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A TOPOLOGICAL APPROACH TO THE PROBLEM OF SEARCHING ON A CONTOUR MAP

Stephen P. Morse
Research Assistant

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NEW YORK UNIVERSITY
SCHOOL OF ENGINEERING AND SCIENCE
DEPARTMENT OF ELECTRICAL ENGINEERING
Laboratory for Electroscience Research

University Heights
Bronx, New York 10453

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ABSTRACT

This report is concerned with the problem of obtaining the ground track of an aircraft, given the elevation below each point of the aircraft during flight and a contour map of the terrain. Problems of this type cannot be solved in a direct manner. Exhaustive searches are impractical. What is needed are efficient search strategies that can yield a solution in a reasonable time. Searches can be facilitated by taking advantage of the topological properties of the map. This is done by using a sequence of numbers that conveys information about the contour lines intersected, the contour lines traveled along, and the inter-contour regions passed through by the ground track. The sequence can be deduced from the known ground profile. An examination of the map topology presented in graphical form will indicate which areas of the map can contain the ground track and which areas cannot, thereby severely narrowing down the search possibilities.

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I. INTRODUCTION

This report is concerned with locating an aircraft ground track on a contour map. Assume a pilot has just completed a flight during which he recorded his flight path (shape of the path that he flew) and ground profile (ground elevation as a function of distance along the path that he flew) but not his ground track (location and orientation of the flight path on a map). If a contour map of the region over which the pilot flew is available, how can the ground track be obtained from the flight path and ground profile?

There are various situations in which this problem may occur. For example, an aircraft lost in flight may wish to locate its position on the basis of radio altimeter data. Spaceships may wish to use a contour map to navigate over the surface of the moon. Ships may desire to navigate by means of contour maps of the ocean floor. This last application is currently being investigated by General Instrument Corporation.[1]

Several variations of the problem exist. In general, the known flight path will need to be translated and rotated over the map to obtain the correct ground track. However, a pilot flying over the Earth will probably know his compass heading and, therefore, only translation of the flight path will be necessary to obtain the ground track. If the flight path is a straight line, certain simplifications may be possible.

Problems of this nature will generally entail large amounts of data and data processing. Solutions will be time consuming and prone to error if carried out manually. The requirement for accuracy and speed of solution suggests the application of digital computers to this problem.

Problems of this type cannot be solved in a direct manner. Thus, some type of searching will be necessary. However, an exhaustive search must be avoided because the amount of time required for such a search would be prohibitive. An efficient search technique, based on the topological properties of the contour map, is the topic of this report.

Some of the terminology to be used here was introduced previously[2,3,4] and will now be summarized. A positive contour line of value e is a line that separates points of elevations greater than e from points of elevations less than or equal to e. Similarly, a negative contour line of value e is a line that separates points of elevations less than e from points of elevations greater than or equal to e. A contour map consists of a collection of all the contour lines of certain specified values. The contour lines are directed so that the points of higher elevation are on the left of the contour lines. The contour lines usually found on a contour map consist of unions of positive and negative contour lines; however, the positive and negative contour lines may separate as at a plain. Even where a union of a positive and negative contour line exists, the positive contour line is considered as being on the left of the negative contour line. It was shown[4] that on a closed surface, such as the surface of the Earth, all positive and negative contour lines are closed curves that do not cross each other. In any specific region of the Earth it is possible to force all the positive and negative contour lines to be closed curves by considering the rest of the Earth to be at a constant elevation.

On a given contour map, consider a contour line that is contained in the interior of a second contour line but is not contained in the interior of any other contour line contained in the interior of this second contour line. The first contour line is called the interior contour line of the second, and the second contour line is called the exterior contour line of the first. A contour line that has no exterior contour line on a given map is called an absolute exterior contour line; a contour line that has no interior contour line on a given map is called an absolute interior contour line.

II. THE GROUND TRACK SEQUENCE

The search for the ground track can be narrowed down by taking advantage of the topological properties of the contour map. Given the ground profile, it is possible to write a sequence of numbers, called a track sequence, which indicates the contour lines intersected, the contour lines traveled along, and the intercontour regions (domains on the contour map that are bounded by contour lines but contain no contour lines) passed through by the ground track. The two major steps for narrowing down the search are:

1. Obtain the track sequence from the given ground profile.
2. From the map topology obtain the loci of all tracks that will have this track sequence. (A track is any line drawn on the contour map and having the shape of the flight path. The ground track is a particular track.)

A. Definition of the Track Sequence

From the ground profile can be obtained an ordered sequence of single numbers that correspond to the values of the contour lines intersected by the ground track and pairs of numbers that correspond to the elevation ranges of the intercontour regions passed through or the values of the contour lines traveled along. Such a sequence is called a track sequence. The track sequence can be obtained as follows:

1. Superimpose a series of horizontal lines over the ground profile such that the heights of the horizontal lines are the same as the values of the contour lines drawn on the map.
2. Trace the ground profile from end to end and form a sequence of the numbers or pairs of numbers associated with the horizontal lines encountered in the order in which they are encountered.

Specifically,

a. Enter a single number for the height of each horizontal line arrived at.

b. Enter a single number for the height of each horizontal line departed from.

c. Enter a pair of numbers for the height of each horizontal line traveled along.

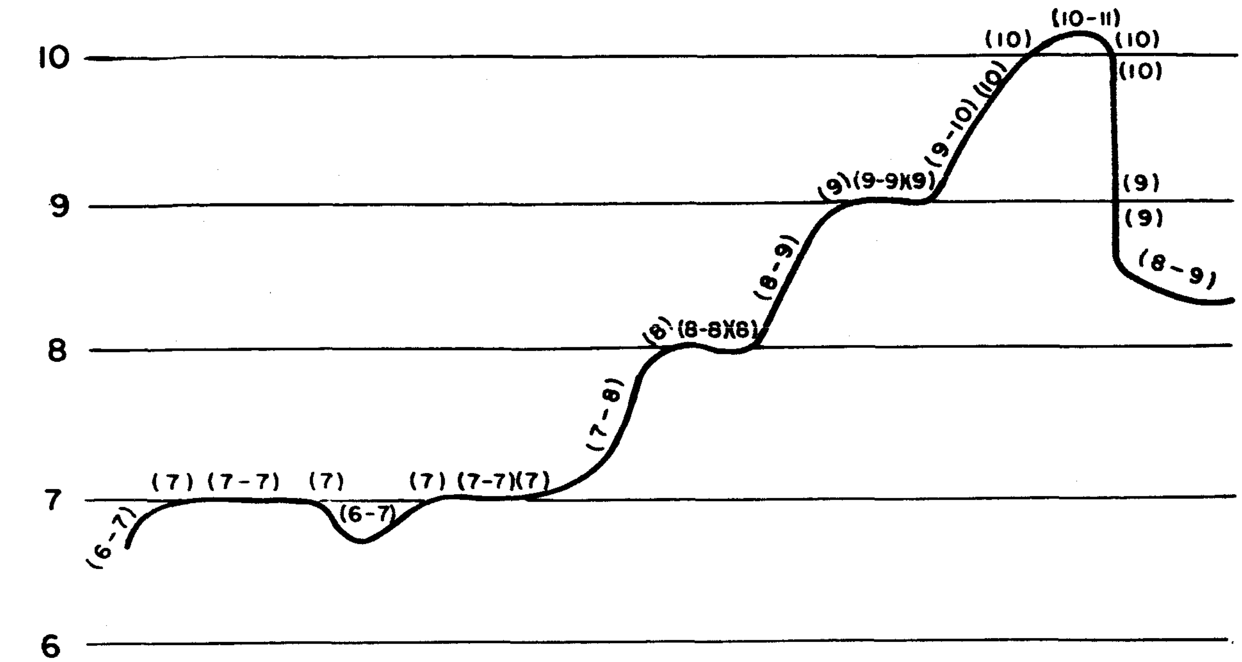
d. Enter a pair of numbers for the heights of each pair of horizontal lines traveled between, provided the horizontal plane projection of the portion of the ground profile between the horizontal lines has a nonzero length. (A horizontal plane projection of zero length corresponds to the ground track crossing a cliff.)

An example of a ground profile and its associated track sequence are shown in Figure 1.

B. <u>Track Sequence and Ground Track</u>

Suppose the ground track were known. If one were then to trace the ground track from end to end and to form a sequence of the numbers or pairs of numbers associated with the contour lines and intercontour regions encountered in the order in which they are encountered, that sequence would bear a one-to-one correspondence to the track sequence. Specifically,

1. Enter a single number for the value of each positive contour line arrived at from the left.
2. Enter a single number for the value of each positive contour line departed from towards the left.
3. Enter a single number for the value of each negative contour line arrived at from the right.
4. Enter a single number for the value of each negative contour line departed from towards the right.



SEQUENCE	RULE USED (SEE TEXT)
6 – 7	2 d
7	2 a
7 – 7	2 c
7	2 b
6 - 7	2 d
7	2 a
7 - 7	2 c
7	2 b
7 - 8	2 d
8	2 a
8 - 8	2 c
8	2 b
8 - 9	2 d
9	2 a
9 – 9	2 c
9	2 b

SEQUENCE	RULE USED (SEE TEXT)
9 – 10	2 d
10	2 a
10	2 b
10 – 11	2 d
10	2 a
10	2 b
9	2 a
9	2 b
8 – 9	2 d

FIG. 1

EXAMPLE OF GROUND PROFILE AND ASSOCIATED TRACK SEQUENCE

5. Enter a pair of numbers for the value of each contour line traveled along.

6. Enter a pair of numbers for the values of the contour lines bounding each intercontour region passed through.

If two or more pairs of numbers are adjacent in the sequence, the pairs must be identical. Delete all but one of such adjacent pairs. An example of a ground track and the associated track sequence is shown in Figure 2.

SEQUENCE	RULE USED (SEE TEXT)
6 - 7	6
7	3
7 - 7	5
7	4
6 - 7	6
7	3
7 - 7	6
~~7 - 7~~	5
7	2
7 - 8	6
8	3
8 - 8	6
8	2
8 - 9	6
9	3
9	2
9 - 10	6
10	3
10	2
10 - 11	6
10	1
10	4
9	1
9	4
8 - 9	6

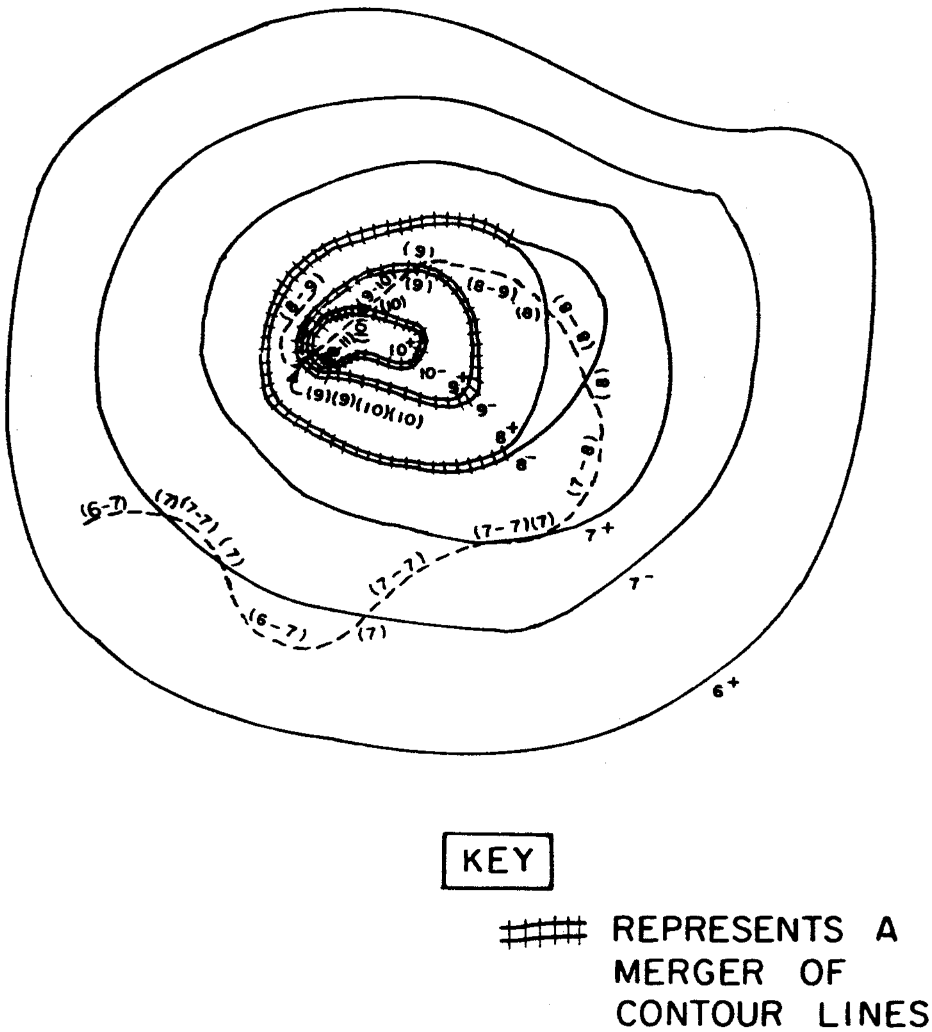


FIG. 2

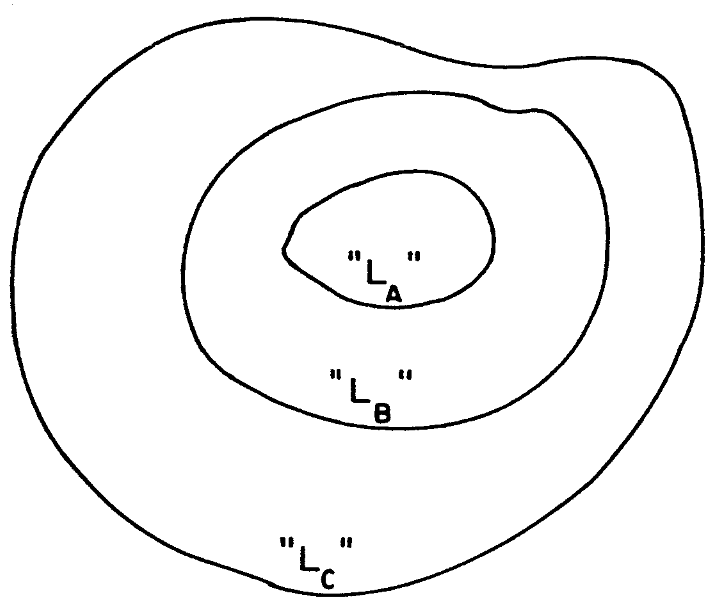
EXAMPLE OF A GROUND TRACK AND THE CORRESPONDING TRACK SEQUENCE

III. GRAPH OF THE MAP TOPOLOGY

A. Map Topology

The map topology is concerned with adjacencies of contour lines. Two contour lines are said to be adjacent if a line can be drawn that connects the two contour lines and intersects no other contour lines. Note that a contour line is adjacent to itself. If two contour lines are adjacent to a third, they need not be adjacent to each other. This is illustrated in Figure 3. However, all the contour lines adjacent to a given contour line form two sets such that all the contour lines in each set are adjacent to all other contour lines in that set. The intersection of the two sets is a set consisting of the given contour line only. The two sets are obviously the set of all contour lines adjacent to the original contour line on its left side (including the original contour line), and the set of all contour lines adjacent to the original contour line on its right side (including the original contour line). These sets are called the left adjacency set, LAS, and the right adjacency set, RAS, of the original contour line. Notice that each adjacency set corresponds to an intercontour region and vice-versa.

If the original contour line is a positive contour line of value e_1, and e_2 is the next higher value of contour line on the map, then the LAS is composed entirely of positive contour lines of value e_1 and negative contour lines of value e_2, and the RAS is composed entirely of positive and negative contour lines of value e_1. Similarly, if the original contour line is a negative contour line of value e_1, and e_0 is



CONTOUR LINE L_A IS ADJACENT TO CONTOUR LINE L_B

CONTOUR LINE L_C IS ADJACENT TO CONTOUR LINE L_B

BUT

CONTOUR LINE L_A IS NOT ADJACENT TO CONTOUR LINE L_C

FIG. 3

EXAMPLE OF ADJACENCIES

the next lower value of contour line on the map, then the LAS is composed entirely of positive and negative contour lines of value e_1, and the RAS is composed entirely of positive contour lines of value e_0 and negative contour lines of value e_1. The LAS of a positive contour line and the RAS of a negative contour line can be composed of contour lines of more than one value, hence such adjacency sets are called heterogeneous. The RAS of a positive contour line and the LAS of a negative contour line must be composed of contour lines of the same value, hence such adjacency sets are called homogeneous. Notice that if an adjacency set of a given contour line is heterogeneous, the other adjacency set of that contour line is homogeneous and vice-versa.

Without knowing whether adjacency sets are LAS or RAS, the adjacency sets can be classified as heterogeneous or homogeneous as follows:

1. Select any adjacency set on the map that contains contour lines of more than one value. Label this set as heterogeneous.
2. Label the other adjacency set of each contour line in this heterogeneous set as homogeneous.
3. Label the other adjacency set of each contour line in these homogeneous sets as heterogeneous.
4. Continue until every adjacency set is labeled.

B. Graph of a Contour Map

A graph of a contour map is a structure that displays the topology of the map. The graph consists of nodes representing contour lines connected by junctors representing adjacency sets. Each junctor connects the nodes representing the contour lines of the adjacency set.

The value and polarity (positive or negative) of a node are the value and polarity of the contour line corresponding to the node and are written alongside the node. Each junctor consists of a circle from which arms emanate, each arm terminating at a node. Clearly every node is connected to two different junctors, the two junctors representing the LAS and RAS of the contour line. A junctor is heterogeneous or homogeneous if its associated adjacency set is heterogeneous or homogeneous. Each junctor is assigned a label, written inside the junctor, according to the following rules:

1. If the junctor is homogeneous and connects to at least one node of value e_1, the junctor is labeled e_1-e_1.

2. If the junctor is heterogeneous and connects to at least one positive node of value e_1, the junctor is labeled e_1-e_2, where e_2 is the next higher value of contour line on the map.

3. If the junctor is heterogeneous and connects to at least one negative node of value e_1, the junctor is labeled e_0-e_1, where e_0 is the next lower value of contour line on the map.

The graph of a contour map is shown in Figure 4.

For identification, each node is labeled with an upper-case letter, each junctor with a lower-case letter, and each arm of a junctor with a number.

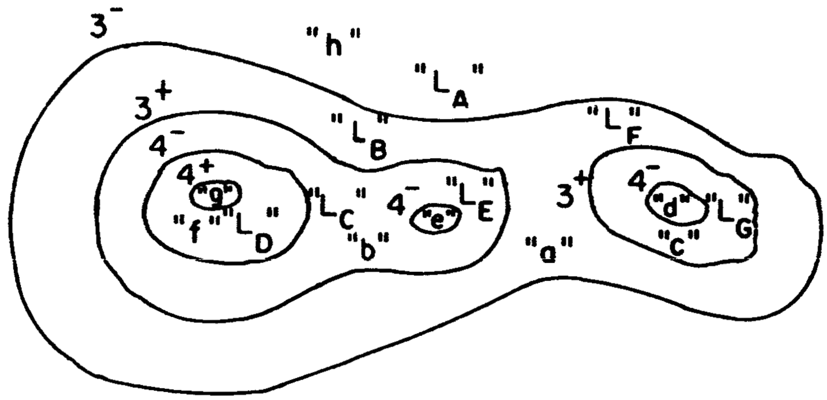
Two adjacent contour lines must do one of the following:

N - <u>N</u>ot touch for any part of their lengths.

T - <u>T</u>ouch for part but not all of their lengths.

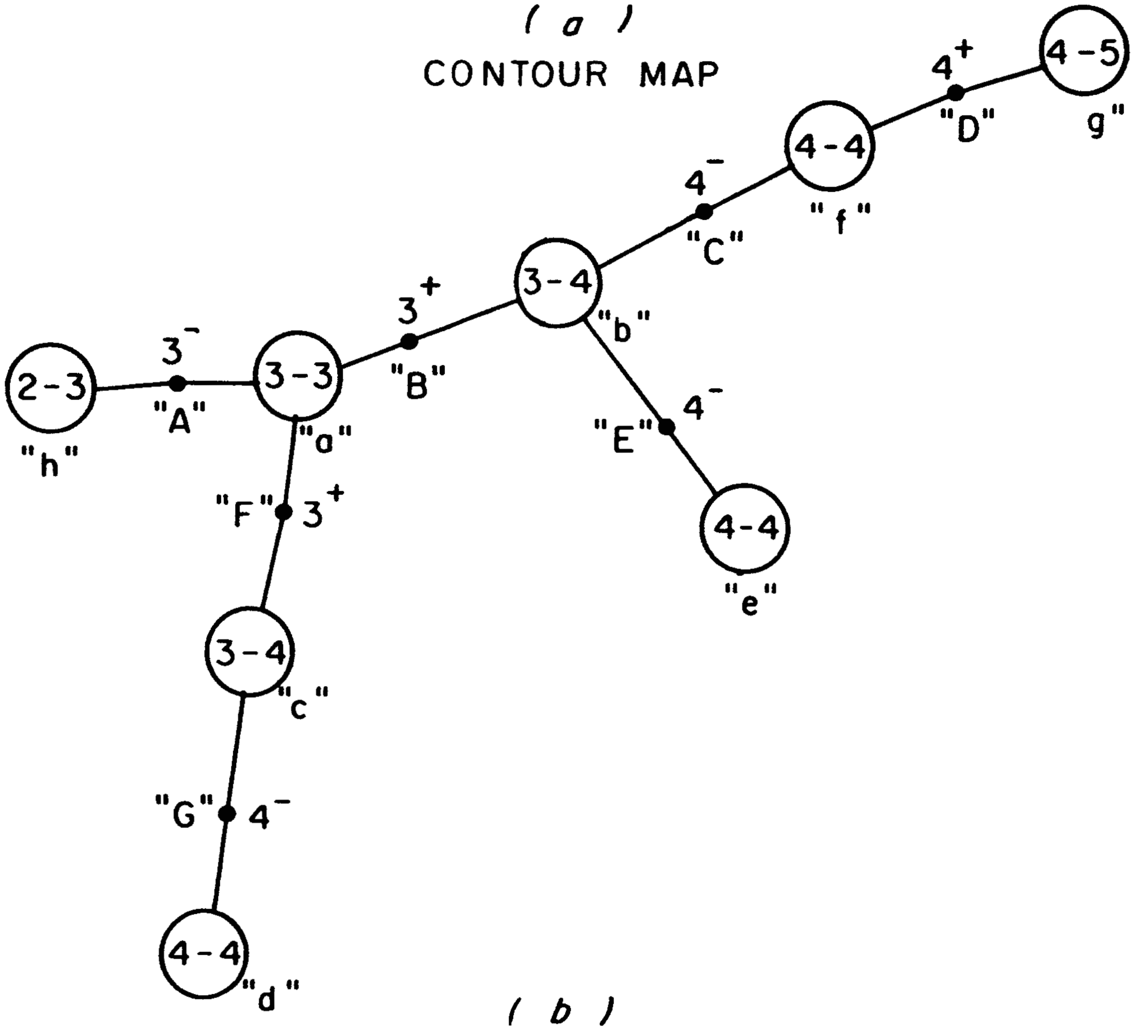
A - touch for <u>A</u>ll of their lengths.

This information can be written directly on the junctor provided the junctor has no more than three arms. This is illustrated in Figure 5.



(a)

CONTOUR MAP

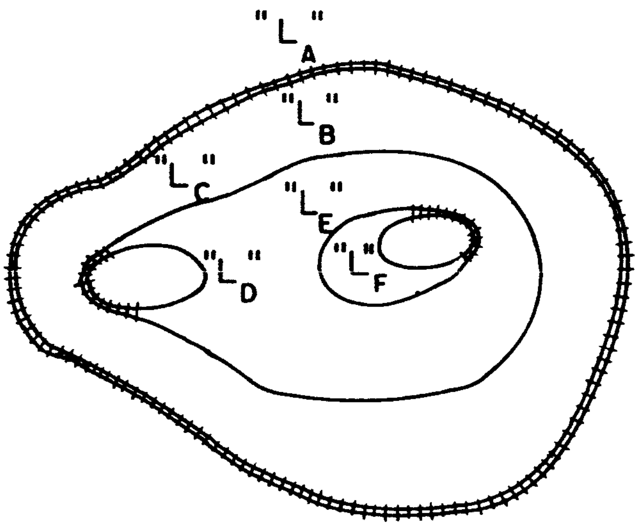


(b)

GRAPH

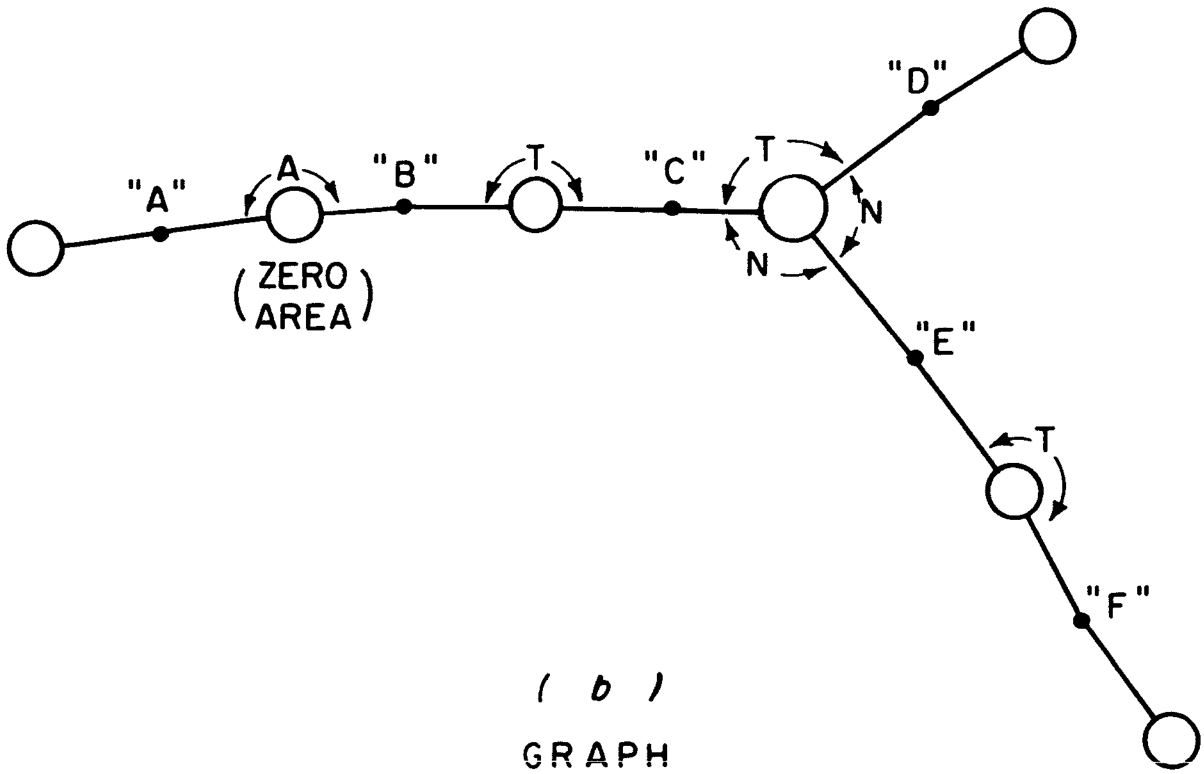
F I G. 4

EXAMPLE OF A CONTOUR MAP AND ITS GRAPH



(a)

CONTOUR MAP



(b)

GRAPH

FIG. 5

EXAMPLE OF STORING N-T-A INFORMATION ON GRAPH

If the junctor has more than three arms the information must be stored in a table associated with the junctor. The letter of the junctor identifies the table for that junctor and the numbers on the arms identify the correct entry in the table. This is illustrated in Figure 6. Also, if the intercontour region represented by a junctor has zero area, this information can be written next to the junctor.

If contour line L_B touches (T) contour line L_A the question arises as to which contour lines, if any, touch L_B over the span that L_B shares with L_A. (See Figure 7.) This type of information is stored as follows. There are two tree structures called $\tau(L_A, L_B)$ and $\tau(L_B, L_A)$. The initial vertex* of $\tau(L_A, L_B)$ has one branch emanating from it for each contour line in the adjacency set of L_B of which L_A is not a member. Let L_C be a contour line in this adjacency set. The branch representing L_C is labeled as follows:

$L_C/\bar{X}$ - L_C cannot be connected to the common span between L_A and L_B by a line that does not intersect other contour lines.

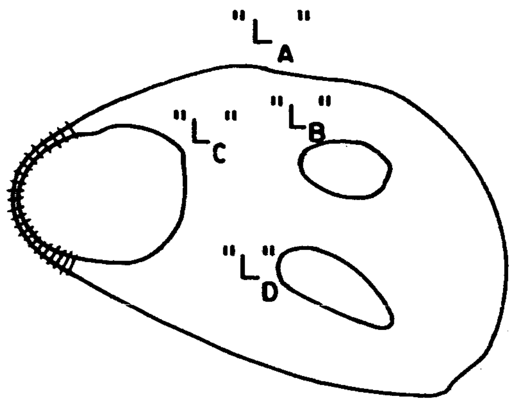
$L_C/\bar{N}$ - L_C does not touch the common span between L_A and L_B but can be connected to this span by a line that does not intersect other contour lines.

$L_C/\bar{T}$ - L_C touches part but not all of the common span between L_A and L_B.

$L_C/\bar{A}$ - L_C touches all of the common span between L_A and L_B.

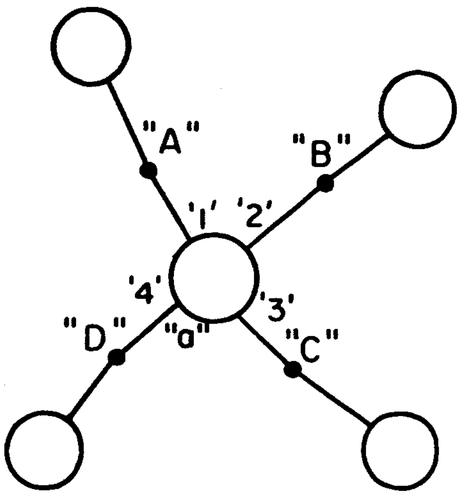
Note that if any branch emanating from a vertex is labeled $L_i/\bar{A}$, all the other branches emanating from that vertex must be labeled $L_j/\bar{X}$. If the branch representing L_C is labeled $L_C/\bar{X}$ or if the relation between contour lines L_A and L_B is N, then the branch representing L_C is considered an <u>endbranch</u> of the tree. If a branch is not an endbranch, it

*To avoid confusion with the nodes of the contour graph, the term <u>vertex</u> is used in connection with trees.



(a)

CONTOUR MAP



(b)

GRAPH

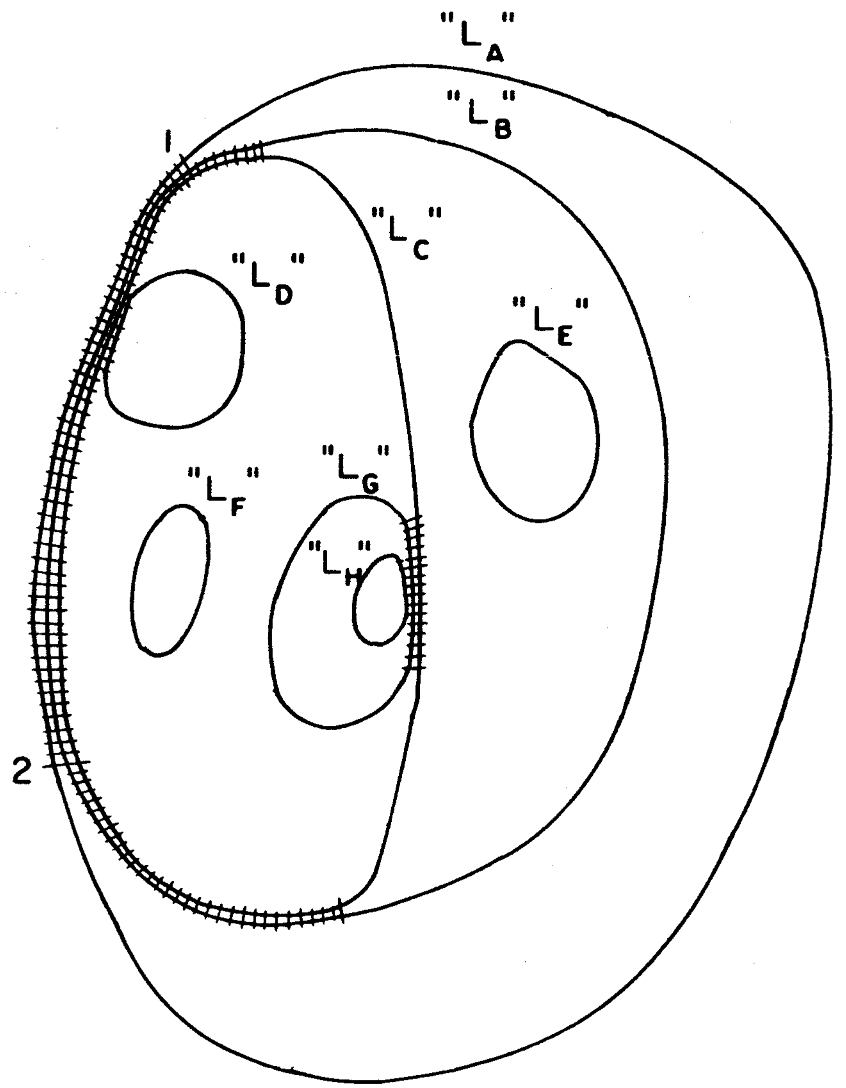
TABLE "a"	
1, 1	–
1, 2	N
1, 3	T
1, 4	N
2, 1	N
2, 2	–
2, 3	N
2, 4	N
3, 1	T
3, 2	N
3, 3	–
3, 4	N
4, 1	N
4, 2	N
4, 3	N
4, 4	–

(c)

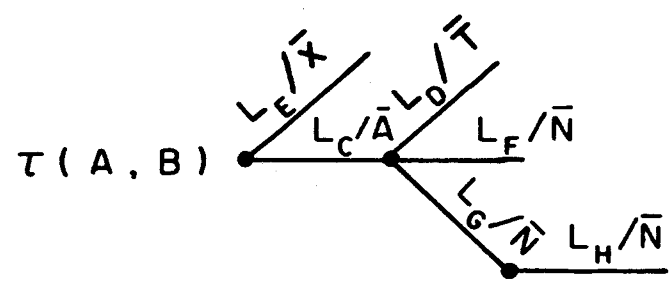
JUNCTOR TABLE

F I G. 6

EXAMPLE OF STORING N-T-A INFORMATION IN JUNCTOR TABLE



(a)
CONTOUR MAP



τ (B , A) = EMPTY

(b)
TREES RELATING "A" AND "B"

F I G . 7
EXAMPLE OF A TREE

feeds into a vertex from which emanate other branches. These branches represent all the contour lines in the adjacency set of L_C of which L_B is not a member. Each of these branches are labeled in accordance with their relation to the common intersection of L_A, L_B and L_C. This process is continued until each outermost branch is an endbranch or its corresponding contour line has an adjacency set consisting of itself only.

An example of a tree structure is shown in Figure 7. The common span that L_A shares with L_B is from point 1 to point 2. Contour line L_C touches all of that common span; therefore, its branch is labeled $L_C/\bar{A}$. Note that the presence of L_C prevents a line from being drawn from L_E to the common span between L_A and L_B without intersecting any other contour line. Thus, the branch for L_E is labeded $L_E/\bar{X}$ and is called an endbranch. Contour line L_D touches part of this common span and hence, is labeled $L_D/\bar{T}$. Because L_D has an empty adjacency set, its branch is an endbranch. Similar comments apply to the labeling of the remaining branches of the tree.

C. Paths on Graph

A path on a graph is a sequence of connected junctor arms. A junctor arm may appear more than once in the sequence and may even appear more than once in succession. A path can be specified by listing each node and junctor passed through.

1. Relation between Paths and Tracks

Many tracks can map into one path. A track is said to map into a path if the following conditions are satisfied:

a. Whenever the path intersects a node, the track intersects the contour line represented by the node.

b. Whenever the path passes through a homogeneous junctor by entering and leaving through two different arms, the track travels through the intercontour region bounded by the adjacency set represented by the junctor.

c. Whenever the path passes through a homogeneous junctor by entering and leaving through the same arm, the track either:

   i. travels along the contour line represented by the node connected to the arm of the junctor or

   ii. travels through the intercontour region bounded by the adjacency set represented by the junctor.

d. Whenever the path passes through a heterogeneous junctor, the track travels through the intercontour region bounded by the adjacency set represented by the junctor.

Note that if the intercontour region bounded by a heterogeneous adjacency set has zero area (i.e., a cliff) it makes no sense for the path to enter and leave the heterogeneous junctor through the same arm. Hence, such behavior of the path is forbidden. Also, if both junctors connected to a node correspond to intercontour regions of zero area, then any path arriving at the node through one of the junctors must depart from the node through the other junctor. In addition, the path must leave the latter junctor through an arm different from the one through which it entered. This corresponds to the track crossing a cliff line.

2. Relation between Paths and Track Sequences

For a given track there is a unique path. If, however, the track is not known and only the track sequence of the track is given, in general a unique path cannot be found. Instead, several paths may be found, each of which represents a family of tracks that yield the given track sequence. These paths all satisfy the following conditions:

a. Whenever the path arrives at a node through the arm of a heterogeneous junctor, the track sequence contains an entry for the value of this node.

b. Whenever the path departs from a node through the arm of a heterogeneous junctor, the track sequence contains an entry for the value of this node.

c. Whenever the path passes through two arms of a junctor that are related by an N in the junctor table, the track sequence contains an entry for the value of the junctor.

d. Whenever the path passes through two arms of a junctor that are related by a T in the junctor table, the track sequence may contain an entry for the value of the junctor. (See note following item f.)

e. Whenever the path passes through two arms of a junctor that are related by an A entry in the junctor table, the track sequence does not contain an entry for the value of that junctor.

f. The value of a homogeneous junctor appearing in the track sequence can correspond to the path passing through the junctor more than once in succession. Conversely, whenever the path passes through a homogeneous junctor more than once in succession, the track sequence contains at most one entry for the value of that junctor.

Note that if a path is selected for which the label of a junctor corresponding to a T entry does not appear in the track sequence, the restriction outlined in Figure 8 must be imposed. Assume the junctor in question connects node A to node B, and assume node A precedes node B on the path. Let C be the node preceding A on the path. A necessary condition for the label of the junctor between A and B not to appear in the track sequence is that the tree $\tau(L_B, L_A)$ not have an $L_C/\overline{X}$ label on the branch corresponding to C. Without this requirement there would be a path on the graph of the contour map of Figure 9a that would give the same track sequences as the ground profile of Figure 9c.

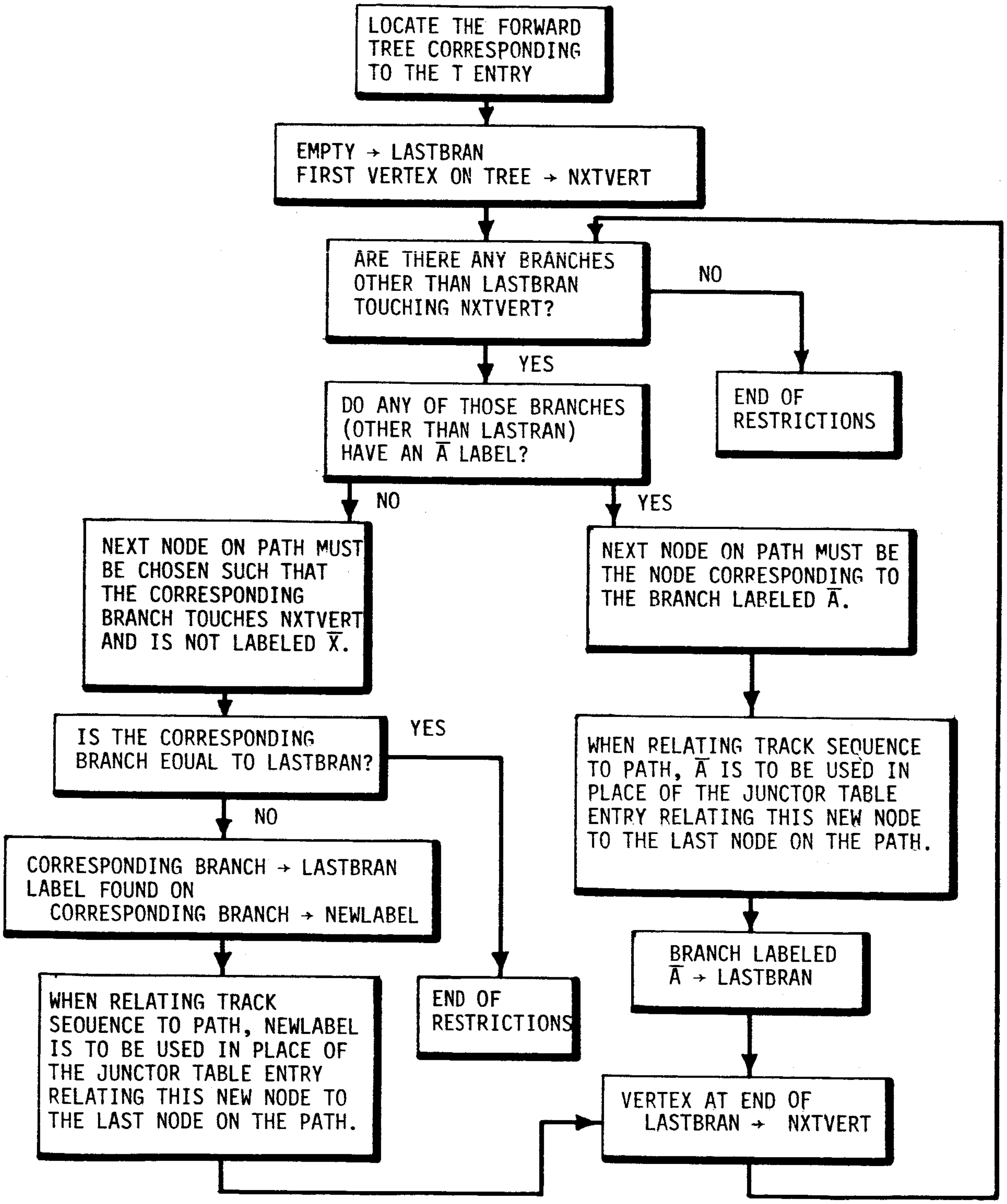
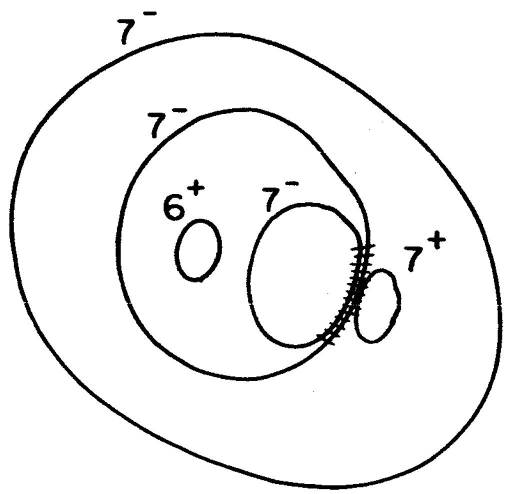
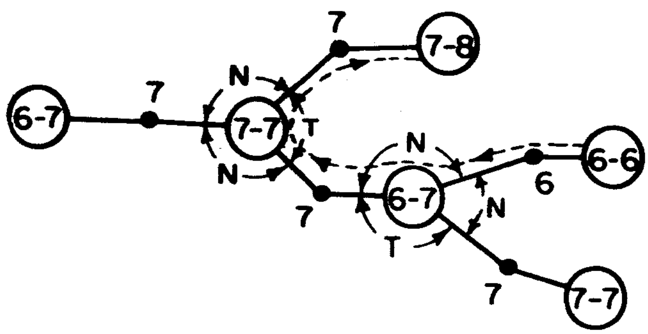


FIG. 8

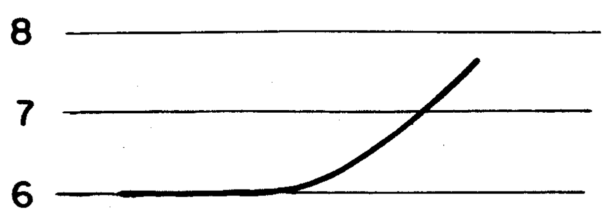
RESTRICTION PLACED ON A PATH WHEN THE OMISSION OPTION FOR A T ENTRY IS USED



(a)
CONTOUR MAP



(b)
PATH ON GRAPH



(c)
GROUND PROFILE

FIG. 9

AN EXAMPLE OF A GROUND PROFILE AND A PATH ON A GRAPH OF A CONTOUR MAP THAT ALMOST YIELDED THE SAME TRACK SEQUENCE. (SEE TEXT)

The question arises as to what entry should the junctor table have when relating an arm to itself. To answer this, one must recall what is implied when a path goes in and comes out the same arm of the junctor. This corresponds to the track either traveling along the contour line represented by the node at the end of the junctor arm (homogeneous junctors only), or passing through the intercontour region bounded by the adjacency set of contour lines represented by the junctor. In either case the label of the junctor will appear in the sequence. Thus, the junctor table entry for two identical arms must be an N.

IV. UTILIZATION OF A GRAPH

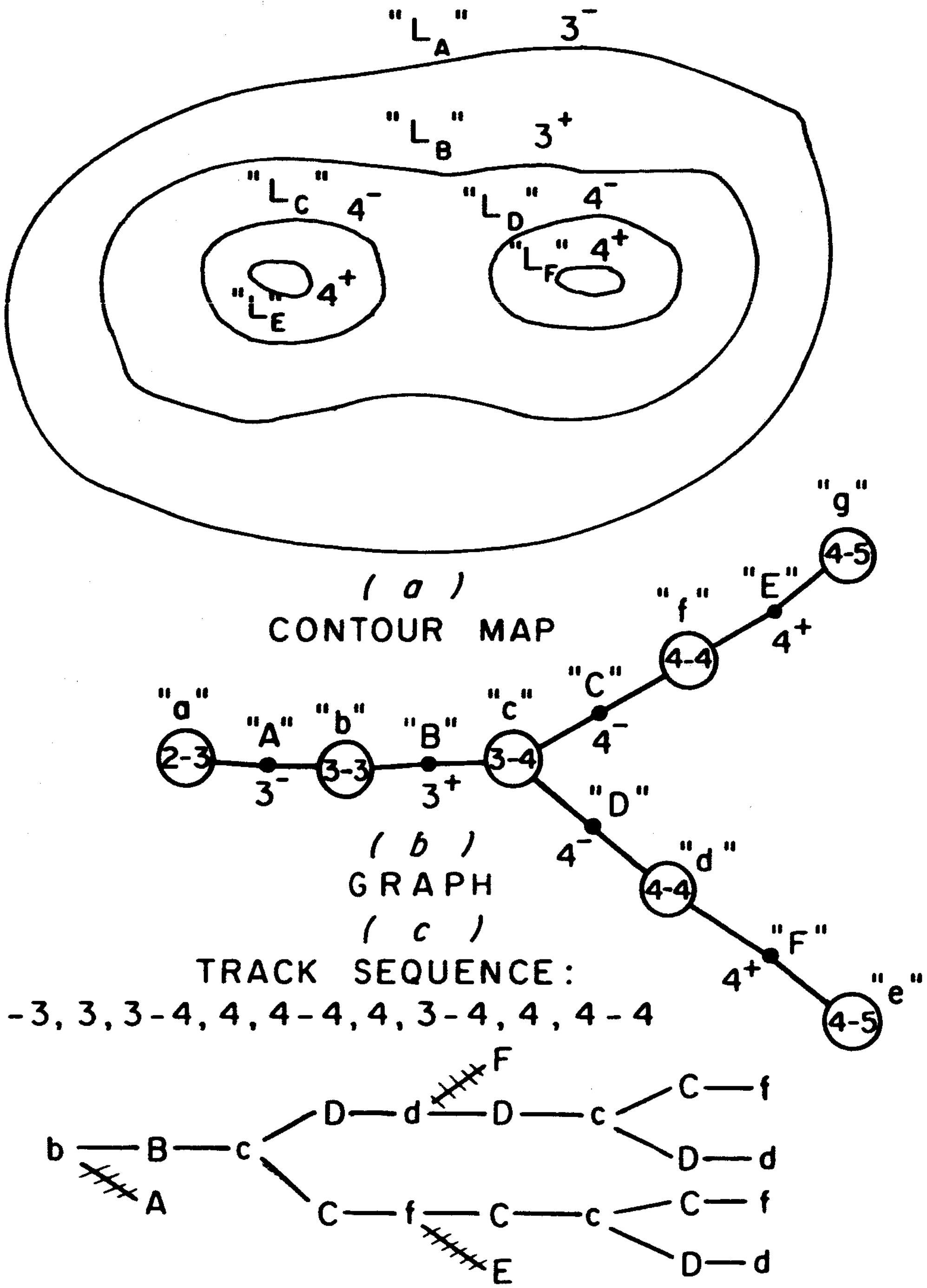
The only step remaining in narrowing down the search is to locate a path that represents tracks with the same track sequence as the ground track. This is done in the following manner. Assume the first entry in the track sequence is a single number, e_1. Pick a node whose value is e_1. Check to see that both junctors connected to the node do not correspond to zero area intercontour regions. If they do, reject the node. Form a list whose first entry is the letter of the node (this list will become the desired path). If there is more than one such node, use some criterion* to select the most likely node and store an indication that alternate choices exist. Assume the next entry in the track sequence is e_1-e_1. See if there is any junctor labeled e_1-e_1 that touches the previous node and has a pair of arms that have an N or T for their entry in the junctor table. If so, add its letter to the list. If there is more than one junctor, use some criterion** to select the most likely one and store an indication that alternate choices exist. If there are no such junctors, back up on the list until an alternate choice indication is found and select the most likely alternate choice placing entries in the list until the track sequence has been completed. Select the entries so that the conditions discussed in Section III.C.2 are satisfied. At all times obey the rules on forming permissible paths which were discussed in Section III.C.1. The final list corresponds to a possible ensemble of ground

*Such a criterion might be to select the node corresponding to the contour line having the greatest area.
**Such a criterion might be to select the junctor corresponding to the intercontour region having the longest perimeter.

tracks. All the possible choices of paths for a given track sequence are shown in Figure 10. Figure 11 shows a track corresponding to each choice of path.

Note that the shape of the contour lines and the flight path have not been used at all. These must now be considered to determine whether the given ground track is a member of the ensemble and, if so, just where the given ground track crosses the various contour lines. This question is beyond the capabilities of the topological analysis.

It must be pointed out that the methods discussed in this report do not utilize all possible topological constraints. For example, the ground profile shown in Figure 12a cannot occur for any ground track on the portion of the contour map shown in Figure 12b. But when the methods presented in this paper are applied, this region of the contour map is not eliminated from consideration. The occurrence of such an example is quite unlikely; therefore, it does not pay to complicate the algorithm to include the additional constraint.



(a)
CONTOUR MAP

(b)
GRAPH

(c)
TRACK SEQUENCE:
3-3, 3, 3-4, 4, 4-4, 4, 3-4, 4, 4-4

#1. b B c D d D c C f
#2. b B c D d D c D f
#3. b B c C f C c C f
#4. b B c C f C c D d

(d)
SET OF PATHS

FIG. 10
PATHS CORRESPONDING TO A GIVEN SEQUENCE

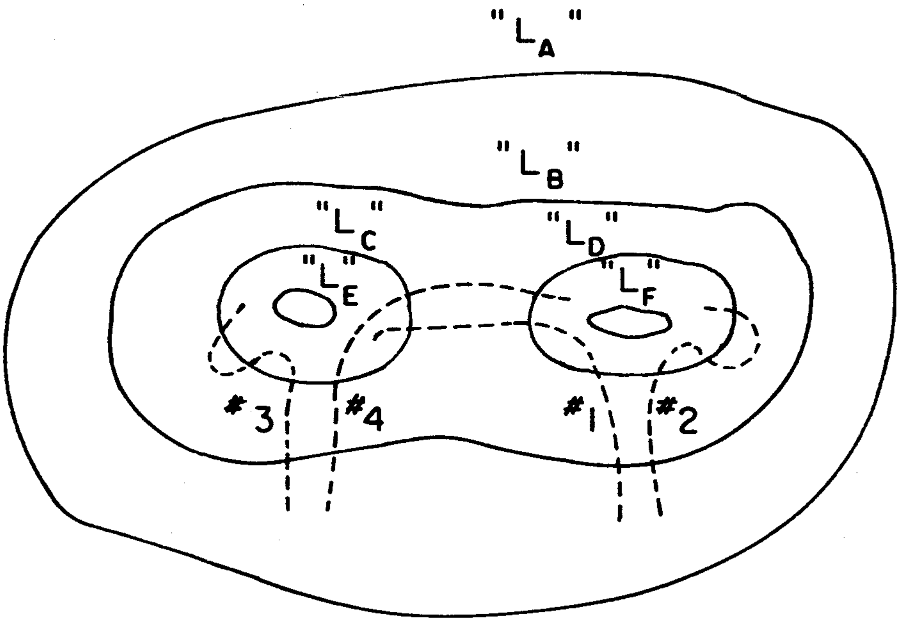
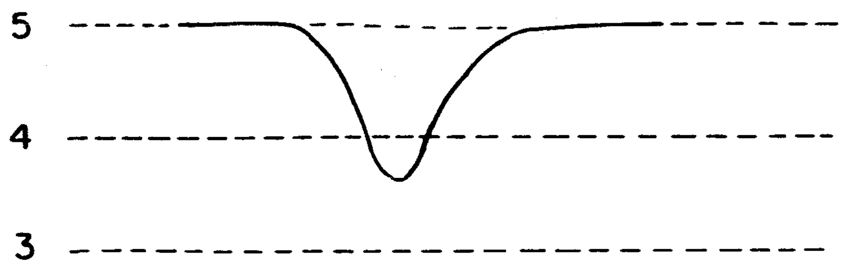
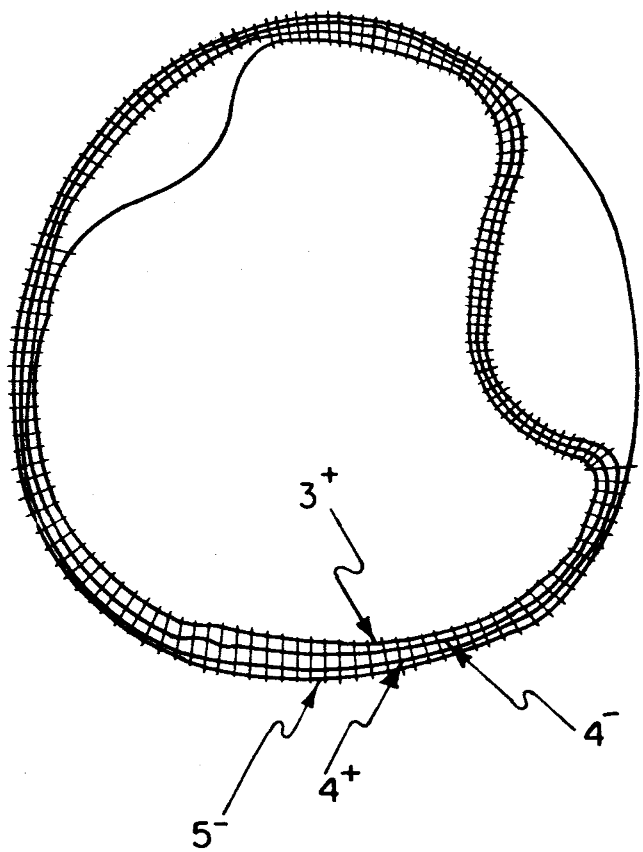


FIG. 11

REPRESENTATIVE PATHS FROM ENSEMBLES OF PATHS IN FIG. 10



(a)
GROUND PROFILE



(b)
CONTOUR MAP

FIG 12

PATHOLOGICAL EXAMPLE (SEE TEXT)

V. DIGITAL COMPUTER TECHNIQUES

The solution of the searching problem as outlined in this report can be readily carried out on a digital computer. The algorithm has already been discussed; the only remaining comments pertain to the storage of the contour graph in the computer memory.

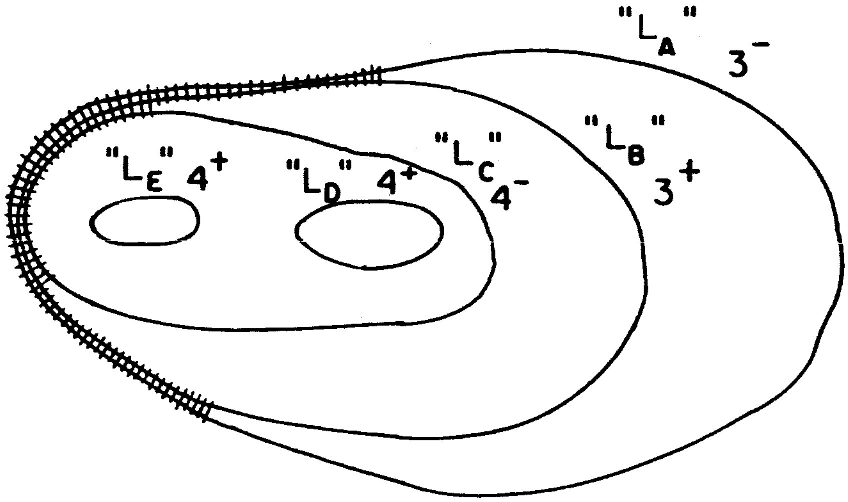
The elements of the contour graph are stored in memory by reserving a block of words for each node and a block of words for each junctor. The storage of the contour graph of Figure 13 is shown in Figure 14. Each node block contains the following pieces of information:

1. Name of node
2. Value of node
3. Address of junctor block for LAS
4. Number of the arm in LAS junctor that touches node
5. Address of junctor block for RAS
6. Number of the arm in RAS junctor that touches node.

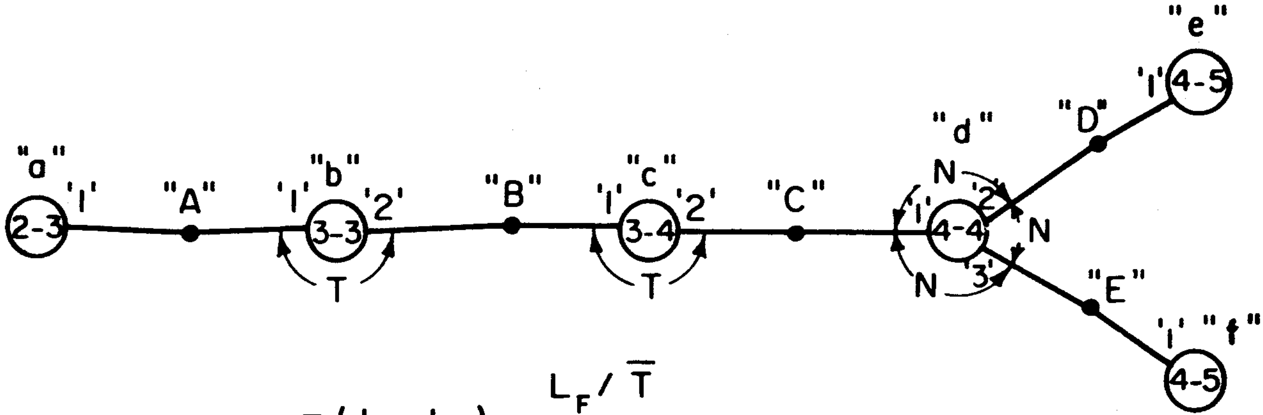
Each junctor block contains the following pieces of information:

1. Name of junctor
2. Label of junctor
3. Zero-area indicator
4. Address of nodes touched by each junctor arm
5. Address of portion of junctor table for each arm
6. Address of tree structure for each T entry in junctor table.

The first step in selecting paths corresponding to the track sequence obtained from the ground profile is to find the nodes whose



(a)
CONTOUR MAP



$\tau(L_A, L_B)$ $\underline{L_F / \overline{T}}$

$\tau(L_B, L_A)$ EMPTY

$\tau(L_B, L_C)$ EMPTY

$\tau(L_C, L_B)$ $\underline{L_A / \overline{A}}$

(b)
GRAPH AND TREES

FIG. 13
EXAMPLE OF A GRAPH

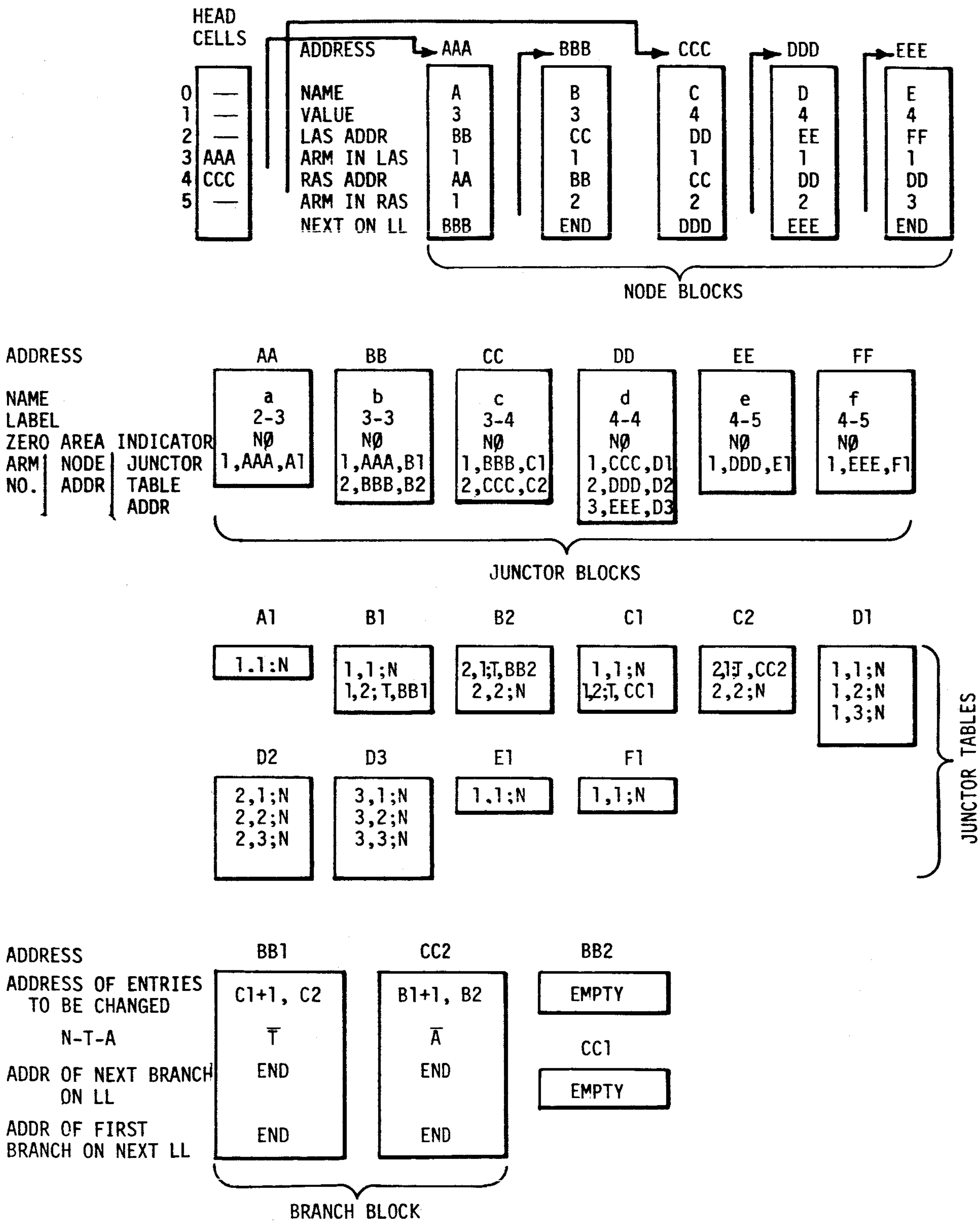


FIG. 14

GRAPH IN COMPUTER MEMORY

values correspond to the first single number in the sequence. Thus, the storage must allow for rapid retrieval of all nodes of a given value. This is done by putting all nodes of the same value on a linked list. The head cells of the linked lists are stored in a table that can be readily indexed for any particular node value.

A tree structure for a T entry in a junctor table can be stored by reserving a block of words for each branch of the tree. The branch blocks of those branches emanating from a common vertex are tied together on a linked list. This list is contained inside the branch blocks. Each branch block contains the following pieces of information:

1. Address of junctor table entries to be changed
2. $\overline{X}$, $\overline{N}$, $\overline{T}$, or $\overline{A}$
3. Address of next branch block on linked list
4. Address of first branch block on linked list of branches emanating from the vertex at which the present branch terminates.

The entire tree can be located if one knows the address of the initial branch block on the linked list of branches emanating from the initial vertex of the tree. A careful study of the tree storage reveals that it is a grid of horizontal and vertical linked lists.

VI. CONCLUSION

The topology of a contour map provides certain constraints that can be used to narrow down a search on a contour map. These constraints are easy to apply and go a long way in simplifying the search.

Work is in progress on a geometric approach which will follow the topological analysis and allow the search to be completed.

VII. REFERENCES

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