LANDAU DAMPING AND DEBYE SHIELDING *

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ABSTRACT

A physical explanation in terms of competition between ion bunching and electron shielding is given for the dependence of the damping of longitudinal ion waves in a collisionless plasma on the electron-ion temperature ratio. The Debye shielding of an externally induced electrostatic perturbation of form $\exp(ikx - i\omega_0 t)$ in a plasma is derived and discussed. The usual procedure of discussing the energetics of Landau damping in terms of only the damped part of the distribution function f is justified.

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I. Introduction

The Landau damping of longitudinal electron oscillations at long wavelengths, when the damping is small, is rather well understood as a resonance between a wave propagating on the main body of the plasma and a small family of particles traveling at nearly the phase velocity of the wave.[1-4] Landau damping in the regions of greater damping (large k, k = wave number) is not physically explained by this resonance picture. This may be seen from the dependence of the resonant-damping formula[2] on $f_o'(u)$, the derivative of the initial velocity distribution evaluated at the phase velocity $u = \omega/k$ of the wave:

$$Im(\omega) = \frac{1}{2}\pi u_r \omega_p^2 |k|^{-1} f_o'(u_r)\left(1 - u_r^{-1} d\omega_r/dk\right) \quad (1)$$

$$\omega_r = Re(\omega), \quad u_r = Re(u)$$

For Maxwellian f_o this formula gives damping tending to zero with u , while in fact the damping is greatest when Re(u) is least. The divergence is most clearly seen in Fig. 1 of reference 5. Superficially, the situation for ion waves seems better understood, but in fact inconsistency is present in the explanation given by Fried and Gould.[5] The difficulty is in understanding why electron Landau damping, which should be <u>large</u> at large k and small u_r fails to bother the ion waves much if $T_i << T_e$. It is not adequate to state[5] that electron Landau damping of the ion waves then becomes small because $f_o'(u_r)$ is small, for that explanation would imply that the damping is also small for electron waves, a false conclusion. The present paper presents an explanation of why electron waves damp heavily at low phase velocity and yet why under some circumstances the ion waves may persist hardly damped. The approach has much more generality, however.

Consider first an electron plasma with fixed positive background. The initial electron distribution will be taken as the Maxwellian

$$f_0(v) = a^{-1}\pi^{-\frac{1}{2}} \exp(-v^2/a^2) \tag{2}$$

and later when ions are included their initial distribution $F_o(v)$ be assumed similar, with a replaced by the ion thermal velocity A. As usual, the problem will be linearized and will be made one-dimensional by integration over directions orthogonal to $\underset{\sim}{k}$, and a single Fourier component proportional to exp (ikx) will be used:

$$E = (2\pi)^{-1}\int_{-\infty}^{\infty} E_k(t)\, e^{ikx}\, dk$$

$$f(x,v,t) = n_0 f_0(v) + (2\pi)^{-1}\int_{-\infty}^{\infty} f_k(v,t)\, e^{ikx}\, dk \tag{3}$$

The correlationless kinetic equation[6] now reads

$$\frac{\partial f_k}{\partial t} + ikv f_k - \frac{e}{m} E_k n_0 \frac{\partial f_0}{\partial v} = 0 \tag{4}$$

(For ions, let e $\rightarrow$ -e, m $\rightarrow$ M, f $\rightarrow$ F). Terms involving powers of E_k or f_k higher than the first are ignored. The most rigorous discussion[3] in the literature of the physical mechanism of Landau damping utilizes the assumption of a time-harmonic dependence of f_k, leading to

$$f_k = \frac{-ie}{m} \frac{E_k(0)}{k(v-u)} \frac{n_0 \partial f_0}{\partial v} e^{-i\omega t} \tag{5}$$

This procedure has well-known dangers; for example, the solution of the initial value problem[7] shows that f_k does not have the above form, even for small or large t. When (5) is substituted in Gauss' law

$$i k E_k = -4\pi e \int f_k \, dv \equiv 4\pi \rho_k \tag{6}$$

a pole is present in the integral. If the Landau prescription[2,7] is used to define the manner of going around this pole, the resulting dispersion relation nevertheless gives a correct description of the damping at long times. It will further be shown in Section III that the energetics may properly be discussed in terms of Eqs. (3) - (6). This justifies Wu's assumption[3] of (5) and permits much of the discussion here to be conducted in terms of the dispersion relation (see Section II). In Section III the shielding mechanism introduced in Section II is put on a firm basis by a rigorous treatment of the initial value problem. As side results, the dynamic form of Debye shielding is derived (this was done for a test particle problem in reference 6) and it is shown that the undamped part of the distribution function[7] has no effect on the energetics. The ultimate motivation is to provide an intuitive picture that can be used to interpret and predict the amount of damping in a wide variety of examples.

II. Analysis in Terms of the Dispersion Relation

The dispersion relation

$$k^2/\omega_p^2 = W(u) = \int_C (v-u)^{-1} f_o'(v)\,dv \tag{7}$$

is obtained by substituting (5) in (6) with the Landau prescription that the contour C must pass below the pole at u. f_o' will be used to designate the derivative of f_o. If (2) is used for f_o, then (7) takes the form[5]

$$k^2/k_D^2 = \tfrac{1}{2} Z'(u/a) \tag{8}$$

where $k_D^2 = 2\omega_p^2/a^2$ and $Z(\zeta) = 2i\int_{-\infty}^{i\zeta} \exp(-t^2)\,dt$ (9)

Since it determines the damping, the dispersion relation should give a clue as to the damping mechanism. The well known resonant damping at long wavelength[1-4] results from the imaginary part of (7) or (8) and needs no further discussion. But at very small Re(u) the salient feature of (8) is that in the region near the origin of the u plane, where one would look for slightly damped waves, W is large and negative. This may be seen from Eq. (8) and the properties[8] of $Z(\zeta)$, especially the value $Z'(0) = -2$. Thus one is forced down to large negative values of Im(u) to find any waves, which exist only where W is real and positive. The negative value of W at the origin may be given a simple physical interpretation as follows:

If an electric field $E = E_k \exp(ikx - i\omega t)$ were present in the plasma, it would produce a perturbation in the distribution of form (5). For values of and k satisfying the dispersion relation, this would result in the correct charge density to satisfy Gauss' law (6), but when W is negative, the resulting charge density is of the wrong sign to fulfill (6). If an additional charge density $\rho_e = -2\rho_k$ were inserted into the plasma by some means, it could drive the oscillation. The plasma charge density ρ_k would shield out half the driving field, leaving the remainder to drive the oscillation. Clearly, the "externally introduced" charge density ρ_e could stand for the ion charge density in a slightly damped wave propagating on the ions. Depending on the details, the electron shielding will either be enough to damp the ion wave heavily or not. This will be discussed further in Section III. The central point here is that the large negative values of W near $u = 0$ correspond to shielding, not resonant damping. No single narrow band of particles in f_o can be isolated and identified as producing the damping. For nonzero frequencies, the shielding must become a dynamic one, leading or lagging the imposed field by some amount. If the lead or lag is nearly 180^o, the shielding becomes a regeneration instead and the wave can propagate with little damping. (Shielding in phase is defined as plasma charge density out of phase with the impressed charge density.)

This kind of damping, due to the impossibility of satisfying the real part (the k^2 part) of the dispersion relation, contrasts strongly with the resonant damping, where the real part is satisfied in a large neighborhood of the solution and the imaginary part, due to resonant particles, determines how far down below the real u axis one must look for the waves. On other physical grounds, it is reasonable that shielding would predominate in the regime of large k . In such a regime, there is plenty of time for static or nearly static Debye shielding to occur, since particles do not have to go far (λ small). As one moves away from this situation, the particles have to go farther to do the shielding. Thus they pick up too much momentum and overshoot their mark, leading to oscillations. By the time one has moved out to the slightly damped region, shielding is nearly impossible, and only a few resonant particles can interfere with the wave.

There is a general method for determining when no positive k^2 values are possible near the real u axis, which seems to correspond to large shielding-type damping. Mathematically, one can see from the little used form[9] of the dispersion relation

$$k^2/\omega_p^2 = -\int_{-\infty}^{\infty} f_o''(v)\, \ln(v-u)\, dv \equiv W(u) \quad (10)$$

that for u nearly real W can be positive only when the phase velocity is in a region of large positive f_o''. This is due to the peak in $-\ln(v-u)$ at $v = u$. Regions of f_o far distant tend to contribute little to k^2 because the logarithm is small and slowly varying and regions with opposite signs for f_o'' tend to cancel. The imaginary part of the integral must also be zero, of course, for oscillations to exist. This is a resonant-particle condition.[1-4] Here we must elucidate the connection between f_o'' and the shielding mechanism, a non-resonant mechanism. A good starting point is the fact that very large values of f_o'' actually lead to instability[9]. The instability is a form of the two stream instability, since it involves an excess of particles moving both faster and slower than the wave. The two-stream instability is well-understood as a bunching process[2], similar to that in klystrons. Hence it could be said that a large positive value of $f_o''(u)$ produces bunching in the right phase to maintain a steady or even a growing oscillation, while $f_o'(u)$ saps energy by a particle wave resonance, and a large negative $f_o''(u)$ represents a deficiency of the two streams on either side of the phase velocity, so that bunching cannot occur and shielding predominates. Since instability can never set in unless f_o' reverses sign due to large positive values of f_o'' (i.e. f_o has a minimum[9,10]), one must conclude that resonant damping overcomes bunching in all cases where f_o has no minimum. The various effects are illustrated in Fig. 1. Cases a, b, and c illustrate forms of f_o leading to bunching, resonant damping, and shielding, respectively. Case d, in which the distribution becomes nearly flat at the wave velocity, may be resolved as shown symbolically into a combination of resonant damping and bunching.

The arguments concerning ion waves depend crucially on the assertion that the nearly undamped behavior found for plasma oscillations at very long λ in a one species plasma result from a combination of bunching with very small resonant damping. Therefore this matter will be pursued further now. It rests on the observation that some bunching is necessary for a plasma oscillation to persist at all. Clearly this must be so, as only bunching of the particles can result in an accumulation of charge at the places necessary to provide the required E_k. If one examines the slightly damped high velocity plasma oscillations in the wave frame of reference, one sees that the central part of the Maxwellian distribution is a fairly monoergetic beam (since $u \gg a$) passing through a sinusoidal potential. Hence it will suffer bunching. Of course, this does not lead to instability unless there is a second stream going the other way to provide proper feedback. If the bunching argument is correct, however, it should be possible to work out the positive values of W at large, nearly real u from this model. This is easily done. First we note from Eqs. (5) - (7) that an electric field $E_k = -ik\phi_k$ in the plasma produces a charge density

$$\rho_k = e^2 n_0 \phi_k W/m \tag{11}$$

On the other hand, when $u \gg a$, one may regard the problem from the <u>wave</u> frame as one of a nearly monoergetic beam of particles with mean streaming velocity u passing through a static sinusoidal potential $\phi = \phi_k e^{ikx}$. The equation of continuity for the stream gives $nv = n_o v_o$, where v_o is identified with u, and energy conservation gives $\frac{1}{2}mv^2 - e\phi = \frac{1}{2}mv_o^2$. If $\delta v = v - v_o$ and $\delta n = n - n_o$, then to first order in the perturbations $n_o \delta v + v_o \delta n = 0$ and $mv_o \delta v = e\phi$. Thus $\delta n = -n_o e\phi/mv_o^2$, and the charge density $-e\,\delta n$ found from the bunching analysis is

$$\rho_k' = n_0 e^2 \phi_k / m v_0^2 \tag{12}$$

Identifying v_o with u and ρ_k' with ρ_k for large u, one sees what W must have the asymptotic form $W \sim 1/u^2$. But this is exactly what has been proven[9] for W

under very general assumptions about f_o. It will be noticed that for very large u, f_o'' is very small and one cannot associate the value of W locally with that of f_o''. This is due to the slow variation of the logarithm in (10).

If the plasma is made up of many streams and the plasma frequency for each stream s is ω_s, while its velocity is V_s, then the bunching of each stream is represented in W by an additive term

$$W_s = \omega_s^2 \omega_p^{-2} (V_s - u)^{-2} \tag{13}$$

where $\omega_p^2 = \sum \omega_s^2$. If there is a portion of f_o that cannot be described in terms of well defined streams, but that possesses large positive values of f_o'' in a small region of width Δv centered on u, with smooth structure elsewhere, its contribution W_f to W may be estimated as

$$W_f = \langle f_o'' \rangle \Delta v \tag{14}$$

where $\langle f_o'' \rangle$ is the mean of f_o'' over the width of the region Δv where it is large. This may be seen by the fact that if f_o'' is fairly constant in a region of width Δv, centered on u one may estimate $\int f_o''(v) \ln|v-u| \, dv$ over that region by

$$\langle f_o'' \rangle \int_{u-\Delta v/2}^{u+\Delta v/2} \ln|v-u| \, dv = 2 \langle f_o'' \rangle \int_0^{\Delta v/2} \ln x \, dx$$

$$= -\langle f_o'' \rangle \Delta v$$

The ion wave damping found by Fried and Gould can now be better understood. For the case of small damping, they obtain the result (their Eq. (33))

$$k^2/k_D^2 \approx -1 - i\pi^{1/2}\rho + \delta/2\rho^2 \qquad (15)$$

where $\rho = u/a \ll 1$ and $\delta = m/M$

The term -1 represents a large electron shielding contribution, the second term is from electron resonant damping (which is much smaller, since $\rho \ll 1$), and the third is ion bunching. Since electron shielding is the main cause of damping, ion bunching must be sufficient to overcome it, which means one needs $\delta/2\rho^2 > 1$. However, one must have $\rho > BA/a$ where B is a constant equal to or exceeding about 2.5 for ion resonant damping to be small. This is just the condition that the wave velocity be several times the ion thermal velocity. With the definition $T_e/T_i = \Theta = \delta a^2/A^2$, the condition for small ion resonant damping is $\rho^2 > B^2\delta/\Theta$, where again $B \approx 2.5$. Then the condition for ion bunching to overcome electron shielding becomes $\Theta/2B^2 > 1$, or roughly $\Theta > 10$. This is precisely where the waves are found to be slightly damped. The third term in (15) may be estimated from (13). The electron shielding term may be found from (9) and the argument showing that in the center of the electron distribution $\rho_k = -2\rho_e$, where ρ_e is the source field impressed on the electrons. A more careful calculation is done in the next section, however.

If the wave is taken to propagate at only about the ion mean thermal speed

($u^2\Theta = a^2 \delta$), the ratio of the ion bunching term as estimated from (14) to the electron shielding term is Θ / e, where $e \approx 2.72$. Thus if $\Theta \gtrsim 3$ the ion bunching predominates, but no ion resonant damping is large. If, on the other hand, u is taken to be comparable to a, electron resonant damping becomes large, as discussed by Fried and Gould. At very large velocities, $u \gg a$, Eq. (14) shows that both ion and electron bunching should help the wave propagate, but here the large electron plasma frequency makes the electron contribution much larger than the ion one, and the oscillations are essentially electron oscillations.

III. The Initial Value Problem for Forced Plasma Oscillations

Consider an electron plasma, with fixed ions, described by Eqs. (3), (4), and (6), but subject to an additional imposed electric field $E_o \exp(ikx - i\omega_o t)$. Clearly, only perturbations of the one fixed wave number k need be considered. In the sequel, the subscript k will be suppressed. E_p will denote the electric field due to the plasma as per (6) and E_e will denote $E_o \exp(-i\omega_o t)$. The Laplace transform

$$\mathcal{L}[A(t)] = \int_0^\infty e^{-pt} A(t)\, dt$$

will be denoted by the same symbol as the original quantity; where confusion might result the argument will be given as either t (for the untransformed quantity) or p (for the transformed one). $f(v, t = 0)$ will be denoted $g(v)$. Taking the Laplace transform of (4) one gets

$$m(p+ikv)f - en_0 E_p(\partial f_0/\partial v) - en_0 E_e(\partial f_0/\partial v) = mg(v) \quad (16)$$

So that

$$f = \frac{1}{p+ikv}\left[\frac{en_0 E_p}{m} f_0' + \frac{en_0 E_e}{m} f_0' + g\right] \quad (17)$$

Substitution of (17) into Gauss' law yields, after some manipulation

$$E_p(p) = \left\{ik\int(p+ikv)^{-1}\left[\omega_p^2 E_e f_0'(v) + 4\pi e g(v)\right]dv\right\} \Big/ D(u,k) \quad (18)$$

where $D(u,k) = k^2 - \omega_p^2 \int (v-u)^{-1} f_0'(v)\,dv \quad (19)$

and $u \equiv ip/k$

The contours of integration in (18) all must pass[7] under u. The perturbation $f(v,p)$ of the velocity distribution may be found from (17), once E_p is found from E_e and g by (18). Both f and E_p contain distinct contributions from E_e and g. Only the relation of E_p to E_e enters the shielding discussion, but first both E_e and g will be kept, as it is desired to justify using the dispersion relation and Eq. (5) to discuss energetics as was done in Section II. When the Laplace transform is inverted, $E_p(t)$ contains contributions from the poles of $E_e(p)$, from the poles of $\int (p + ikv)^{-1} g(v)\, dv$, and from the zeros of the denominator in

(18). Denavit has shown[11] that Landau damping is obtained if the Fourier transform $H(q)$ in velocity space of $g(v)$ falls off at large q faster than $\exp(-\lambda_D \gamma |q|)$ where $-\gamma$ is the imaginary part of the least damped root of D as a function of u. This assumption will be made, which then allows neglect of any contribution of $\int (p + ikv)^{-1} g(v)\, dv$ to $E_p(t)$ other than through its appearance in the residue of (18) at the zeros of D. Thus only the poles of $E_e(p)$ and the zeros of $D(p,k)$ will contribute to $E_p(t)$. One must also remember that the contour of the integrals over v is explicitly deformed so that in inverting the Laplace transforms no singularity is encountered from the factor $(p + ikv)^{-1}$. This can be done only when that factor is inside an integration, of course, so that when f as found from (17) is Laplace-inverted, one cannot avoid terms from the singularity at $p = -ikv$.

By hypothesis, $E_e(p) = E_o / (p + i\omega_o)$. Thus $E(t)$ contains a contribution from the residue at $p = -i\omega_o$. There would also be contributions from E_e due to its residue at the zeros of D. These represent an effect of E_e on the naturally damped oscillations of the undriven plasma, and would damp at the plasma's free Landau damping rate. It does not seem to be of much interest to consider driving a plasma with a forcing field E_e which is more highly damped than the free oscillations. Therefore it will be assumed that $0 \geq \mathrm{Im}(\omega_o) \geq -\gamma$. Then the forced oscillations at $\omega = \omega_0$ will dominate at long times, and we obtain

$$E_p \sim E_0 e^{-i\omega_0 t} I(u_0)/D(u_0, k) \qquad (20)$$

where

$$I(u) = \omega_p^2 \int (v-u)^{-1} f_0'(v)\, dv \tag{21}$$

and $u_o = \omega_o/k$

When I is real and negative for some u one can recover the case described in Section I where $E_e = -2E_p$ by setting $k^2 = -I$, thus justifying the discussion there which skirted the initial-value problem.

The other method to be justified here is that of the neglect by Wu[3] of the undamped part[7] f_u of f in considering energetics, for the case $E_e = 0$. f_u is found from the residue of (17) at $p = -ikv$, and is

$$f_u = e^{-ikvt}\left[g(v) + h(v)\right] \tag{22}$$

where

$$h(v) = \left[\omega_p^2 f_0'(v) \int (v'-v)^{-1} g(v')\, dv'\right] / D(v,k) \tag{23}$$

The general formula for average transfer of energy to the particles from the wave is[3]

$$-\Delta A = -e \left\langle \int E\, v f(v)\, dv \right\rangle_{av} \tag{24}$$

The contribution of f_u to ΔA is

$$(-\Delta A)_u = -e \left\langle \int E_p(t)\, v[g(v)+h(v)]\, e^{-ikvt}\, dv \right\rangle_{av} \tag{25}$$

However, at large times E_p is of the form $E_p = E_1 \exp(-\gamma t)$, so $(-\Delta A)_u$ takes the form

$$(-\Delta A)_u = -eE_1 \left\langle \int v[g(v)+h(v)]\, e^{-ikvt-\gamma t}\, dv \right\rangle_{av}$$

But any physically reasonable $f_o'(v)$ obeys the same conditions of Denavit[11] on its Fourier transform that were assumed for g, and this causes h to have the same asymptotic behavior. Thus $S(t) \equiv \int [g(v) + h(v)]\, e^{-ikvt}\, dv$ must fall off at large t faster than $e^{-\gamma t}$. But $\int v[g(v) + h(v)]\, e^{-ikvt} dv = ik^{-1}\,(dS/dt)$ which must therefore also drop off faster than $e^{-\gamma t}$. Thus $(\Delta A)_u$ falls off faster than $e^{-2\gamma t}$ at large t, and is negligible compared to the energy transfer from the portions of f that are Landau damped.

Finally, it is worthwhile to consider some special cases of Eq. (20), which gives the shielding of any sinusoidal perturbation in the plasma that is steady or is damped at a rate less than the free Landau damping rates. The case I real and negative has been discussed only if $k^2 = -I$, which results in 50 percent shielding of E_e. If u_o is kept fixed and k^2 made very large, $D \rightarrow k^2$ and the shielding disappears like $1/k^2$, a strange result at first sight. However, if $k \rightarrow \infty$ while u_o is constant, $\omega_o \rightarrow \infty$, in which case it is quite reasonable that the finite inertia of the particles prevents them from following the oscillation and shielding it.

To do more interesting cases, one must allow u_o to vary, which means that the form of $I(u_o)$ must be known. If f_o is any reasonable distribution[9], however, $I(u_o)$ is asymptotic to ω_p^2/u_o^2 at large u_o, so $E_p \sim E_e\,\omega_p^2/(\omega_o^2-\omega_p^2)$, where the limit ($u_o \rightarrow \infty$, $k \rightarrow 0$, ω_o = constant) has been taken. If ω_o is very small, the shielding is perfect, while if it is large enough the field can actually be enhanced by resonance of the plasma. To do the limiting case $\omega_o \rightarrow 0$, k^2 fixed, one must find the value of $I(0)$. For distribution (2) this is $-k_D^2$, leading to $-E_p = E_e k_D^2/(k_D^2 + k^2)$. Thus if k/k_D is small, shielding is nearly perfect, while if it is large, there is hardly any shielding. This behavior is exactly the opposite of what has been said earlier about Debye shielding being large at short wavelength. One must remember, however, that one has here the static limit, and that E_e is a sinusoidal function of x. Each crest in E_e tends to get shielded out by the plasma in a distance λ_D, but if the crests are too close, the shielding is not complete by the time the next one is reached. If k is very small, however, complete shielding can occur before the next crest intervenes. This static behavior contrasts strongly with the dynamic behavior, near ω_p, in which shielding is largest at large k.

In the case that E_e is interpreted as an ion wave in the electron plasma, it can be verified that shielding is rather good even in the slightly damped waves. But if T_i/T_e is made small enough, the arguments in connection with Eq. (15) show that there is enough unshielded ion charge to allow bunching.

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Figure Captions

Fig. 1. Initial Velocity Distributions for bunching (a), resonant damping (b), and shielding-type damping (c). Part (d) shows how a point of inflection in f_o may be regarded as producing a competition between bunching and damping, if f_o is split into parts $f_o^{(2)}$ and $f_o^{(3)}$.

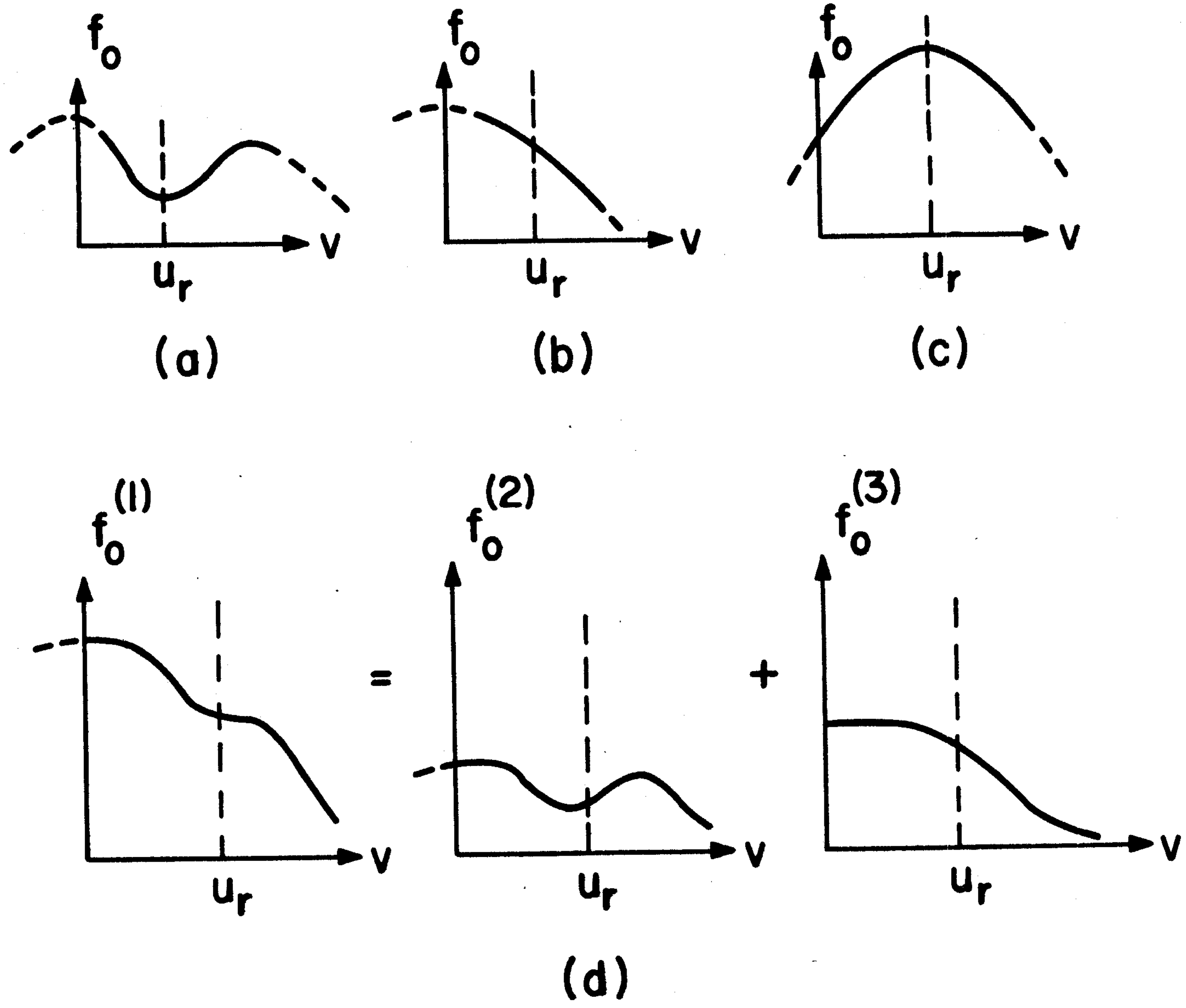

Fig. 1