THE SPECTRAL SHIFTS OF AN ENSEMBLE OF RELATIVISTIC EMITTERS*

Peter D. Noerdlinger
Enrico Fermi Institute for Nuclear Studies and Department of Physics
The University of Chicago
and
J. R. Jokipii
Enrico Fermi Institute for Nuclear Studies
The University of Chicago
Chicago, Illinois

Laboratory for Astrophysics and Space Research

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ABSTRACT

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A Doppler shift toward the red is found for the radiation from most members of an ensemble of relativistically moving emitters. This effect would make it difficult to observe blue shifts from quasars if they are relatively nearby objects in highly relativistic motion.

It is generally accepted that quasars are objects exhibiting gravitational or cosmological red shifts since no blue shifts are observed. However, we wish to point out that even if quasars were nearby objects in highly relativistic motion, most would exhibit red shifts. This can be seen from the following analysis.

Consider a number of objects emitting radiation which move in random directions at relativistic velocities. Two physical effects may be identified as producing a preferential Doppler shift to the red as compared to the violet: The transverse part of the Doppler effect and the effect of the Doppler shifts on the apparent lifetimes of the objects. The first effect may be found from the formula (Jackson 1962) for the Doppler effect for light

$$\omega = \frac{(1-\beta^2)^{1/2}}{1-\beta\cos\theta'}\,\omega_0 \tag{1}$$

where ω is the observed frequency, ω_0 is the emitted frequency, $\beta = v/c$, and θ' is the angle between the velocity of source relative to the observer and the direction of the emitted light in the <u>observer's</u> frame of reference.[1] Since the objects are taken to have an isotropic velocity distribution in the observer's frame, their distribution of velocities with respect to θ' must be uniform over all solid angles. Thus the number of objects aimed at angles between θ' and $\theta' + d\theta'$ will be

$$N(\theta')d\theta' = \tfrac{1}{2} N_0 \sin\theta' d\theta' \tag{2}$$

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[1] Here $\theta' = 0$ corresponds <u>to approach.</u>

where the total number of objects is N_o. When β is near unity, the small factor $(1 - \beta^2)^{\frac{1}{2}}$ in the numerator of Eq. (1) tends to produce red shifts even for θ' rather less than $\pi/2$. The case of $\theta' = \pi/2$ is the well known transverse Doppler effect, which may be understood as a time dilation effect. In the sequel, the notation

$$\gamma = (1-\beta^2)^{-1/2}, \quad r = 1+z = \omega_0/\omega, \quad \rho = \frac{(1-\beta)^{1/2}}{(1+\beta)^{1/2}} \tag{3}$$

will be used. Clearly ρ and ρ^{-1} are the extreme values taken by r. The objects will be taken to have equal speeds and will be presumed to be created in groups at random times, all the objects having intrinsic lifetimes that are equal or distributed randomly. The term "intrinsic" here is in contrast to "observed," since the life of an object of redshift r will be multiplied by r when seen by the observer (Noerdlinger 1966). This introduces an additional preponderance of redshifted objects as compared to blue shifted ones. The total number of objects observed on the average will no longer be the time average of N_o, will be increased, since the average of r exceeds unity. This may be thought of as an average tendency of clocks on the emitting objects to run slower than ours, from our viewpoint. Having discussed the qualitative features of the problem, we shall proceed to the calculation of precise results.

Solving Eq. (1) for cos θ', one obtains $\theta' = (\gamma - r)/\gamma\beta$, so that

$$\sin\theta' \qquad d\theta' = (1/\gamma\beta)\, dr \tag{4}$$

Therefore, the distribution of emitters as a function of r is

$$N(r)dr = \frac{1}{2} N_0 \gamma^{-1} \beta^{-1} dr \tag{5}$$

where no allowance for the lifetime effect has been made. With such an allowance, one obtains

$$\langle N(r) \rangle dr = \frac{1}{2} N_0 \gamma^{-1} \beta^{-1} r \, dr \tag{6}$$

The expectation brackets are used to indicate that distribution (6) is applicable in a time-average sense. The lower and upper limits on r are obviously the values at $\theta' = 0$ or π, and are respectively ρ and ρ^{-1}. The mean observed numbers R of redshifted and B of blueshifted objects respectively are found from

$$R = \int_1^{1/\rho} \langle N(r) \rangle dr \quad \text{and} \quad B = \int_\rho^1 \langle N(r) \rangle dr \tag{7}$$

The integrals are done easily, with the results

$$R = \tfrac{1}{2} N_0 / \rho \quad , \quad B = \tfrac{1}{2} \rho N_0 \tag{8}$$

The sum of these exceeds N_o, as predicted from the mean lifetime increase. If the distribution (3) were used, as would be appropriate for emitters created a much shorter time ago than one standard intrinsic lifetime, the values

$$R' = \frac{N_0}{2\beta\gamma}\left(\frac{1}{\rho} - 1\right) \quad , \quad B' = \frac{N_0}{2\beta\gamma}(1 - \rho) \tag{9}$$

would be obtained instead. These add to N_o.

The mean values of r for the two classes of emitters (R and B) may be found from

$$r_R = R^{-1}\int_1^{1/\rho} r\langle N(r)\rangle dr \quad , \quad r_B = B^{-1}\int_\rho^1 r\langle N(r)\rangle dr \tag{10}$$

The resulting values are

$$r_R = \frac{1-\beta}{3\beta}\left(\frac{1}{\rho^3} - 1\right) \quad , \quad r_B = \frac{1+\beta}{3\beta}\left(1-\rho^3\right) \tag{11}$$

It is interesting to note that in the highly relativistic limit ($\beta \to 1$, $\gamma \gg 1$), the mean blue shift tends to the value 2/3, whilst the mean red shift is asymptotic to $2^{3/2}\gamma/3$, which is unbounded as $\gamma \to \infty$. The reason for the limit on the mean blue shift seems to be that most of the blue shifted light comes from emitters near the cone of zero shift ($r = 1$), on account of the comparatively large solid angle available there. This bunching near $r = 1$ becomes more severe, the closer β is to unity, with the result that r_B is bounded.

Of particular interest is the fraction

$$f \equiv B/(R+B) = \rho^2/(\rho^2+1) \tag{12}$$

of objects seen to be blue shifted. If quasars are to be interpreted as objects at less than cosmological distances (Arp, 1966), one must understand why no blue shifts are observed. Values of f are shown in Table 1 for various values of the mean observed redshift r_R. This redshift would apply to single groups of emitters with a common origin only if their lifetimes were broadly distributed and we were observing them at times after creation roughly comparable to their mean lifetime. If detailed models are proposed for the emitters, the lifetime information from such models would have to be used to modify the factor r inserted in Eq. (6). The qualitative effects would not vary much. The present calculations show at least that the observed redshifts could possibly be special relativistic Doppler shifts rather than being cosmological or gravitational. It is evident from the entries in Table 1 that when $\Delta\lambda/\lambda_o = z$ is of order unity, only a tenth of the objects should show blue shifts.

For completeness, we briefly consider the question of relative intensity. Although most of the objects will show red shifts, it is also true that the usual relativistic forward peaking of radiation occurs, enhancing the intensity of objects with smaller r (that is, objects aimed more nearly at the observer). Assume that the emission of an object is isotropic in its rest frame, so that

$$F(\theta, f_o)\,d\theta\,df_o = \frac{1}{2} F_o(f_o) \sin\theta \, d\theta \, df_o \tag{13}$$

is the energy flux emitted at angles between θ and $\theta + d\theta$, with respect to the velocity of the object and per unit bandwidth at frequency f_o. The observer sees the radiation at frequency $f_o/r = f'$, and at the angle θ' related to θ by

$$\tan\theta' = \frac{\gamma^{-1}\sin\theta}{\beta + \cos\theta} \tag{14}$$

(Jackson, 1962). Making use of Eqs. (1), (4), and the relation $E = h\nu$ for photons, whose number must be conserved under any transformation, one eventually obtains

$$F' (\theta', f') d\theta' df' = \frac{F_o \sin \theta' d\theta' df'}{2\gamma^3 (1 - \beta \cos \theta')^3} \tag{15}$$

as the observed energy flux in the solid angle between θ' and $\theta' + d\theta'$ and in the band of frequencies between f' and $f' + df'$. If averaged over a distribution (Eq. 6) of nearby emitters, Eq. (15) leads to the conclusion that more energy is received from the relatively few blue-shifted objects. So long as none of the reddened objects falls below the threshold of intensity required for observation; however, the conclusions as to the relative number of red and blue-shifted objects are unchanged by intensity considerations. If some objects are so distant that they fall below the threshold of observation, these conclusions must be modified, since the red-shifted ones will fall below threshold first. At sufficiently large distances, however, cosmological redshifts will become important.

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TABLE I

THE FRACTION OF RED-SHIFTED OBJECTS VS. THEIR MEAN REDSHIFT

f	$r_R = \langle (1 + z) \rangle$	ρ^2	β
0.500	1.00	1.00	0.00
0.475	1.04	0.9	0.053
0.445	1.06	0.8	0.111
0.375	1.15	0.6	0.250
0.286	1.31	0.4	0.429
0.231	1.46	0.3	0.538
0.167	1.74	0.2	0.667
0.136	1.91	0.15	0.739
0.091	2.27	0.1	0.819
0.048	3.1	0.05	0.905
0.0196	4.7	0.02	0.961
0.01	6.74	0.01	0.990

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