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THE DETECTION OF THE PLASMA COMPONENT OF MAGNETOHYDRODYNAMIC WAVES IN SPACE

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ABSTRACT

The fundamental structure of low-frequency magnetohydrodynamic waves in interplanetary space is analyzed with respect to variations of the magnetic field, the plasma velocity, and the plasma density. The analyses have been conducted in order to (a) determine the time profile of such waves as seen by the types of plasma detectors presently used or planned for use in the direct measurement of the properties of the interplanetary plasma, and (b) aid in the design of second-generation detectors specifically suited to the analysis of magnetohydrodynamic waves in space. It is concluded that plasma detectors with flat resolution functions allow a reasonably faithful reproduction of the waveform of the plasma motion, but that this type of instrument suffers from blind spots for the detection of some transverse hydromagnetic waves. The type of instrument with a peaked resolution function, on the other hand, is not troubled by blind spots but creates a large amount of distortion of the plasma waveform. It is possible, however, to determine fluid velocity and density of the plasma by simultaneously monitoring each member of a group of at least four plasma detectors of conventional design.

I. INTRODUCTION

It is the purpose of this Memorandum to analyze the fundamental structure of low-frequency magnetohydrodynamic waves in interplanetary space with respect to variations of the magnetic field, the plasma velocity, and the plasma density. The Memorandum includes the determination of the time profiles of such waves, as seen by plasma detectors presently in use (or planned) to obtain direct measurement of the properties of the interplanetary plasma. The analysis will aid in the interpretation of data from space-probe plasma detectors, and in the design of second-generation detectors specifically suited to analysis of magnetohydrodynamic waves in space.

II. BASIC EQUATIONS GOVERNING MAGNETOHYDRODYNAMIC WAVES

We wish to consider a collisionless (η = resistivity = 0) plasma made up of protons and electrons. Because of the limitations of existing plasma detectors, we are interested in oscillations of such a low frequency that (1) inertial effects of the protons are important and, (2) that the electrons can be assumed to be distributed at all times so no net space charge is accumulated. Furthermore, the displacement current can be neglected with respect to the material currents. Electromagnetic units are employed.

With these assumptions, we have Maxwell's equations[1]:

$$\nabla \times \mathbf{B} = 4\pi \mathbf{J} \tag{1}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2}$$

We have also the equation of the conservation of mass:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{V} = 0 \tag{3}$$

the momentum equation:

$$\rho \frac{d\mathbf{V}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla p \tag{4}$$

and Ohm's Law:

$$\eta \mathbf{J} = \mathbf{E} + \mathbf{V} \times \mathbf{B} = 0 \tag{5}$$

[1] The notation is fairly standard, but definitions are given in the Nomenclature Section.

III. TRANSVERSE WAVES

For transverse hydromagnetic waves, we define a coordinate system as follows (Fig. 1):

$\mathbf{B} = B_1\hat{x} + B_0\hat{z}$, where B_0 is the undisturbed magnetic field and B_1 is the perturbation in the field due to the wave.

$\mathbf{V} = \mathbf{V}_0 + V_1\hat{x}$, where $\mathbf{V}_0$ is the undisturbed velocity (arbitrary direction) and V_1 is the perturbation of velocity.

$\hat{n} = +\hat{z}$ = direction of propagation of wave.

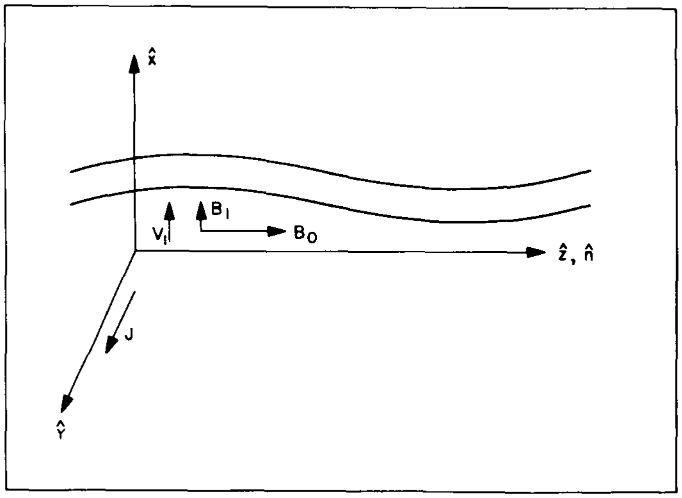

Fig. 1. Representation of a purely transverse hydromagnetic wave

With these assumptions, Maxwell's equations become:

$$\left.\begin{aligned} \frac{\partial B_1}{\partial z} &= 4\pi J_y \\ J_x &= J_z = 0 \end{aligned}\right\} \text{From Eq. (1)} \qquad (6)$$

$$\left.\begin{aligned} &\nabla \times \mathbf{E} = -\nabla \times (\mathbf{V} \times \mathbf{B}) = -\frac{\partial \mathbf{B}}{\partial t} \\ &\frac{\partial B_1}{\partial t} = -V_{0z}\frac{\partial B_1}{\partial z} + B_0 \frac{\partial V_1}{\partial z} \end{aligned}\right\} \text{From Eq. (2) and (5)} \qquad (7)$$

For a plane-transverse wave propagating in the $\hat{z}$ direction, dependent variables vary only with t and z. Since the velocity perturbation is parallel to the wave front, $\nabla \cdot \mathbf{V} = 0$. It then follows from Eq. (3) that the density ρ is constant. The momentum equation (4) is:

$$\rho_0 \frac{\partial V_1}{\partial t} + \rho_0 V_{0z} \frac{\partial V_1}{\partial z} = J_y B_0 \qquad x \text{ component equation} \qquad (8)$$

$$\rho_0 \frac{\partial V_z}{\partial t} + \rho_0 V_{0z} \frac{\partial V_z}{\partial z} = -J_y B_1 - \frac{dp}{dz} \qquad z \text{ component equation} \qquad (9)$$

Equation (9) tells us that unless B_1 is much smaller than B_0, appreciable longitudinal components of the velocity and pressure gradients will build up; the wave would then no longer be wholly transverse; and density variations would be important.

We now have three linear, first order equations, (6), (7), and (8), in B_1, J_y, and V_1. If we assume solutions of the form:

$$B_1 = B_{10}\, e^{i(\kappa z - \omega t)}$$

$$J_y = J_{10}\, e^{i(\kappa z - \omega t)}$$

$$V_1 = V_{10}\, e^{i(\kappa z - \omega t)}$$

we obtain the following algebraic relations from Eqs. (6), (7) and (8):

$$i\kappa B_{10} = 4\pi J_{10}$$

$$-i\omega B_{10} = -i\kappa V_{0z} B_{10} + i\kappa B_0 V_{10}$$

$$-i\omega \rho_0 V_{10} + i\kappa \rho_0 V_{0z} V_{10} = J_{10} B_0 \qquad (10)$$

The relations between J_{10}, V_{10}, and B_{10} found from the first two of these equations are:

$$J_{10} = \frac{i\omega}{4\pi V_\phi} B_{10}$$

$$V_{10} = -(V_\phi - V_{0z}) \frac{B_{10}}{B_0} \qquad (11)$$

where $V_\phi = \omega/\kappa$ = phase velocity. Note that the velocity oscillation is 180 deg out of phase with the magnetic field oscillation. If B_0 had been opposite to the direction of propagation instead of along it, V_{10} and B_1 would have been in phase with each other.

V_ϕ can be found by substitution of J_{10} and V_{10} into Eq. (10):

$$V_\phi - V_{0z} = \frac{B_0}{\sqrt{4\pi\rho_0}} \qquad (12)$$

Transverse hydromagnetic waves can also be pictured as oscillations of the magnetic lines of force to which the ions are attached (see Alfvén, Ref. 1).

IV. LONGITUDINAL WAVES

Again let $+\hat{z}$ be the direction of propagation of the wave. Only in this instance,

$$\mathbf{B} = (B_0 + B_1)\hat{x}$$

$$\mathbf{V} = \mathbf{V}_0 + V_1\hat{z}$$

A pictorial representation of longitudinal oscillations is given in Fig. 2.

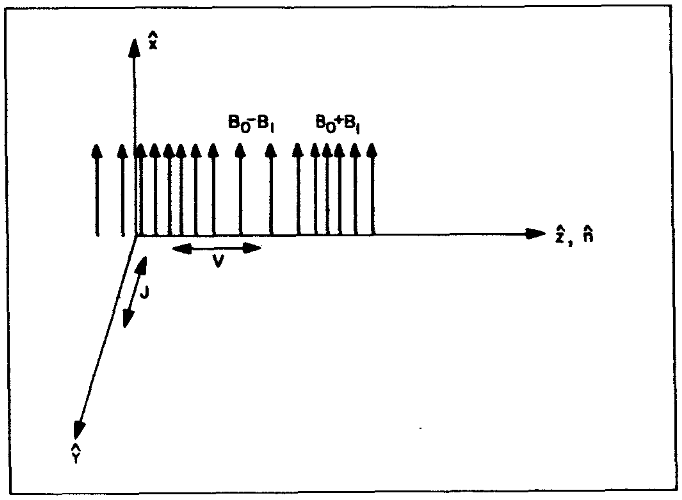

Fig. 2. Representation of a purely longitudinal hydromagnetic wave

The density and pressure will similarly be written as the sums of the undisturbed and perturbed parts:

$$\rho = \rho_0 + \rho_1(z,t)$$

$$p = p_0 + p_1(z,t)$$

If we assume that the oscillations are adiabatic ($p \propto \rho^{\gamma}$) and of small amplitude, then the equation of state ($p_i = \rho_i k T_i / m_i$) can be approximated by:

$$p_1 = \frac{\rho_1 k(\gamma_i T_i + \gamma_e T_e)}{m_i}$$

where in the absence of collisions $\gamma_i = \gamma_e = 2$.

For these longitudinal oscillations, Maxwell's equations can be written as:

$$\left.\begin{aligned} \frac{\partial B_1}{\partial z} &= 4\pi J_y \\ J_x = J_z &= 0 \end{aligned}\right\} \text{ from Eq. (1)} \tag{13}$$

$$\left.\begin{aligned} \nabla \times \mathbf{E} = -\nabla \times (\mathbf{V} \times \mathbf{B}) &= -\frac{\partial \mathbf{B}}{\partial t} \\ -V_{0z}\frac{\partial B_1}{\partial z} - B_0 \frac{\partial V_1}{\partial z} - \frac{\partial (V_1 B_1)}{\partial z} &= \frac{\partial B_1}{\partial t} \end{aligned}\right\} \text{ from Eqs. (2) and (5)}$$

We shall limit outselves again to oscillations of small amplitude so the second order term involving the product of V_1 and B_1 can be neglected; thus:

$$-V_{0z}\frac{\partial B_1}{\partial z} - B_0 \frac{\partial V_1}{\partial z} = \frac{\partial B_1}{\partial t} \tag{14}$$

The conservation of mass, Eq. (3), and momentum, Eq. (4), for small oscillations can be written as:

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \frac{\partial V_1}{\partial z} + V_{0z}\frac{\partial \rho_1}{\partial z} = 0 \tag{15}$$

and

$$\rho_0 \frac{\partial V_1}{\partial t} + \rho_0 V_{0z} \frac{\partial V_1}{\partial z} = -J_y B_0 - \frac{k}{m_i} (\gamma_i T_i + \gamma_e T_e) \frac{\partial \rho_1}{\partial z} \tag{16}$$

Equations (13) through (16) constitute a set of four linear first-order equations for the variables B_1, V_1, ρ_1 and J_y. If, as in the case of transverse waves, we assume that each variable is proportional to $\exp i(\kappa z - \omega t)$, we obtain the following relations between the amplitudes of the variables:

$$J_{10} = \frac{i\kappa}{4\pi} B_{10}$$

$$V_{10} = (V_\phi - V_{0z}) \frac{B_{10}}{B_0} \tag{17}$$

$$\rho_{10} = \frac{\rho_0 B_{10}}{B_0} = \frac{V_{10}}{V_\phi - V_{0z}} \rho_0$$

$$V_\phi - V_{0z} = \sqrt{\frac{B_0^2}{4\pi\rho_0} + \frac{k(\gamma_i T_i + \gamma_e T_e)}{m_i}} \tag{18}$$

Note that V_1, B_1 and ρ_1 are all in phase.

V. INSTRUMENTAL CONSIDERATIONS

The detectors presently planned for direct plasma measurements in interplanetary space measure a current I which can be expressed as:

$$I(\Gamma, \mathscr{E}) = nV \cos \Gamma \, F(\Gamma, \mathscr{E}) \tag{19}$$

where Γ is the angle between the normal to the detector aperture and the direction of incidence of the ionized particles, $\mathscr{E}$ is the particle kinetic energy, n is the number of particles (angle Γ, energy $\mathscr{E}$) per cm^3, and $F(\Gamma, \mathscr{E})$ is the fraction of the incident flux which gets through the analysis part of the instrumentation to the detector or current measurement device. The nature of $F(\Gamma, \mathscr{E})$ differs from one type of instrument to another.

We consider first the Faraday-cup type of instrument with grids which, (a) serve to keep plasma electrons from reaching the collector, and (b) suppress the photoelectric current by modulating the incoming protons without modulating the photoelectrons (Ref. 2). Using proper geometrical design, such an instrument could serve as an example of a detector with $F(\Gamma, \mathscr{E})$ nearly independent of Γ for small Γ. The dependence on $\mathscr{E}$ also could be made negligible by choosing the amplitude of the square-wave modulating potential sufficiently large.

A very different form of $F(\Gamma, \mathscr{E})$ can be illustrated by instruments of the curved-plate-electrostatic analyzer type (Ref. 3). Figure 3 is a plot of $F(\Gamma, \mathscr{E})$ (with $\mathscr{E}$ in units of $\mathscr{E}_c = (e\Phi/2)/ln\,(b/a)$ where Φ is the potential difference across the curved plates and b/a is the ratio of the radii of curvature of the two plates), for an analyzer 120 deg long with b/a equal to 1.25 (geometry planned for a Surveyor Lunar landing mission). With proper choice of geometric factors and a small amplitude of the modulating voltage, a Faraday-cup type of instrument, as discussed above, could also have such a sharply peaked resolution function.

We now wish to consider the appearance of the hydromagnetic waves discussed in Sections III and IV, as seen by the two general types of instruments, (a) $F(\Gamma, \mathscr{E})$ = constant, and (b) $F(\Gamma, \mathscr{E})$ sharply peaked.

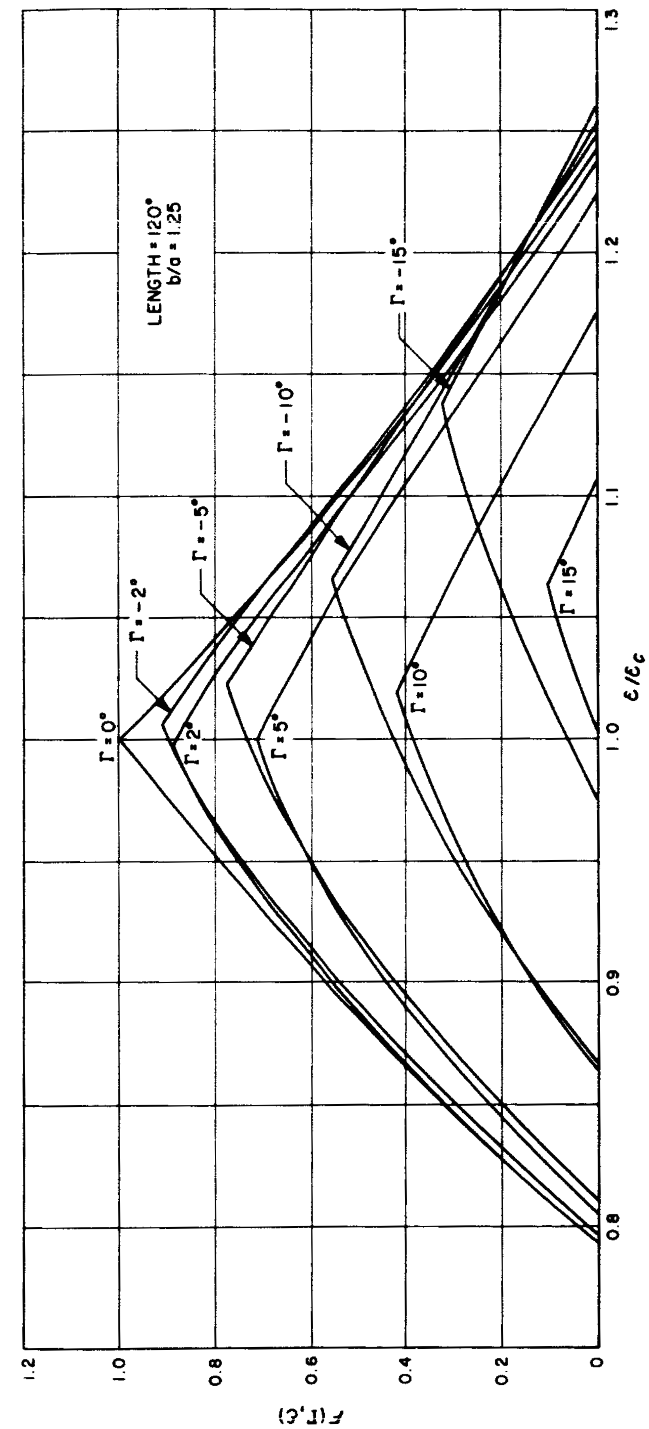

Fig. 3. $F(\Gamma, \mathscr{E})$ for curved plate analyzers with $b/a = 1.25$ and length 120°

A. Appearance of Transverse Hydromagnetic Waves

Let the direction of propagation and of $\mathbf{B}_0$, the undisturbed magnetic field vector, be at an angle θ to $\mathbf{V}_0$ (assumed radial away from the Sun). The oscillation will be assumed polarized such that V_1 and B_1 are in the plane defined by $\mathbf{V}_0$ and $\mathbf{B}_0$ (Fig. 4). If $\mathbf{V}$ is the resultant velocity vector which makes an angle Γ with $\mathbf{V}_0$, then

$$V \cos \Gamma = V_0(1 + v \sin \theta) \tag{20}$$

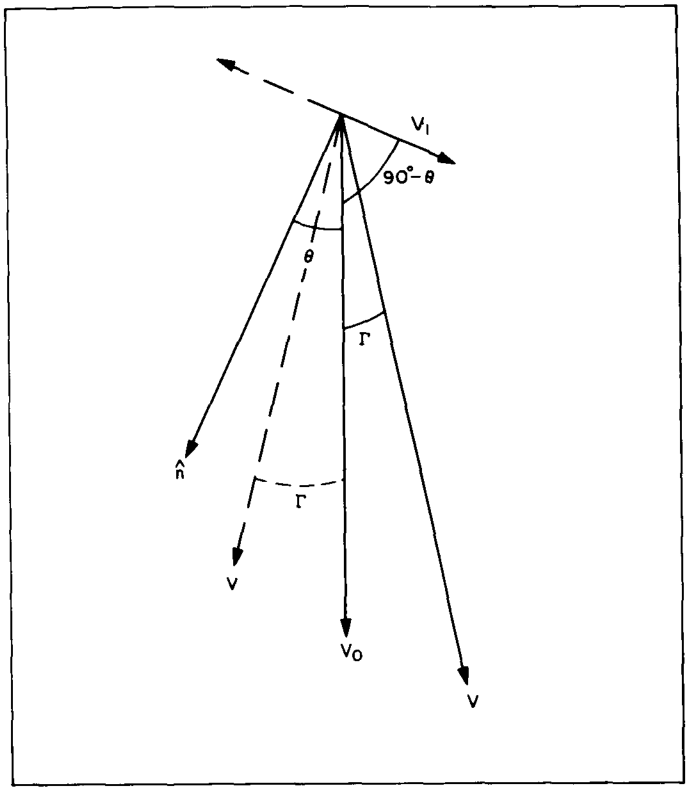

Fig. 4. Angles involved in discussion of transverse hydromagnetic wave detection

where

$$v = \frac{V_1}{V_0} = \frac{V_{10}}{V_0} e^{i(\kappa z - \omega t)} \tag{21}$$

We can also show that for small Γ

$$\tan \Gamma \approx \Gamma = \frac{v \cos \theta}{1 + v \sin \theta} \tag{22}$$

Also,

$$\frac{\mathcal{E}}{\mathcal{E}_0} = v^2 + 2v \sin \theta + 1. \tag{23}$$

For a detector pointing toward the Sun, the ratio of the measured current I, in the presence of a transverse hydromagnetic wave to the measured current I_0, for the undisturbed medium ($\mathbf{V} = \mathbf{V}_0$, $\Gamma = 0$), is given by:

$$\frac{I}{I_0} = (1 + v \sin \theta) \frac{F(\Gamma, \mathcal{E})}{F(O, \mathcal{E}_0)} \tag{24}$$

as seen from Eq. (19) and (20).

We shall take the examples of $\theta = 0$, 45 deg and 90 deg. For $\theta = 0$,

$$\frac{I}{I_0} = \frac{F(\Gamma, \mathcal{E})}{F(0, \mathcal{E}_0)}$$

Thus the first type of instrument with $F(\Gamma, \mathcal{E})$ approximately constant will not be able to detect the presence of a transverse hydromagnetic wave propagating straight toward it.

Equations (22), (23), and (24), together with the curve for $F(\Gamma, \mathcal{E})$ (Fig. 3), allow us to draw Fig. 5a, b, c, which show the appearance of representative hydromagnetic waves. For illustrative purposes, it was assumed in drawing these figures that $\mathcal{E}_0/\mathcal{E}_c = 1.05$ (*i.e.*, the energy of the undisturbed plasma is slightly off-center of the maximum sensitivity of the plasma detector). It was also assumed that the amplitude of the velocity oscillation was $V_{10}/V_0 = 0.1$. This last corresponds to an oscillation of the magnetic field of magnitude

$$\frac{B_{10}}{B_0} = \frac{V_0}{-(V_\phi - V_{0z})} \frac{V_{10}}{V_0} = -\frac{V_0\sqrt{4\pi\rho_0}}{B_0} \frac{V_{10}}{V_0}$$

as is seen from Eq. (11) and (12). If $V_0 = 300$ km/sec and $n_0 = 20$ ion pairs per cm^3, $B_{10} = 6$ gamma for $V_{10}/V_0 = 0.1$.

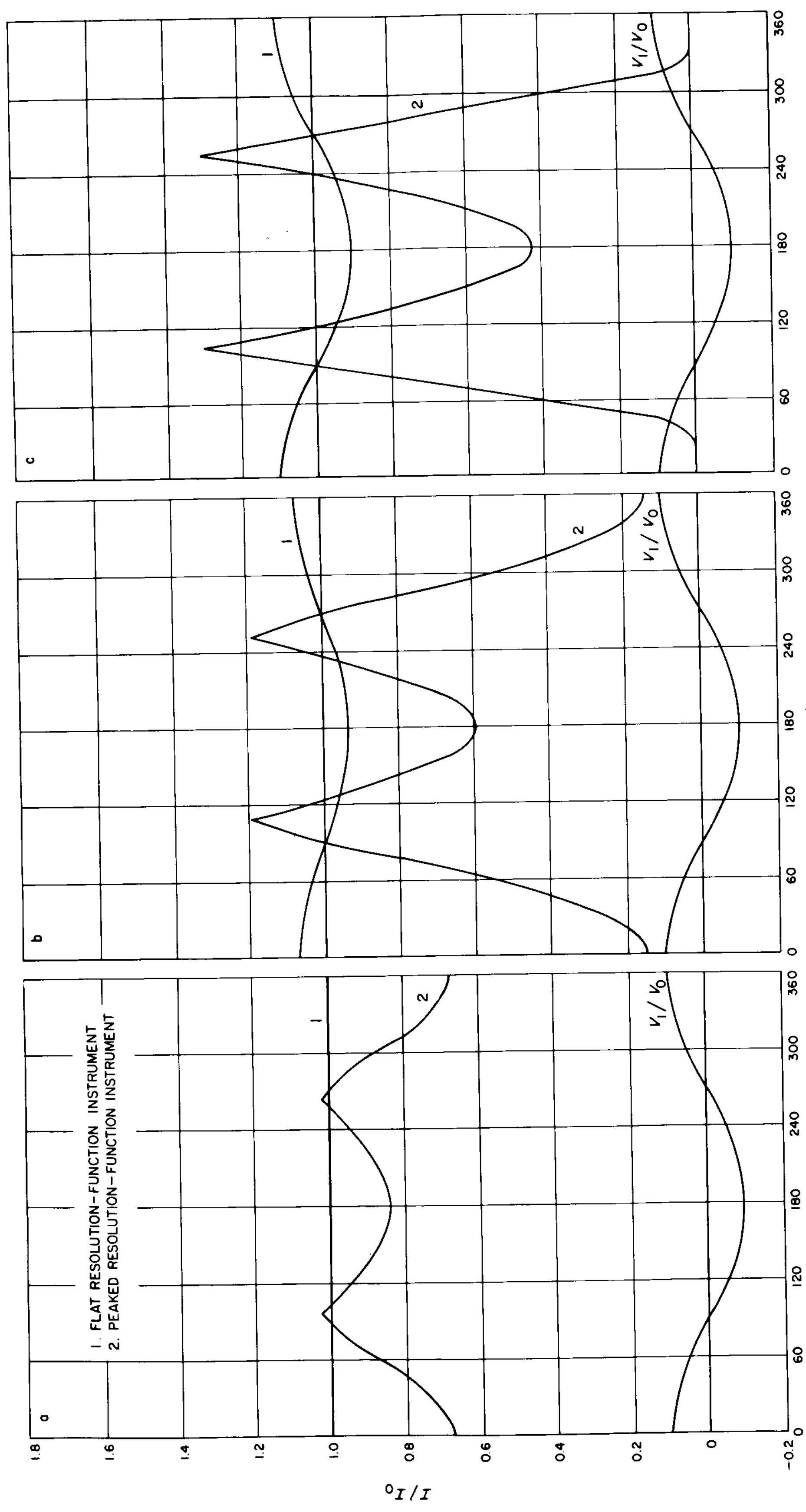

Fig. 5. Appearance of plasma component of transverse hydromagnetic waves, (a) $\theta = 0$, (b) $\theta = 45°$, (c) $\theta = 90°$

The following features of Fig. 5a, b, and c call for further comment:

1. For the type of plasma analyzer with a sharply peaked resolution function, the frequency of I/I_0 may be twice the frequency of both V_1 and B_1; whereas, for plasma analyzers with flat resolution functions, when these can detect the wave at all, the measured oscillation is always at the same frequency as V_1 and B_1.
2. The amplitude of the variation of I/I_0 may be much greater than the amplitude of V_1/V_0 or B_1/B_0 due to the possible multiplicative effects of $F(\Gamma, \mathcal{E})/F(O, \mathcal{E}_0)$.
3. With respect to the peaked-resolution-function instrument, if the vector velocities at the extremes of the plasma oscillation happen to correspond to values of Γ and $\mathcal{E}$ in the wings of the resolution function, then the current may be a minimum when the flux is a maximum.
4. Due principally to the fact that $\mathcal{E}_0/\mathcal{E}_c \neq 1$, the two current minima per cycle of the magnetohydrodynamic oscillation are of different amplitudes, even for a perfectly symmetric plasma waveform.

The response of plasma analyzers to transverse hydromagnetic waves polarized perpendicular to the plane defined by $\mathbf{V}_0$ and the direction of propagation is exactly the same as to the $\theta = 0$ deg case discussed earlier. The response to a circularly polarized transverse oscillation can thus be found by a combination of diagrams such as Fig. 5a, b, c. Note that neither type of instrument is able to detect a circularly polarized wave propagating straight toward it.

B. Appearance of Longitudinal Hydromagnetic Waves

Again let θ be the angle between $\mathbf{V}_0$ (radially away from the Sun) and the direction of propagation $\hat{n}$. For these waves, V_1 is along $\hat{n}$, and the polarization of B_0 and B_1, in the plane of the wave front, is irrelevant. See Fig. 6 for details.

$$n = n_0 \left(1 + \frac{V_0 v}{V_\phi - V_{0z}}\right) \tag{25}$$

$$V \cos \Gamma = 1 + v \cos \theta \tag{26}$$

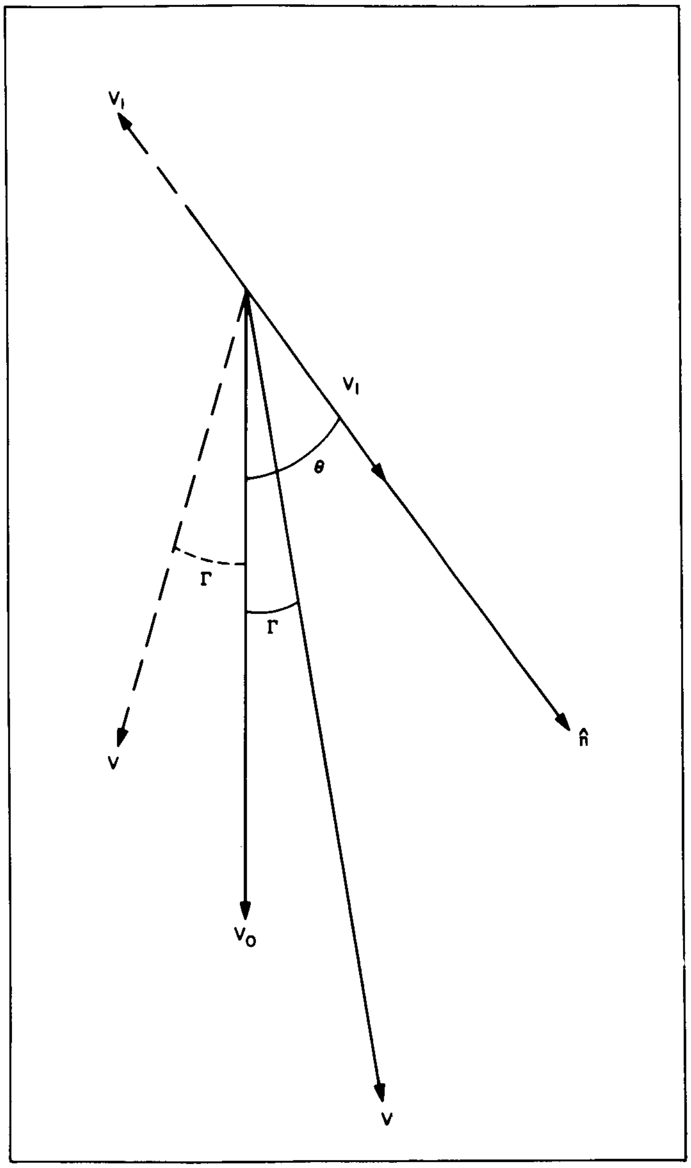

Fig. 6. Angles involved in discussion of longitudinal hydromagnetic wave detection

$$\tan \Gamma \approx \Gamma \approx \frac{v \sin \theta}{1 + v \cos \theta} \tag{27}$$

$$\frac{\mathscr{E}}{\mathscr{E}_0} = 1 + 2v \cos \theta + v^2 \tag{28}$$

By combining Eq. (19), (25) and (26), we obtain

$$\frac{I}{I_0} = \left(1 + \frac{V_0 v}{V_\phi - V_{0z}}\right)(1 + v \cos \theta) \frac{F(\Gamma, \mathscr{E})}{F(O, \mathscr{E}_0)} \tag{29}$$

From the combination of Eq. (27), (28) and (29) with Fig. 3, we can obtain Fig. 7a, b, and c, which show the appearance of longitudinal waves for $\theta = 0$, 45 deg and 90 deg. If it is assumed that $V_1/V_0 = 0.1$, $\gamma_e = \gamma_i = 2$, $T_e = T_i = 5 \times 10^4$ deg K, $V_0 = 300$ km/sec, $n_0 = 20$ cm^{-3}, and $B_0 = 10$ gamma, Eq. (18) tells us $V_0/V_\phi - V_{0z}) = 4.7$ and Eq. (17) gives $B_{10} = 0.47\ B_0 = 4.7$ gamma.

Note that

$$V_\phi - V_{0z} = \left[\frac{B_0^2}{4\pi \rho_0} + \frac{k(\gamma_i T_i + \gamma_e T_e)}{m_i}\right]^{1/2}$$

If

$$\frac{B_0^2}{4\pi \rho_0} >> \frac{k(\gamma_i T_i + \gamma_e T_e)}{m_i}$$

the wave is purely hydromagnetic; whereas if

$$\frac{B_0^2}{4\pi \rho_0} << \frac{k(\gamma_i T_i + \gamma_e T_e)}{m_i}$$

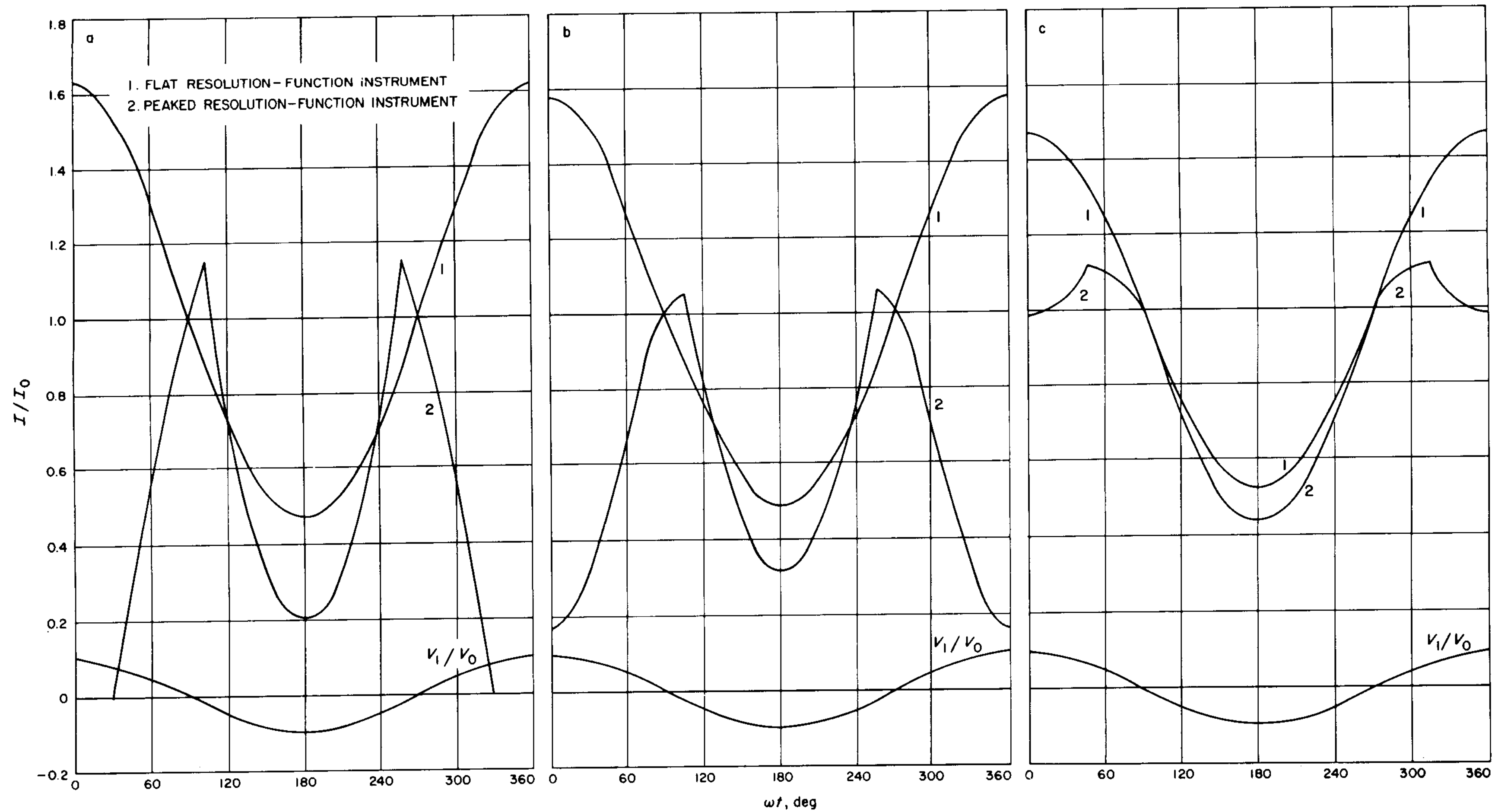

Fig. 7. Appearance of plasma component of longitudinal hydromagnetic wave, (a) $\theta = 0$, (b) $\theta = 45°$, (c) $\theta = 90°$

the wave is an ordinary acoustic wave. For the numbers chosen above to represent the interplanetary medium,

$$\frac{B_0^2}{4\pi\rho_0} = (49 \text{ km/sec})^2$$

and

$$\frac{k(\gamma_i T_i + \gamma_e T_e)}{m_i} = (41 \text{ km/sec})^2$$

Thus, the interplanetary medium should have waves intermediate between the two types. Time variations in the properties of the medium may, in fact, cause a fluctuation between the predominance of one type over the other. The problem of measurement and analysis of longitudinal waves in the interplanetary medium is thus more complicated than in other regions where one type of wave is clearly predominant.

VI. DISCUSSION

From the discussion and Figures of the last section, we conclude that plasma detectors with flat resolution functions allow a reasonably faithful reproduction of the waveform of the plasma motion, but that this type of instrument is insensitive to normally incident transverse hydromagnetic waves. The type of instrument with a peaked resolution function, on the other hand, is not troubled by blind spots, but creates a large amount of distortion of the plasma waveform. Thus the simultaneous use of these two types of plasma detectors will tell more about the plasma oscillation than just one type alone. A complete description of the plasma motion requires a knowledge of $\mathscr{E}$, Γ, n, and the azimuthal angle ψ as a function of time. The ratio of the currents measured by the two types of detectors is $F(\Gamma, \mathscr{E})/F(\Gamma, \mathscr{E}_0)$ which is independent of n and assumed independent of ψ. A family of permissible detailed plasma motion descriptions could be found if both types of detectors were used simultaneously. That is, for a trial value of $\mathscr{E}$, we could compute a Γ and n which correspond to the measured current.

Not even such a combination of detectors, however, would give enough information to reconstruct the plasma motion uniquely. In a space-probe experiment, if one determines the form of the magnetic component of the magnetohydrodynamic disturbance with a magnetometer, one can compute on the basis of the simple models of the waves discussed in Sections II, III, and IV, what the currents in the plasma detectors should be as a function of time, and then compare with the measured currents. From any disagreement, new models of the structure of the magnetohydrodynamic disturbances could perhaps be constructed.

Two possible methods of obtaining more detailed information on plasma motions present themselves:

1. If three detectors, each with a different resolution function $F(\Gamma, \mathscr{E})$ were used, then unique values of Γ, $\mathscr{E}$, and n could be found. To find ψ also, four detectors would be needed, each with a different "three-dimensional" resolution function $F(\psi, \Gamma, \mathscr{E})$.
2. It would also be possible to use a group of four instruments with identical resolution functions $[F(\psi, \Gamma, \mathscr{E}) \neq \text{constant}]$ but with slightly different orientations, so that ψ and Γ differ from one instrument to the next. Two or three instruments whose view angles sweep across space as a function of time would also serve in the place of four or more fixed instruments if the frequency of the waves were much less than the view angle sweep frequency.

VII. CONCLUSIONS

For a complete description of the hydromagnetic structure of any disturbance it is necessary to know the behavior of **B**, **E**, **J**, **V**, ρ, p, and T. Magnetometers have already successfully detected oscillations of **B** in space (Refs. 4 and 5). In this document, we have pointed to the possibility of determining **V** and ρ by simultaneously monitoring each member of a group of at least four plasma detectors of conventional design. The limitations of currently planned plasma detection space-probe experiments can be deduced from the discussion of Sections V and VI.

ACKNOWLEDGEMENT

The author would like to thank Prof. H. Alfvén for several helpful and enlightening discussions on the expected form of hydromagnetic waves in space and the problems involved in wave detection.

NOMENCLATURE

a radius of curvature of inner plate of a curved-plate electrostatic analyzer

b radius of curvature of outer plate of a curved-plate electrostatic analyzer

$\mathbf{B}$ total magnetic field vector $= \mathbf{B}_0 + \mathbf{B}_1$

$\mathbf{B}_0$ magnetic field when no oscillations are present

$\mathbf{B}_1$ disturbance magnetic field $= B_{10} e^{i(\kappa z - \omega t)}$ for wave propagation in the z direction

e 1) electronic charge
2) base of natural logarithm system

$\mathbf{E}$ electric field vector

$\mathscr{E}$ kinetic energy of plasma particles

$\mathscr{E}_0$ kinetic energy of plasma particles when no oscillations are present

$\mathscr{E}_c$ kinetic energy at peak sensitivity of curved-plate electrostatic analyzers

F resolution function, or fraction of an incident plane-parallel, monoenergetic beam of particles which passes through the analysis part of a plasma instrument to the detector or current-measuring device

i $\sqrt{-1}$

I current measured by a plasma detector

I_0 current measured by a plasma detector when no oscillations are present

$\mathbf{J}$ current density in plasma

k Boltzmann constant

m_i mass of positive ions (assumed to be protons)

n number of ions or electrons per cm^3

n_0 number of ions or electrons per cm^3 when no oscillations are present

$\hat{n}$ unit vector in direction of propagation of wave

p total gas pressure of plasma $= p_0 + p_1$

p_0 total gas pressure of plasma when no oscillations are present

NOMENCLATURE (Cont'd)

p_1 perturbation of plasma pressure due to presence of waves; $p_1 = p_{10}\, e^{i(\kappa z - \omega t)}$ for a wave propagating in the z direction

t time

T_e electron temperature

T_i ion temperature

v ratio of V_1 to V_0

$\mathbf{V}$ fluid velocity $= \mathbf{V}_0 + \mathbf{V}_1$

$\mathbf{V}_0$ fluid velocity when no oscillations are present

$\mathbf{V}_1$ perturbation of fluid velocity due to waves; $V_1 = V_{10}\, e^{i(\kappa z - \omega t)}$ for wave propagation in the z direction

V_ϕ phase velocity of wave

$\hat{x}, \hat{y}, \hat{z}$ unit coordinate vectors

γ_e ratio of specific heats for electrons

γ_i ratio of specific heats for positive ions

Γ polar angle of incidence of plasma particle on detector

η electrical resistivity, assumed $= 0$

κ wave number, $2\pi/\lambda$

ρ plasma density $= \rho_0 + \rho_1 \approx m_i n$

ρ_0 density of unperturbed plasma

ρ_1 density perturbation $= \rho_{10}\, e^{i(\kappa z - \omega t)}$ for propagation of wave in z direction

θ angle between $\mathbf{V}_0$ (unperturbed plasma velocity) and $\hat{n}$ (direction of propagation of wave)

Φ potential difference across electrodes of a curved-plate electrostatic analyzer

ψ azimuthal angle of incidence of plasma particle on detector

ω angular frequency of wave

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