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# ANALYSIS OF THE PRESSURE LOSS CHARACTERISTICS OF THE GAS FEEDING REGIONS OF THE NASA AB-5 GYRO GIMBAL BEARING

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INTRODUCTION

One of the most difficult and critical problem in the analysis of externally pressurized gas bearings is that of analyzing the flow in the vicinity of the point where the gas is fed into the bearing. The principal features of this entrance flow are illustrated in Figure 1a. Gas is fed to the bearing from a supply reservoir at pressure $P_s$ through an orifice into a circular feeder hole in the top plate of the bearing. In passing through the orifice, the static pressure of the gas is reduced to a value $P_c$ at the point of the vena contracta downstream of the orifice. As the high velocity jet issuing from the orifice impinges on the thrust plate at the bottom of the feeder hole, some of the dynamic pressure of the jet will be recovered, so that the static pressure $P_r$ at the bottom of the feeder hole will be somewhat greater than $P_c$. From the feeder hole the gas enters into the narrow clearance gap between the two bearing plates, accelerating in velocity as it does so. Associated with this entrance flow there will be another vena contracta point at which the static pressure $P_v$ will reach a minimum. Downstream of this vena contracta the flow will decelerate and expand to fill the entire bearing gap. Due to the turbulent dissipation associated with this expansion, the pressure recovered, $P_B - P_v$, will be only a fraction of the pressure difference $P_r - P_v$ so that there will be a net pressure "loss", $P_r - P_B$, accompanying the entrance of the flow into the bearing clearance. At the point $r_B$ in the bearing clearance, laminar flow will have been established so that the pressure distribution in the rest of the bearing can be calculated from the Navier-Stokes equations for laminar flow.

In addition to the physical situation described above, a more complicated physical situation can arise when the mass flow rate through the bearing is sufficiently great so that the flow in the entrance region of the bearing clearance becomes supersonic. In this case, the transition to laminar flow in the bearing clearance takes place via a normal shock or shocks as is shown in Figure 1b. Although this is a physically more complex situation than that occurring when flow is everywhere subsonic, it is easier to treat analytically as will be shown later in this report. First, however, we shall

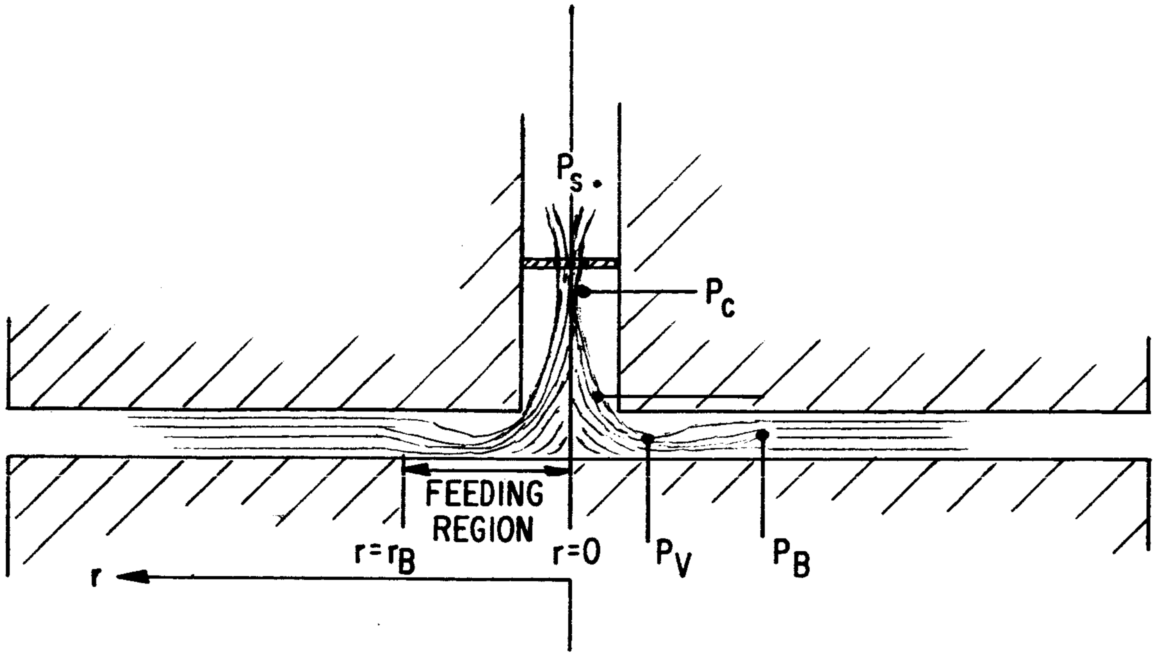



FIG. 1a EXTERNALLY PRESSURIZED CIRCULAR THRUST BEARING IN SUBSONIC FLOW

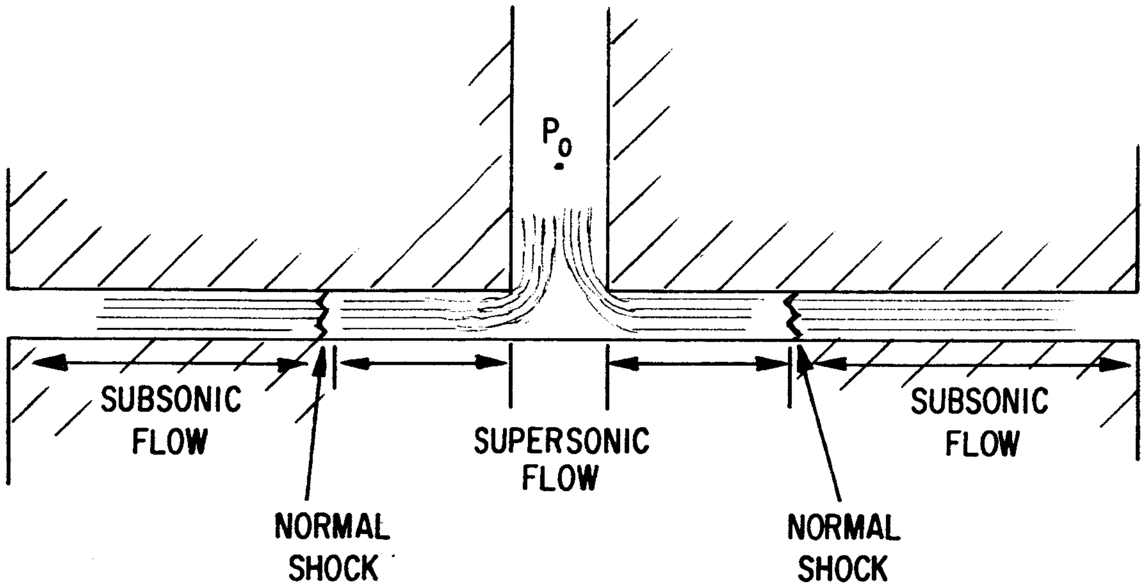



FIG. 1b EXTERNALLY PRESSURIZED CIRCULAR THRUST BEARING IN SUPERSONIC FLOW

consider the problem of determining the feeding characteristics of hydrostatic bearings for the case of subsonic flow. In this discussion we will consider the specific case of the circular thrust bearing shown in Figure 1a The results obtained, however, apply generally to any bearing in which there is radial symmetry about the feeding hole and can be applied to the bearings lacking such symmetry by appropriate geometrical averaging. The specific application of the results to the NASA AB-5 bearing will be considered in a separate section. .

## SUBSONIC ENTRANCE FLOW

The bearing region we will be considering in the discussion of subsonic entrance flow is the region from $r = 0$ to $r = r_B$ shown in Figure 1a. We shall call this the feeding region. Since this region occupies generally only a negligible fraction of the total bearing area, the details of the distribution of pressure in this region are not important with respect to the load carrying capacity of the bearing. What is very important, however, is the magnitude of the overall feeding pressure loss, $P_s - P_B$, as a function of mass flow rate $\dot{m}$, bearing clearance h, and the geometry of the feeding region. This is critically important because the magnitude of the pressure level throughout the bearing is established, essentially, by the magnitude of $P_B$, the downstream pressure of the feeding region.

In the case where the diameter d, of the orifice hole is very small compared with the diameter D and length L of the feeding hole, and the area a of the orifice hole is small compared with $A_o$, the flow entrance area for the bearing clearance, then the overall feeding region pressure loss is due principally to the orifice and can be readily determined. In this case the recovered pressure $P_r - P_c$ and the clearance entrance loss $P_r - P_B$ would be negligible compared with the orifice loss $P_s - P_c$.

If, on the other hand, the areas a and $A_o$ are of the same order of magnitude, then $P_r - P_B$ will be comparable to $P_s - P_B$ and the overall feeding region pressure loss can not be determined by considering the orifice pressure loss alone. Also, if the feeder hole dimensions D and L are comparable to the orifice dimension d, then the recovered pressure $P_r - P_c$ can be significant.

Generally speaking, the pressure loss $P_s - P_c$ occurring across the bearing orifice can be easily predicted. For one thing, there are extensive empirical data available for the mass flow characteristic of standard orifices. Also, it is a fairly easy matter to calibrate particular orifices separate from their assembly. On the other hand, to predict the extent of pressure recovered downstream of a bearing orifice and to predict the pressure loss for the flow entering the bearing clearance are relatively difficult tasks. These quantities can not be theoretically calculated, and generally they must be measured for the particular bearing assembly of interest. Since, however, the ultimate objective of analyzing the feeder region of a gas bearing is to be able to determine the relation between the overall feeding region pressure loss and the mass flow rate, it is obviously more straight forward to measure and correlate $P_s - P_B$ directly than to try to separately determine $P_s - P_c$, $P_r - P_c$, and $P_r - P_B$. One difficulty associated with direct measurements of $P_s - P_B$ vs $\dot{m}$ is the question of whether these measurements can be meaningfully correlated in terms of the significant geometrical parameters of the feeding region. The rest of this section will be devoted to deriving an equation which, it is hoped, can be effectively used to correlate values of $P_s - P_B$ vs $\dot{m}$.

First, we write an expression for the orifice pressure loss $P_o - P_c$. This can be done by writing the ideal isentropic equation for flow through an orifice and multiplying this expression by an empirical vena contracta coefficient $\nu$ and an empirical "efficiency" coefficient $\eta$. The latter coefficient accounts for the fact that the actual flow would not be exactly isentropic. The expression obtained is:

$$\frac{\dot{m}}{g_c} = \nu\sqrt{\eta}\, a \left\{ P_s \frac{\rho_s}{g_c}\left(\frac{P_c}{P_s}\right)^{2/K}\left(\frac{K}{K-1}\right)\left[1 - \left(\frac{P_c}{P_s}\right)^{\frac{K-1}{K}}\right]\right\}^{1/2} \qquad (1)$$

where
- $\dot{m}$ = mass flow rate lbs/sec
- $a$ = orifice area $in^2$
- $P_s$ = supply pressure $lb/in^2$
- $\rho_s$ = supply density $lbs/in^3$
- $P_c$ = static pressure at vena contracta $lb/in^2$
- $k$ = ratio specific heats = 1.4 for air
- $g_c$ = gravitational constant = 386 $in/sec^2$

From the viewpoint of physical understanding, Equation (1) is the best way of relating the flow through the orifice to the parameters affecting it. For simplicity, however, it is better to use the following "working" equation for compressible flow through an orifice

$$\frac{\dot{m}}{g_c} = 1.43\, a K Y_1 \sqrt{\frac{\rho_s}{g_c}\left(P_s - P_c\right)} \tag{2}$$

The factor K in Equation (2) is the vena contracta orifice coefficient for incompressible flow while $Y_1$, called the expansion factor, is an empirical factor to adjust the equation for compressible flow. For square edged orifices K is usually about 0.61 while $Y_1$ is a function of the pressure ratio across the orifice, varying between 1 for $P_c/P_s = 1$ to about .87 for $P_c/P_s = 0.6$.

Next we consider the pressure recovered as a result of the high velocity orifice jet impinging on the bottom thrust plate of the bearing. This can be expressed in terms of a recovery factor r defined by

$$r \equiv \frac{P_r - P_c}{P_s - P_c} \tag{3}$$

r, in general, will depend in some complex way on the dimensions L, D, and d and probably also upon the ratio $P_s/P_c$. For fixed geometry it could be determined as an empirical function of mass flow rate.

Finally, we consider the pressure loss $P_r - P_B$ associated with the flow entering into the bearing clearance. As discussed earlier, this pressure loss occurs as a result of the dissipation of the kinetic energy of the entering flow into heat by turbulence. This pressure loss should, therefore, be able to be expressed in terms of an empirical factor, $Z^2$, multiplying the dynamic pressure of the flow entering the bearing clearance. That is

$$P_r - P_B = \frac{Z^2 \dot{m}^2}{2 g_c \rho A_o} \tag{4}$$

The density $\rho$ to be used in Equation (4) should be the density of the flow at the point of maximum pressure (point of vena contracta). However, for convenience, the value $\rho_B$ can be used in Equation (4) and the difference accounted for in the empirical factor $Z^2$. One should note that the area $A_o$ in Equation (4) should be multiplied by some coefficient to take account of the vena contracta effect in the entrance to the bearing clearance. This coefficient is also included in the value of $Z^2$, which could result in $Z^2$ having magnitude greater than 1.

Combining Equations (2), (3) and (4) we obtain an expression for $\dot{m}$ in terms of $P_s - P_B$.

$$\frac{\dot{m}}{g_c} = \frac{1.43\, a K Y_1 \sqrt{\frac{\rho_s}{g_c}\left(P_s - P_B\right)}}{\sqrt{1 - r + \left(Z K Y_1\right)^2 \left(\frac{a}{A_o}\right)^2 \frac{\rho_s}{\rho_B}}} \qquad (5)$$

Equation (5) can be used to correlate experimental measurements for $\dot{m}$ vs $P_s - P_B$. In using Equation (5) one would generally evaluate the orifice coefficient $KY_1$ by separate experimental measurement on the orifice alone. This would then leave the two coefficients r and $Z^2$ as adjustable parameters to fit Equation (5) to the data for $\dot{m}$ vs $P_s - P_B$.

## APPLICATION OF FEEDING REGION ANALYSIS TO AB-5 BEARING

The geometry of the NASA AB-5 bearing is such that the overall pressure loss in the feeding region should be influenced significantly by the effects of pressure recovery in the feeder hole and pressure loss in the entrance region to the bearing clearance. However, in the analysis of the AB-5 bearing recently performed by MTI (Ref.1), the overall feeding region pressure losses were assumed to be those due to the orifice alone, an assumption made necessary by the lack of knowledge concerning the actual feeding characteristics of this bearing. With the above simplifying assumption, matching feeder region downstream with the upstream pressure at the start of the laminar clearance flow required using the unrealistic procedure of matching the pressures at a point that was actually within the feeder hole. It was concluded by MTI that to further improve the analysis of the AB-5 bearing, it would be necessary to take more accurate account of the behavior of the flow in the feeding regions of this bearing.

To try to obtain some idea of the relative magnitude of the different pressure losses in the feeder region of the AB-5 bearing, an attempt was made to apply equation (5) to available experimental data on the AB-5 bearing. This was done in the following way. From the NASA data 2588-10, -12, -14 and -16 particular operating conditions of mass flow rate and bearing clearance were selected. Corresponding to each mass flow rate and clearance a particular value of the orifice source strength, C, was calculated. C is defined in Reference 1 as

$$C = \frac{\dot{m}\, 12\mu}{\pi \rho_a P_a h^3}$$

Given a value of C, the laminar flow pressure profile in the bearing could be determined by the MTI analysis presented in Reference 1. In particular, Figure 16 in this reference gives values of, $P_i$, the pressure in the bearing clearance at the edge of the orifice feeding hole, from which one could obtain $P_s - P_i$, the overall pressure loss across the feeding region.

When values of $P_i$ were determined in the above manner, it was found that for small clearances, ($h \leq 6 \times 10^{-4}$ in.), the values of $P_i$ were greater than the supply pressure. In view of the generally excellent accuracy of theoretically predicted pressure profiles for gas bearings with laminar flow, it is believed that the unreasonably large values obtained for $P_i$ were the result of error in the measurement of either $\dot{m}$ or h. Some evidence supporting this belief is provided by the fact that the pressure profiles calculated by MTI analysis agree exactly with the pressure profiles measured by NASA if one assumes a value for C which is less than that corresponding to the NASA measurements of $\dot{m}$ and h. In any case, the fact remains that reliable values of the pressure loss across the AB-5 feeding regions cannot be obtained from the presently available data on this bearing. Since it is of demonstrated importance to know the AB-5 feeding characteristics, it is recommended that specific measurements of the overall pressure loss over the feeding region be made for the AB-5 bearing at different mass flow rates, pressures and clearances and that equation (5) be used to correlate the measurements. An experimental program for the purpose is outlined in Appendix A.

## SUPERSONIC ENTRANCE FLOW

We now turn our attention to the case in which supersonic flow occurs in the bearing clearance For ease in discussing this phenomenon, we will consider the specific case of a circular thrust bearing such as is shown in Figure 2. However, the physical concepts discussed will apply, in general, to all externally fed bearings. To gain an understanding of the conditions under which supersonic flow will occur in the entrance region of a bearing, let us consider what happens to the flow through the circular thrust bearing shown in Figure 2, when the ambient Pressure $P_a$ at the exit of the bearing issteadily reduced while the feeder hole supply pressure $P_o$ and the bearing clearance h are kept constant. The first curve (curve 1) shows a situation in which $P_a$ is only slightly less than $P_o$. In this case the flow is subsonic everywhere and the pressure distribution has the following characteristics. There is a decrease in pressure to the point $a_1$, corresponding to the acceleration of the flow entering the clearance. Downstream of $a_1$, the flow rapidly decelerates and some of the dynamic pressure of the flow is recovered. At the point $b_1$, the dynamic pressure of the flow is negligible and laminar flow is established in the bearing. Downstream of $b_1$ the pressure distribution curve is that corresponding to laminar flow.

As the exit pressure $P_a$ is steadily lowered below the point $C_1$, the mass flow rate through the bearing increases until the point is reached where the pressure at $a_2$ in the inlet to the bearing clearance is 0.528 times $P_o$, the feeder hole pressure. At this point, the flow in the inlet to the bearing clearance will be at sonic velocity and the flow will be "choked" , that is, the maximum flow rate corresponding to the given $P_o$ and flow entrance area will have been obtained. Downstream of the point $a_2$ the flow still decelerates subsonically, recovering some of the dynamic pressure of the flow, until laminar flow is established.

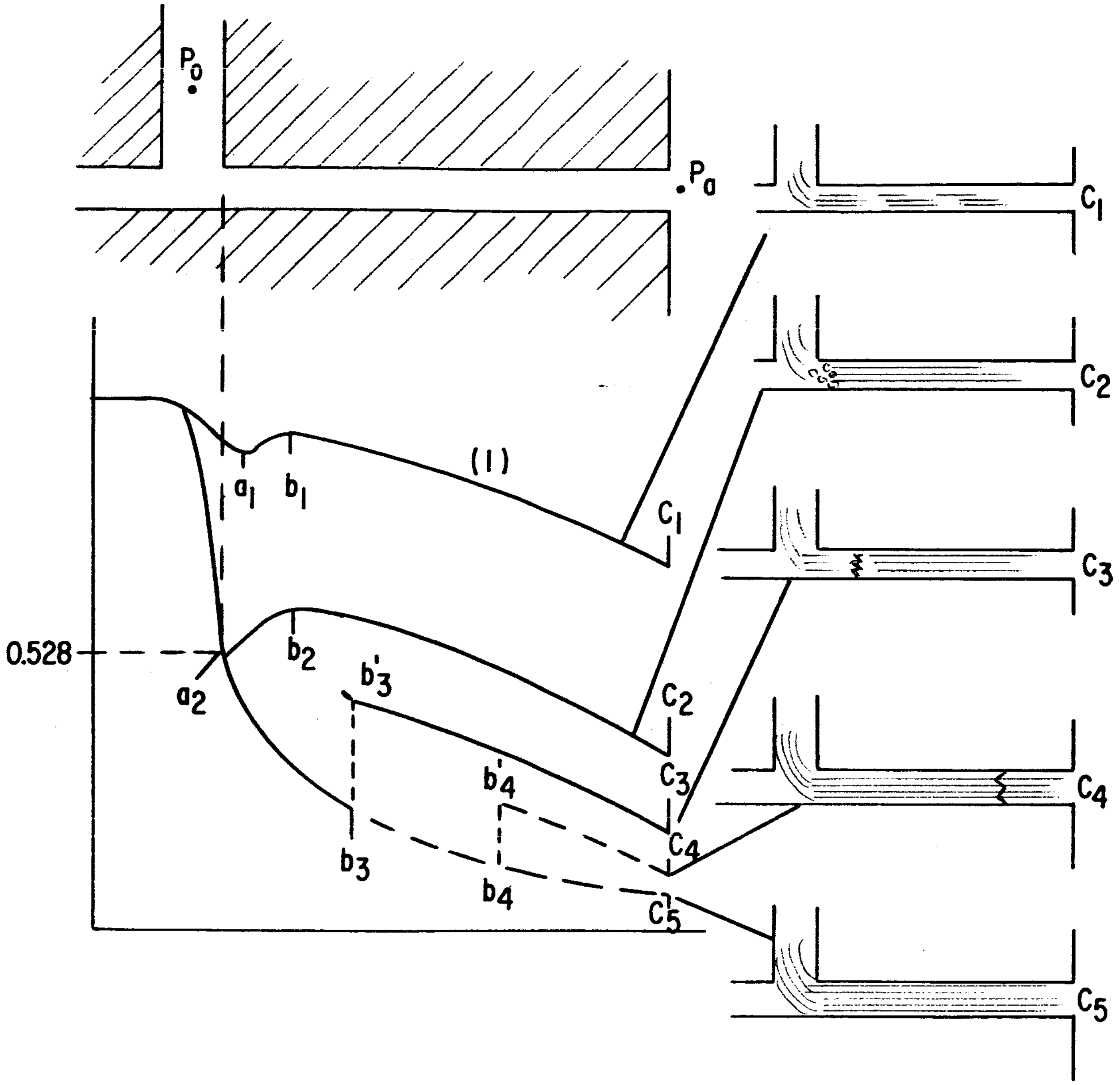



FIG. 2 SUPERSONIC OPERATION OF CIRCULAR THRUST BEARING

Now, if the bearing exit pressure is reduced still further below that corresponding to point $b_2$, the mass flow and the pressure at the point $a_2$ will both remain unchanged. However, in order that the exit pressure condition be satisfied, the flow downstream of the point $a_2$ must now become supersonic and its eventual transition to subsonic laminar flow must now take place by means of a normal shock. This situation is illustrated by curve $a_2 b_3 b_3' C_4$. From $a_2$ to $b_3$ the flow is supersonic. At $b_3$ a normal shock occurs and the pressure increases discontinuously to the point $b_3'$. From $b_3'$ to $C_3$ the flow us subsonic and laminar.

As the exit pressure is reduced still further below the point $C_3$, the position of the shock in the bearing clearance will move steadily toward the bearing exit as indicated by the dashed line $a_2 b_3 b_4 b_4' C_4$. When the shock reaches the bearing exit, the entire flow in the bearing will be supersonic and further decrease in exit pressure will have no effect on the pressure distribution in the bearing. The pressure distribution for this final situation is indicated by the dashed line $a_2 b_4 C_5$.

Although the flow in a bearing is more complex when supersonic flow occurs, it actually is much easier to analytically calculate the performance of bearings under these conditions than when the flow is entirely subsonic. The reason for this is that under supersonic flow conditions, the mass flow rate is determined by the sonic conditions at the minimum corss-section, flow area in the bearing i.e. at the entrance to the bearing clearance. Theoretically, this mass flow rate is given by Fleigner's formula (Ref.2).

$$\dot{m} = 0.532 \frac{P_o A_o}{\sqrt{T_o}} \tag{6}$$

where

$P_o$ = Feeder hole stagnation pressure PSIA

$T_o$ = Feeder hole stagnation temperature $^oR$

$A_o$ = Flow cross-section area at bearing clearance entrance $in^2$

If the mass flow rate through the bearing is known, then the pressure distribution in the laminar flow region downstream of the shock can be readily calculated. Also, as will be shown later, the pressure distribution in the supersonic flow region appears to be predicted remarkably well by means of the equations governing one-dimension adiabatic flow in ducts with friction and area change. The only details of pressure distribution which cannot be readily predicted are the details of the pressure rise through the shock system in the bearing. This shock system, however, generally occupies only a small fraction of the radial distance in a bearing, and thus the details of pressure distribution associated with it are not too important with respect to the overall load characteristics of the bearing.

Let us now consider the equations by which the pressure distribution in a bearing can be calculated under conditions of supersonic flow in the entrance region of the bearing. First, it is assumed that at the very entrance to the bearing clearance, i.e. at the point of minimum area for the flow, the flow is at sonic velocity. Furthermore, it is assumed that the acceleration of the flow from stagnation conditions in the feeder hole to sonic conditions at the bearing entrance takes place isentropically. This assumption establishes that the pressure at the bearing entrance would be 0.528 times the stagnation pressure in the feeder hole. This result is obtained from the equations governing one dimensional isentropic, compressible flow.

Once in the bearing clearance, the flow continues to accelerate to a supersonic velocity.In analyzing this flow it is assumed that it is adiabatic and that it can be adequately described as a one dimensional flow. The latter assumption is probably reasonable since, for practical bearing clearances, the flow in the supersonic region will be turbulent. The assumption of adiabatic conditions is more questionable, since the bulk static temperature of the flow with no external heat transfer will be very much lower than the bearing temperature. However, frictional dissipation in the boundary layer of the flow will create a region of high temperature near the bearing surfaces which would limit the heat transferred to the flow from the bearing. The net effect could very well be a nearly adiabatic flow condition.

With the above assumptions, the equations governing the supersonic flow in the bearing are:

$$\frac{dP}{P} = \frac{KM^2}{(1-M^2)}\frac{dA}{A} - \frac{KM^2\left[1+(K-1)M^2\right]}{2(1-M^2)} fr_o/D_B \, d\bar{r} \qquad (7)$$

$$\frac{dM^2}{M^2} = \frac{-2\left(1+\frac{K-1}{2}M^2\right)}{(1-M^2)}\frac{dA}{A} + \frac{KM^2\left(1+\frac{K-1}{2}M^2\right)}{(1-M^2)} fr_o/D_B \, d\bar{r} \qquad (8)$$

where

| | | |
|---|---|---|
| $M$ | = | Mach Number |
| $D_B$ | = | Hydraulic diameter in bearing clearance |
| $f$ | = | Friction factor for turbulent |
| $A$ | = | Flow cross section area in bearing |
| $\bar{r}$ | = | $r/r_o$ |
| $r_o$ | = | radial point at which minimum flow cross section occurs (sonic throat). |

To obtain the pressure distribution in the supersonic flow region, it is first necessary to integrate Equation (8) to obtain values of $M^2$ which then can be used in Equation (7) to obtain values of P. One can note that, for $M > 1$, the effect of a radial increase in flow area is to increase the flow Mach Number and decrease the static pressure while the effect of friction is exactly opposite to this. Equations (7) and (8) were integrated for four different values of the parameter $\frac{fr_o}{D_B}$. The results are shown in Figures 3 and 4.

Unless the stagnation pressure in the feeder hole of the bearing is extremely high or bearing clearance is very large, the supersonic flow in the entrance region of the bearing clearance will eventually go through a transition to subsonic laminar flow. Although, in actuality, this transition takes place by means of a complicated system of oblique and normal shocks, it can, for analytical purposes, be considered to take place by means of a single normal shock. $P_y$, the static pressure obtained following the normal shock is expressed in terms of $M_x$ and $P_x$, the Mach Number and pressure preceding the shock, by means of the following equation -

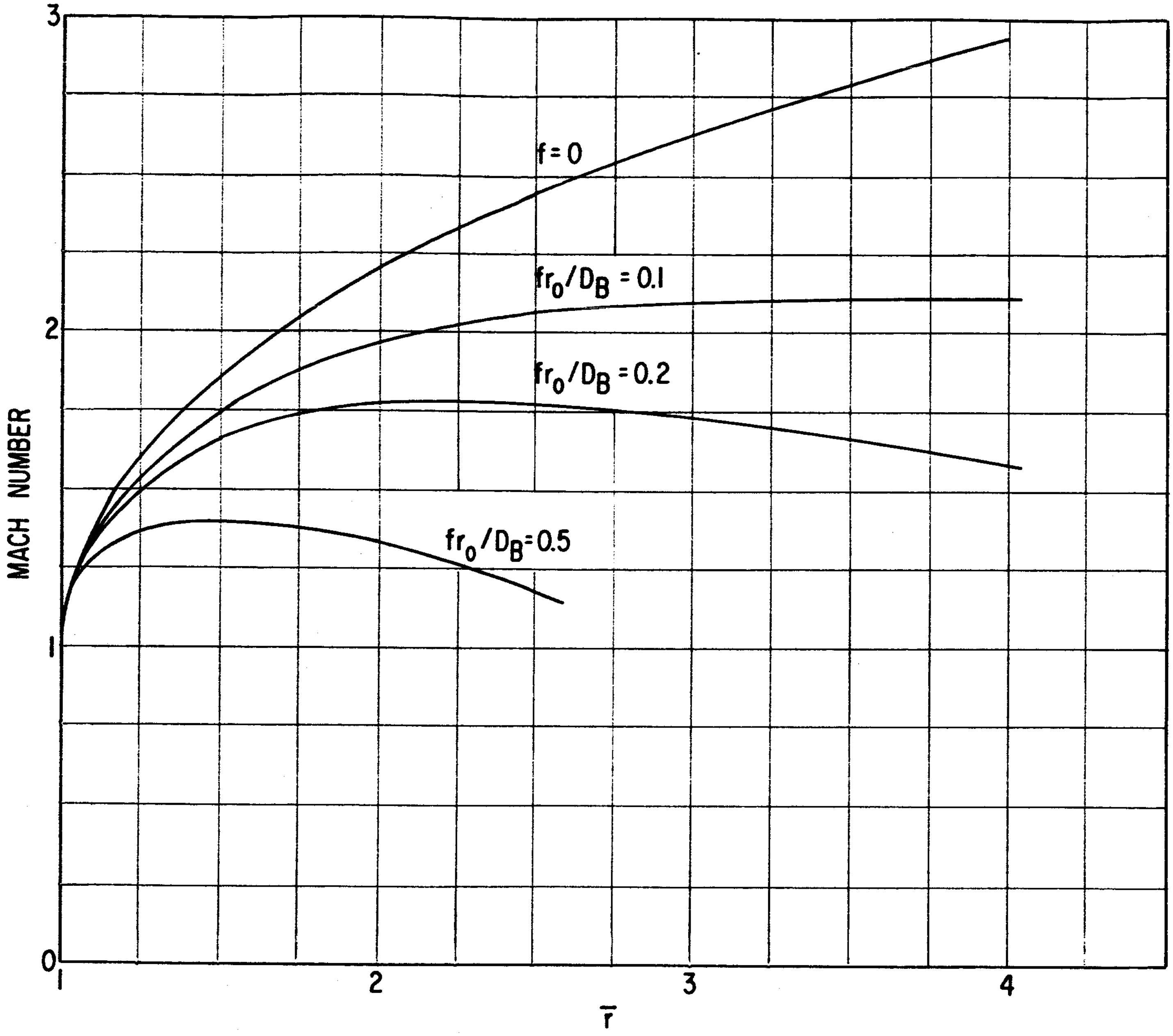



FIG. 3 MACH NUMBER vs RADIAL POSITION FOR SUPERSONIC FLOW
(K = 1.4)

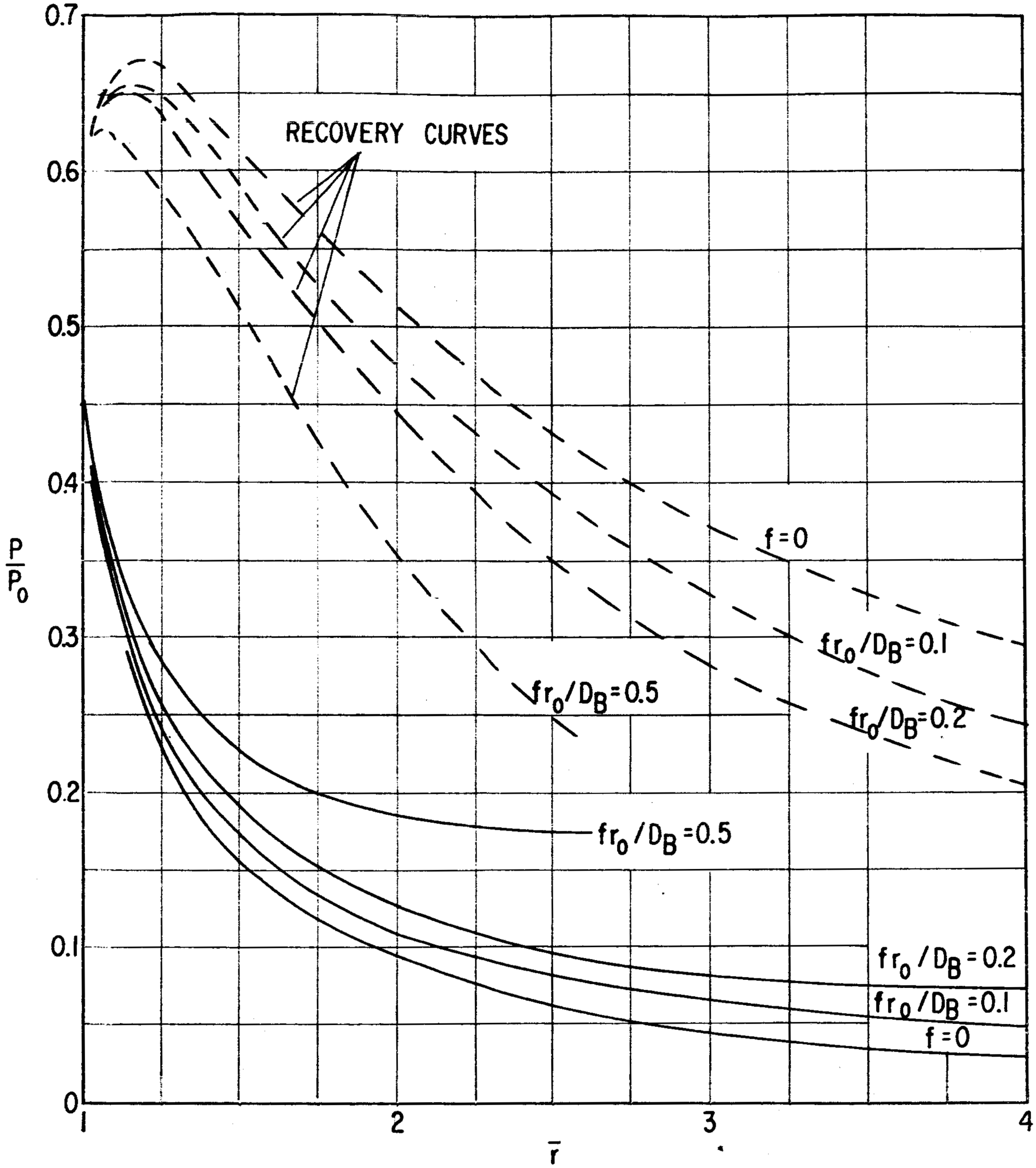



FIG. 4 $P/P_0$ vs RADIAL POSITION FOR SUPERSONIC FLOW (K= 1.4)

$$P_y/P_x = \frac{2K}{K+1} M_x^2 - \frac{K-1}{K+1} \qquad (9)$$

By means of Equation (9), a curve of $P_y$ vs. $\bar{r}$ can be obtained corresponding to each of the curves of P vs. $\bar{r}$ for supersonic flow shown in Figure 4. These curves of $P_y$, labeled recovery curves, are shown in Figure 4. They give the initial value of the static pressure for the subsonic, laminar flow downstream of normal shock occurring at different values of $\bar{r}$.

If the subsonic, laminar flow in the bearing is assumed to be isothermal, and if inertia effects are neglected, then the pressure distribution for subsonic laminar flow can be calculated from the expression:

$$P^2(r) - P_a^2 = \frac{12\mu \dot{m} R_c T_a}{\pi h^3} \ln \frac{R}{r} \qquad (10)$$

where
$\dot{m}$ = Mass flow rate lb/sec
$\mu$ = Viscosity at ambient temperature $\frac{\text{lb-sec}}{\text{in}^2}$
$T_o$ = Ambient Temperature-$^o$R
$R_c$ = Gas constant -in/$^o$R
h = Bearing Clearance -in.
R = Radial of bearing - in.
$P_a$ = Ambient Pressure - lb/in$^2$

As noted earlier, $\dot{m}$, the mass flow rate through the bearing, is determined by the condition of sonic or "choked" flow at the entrance to the bearing clearance and is given theoretically by Fleigner's formula

$$\dot{m} = 0.532 \frac{P_o A_o}{\sqrt{T_o}} \qquad (11)$$

Now, Equation (10) is derived by neglecting the inertial terms in the equations of motion for viscous laminar flow. In close to the center of a circular thrust bearing, however, inertial effects can have a significant effect on the pressure gradient accompanying the flow. This effect can be important when one is attempting to match the calculated static pressure in the laminar flow regime to the calculated pressure downstream of the shock transition from supersonic to subsonic flow. To solve the equations of motion with both the viscous and non-linear inertial terms included is a very difficult task. One can, however, obtain a first order inertial correction to equation (10) quite easily by means of the following procedure. Assuming a parabolic velocity distribution across the bearing clearance and uniform pressure across the clearance, and letting $V_m$ represent the mean velocity in the radial direction, the pressure gradient in the radial direction due to inertia effects is given by:

$$\left(\frac{dP}{dr}\right)_{inertia} = -1.2 \frac{\rho}{2g_c} \frac{d\left(V_m^2\right)}{dr} \qquad (12)$$

where the factor 1.2 is obtained as a result of the parabolic velocity distribution. The difference in pressure between the point r and R which arises because of inertia effects can be obtained by integrating equation (12) between those points. Using the mean value theorem of integral valculus we obtain for the integral of (12)

$$\left[P(r) - P_a\right]_{inertia} = -0.6 \frac{\bar{\rho}}{g_c} \left[V_m^2(r) - V_m^2(R)\right] \qquad (13)$$

where $\bar{\rho}$ is some mean value of the density between the points r and R.

Equation (13) can now be used to obtain a first order inertial correction to the pressure distribution given by Equation (10). The procedure is as follows. $V_m^2$ (R) is calculated exactly since the mass flow rate through the bearing is known and the density at R is the ambient density. Next, $V_m^2(r)$ is calculated approximately by assuming that the pressure at r is that predicted by Equation (10), the density at r being determined from the pressure by means of the condition of isothermal flow. Now, $\bar{\rho}$ the mean value of density in Equation (13), must lie between the maximum and minimum value of $\rho$ in the region between r and R. The minimum value of $\rho$ is $\rho_a$ . The maximum of $\rho$ will be approximately the value of $\rho$ at r. Corresponding to these maximum and minimum values of one can obtain maximum and minimum first order evaluation of $\left[P(r) - P_a\right]_{inertia}$.

It should be obvious from the way in which the above inertia correction is calculated that it will be accurate only when the correction amounts to a small percentage of the uncorrected pressure. In all cases, however, the correction does provide a simple way of estimating the magnitude of the effect of inertia terms on the laminar flow pressure distribution, and undoubtedly the correction always does bring the purely viscous pressure distribution closer to the exact pressure distribution.

In the above paragraphs are briefly presented the equations and procedures for the calculation of pressure distribution in a circular thrust bearing with supersonic flow. The analysis is not new or original but is, basically, the approach suggested by Mori (Ref.3) for analyzing the supersonic pressure depression in externally pressurized bearings. One difference between the analysis presented here and that presented by Mori is the use in the present analysis of computer integrated supersonic flow equations. A second difference is the suggested first order inertial correction for the pressure distribution in the subsonic flow regime.

In order to see exactly how the above equations are used, let us consider a sample problem. Let us calculate the pressure distribution for the circular thrust bearing shown in Figure 5 under the following conditions with air as the lubricant.

| | | |
|---|---|---|
| $R$ | - | Radius of bearing - 3 in. |
| $r_o$ | - | Radius of feeder hole - .0625 in. |
| $h$ | - | Bearing clearance - .004279 in. |
| $A_o$ | - | Flow area at entrance to bearing ≅ 2 $r_o h = 1.68 \times 10^{-3}$ in$^2$ |
| $P_o$ | - | Stagnation pressure in feeder hole = 85 psia |
| $T_o$ | - | Stagnation temperature in feeder hole = 540$^o$R |
| $P_a$ | - | Ambient pressure 14.7 psia |
| $T_a$ | - | Ambient temperature 540$^o$R |

At present, we shall simply assume that the above conditions would result in supersonic flow in the bearing. Later there will be discussed a simple criterion which can be applied to indicate whether supersonic flow is or is not likely to occur under a given set of conditions. In any case, if one assumed supersonic flow to occur, the resulting calculation would indicate whether the assumption is correct.

The first thing to be calculated is the Reynolds number for the flow in the bearing clearance. At the entrance to the bearing clearance it is assumed that sonic conditions are reached isentropically. Therefore, conditions at this point can be calculated from tables of one dimensional isentropic compressible-flow functions*.

We get -

$$M = 1$$
$$T = .8333\, T_o = 450^{o}R$$
$$V = 1040 \text{ ft/sec.}$$
$$\rho = .634\, \rho_o = 0.27 \text{ lb/ft}^3$$
$$\mu = 1.075 \times 10^{-5} \text{ lb/sec.ft.}$$
$$D = 2h = .713 \times 10^{-3} \text{ ft.}$$
$$\frac{\rho V D}{\mu} = 1.85 \times 10^4$$

Now, in the bearing clearance the quantity $\rho Vr$ is a constant. Therefore $\rho V$ varies as $1/r$. However, with supersonic flow, the temperature of the flow, and hence $\mu$, decreases with r. Therefore, one can say that the Reynolds Number decreases in the bearing clearance at a rate somewhat less than $1/r$. Since f, the friction factor, depends on Reynolds Number to only approximately the minus one-fourth power, one can reasonably consider the friction factor to be constant over a range for which the radius changes by only a factor of two or three.

Based on the Reynolds Number calculated above for flow at the bearing entrance, the friction factor f is determined from standard plots of f vs. $N_{Re}$ to be

$$f = .026 \qquad (14)$$

from which we get

$$\frac{fr_o}{D} = 0.19 \cong 0.2 \qquad (15)$$

* for example, Reference 4.

Referring to Figure 4, where solutions of equations (8) and (9) are plotted for various values of $\frac{fr_o}{D}$, we obtain our solution for the pressure distribution in the supersonic flow region for the case $fr_{o/D}=0.2$. We also obtain the corresponding curve for $P_y$, the recovered pressure following a normal shock. These two curves are plotted in Figure 5.

Next one calculates the mass flow rate through the bearing. This can be done using Equation (11) or can be done directly from the already calculated values for $\rho$ and V at the bearing entrance by using the relation

$$\dot{m} = \rho V 2\pi r h = 3.28 \times 10^{-3} \ \text{lb/sec} \qquad (16)$$

Using this value of $\dot{m}$ in Equation (10) together with the appropriate value of $\mu$ for air at ambient temperature we get

$$P^2(r) = 216 + 148 \ln R/r \qquad (17)$$

Equation (17) gives the pressure distribution in the laminar regime downstream of the normal shock neglecting inertia terms. This distribution is plotted in Figure 5. The first order inertia correction for this pressure distribution is obtained from Equation (13). The calculations are as follows. First $V_m(R)$ is determined from the relationship

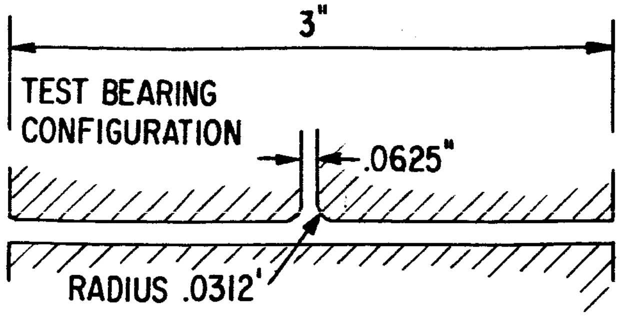
$$\rho_a V_m(R) 2\pi R = \dot{m} = 3.28 \times 10^{-3} \ \text{lb/sec} \qquad (18)$$

from which we get

$$V_m(R) = 158 \ \text{ft/sec} \qquad (19)$$

Next, $V_m(r)$ is calculated in the same way, i.e.

$$\rho(r) V_m(r) 2\pi r = \dot{m} = 3.28 \times 10^{-3} \ \text{lb/sec} \qquad (20)$$



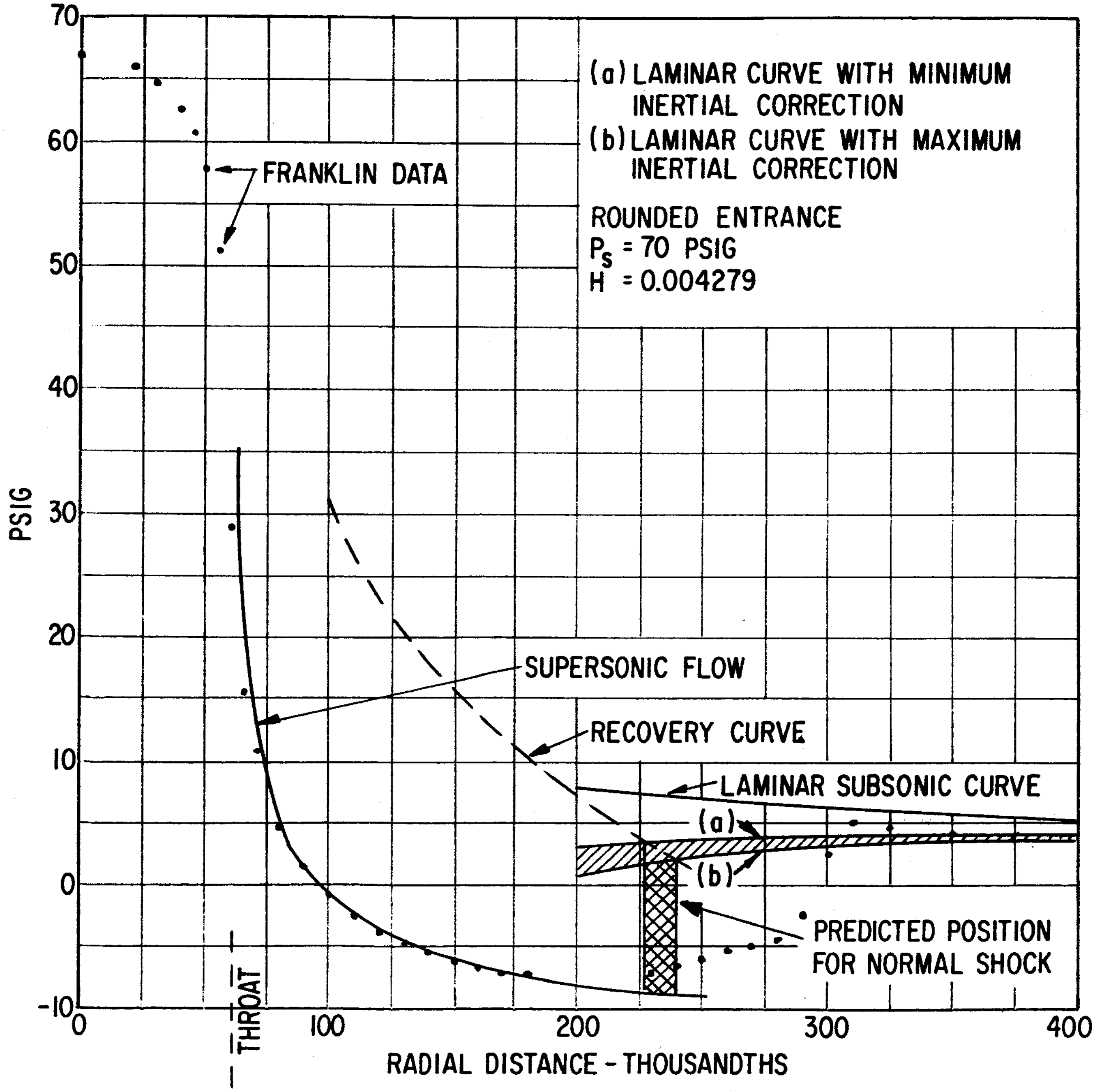




FIG. 5 SUPERSONIC PRESSURE DISTRIBUTION

The value of $\rho(r)$ is obtained from the values of P(r) given by Equation (17). For example at r = .300 in., $\rho(r)$ from Equation (17) is 21.3 psia. Since

$$\frac{\rho_a}{P_a} = \frac{\rho(r)}{P(r)} \tag{21}$$

we get -

$$\rho(r) = 0.105 \ \mathrm{lb/ft^3} \tag{22}$$

Substituting $\rho(r)$, $\dot{m}$ and r in (20) we get

$$V_m(r) = 554 \ \mathrm{ft/sec} \ ; \quad r = .30 \ \mathrm{in} \tag{23}$$

Other values of $V_m(r)$ are obtained in the same way. To evaluate the minimum value of the inertia correction we use the value $\bar{\rho} = \rho_a = .0735 \ \frac{lb}{ft^3}$ in equation (13) to get

$$\left[P(r=.3) - P_a\right]_{inertia} = -2.1 \ \mathrm{psi} \quad \text{(minimum correction)} \tag{24}$$

To evaluate the maximum value we use $\bar{\rho} = \rho(r=.3) = 0.105 \ \mathrm{lb/ft^3}$ in equation (13) to get -

$$\left[P(r=.3) - P_a\right]_{inertia} = -3.2 \ \mathrm{psi} \quad \text{(maximum correction)} \tag{25}$$

The corrections are applied by adding them to the initial value of P(r) obtained neglecting inertia terms. The two corrected curves of P(r) are shown in Figure 5. The area between the two curves is filled in with single-hatched shading and represents the region in which the actual pressure distribution would be predicted to lie.

Finally we come to the prediction of the point at which the normal shock would occur in the bearing. In order that the supersonic flow region by correctly linked to the subsonic flow region, the normal shock should occur at the point where the inertia-corrected subsonic pressure curve intersects the recovered pressure curve. Since the former curve is given as a bounded region. The predicted position for the normal shock will also be determined as a region. This region is shown filled in with cross-hatching in Figure 5.

From the above analysis, the total predicted pressure distribution for the case under consideration would be as follows. From the point $r = r_o$ to the region between r = .227 in. and r = .240 in., the pressure would follow the supersonic flow curve. In the region between r = .227 in. and r = .240 in. a normal shock should occur and the pressure should jump up to the recovery curve. Downstream of this the pressure would lie in the single hatched region i.e. the predicted region for laminarsubsonic flow.

## CRITERIA FOR DETERMINING IF SUPERSONIC FLOW OCCURS IN AN EXTERNALLY PRESSURIZED BEARING

Determining if supersonic flow will occur in an externally pressurized bearing involves, essentially, determining whether a solution of the kind just calculated above exists for the flow through the bearing. To put it another way, supersonic flow will exist through the bearing if the pressure curve for laminar, subsonic flow in the bearing and the pressure curve for supersonic flow in the entrance to the bearing can be joined by means of a normal shock relation. Referring to Figure 5 we see that this joining of the subsonic and supersonic flow regions will be theoretically possible if the subsonic flow curve intersects the recovery curve which gives the downstream pressures following a normal shock in the bearing clearance. Developing an exact yet simple criteria for when this occurs, however, appears impossible. The principle difficulty lies in the fact that, in order to calculate the subsonic flow pressure distribution in near the center of a circular thrust bearing, it is necessary to take account of inertia effects. In very close to the center of such a bearing, a first order corrections for these inertia terms suggested above would no longer be adequate. Solving the exact Navier-Stokes equations with inertia term included, however, would be a very complicated task.

Although an exact yet simple criterion for the existence of supersonic flow in the bearing may not be possible, it would still be of considerable use to have a rough criterion to indicate whether one is near the region of supersonic operation. In the following paragraphs there is developed a rather simple yet conservative criterion to indicate whether a bearing is still securely within the region of entirely subsonic operation. This criterion is based on the following physical reasoning. Basically, supersonic flow will not occur in an externally pressurized bearing when the difference between the supply pressure in the feeder hole and the exit pressure is less than the pressure drop attainable through the bearing with entirely subsonic flow. Now, it can be shown that for a fixed mass flow rate, the pressure drop through the bearing neglecting inertia terms will always be less than the pressure drop calculated including inertia terms. Combining the above two inequality conditions, it follows that if the difference between the feeder hole pressure and the ambient pressure is less than the pressure drop through the neglecting inertia terms assuming a "choked" mass flow rate, then flow in the bearing will be entirely subsonic. This is the criterion suggested for predicting a conservative limit to subsonic operation. This criterion can be expressed in algebraic form in the following way. The calculated pressure drop through the bearing neglecting inertia terms is obtained from Equation (10).

$$P(r_o) - P_a = \left[ \frac{12 \mu \dot{m} R_c T_a}{\pi h^3} \ln R/r_o + P_a^2 \right]^{1/2} - P_a \quad (26)$$

For "choked" flow $\dot{m}$ is given by Equation (11).

$$\dot{m} = 0.532 \frac{P_o A_o}{\sqrt{T_o}} \quad (27)$$

Now, our limit for subsonic flow operation is taken to be the point at which

$$P_o - P_a = P(r_o) - P_a \quad (28)$$

where $P(r_o)$ is calculated from Equation (26) combined with Equation (27). Equations (26), (27) and (28) taken together serve to define a value of $(P_o)_{critical}$ below which subsonic flow should occur. Solving (26), (27) and (28) for $(P_o)_{critical}$ we get

$$(P_o)_{critical} = \phi + \sqrt{\phi + P_a^2} \qquad (29)$$

$$\text{where } \phi = \frac{12\mu\,(0.532)\, r_c\, R_c\, T_a}{h^2\sqrt{T_o}} \ln r/R_c \qquad (30)$$

We can now define an index N as

$$N = \frac{P_o - P_a}{(P_o)_{critical} - P_a} \qquad (31)$$

so that our criterion for subsonic flow becomes:

| | |
|---|---|
| $N < 1$ | Bearing has subsonic flow throughout. |
| $N > 1$ | Flow may be supersonic in part of bearing. |

As noted above, the criterion that N be less than unity for subsonic flow is a conservative one. From experimental evidence to be discussed shortly, it appears that $N_c$, the value of N for which supersonic flow will actually occur in a bearing, lies somewhere in the range

$$1.28 < N_c < 1.76 \qquad (32)$$

Therefore, our criterion does not appear to be so conservative as to be uesless. On the other hand, it is firmly believed that supersonic flow will never occur in a circular thrust bearing under conditions for which N is less than one.

## COMPARISON OF THEORY WITH EXPERIMENT

In Figures 5,6 and 7 the supersonic flow theory outlined in the previous pages is compared with some unpublished experimental data taken in 1958 at the Franklin Institute Laboratories in Philadelphia under ONR Research contract Nonr - 2342(00). The data were taken on a circular thrust bearing of three inch radius having a rounded entrance hole as shown at

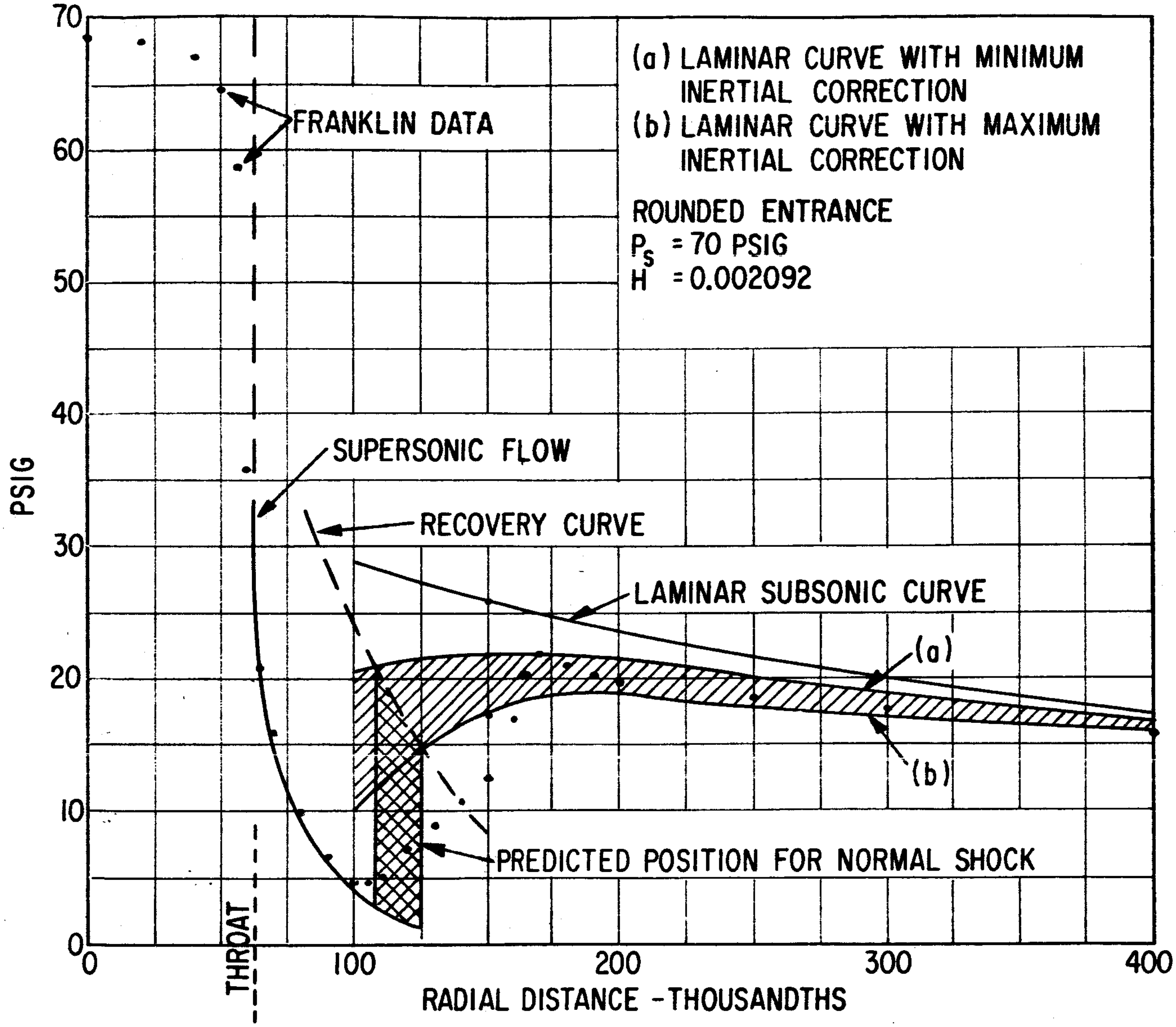



FIG. 6 SUPERSONIC PRESSURE DISTRIBUTION

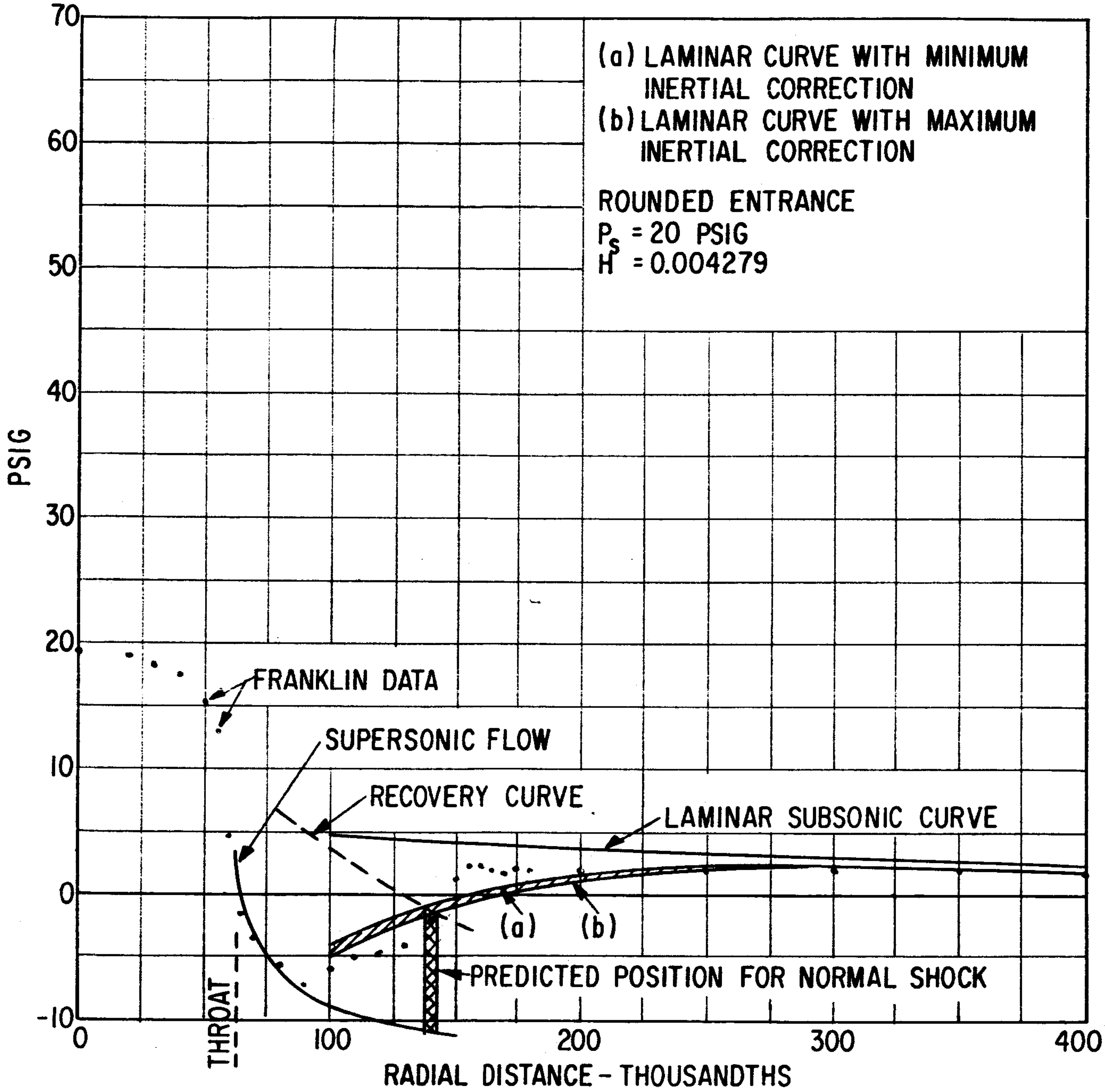



FIG. 7 SUPERSONIC PRESSURE DISTRIBUTION

the top of Figure 5. The minimum flow area (sonic throat) occurs at the point $r_o$ = 0.625 inches.

The data are considered to be preliminary in nature as the Franklin Institute is in the process of reproducing and expanding the data on a redesigned test rig. However, these preliminary measurements appear to have been made with exceptional care and accuracy, and the writer is indebted to Mr. John T. McCabe of the Franklin Institute for granting permission to the writer to show the data.

The theoretical calculations and the experimental measurements in Figures 5 through 7 are in generally excellent agreement. Particularly striking is the very close agreement in the region of supersonic flow. The theoretical supersonic flow curves shown in these figures well all calculated by using a constant friction factor based on the Reynolds Number at the entrance to the bearing. A comparison of the curves in Figures 5 and 6 reveals that the value of the friction factor does have a significant effect on the pressure distribution curves.

The experimentally measured pressure rise from the supersonic flow curve to the subsonic flow curve is seen to take place over a radial distance of about 0.1 inches rather than abruptly and discontinuously as would be predicted for an ideal normal shock. This indicates that the shock system in the bearing probably consists of a series of partially oblique and partially normal shocks rather than consisting of a single normal shock. This would be expected to occur,since a single normal shock would interact with the boundary layer in the bearing giving rise to oblique shocks. In any case, the predicted location for a single normal shock to occur does lie within the range over which the actual shock system appears to be spread.

Downstream of the shock system, the measured laminar subsonic pressure distribution is seen to agree in all cases with the predicted subsonic pressure distribution including the first order correction for inertia.

This indicates that the flow rate through the bearing is correctly predicted by expression (11) and also that the first order correction for inertia is fairly accurate (if the correction is small!) In connection with the use of expression (11) to predict "choked" mass flow rate, it was also observed by P. F. Carrothers (Ref. 2) that this expression was in agreement with experiment to within 3% in the case of circular thrust bearings with sharp edged entrance regions.

Of the various pressure distribution curves measured at the Franklin Institute, only two appeared to be near the condition where supersonic flow was just starting to occur in the bearing. This condition would be characterized by a pressure distribution curve which decreased to 0.528 $P_o$ at the entrance to the bearing and then immediately began to increase again in the bearing clearance. One of the Franklin curves, corresponding to all subsonic flow, decreased to a value of 0.65 $P_o$ at the bearing entrance, which corresponds to an entrance Mach Number of 0.8. For this condition the calculated value for N was 1.28. A second of Franklin's curves decreased to a minimum value of .435 $P_o$ in the entrance region of the bearing. This corresponds to an estimated Mach Number of 1.1. The value for N in this case was 1.76. For the Franklin Bearing, therefore, supersonic flow would apprently start at a value of N somewhere in the range $1.28 < N_c < 1.76$.

APPLICATION OF SUPERSONIC FLOW THEORY TO AB-5 BEARING

The discussion of supersonic flow in bearings presented thus far has related specifically to the case of a circular thrust bearing in which there was radial symmetry about the feeder hole. In the case of the NASA AB-5 bearing, none of the orifices is positioned so as to have a radially symmetric geometry about it. However, in the analysis of this bearing by MTI, it was found that the calculated pressure and flow distributions for the bearing did demonstrate a considerable degree of radial symmetry in the vicinity of each orifice. On the basis of this finding, therefore, it appears that the analysis of supersonic flow pressure distribution presented above for a circular thrust bearing could be applied directly to the NASA AB-5 bearing without incurring very much error, provided the supersonic flow regime did not extend too far from each orifice i.e. beyond a distance of, say, 0.25 inches from the orifice. It should be realized, however, that the pressure distribution in the subsonic laminar regime for the AB-5 bearing would not be given by equation (10) but would be obtained from the MTI computer program for the AB-5 bearing with point source feeding. Normally, this program determines the mass flow rate from each orifice as part of its calculation duties. In the case of supersonic flow through each orifice, the mass flow rate is fixed by the condition of "choked" sonic flow at the entrance to the bearing. Therefore, in this case, the MTI computer program would simply calculate the subsonic flow pressure distribution in the AB-5 bearing using the given mass flow rates for each orifice. Unfortunately, in its present form, the MTI computer program cannot calculate pressure distributions for a "mixed" operating condition, i.e. one in which some orifices are feeding with subsonic flow and some orifices are feeding with supersonic flow.

Considerable difficulties remain in calculating the performance of the AB-5 bearing under supersonic flow conditions, even using the simplified approach suggested above. One difficulty is that of calculating the "choked" mass flow rate through each orifice. This can be done by using Equation (6 )i.e.

$$\dot{m} = 0.532 \frac{P_o A_o}{\sqrt{T_o}} \tag{33}$$

In using expression (33), however, the problem arises as to how to determine the proper value for $P_o$. $P_o$ is the stagnation pressure in the feeder hole. To determine this it is necessary to know not only the mass flow vs pressure difference characteristics for the bearing orifices, but it is also necessary to know the extent to which the dynamic pressure of the flow through the orifice is recovered in the feeder hole. As noted earlier in this report, the extent to which pressure is recovered in the feeder hole can only be determined experimentally.

Assuming that the stagnation pressure in each orifice feeding hole can be determined as a function of mass flow rate, the total calculation of pressure distribution in the AB-5 bearing under supersonic flow conditions would involve the following steps. First, the mass flow rate through each orifice would be determined using Equation (6 ) plus the $P_o$ vs $\dot{m}$ relation for the orifice. Determing $\dot{m}$ in this way would probably require several iterations. Next, the supersonic pressure distribution around each orifice would be determined. To do this it would be necessary to evaluate the parameter $fr_{o/D}$. If the journal is eccentrically located within the bearing, then it is suggested that a linear average of the clearance at the rim of the feeder hole be used to obtain a value for D. In calculating the supersonic pressure distribution around an orifice, it is assumed that the flow pattern is radially symmetric in the vicinity of each orifice so that the curves presented in Figure 4 could be used.

With the mass flow rate from each orifice known, the subsonic flow pressure distribution throughout the bearing would be determined from the MTI analysis of the AB-5 bearing with point source feeding. It should be noted that this analysis assumes that laminar flow exists right up to the edge of each orifice feeder hole. However, it seems likely that the flow pattern in the subsonic region of the bearing would be nearly the same regardless of whether supersonic or subsonic flow were occurring in the immediate vicinity of each orifice. This conclusion is based on the fact that the mass flow distribution tends to be radially symmetric in the vicinity of each orifice under subsonic, laminar flow conditions and would be expected to tend to be equally symmetric under supersonic, turbulent flow conditions.

The calculated pressure distribution in the subsonic and supersonic regions of flow in the AB-5 bearing would be joined or matched at the points at which the subsonic pressure curves intersect the supersonic recovery curves (see Figure 5 ). The extent of radial symmetry of pressure in the subsonic flow regime at these radii would give an indication of how reasonable the assumption of radial symmetry was for the supersonic regime.

## SUMMARY AND CONCLUSIONS

The total pressure "loss" occurring in the entrance region of an externally pressurized gas bearing is a complicated phenomenon consisting of loss due to the feeding orifice, loss due to imperfect recovery of the flow dynamic pressure in the feeder hole, and loss associated with the flow entering the bearing clearance from the feeder hole. These losses cannot be predicted exactly analytically; however, a semi-empirical equation was derived which relates the total pressure loss over the entrance region to the mass flow rate through the bearing. The application of this equation to the AB-5 bearing is discussed, and some experimental measurements are recommended which, when correlated by means of the above mentioned equation, should serve to determine the feeding characteristics of the AB-5 bearing. These recommended measurements are described in Appendix A.

The phenomenon of supersonic flow occurring in the entrance region of a externally pressurized, circular thrust bearing appears to be susceptible to quite accurate analytical prediction. The mass flow rate through the bearing under these conditions appears to agree quite closely with that predicted assuming "choked" (sonic) flow at the entrance to the bearing clearance. Also, the pressure distribution in the supersonic region seems to be predicted very well by the equations governing one-dimensional compressible flow in ducts with friction and area change. One complication that arises when analyzing supersonic flow in bearings is that in the subsonic flow regime just downstream of shock transition, inertia forces may still be quite significant in the flow. A first order correction to account for these was suggested in this report. This correction appears to be fairly accurate provided it does not amount to more than say, 10% of the calculated pressure.

For bearings other than circular thrust bearings, the problem of analyzing supersonic flow operation is made considerably more complicated by the lack of geometrical symmetry about the feeding holes. However, it appears that even in non-circular bearing geometries, the flow fields tend toward radial symmetry about feeding holes. Therefore, it is concluded that supersonic flow analysis presented in this report can be applied directly to the AB-5 bearing geometries provided the supersonic flow regions are confined to the immediate vicinity of the feeding holes.

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APPENDIX A

Experimental Measurements to Determine AB-5 Bearing Feeding Characteristics.

Equation (5), derived in the text of this report,

$$\frac{\dot{m}}{g_c} = \frac{1.43\, a K Y_1 \sqrt{\frac{\rho_s}{g_c}\left(P_s - P_B\right)}}{\sqrt{1 - r + (z K Y_1)^2 \left(\frac{a}{A_0}\right)^2 \frac{\rho_s}{\rho_B}}} \qquad (5)$$

expresses $\dot{m}$, the mass flow rate through a single feeding hole, as a function of $P_s - P_B$, the overall pressure drop over the feeding region ( See. Fig. 1). Equation (5) contains three empirical factors, $KY_1$, r, and $z^2$, which are to be determined by experiment. The first of these to be determined should be $KY_1$, the orifice coefficient. $KY_1$ should be evaluated by having the orifice of interest discharge directly to ambient pressure and measuring the mass flow rate vs $P_s - P_a$ where $P_s$ is the upstream pressure for the orifice and $P_a$ is the ambient pressure. Values of $KY_1$ would then be calculated by means of equation (2) below

$$\frac{\dot{m}}{g_c} = 1.43\, a K Y_1 \sqrt{\frac{\rho_s}{g_c}\left(P_s - P_a\right)} \qquad (2)$$

The values of $KY_1$ obtained should be correlated vs $\frac{P_s - P_a}{P_s}$. Values of $KY_1$ should be determined for the whole range of mass flow rates from near zero up to critical mass flow rate.

To evaluate the coefficients r and $z^2$ one should measure $P_s - P_B$ vs $\dot{m}$ for the test conditions to be enumerated later. Values of $P_s - P_B$ could be measured with the same experimental equipment as was used previously by NASA to measure pressure profiles in the AB-5 bearing i.e. the pressure distribution curves measured April 11, 1962. It should be emphasized that the pressure measurement sensing hole should be kept as small as possible i.e. 2 - 3 mils in diameter. By obtaining pressure profiles in the vicinity of the bearing feeding holes one can obtain a qualitative "picture" of the flow in the vicinity of the orifice as well as obtain a value of $P_s - P_B$. Also, more importantly, one can compare the measured pressure profiles with those analytically predicted by MTI analysis. This should help to resolve the discrepancy between theory and experiment discussed on page 6a in the text.

The experimental conditions for which the pressure profiles should be measured are as follows:

a/Ao - 0.10, 0.20, 0.40, 0.6,1.0
Ambient Pressure - Atmospheric
Mass Flow Rate - zero to critical (i.e. critical mass flow through orifice)

One should note that, for a given orifice area a, each particular value of the ratio a/Ao corresponds to a particular value of the bearing clearance. Five different mass flow rate conditions should suffice to cover the whole range of mass flow rates to be investigated, i.e. five different mass flow rate measurements should be made for each value of a/Ao.