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Summary

A technique for synthesising high-efficiency, low-noise feeds for large circular paraboloids has been developed from a study of the focal-region fields. This has led to the concept of hybrid modes propagated inside cylindrical waveguides with anisotropic boundaries. These boundaries may be realised physically with corrugated structures.

The radiation patterns of hybrid-mode feeds have inherent axial symmetry and produce zero cross-polarization at the paraboloid aperture. Radially-shaped patterns may be constructed by superimposing higher-order hybrid modes. This requires only half the number of independently-generated modes as existing synthesis techniques.

1. INTRODUCTION

Large sums of money are now being spent on radio telescopes with very large apertures and precision surfaces. It is obviously wasteful to use feeds for these dishes which can reject up to 40% of the received energy because of poor response to the outer areas. The lag in feed design is even more striking when unwanted response to noise sources outside the dish is considered.

Although low efficiency is often tolerated for the sake of low side lobes, there are many applications in which maximum energy collection is the prime requirement. What is needed is a synthesis technique for designing an appropriate compromise for each specific task. Control of feed patterns would allow the designer to balance efficiency, side-lobes and aerial temperature and to produce symmetrical beams with low cross-polarization.

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Some years ago a theory of synthesis was developed for application to the 210-ft prime-focus paraboloid at Parkes. The concepts, however, are applicable also to Cassegrain feed systems. This method is related to the synthesis technique using multi-modes in circular pipes developed independently at the Jet Propulsion Laboratory, Pasadena¹, but is more general. Thus the patterns have inherent axial symmetry and zero cross polarization and this property is not restricted to small angles to the axis. In addition, only half the number of modes have to be independently generated.

The method of analyzing the problem provides a useful physical picture of feed operation and defines a theoretically ideal solution which can be used as a guide for practical development. In this short paper the treatment is necessarily qualitative; details will be published elsewhere.

2. ANALYTICAL APPROACH

Theoretical work was based on the concept of a feed capable of absorbing all of the focal plane energy produced when a linearly-polarized, uniform plane wave is incident normally on the dish aperture. Such a feed would have zero response to noise sources outside the dish for, by reciprocity, it must also be perfectly efficient when radiating and spillover must therefore be zero. The fields produced in the dish aperture would be uniform in amplitude, phase and polarization.

At optical wavelengths the intensity in the focal plane of a uniformly illuminated paraboloidal mirror (or lens) has the well known disk and ring pattern. It is interesting to speculate on the possibility (at micro-wavelengths) of introducing a boundary along one of the dark rings to trap the energy contained within that radius. Optical analyses, however, are not concerned with polarization and to answer the question the vector structure of the focal plane fields must be derived.

3. FOCAL REGION FIELDS

Figure 1 shows a uniform plane wave, polarized along OY incident normally on a paraboloid aperture. The field at a point in the focal region is the vector sum of the spherical wavelets radiated by the induced currents on the dish surface. For large dishes and short wavelengths the wavelets are nearly plane over a large region near the focus. Thus at Parkes, $f = 26.2$ m and at $\lambda = 10$ cm, the phase error is less than $\lambda/16$ within a 12λ -diameter area of the focal plane.

Plane waves arriving at the focus are polarized either parallel or normal to the plane of incidence (Fig. 1). Those in the first group generated by an annular ring of the dish at an angle θ combine to form the equiphase TM pattern of Fig. 2a in any transverse plane. In this diagram u is the generalized radius $2\pi\rho \sin \theta/\lambda$. The pattern propagates along the dish axis with velocity $v/\cos \theta$, where v is the free-space velocity. Similarly, the second group of waves generate the TE plane wave of Fig. 2b, moving in phase with the TM wave.

These patterns will be recognized as the TM_{1n} and TE_{1n} plane-wave solutions of Maxwell's equations appropriate for cylindrical waveguides. They have one circumferential periodicity because of the linearly-polarized excitation and n radial periodicities (to be determined when the boundaries are specified). However, the circles of Fig. 2 on which tangential E is zero do not coincide and the fields cannot both be bounded by the same conducting tube.

The total field generated by the ring is the sum of (a) and (b) and is shown in Fig. 2c. We call this the hybrid mode EH_{1n} , since both E and H have axial components. The power flow in a hybrid mode is axially-symmetric in bands between "dark" rings (shown dotted) on which the fields are entirely longitudinal. The transverse fields are linearly polarized for large f/D (common in optics), but curve in a complex way near the dark rings for f/D values usual in radio telescopes.

The total focal-region field is the sum of the hybrid modes generated by all the rings of the dish. The modes have different axial velocities, since θ is different for each ring. However,

all modes coincide in phase as they pass through the focal plane where, for large f/D , they combine into a linearly-polarized pattern. The field amplitudes are distributed in the well-known Airy pattern $J_1(u_0)/u_0$, where $u_0 = 2\pi\rho \sin \theta_0/\lambda$ and θ_0 is the semi-angle of the dish. This expression is, of course, the Fourier transform of the uniform aperture amplitude produced by the incident wave.

For the relatively low f/D values of radio telescopes, these simple relations do not hold. The focal-plane pattern produced by combining the in-phase hybrid modes has curved field lines and the amplitude in the bright rings varies circumferentially. The rings are also displaced from the radial positions defined by the above equation.

4. PATTERN SYNTHESIS WITH HYBRID MODES

The radiation fields of a hybrid mode have the form $E_\theta = F(\theta) \sin \phi$, $E_\phi = F(\theta) \cos \phi$ everywhere in the forward hemisphere ($0 \leq \theta \leq \frac{\pi}{2}$). Thus the radiation pattern has exact axial symmetry and produces zero cross-polarization as a paraboloid feed. This is a property shared with the elementary Huygen source², but the hybrid field relations are more general.

An ideal feed would need to extend over the entire focal plane to interact with all the received energy. Fortunately, energy concentration near the focus permits reasonable approximations with feed apertures several wavelengths in diameter. The symmetry property is not affected by this restriction.

Each hybrid mode radiating through a finite aperture produces a conical beam with phase centre in the centre of the aperture. Figure 3(a) shows $F(\theta)$ for the EH_{15} mode in a 7λ aperture; the semi-angle $\bar{\theta}$ of the cone is 48° and there is an axial secondary lobe. For a given $\bar{\theta}$, this lobe decreases with aperture size and in the limit, one mode may be said to illuminate one elementary annular ring of the dish.

A given radial illumination function over the dish can therefore be synthesized by superimposing in-phase hybrid modes in the feed aperture with appropriate relative amplitudes. The

required radial function will, of course, be specified by the Fourier transform of the desired response pattern of the dish. Figure 3(b) shows the synthesis of the flat-top beam for maximizing efficiency using the first 5 hybrid modes. In Fig. 3(c) the mode amplitudes have been adjusted to produce a tapered illumination for lower side lobes. The low spillover in both cases ensures low-noise performance.

By varying the phase relations of the modes in the feed aperture, dishes of non-parabolic shape could be correctly fed provided the dish concentrates the received energy into a reasonably small region. Examples are the spherical dish and a plane zone plate.

5. GUIDES FOR HYBRID MODES

For practical synthesis, a method of bounding the hybrid modes and guiding them to the radiating aperture is required. The solution to this problem can be developed in terms of circumferential and longitudinal surface reactances defined respectively as $X_z = -E_z/H_\phi$ and $X_\phi = E_\phi/H_z$, directed radially outwards. In a hybrid mode these satisfy the relation $X_z X_\phi = -Z_0^2$, where $Z_0 = 377$ ohms. Figure 4(a) illustrates the variation of X_z and X_ϕ with $\sin \bar{\theta}$ for a radius $\rho = \lambda$.

To enclose the fields at this radius, a cylindrical boundary must match both X_z and X_ϕ at every point and the boundary material must therefore be anisotropic. For example, material with X_z, X_ϕ values given by the dotted lines would support two hybrid modes specified by the discrete values of $\sin \bar{\theta}$ shown. Two other reactance combinations having more practical significance would support modes specified by the points A and B respectively.

In the former case, the boundary surface must have $X_z = \infty$ and $X_\phi = 0$ which can be satisfied with the circumferentially slotted structure of Fig. 4(b). The slots are approximately $\lambda/4$ deep and permit E_z , but prevent longitudinal current, so that $H_\phi = 0$. The edges of the flanges separating the slots allow circumferential current (and hence H_z) but require $E_\phi = 0$. For sufficient slots per wavelength the structure thus behaves like a continuous anisotropic surface with $X_z = \infty, X_\phi = 0$. Similarly the

points B represent a boundary with $X_z = 0$, $X_\phi = \infty$. This may be realized with the $\lambda/4$ longitudinal slot structure of Fig. 4(c) which requires $E_z = 0$ and $H_z = 0$. It will be noted that both structures treat E and H fields the same. This is consistent with the fact that the patterns of the E and H fields in hybrid modes are identical, apart from a 90° rotation circumferentially, and at corresponding points are related by $E = Z_0 H$.

Pattern synthesis with hybrid modes has the advantage that the anisotropic boundary automatically ensures symmetrical radiation patterns. Independent generation of TE and TM modes is unnecessary and the practical problem of mode control is substantially eased.

6. EXPERIMENTAL WORK

Figure 5 shows a circumferentially slotted structure for $\lambda = 11$ cm in which only the EH_{11} can propagate. The structure was made by bolting together a stack of rings with different internal diameters alternating. Different combinations of ring thickness allowed the slot/flange ratio and pitch of the structure to be varied.

Cavity-type measurements with a plunger in one end were made to determine guide wavelength $\lambda_g = \lambda / \cos \theta$ over a range of wavelengths. Over a 1.5 to 1 range in which the EH_{11} mode alone was propagated, agreement with values computed from the reactances of the structures was better than 2%. The effect of different slot/flange ratios and pitches was small except approaching cut-off when the predicted trend was observed.

Measured radiation patterns were also virtually independent of these parameters. Pattern symmetry was obtained at a frequency higher than the resonant frequency of the slots ($X_z = \infty$). It is thought that this effect is probably due to currents on the exterior face of the $\lambda/4$ flange surrounding the aperture. To preserve the symmetrical character of the fields outside the aperture, the anisotropic boundary should be extended to the exterior surfaces of the guide. Experiments on such exterior surfaces and also on various internal structures supporting multiple hybrid modes are in progress.

7. ACKNOWLEDGEMENT

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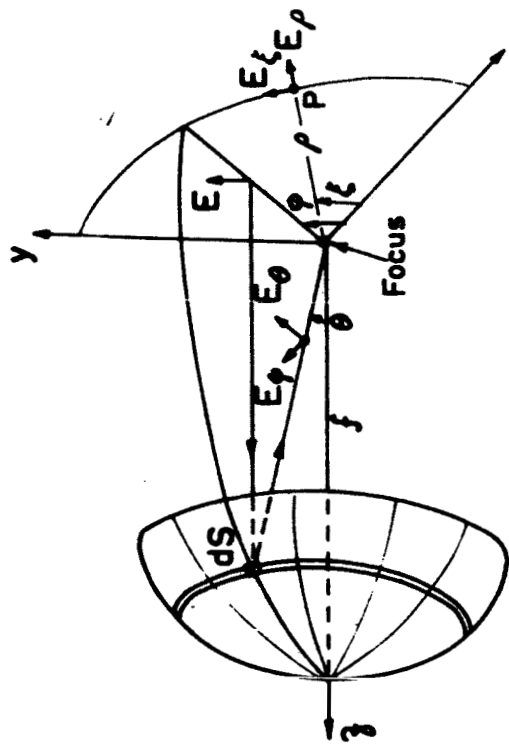
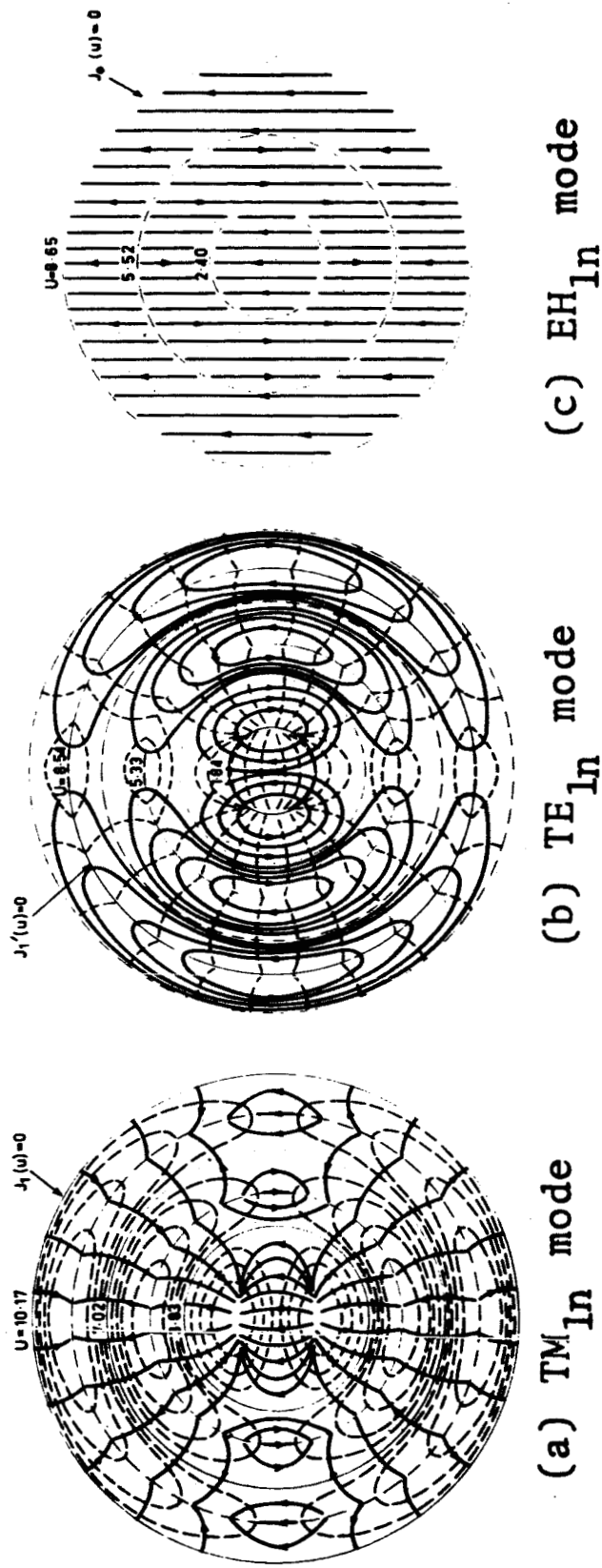


Fig. 1 Geometry for focal region field analysis



(a) TM_{1n} mode (b) TE_{1n} mode (c) EH_{1n} mode
 Fig. 2 Focal-plane fields produced by annular zone of dish

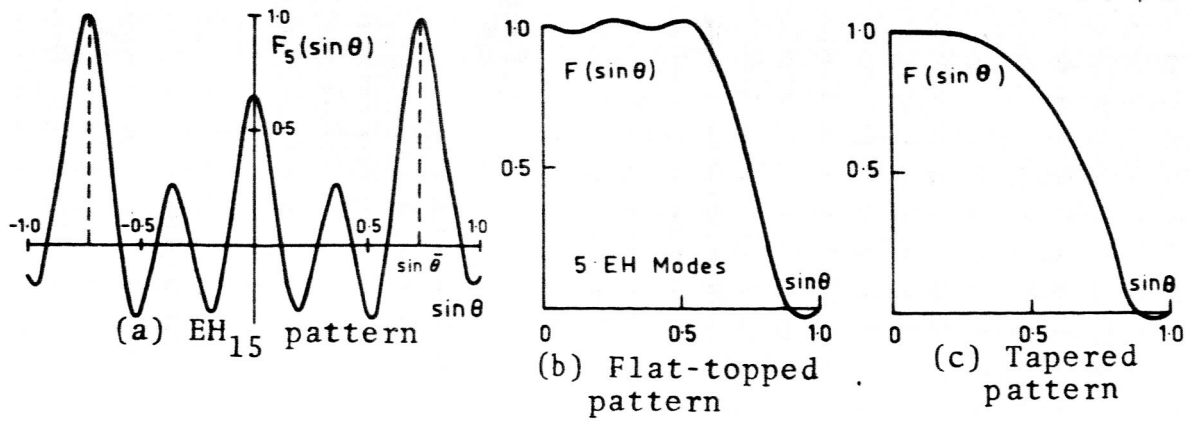
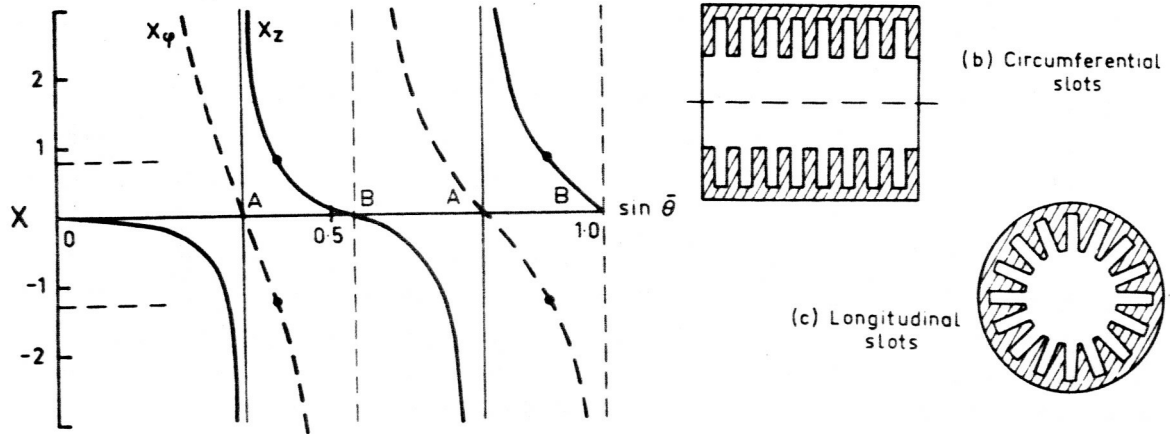


Fig. 3 Pattern syntheses with hybrid modes



(a) Surface reactance curves

Fig. 4 Anisotropic boundary surfaces

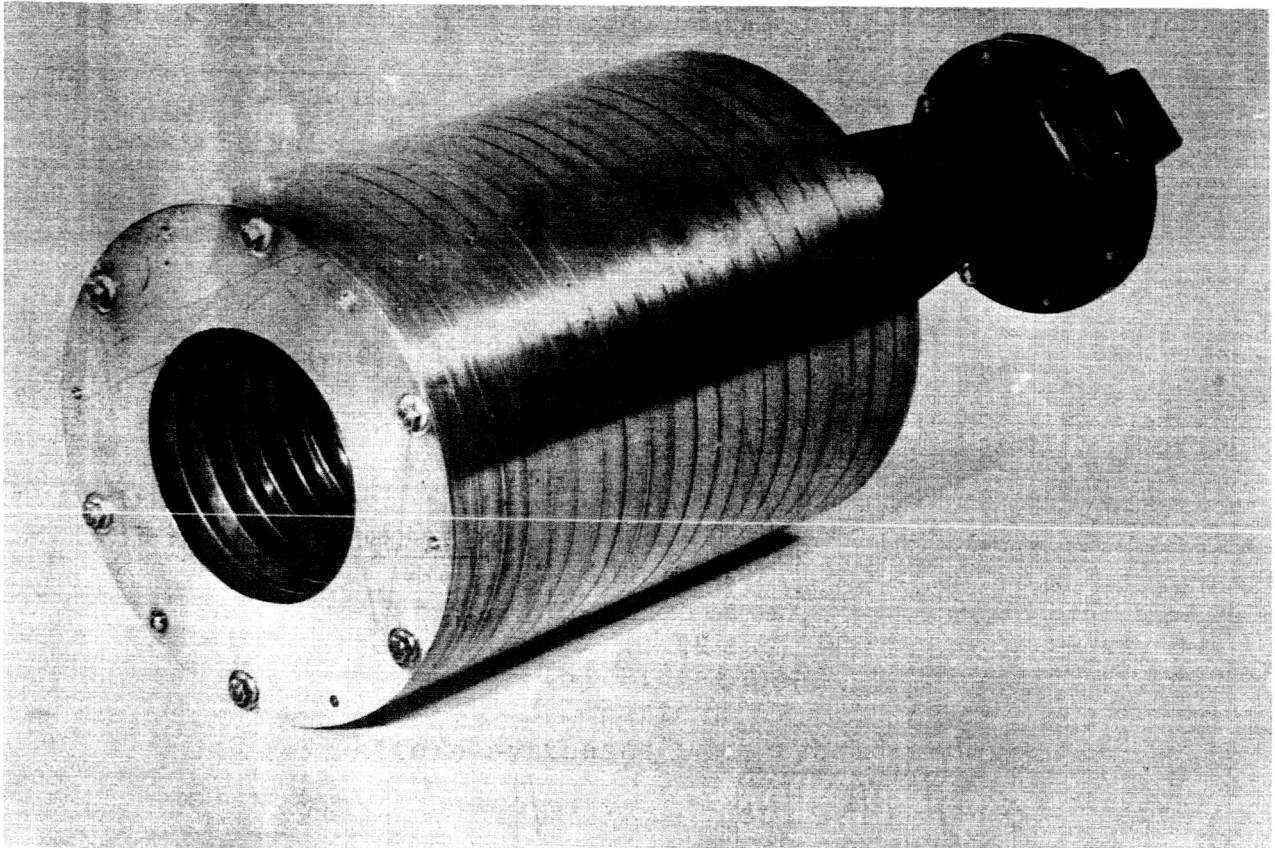


Fig. 5 Experimental slotted structure