DRI #2337

FINAL REPORT

ELECTROMAGNETIC WAVE CHARACTERISTICS OF A FULLY IONIZED GAS

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ABSTRACT

In this report, the equations for incident, reflected and transmitted waves are given for a magneto-plasma slab of a thickness which is of the order of a wavelength of the incident signal. The transmitted power term is derived and compared, in the cold, collisionless approximation, to data taken from microwave interferometer measurements on a quiescent cesium magneto-plasma. A theoretical development of the MHD energy and momentum equations is carried out in terms of the electromagnetic stress tensor to give expressions in terms of the field terms only. This coupled with the wave solution gives another way to attack the problem of plasma motion due to the forces of an incident EM field.

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LIST OF SYMBOLS

A	-	incident magnetic vector amplitude
a	-	plasma slab thickness
$\overline{B}$	-	magnetic flux density vector
$\overline{B}_0$	-	static magnetic flux density vector
$\overline{D}$	-	electric flux density vector
$\overline{E}$	-	electric field intensity vector
e	-	electronic charge, natural log base
f	-	distribution function, frequency
$\overline{H}$	-	magnetic field intensity vector
I	-	unit matrix
$\overline{J}$	-	electric current density
j	-	imaginary unit
K	-	Boltzmann's constant, off diagonal element of inverse dielectric tensor
K_0	-	free space propagation constant
K^*	-	plasma propagation constant
M	-	diagonal element of universe dielectric tensor
m	-	mass
N	-	plasma relative permeability
$\overline{n}$	-	normal unit vector
P	-	normalized plasma resonance
P_T	-	transmitted power
^2P	-	pressure tensor
P_c	-	momentum transfer due to collisions
p	-	scalar pressure
q_i	-	species charge
q	-	off diagonal element of dielectric tensor
$\overline{S}$	-	poynting vector

^2S - electromagnetic stress tensor

T - temperature in degrees Kelvin

T_c - energy transfer due to collisions

$\overline{v}$ - velocity vector

$\overline{v}_r$ - random velocity vector

x, y, z - cartesian coordinates

ϵ_0 - free space dielectric constant

ϵ', ϵ'' - diagonal elements of dielectric tensor

η_0 - free space characteristic impedence

λ - wavelength

μ_0 - free space permeability

π - 3.1416

ρ - mass density

ρ_e - charge density

σ^* - effective surface charge density

τ - kinetic stress tensor

ω - angular frequency

ω_p - plasma resonance frequency

ω_b - magnetic resonance frequency

I. INTRODUCTION

A "plasma" is often referred to in physics as a fourth state of matter since it has characteristics which are not well covered by either solid, liquid or gas theory. Most simply stated - a "plasma" is a high temperature gaseous state in which an appreciable number of the gas atoms are ionized. The importance of the study of plasmas is indicated by the fact that virtually the entire universe is made up of matter in the plasma state - i.e., the sun, stars, space and the earth's ionosphere are all in the plasma state. With respect to man made plasmas; flame, gas filled vacuum tubes, neon and florescent lamps, ion engines, MHD converters, and laboratory plasmas for study purposes are all examples of plasmas.

Viewed as a media for electromagnetic propagation, a plasma in a magnetic field is refractive, lossy, dispersive, resonant, anisotropic, nonreciprocal, nonlinear and inhomogeneous. Thus, any useful approach to the study of electromagnetic wave properties of a plasma must necessarily entail many simplifying assumptions.

Any completely adequate treatment of a plasma must necessarily be statistical in nature. The most satisfactory approach to date has been through the use of the Boltzmann equation;

$$(1)\quad \frac{\partial f_k}{\partial t} + \overline{v}\cdot \nabla_r f_k + q_k \ (\overline{E} + \overline{v} \times \overline{B})\cdot \nabla_v f_k = \left(\frac{\partial f_k}{\partial t}\right) \quad \text{collisions}$$

which describes the rate of change of the distribution function f_k for the kth species, of charge q_k and velocity $\overline{v}$. Equation (1) together with Maxwell's equations;

$$(2)\quad \nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t} \qquad\qquad (4)\quad \nabla \cdot \overline{B} = 0$$

$$(3)\quad \nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t} \qquad\qquad (5)\quad \nabla \cdot \overline{D} = \rho$$

along with definitions for macroscopic quantities such as charge density ρ and current density $\overline{J}$ i.e.,

$$(6)\quad \rho = \sum_k q_k \oint_v f_k d\overline{v}_k \qquad\qquad (7)\quad \overline{J} = \sum_k q_k \oint_v \overline{v} f_k dv_k$$

can, in theory, be solved for any particular set of boundary conditions to give a knowledge of the nature of the field - plasma interactions.

Examination of equation (1) reveals the major difficulty in any solution of a plasma problem; the Boltzmann equation combines the effects of external and internal forces through the use of the third and fourth terms respectively of equation (1). The big problem is the distinction in the terms, with respect to the long range particle interactions i.e., coulomb interactions; here, the distinction becomes somewhat vague since the collisions are no longer isolated binary events.

The production of highly ionized, low temperature plasmas can be of considerable assistance in the quest for correlation between theory and experiment provided the coulomb collision terms can be better understood. It is the coulomb scattering aspect of collision theory that has become the major microscopic area of interest in the work we are conducting.

In order to attempt a complete solution of the problem without a cumbersome solution of Boltzmann's equation, we will conduct most of the initial effort in the macroscopic world, i.e., a solution of the wave equation with respect to the given boundary conditions and a solution of the energy equation. This approach should provide the most information with regard to correlation between experiment and theory.

Many people have approached similar problems from almost as many different view points. Our basic approach to the solution of the wave equation is to treat the plasma as an anisotropic media the account for which is taken up in the dielectric tensor.

$$(7) \quad \overline{D} = [\epsilon]\ \overline{E} \quad \text{where } [\epsilon] = \begin{bmatrix} \epsilon' & -iq & 0 \\ iq & \epsilon' & 0 \\ 0 & 0 & \epsilon'' \end{bmatrix}$$

where the anisotrophy is produced by a uniform magnetic field in the z direction. The use of the above tensor relation and its reciprocal enable us to solve Maxwell's equations to give us the proper electromagnetic equations. However, due to the mechanics of particle interactions, there are other types of waves produced which can be considered only through the solution of the momentum equation which also (along with the continuity equation) completes the set of equations that must be satisfied at the boundaries i.e., the boundary conditions. This indicates that for a complete macroscopic solution, we must solve the combine set of Maxwell's and the mechanical equations that describe the wave plasma interactions.

Some of the currently accepted theory for plasma distribution functions, and waves in plasmas and their boundary conditions is covered in appendices A and B respectively.

The theoretical treatments which follow are subject to the following simplifying assumptions in accordance with the properties of the plasma actually produced in our laboratory;

(1) The plasma is of two species - (1) electrons and (2) singly ionized positive ions.

(2) The plasma is completely ionized. ($X > 90\%$)

(3) The plasma is "cool" (Te and Ti in the range of 2000 to 2500°K).

(4) The plasma is in equilibrium except for EM perturbations.

(5) There are no work producing fields other than the incident EM wave.

(6) The static magnetic field is spatially and temporarily uniform throughout the region of interest.

(7) The amplitude of the incident EM wave is sufficiently small to allow the linearization of the Boltzmann equation.

(8) The boundaries of the plasma are sufficiently well defined to enable the use of boundary conditions as outlined in Appendix B.

These assumptions, although "unreal" in some instances, make the problem at least solvable and produce amazingly useful results in the gross sense.

II. THE BOUNDARY VALUE PROBLEM OF A MAGNETO-PLASMA SLAB IN THE PRESENCE OF A EM PLANE WAVE

In the treatment that follows, we are considering the solution of Maxwell's equation in a lossy anisotropic plasma. The approach used here is to assume that any effects due to the media are taken into account in the dielectric matrix plus the modification of the boundary conditions due to the additional equations that must be satisfied. The approach to be used in this solution is that of Chapman and Enskog[3] where the moment equations of Boltzmann's equation are used to give the so called MHD or macroscopic equations for continuity, momentum and energy, along with Maxwell's equations.

In most plasmas, the negative charges are electrons and the positive charges are ionized atoms which indicated that the negative particles (electrons) are in general much more mobil with respect to their interaction with an electromagnetic wave. Thus, it is common practice to omit the motion of ions in the study of EM wave - plasma interactions unless the frequency of the wave is near one of the ion resonance frequencies. In the particular case we are treating, the omission of ion effects is justified because for a density of 10^{12} ions per cm^3, the ion gyro frequency f_{bi} = 11.5 kcps and the ion plasma frequency f_{pi} = 18 mcps which compares to the electron values of f_{be} = 5.67 mcps and f_{pe} = 8.9 gcps and since we are operating in the x band frequency range (8.4 - 12 gcps) it is apparent that the ion effects can be ignored.

Now, from Appendix II, we will write the dielectric matrix as:

$$(1.1)\quad [\epsilon] = \begin{bmatrix} \epsilon' & -iq & 0 \\ iq & \epsilon' & 0 \\ 0 & 0 & \epsilon'' \end{bmatrix}$$

For $\overline{B}_0 = (0, 0, B_0)$

Now consider the form of Maxwell's equations:

$$(1.2)\quad \nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$(1.3)\quad \nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t}$$

where (1.3) has rolled ohmic losses up in the $\frac{\partial \bar{D}}{\partial t}$ term. Now within the plasma, this means that;

$$(1.4)\quad \bar{D} = [\epsilon]\ \bar{E}$$

Now consider the case for a slab geometry in the time harmonic case (see Figure 1).

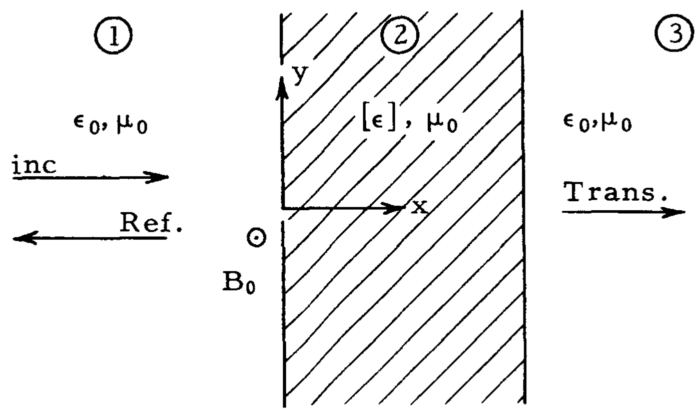

Figure 1. Plasma Slab Geometry

where B_0 is out of the paper and E_{inc} is in the y direction this implies (see Appendix B) that inside the plasma region;

$$(1.5)\quad \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{k^2}{MN}\right) H_z = 0$$

This along with equation (1.2) and the boundary conditions of Appendix B, i.e.,

$$(1.6)\quad \begin{aligned} &a)\ \bar{n} \times (\bar{E}_p - \bar{E}_v) = 0 \\ &b)\ \bar{n} \cdot (\bar{E}_p - E_v) = \sigma_s^* \\ &c)\ \bar{n} \times (\bar{H}_p - \bar{H}_v) = \bar{j}^* \\ &d)\ \bar{n} \cdot (\bar{H}_p - \bar{H}_v) = 0 \\ &e)\ \bar{j}^* \times \langle B \rangle_{avg} + \sigma^* \langle E \rangle_{avg} - \bar{n}\,(P_p - P_v) = 0 \end{aligned}$$

Since the magnetic field obeys the simple wave equation (1.5) above, the easiest approach is to solve for $\overline{H}$, then use Maxwell's equations to find $\overline{E}$. The case considered here will be that of normal incidence with respect to the plasma slab. Also, the electric field vector is oriented such that it is perpendicular to the static magnetic field. This would imply a solution of the form;

$$(1.7)\quad H_{z1} = Ae^{-jk_0x} + Be^{jk_0x}$$

Outside the plasma in Region ①.

Now within the plasma equation (1.5) is satisfied which implies;

$$H_{z2} = X(x)\; Y(y)$$

$$(1.8)\quad Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y} + XY\frac{k_0^2}{MN} = 0$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \frac{k_0^2}{MN} = 0$$

Upon separating the variables and imposing the boundary condition that $H_{z2}\Big|_{x=0} = H_{z1}\Big|_{x=0}$ for all y then;

$$(1.9)\quad H_{z2} = Ce^{-j\frac{k_0}{(MN)^{\frac{1}{2}}}x} + De^{j\frac{k_0^2}{(MN)^{\frac{1}{2}}}x}$$

and in region (3);

$$(1.10)\quad H_{z3} = Ee^{-jk_0x} + Fe^{jk_0x}$$

Outside the plasma in region① we have hypothesized a uniform plane wave with $\overline{E} = (0,\ E,\ 0)$, thus through the use of the curl relation;

$$(1.11)\quad E_{y1} = \frac{k_0}{\omega\epsilon_0}\left(Ae^{-jk_0x} - Be^{jk_0x}\right)$$

while inside the plasma region the anisotrophy causes the $\overline{E}$ field to rotate in the x-y plane;

$$(1.12)\quad E_{x_2} = \frac{jKk_0}{\epsilon_0\,\omega(MN)^{\frac{1}{2}}}\left[-\,Ce^{-j\,\frac{k_0}{(MN)^{\frac{1}{2}}}x} + De^{j\,\frac{k_0}{(MN)^{\frac{1}{2}}}x}\right]$$

$$E_{y_2} = \frac{Mk_0}{\epsilon_0\,\omega(MN)^{\frac{1}{2}}x}\left[Ce^{-j\,\frac{k_0}{(MN)^{\frac{1}{2}}}x} - De^{j\,\frac{k_0}{(MN)^{\frac{1}{2}}}x}\right]$$

and in region (3) which is again free space;

$$(1.13)\quad E_{y_3} = \frac{k_0}{\epsilon_0\omega}\left[Ee^{-jk_0x} - Fe^{jk_0x}\right]$$

Defining the terms;

$$k^* = \frac{k_0}{(MN)^{\frac{1}{2}}} \quad ; \quad \left(\frac{M}{N}\right)^{\frac{1}{2}} = R^*$$

and imposing the boundary conditions from equations (1.6) we get the relations

a) $A + B = C + D$

b) $F = O$ (since we must have an outgoing wave)

c) $A - B = R^*(C - D)$

d) $Ce^{-jk^*a} + De^{jk^*a} = Ee^{-jk_0a}$

e) $R^*\left[Ce^{-jk^*a} - De^{jka^*}\right] = E^{-jk_0a}$

Solving for B, C, D and E in terms of A gives the results;

$$(1.15)\quad B = \frac{A(R^{*2}-1)(1 - e^{j2k^*a})}{(R^* + 1)^2\, e^{j2k^*a} - (R^*-1)^2}$$

$$(1.16)\quad C = \frac{2A(R^*+1)\, e^{j2k^*a}}{(R+1)^2\, e^{j2k^*a} - (R^*-1)^2}$$

$$(1.17)\quad D = \frac{2A(R^*-1)}{(R^*+1)^2\, e^{j2k^*a} - (R^*-1)^2}$$

$$(1.18) \qquad E = \frac{4AR^* e^{jk_0a} e^{jk^*a}}{(R^*+1)^2 e^{j2k^*a} - (R^*-1)^2}$$

Inserting the expressions above into the field relations (1.7), (1.9), (1.10), (1.11), (1.12), and (1.13) gives the expressions for the fields in terms of the plasma physical properties and its boundary conditions;

$$(1.19) \qquad H_z^1 = A \left\{ e^{-jk_0x} + \frac{(R^{*2}-1)(1-e^{j2k^*a})\, e^{jk_0x}}{(R^*-1)^2 e^{j2k^*a} - (R^*-1)^2} \right\}$$

$$(1.20) \qquad H_z^2 = \frac{2A\,(R^*+1)\, e^{jk^*a}}{(R^*+1)^2 e^{j2k^*a} - (R^*-1)^2} \left\{ e^{-jk^*x} + e^{-jk^*a} \left(\frac{R^*-1}{R^*+1}\right) e^{jk^*x} \right\}$$

$$(1.21) \qquad H_z^3 = \frac{4AR^* e^{j(k_0a+k^*a)} e^{-jk_0x}}{(R^*+1)^2 e^{j2k^*a} - (R^*-1)^2}$$

$$(1.22) \qquad E_y^1 = \eta_0 A \left\{ e^{-jk_0x} - \frac{(R^{*2}-1)(1-e^{j2k^*a})}{(R^*+1)^2 e^{j2k^*a} - (R^*-1)^2} e^{jk_0x} \right\}$$

$$(1.23) \qquad E_x^1 = 0$$

$$(1.24) \qquad E_y^2 = \frac{(R^*+1)2AR^*\eta_0 e^{jk^*a}}{(R+1)^2 e^{j2k^*a} - (R^*-1)^2} \left\{ e^{-jk^*x} - e^{-jk^*a} \frac{R^*-1}{R^*+1} e^{jk^*x} \right\}$$

$$(1.25) \qquad E_x^2 = \frac{jK\eta_0 2A(R^*+1) e^{j2k^*a}}{R^*[(R^*+1)^2 e^{j2k^*a} - (R^*-1)^2]} \left\{ -e^{-jk^*x} + e^{-jk^*a} \left(\frac{R^*-1}{R^*+1}\right) e^{jk^*x} \right.$$

$$(1.26) \qquad E_y^3 = \frac{\eta_0 4AR^* e^{j(k_0+k^*)a} e^{-jk_0x}}{(R^*+1)^2 e^{j2k^*a} - (R^*-1)^2} = \eta_0 H_z^3$$

(1.27) $E_x^3 = 0$

These equations can be used to describe the nature of the field inside the plasma provided some previous knowledge of the dielectric tensor is given, i.e. The elements of (1.1). Now in the matching of boundary conditions for the magnetic field, it was assumed that there was negligible surface current. However, there is an effective surface charge density σ_i* on each of the surfaces normal to incidence such that;

$$(1.28) \quad E_x^2 \Big|_{x=0} = \sigma_1^* = \frac{jK\eta_0 2A(R^*+1)\,e^{j2k^*a}}{R^*[(R^*+1)^2 e^{j2k^*a} - (R^*-1)^2]} \left[e^{-jk^*a}\left(\frac{R^*-1}{R^*+1}\right) - 1 \right]$$

$$(1.29) \quad E_x^2 \Big|_{x=a} = \sigma_2^* = \frac{jK\eta_0 2A(R^*+1)e^{j2k^*a}}{R^*[(R^*+1)^2 e^{j2k^*a} - (R^*-1)^2]} \left[\left(\frac{R^*-1}{R^*+1}\right) - e^{-jk^*a} \right]$$

This implies that

$$(1.30) \quad \sigma_1^* - \sigma_2^* = \frac{jK\eta_0 4Ae^{j2k^*a}(1-e^{-jk^*a})}{R^*[(R^*+1)^2 e^{j2k^*a} - (R^*-1)^2]}$$

By imposing the boundary condition (1.6e) and assuming $P_v = 0$; along with the condition that $\langle E \rangle_{avg} = \frac{\overline{E}_p + \overline{E}_v}{2}$

so that at the plasma boundaries;

$$(1.31) \quad P_x \Big|_{x=0} = \sigma_1^* E_x \Big|_{x=0} = \left[E_x \Big|_{x=0} \right]^2$$

$$(1.32) \quad P_x \Big|_{x=a} = \sigma_2^* E_x \Big|_{x=a} = \left[E_x \Big|_{x=a} \right]^2$$

Thus a pressure gradient due to the electric field exists across the plasma,

$$(1.33) \quad P_x \Big|_{x=0} - P_x \Big|_{x=a} = \frac{16K^2\eta_0^2 A^2 e^{j4k^*a}(1-e^{-j2k^*a})}{R^{2*}[(R^*+1)^2 e^{j2k^*a} - (R^*-1)^2]^2}$$

This pressure gradient, which is a function of the distance a, is a result of the longitudinal field variations which create a longitudinal motion of particles as will be discussed in the next section.

Equations (1.19) through (1.27) demonstrate the fact that microwave diagnostics of a plasma using the relations given in Appendix B for microwaves in a plasma can be erroneous if the dimension of the plasma are not very large as compared to the wavelength of the incident wave. Since the microwave measurement scheme consists primarily of comparing the transmitted wave's attenuation and phase shift against a known standard, it is obvious from equations (1.21) and (1.27) that the direct interpretation of the measured attenuation and phase shift as being the sole results of the plasma dielectric properties will be erroneous unless a is very large. The same sort of problem arises in the transverse directions, if these dimensions are small compared to a wavelength; however, this problem will not be treated here.

The plasma we have succeeded in generating in our laboratory has large characteristic dimensions in the transverse directions but the longitudinal length is approximately equal to 1.5 cm. which is one-half wavelength at 10 gc, this being the median frequency of our microwave signal generator. Thus, any conclusions we make in correlating theory and experiment will have to include the limitations imposed by the boundaries.

In our measurements, we are at present concerned mostly with the transmitted power. Now from equations (1.21) and (1.26) we can calculate the power flow. The expression for transmitted power is written as;

$$(1.34) \qquad \overline{E} \times \overline{H} = \eta_0 \left\{ \frac{16A^2R^{*2}\left(\mathrm{Re}\; e^{j[(k_0+k^*)a-k_0z)]}\right)^2}{[(R^*+1)^2 e^{j2k^*a} - (R^*-1)^2]^2} \right.$$

Now the phase term due to the plasma is given by the term e^{jk^*a}; since in this case it has very little effect on the amplitude, the term e^{j2k^*a} can be replaced by a magnitude term. In this development, therefore, e^{j2k^*a} will be replaced by -1. Since for our experiments $k^* \approx \frac{k_0}{2}$ for the cold collisionless case, i.e.;

(1.35) $2k^{*}a \approx \pi$ where $a \approx \frac{\lambda_0}{z}$ at 10 gcps.

i.e., assume $\omega > \omega p$. Then;

$$(1.36) \quad \overline{E} \times \overline{H} \approx \frac{\eta_0 4A^2M}{(M+1)^2} \cos^2 \left\{ k_0 \left[x - a \left(\frac{M^{\frac{1}{2}}+1}{M^{\frac{1}{2}}} \right) \right] \right\}$$

where $M = \frac{\epsilon' \epsilon_0}{(\epsilon')^2 - g^2}$

ϵ' being the low temperature approximation for k_{xx} and k_{yy} as defined in equation (A-15) and iq being the low temperature approximation for $k_{yx} = - k_{xy}$.

Using the above relations it follows that;

$$(1.37) \quad \frac{M}{(M+1)^2} = \frac{\omega(\omega L^2 - \omega\omega_b^2 - \omega_p^2 L)(L^2 - \omega_b^2)[(\omega L^2 - \omega\omega_b^2 - \omega_p^2 L)^2 - \omega_p^4 \omega_b^2]^2}{[\omega(\omega L^2 - \omega\omega_b^2 - \omega_p^2 L)(L^2 - \omega_b^2) + (\omega L^2 - \omega\omega_b^2 - \omega_p^2 L)^2 - \omega_p^4 \omega_b^2]^2}$$

where $L = \omega - j\nu_c$

In order to make a theoretical curve with which to compare experimental data, we shall assume the cold, collisionless case;

$$(1.38) \quad \frac{M}{(M+1)^2} = \frac{\omega^2[\omega^2 - (\omega_b^2 + \omega_p^2)][\omega^2 - \omega_b^2]\{\omega^2[\omega^2 - (\omega_b^2 + \omega_p^2)]^2 - \omega_p^4 \omega_b^2\}^2}{\{\omega^2[\omega^2 - (\omega^2 + \omega_p^2)](\omega^2 - \omega_b^2) + \omega^2[\omega^2 - (\omega_b^2 + \omega_p^2)] - \omega_p^4 \omega_b^2\}^2}$$

Now in our experiments where $\omega > \omega_p$ it is possible to ignore $\omega_p^4 \omega_b^2$ in comparison to $\omega^2[\omega^2 - (\omega_b^2 + \omega_p^2)]$ which gives;

$$(1.39) \quad \frac{M}{(M+1)^2} \approx \frac{\left(1 - \frac{\omega_b^2}{\omega^2}\right)\left[1 - \frac{\omega_b^2 + \omega_p^2}{\omega^2}\right]}{4\left[1 - \frac{2\omega_b^2 + \omega_p^2}{2\omega}\right]^2}$$

Thus it is shown that at frequencies where $\omega \gg \omega_p$ the term ω_b may be discounted in the magnitude term above giving an approximate function;

$$(1.40)\quad \frac{M}{(M+1)^2} \approx \frac{\left(1-\frac{\omega_p^2}{\omega^2}\right)}{\left(2-\frac{\omega_p^2}{\omega^2}\right)}$$

This same assumption gives an expression for

$$(1.41)\quad \frac{M^{\frac{1}{2}}+1}{M^{\frac{1}{2}}} \approx 1+\left[1-\frac{\omega_p^2}{\omega^2}\right]^{\frac{1}{2}}$$

Now let $\frac{\omega_p^2}{\omega^2} = P$

then;

$$(1.42)\quad \overline{E}\times\overline{H} \approx \frac{\eta_0 4A^2(1-P)}{(2-P)} \cos^2 \{k_0 \{x - a\ (1-P)^{\frac{1}{2}}]\}$$

Now $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

And $\cos(A-B) = \cos A \cos B + \sin A \sin B$

So that

$$(1.43)\quad \overline{E}\times\overline{H} \approx \frac{\eta_0 2A^2(1-P)}{(2-P)} \{1+\cos 2k_0x \cos 2ak_0 [1+(1-P)^{\frac{1}{2}}] + \sin 2k_0x \sin 2ak_0[1+(1-P)^{\frac{1}{2}}]\}$$

for convenience, let $2k_0x = \pi$ so that

$$(1.44)\quad P_T = \overline{E}\times\overline{H}\Big|_{2k_0x=\pi} = \frac{2\eta_0 A^2(1-P)}{(2-P)} \{1+\cos 2ak_0 [1+(1-P)^{\frac{1}{2}}]\}$$

where $P = \frac{\omega_p^2}{\omega^2}$

Based on equation (1. 44) a set of curves has been plotted for various resonant frequencies and plasmas dimensions. This is discussed further in Section III.

III. ELECTROMAGNETIC FIELD FORCES IN THE ENERGY AND MOMENTUM EQUATIONS

We now take a look at the effects that the incident electromagnetic energy may have on the momentum and energy distribution of the microscopic plasma.

In the microscopic approximation i. e. continuum theory, it has been shown that the plasma can be described by the momentum equation:

$$(2.1)\quad \frac{\partial}{\partial t}(\rho\overline{v}) + \nabla \cdot (\rho\overline{v}\overline{v}) + \nabla \cdot {}^2P = \rho_e \overline{E} + \overline{J} \times \overline{B} + \overline{P}_c$$

where $\overline{v}$ is the mass average velocity, ρ the total mass density, ${}^2P$ the total pressure tensor, ρ_e the charge density and $\overline{P}_c$ the time variation of the momentum transferred in the collision process. The electromagnetic stress tensor relation from EM field theory gives the relation:

$$(2.2)\quad \nabla \cdot {}^2S = \rho\overline{E} + \overline{J} \times \overline{B} + \mu_0 \frac{\partial}{\partial t}(\epsilon\, \overline{E} \times \overline{H})$$

where ${}^2S$ is the total electromagnetic stress tensor made up of the sum of the electric and magnetic stress tensors.

Substituting (2. 2) into (2. 1) gives the result:

$$(2.3)\quad \frac{\partial}{\partial t}(\rho\overline{v}) + \nabla \cdot (\rho\overline{v}\overline{v}) + \nabla \cdot {}^2P = \nabla \cdot {}^2S - \frac{\partial}{\partial t}(\overline{D} \times \overline{B}) + \overline{P}_c$$

This gives a form of the momentum equation which relates the kinetic motion of the particles to the electromagnetic momentum.

Similarly, the energy equation is normally expressed in the form:

$$(2.4)\quad \frac{\partial}{\partial t}\left(\frac{\rho}{2}\overline{v}^2\right) + \nabla \cdot \left(\frac{\rho}{2}\overline{\overline{v}v^2}\right) = \overline{J} \cdot \overline{E} + T_c$$

where T_c is the time rate of change of the energy due to collisions.

From Poynting's theorem;

$$(2.5)\quad \nabla \cdot (\overline{E} \times \overline{H}) + \overline{E} \cdot \overline{J} = -\overline{E} \cdot \frac{\partial \overline{D}}{\partial t} - \overline{H} \cdot \frac{\partial \overline{B}}{\partial t}$$

Substituting (2. 5) into (2. 4) gives;

$$(2.6)\quad \frac{\partial}{\partial t}\left(\frac{\rho}{2}v^2\right) + \nabla \cdot \left(\frac{\rho}{2}\overline{\overline{v}v^2}\right) = T_c - \left\{\nabla \cdot (\overline{E} \times \overline{H}) + \overline{E} \cdot \frac{\partial \overline{D}}{\partial t} + \overline{H} \cdot \frac{\partial \overline{B}}{\partial t}\right\}$$

This form of the energy equation relates the kinetic energy of the charged particles to the energy of the electromagnetic field. It is the treatment of these two equations, (2.3) for momentum and (2.6) for energy, and the associated relations that is the concern of this section.

Using the conservation of mass equation;

$$(2.7)\quad \frac{\partial\rho}{\partial t} + \nabla\cdot\rho\overline{v} = 0$$

The momentum equation take the form,

$$(2.8)\quad \rho\frac{D\overline{v}}{Dt} + \nabla\cdot{}^{2}P = -\frac{\partial}{\partial t}(\overline{D}\times\overline{B}) + \nabla\cdot{}^{2}S + \overline{P}_c$$

which shows that for a given element of the plasma, the sum of the time rate of change of the kinetic and electromagnetic momentum is equal to the net flux across the boundary plus the time rate of momentum created due to collisions.

In order to make use of the relations between the energy equation and the momenta equation, it is desirable to write the energy equation in terms of an average drift velocity plus a random component, i.e. $\overline{v} = \overline{v} + v_r$. This implies that;

$$(2.9)\quad \frac{1}{2}\rho<\overline{v}^2> = \frac{1}{2}\rho<v^2> + \frac{1}{2}\rho<v_r{}^2>$$

and also;

$$(2.10)\quad \frac{1}{2}\rho<\overline{v}v^2> = \frac{\rho}{2}\Big\{<\overline{v}>^2<v> + 2\,\overline{<\overline{v}>\cdot\ \overline{v}_r<\overline{v}>} + \overline{v_r{}^2<\overline{v}>}$$

$$+ <\overline{v}^2\overline{v}_r> + \overline{2<\overline{v}>\cdot\overline{v}_r\overline{v}_r} + \overline{v_r{}^2\overline{v}_r}$$

$$= \frac{\rho}{2}\Big\{<\overline{v}>^2<v> + \overline{v_r{}^2<\overline{v}>} + \overline{2<\overline{v}>\cdot\ \overline{v}_r\overline{v}_r}$$

Substituting eq. (2.10) into (2.6) gives;

$$(2.11)\quad \frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \frac{1}{2}\rho<v_r{}^2>\right) + \nabla\cdot\frac{\rho v^2}{2}\overline{v} + \nabla\cdot\frac{\rho<v_r{}^2>}{2}\overline{v}$$

$$+ \nabla\cdot(\rho\overline{v}\cdot<v_r v_r>) + \nabla\cdot(\overline{E}\times\overline{H}) + \overline{E}\cdot\frac{\partial\overline{D}}{\partial t}$$

$$+ \overline{H}\cdot\frac{\partial\overline{B}}{\partial t} - T_c = 0$$

where internal energies have been ignored since they may be considered as constant in the subsequent development. It can be shown that $\rho<v_r v_r>$ is the total kinetic stress tensor ${}^{2}P$ which gives;

$$(2.12)\quad \frac{D}{Dt}\left(\frac{1}{2}\rho v^2 + \frac{1}{2}\rho < v_r^2 >\right) + \nabla \cdot (\overline{v} \cdot {}^2P) + \nabla \cdot (\overline{E} \times \overline{H})$$
$$+ \overline{E} \cdot \frac{\partial \overline{D}}{\partial t} + \overline{H} \cdot \frac{\partial \overline{B}}{\partial t} - T_c = 0$$

Scalarly multiplying equation (2. 8) by $\overline{v}$ and assume that the mass density, ρ, is not a function of time or position in coordinate space results in;

$$(2.13)\quad \frac{D}{Dt}\frac{\rho v^2}{2} + \overline{v} \cdot (\nabla \cdot {}^2P) = -\overline{v} \cdot \frac{\partial}{\partial t}(\overline{D} \times \overline{B}) + \overline{v} \cdot (\nabla \cdot {}^2S)$$
$$+ \overline{v} \cdot \overline{P}_c$$

Substituting (2. 13) from (2. 12) and taking into account equation (2. 2) and (2. 5) plus the conditions;

$$(2.14)\quad \overline{v} \cdot \overline{J} \times \overline{B} = 0 \qquad \text{and} \qquad \overline{v} \cdot \rho_e \overline{E} = \overline{J} \cdot \overline{E}$$

Results in the equation;

$$(2.15)\quad \frac{D\left(\frac{1}{2}\rho < v_r^2 >\right)}{Dt} + \nabla \cdot (\overline{v} \cdot {}^2P) - \overline{v} \cdot (\nabla \cdot {}^2P) = T_c - \overline{v} \cdot \overline{P}_c$$

The first term on the left side is the total derivative of the "random" energy, the second term represents the flux of energy across the boundary, the third term represents the flux of drift in the direction of drift. The first term on the right side is the rate of change of collision energy and the second term represents the rate of change of collision energy in the direction of drift.

Next, the pressure dyad will be considered, this may be separated into (1) the hydrostatic pressure and stress tensor;

$$(2.16)\quad {}^2\overline{P} = {}^2\overline{\tau} + p\,\overline{I}$$

where ${}^2\overline{\tau}$ is the stress tensor, $p\,\overline{I}$ is the hydrostatic pressure term. By using the linearized perturbation solution to Boltzmann's equation as outlined Appendix B, an expression for ${}^2\overline{\tau}$ can be obtained by taking the moments of the term $\sum_s \rho_s < cc >_s$ over all species, s, with respect to the distribution function $f_1(\overline{r}, \overline{v}, t)$. This gives the Navier-Stokes relations.

$$(2.17)\quad \tau_{ij} = -\eta\left(\frac{\partial v_i}{\partial_{xj}} + \frac{\partial v_j}{\partial_{xi}}\right) + \frac{2}{3}\eta\,\partial_{ij}\,\nabla \cdot \overline{v}$$

where η is the viscosity term. Since this report deals primarily with a fully ionized gas, electrostatic forces will play the dominant role in the collisions between (or "among") particles. As a result, we shall use the first approximation to the coefficient of viscosity as developed by Chapman;[3]

$$(2.18)\quad [\eta] = \frac{5}{8}\left(\frac{km\,T}{\pi}\right)^{\frac{1}{2}}\left(\frac{2k\,T}{e^2}\right)^2 \quad A_2^{\,2}$$

where $A_2\,(2) = 2\,\ell n(1 + v_{01}^{\,2}) - 2\,v_{01}^{\,2}/1 + v_{01}^{\,2}$ v_{01} being approximated by $v_{01}^{\,2} - 4\,d\,kT/e^2$ where d is the mean free path between positive ions and electrons.

The solution of the momentum and energy equations can now be carried out by using the values of $\overline{E}$, $\overline{D}$, $\overline{H}$, and $\overline{B}$ taken from the previous solution of the wave equation as in Section I, a knowledge of temperature T, and the use of equations (2.17) and (2.18). This will give expressions for the macroscopic motion of the plasma. The approach to be used here is to (1) write out the equations in component form, (2) apply the simplifications which arise from the physical geometry of the particular problem (3) solve this simplified set of equations simultaneously.

The electromagnetic stress tensor may be written as,

$$(2.19)\quad {}^2S = {}^2S^{(e)} + {}^2S^{(m)} = \begin{bmatrix} E_xD_x + B_xH_x - \frac{1}{2}(\overline{E}\cdot\overline{D}\cdot + \overline{B}\cdot\overline{H}) & E_xD_y + B_xH_y & E_xD_z + B_xH_z \\ E_yD_x + B_yH_x & E_yD_y + B_yH_y - \frac{1}{2}(\overline{E}\cdot\overline{D} + \overline{B}\cdot\overline{H}) & E_yD_z + B_yD_z \\ E_zD_x + B_zH_x & E_zD_y + B_zH_y & E_zD_z + B_zH_z - \frac{1}{2}(\overline{E}\cdot\overline{D} + \overline{B}\cdot\overline{H}) \end{bmatrix}$$

where ${}^2S^{(e)}$ is the electric stress tensor and ${}^2S^{(m)}$ is the magnetic stress tensor. Also, the tensor expression for the total stress is;

$$(2.20)\quad {}^2P = {}^2\tau + p\,I = \begin{bmatrix} \tau_{xx} - p & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} - p & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} - p \end{bmatrix}$$

Using the above expressions, (2.19) and (2.20), we can write the component equations for momentum and the one for energy;

(2.21)

(.1)
$$\rho\frac{\partial v_x}{\partial t} + \frac{\partial}{\partial t}(D_yH_z - D_zH_y) + \frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}$$
$$- \frac{\partial}{\partial x}\left(\frac{1}{2}E_xD_x - \frac{1}{2}E_yD_y - \frac{1}{2}E_zD_z + E_xD_y + E_xD_z + \frac{1}{2}B_xH_x - \frac{1}{2}B_yH_y - \frac{1}{2}B_zH_z + B_xH_y + B_xH_z\right)$$
$$- \frac{\partial}{\partial y}\left(\frac{1}{2}E_xD_x - \frac{1}{2}E_yD_y - \frac{1}{2}E_zD_z + E_xD_y + E_xD_z + \frac{1}{2}B_xH_x - \frac{1}{2}B_yH_y - \frac{1}{2}B_zH_z + B_xH_y + B_xH_z\right)$$
$$- \frac{\partial}{\partial z}\left(\frac{1}{2}E_xD_x - \frac{1}{2}E_yD_y - \frac{1}{2}E_zD_z + E_xD_y + E_xD_z + \frac{1}{2}B_xH_x - \frac{1}{2}B_yH_y - \frac{1}{2}B_zH_z + B_xH_y + B_xH_z\right) = P_{cx}$$

(.2)
$$\rho\frac{\partial v_y}{\partial t} + \frac{\partial}{\partial t}(H_xD_z - D_xH_z) + \frac{\partial p}{\partial y} + \frac{\partial(\tau_{yx})}{\partial x} + \frac{\partial(\tau_{yy})}{\partial y} + \frac{\partial(\tau_{yz})}{\partial z}$$
$$- \frac{\partial}{\partial x}\left(E_yD_x + \frac{1}{2}E_yD_y - \frac{1}{2}E_xD_x - \frac{1}{2}E_zD_z + E_yD_z + B_yH_x + \frac{1}{2}B_yH_y - \frac{1}{2}B_xH_x - \frac{1}{2}B_zH_z + B_yH_z\right)$$
$$- \frac{\partial}{\partial y}\left(E_yD_x + \frac{1}{2}E_yD_y - \frac{1}{2}E_xD_x - \frac{1}{2}E_zD_z + E_yD_z + B_yH_x + \frac{1}{2}B_yH_y - \frac{1}{2}B_xH_x - \frac{1}{2}B_zH_z + B_yH_z\right)$$
$$- \frac{\partial}{\partial z}\left(E_yD_x + \frac{1}{2}E_yD_y - \frac{1}{2}E_xD_x - \frac{1}{2}E_zD_z + E_yD_z + B_yH_x + \frac{1}{2}B_yH_y - \frac{1}{2}B_xH_x - \frac{1}{2}B_zH_z + B_yH_z\right) = P_{cy}$$

$$(.3) \quad \rho \frac{\partial v_z}{\partial t} + \frac{\partial}{\partial t}(D_x H_y - H_x D_y) + \frac{\partial p}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}$$

$$- \frac{\partial}{\partial x}\Big(E_z D_x + E_z D_y + \frac{1}{2} E_z D_z - \frac{1}{2} E_x D_x - \frac{1}{2} E_y D_y$$

$$+ B_z H_x + B_z H_y + \frac{1}{2} B_z H_z - \frac{1}{2} B_x H_x - \frac{1}{2} B_y H_y\Big)$$

$$- \frac{\partial}{\partial y}\Big(E_z D_x + E_z D_y + \frac{1}{2} E_z D_z - \frac{1}{2} E_x D_x - \frac{1}{2} E_y D_y$$

$$+ B_z H_x + B_z H_y + \frac{1}{2} B_z H_z - \frac{1}{2} B_x H_x - \frac{1}{2} B_y H_y\Big)$$

$$- \frac{\partial}{\partial z}\Big(E_z D_x + E_z D_y + \frac{1}{2} E_z D_z - \frac{1}{2} E_x D_x - \frac{1}{2} E_y D_y$$

$$+ B_z H_x + B_z H_y + \frac{1}{2} B_z H_z - \frac{1}{2} B_x H_x - \frac{1}{2} B_y H_y\Big) = P_{cz}$$

For the energy equation;

$$(2.22) \quad \frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2\right) + \frac{\partial}{\partial t}\left(\frac{1}{2}\rho \langle v_r^2 \rangle\right) + v_x \nabla_x \langle v_r^2 \rangle + v_y \nabla_y \langle v_r^2 \rangle$$

$$+ v_z \nabla_z \langle v_r^2 \rangle + E_x \frac{\partial D_x}{\partial t} + E_y \frac{\partial D_y}{\partial t} + E_z \frac{\partial D_z}{\partial t}$$

$$+ H_x \frac{\partial B_x}{\partial t} + H_y \frac{\partial B_y}{\partial t} + H_z \frac{\partial B_z}{\partial t} + \frac{\partial}{\partial x}(E_y H_z - H_y E_z)$$

$$+ \frac{\partial}{\partial y}(E_z H_x - E_x H_z) + \frac{\partial}{\partial z}(E_x H_y - E_y H_x)$$

$$+ \frac{\partial}{\partial x}(v_x \tau_{xx} + v_y \tau_{yx} + v_z \tau_{zx})$$

$$+ \frac{\partial}{\partial y}(v_x \tau_{xy} + v_y \tau_{yy} + v_z \tau_{zy})$$

$$+ \frac{\partial}{\partial z}(v_x \tau_{xz} + v_y \tau_{yz} + v_z \tau_{zz}) = 0$$

As was previously defined;

$$\tau_{ij} = -\eta\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) + \frac{2}{3}\eta\, \delta_{ij}\, \nabla \cdot \bar{v}$$

where $\eta \neq \eta(t, \overline{r})$

This gives the elements of τ_{ij};

$$(1)\quad \tau_{xx} = -\frac{4}{3}\eta\frac{\partial v_x}{\partial x} \qquad (2)\quad \tau_{xy} = -\eta\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$$

$$(3)\quad \tau_{xz} = -\eta\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right) \qquad (4)\quad \tau_{yx} = -\eta\left(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial x}\right) = \tau_{xy}$$

$$(5)\quad \tau_{yy} = -\frac{4}{3}\eta\frac{\partial v_y}{\partial y} \qquad (6)\quad \tau_{yz} = -\eta\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)$$

$$(2.23)\quad (7)\quad \tau_{zx} = -\eta\left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right) = \tau_{xz}$$

$$(8)\quad \tau_{zy} = -\eta\left(\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}\right) = \tau_{yz}$$

$$(9)\quad \tau_{zz} = -\frac{4}{3}\eta\frac{\partial v_z}{\partial z}$$

The above equations; (2.21), (2.22) and (2.23); although not difficult present a very cumbersome set. Therefore, it is necessary to simplify the problems through the application of the physical simplifications.

Now in Section I we had a wave-plasma interaction with the conditions;

(1) The static magnetic field is given by $B_0 = (0, 0, B_0)$

(2) The incident wave is time harmonic and has the form
$\overline{E} = E_0\, e^{j(\omega t - k_0 x)}\, \overline{j} \Rightarrow \frac{\partial}{\partial t} = j\omega$

(3) There is no z variation in $\overline{E}$ and as a result no z component of $\overline{E}$.

(4) There are no H_x or H_y components of $\overline{H}$

(5) From physical arguments it is apparent that

$\overline{v} = \overline{v}(x, t)$

with the above conditions, the kinetic stress tensor components become;

$$\tau_{xx} = -\frac{4}{3}\eta\frac{\partial v_x}{\partial x}$$

$$\tau_{xz} = -\eta\frac{\partial v_z}{\partial x} = \tau_{zx}$$

$$\tau_{yy} = \tau_{yz} = \tau_{zy} = \tau_{zz} = 0$$

$$\tau_{xy} = -\eta\frac{\partial v_y}{\partial x} = \tau_{yx}$$

As result of the above restrictions the momentum equations become;

$$(2.24)\quad \rho\frac{\partial v_x}{\partial t} + \frac{\partial}{\partial t}(D_yH_z) + \frac{\partial P}{\partial x} - \frac{4}{3}\eta\frac{\partial^2 v_x}{\partial x^2} - \frac{\partial}{\partial x}\left(\frac{1}{2}E_xD_x - \frac{1}{2}E_yD_y + E_xD_y - \frac{1}{2}B_zH_z\right) = P_{cx}$$

$$(2.25)\quad \rho\frac{\partial v_y}{\partial t} - \frac{\partial}{\partial t}(D_xH_z) + \frac{\partial P}{\partial y} - \eta\frac{\partial^2 v_y}{\partial x^2} - \frac{\partial}{\partial x}\left(E_yD_x + \frac{1}{2}E_yD_y - \frac{1}{2}E_xD_x - \frac{1}{2}B_zH_z\right) = P_{cy}$$

$$(2.26)\quad \rho\frac{\partial v_z}{\partial t} + \frac{\partial P}{\partial z} - \eta\frac{\partial^2 v_z}{\partial z^2} - \frac{\partial}{\partial x}\left(\frac{1}{2}B_zH_z - \frac{1}{2}E_xD_x - \frac{1}{2}E_yD_y\right) = P_{cz}$$

Now, since there is no electric field in the z direction and no magnetic field in the x or y direction, we will assume that $v_z = 0$ which implicitly states also that there is no pressure gradient in the z direction thus the last equation becomes;

$$(2.27)\quad -\frac{\partial}{\partial x}\left(-\frac{1}{2}B_zH_z + \frac{1}{2}E_xD_x + \frac{1}{2}E_yD_y\right) = -P_{cz}$$

Substituting this into equations (2.23) and (2.24);

$$(2.28)\quad \rho\frac{\partial v_x}{\partial t} + \frac{\partial}{\partial t}(D_yH_z) + \frac{\partial p}{\partial x} - \frac{4}{3}\eta\frac{\partial^2 v_x}{\partial x^2} - \frac{\partial}{\partial x}(E_xD_x + E_xD_y - B_zH_z) = P_{cx} - P_{cz}$$

$$(2.29)\quad \rho \frac{\partial v_y}{\partial t} - \frac{\partial}{\partial t}(D_x H_z) + \frac{\partial p}{\partial y} - \eta \frac{\partial^2 v_y}{\partial x^2} - \frac{\partial}{\partial x}(E_y D_x + E_y D_y - B_z H_z) = P_{cy} - P_{cz}$$

For the energy equation;

$$(2.30)\quad \frac{\partial}{\partial t}\left(\frac{1}{2}\rho v_x^2 + \frac{1}{2}\rho v_y^2\right) + \frac{\partial}{\partial t}\left(\frac{1}{2}\rho < v_r^2 >\right) + v_x \nabla_x < v_r^2 > + v_y \nabla_y < v_r^2 > + E_x \frac{\partial D_x}{\partial t} + E_y \frac{\partial D_y}{\partial t} - \frac{4}{3}\eta \frac{\partial^2 \left(\frac{v_x^2}{2}\right)}{\partial x^2} - \eta \frac{\partial^2 \left(\frac{v_y^2}{2}\right)}{\partial x^2} + \frac{\partial (E_y H_z)}{\partial x} = T_c$$

Now, it can be shown that for non-relativistic velocities, the force exerted on a charge particle by the magnetic component is negligible to that of the electric vector. Thus it is possible to ignore the time varying part of the electric field i. e.

$$\overline{B} = B_1 C^{j(\omega t - kx)} + B_0 \approx B_0$$

for our purposes and since our system is close to isothermic we assume P = constant. Also, since the effects of the EM field are treated as perturbations of the normal (Maxmillian) distribution we can assume that $P_{cx} - P_{cz} \approx 0$ and $P_{cy} - P_{cz} \approx 0$ thus;

$$(2.31)\quad j\omega\rho v_x + j\omega \frac{B_0}{\mu} D_y + \frac{4}{3}\eta k^2 v_x - 2\,jk\,E_x D_x - 2\,jk\,E_x D_y = 0$$

$$(2.32)\quad j\omega\rho v_y - j\omega \frac{B_0}{\mu_0} D_x + \eta k^2 v_y - 2\,jk\,E_y D_x - 2\,jk\,E_y D_y = 0$$

$$(2.33)\quad \frac{B_0^2}{2\mu_0} - jk\,E_x D_x - jk\,E_y D_y = P_{cz} \approx P_{cx} \approx P_{cy}$$

$$j\omega\rho(v_x^2 + v_y^2) + \frac{D}{Dt}\left(\frac{1}{2}\rho < v_r^2 >\right) + j\omega E_x D_x + j\omega E_y D_y + \frac{8}{3}\eta k^2 v_x^2 + 2\,\eta k^2 v_y^2 + \frac{j\omega B_0}{\mu_0} E_y = T_c$$

From (2.31)

$$(2.34)\quad v_x = \frac{j\left[\frac{-\omega B_0}{\mu_0} D_y + 2\,k\,E_x\,(D_x + D_y)\right]}{j\omega\rho + \frac{4}{3}\eta\,k^2}$$

and (2.32)

$$(2.35)\quad v_y = \frac{j\left[\frac{\omega B_0}{\mu_0} D_x + 2\,k\,E_y\,(D_x + D_y)\right]}{j\omega\rho + \eta k^2}$$

Rearranging (2.33)

$$(2.36)\quad (j\omega\rho + \frac{8}{3}\eta\,k^2)\,v_x^2 + j\omega\rho + 2\eta k^2)\,v_y^2 + j\omega\left[E_x D_x + E_y D_y + \frac{B_0 E_y}{\mu_0}\right] = T_c - \frac{D}{Dt}\,\frac{1}{2}\,\rho < v_r^2 >$$

From Section I;

$$(2.37)\quad \begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \epsilon' & -iq & 0 \\ iq & \epsilon' & 0 \\ 0 & 0 & \epsilon'' \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

$$(2.38)\quad \begin{aligned} D_x &\Rightarrow \epsilon' E_x - iq\,E_y \\ D_y &\Rightarrow iq\,E_x + \epsilon' E_y \\ D_z &\Rightarrow \epsilon'' E_z \end{aligned}$$

So that;

$$(2.39)\quad v_x = j\left\{\frac{\epsilon_0\,\omega\,\beta_0}{\mu_0}\,(iq\,E_x + \epsilon' E_y) - \frac{2\,k_0\epsilon_0\,E_x}{(M)^{\frac{1}{2}}}\left[(\epsilon' + iq)\,E_x + (\epsilon' - iq)\,E_y\right]\right\} \Big/ j\omega\rho + \frac{4}{3}\,\eta\,\frac{k_0^2}{M}$$

$$(2.40)\quad v_y = j\left\{\frac{\epsilon_0\omega B_0}{\mu_0}(\epsilon' E_x - iq E_y) + \frac{2 k_0\epsilon_0 E_y}{M^{\frac{1}{2}}}\left[(\epsilon' + iq) E_x + (\epsilon' - iq) E_y\right]\right\} \Big/ j\omega\rho + \frac{\eta k_0^2}{\mu}$$

These two equations describe the drift motion of the elemental volume while equation (2. 36) in conjunction with eq. (2. 39) and (2. 40) gives the random energy flux i. e.

$$T_c - \frac{D}{Dt}\left(\frac{1}{2}\rho < v_r^2 >\right)$$

These equations are extremely cumbersome; however, a cold collisionless approximation gives some valuable insight. In the approximation,

(1) $\nu_c \approx 0$

(2) $k_T \approx 0$

This implies that the viscous terms are negligible and that the elements of the dielectric tensor are real and the propagation constant $\frac{k_0}{M^{\frac{1}{2}}}$ is pure real or pure imaginary depending on the frequency, ω.

This gives;

$$(2.41)\quad \omega\rho v_x = -\frac{\epsilon_0\omega B_0\epsilon'}{\mu_0} E_y + \frac{2 k_0\epsilon_0\epsilon'}{(M)^{\frac{1}{2}}}(E_x^2 + E_x E_y) + j\left\{\frac{\epsilon_0\omega B_0 q}{\mu_0} E_x - \frac{2 k_0 \epsilon_0 q}{M^{\frac{1}{2}}}(E_x^2 - E_x E_y)\right\}$$

or

$$\omega\rho v_x + \frac{\epsilon_0\omega B_0}{\mu_0}(\epsilon' E_y - j q E_x) = \frac{2 k_0\epsilon_0\epsilon' E_x}{(M)^{\frac{1}{2}}}(E_x + E_y) - j\frac{2 k_0\epsilon_0 q E_x}{(M)^{\frac{1}{2}}}(E_x - E_y)$$

and;

$$(2.42)\quad \omega\rho v_y - \frac{\epsilon_0 \omega B_0}{\mu_0}(\epsilon' E_x - iq E_y) = \frac{2 k_0 \epsilon_0 E_y \epsilon'}{M^{\frac{1}{2}}}(E_x + E_y)$$

$$+ j \frac{2 k_0 \epsilon_0 q}{M^{\frac{1}{2}}} E_y (E_x - E_y)$$

Equation (2.41) shows that the momentum due to the mass motion plus the EM wave momentum; which is made up of two terms, the first which is real and represents motion as experienced in an isotropic media, the second (which is in quadrature) is momentum due to the y direction motion which is couple via the dielectric tensor into the x direction; is equal to the momentum flow across the boundary in the "normal" sense plus the y direction momentum flux coupled into the x direction by the dielectric tensor. This analysis applies to Eq. (2.42) by reversing the unknown.

This approach provides the possibility of a greater insight to the subsequent mass motion of a plasma resulting from the incidence of an electromagnetic field. It side-steps the problem of charge density and current since they are eliminated from the equations. This approach has definite merit with regard to plasma diagnostics since the measurement of field quantities without perturbing the plasma is possible. It is our feeling that the integral solution of the momentum equation as treated here should be solved in conjunction with the wave equations for a particular bounded media in order to demonstrate the possibility of studying macroscopic motion of plasmas by measuring the external fields.

IV. EXPERIMENTAL PROGRESS

Unfortunately, the experimental phase of this investigation has proceeded much slower than expected. The principle drawbacks being (1) the delivery of capital equipment and (2) the development of proper laboratory techniques to achieve the desired plasma.

The plasma generated in the experiment is a thermally generated cesium plasma; this particular type of plasma was chosen because:

(1) It is a "cool" plasma with the electron temperature, Te, and ion temperature, Ti, approximately equal to the filament temperature which is normally varied over the range of 2000°K to 2500° K.

(2) There are no external force fields other than the uniform magnetic field, Bo, which is used to produce the anisotrophy as well as to contain the charged particles.

(3) A high degree of ionization can be obtained; reports of ionization percent greater than 90 are in the literature.[7,11]

(4) Plasma density can be varied as a direct function of known temperature and pressure.

A picture of the apparatus is shown in Fig. 2. The plasma is generated by releasing cesium vapor into the large pyrex tube containing the two tungsten filaments which are heated to electron emission temperature. The cesium, under proper pressure conditions, will be ionized by contacting the heated filaments. Both, the electrons and ions are then expelled from the filaments. Those charged particles with velocity vectors parallel to the magnetic field, Bo, are contained to form a plasma in the region between the filaments. This is shown in Fig. 3. The applied magnetic field is produced by the Helmholtz coil which gives a constant field ± .07% over the region of interest.

Since cesium is a very active element and oxidizes quite rapidly, it can never be exposed to atmospheric pressures. To avoid the problem of handling the cesium metal directly, we have employed the use of a cesium generator which consists of a vile of a cesium chloride-calcium mixture which upon being heated to 400°C gives off cesium; i.e., $2C_SCl + Ca \xrightarrow[400°C]{} CoCl_2 + 2C_S\uparrow$ The calcium chloride residue also acts as an excellent H_2O "getter".

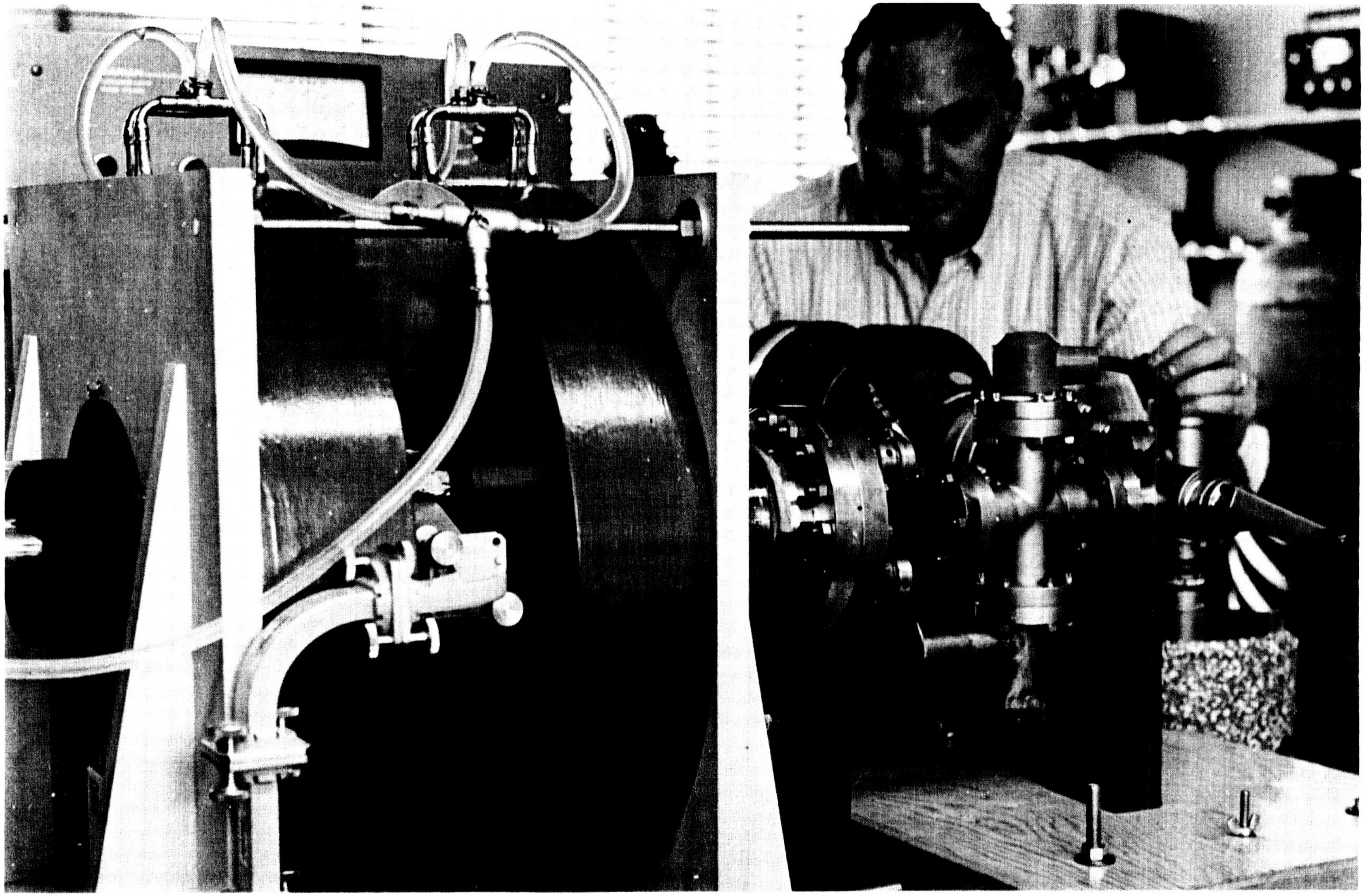

Figure 2. Cesium plasma generating apparatus.

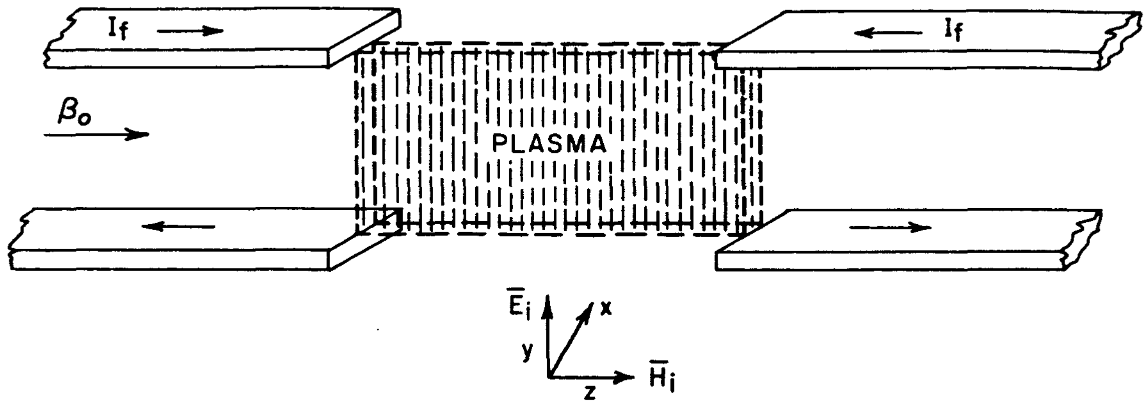

Figure 3. Plasma generation scheme; the regions near the filaments will have an excess of electrons or positive ions depending on the filament temperature but the region in the center is electrically neutral. I_f is of the order of 10^2 amps and Bo is of the order of 10^3 Guass.

The filaments are each heated by 2KVA AC power supplies capable of delivering 400 amps at 5 volts. A temperature calibration was made through the use of an optical pyrometer. Both voltage and current calibration curves were made so that a change in filament characteristics can be detected by cross checking.

Pressure in the system is measured with a hot ion gauge which has an accuracy of ±50%. At present, we have no means of measuring the plasma density other than the microwave interferometer; however, in the near future we hope to install a langmuir probe to give a cross reference on our density measurements.

The cesium ions in the plasma which we have been taking data on are almost entirely singly ionized. The emission for this case falls in the infrared region; hence, there is no visible indication of a plasma. By increasing the filament temperature to 2600°K +, we have been able to achieve double ions; this results in a visible plasma. In this visible plasma the boundaries are clearly defined and conform to geometry of the volume defined by the space between the electrodes. Wada and Knechtli[11] have published results on their plasma, which was generated in a similar manner, which also shows a well defined boundary.

Fig. 4 shows a semi-schematic diagram of the microwave interferometer as used in these measurements. The original data was taken by establishing a reference of 0.1 mw transmitted power and 0° phase shift for each chosen frequency with no plasma present. This procedure proved to be inconsistent in its results because of source variations and mismatches in the line. The difficulty arose from the fact that we had to return to exactly the same incident frequency and amplitude for the data "run" as was used in the reference "run" this being further complicated by the fact that once a plasma is generated it is desirable to perform all the necessary measurements without interrupting the plasma. As result of the afore-mentioned difficulties it was decided to operate at a fixed frequency and vary the plasma properties by changing the filament temperature. Thus, it is possible to establish a reference for each frequency, collect a set of data and then move to another frequency and repeat the process. Unfortunately the data taken by this method is incomplete at present. We, therefore, feel that the apparent inconsistencies in our data can be overcome by better experimental technique.

In spite of the afore-mentioned difficulties, we have been able to achieve fairly good correlation between theory and experiment. A set of normalized curves for transmitted power, P_T, versus frequency and phase shift, ϕ, versus frequency where calculated from equation (1.44) for various valves of the parameter $\underline{a}$ from 0.25 to 1.5 cm and for ωp = 8qcps to $\omega p \ll \omega$. These plots show pronounced dependence of P_T on the plasma dimensions due to phase shift. From the plots examined, only three showed any correlation with our experimental data these results are shown in Figs. 5, 6 and 7.

These results show that with a firm knowledge of the plasma dimensions (which we have not as yet achieved) it is possible to determine the plasma resonance and, eventually, the collision frequency from microwave measurements on a bounded slab where the wavelength of the signal is of the order of the slab thickness. The inclusion of ν_c and ω_b in the theory will naturally make the calculations a great deal more cumbersome and probably make numerical calculations a necessity.

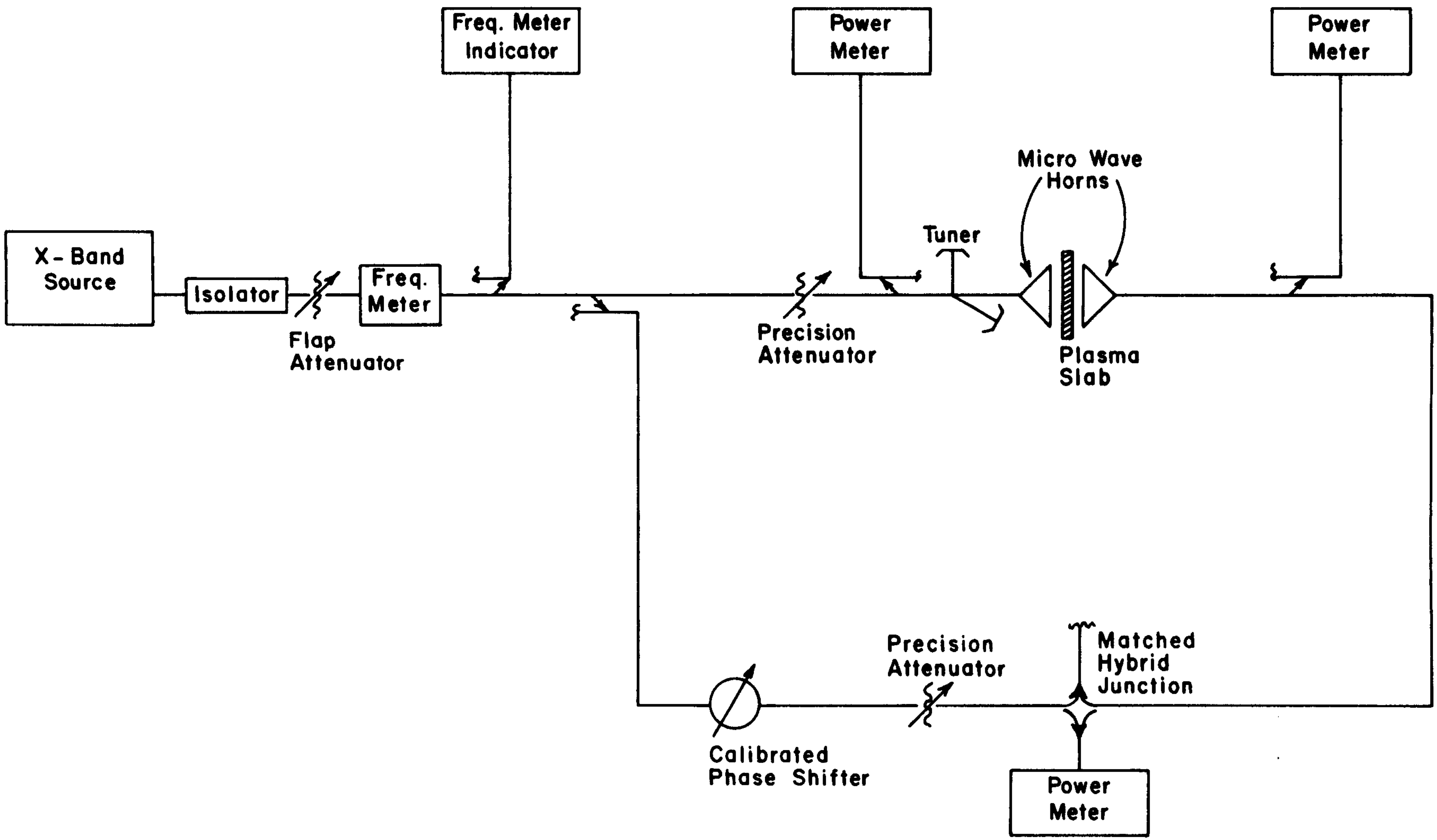

Figure 4. Semi-schematic diagram of the microwave interferometer.

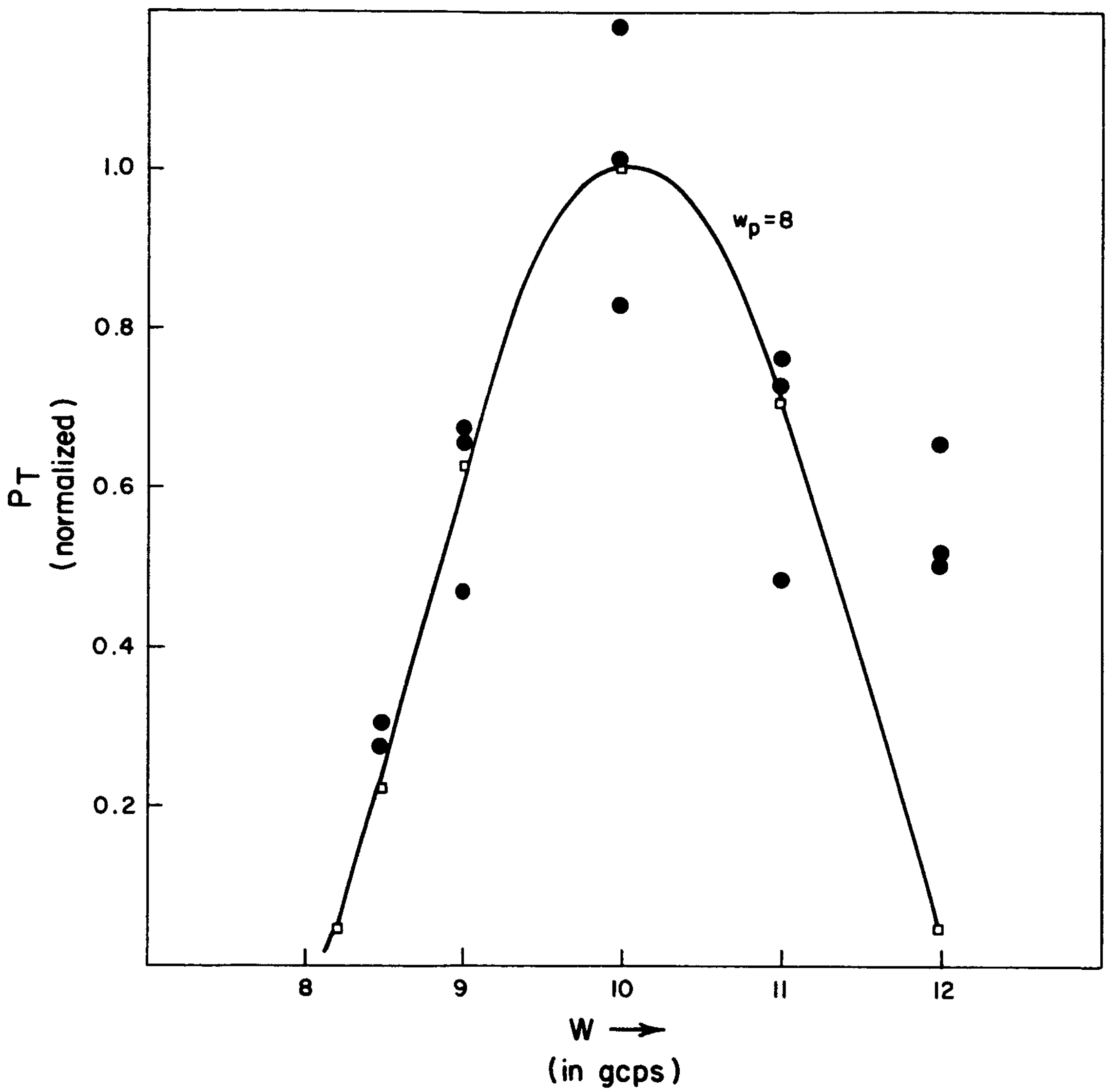

Figure 5. Plot shows the distribution of experimental points about the theoretical curve taken from equation (1.44) for a = 1 cm, ωp = 8 gcps. The curves have been normalized to 1.0 at 10 gcps.

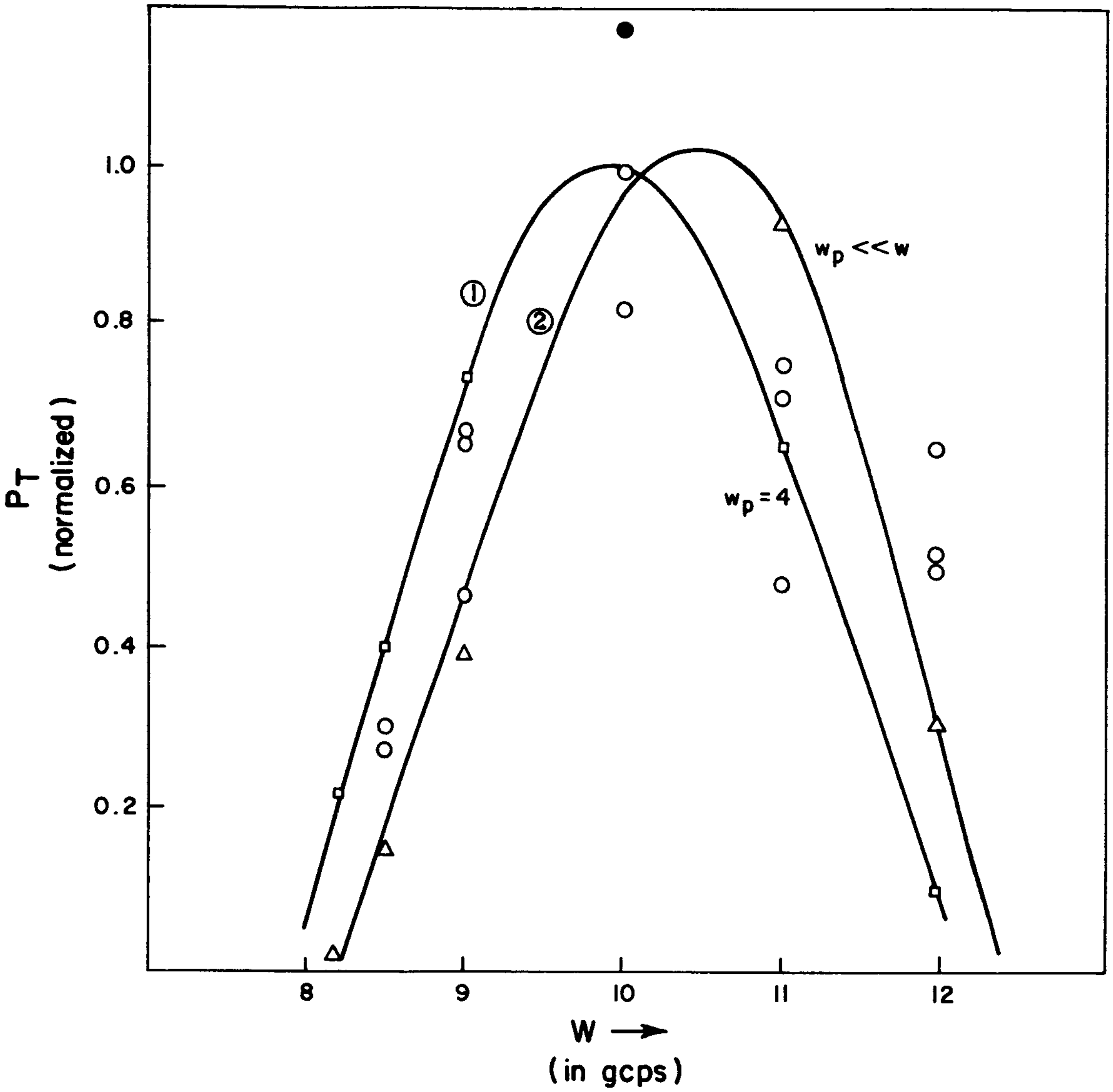

Figure 6. Plot shows the distribution of experimental points about two theoretical curves taken from equation (1.44) for ① a = 1.5 cm, $\omega\rho$ = 4 gcps and ② a = 1.5 cm, $\omega\rho << \omega$. The curves are normalized to a value of 1.0 at 10 gcps.

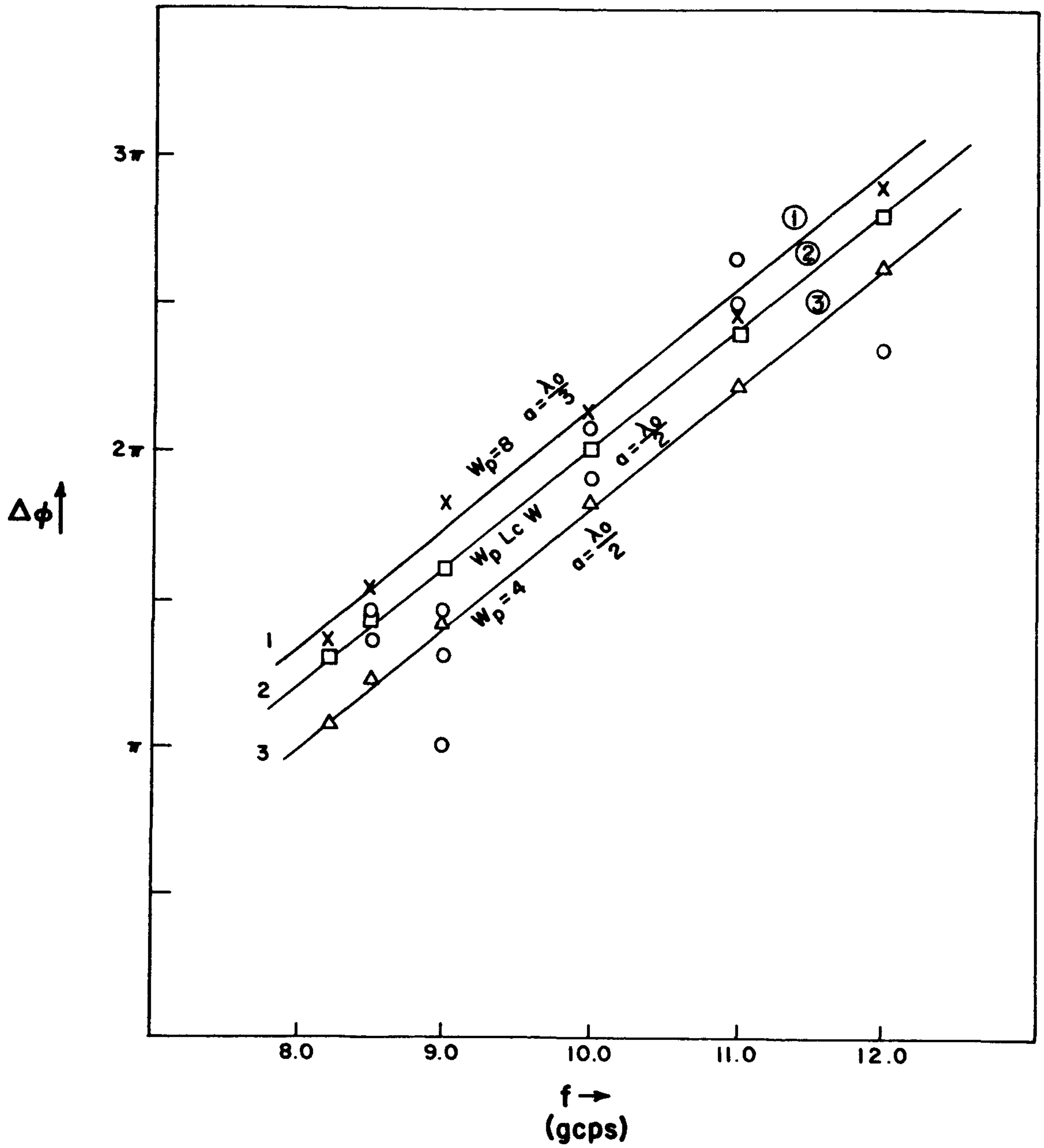

Figure 7. Plot of phase shift versus frequency showing the distribution of experimental points about the theoretical curves taken from equation (1.44) where $\phi = 2\,a\,k_0\,[1 + (1 - P)^{\frac{1}{2}}]$ reference to 2 II. Parameters for the curves are ① a = 1 cm, $\omega\rho$ = 8 gcps ② a = 1.5 cm, $\omega\rho << \omega$ ③ a = 1.5 cm, $\omega\rho$ = 4 gcps.

V. CONCLUSIONS

Our initial investigations have shown that it is possible to treat forces exerted on a plasma in terms of the EM field quantities that may be incident upon a particular point in the media. Solutions for the wave equation when the thickness of the slab is of the order of the wavelength make it necessary to include the reflections at the boundaries in the solution. It is felt that the additional knowledge of the thickness offers a means to determine additional information about the plasma and this should be pursued further in conjunction with integral solutions to the energy and momentum equations covered in Section II.

Also, in order to substantiate any future date, we feel that additional measuring techniques should be employed, such as langmuir probes or charged particle probes.

Of utmost importance in this study are these facts:

(1) We have succeeded in generating a low temperature quiescent plasma of relatively large dimensions in a slab geometry.

(2) We have achieved a degree of correlation between theory and experiment and furthermore, we feel that with better techniques we can achieve a high degree of correlation.

(3) The theoretical aspects have been developed sufficiently to give an insight to some of the more pressing problems with respect to the interpretation of experimental data.

(4) We have shown that the stress tensor approach to the energy and momentum equations offers another method to interpret plasma motion in terms of electromagnetic fields.

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VII. APPENDIX A

Boltzmann's Equation and the Dielectric Tensor

The statistical features of a plasma are treated in terms of a distribution function f(r, v, t) defined such that at time, t, there are;

$$(A.\ 1) \qquad f(\bar{r}, \bar{v}, t)\ d\bar{r}\, d\bar{v}$$

particles located in the element of phase space between (r, v) and (r + dv, v + dv); now at a time t + dt these same particles will occupy a space (r + v dt, v + a dt) thus, we can write a rate of change equation;

$$(A.\ 2) \qquad \frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x_i}\frac{\partial x_i}{\partial t} + \frac{\partial f}{\partial v_i}\frac{\partial v_i}{\partial t}$$

or in vector rotation;

$$(A.\ 3) \qquad \frac{df}{dt} = \frac{\partial f}{\partial t} + \bar{v} \cdot \nabla_{\bar{r}} f + \bar{a} \cdot \nabla_{\bar{v}} f$$

where v is $\frac{\partial \bar{r}}{\partial t}$ and $\bar{a} = \frac{\bar{F}}{M}$ where $\bar{F}$ is an external force $\nabla_{\bar{r}}$ is a coordinate space operator and $\nabla_{\bar{v}}$ is a velocity space operator.

Now, equation A. 3 takes into account all continous changes in $f(\bar{r}, \bar{v}, t)$ but collisions can produce abrupt changes in velocity space so that in order to write a complete generalized equation of continuity;

$$(A.\ 4) \qquad \frac{\partial f}{\partial t} + \bar{v} \cdot \nabla_r f + \bar{a} \cdot \nabla_{\bar{v}} f = \left(\frac{\partial f}{\partial t}\right)_{\text{collisions}}$$

Unfortunately, not all collisions are well defined binary events which produce abrupt velocity changes as in classical collision theory. Some collisions are the result of coulomb interactions which can take place over large distances. Coulomb interactions in an ionized gas are usually catagorized according to the deflection angle;

(1) If the impact parameter b is < impact parameter for 90° deflection, b_{90} ; then this is a situation of close incounters and the electron-ion collision may be treated as discrete two particle collisions.

(2) If the impact parameter, b, is such that, $b_{90°} < b <$ the Debye length, λ_0; then this is a situation of many body encounters producing a succession of uncorrelated small-angle deflections. This case is usually handled by using the Fokker-Plank equation which is actually a specialized version of Boltzmann's equation developed by assuming small velocity changes and, as a result, permitting the expansion of the collision term into a Taylor series (in velocity).

(3) If $b > \lambda_0$ then the particle encounters are no longer statistical in nature and the collision concept becomes one of correlated motion (such as wave motion).

Regardless of the dominate collision process, the primary problem is to solve Boltzmann's equation. For the case of small perturbations away from Maxwellian an expansion technique is used; called the Enskog - Chapman techniques[3];

$$\text{(A. 5)} \qquad f(\overline{r}, \overline{v}, t) = f_0(\overline{r}, v^2) + \alpha f_1(\overline{r}, \overline{v}, t) + \alpha^2 f_2(\overline{r}, \overline{v}, t) + \ldots$$

where

$$\text{(A. 6)} \qquad f_0(r, v^2) = n(\overline{r}) \left(\frac{m}{2\pi k T}\right)^{\frac{3}{2}} \exp - \left(\frac{Mv^2}{2kT}\right)$$

the Maxwellian distribution. If the media is anisotropic as in the case of an applied magnetic field, it is useful to expand f in terms of spherical harmonics;

$$\text{(A. 7)} \qquad f(\overline{r}, \overline{v}, t) = f_0(v^2) + f_1(\overline{r}, v^2) \cos\theta + f_2(\overline{r}, v^2, t) \frac{3\cos^2\theta - 1}{2} + \ldots$$

if the problem is not symetrical with respect to the azmuthal angle;

$$\text{(A. 8)} \qquad f(\overline{r}, \overline{v}, t) = \sum_{\ell=o}^{\infty} \sum_{m=o}^{\ell} (f_{\ell 1}(\overline{r}, \overline{v}, t) \cos m\phi + $$

$$+ f_{\ell_2}(\overline{r}, \overline{v}, t) \sin m\phi) P_\ell^{\,m}(\cos\theta)$$

By using the expansion techniques as above, a linearized Boltzmann's equation may be obtained from which a solution for $f(\overline{r}, \overline{v}, t)$ is taken.

Now

$$\text{(A. 9)} \qquad \int_{vol} f(\overline{r}, \overline{v})\, d^3v = n(r)$$

which is the basis for normalization. This results in

$$\text{(A. 10)} \qquad \int_{vol} f(\overline{v})\, d^3v = 1$$

so that;

$$\text{(A. 11)} \qquad \int_{vol} \overline{v}\, f(\overline{v})\, d^3v = \langle v \rangle$$

Therefore, with a knowledge of $f(\overline{r}, \overline{v}, t)$ it is possible to get an expression for current;

$$\text{(A. 12)} \qquad J_i = ne \langle v_i \rangle$$

so that along with the relations;

$$\text{(A. 13)} \qquad J_i = \sigma_{ij} E_j$$

and

$$\text{(A. 14)} \qquad \epsilon_{ij} = 1 - j \frac{\sigma_{ij}}{\epsilon_0 \omega}$$

It is possible to develop the dielectric tensor $[\epsilon_{ij}]$. Needless to say, this development can be made as difficult and cumbersome as one desires depending on the assumptions made with regard to (1) the collision term $\left(\frac{\partial f}{\partial t}\right)_{collisions}$ (2) the linearization of the distribution function and (3) the symetry of the problem.

Assuming a characteristic relaxation time constant such that $\left(\frac{\partial f}{\partial t}\right)_{collisions} = -\nu(v)\,(f - f_0)$ and expanding the distribution function such that Sitenko and Sepanov[7] developed the following expressions for the dielectric tensor elements for the case of propagation normal to the static magnetic field;

$$k_{xx} = 1 - \frac{\omega_p^2}{\omega} \frac{\omega - j\nu}{(\omega - j\nu)^2 - \omega_b^2} \left\{ 1 + \frac{\mu^2 kT}{mc^2} \frac{3\omega^3}{(\omega - j\nu)^2 - 4\omega_b^2} \right\}$$

$$k_{yy} = 1 - \frac{\omega_p^2}{\omega} \frac{\omega - j\nu}{(\omega - j\nu)^2 - \omega_b^2} \left\{ 1 + \frac{\mu^2 kT}{mc^2} \frac{\omega^2 [(\omega - j\nu)^2 + 8\omega_b^2]}{(\omega - j\nu)^2 [(\omega - j\nu)^2 - 4\omega_b^2]} \right\}$$

$$k_{zz} = 1 - \frac{\omega_p^2}{\omega} \frac{1}{\omega - j\nu} \left\{ 1 + \frac{\mu^2 kT}{mc^2} \frac{\omega^2}{(\omega - j\nu)^2 - \omega_b^2} \right\}$$

$$k_{xy} = -k_{yx} = -j \frac{\omega_p^2}{\omega} \frac{\omega_b}{(\omega - j\nu)^2 - \omega_b^2} \left\{ 1 + \frac{\mu^2 kT}{mc^2} \frac{6\omega^2}{(\omega - j\nu)^2 - 4\omega_b^2} \right\}$$

$$k_{xz} = k_{zx} = 0$$

$$k_{yz} = k_{zy} = 0$$

Neglecting temperature effects, these terms reduce to those found in most elementary works on wave interactions with plasmas.

VIII. APPENDIX B

Waves in a Plasma Medium and Their Boundary Conditions For Well Defined Boundaries

This development is confined to the case of EM wave incidence normal to the applied magnetic field, $B_0 \overline{A_z}$. For the time harmonic case; following the development of wait;

(B. 1) $$\overline{D} = [\epsilon]\,\overline{E}$$

where

(B. 2) $$[\epsilon] = \begin{bmatrix} \epsilon' & -iq & 0 \\ iq & \epsilon' & 0 \\ 0 & 0 & \epsilon'' \end{bmatrix}$$

where the elements of the dielectric tensor are the same as those developed in Appendix A. Now, wave properties are governed by Maxwell's equations;

(B. 3) $$j\omega\,[\epsilon]\,\overline{E} = \nabla \times \overline{H}$$

(B. 4) $$-j\omega\mu_0\,\overline{H} = \nabla \times \overline{E}$$

For plasma work, it is desirable to write (B. 3) in the form

(B. 5) $$j\omega\overline{E} = [\epsilon^{-1}]\;\nabla \times \overline{H}$$

where $[\epsilon^{-1}]$ is the inverse of $[\epsilon]$ and has the elements;

(B. 6) $$[\epsilon^{-1}] = \frac{1}{\epsilon_0} \begin{bmatrix} M & -jK & 0 \\ jK & M & 0 \\ 0 & 0 & \epsilon_0/\epsilon'' \end{bmatrix}$$

where

(B. 7) $$M = \frac{\epsilon'\epsilon_0}{(\epsilon')^2 - q^2} \quad \text{and} \quad K = \frac{-q\epsilon_0}{(\epsilon')^2 - q^2}$$

Now using equations (B. 4) and (B. 5) and the inverse dielectric tensor (B. 6), the component equations are;

(B. 8) $$j\epsilon_0\omega E_x = M\frac{\partial H_z}{\partial y} + jK\frac{\partial H_z}{\partial x}$$

(B. 9) $$j\epsilon_0\omega E_y = iK\frac{\partial H_z}{\partial y} - M\frac{\partial H_z}{\partial x}$$

(B. 10) $$j\epsilon_0\omega E_z = (\epsilon_0/\epsilon'')\left[\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial y}\right]$$

(B. 11) $$-j\mu_0\omega H_x = \frac{\partial E_z}{\partial y}$$

(B. 12) $$-j\mu_0\omega H_y = \frac{\partial E_z}{\partial x}$$

(B. 13) $$-j\mu_0\omega H_z = \left[\frac{\partial Ey}{\partial x} - \frac{\partial Ex}{\partial y}\right]$$

Elimination of x and y components of the H and E fields gives the results;

(B. 14) $$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{k_0^2}{M}\right] H_z = 0$$

and $$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{k_0^2\epsilon''}{\epsilon_0}\right] E_z = 0$$

(B. 15)

A complete solution will be a linear combination of the results obtained from solving equations (B. 14) and (B. 15). In this paper we are concerned only with the case for $E_z = 0$ so that we are only concerned with the solution of equation (B. 14) and as result, equations (B. 8), (B. 9) and (B. 10).

When the EM wave is incident upon a bounded plasma there are certain boundary conditions that must be fulfilled resulting from the necessity of a continuous solution to Maxwell's equations as well as the momentum and continuity equation across the boundary. These conditions are merely stated here, a deriviation may be found in the literature.[12] For a plasma vacuum boundary;

(B. 16) $$\bar{n} - [(\rho\bar{v})_\rho - (\rho v)_v] = u(\rho_\rho - \rho_v)$$

(B. 17) $\bar{n} \times (\bar{B}_\rho - \bar{B}_v) = \bar{J}^* - u(\bar{E}_\rho - \bar{E}_v)$

(B. 18) $\bar{n} \cdot (\bar{B}_\rho - \bar{B}_v) = 0$

(B. 19) $\bar{n} \times (\bar{E}_\rho - \bar{E}_v) = u(\bar{B}_\rho - B_v)$

(B. 20) $\bar{n} \cdot (\bar{E}_\rho - \bar{E}_v) = \sigma^*$

(B. 21) $\bar{J}^* + \langle \bar{B} \rangle_{avg} + \sigma^* \langle \bar{E} \rangle_{avg} - \bar{n}(P_\rho - V_v) = 0$

where;

$\bar{n}$ is the surface normal

$\bar{v}$ is the drift velocity at the surface

$u = \bar{v} \cdot \bar{n}$

P is the pressure

$\bar{J}^* = \int_0^\delta \bar{J} ds$

$\sigma^* = \int_0^\delta \sigma ds$

where δ is the boundary layer thickness.