overline{M}$	Normalized moment.
P	Normalized pressure, p/p_a.
p	Gas Film pressure, psia.
p_a	Ambient pressure, psia.
Q_o	$\Psi_\infty{}^2$ for the reference concentric position.
Q_R	Perturbation of $\Psi_\infty{}^2$ due to η_R.
Q_Γ	Perturbation of $\Psi_\infty{}^2$ due to η_Γ.
R	Radius of sphere or larger base of cone frustum, in.
R_1	Radius of smaller base of cone frustum, in.
r	Conical radial coordinate, in.
t	Time, sec.
z	Axial dimension, in.

β	$\theta-\theta_R$.
Γ	Half cone angle.
δ_p = C.P.	Normalized axial location of center of pressure, measured from mid-point away from the cone apex (See eq. 35).
$\varepsilon = e/c$	Normalized excursion, or excursion ratio .
ε_o	Cylindrical bearing axial mid-plane excursion ratio .
ε_ζ	Conical bearing axial excursion ratio.
ζ	Conical bearing, r/R; Cylindrical bearing, z/(L/2).
η	Normalized eccentricity, or eccentricity ratio.
$\dot{\Theta}$	$\theta_R-\theta_\Gamma$.
θ	Angular coordinate in describing conical, cylindrical, or spherical (meridian) bearing .
τ	$t\Omega$.
ϕ	Azimuthal angle of spherical bearing.
Ψ	PH.
Ω	Squeeze frequency.

Subscripts - Superscripts

1,2	refers to ambient pressure ends of bearing.
∞	refers to asymptotic approximation, i.e. Ω very large .
R	refers to radial direction .
z	refers to axial direction.
T	refers to tangential forces.
Γ	refers to angular motions.
Bar(—)	refers to normalized variable.

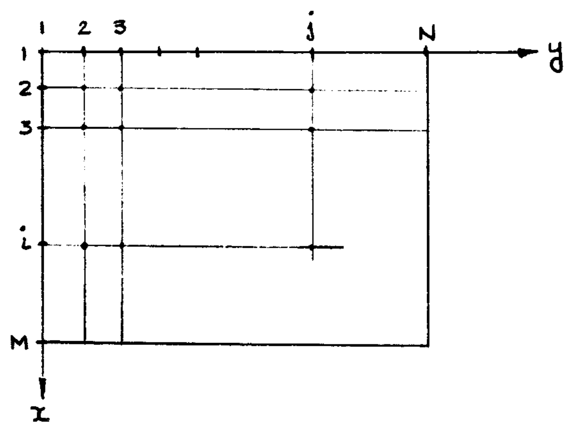
APPENDIX I

NUMERICAL INTEGRATION SCHEME

Given D.E.,

$$f_1 \frac{\partial^2\Phi}{\partial x^2} + f_2 \frac{\partial^2\Phi}{\partial y^2} + f_3 \frac{\partial\Phi}{\partial x} + f_4 \frac{\partial\Phi}{\partial y} + f_5\Phi = f_6 \quad \ldots\ldots\ldots\ldots \quad (A.1)$$

Grid is represented by the sketch below :



Each column (j) of equations can be written in the form

$$[A_j]\,\Phi_j + [B_j]\,\Phi_{j-1} + [C_j]\,\Phi_{j+1} = \mathbb{F}_j \quad \ldots\ldots\ldots\ldots \quad (A.2)$$

Discussion on elements of A, B, C, $\mathbb{F}$ (Central difference approximation)

1. If i,j is a regular field point

$$A_{j,i,i-1} = f_{1_{ij}} \frac{1}{\Delta x^2} - f_{3_{ij}} \frac{1}{2\Delta x}$$

$$A_{j,i,i} = f_{1_{ij}} \frac{(-2)}{\Delta x^2} - f_{2_{ij}} \frac{2}{\Delta y^2} + f_{5_{ij}}$$

$$A_{j,i,i+1} = f_{1_{ij}} \frac{1}{\Delta x^2} + f_{3_{ij}} \frac{1}{2\Delta x}$$

all other $A_{j,i,r} \equiv 0$ for any "r" different from i, i-1, i+1.

$$B_{j,i,i} = f_{2_{ij}} \frac{1}{\Delta y^2} - f_{4_{ij}} \frac{1}{2\Delta y}$$

$$B_{j,i,r} \equiv 0 \text{ for } r \neq i$$

$$C_{j,i,i} = f_{2_{ij}} \frac{1}{\Delta y^2} + f_{4_{ij}} \frac{1}{2\Delta y}$$

$$C_{j,i,r} \equiv 0 \text{ for } r \neq i$$

$$F_{ji} = f_{6_{ij}} = F_j$$

2. If i,j is a point where Φ_{ij} is specified, (boundary point),

$$A_{j,i,i} = 1$$

$$A_{j,i,r} = 0 \text{ for } r \neq i$$

$$B_{j,i,r} \equiv C_{j,i,r} \equiv 0 \text{ for all } r$$

$$F_{j,i} = \Phi_{ij}$$

This specification applies to the axial extremes of the cylindrical and conical squeeze films, and the azimuthal angular extremes of the spherical squeeze film.

3. If i,j is on a line of symmetry

   a) Left line of symmetry (j=1)

$$B \equiv 0$$

$$C_{j,i,i} = f_{2_{ij}} \frac{2}{\Delta y^2}$$

All other elements as in (1) above.

b) Right line of symmetry (j=N)

$$B_{j,i,i} = f_{2_{ij}} \frac{2}{\Delta y^2}$$

$$C \equiv 0$$

All other elements as in (1) above.

This specification applies to the spherical squeeze film at $y = 0,\pi$. (Meridianal angular speed of rotation is zero).

4. If i,j is on a butt joint.

at $i = 1$

$$A_{j,i,M} = f_{1_{11}} \frac{1}{\Delta x^2} - f_{3_{11}} \frac{1}{2\Delta x}$$

$A_{j,i,i-1}$ does not appear.

All other elements as in (1) above.

At $i = M$

$$A_{j,i,1} = f_{1_{MM}} \frac{1}{\Delta x^2} + f_{3_{MM}} \frac{1}{2\Delta x}$$

$A_{j,i,i+1}$ does not appear.

All other elements as in (1) above.

For this specification the last points in the x direction are then $\Phi_{i=M-1,j}$

This specification applies to both the cylindrical and conical squeeze film in the circumferential direction.

Solution of Equations

There exist relations such as

$$\Phi_{j-1} = [D_j]\ \Phi_j + \mathbb{E}_j \quad \text{(A.3)}$$

Substituting this Eq. into (A.2),

$$[A_j]\ \Phi_j + [B_j]\ [D_j]\ \Phi_j + [B_j]\ \mathbb{E}_j + [C_j]\ \Phi_{j+1} = \mathbb{F}_j$$

Let,

$$[K_j] = \Big[[A_j] + [B_j]\ [D_j]\Big]^{-1}. \quad \text{(A.3a)}$$

Then,

$$\Phi_j = -[K_j]\ [C_j]\ \Phi_{j+1} + [K_j]\ \{\mathbb{F}_j - [B_j]\ \mathbb{E}_j\} \quad \text{(A.4)}$$

Comparing (A.4) with (A.3),

$$[D_{j+1}] = -[K_j]\ [C_j] \quad \text{(A.5)}$$

and

$$\mathbb{E}_{j+1} = [K_j]\ \{\mathbb{F}_j - [B_j]\ \mathbb{E}_j\}. \quad \text{(A.5)}$$

On j = 1, B is always zero and Eq. (A.2) is

$$[A_1]\ \Phi_1 + [C_1]\ \Phi_2 = \mathbb{F}_1. \quad \text{(A.6)}$$

Putting (A.6) in the form of (A.3)

$$\Phi_1 = -[A_1]^{-1}\ [C_1]\ \Phi_2 + [A_1]^{-1}\ \mathbb{F}_1 \quad \text{(A.7)}$$

Comparing (A.7) with (A.3)

$$[D_2] = -[A_1]^{-1}\ [C_1] \quad \text{(A.8)}$$

and

$$\mathbb{E}_2 = [A_1]^{-1}\ \mathbb{F}_1. \quad \text{(A.9)}$$

<u>Procedure for Calculation</u>

a) Set $[D_1] = \mathbb{E}_1 = \Phi_{N+1} = 0$

b) Use Eqs. (A.3) and (A.5)

for $j = 1 \rightarrow N$,

and save $[D_{j+1}]$, $\mathbb{E}_{j+1}$.

c) Use Eq. (A.3) to calculate $\Phi_{i,j}$,

for $j = N + 1 \rightarrow 2$.

Save Φ_{j-1}.

This procedure has been programmed, and is called SUBROUTINE RINO1. (See Appendix II for listing).

APPENDIX II

COMPUTER PROGRAM DISCUSSION AND LISTING

The computer program is comprised of an executive program and five subroutines.

The executive program performs some preliminary calculations. However, its primary functions are to read input and punch output besides calling on the subroutines to perform the bulk of the computation.

The five subroutines are titled RINO1, MATINV, STORE, TOTAL, and SUM.

RINO1 performs the numerical integration as described in Appendix I.

MATINV is called upon by RINO1 to invert the necessary matrices.

STORE is called upon by the executive program to define and store in COMMON storage the coefficients "f_n".

SUM is the Simpson's rule integration scheme.

TOTAL is used in conjuction with SUM to perform a two-dimensional integration by Simpson's rule.

INPUT

The input consists of only three cards, namely: a title card, control card and bearing specification card. These three cards are defined as follows:

1. TITLE CARD

   Anything may be punched in columns 2-72.

2. CONTROL CARD Format (16I5)

   ITEM # (6 items)

   1. NBRG, Control number specifying bearing to be calculated.

      1 - SPHERICAL

      2 - CYLINDRICAL

      3 - CONICAL

   2. MDIAG, Control number used for "debugging" program.

      0 - No diagnostic output - used for production runs

      1 - Diagnostic output - used for "debugging" runs

3. M, Total number of grid points in 'x' direction. Maximum = 25
4. N, Total number of grid points in 'y' direction. Maximum = 19

Suggested values to be assigned 'M' and 'N':

BEARING	NBRG	M	N
SPHERICAL	1	$\Phi = 9$	$\theta = 19$
CYLINDRICAL	2	$\theta = 25$	$\zeta = 19$
CONICAL	3	$\theta = 25$	$\zeta = 9$

ITEM # (Continuation of card 2)

5. LC, Control number specifying last case to be run.
   0 = Program control returns for 3 new input cards.
   1 = Last case.
6. ISIG, Specifies shape of excursion desired in conical bearing calculation. Not used in spherical or cylindrical bearing calculations.
   1 - Drooping
   2 - Linear
   3 - Bulging

3. SPECIFICATION CARD Format (8E10.3)

This card is different for each type of bearing.

SPHERICAL BEARING (5 items)

ITEM #

1. ETAZ; η_z, Axial eccentricity ratio
2. ETAR; η_R, Radial eccentricity ratio
3. PHI1, Φ_1, Polar angle in degrees ($\Phi_1 > 0$)
4. PHI2; Φ_2, Equatorial angle in degrees
5. EPS; ε, Axial excursion ratio

CYLINDRICAL BEARING (6 items)

ITEM #

1. ETAR, η_R, Radial eccentricity ratio
2. ETAD, η_Γ, Angular misalignment ratio
3. ELD, L/D, Slenderness ratio

4. THET; Θ, Reference angle in degrees between radial displacement plane and angular misalignment plane
5. A; a, Degree of axial non-uniformity of excursion
6. EPSO; ε_o, Radial excursion ratio at bearing axial center plane

CONICAL BEARING (8 items)

ITEM #

1. ETAZ; η_z, Axial eccentricity ratio
2. ETAR; η_R, Radial eccentricity ratio
3. ETAT; η_Γ, Angular misalignment ratio
4. RR; R_1/R, Inner to outer radius ratio
5. TAU; Γ, One-half apex angle of cone
6. EPS1; ε_{ζ_1}, Radial excursion ratio at $\zeta = \zeta_1$
7. A; a, Degree of axial non-uniformity of excursion
8. THET, Θ, Reference angle in degrees between radial displacement plane and angular misalignment plane.

OUTPUT

The output is self explanatory. It consists of a print-out of the input and also the calculated results which are the static performance parameters explained in the analysis section. See pages 39-41 for typical output sheets.

```
C     ANALYSIS OF   SQUEEZE FILM BRGS USING  RINOS METHOD
C     ANALYSIS- S.B. MALANOSKI      10/18/65
C     NBRG=1  ,SPHERICAL BEARING
C         =2  ,CYLINDRICAL BEARING
C         =3  ,CONICAL BEARING
      DIMENSION  Q (20,25),SUM1(20,25),FZ1(25),FZ2(25   )
      COMMON   Q,N,M,DELTI,DELTJ
      DIMENSION AF1(20,25),AF2(20,25),AF3(20,25),AF4(20,25),AF5(20,25),
     1AF6(20,25)
      COMMON I,J,T11,T12,T13,T14,T15,AF1,AF2,AF3,AF4,AF5
      DIMENSION NP(20,25),CSJ(25),SIJ(25),SIP2(25),SUM2(20,25),SUM3(20,2
     15),SUM4(20,25),SUM5(20,25),SUM6(20,25),SUM7(20,25),SUM8(20,25),EZ(
     125)
      COMMON NP,CSJ,SIJ,SIP2,SUM2,SUM3,SUM4,SUM5,SUM6,SUM7,SUM8,EZ
    1 READ 100
      READ 101,NBRG,MDIAG,M,N,LC,ISIG
      DO 2 I=1,M
      DO 2 J=1,N
      AF6(J,I)=0.0
      Q(J,I)=1.0
    2 NP(J,I)=0
      PRINT 100
      PRINT 103
      PRINT 101,NBRG,MDIAG,M,N,LC,ISIG
      T1=M-1
      T2=N-1
      GO TO (3,17,33),NBRG
C     SPHERICAL BEARING
    3 READ 102,ETAZ,ETAR,PHI1,PHI2,EPS
      PRINT 104
      PRINT 102,ETAZ,ETAR,PHI1,PHI2,EPS
      PH1=PHI1*0.01745329
      PH2=PHI2*0.01745329
      DELTI=(PH2-PH1)/T1
      DELTJ= 3.1415927/T2
      ANG=0.0
      EPSQ= EPS*EPS
      EPS2= 1.5*EPSQ
      DO 4 J=1,N
      CSJ(J)=COS (ANG)
      SIJ(J)=SIN (ANG)
    4 ANG=ANG+DELTJ
      ANG= PH1
    5 DO 12 I=1,M
      CS=COS (ANG)
      SI=SIN (ANG)
      T3= ETAZ*CS+1.0
      T4=ETAR*SI
      T5= SI*SI
      T7= 0.5*SI
      T6= (CS+ETAZ)*T7
      T7= T7*ETAR
      T8= 2.0*T5
      SI2P=CS*SI
      SIP2(I)=T5
      EPCS=EPSQ*CS*CS
    6 DO 10 J=1,N
      T9=CSJ(J)
      T10=SIJ (J)
      T16=(T9*T4+T3)
      T12=0.5*T16
```

```
      T11=T5*T12
      T13=T6
      T16=T16*T16
      T14=T7*T10
      T15=(2.0*T12-1.0)*T8
      IF(MDIAG) 7,8,7
    7 PRINT 105,T11,T12,T13,T14,T15,T16
    8 IF(I-1)110,110,9
    9 IF(I-M)111,110,111
  110 Q(J,I)= 1.5*EPCS+T16
      NP(J,I)=1
  111 I=I
      J=J
      CALL STORE
      T11=T16-EPCS
      T11=SQRT (T11)
      SUM1(J,I)=SI2P/T11
      SUM2(J,I)=T5*T9/T11
   10 CONTINUE
      ANG=ANG+DELTI
      IF(MDIAG) 11,12,11
   11 PRINT 105,T3,T4,T5,T6,T7,T8
   12 CONTINUE
      KLUE=0
   13 IF(MDIAG) 312,313,312
  312 DO 314 J=1,N
  314 PRINT 101,(NP(J,I),I=1,M)
      DO 315 J=1,N
  315 PRINT 105,(Q(J,I),I=1,M)
  313 CALL RINO1(AF1,AF2,AF3,AF4,AF5,AF6,DELTI,DELTJ,Q,M,N,NP,KLUE)
      IF(MDIAG) 14,116,14
   14 DO 15  J=1,N
   15 PRINT 105,(Q(J,I),I=1,M)
  116 GO TO (16,31,45),NBRG
C     SPHERICAL BEARING  FORCE   COMPUTATION
   16 T4= SIP2(1)-SIP2(M)
      CALL TOTAL(SUM1, FZ)
      CALL TOTAL (SUM2, FR)
      FZ=2.0*FZ/3.1415927+T4
      FR= -2.0*FR/3.1415927
      PRINT 106
      PRINT 500,FZ,FR
      GO TO 99
C     END OF SPHERICAL BRG COMPUTATION
C     BEGIN CYLINDRICAL BEARING
   17 READ 102,ETAR,ETAD,ELD,THET,A,EPSO
      PRINT 107
      PRINT 102,ETAR,ETAD,ELD,THET,A,EPSO
      DELTI=6.283185/T1
      DELTJ=2.0/T2
   18 ELD2=ELD*ELD
      ELD2=1.0/ELD2
      ELET=-ELD2*ETAD*0.5
      EPS2= A+1.0
      EPS2=EPS2*EPS2*EPSO*EPSO*1.5
      SI2P=-1.0
      THETR=THET*0.01745329
   19 DO 21 J=1,N
      CSJ(J)=SI2P
      T3=(SI2P*SI2P*A+1.0)*EPSO
      SIP2(J)=T3*T3
```

```
      SIJ(J)=T3
      SI2P=SI2P+DELTJ
      IF(MDIAG) 20,21,20
   20 PRINT 105,SI2P,T3
   21 CONTINUE
      ANG=0.0
   22 DO 30 I=1,M
      CS=COS (ANG)
      SI=SIN (ANG)
      T3 =ANG+THETR
      T4=COS (T3)
      T5=SIN (T3)
      T6= ETAR*CS+1.0
   23 T7=ETAD*T4
      T8= ETAR*SI
      T9= ETAD*T5
      T10=ELET*T4
   24 DO 29 J=1,N
      SI2P=CSJ(J)
      T16= SI2P*T7+T6
      T11= 0.5*T16
      T12=T11*ELD2
      T13=(SI2P*T9+T8)*0.5
      T14=T10
      T15= 2.0*T11-1.0
      T16=T16*T16
      IF(MDIAG) 25,26,25
   25 PRINT 105,T11,T12,T13,T14,T15,T16
   26 I=I
      J=J
      CALL STORE
      IF(J-1) 27,27,270
  270 IF(J-N) 28,27,28
   27 Q(J,I)=T16+EPS2
      NP(J,I)=1
   28 T11=-SIP2(J)+T16
      T11=SQRT (T11)
      T11=1.0/T11
      SUM1(J,I)=-CS*T11
      SUM2(J,I)=SI*T11
      T11=SI2P*T11
      SUM3(J,I)=-T4*T11
      SUM4(J,I)=T5*T11
   29 CONTINUE
      ANG=ANG+DELTI
   30 CONTINUE
      KLUE= 1
      M=M-1
      GO TO 13
   31 M=M+1
      DO  32 J=1,N
   32 Q(J,M)=Q(J,1)
      IF(MDIAG) 316,317,316
  316 DO 318 J=1,N
  318 PRINT 105,(Q(J,I),I=1,M)
  317 CALL TOTAL(SUM1,FR)
      FR=0.25*FR
      CALL TOTAL(SUM2,FT)
      FT=0.25* FT
      CALL TOTAL (SUM3,EMDR)
      EMDR=0.125*EMDR
```

```
      CALL TOTAL(SUM4,EMDT)
      EMDT=0.125*EMDT
      PRINT 108
      PRINT 500,FR,FT,EMDR,EMDT
      GO TO 99
C     BEGIN CONICAL COMPUTATION
   33 READ 102,ETAZ,ETAR,ETAT,RR,TAU,EPS1,A   ,THET
      PRINT 109
      PRINT 102,ETAZ,ETAR,ETAT,RR,TAU,EPS1,A   ,THET
      TAUR=TAU*0.01745329
      THETR=THET*0.01745329
      CC=COS (TAUR)
      SC=SIN (TAUR)
      Z2=1.0/SC
      ALOR=(1.0-RR)/SC*CC
      Z1=Z2-ALOR/CC
      DELTI=6.283185/T1
      DELTJ=(Z2-Z1)/T2
      SC2=SC*SC
      FAC= SC2/CC/(1.0-RR*RR)
      FACT=0.5*CC/SC*(1.0+RR)
      Z=Z1-DELTJ
      DO 341 J=1,N
      Z=DELTJ+Z
      X=Z-Z1
      IF(X) 351,350,350
  351 X=0.0
  350 GO TO (35,36,37),ISIG
   35 E     =EPS1*(1.0+A*SQRT (X))
      GO TO 34
   36 E     =EPS1*(1.0+A*X)
      GO TO 34
   37 E     =EPS1*(1.0+A*X*X)
   34 EZ(J)=E     *E
      IF(MDIAG) 340,341,340
  340 PRINT 105,X,E,EZ(J)
  341 CONTINUE
      ANG=0.0
      T3=1.0+ETAZ*SC
      T60=ETAR*CC
      DO 38 I=1,M
      CS=COS (ANG)
      SI=SIN (ANG)
      S3=ANG+THETR
      T4=COS (S3)
      T5=SIN (S3)
      T7=T60*CS
      T6=T60*SI
      Z=Z1-DELTJ
      DO 39 J=1,N
      EP2 =EZ(J)
      Z=Z+DELTJ
      ZO2=Z/Z2*ETAT
      T16=T3+T7+ZO2*T4
      T11=0.5*T16
      T12=Z*Z*SC2*T11
      T13=0.5*(T6+ZO2*T5)
      T14=Z*SC2*(T11-0.5*ZO2*T4)
      T15=2.0*T11-T3-ZO2*SC2*T4
      T16=T16*T16
      IF(MDIAG) 40,41,40
```

```
 40 PRINT 105,T11,T12,T13,T14,T15,T16
 41 I=I
    J=J
    CALL STORE
    IF(J-1) 42,42,43
 43 IF(J-N) 44,42,44
 42 Q(J,I)=T16+1.5*SC2*EP2
    NP(J,I)=1
 44 T11=T16-EP2*SC2
    T11=SQRT (T11)
    T11=1.0/T11*Z
    SUM1(J,I)=-CS*T11
    SUM2(J,I)=SI*T11
    SUM3(J,I)=-T4*T11
    SUM4(J,I)= T5*T11
    T11= T11*Z
    SUM5(J,I)=-CS*T11
    SUM6(J,I)=SI*T11
    SUM7(J,I)=-T4*T11
    SUM8(J,I)=T5*T11
 39 CONTINUE
    ANG=ANG+DELTI
 38 CONTINUE
    KLUE=1
    M=M-1
    GO TO 13
 45 M=M+1
    DO 46 J=1,N
 46 Q(J,M)=Q(J,1)
    IF(MDIAG) 319,320,319
319 DO 321 J=1,N
321 PRINT 105,(Q(J,I),I=1,M)
320 CALL TOTAL(SUM1,FR)
    FR=FR*FAC
    CALL TOTAL(SUM2,FT)
    FT=FT*FAC
    CALL TOTAL(SUM3,EMDR)
    EMDR=EMDR*FAC*CC
    CALL TOTAL(SUM4,EMDT)
    EMDT=EMDT*FAC*CC
    PRINT 208
    PRINT 500,FR,FT,EMDR,EMDT
    CALL TOTAL(SUM5,CPR)
    CPR=CPR*FAC/FR
    CPR=(CPR*CC-FACT)/ALOR
    CALL TOTAL(SUM6,CPT)
    CPT=CPT*FAC/FT
    CPT=(CPT*CC-FACT)/ALOR
    CALL TOTAL(SUM7,CPRM)
    CPRM=CPRM*FAC/EMDR*CC
    CPRM=(CPRM*CC-FACT)/ALOR
    CALL TOTAL(SUM8,CPTM)
    CPTM=CPTM*FAC/EMDT*CC
    CPTM=(CPTM*CC-FACT)/ALOR
    PRINT 211
    PRINT 102,CPR,CPT,CPRM,CPTM
 99 IF(LC) 1,1,999
999 STOP
100 FORMAT(72H1
  1
101 FORMAT(16I5)
```

```
102 FORMAT(8E10.3)
103 FORMAT(32H0NBRG   MDI      M     N    LC    ISIG)
104 FORMAT(46H0   ETA-Z      ETA-R     PHI-1       PHI-2       EPS)
105 FORMAT(6(1X1PE11.4))
106 FORMAT(27H0     FORCE-Z            FORCE-R)
107 FORMAT(58H0   ETA-R      ETA-M      L/D        PHASE-M      A          EPS
   1-0)
108 FORMAT(57H0     FORCE-R            FORCE-T           TORQUE-R          TORQUE-
   1T)
208 FORMAT(57H0     FORCE-RR           FORCE-TR          FORCE-RA          FORCE-T
   1A)
109 FORMAT(79H0   ETA-Z      ETA-R      ETA-M      RRAT       GAMMA      EPS-
   11        A          PHASE-M)
211 FORMAT(40H0      CPR      CPT         CPRM        CPTM)
500 FORMAT(4(1PE15.6))
    END
```

```
      SUBROUTINE RINO1  (AF1,AF2,AF3,AF4,AF5,AF6,DELX,DELY,PHI,M,N,NP,KL
     1UE)
C       NOTATION AND NOTES
C      A= A TRI DIAGONAL MATRIX
C      B= A DIAGONAL MATRIX
C      C= A DIAGONAL MATRIX
C       I=VARIABLE IN THE X DIRECTION, THE NUMBER OF I POINTS = M
C       J=VARIABLE IN THE Y DIRECTION, THE NUMBER OF J POINTS = N
C       I=THE SHORTEST VARIABLE SINCE THIS IS THE INVERTED VARIABLE
C       J=THE OUTER LOOP VARIABLE
C      KLUE=0, NORMAL LINE OF SYMMETRY
C      KLUE=1, BUTT JOINT (AS IN A JOURNAL BEARING)
      COMMON DUM1(500),NDUM2(2),DUM4(2),NDUM5(2),DUM6(2505),NDUM7(500),
     1DUM8(3600)
      DIMENSION AF1(20,25),AF2(20,25),AF3(20,25),AF4(20,25),AF5(20,25),A
     1F6(20,25),PHI(20,25),NP(20,25)
      COMMON C(25),D(20,25,25),E(20,25),AK(25,25),G(25),AFJI(25),A(25,25
     1),B(25)
      DELX2=0.5/DELX
      DELY2=0.5/DELY
      DELX3=(1.0/DELX)**2
      DELY3=(1.0/DELY)**2
      DO 10  I=1,M
C      ZERO E(1,I) ARRAY
      E(1,I)=0
C      ZERO PHI(N+1,I) ARRAY
      PHI(N+1,I)=0
      DO 10 I1=1,M
C      ZERO D(1,I,I) ARRAY
   10 D(1,I,I1)=0
C       THE EXTERNAL LOOP IN J
      DO 440 J=1,N
C      ZERO THE A, B, AND C ARRAYS
      DO 100 I=1,M
      B(I)=0
      C(I)=0
      DO 100 II=1,M
  100 A(I,II)=0
C      THE INTERNAL LOOP IN I
      DO 400 I=1,M
      IF(NP(J,I)) 190,210,190
C      I,J IS A POINT WHERE PHI(I,J) IS SPECIFIED
  190 A(I,I)=1
      AFJI(I)=PHI(J,I)
      GO TO 400
  210 A(I,I)=-2.0*(AF1(J,I)*DELX3+AF2(J,I)*DELY3)+AF5(J,I)
      AFJI(I)=AF6(J,I)
      IF(J-1) 350,350,230
  230 IF(J-N) 240,330,330
  240 B(I)=AF2(J,I)*DELY3-AF4(J,I)*DELY2
      C(I)=AF2(J,I)*DELY3+AF4(J,I)*DELY2
  255 IF(I-1) 310,310,260
  260 IF(I-M) 270,290,290
C      I,J= A REGULAR FIELD POINT
  270 A(I,I-1)=AF1(J,I)*DELX3-AF3(J,I)*DELX2
      A(I,I+1)=AF1(J,I)*DELX3+AF3(J,I)*DELX2
      GO TO 400
C      I,J IS A POINT ON A LINE OF SYMMETRY
  290 IF(KLUE) 291,291,292
C      B.) BOTTOM LINE OF SYMMETRY,I=M
  291 A(M,M-1)=2.0*AF1(J,M)*DELX3
```

```
      GO TO 400
C     BOTTOM JOINT
  292 A(M,M-1)=AF1(J,I)*DELX3-AF3(J,I)*DELX2
      A(M,1)=AF1(J,I)*DELX3+AF3(J,I)*DELX2
      GO TO 400
C     I,J IS A POINT ON A LINE OF SYMMETRY
  310 IF(KLUE) 311,311,312
C     A.) TOP LINE OF SYMMETRY, I=1
  311 A(1,2)=2.0*AF1(J,1)*DELX3
      GO TO 400
C     TOP JOINT
  312 A(1,2)=AF1(J,I)*DELX3+AF3(J,I)*DELX2
      A(1,M)=AF1(J,I)*DELX3-AF3(J,I)*DELX2
      GO TO 400
C     I,J IS A POINT ON A LINE OF SYMMETRY
C     D.) RIGHT LINE OF SYMMETRY, J=N
  330 B(I)=2.0*AF2(N,I)*DELY3
      GO TO 255
C     I,J IS A POINT ON A LINE OF SYMMETRY
C     C.) LEFT LINE OF SYMMETRY, J=1
  350 C(I)=2.0*AF2(1,I)*DELY3
      GO TO 255
  400 CONTINUE
C     END OF THE INTERNAL LOOP IN I
      DO 410 I=1,M
      DO 410 II=1,M
      AK(I,II)=A(I,II)+B(I)*D(J,I,II)
  410 CONTINUE
      CALL MATINV(AK,M,G,0,DETERM)
C     K(J,I,I) IS NOW DEFINED AS IN EQUATION 3A
      DO 435 I=1,M
C     THE MATRIX G(I) IS A DUMMY MATRIX FOR  E(J+1,I)
      G(I)=AFJI(I)-B(I)*F(J,I)
      DO 430 II=1,M
C     EQUATIONS 5, THE FORMATION OF D(J+1,I,I)
C     POSTMULTIPLICATION OF A SQUARE MATRIX K BY THE DIAGONAL MATRIX C
  430 D(J+1,II,I)=-AK(II,I)*C(I)
  435 CONTINUE
C     D(J+1,I,I) IS NOW DEFINED AS IN EQUATION 5
C     THE MATRIX MULTIPLICATION OF THE MATRICES K AND F-B*E TO FORM THE
C     RIGHT HAND SIDE OF EQUATION 5,I.E.,E(J+1,I)
      DO 440 I=1,M
      E(J+1,I)=0
      DO 440 II=1,M
  440 E(J+1,I)=E(J+1,I)+AK(I,II)*G(II)
C     END OF THE EXTERNAL LOOP IN J
C     THE RECURSION FORMULA 3
      DO 460 J=1,N
      N1=N-J+1
      N2=N-J+2
C     THE MATRIX MULTIPLICATION OF THE MATRICES D(J,I,I) AND PHI(J)
      DO 460 I=1,M
      IF(NP(N1,I)) 460,445,460
  445 PHI(N1,I)=0
      DO 450 II=1,M
  450 PHI(N1,I)=PHI(N1,I)+D(N2,I,II)*PHI(N2,II)
      PHI(N1,I)=PHI(N1,I)+E(N2,I)
  460 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MATINV(A,N,B,M,DETERM)
CMATINV
C     MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS
C     ANF402
C     MATINV COMMENTS
C     MAXIMUM DIMENSION ON IPIVOT = N
C     MAXIMUM DIMENSION ON A = N*N
C     MAXIMUM DIMENSION ON B = N*M
C     MAXIMUM DIMENSION ON INDEX = N*2
C     MAXIMUM DIMENSION ON PIVOT = N
      DIMENSION IPIVOT(25 ),A(25,25),B(25,1),INDEX(25,2),PIVOT(25)
      EQUIVALENCE(IROW,JROW),(ICOLUM,JCOLUM),(AMAX,T,SWAP)
C     INITIALIZATION
      DETERM =1.0
      DO 20 J=1,N
   20 IPIVOT(J)=0
      DO 550 I=1,N
C     SEARCH FOR PIVOT ELEMENT
      AMAX=0.0
      DO 105 J=1,N
      IF(IPIVOT(J) -1) 60,105,60
   60 DO 100 K =1,N
      IF(IPIVOT(K)-1) 80,100,740
   80 IF(ABS (AMAX)-ABS (A(J,K))) 85,100,100
   85 IROW=J
      ICOLUM=K
      AMAX=A(J,K)
  100 CONTINUE
  105 CONTINUE
      IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
      IF(IROW-ICOLUM) 140,260,140
  140 DETERM=-DETERM
      DO 200 L=1,N
      SWAP=A(IROW,L)
      A(IROW,L)=A(ICOLUM,L)
  200 A(ICOLUM,L)=SWAP
      IF(M) 260,260,210
  210 DO 250 L=1,M
      SWAP=B(IROW,L)
      B(IROW,L)=B(ICOLUM,L)
  250 B(ICOLUM,L)=SWAP
  260 INDEX(I,1)=IROW
      INDEX(I,2)=ICOLUM
      PIVOT(I)=A(ICOLUM,ICOLUM)
      DETERM=DETERM*PIVOT(I)
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT
      A(ICOLUM,ICOLUM)=1.0
      DO 350 L=1,N
  350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT(I)
      IF(M) 380,380,360
  360 DO 370 L=1,M
  370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT(I)
C     REDUCE NON-PIVOT ROWS
  380 DO 550 L1=1,N
      IF(L1-ICOLUM) 400,550,400
  400 T=A(L1,ICOLUM)
      A(L1,ICOLUM)=0.0
      DO 450 L=1,N
      A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
```

```
      IF(M) 550,550,460
  460 DO 500 L=1,M
  500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T
  550 CONTINUE
C     INTERCHANGE COLUMNS
      DO 710 I=1,N
      L=N+1-I
      IF(INDEX(L,1)-INDEX(L,2)) 630,710,630
  630 JROW=INDEX(L,1)
      JCOLUM=INDEX(L,2)
      DO 705 K=1,N
      SWAP=A(K,JROW)
      A(K,JROW)=A(K,JCOLUM)
      A(K,JCOLUM)=SWAP
  705 CONTINUE
  710 CONTINUE
  740 RETURN
      END
```

```
      SUBROUTINE STORE
      DIMENSION  Q (20,25)
      COMMON    Q,N,M,DELTI,DELTJ
      DIMENSION AF1(20,25),AF2(20,25),AF3(20,25),AF4(20,25),AF5(20,25)
      COMMON I,J,T11,T12,T13,T14,T15,AF1,AF2,AF3,AF4,AF5
      AF1(J,I)=T11
      AF2(J,I)=T12
      AF3(J,I)=T13
      AF4(J,I)=T14
      AF5(J,I)=T15
      RETURN
      END
```

```
      SUBROUTINE TOTAL(SUM1,ANS)
      DIMENSION  Q (20,25),SUM1(20,25),FZ1(25),FZ2(25   )
      COMMON    Q,N,M,DELTI,DELTJ
      NM=N-1
      MM=M-1
      DO 18 I=1,M
      DO 17  J=1,N
      T5= SQRT (Q(J,I))
   17 FZ1(J)= T5*SUM1(J,I)
      CALL SUM(FZ1,FZ,N,NM)
      FZ2(I)=FZ*DELTJ
   18 CONTINUE
      CALL SUM(FZ2,FZ,M,MM)
      ANS= FZ*DELTI
      RETURN
      END
```

```
      SUBROUTINE SUM(X,Y,NP,NPM)
      DIMENSION X(201)
    1 Y=0.0
      N1=1
      N2=NP
      N3=NPM
      M=1
      CON =1.0
    2 DO 3I=N1,N2,N3
      Y= X(I)*CON+Y
    3 CONTINUE
      GO TO (4,5,6),M
    4 M=2
      N1=2
      N2=NPM
      N3=2
      CON=4.0
      GO TO 2
    5 M=3
      N1=3
      N2=NPM-1
      N3=2
      CON=2.0
      GO TO 2
    6 Y=Y*0.3333333
      RETURN
      END
```

SPHERICAL BEARING LARGE ETA-R 11JAN66

NBRG	MDI	M	N	LC	ISIG
1	0	9	19	0	1

ETA-Z	ETA-R	PHI-1	PHI-2	EPS
0.	0.400E-01	0.415E 02	0.680E 02	0.500E 00

FORCE-Z	FORCE-R
4.810299E-02	2.019075E-03

NBRG	MDI	M	N	LC	ISIG
1	0	9	19	0	1

ETA-Z	ETA-R	PHI-1	PHI-2	EPS
0.	0.500E-01	0.415E 02	0.680E 02	0.500E 00

FORCE-Z	FORCE-R
4.814445E-02	2.526229E-03

NBRG	MDI	M	N	LC	ISIG
1	0	9	19	0	1

ETA-Z	ETA-R	PHI-1	PHI-2	EPS
0.	0.600E-01	0.415E 02	0.680E 02	0.500E 00

FORCE-Z	FORCE-R
4.819521E-02	3.034982E-03

NBRG	MDI	M	N	LC	ISIG
1	0	9	19	0	1

ETA-Z	ETA-R	PHI-1	PHI-2	EPS
0.	0.900E-01	0.415E 02	0.680E 02	0.500E 00

FORCE-Z	FORCE-R
4.840392E-02	4.574101E-03

NBRG	MDI	M	N	LC	ISIG
1	0	9	19	0	1

ETA-Z	ETA-R	PHI-1	PHI-2	EPS
0.	0.100E 00	0.415E 02	0.680E 02	0.500E 00

FORCE-Z	FORCE-R
4.849255E-02	5.092549E-03

NBRG	MDI	M	N	LC	ISIG
1	0	9	19	0	1

ETA-Z	ETA-R	PHI-1	PHI-2	EPS
0.	0.110E 00	0.415E 02	0.680E 02	0.500E 00

```
  CYLINDRICAL BEARING TEST 11JAN66

NBRG   MDI     M     N    LC    ISIG
   2     0    25    19     0     1

  ETA-R       ETA-M       L/D       PHASE-M       A          EPS-0
0.500E-01 0.         0.200E 01 0.          -0.400E 00 0.400E 00

    FORCE-R          FORCE-T        TORQUE-R        TORQUE-T
  1.553928E-02  -3.094884E-07  -4.364673E-08  -2.829538E-10

NBRG   MDI     M     N    LC    ISIG

   2     0    25    19     0     1

  ETA-R       ETA-M       L/D       PHASE-M       A          EPS-0
0.100E 00 0.         0.200E 01 0.          -0.400E 00 0.400E 00

    FORCE-R          FORCE-T        TORQUE-R        TORQUE-T
  3.150929E-02  -2.795799E-07  -3.110657E-08  -2.414118E-09

NBRG   MDI     M     N    LC    ISIG
   2     0    25    19     0     1

  ETA-R       ETA-M       L/D       PHASE-M       A          EPS-0
0.150E 00 0.         0.200E 01 0.          -0.400E 00 0.400E 00

    FORCE-R          FORCE-T        TORQUE-R        TORQUE-T
  4.836489E-02  -3.615033E-07  -3.497759E-08  -1.252213E-09
```

```
CONICAL BEARING    19JAN66   DWG NO.  SK-C-2112

NBRG  MDI    M     N   LC   ISIG
   3    0   25     9    0     3

  ETA-Z      ETA-R      ETA-M      RRAT      GAMMA      EPS-1         A        PHASE
0.          0.550E 00 0.           0.429E 00 0.450E 02 0.707E 00-0.900F 00 0.
    FORCE-RR         FORCE-TR         FORCE-RA         FORCE-TA
  5.067533E-01  -4.310378E-07   3.583287E-01  -3.047898E-07

      CPR       CPT        CPRM        CPTM
-0.102E 00 0.324E-01-0.102E 00 0.324E-01

NBRG  MDI    M     N   LC   ISIG
   3    0   25     9    0     3

  ETA-Z      ETA-R      ETA-M      RRAT      GAMMA      EPS-1         A        PHASE
0.          0.500E-01 0.           0.429F 00 0.450E 02 0.707E 00 0.              0.
    FORCE-RR         FORCE-TR         FORCE-RA         FORCE-TA
  6.171215E-02  -5.758440F-07   4.363709E-02  -4.071832F-07

      CPR       CPT        CPRM        CPTM
0.697E-01 0.694E-01 0.697E-01 0.694E-01

NBRG  MDI    M     N   LC   ISIG
   3    0   25     9    0     3

  ETA-Z      ETA-R      ETA-M      RRAT      GAMMA      EPS-1         A        PHASE
0.          0.100E 00 0.           0.429E 00 0.450E 02 0.707E 00 0.              0.
    FORCE-RR         FORCE-TR         FORCE-RA         FORCE-TA
  1.245513E-01  -5.510523E-07   8.807108E-02  -3.896528E-07

      CPR       CPT        CPRM        CPTM
0.698E-01 0.984E-02 0.698E-01 0.984E-02

NBRG  MDI    M     N   LC   ISIG
   3    0   25     9    0     3

  ETA-Z      ETA-R      ETA-M      RRAT      GAMMA      EPS-1         A        PHASE
0.          0.150E 00 0.           0.429E 00 0.450E 02 0.707E 00 0.              0.
    FORCE-RR         FORCE-TR         FORCE-RA         FORCE-TA
  1.897236E-01  -5.544330E-07   1.341548E-01  -3.920433E-07

      CPR       CPT        CPRM        CPTM
0.698E-01 0.575E-01 0.698E-01 0.575F-01

NBRG  MDI    M     N   LC   ISIG
   3    0   25     9    0     3
```

TABLE I

RESULTS

SPHERICAL BEARING

$\Phi_1 = 41.5^\circ$ $\Phi_2 = 68.0^\circ$ $\varepsilon = 0.5$ $\eta_Z = 0.0$

η_R	$\overline{F}_z$	$\overline{F}_R$	$\overline{K}_R$
.04	.048103	.0020191	
.05	.048144*	.0025262	.050795*
.06	.048195	.0030350	
.09	.048404	.0045741	
.10	.048492	.0050925	.05201
.11	.048591	.0056143	
.19	.049742	.0099499	
.20	.049933	.010518	.05716
.21	.050137	.011093	
.29	.052204	.016017	
.30	.052523	.016681	.06705
.31	.052857	.017358	
.39	.056119	.023333	
.40	.056611	.024164	.08925
.41	.057124	.025018	
.49	.062117	.032825	
.50	.062871	.033951	.1146
.51	.063660	.035117	
.59	.071490	.046293	
.60	.072703	.047983	.1732
.61	.073982	.049757	

*Small η_R Analysis $\overline{K}_R = .05047$ $\overline{F}_z = .04803$

TABLE II

RESULTS

CYLINDRICAL BEARING

$\frac{L}{D} = 2 \qquad \eta_\Gamma = 0 \qquad a = -.4 \qquad \varepsilon_o = .4$

η_R	$\overline{F}_R$	$\overline{K}_R$*
.05	.015539	
.10	.031505	.32826
.15	.048365	

*Small η_R Analysis $\overline{K}_R = .31$

$\frac{L}{D} = 2 \qquad \eta_R = 0 \qquad a = -.4 \qquad \varepsilon_o = .4$

η_Γ	$\overline{M}_R$**
.1	.0048710

**Small η_Γ Analysis $\overline{M}_R/\eta_\Gamma = .049$

TABLE III

RESULTS

CONICAL BEARING

$\frac{R_1}{R} = 0.5$ $\Gamma = 45^o$ $\varepsilon \sin \Gamma = .5$ $a = 0$ $\eta_z = 0$

$\eta_\Gamma = 0$

η_R	$\bar{F}_R$	C.P.
.1	0.126	.0574

Small η_R Analysis

$\bar{F}_R/\eta_R = 1.26$, C.P. = .0575

$\eta_R = 0$

η_Γ	$\bar{G}_R$	C.P.
.1	0.100	0.110

Small η_Γ Analysis

$\bar{G}_R/\eta_\Gamma = 0.995$, C.P. = 0.1085

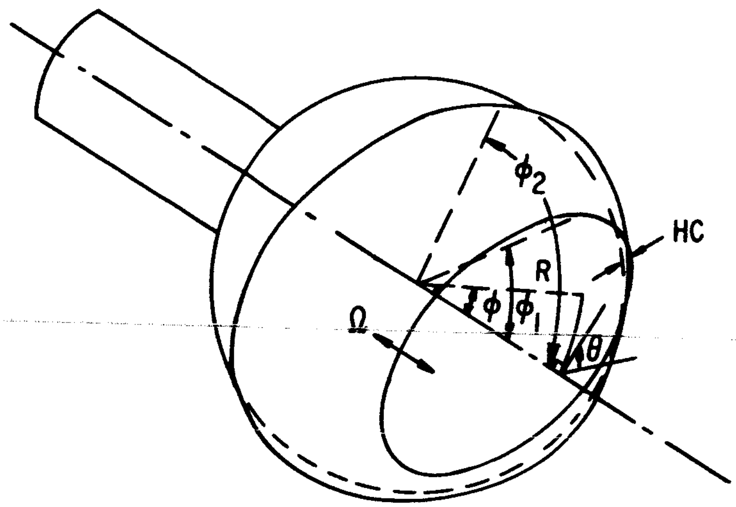


Fig. 1   Spherical Squeeze-Film Bearing

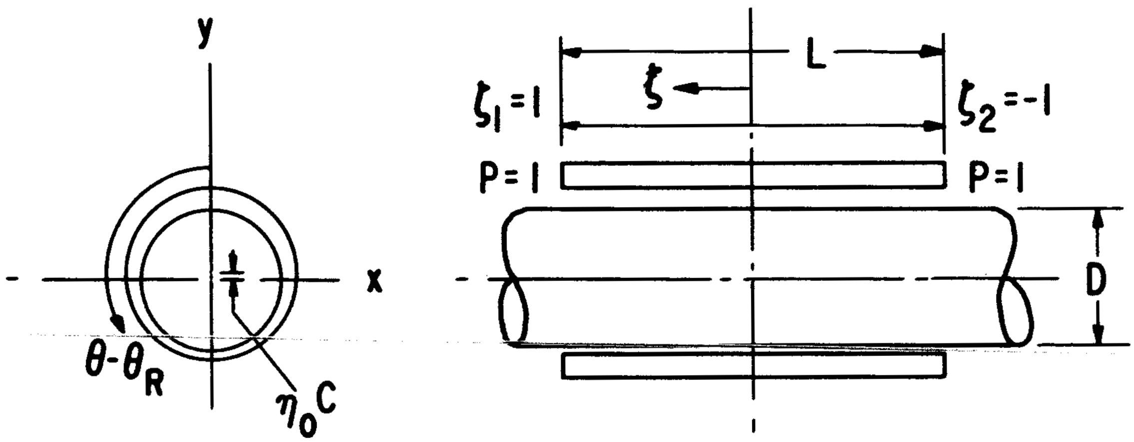
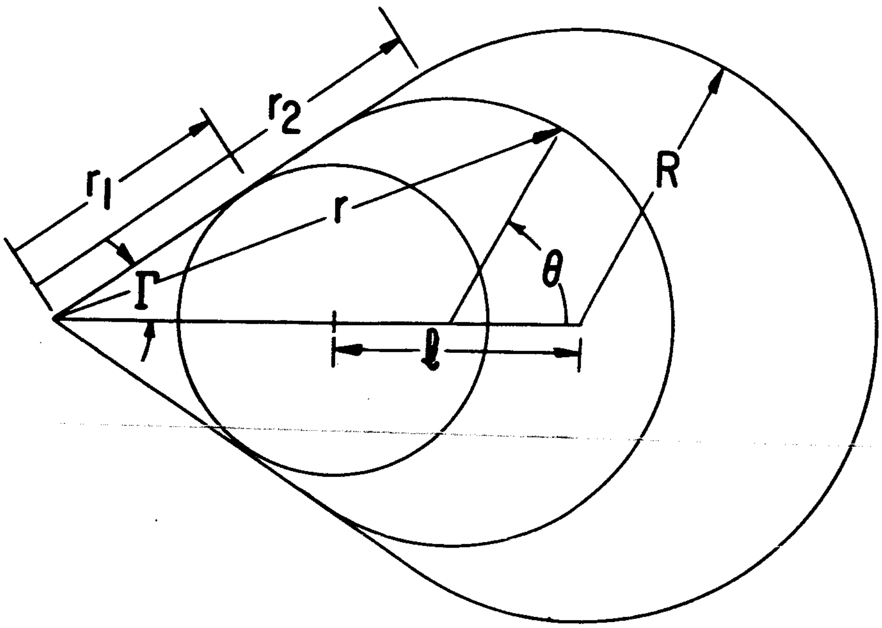


Fig. 2 Cylindrical Squeeze-Film Bearing



$$r_1 = R \csc \Gamma - \ell \sec \Gamma$$

$$r_2 = R \csc \Gamma$$

$$\zeta = \frac{r}{R}$$

Fig. 3 Conical Squeeze-Film Bearing

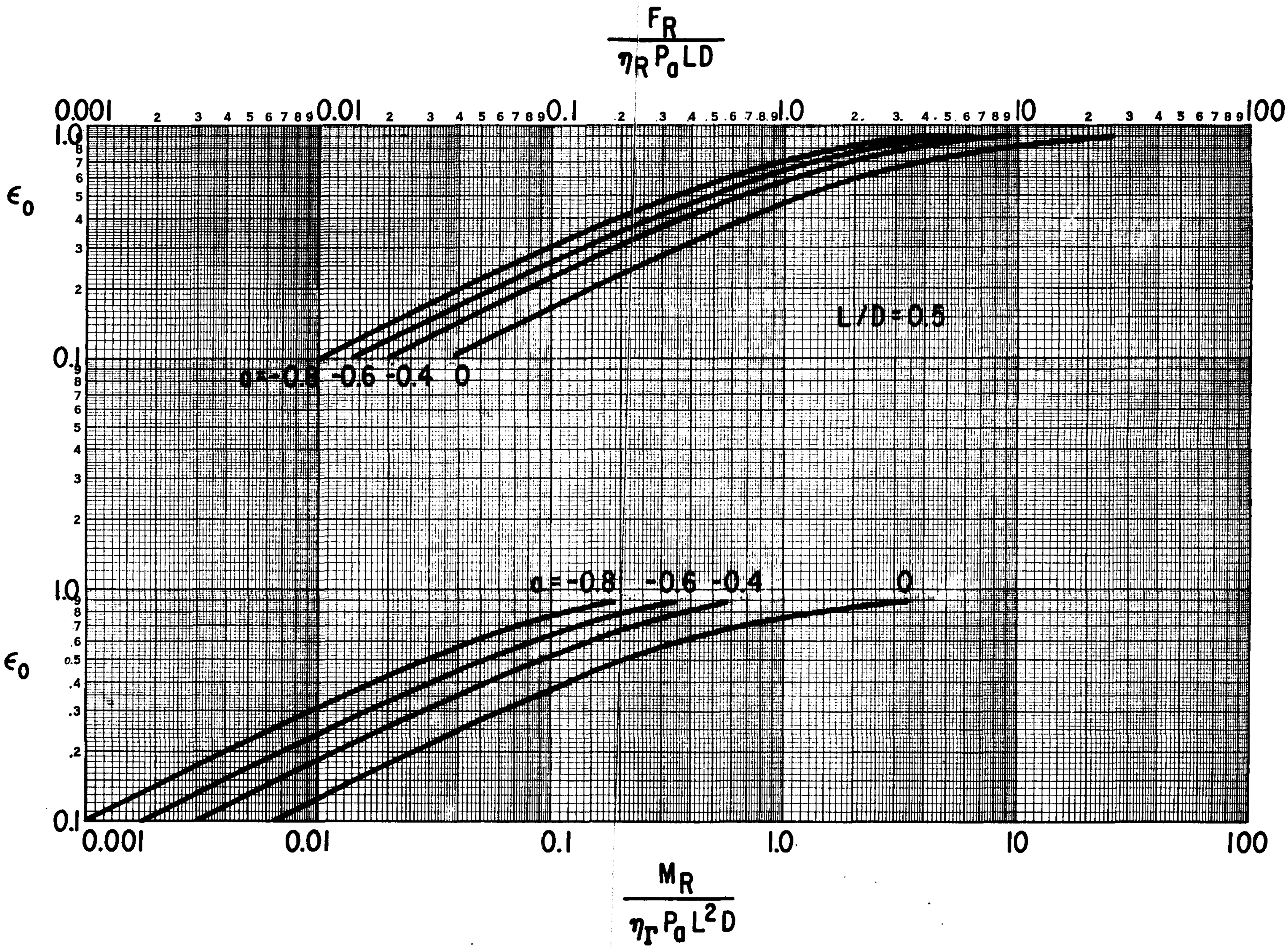


Fig. 4 Normalized Radial Force and Moment Versus Mid-Plane Excursion Ratio for Cylindrical Bearing with L/D = 0.5

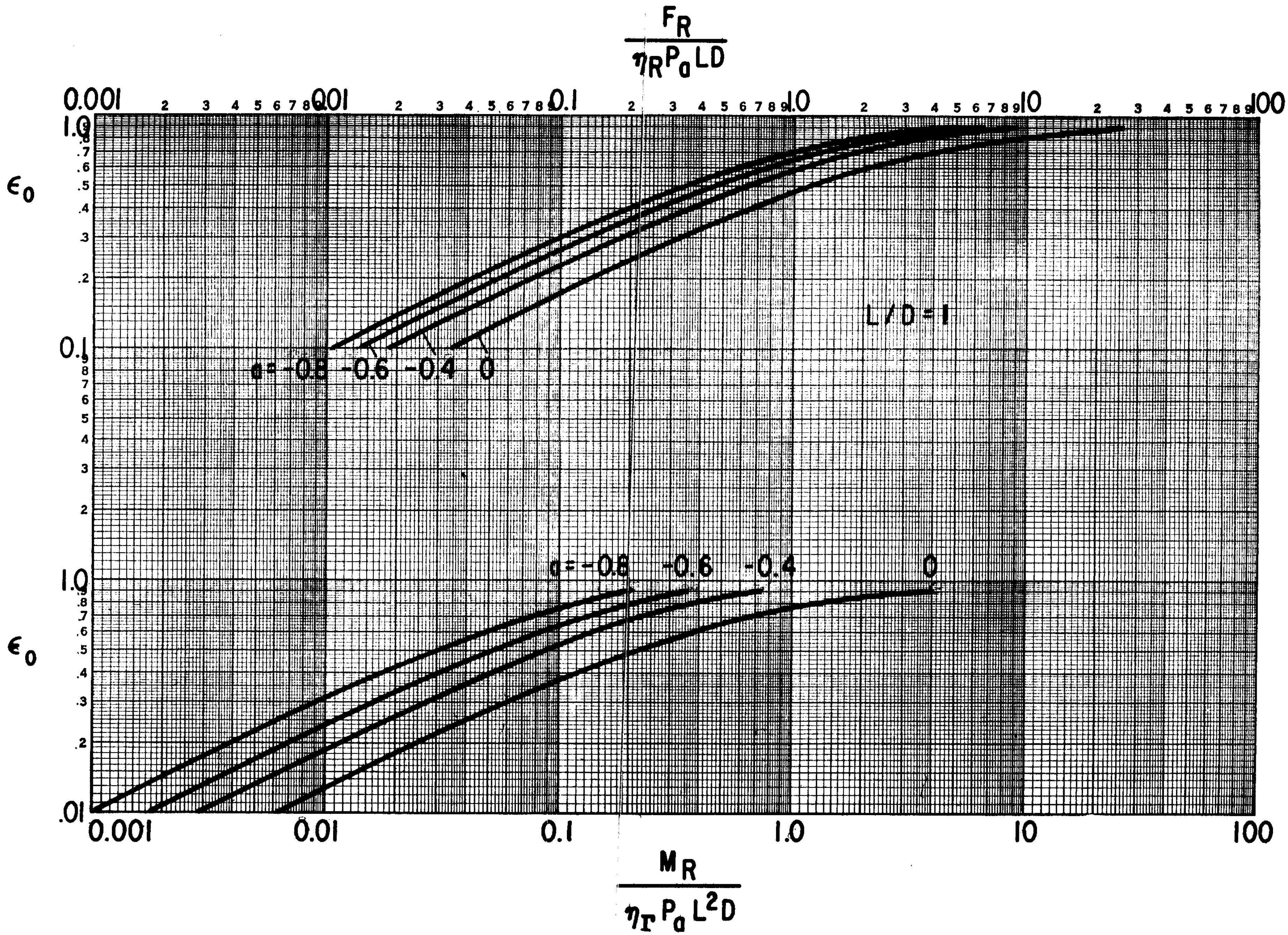


Fig. 5 Normalized Radial Force and Moment Versus Mid-Plane Excursion Ratio for Cylindrical Bearing with L/D = 1.0

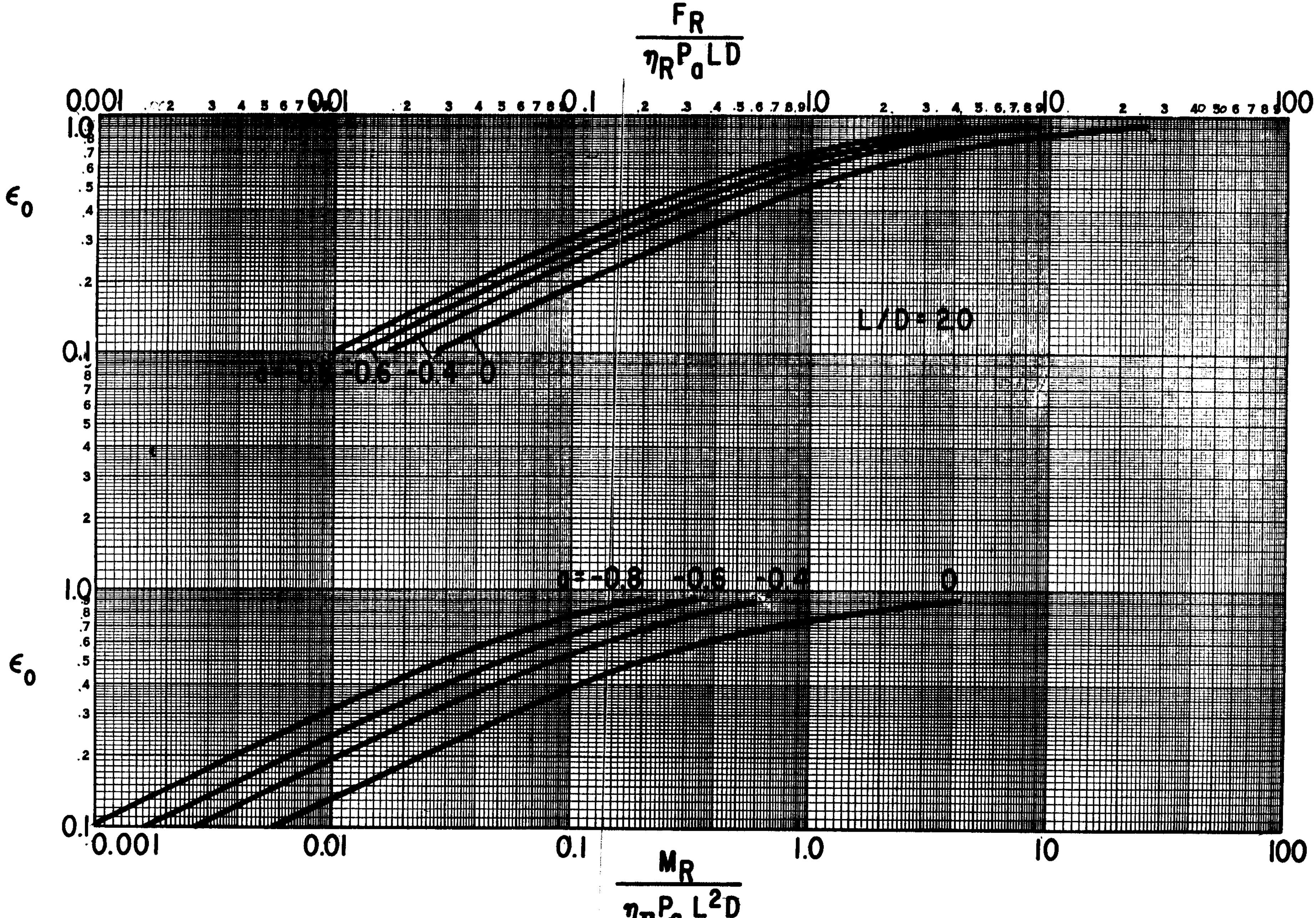


Fig. 6 Normalized Radial Force and Moment Versus Mid-Plane Excursion Ratio for Cylindrical Bearing with L/D = 2.0

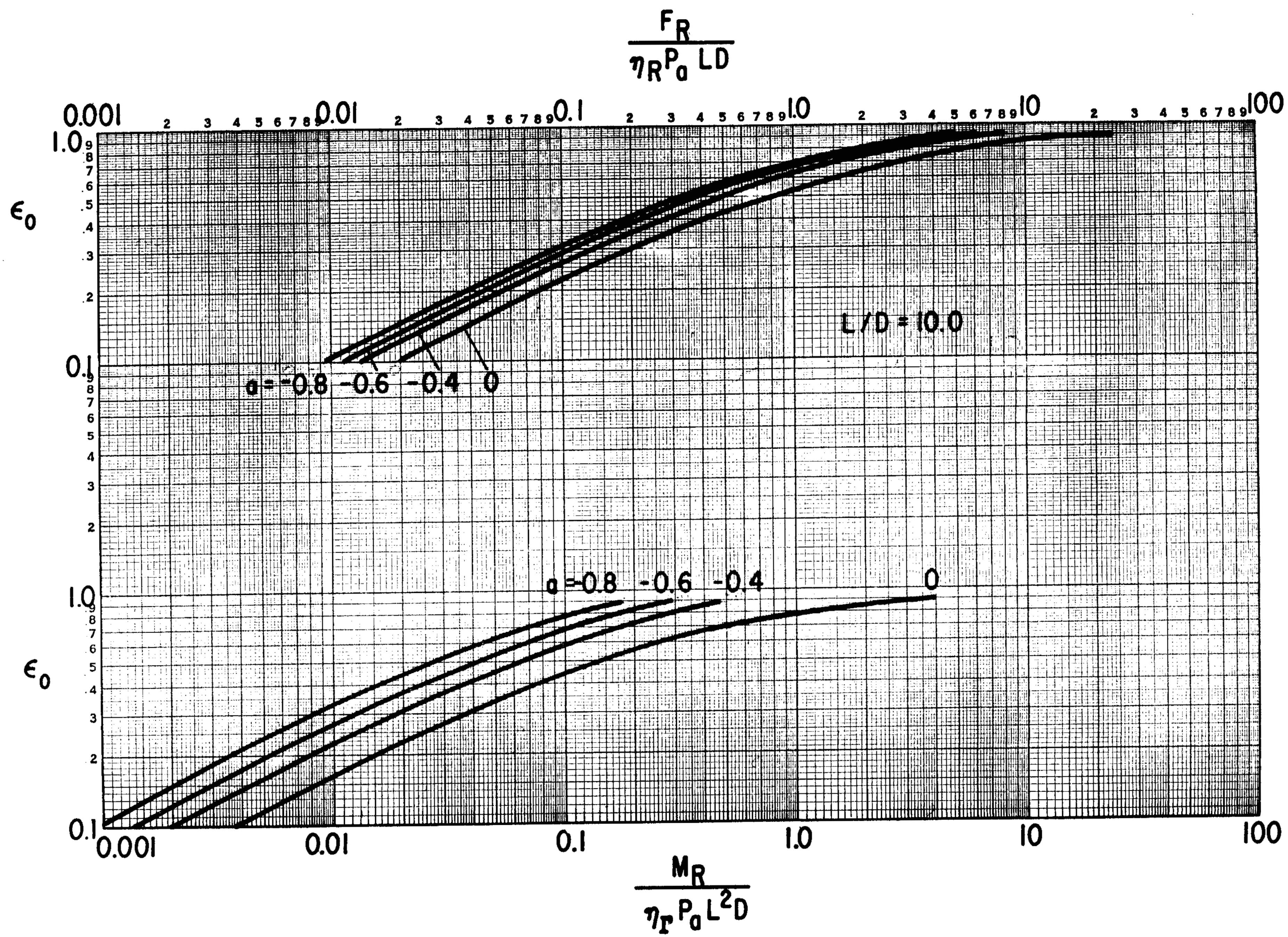


Fig. 7 Normalized Radial Force and Moment Versus Mid-Plane Excursion Ratio for Cylindrical Bearing with L/D = 10.0

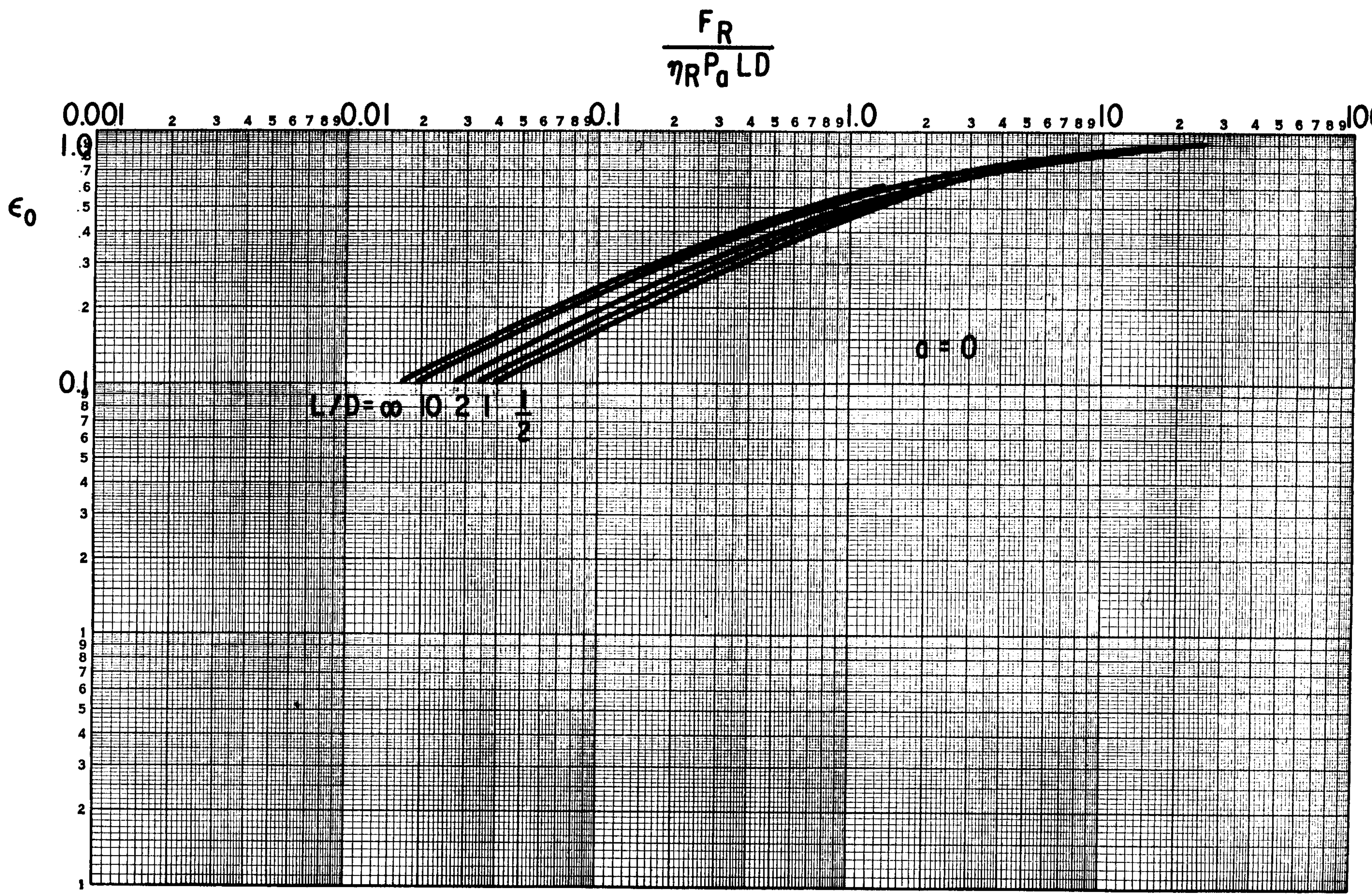


Fig. 8 Normalized Radial Force Versus Mid-Plane Excursion Ratio for Cylindrical Bearing with Various Slenderness Ratios and a = 0.