RADIATIVE TRANSFER FOR PARALLEL STREAMS OF RADIATING GASES

by

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ABSTRACT

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The radiation heat transfer between two parallel streams of absorbing and emitting radiating gases is studied. The problem is described by the Rosseland diffusion approximation together with the radiation slip boundary condition. Considering that the flow is incompressible and invidcid, the exact solution of the problem is obtained. The solution shows a smooth transition regime between the optically thin and the optically thick regions. The thickness of radiation layers is calculated.

1. INTRODUCTION

This paper presents an exact solution for the radiative and convective heat transfer between two parallel streams of absorbing and emitting radiating gases. The Rosseland diffusion approximation and the radiation slip boundary condition are used to describe the problem. Probstein[1] calculated the radiative heat transfer between two parallel plates containing a radiating as in the gap by this method. The calculated results based on the radiation slip boundary condition are in good agreement with the numerical results[2-4] obtained by solving the governing integro-differential equations. Probstein suggested that the difficult problems of solving the integro-differential equations may be circumvented by the use of the "radiation slip" method and that this method may be applied to other gas flow problems where radiation heat transfer is of importance. Some applications of the radiation slip boundary condition to gas flow problems have been carried out in references 5-7.

In this paper the radiation slip boundary condition is used to study the radiative transfer between two streams of incompressible inviscid flow. In order to investigate the main features of the radiation transfer in the transition regime, we consider that the conductive heat flux is neglegible compared with the radiative flux. The exact solution of the problem in a closed form is obtained. A special case of this solution gives the exact solution for the radiative transfer in an incompressible inviscid flow over a flat plate. The physical significance of the solution is discussed and the thickness of radiation layers is calculated.

2. RADIATIVE TRANSFER IN TWO PARALLEL STREAMS

We shall study the radiative transfer between two parallel streams of incompressible invicid gases, moving horizontally in the same direction. The x–axis is drawn horizontally in the direction of motion and y - axis vertically upwards. At $x = 0$ the two flows come into contact and have different temperatures, densities and velocities. The energy equation is

$$\rho_i u_i c_{pi} \frac{\partial T}{\partial x} = - \frac{\partial q^R}{\partial y} \qquad (1)$$

where the suffix $i = 1$ refers to the upper flow and $i = 2$ to the lower. Using the Rosseland diffusion approximation, the radiative flux q^R may be expressed as the product of the temperature gradient and the radiative thermal conductivity, namely,

$$q^R = - \frac{16 \sigma T^3}{3 k_R} \frac{\partial T}{\partial y}$$

where σ is the Stefan - Boltzmann constant and k_R the volumetric absorption coefficient. Assume that k_R varies as a third power of the temperature, then we can write (1) as

$$\rho_i u_i c_{pi} \frac{\partial T}{\partial x} = \frac{16 \sigma T_i^3}{3 k_{Ri}} \frac{\partial^2 T}{\partial x^2} \qquad (2)$$

The radiation slip boundary condition will be applied to the interface between the two streams. On the interface there is a temperature jump that is proportional to the radiation temperature gradient. Thus we can write the boundary conditions

(i) As $x = 0$ $\qquad T = T_i$ $\qquad$ for all y

(ii) As $x > 0$ $\qquad T = T_i$ $\qquad$ for $|y| \to \infty$

$$(T)_{y=0_+} - (T)_{y=0_-} = c_1 \left(\frac{\partial T}{\partial y}\right)_{y=0_+} = c_2 \left(\frac{\partial T}{\partial y}\right)_{y=0_-}$$

and

$$\frac{16\sigma T_1^3}{3 k_{R1}} \left(\frac{\partial T}{\partial y}\right)_{y=0_+} = \frac{16\sigma T_2^3}{3 k_{R2}} \left(\frac{\partial T}{\partial y}\right)_{y=0_-}$$

where the constants c_1 and c_2 are to be determined later.

We introduce the following dimensionless quantities:

$$\theta = (T - T_2) / (T_1 - T_2)$$

$$\xi = 16\sigma T_1^3 x / 3 c_1^2 k_{R1} \rho_1 u_1 c_{p1}$$

$$\eta = y / c_1$$

$$\alpha^2 = \rho_1 u_1 c_{p1} / \rho_2 u_2 c_{p2}$$

and

$$\beta^2 = T_1^3 k_{R2} / T_2^3 k_{R1}$$

In terms of the dimensionless quantities, we can write equation (2) and the boundary conditions as

$$\left.\begin{aligned} \frac{\partial \theta}{\partial \xi} &= \frac{\partial^2 \theta}{\partial \eta^2} \qquad \text{for} \quad \eta > 0 \\ \frac{\partial \theta}{\partial \xi} &= \frac{\alpha^2}{\beta^2} \frac{\partial^2 \theta}{\partial \eta^2} \qquad \text{for} \quad \eta < 0 \end{aligned}\right\} \tag{3}$$

As $\xi = 0$ $\quad \theta = 1$ for $\eta > 0$

$\theta = 0$ for $\eta < 0$

As $\xi > 0$ $\quad \theta = 1$ for $\eta \to +\infty$

$\theta = 0$ for $\eta \to -\infty$

and

$$(\theta)_{\eta=0_+} - (\theta)_{\eta=0_-} = \left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0_+} = \frac{1}{\beta^2}\left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0_-}$$

The exact solution of this problem can now be obtained by means of the Laplace transformation method. Let $f(s,\eta)$ be the Laplace transform of $\theta(\xi,\eta)$, then we can find the Laplace transformation of the solution of (3) subject to the given boundary conditions

$$f = \frac{1}{s} - \frac{\exp(-\eta s^{1/2})}{s(1+\alpha\beta+s^{1/2})} \qquad \text{for } \eta > 0$$

and

$$f = \frac{\alpha\beta \exp(\beta\eta s^{1/2}/\alpha)}{s(1+\alpha\beta+s^{1/2})} \qquad \text{for } \eta < 0 \tag{4}$$

The inverse transform of $f(s,\eta)$ gives

$$\theta = 1 - \frac{\exp(-\eta^2/4\xi)}{1+\alpha\beta}\left\{F[\eta/(2\xi^{1/2})] - F[\eta/(2\xi^{1/2}) + (1+\alpha\beta)\xi^{1/2}]\right\} \quad \text{for } \eta > 0$$

$$\theta = \frac{\alpha\beta}{1+\alpha\beta}\exp[-\beta^2\eta^2/(4\alpha^2\xi)]\left\{F[-\beta\eta/(2\alpha\xi^{1/2})] - F[-\beta\eta/(2\alpha\xi^{1/2}) + (1+\alpha\beta)\xi^{1/2}]\right\} \quad \text{for } \eta < 0 \tag{5}$$

where

$$F(z) = \exp(z^2)\,\text{erfc}\, z \tag{6}$$

From (5) we can obtain the radiative heat flux across the interface.

$$(-q^R)_{y=0} = \frac{16\sigma T_1^3}{3k_{R1}} \frac{T_1 - T_2}{c_1} F[(1+\alpha\beta)\xi^{1/2}]$$

Now we can determine the two constants c_1 and c_2 by requiring that the solution reduces to the black-body result in the limit of the optically thin regime, namely

$$\lim_{\xi \to 0} (-q^R)_{y=0} = \sigma(T_1^4 - T_2^4)$$

This condition yields

$$C_1 = \frac{16 T_1^3}{3 k_{R1}} \frac{T_1 - T_2}{T_1^4 - T_2^4}$$

and

$$C_2 = \frac{16 T_2^3}{3 k_{R2}} \frac{T_1 - T_2}{T_1^4 - T_2^4}$$

Therefore we obtain the exact solution for the radiative flux between the two streams of incompressible, inviscid gases as

$$(-q^R)_{y=0} = \sigma (T_1^4 - T_2^4) F[(1+\alpha\beta)\xi^{1/2}] \quad (7)$$

The temperature changes on the two sides of the interface are related by

$$(T_{0-} - T_2)/(T_1 - T_{0+}) = \alpha\beta$$

Some solution for θ and $(-q^R)_{y=0}$ are plotted in Figures 1 - 7 for various values of α and β. Figure 1 shows that the exact solution processes a smooth transition between the black body formulation for the optically thin regime and the Rosseland diffusion formulation. When $0 \leq (1 + \alpha\beta)^2 \xi \leq 7.85 \times 10^{-5}$, equation (7) gives $0.99 < (-q^R)_{y=0} / [\sigma (T_1^4 - T_2^4)] \leq 1$. Physically this result means that the accuracy of the black body formulation is within one percent for the region

$$x < 4.2 \times 10^{-4} B_1 \ell_{R1} /(1 + \alpha\beta)^2$$

where $B_1 = \rho_1 u_1 c_{p1} / \sigma T_1^3$ is the Boltzmann number, and $\ell_R = 1/ k_R$ is the photon mean free path. If the "radiation slip" boundary condition is not used, the heat flux across the interface calculated by the Rosseland approximation is

$$(-q^R)_{y=0} / [\sigma (T_1^4 - T_2^4)] = (\pi)^{-1/2} [(1+\alpha\beta)\xi^{1/2}] \quad (8)$$

Equation (7) approaches to (8) asymptotically as $(1+\alpha\beta)^2 \xi \to \infty$. When $(1+\alpha\beta)^2 \xi > 48.5$ the error introduced by equation (8) is less than one percent. This means the no-slip solution (8) is accurate to within one percent in the region

$$x > 260\, B_1\, \ell_{R1}/(1+\alpha\beta)^2$$

In this region the influence of radiation slip can be neglected.

3. RADIATIVE TRANSFER FOR AN INCOMPRESSIBLE INVISCID FLOW OVER A FLAT PLATE

The radiative heat transfer for an incompressible inviscid flow over a plate of constant temperature has been studied by Goulard[8] and Tien and Greif[5]. Its exact solution can actually be obtained directly from (5) and (7) by taking $k_{R2} \to 0$ or $\beta = 0$, namely

$$\theta = 1 + \exp(\eta + \xi)\, \mathrm{erfc}\left[\xi^{1/2} + \eta/(2\xi^{1/2})\right] \tag{9}$$

and

$$(-q^R)_{y=0} = \sigma(T_1^4 - T_2^4)\, \exp(\xi)\, \mathrm{erfc}\,(\xi^{1/2}) \tag{10}$$

The exact solution (10) can be expanded as

$$\frac{(-q^R)_{y=0}}{\sigma(T_1^4 - T_2^4)} = \frac{1}{(\pi\xi)^{1/2}}\left[1 - \frac{1}{2\xi} + \frac{1\cdot 3}{(2\xi)^2} - \frac{1\cdot 3\cdot 5}{(2\xi)^3} \pm \cdots\right]$$

While the solution by Tien and Greif can be expressed as

$$\frac{(-q^R)_{y=0}}{\sigma T_1^4} = \frac{1}{(\pi\xi)^{1/2}}\left(1 + \frac{1}{\pi\xi}\right)^{-1/2}$$

$$= \frac{1}{(\pi\xi)^{1/2}}\left[1 - \frac{1}{2\pi\xi} + \frac{1\cdot 3}{2!(2\pi\xi)^2} - \frac{1\cdot 3\cdot 5}{3!(2\pi\xi)^3} \pm \cdots\right]$$

These solutions are also plotted in figures 1 and 2. For this case the black body solution is accurate to within one percent in the region

$$x < 4.2 \times 10^{-4} \quad B_1 \ell_{R1}$$

and the effect of the radiation slip condition can be neglected in the region

$$x > 260 \; B_1 \ell_{R1}$$

4. THICKNESS OF RADIATION LAYERS

The results obtained in section 2 show that the temperature changes rapidly from the temperature at $y = 0_+$ (say T_o) to the main stream temperature at $y = \infty$, T_1 , within a layer beyond which the temperature changes asymptotically and the influence of radiation is imperceptible. If we define the radation layer thickness as the distance δ from the interface of two streams(or from the surface of a flat plate) for which

$$T - T_o = 0.99(T_1 - T_o)$$

then from (5) we obtain that when $y = \delta$

$$(0.01)\exp[\eta^2/4\xi] = \frac{F[\eta/(2\xi^{1/2})] - F[\eta/(2\xi^{1/2}) + (1+\alpha\beta)\xi^{1/2}]}{F(0) - F[(1+\alpha\beta)\xi^{1/2}]} \qquad (11)$$

where F is defined by equation (6)

when ξ tends to infinity, equation (10) reduces to

$$0.01 = \operatorname{erfc}[\eta/(2\xi^{1/2})]$$

or

$$\eta/\xi^{1/2} = 2.58 \qquad (12)$$

when ξ tends to zero, equation (11) reduces to

$$0.01 = \exp(-\eta^2/4\xi) - \pi^{1/2}[\eta/(2\xi^{1/2})]\,\mathrm{erfc}[\eta/(2\xi^{1/2})]$$

or

$$\eta/\xi^{1/2} = 2.27 \quad (13)$$

From (12) and (13), we can express the thickness of radiation layers as

$$\delta = \epsilon(D_{R1}\,x/u_1)^{1/2} \quad (14)$$

or

$$\delta/x = \epsilon(u_1 x/D_{R1})^{-1/2}$$

where $D_R = (16\,\sigma T^3)/(3\,\rho\,C_p\,k_R)$ may be called the radiation diffusivity, and the value of ϵ is between 2.27 and 2.58, ϵ equals to 2.27 at the optically thin limit and 2.58 the optically thick limit. The variation of ϵ shows that the thickness of radiation layer is slightly reduced near the leading edge of the plate due to the effect of the radiation slip. This phenomenon is quite similar to the effect of the mass velocity slip on the momentum thickness of boundary layers [9]. In terms of the Boltzmann number and the radiation mean free path, the thickness of radiation layers becomes

$$\delta/x = \epsilon'(\ell_R/B_1 x)^{1/2}$$

where $\epsilon' = 4\epsilon(3)^{-1/2} = 5.25$—$5.95$. The thickness ratio of the two radiation layers for the two parallel streams of incompressible inviscid gases is

$$(\delta_1/\delta_2)^2 = (B_2\,\ell_{R1})/(B_1\,\ell_{R1}) \quad (13)$$

or

$$\delta_1/\delta_2 = \beta/\alpha$$

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Table 1. Radiation Heat Flux calculated by equation (7) compared with the approximate solution by Tien and Greif for $\beta = o$.

$(1 + \alpha\beta)^2 \xi$	$(-q^R)_{y=o} / (T_1^4 - T_2^4)$	
	Equation (7)	Tien and Greif, $\beta = o$
0.01	0.89646	0.98464
0.02	0.85848	0.97003
0.05	0.79038	0.92885
0.10	0.72358	0.87161
0.20	0.64379	0.78370
0.50	0.52316	0.62367
1.00	0.42758	0.49138
2.00	0.33620	0.37054
5.00	0.23233	0.24391
10.00	0.17058	0.17564
20.00	0.12321	0.12516
50.00	0.07901	0.07954
100.00	0.05614	0.05624

Table 2. Calculated temperature for $\beta = o$

η	$\xi = 0.1$	$\xi = 1$	$\xi = 10$	$\xi = 100$
100				1.00000
50				0.99968
30				0.97126
20			1.00000	0.86149
10		1.00000	0.98421	0.56216
5		0.99989	0.81060	0.32779
3		0.98831	0.61732	0.22208
2	1.00000	0.93666	0.48660	0.16750
1	0.99655	0.77095	0.33599	0.11210
0.5	0.95059	0.62186	0.25473	0.08417
0.3	0.89084	0.54915	0.22137	0.07297
0.2	0.84620	0.51020	0.20453	0.06736
0.1	0.79049	0.46964	0.18760	0.06175
0.05	0.75841	0.44879	0.17910	0.05895
0.03	0.74480	0.44035	0.17570	0.05783
0.02	0.73784	0.43611	0.17399	0.05726
0.01	0.73076	0.43185	0.17229	0.05670
0	0.72358	0.42758	0.17058	0.05614

Table 3. Calculated temperature for $\alpha = 1$, $\beta = 1$.

η	$\xi = 0.1$	$\xi = 1$	$\xi = 10$	$\xi = 100$
100				1.00000
50				0.99982
30				0.98443
20			1.00000	0.92629
10		1.00000	0.99023	0.77095
5		0.99991	0.88924	0.65123
3		0.99124	0.78162	0.59767
2	1.00000	0.95428	0.71065	0.57011
1	0.99695	0.84230	0.63042	0.54220
0.5	0.95779	0.74671	0.58777	0.52816
0.3	0.90857	0.70146	0.57039	0.52254
0.2	0.87262	0.67757	0.56164	0.51972
0.1	0.82858	0.65295	0.55287	0.51690
0.05	0.80361	0.64040	0.54847	0.51550
0.03	0.79310	0.63534	0.54671	0.51493
0.02	0.78774	0.63280	0.54583	0.51465
0.01	0.78231	0.63025	0.54495	0.51437
0	0.77680	0.62770	0.54407	0.51409
-0	0.22320	0.37230	0.45593	0.48591
-0.01	0.21769	0.36975	0.45505	0.48563
-0.02	0.21226	0.36720	0.45417	0.48535
-0.03	0.20690	0.36466	0.45329	0.48507
-0.05	0.19639	0.35960	0.45153	0.48450
-0.1	0.17142	0.34704	0.44713	0.48310
-0.2	0.12738	0.32243	0.43836	0.48028
-0.3	0.09143	0.29854	0.42961	0.47746
-0.5	0.04221	0.25329	0.41223	0.47184
-1	0.00305	0.15770	0.36958	0.45780
-2	0	0.04572	0.28935	0.42989
-3		0.00876	0.21838	0.40233
-5		0.00009	0.11076	0.34877
-10		0	0.00977	0.22905
-20			0	0.07371
-30				0.01557
-50				0.00018
-100				0

Table 4. Calculated temperature for $\alpha = 0.1$, $\beta = 1$

η	$\xi = 0.1$	$\xi = 1$	$\xi = 10$	$\xi = 100$
100				1.00000
50				0.99970
30				0.97350
20			1.00000	0.87267
10		1.00000	0.98512	0.59870
5		0.99990	0.82308	0.38478
3		0.98867	0.64390	0.28843
2	1.00000	0.93898	0.52323	0.23871
1	0.99660	0.78071	0.38462	0.18827
0.5	0.95141	0.63928	0.31002	0.16285
0.3	0.89288	0.57058	0.27942	0.15266
0.2	0.84928	0.53385	0.26400	0.14756
0.1	0.79496	0.49566	0.24848	0.14245
0.05	0.76374	0.47605	0.24070	0.13990
0.03	0.75051	0.46812	0.23758	0.13888
0.02	0.74373	0.46413	0.23602	0.13837
0.01	0.73686	0.46013	0.23446	0.13786
0	0.72988	0.45612	0.23290	0.13735
-0	0.02701	0.05439	0.07671	0.08627
-0.01	0.02050	0.05043	0.07515	0.08575
-0.02	0.01507	0.04662	0.07360	0.08524
-0.03	0.01071	0.04294	0.07206	0.08473
-0.05	0.00486	0.03607	0.06900	0.08371
-0.1	0.00034	0.02193	0.06154	0.08117
-0.2	0	0.00610	0.04768	0.07613
-0.3		0.00113	0.03561	0.07116
-0.5		0.00001	0.01769	0.06152
-1		0	0.00149	0.04013
-2			0	0.01273
-3				0.00265
-5				0.00003
-10				0
-20				
-30				
-50				
-100				

Table 5. Calculated temperature for $\alpha = 10$, $\beta = 1$

η	$\xi = 0.1$	$\xi = 1$	$\xi = 10$	$\xi = 100$
100				1.00000
50				0.99996
30				0.99697
20			1.00000	0.98587
10		1.00000	0.99781	0.95677
5		0.9997	0.97681	0.93465
3		0.99735	0.95549	0.92482
2	1.00000	0.98726	0.94180	0.91978
1	0.99851	0.95987	0.92661	0.91468
0.5	0.98231	0.93848	0.91865	0.91212
0.3	0.96439	0.92884	0.91542	0.91110
0.2	0.95232	0.92387	0.91380	0.91058
0.1	0.93846	0.91883	0.91219	0.91007
0.05	0.93100	0.91629	0.91138	0.90981
0.03	0.92794	0.91527	0.91105	0.90971
0.02	0.92640	0.91476	0.91089	0.90966
0.01	0.92485	0.91425	0.91073	0.90961
0	0.92329	0.91373	0.91056	0.90956
-0	0.76710	0.86265	0.89435	0.90443
-0.01	0.76554	0.86214	0.89419	0.90438
-0.02	0.76398	0.86163	0.89403	0.90433
-0.03	0.76242	0.86112	0.89387	0.90427
-0.05	0.75930	0.86010	0.89354	0.90417
-0.1	0.75152	0.85755	0.89273	0.90392
-0.2	0.73601	0.85244	0.89111	0.90340
-0.3	0.72058	0.84734	0.88949	0.90289
-0.5	0.68998	0.83715	0.88625	0.90186
-1	0.61538	0.81173	0.87814	0.89930
-2	0.47677	0.76129	0.86195	0.89417
-3	0.35610	0.71157	0.84578	0.88904
-5	0.17692	0.61522	0.81355	0.87879
-10	0.01488	0.40130	0.73393	0.85319
-20	0	0.12733	0.58198	0.80224
-30		0.02650	0.44506	0.75181
-50		0.00030	0.23188	0.65351
-100		0	0.02189	0.43229

Table 6. Calculated temperature for $\alpha = \sqrt{10}$, $\beta = 1/\sqrt{10}$

η	$\xi = 0.1$	$\xi = 1$	$\xi = 10$	$\xi = 100$
100				1.00000
50				0.99982
30				0.98443
20			1.00000	0.92629
10		1.00000	0.99023	0.77095
5		0.99991	0.88924	0.65123
3		0.99124	0.78162	0.59767
2	1.00000	0.95269	0.71065	0.57011
1	0.99695	0.84233	0.63042	0.54220
0.5	0.95779	0.74671	0.58777	0.52816
0.3	0.90856	0.70146	0.57039	0.52254
0.2	0.87262	0.67757	0.56164	0.51972
0.1	0.82858	0.65295	0.55287	0.51690
0.05	0.80361	0.64040	0.54847	0.51550
0.03	0.79310	0.63534	0.54671	0.51493
0.02	0.78774	0.63280	0.54583	0.51465
0.01	0.78231	0.63025	0.54495	0.51437
0	0.77680	0.62770	0.54407	0.51409
-0	0.22320	0.37230	0.45593	0.48591
-0.01	0.22264	0.37205	0.45585	0.48588
-0.02	0.22209	0.37179	0.45576	0.48586
-0.03	0.22154	0.37154	0.45567	0.48583
-0.05	0.22044	0.37102	0.45549	0.48577
-0.1	0.21769	0.36975	0.45505	0.48563
-0.2	0.21226	0.36720	0.45417	0.48535
-0.3	0.20690	0.36466	0.45329	0.48507
-0.5	0.19639	0.35960	0.45153	0.48450
-1	0.17142	0.34705	0.44713	0.48310
-2	0.12738	0.32263	0.43836	0.48028
-3	0.09143	0.29854	0.42961	0.47746
-5	0.04221	0.25329	0.41223	0.47184
-10	0.00305	0.15767	0.36958	0.45780
-20	0	0.04731	0.28935	0.42989
-30		0.00876	0.21838	0.40233
-50		0.00009	0.11076	0.34877
-100		0	0.00977	0.22905

Table 7. Calculated temperature for $\alpha = \sqrt{10}$, $\beta = \sqrt{10}$

η	$\xi = 0.1$	$\xi = 1$	$\xi = 10$	$\xi = 100$
100				1.00000
50				0.99996
30				0.99697
20			1.00000	0.98587
10		1.00000	0.99781	0.95677
5		0.99997	0.97681	0.93465
3		0.99735	0.95549	0.92482
2	1.00000	0.98727	0.94180	0.91978
1	0.99851	0.95987	0.92661	0.91468
0.5	0.98231	0.93848	0.91865	0.91212
0.3	0.96439	0.92884	0.91542	0.91110
0.2	0.95232	0.92387	0.91380	0.91058
0.1	0.93846	0.91883	0.91219	0.91007
0.05	0.93100	0.91629	0.91138	0.90981
0.03	0.92794	0.91527	0.91105	0.90971
0.02	0.92640	0.91476	0.91089	0.90966
0.01	0.92485	0.91425	0.91073	0.90961
0	0.92329	0.91373	0.91056	0.90956
−0	0.76710	0.86265	0.89435	0.90443
−0.01	0.75152	0.85755	0.89273	0.90392
−0.02	0.73601	0.85244	0.89111	0.90340
−0.03	0.72058	0.84734	0.88949	0.90289
−0.05	0.68998	0.83715	0.88625	0.90186
−0.1	0.61538	0.81173	0.87814	0.89930
−0.2	0.47677	0.76129	0.86195	0.89417
−0.3	0.35610	0.71157	0.84578	0.88904
−0.5	0.17692	0.61522	0.81355	0.87879
−1	0.01488	0.40130	0.73393	0.85319
−2	0	0.12733	0.58198	0.80224
−3		0.02650	0.44506	0.75181
−5		0.00030	0.23188	0.65351
−10		0	0.02189	0.43229
−20			0	0.14130
−30				0.03033
−50				0.00036
−100				0

Figure Captions

Figure 1 Radiant heat flux between two parallel streams ($\beta = o$ for flow past a flat plate)

Figure 2 Temperature profile for $\beta = o$

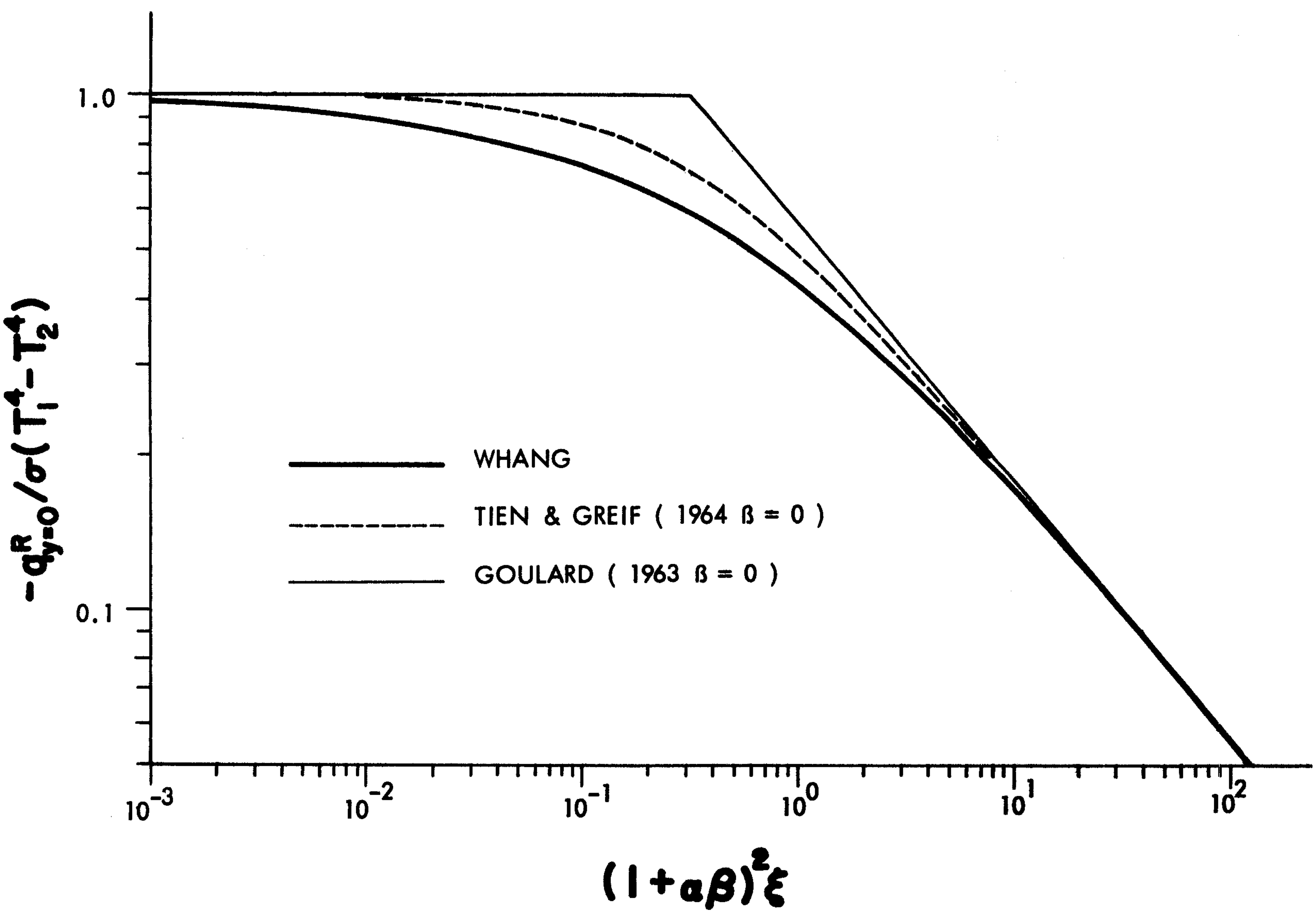
Figure 3 Temperature profile for $\alpha = 1$, $\beta = 1$

Figure 4 Temperature profile for $\alpha = 0.1$, $\beta = 1$

Figure 5 Temperature profile for $\alpha = 10$, $\beta = 1$

Figure 6 Temperature profile for $\alpha = (10)^{1/2}$, $\beta = (10)^{-1/2}$

Figure 7 Temperature profile for $\alpha = (10)^{1/2}$, $\beta = (10)^{1/2}$



1.0
0.1
$-q^R_{y=0}/\sigma(T_1^4 - T_2^4)$
WHANG
TIEN & GREIF (1964 ß = 0)
GOULARD (1963 ß = 0)
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
$(1+\alpha\beta)^2\xi$

Figure 1

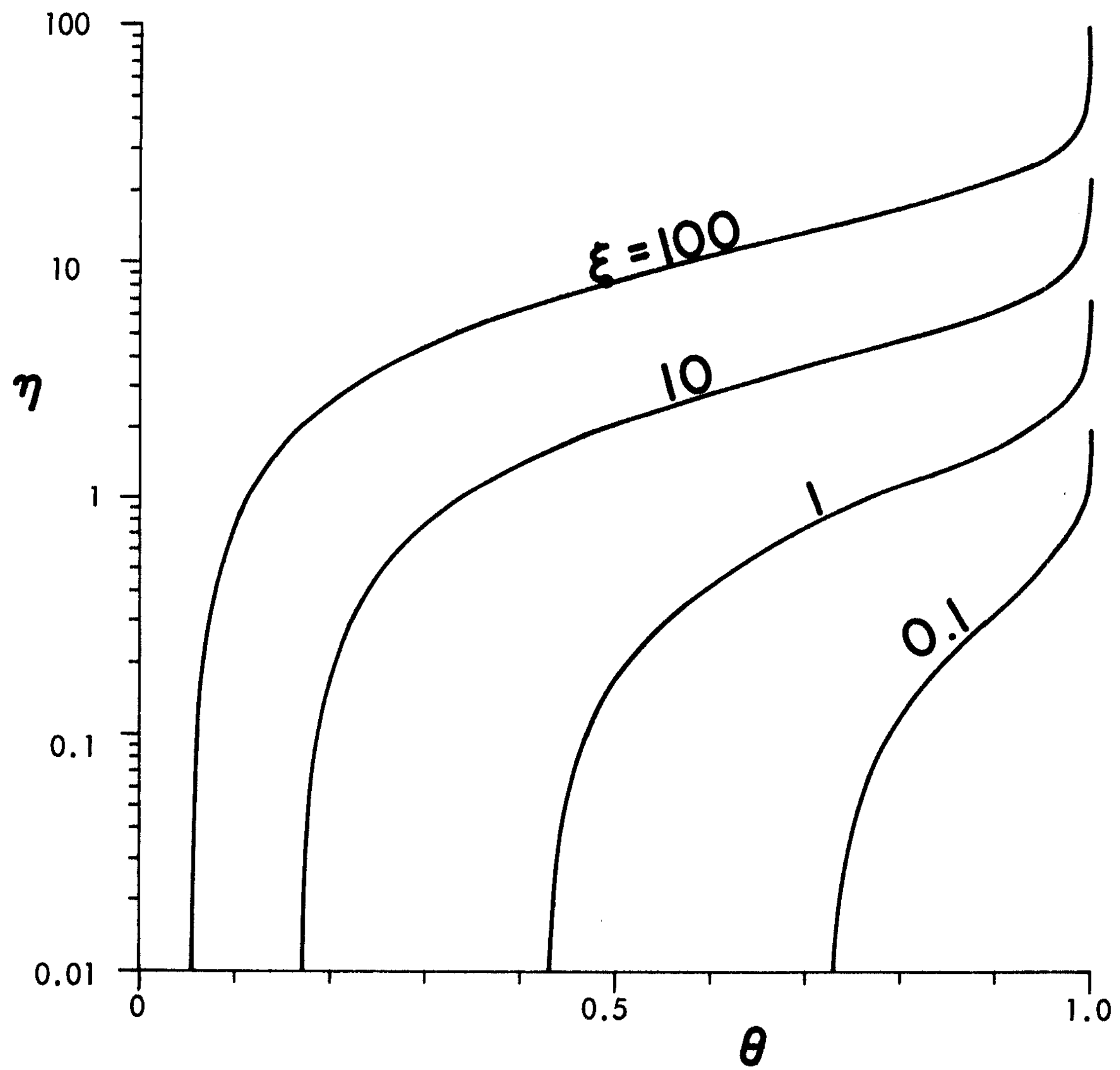


Figure 2

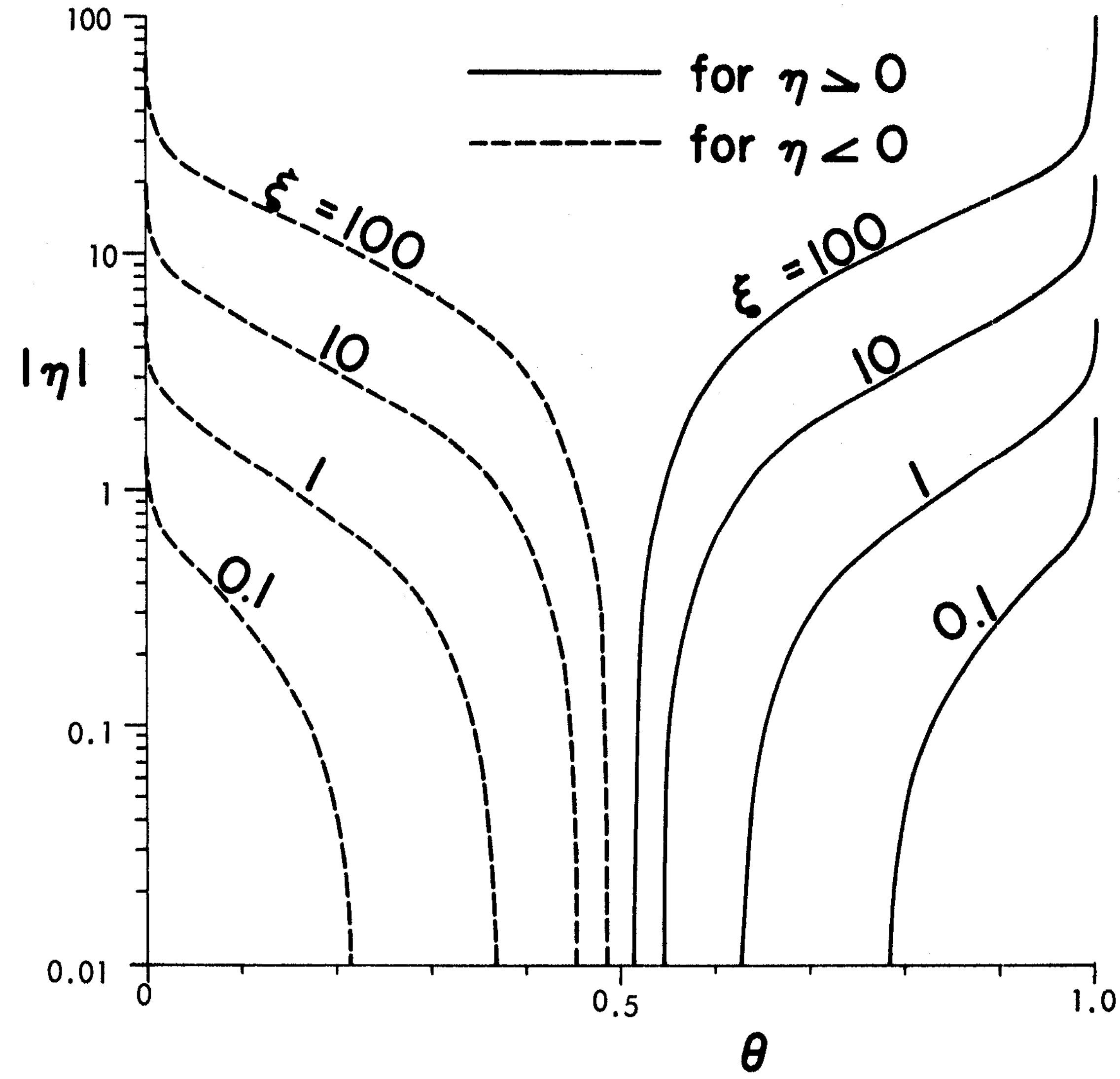


Figure 3

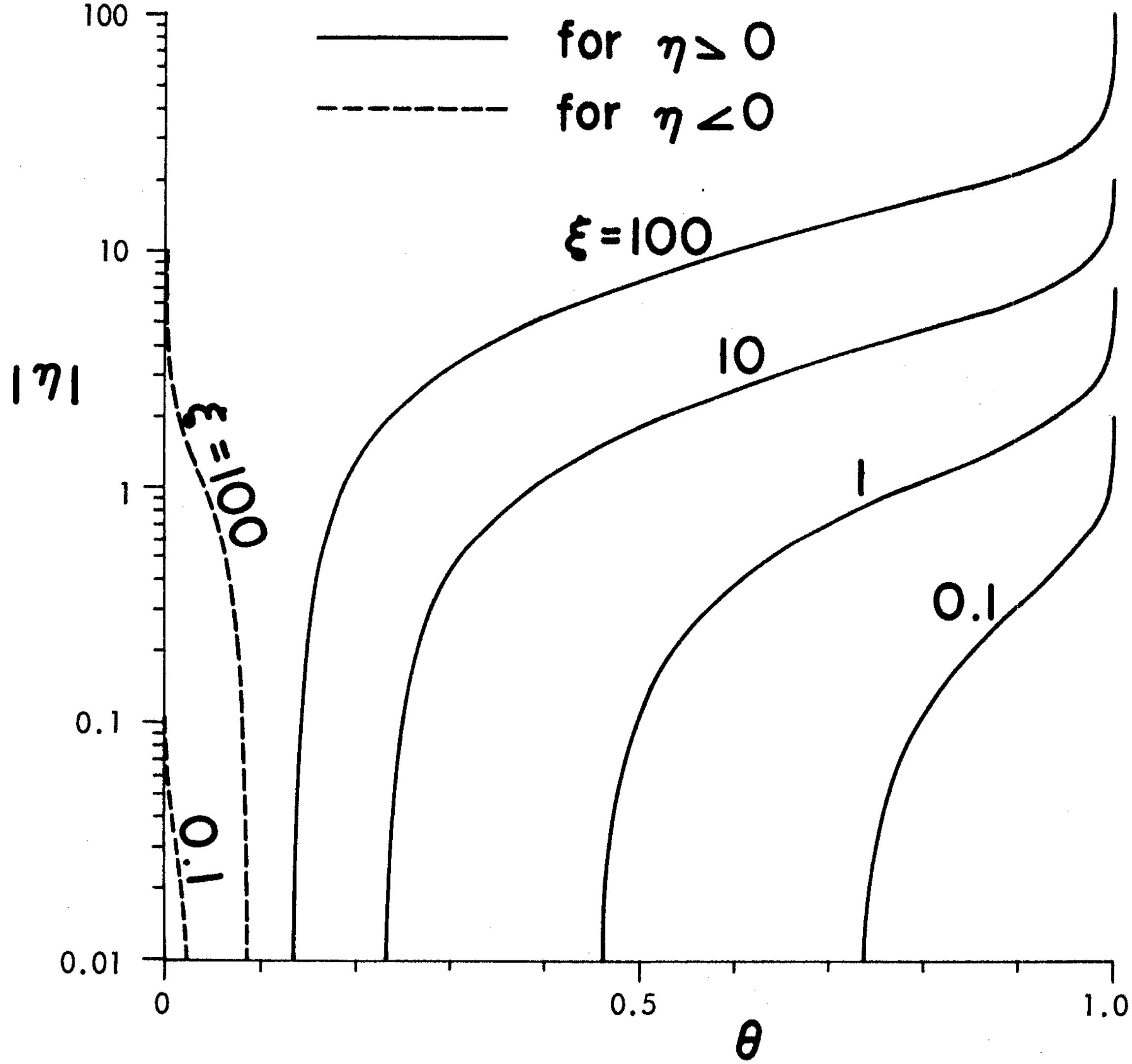


Figure 4

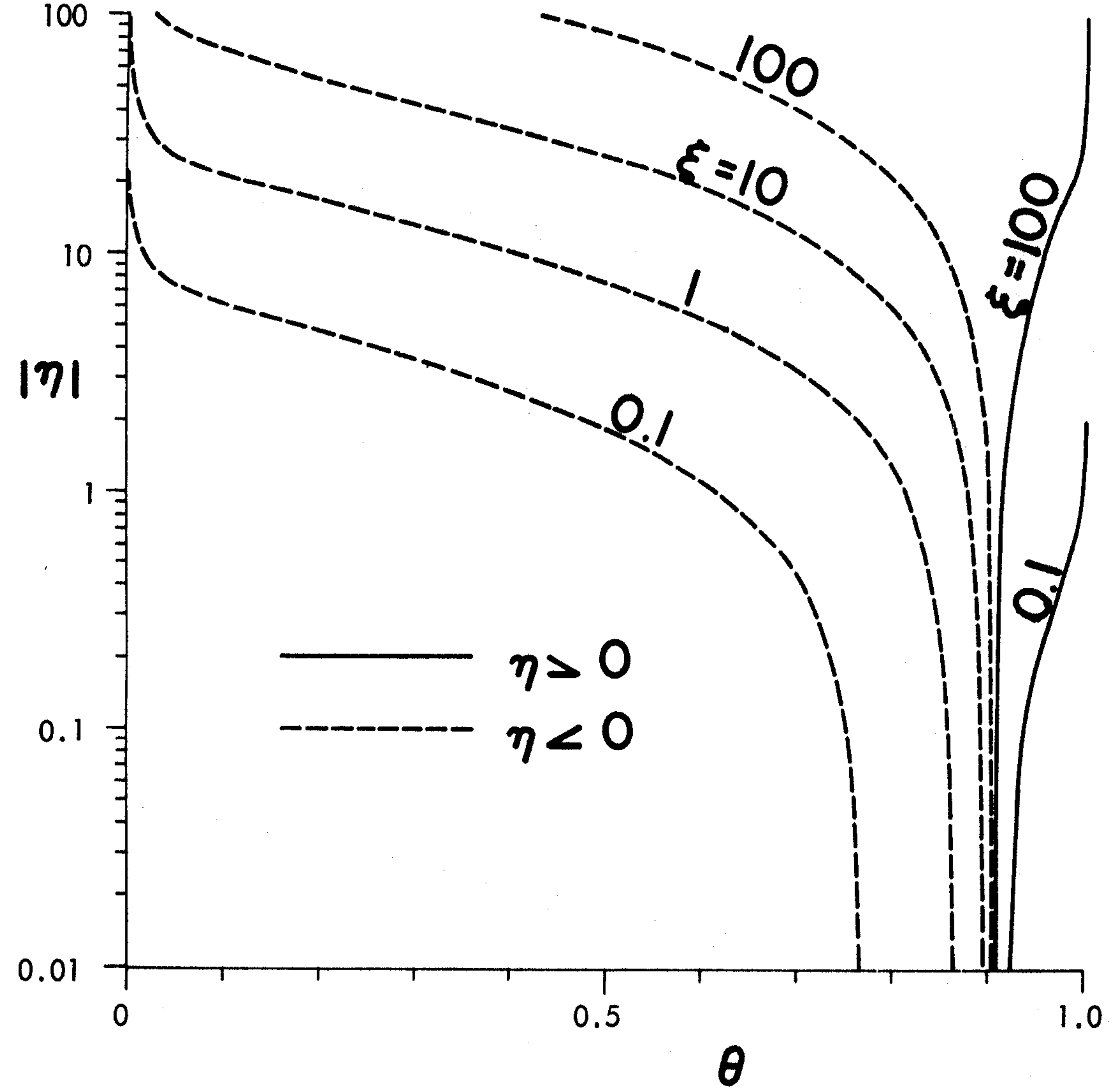


Figure 5

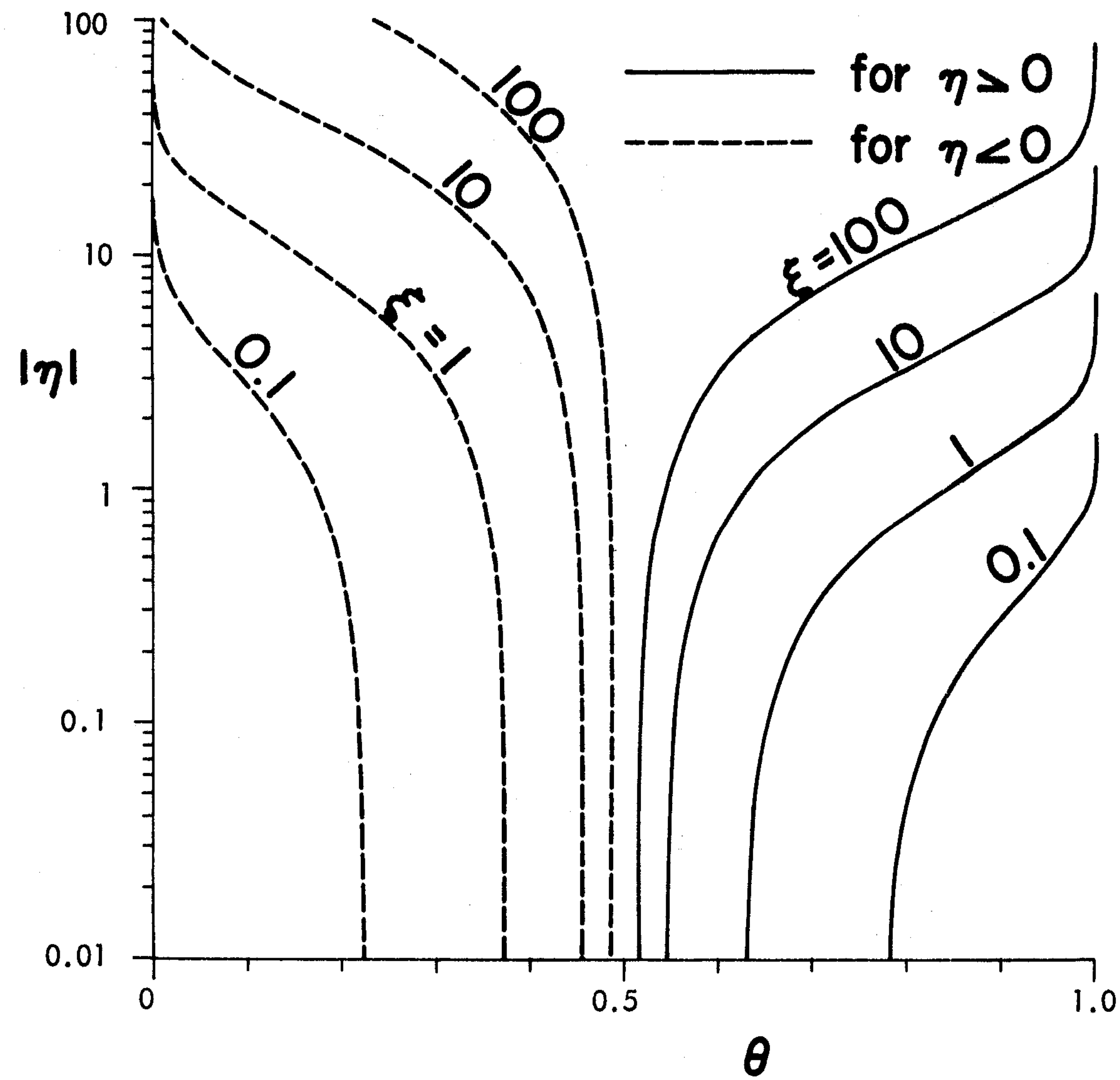


Figure 6

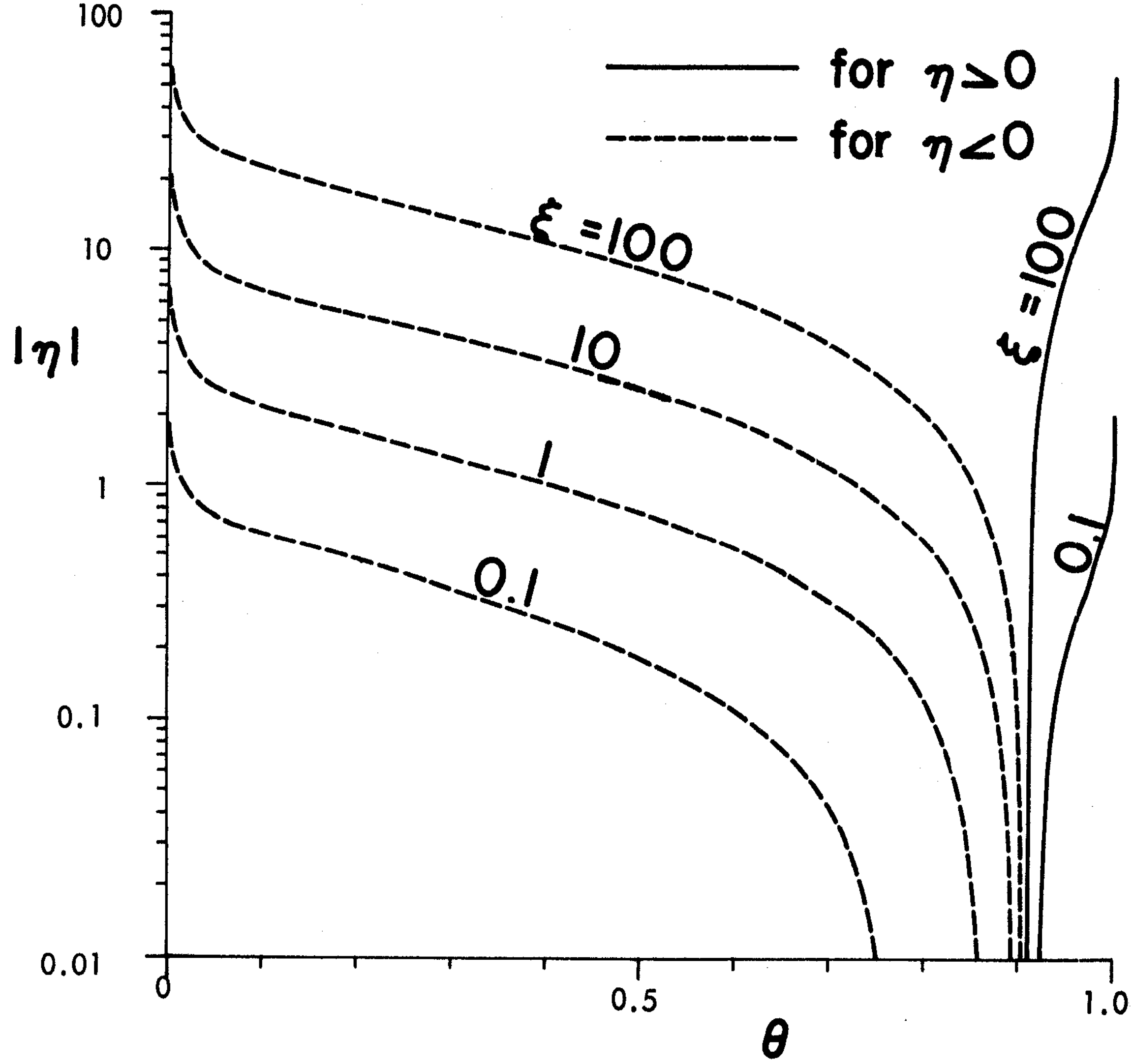


Figure 7