DC-FR-1047

COMPUTER PROGRAMS FOR THE GEOMAGNETIC FIELD

GPO PRICE $ ______

CFSTI PRICE(S) $ ______

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Hard copy (HC) 4.00

Microfiche (MF) .75

Final Report

Contract No. NAS 5-3944

ff 653 July 65

N66 30298	
(ACCESSION NUMBER)	(THRU)
126	1
(PAGES)	(CODE)
CR 67598	13
(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)

FACILITY FORM 602

Prepared for

National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland

August 13, 1965

THE Dikewood CORPORATION [redacted]

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ACKNOWLEDGEMENTS

The author wishes to express thanks for the assistance and suggestions received from Dr. Sam Stearns, Kenneth Granzow, and Duane Jensen, all of The Dikewood Corporation. Mr. Jensen supplied much needed background information on geomagnetics in general and on his Fit program. Mr. Granzow was instrumental in formalizing the external magnetic field. Dr. Stearns reviewed the understanding of the author and the progress of his work many times during the past eight months. Their contribution to the contractual effort was significant.

The author also wishes to thank Dr. Joseph Cain of the Goddard Space Flight Center for patience and understanding of our efforts to complete these tasks satisfactorily.

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1.0 INTRODUCTION

The geomagnetic potential function is usually represented by a spherical-harmonic expansion. The Dikewood Corporation has prepared two computer programs related to this spherical-harmonic expansion for the Goddard Space Flight Center. The first program, Jensen's Fit, is a least squares program for improving the precision of the parameters in the spherical-harmonic expansion. The second program, Wall's Error, is a statistical program for mapping the field from the spherical-harmonic expansion produced by Jensen's Fit and then computing the random error in the mapping.

These two programs are similar in much of the basic formulation. Hence, Section 2.0 of this report on basic formulation applies to both programs. Section 3.0 discusses in some detail Jensen's Fit program while Section 4.0 discusses in a similar manner Wall's Error program.

Most if not all of the information presented in this report can be found in the documents and books listed in the bibliography.

This document is a final report on work performed at The Dikewood Corporation and does not cover any program modifications later made at Goddard Space Flight Center for the accommodation of the Goddard computer system.

2.0 BASIC FORMULATION

We begin with the "terms of internal origin" in the Chapman and Bartels (1940, p. 639) formulation of the geomagnetic potential function to write the following equation for the magnetic potential V:

$$V = a \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+1} \left(g^{n,m} \cos m\phi + h^{n,m} \sin m\phi\right) P^{n,m}(\theta) \tag{1}$$

where

a = radius of the earth

r = geocentric distance

g, h = Gauss coefficients

ϕ = longitude

θ = colatitude

$P^{n,m}(\theta)$ = associated Legendre functions, Gauss normalized

The three orthogonal components are derived by taking the gradient $\vec{B} = +\nabla V$ to give

$$B_\theta = \frac{1}{r}\frac{\partial V}{\partial \theta} = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+2} (g^{n,m} \cos m\phi + h^{n,m} \sin m\phi) \frac{\partial P^{n,m}(\theta)}{\partial \theta} \tag{2}$$

$$B_\phi = \frac{1}{r \sin\theta}\frac{\partial V}{\partial \phi} = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+2} \frac{m}{\sin\theta} (-g^{n,m} \sin m\phi + h^{n,m} \cos m\phi) P^{n,m}(\theta) \tag{3}$$

$$B_r = \frac{\partial V}{\partial r} = -\sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+2} (n+1)(g^{n,m} \cos m\phi + h^{n,m} \sin m\phi) P^{n,m}(\theta) \tag{4}$$

and, of course,

$$B = (B_\theta^2 + B_\phi^2 + B_r^2)^{1/2} \tag{5}$$

There have been several versions of Jensen's Fit. Basic formulation for the original versions ended with the above equations. However, the most recent version included provision for a field of external origin. The field specified was a simple magnetic field of intensity B at infinity (denoted by B_∞) and parallel to an arbitrary axis z' in the (x', y', z') coordinate system, and positive in the increasing z' direction.

To develop the formulation for this field, we begin with

$$\vec{B}_\infty = +\nabla V = +\nabla(+B_\infty z')$$

$$V = +B_\infty z'$$

$$= +B_\infty r(\cos\gamma)\, P^{1,0}(\theta)$$

$$= +B_\infty r P^{1,0}(\alpha)\, P^{1,0}(\theta) + B_\infty r P^{1,1}(\alpha)\, P^{1,1}(\theta)(\cos(\beta-\phi)) \tag{6}$$

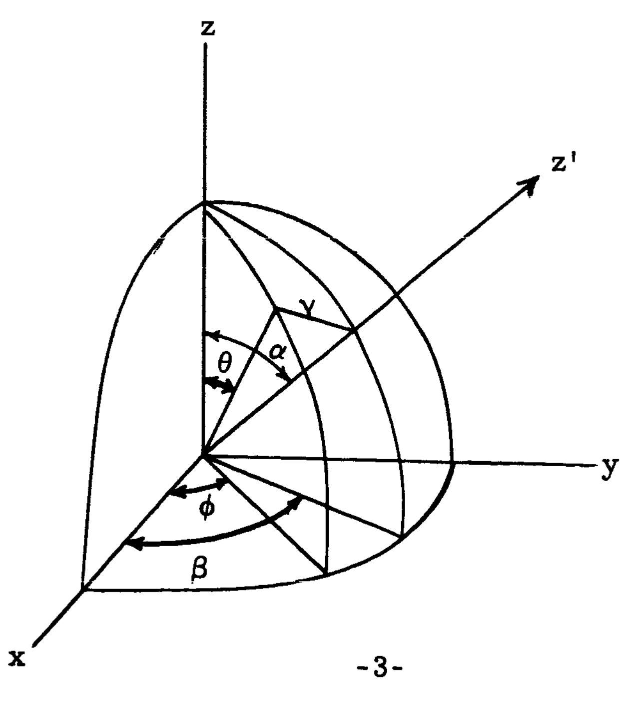


Fig. 1

Let $B_\infty \cos\alpha = E_1$

$B_\infty \sin\alpha \cos\beta = E_2$

$B_\infty \sin\alpha \sin\beta = E_3$.

Then

$$V = rE_1 P^{1,0}(\theta) + rE_2 \cos\phi P^{1,1}(\theta) + rE_3 \sin\phi P^{1,1}(\theta) \tag{8}$$

The three orthogonal components are derived by taking the gradient $\vec{B} = +\nabla V$ to give

$$B_\theta = \frac{1}{r}\frac{\partial V}{\partial\theta} = E_1 \frac{\partial P^{1,0}(\theta)}{\partial\theta} + E_2 \cos\phi \frac{\partial P^{1,1}(\theta)}{\partial\theta} + E_3 \sin\phi \frac{\partial P^{1,1}(\theta)}{\partial\theta} \tag{9}$$

$$B_\phi = \frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi} = -\frac{E_2 \sin\phi}{\sin\theta} P^{1,1}(\theta) + \frac{E_3 \cos\phi}{\sin\theta} P^{1,1}(\theta) \tag{10}$$

$$B_r = \frac{\partial V}{\partial r} = E_1 P^{1,0}(\theta) + E_2 \cos\phi P^{1,1}(\theta) + E_3 \sin\phi P^{1,1}(\theta) \tag{11}$$

Substituting for the Legendre polynomials,

$P^{1,0}(\theta) = \cos\theta$

and

$P^{1,1}(\theta) = \sin\theta$,

we have

$$B_\theta = -E_1 \sin\theta + E_2 \cos\phi \cos\theta + E_3 \sin\phi \cos\theta \tag{12}$$

$$B_\phi = -E_2 \sin\phi + E_3 \cos\phi \tag{13}$$

$$B_r = E_1 \cos\theta + E_2 \cos\phi \sin\theta + E_3 \sin\phi \sin\theta \quad . \tag{14}$$

Hence, the complete basic formulation including the simple external field is given by

$$V = a \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+1} (g^{n,m} \cos m\phi + h^{n,m} \sin m\phi)\ P^{n,m}(\theta)$$

$$+ rE_1\ P^{1,0}(\theta) + rE_2 \cos\phi\ P^{1,1}(\theta) + rE_3 \sin\phi\ P^{1,1}(\theta) \quad . \tag{15}$$

The orthogonal components including the simple external field are

$$B_\theta = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+2} (g^{n,m} \cos m\phi + h^{n,m} \sin m\phi)\ \frac{\partial P^{n,m}(\theta)}{\partial\theta}$$

$$- E_1 \sin\theta + E_2 \cos\phi \cos\theta + E_3 \sin\phi \cos\theta \tag{16}$$

$$B_\phi = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+2} \frac{m}{\sin\theta}\ (-g^{n,m} \sin m\phi + h^{n,m} \cos m\phi)\ P^{n,m}(\theta)$$

$$- E_2 \sin\phi + E_3 \cos\phi \tag{17}$$

$$B_r = -\sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+2} (n+1)\,(g^{n,m} \cos m\phi + h^{n,m} \sin m\phi)\ P^{n,m}(\theta)$$

$$+ E_1 \cos\theta + E_2 \cos\phi \sin\theta + E_3 \sin\phi \sin\theta \quad . \tag{18}$$

The external field is defined as positive if the direction of force is toward the positive z' axis. Hence from Fig. 1, E_1 is positive if the direction of force is out of the northern hemisphere. E_2 is positive if the direction of force is out of the hemisphere bisected by the Greenwich meridian (i. e., the half-circle passing through Greenwich) and E_3 is positive if the direction of force is out of the hemisphere to the right of the Greenwich meridian (i. e., east longitude).

In the above, $P^{n,m}(\theta)$ has been used to denote Gauss normalized associated Legendre polynomials. To simplify the computer program, the Legendre polynomials have been adjusted so that the coefficients of the highest order term in θ is one. Generating functions for these polynomials and their derivatives are given by

$$P^{0,0}(\theta) = 1.0$$

$$\frac{\partial P^{0,0}(\theta)}{\partial \theta} = 0.0$$

$$P^{n,n}(\theta) = \sin\theta\, P^{n-1,n-1}(\theta) \qquad n \geq 2$$

$$\frac{\partial P^{n,n}(\theta)}{\partial \theta} = \sin\theta \frac{\partial P^{n-1,n-1}(\theta)}{\partial \theta} + \cos\theta\, P^{n-1,n-1}(\theta) \qquad n \geq 2$$

$$P^{n,m}(\theta) = \cos\theta\, P^{n-1,m}(\theta) - K_{n,m}\, P^{n-2,m}(\theta) \qquad n \geq 2,\ m \neq n$$

$$\frac{\partial P^{n,m}(\theta)}{\partial \theta} = \cos\theta \frac{\partial P^{n-1,m}(\theta)}{\partial \theta} - \sin\theta\, P^{n-1,m}(\theta) - K_{n,m} \frac{\partial P^{n-2,m}(\theta)}{\partial \theta} \qquad (19)$$

where $K_{n,m} = \dfrac{(n-1)^2 - m^2}{(2n-1)(2n-3)}$.

These Gauss normalized polynomials are then converted to the Schmidt quasi-normalized functions $P_n^m(\theta)$ via the relationship:

$$P_n^m(\theta) = S^{n,m} P^{n,m}(\theta)$$

where

$$S^{0,0} = -1.0$$

$$S^{n,0} = S^{n-1,0} \left(\frac{2n-1}{n}\right)$$

$$S^{n,1} = S^{n,0} \left(\frac{2n}{n+1}\right)^{1/2}$$

$$S^{n,m} = S^{n,m-1} \left(\frac{n-m+1}{n+m}\right)^{1/2} . \qquad m \geq 3 \qquad (20)$$

To introduce secular change coefficients into the equations for the potential function and its three orthogonal components, each coefficient in the terms of internal origin are expressed as follows:

$$g^{n,m} = g^{n,m,0} + g^{n,m,t}t + g^{n,m,tt}t^2$$

and

$$h^{n,m} = h^{n,m,0} + h^{n,m,t}t + h^{n,m,tt}t^2 \qquad (21)$$

Hence, the complete basic formulation including the simple external field using the Schmidt quasi-normalized functions $P_n^m(\theta)$ is given by

$$V = a \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+1} \Big((g^{n,m,0} + g^{n,m,t}t + g^{n,m,tt}t^2) \cos m\phi$$

$$+ (h^{n,m,0} + h^{n,m,t}t + h^{n,m,tt}t^2) \sin m\phi \Big) P_n^m(\theta)$$

$$- rE_1 P_1^0(\theta) - rE_2 \cos\phi P_1^1(\theta) - rE_3 \sin\phi P_1^1(\theta) \quad . \qquad (22)$$

The orthogonal components are given by

$$B_\theta = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+2} \Big((g^{n,m,0} + g^{n,m,t}t + g^{n,m,tt}t^2) \cos m\phi$$

$$+ (h^{n,m,0} + h^{n,m,t}t + h^{n,m,tt}t^2) \sin m\phi \Big) \frac{\partial P_n^m(\theta)}{\partial\theta}$$

$$+ E_1 \sin\theta - E_2 \cos\phi \cos\theta - E_3 \sin\phi \cos\theta \qquad (23)$$

$$B_\phi = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+2} \frac{m}{\sin\theta} \Big((-g^{n,m,0} - g^{n,m,t}t - g^{n,m,tt}t^2) \sin m\phi$$

$$+ (h^{n,m,0} + h^{n,m,t}t + h^{n,m,tt}t^2) \cos m\phi \Big) P_n^m(\theta)$$

$$+ E_2 \sin\phi - E_3 \cos\phi \qquad (24)$$

$$B_r = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+2} (n+1) \Big((g^{n,m,0} + g^{n,m,t}t + g^{n,m,tt}t^2) \cos m\phi$$

$$+ \left(h^{n,m,0} + h^{n,m,t}t + h^{n,m,tt}t^2\right) \sin m\phi \Big) P_n^m(\theta)$$

$$- E_1 \cos\theta - E_2 \cos\phi \sin\theta - E_3 \sin\phi \sin\theta \qquad (25)$$

Secular change coefficients were not included for the external field. The Dikewood Corporation has proposed to modify the formulation to include secular change coefficients for the external field in a future contractual effort.

To provide compatibility with the FORTRAN programming system, the spherical-harmonic expansion of the geomagnetic potential function (Eq. (22)) is redefined in terms of subscripts starting at one instead of zero, by letting $N = n+1$ and $M = m+1$. Thus,

$$V = a \sum_{N=2}^{\infty} \sum_{M=1}^{N} \left(\frac{a}{r}\right)^N \Big(\left(g_{N,M,0} + g_{N,M,t}t + g_{N,M,tt}t^2\right) \cos(M-1)\phi$$

$$+ \left(h_{N,M,0} + h_{N,M,t}t + h_{N,M,tt}t^2\right) \sin(M-1)\phi \Big) P_{N,M}(\theta)$$

$$- rE_1 P_{2,1}(\theta) - rE_2 \cos\phi P_{2,2}(\theta) - rE_3 \sin\phi P_{2,2}(\theta) \quad . \qquad (26)$$

The orthogonal components of the magnetic field are given by

$$B_\theta = \sum_{N=2}^{\infty} \sum_{M=1}^{N} \left(\frac{a}{r}\right)^{N+1} \Big(\left(g_{N,M,0} + g_{N,M,t}t + g_{N,M,tt}t^2\right) \cos(M-1)\phi$$

$$+ \left(h_{N,M,0} + h_{N,M,t}t + h_{N,M,tt}t^2\right) \sin(M-1)\phi \Big) \frac{\partial P_{N,M}(\theta)}{\partial\theta}$$

$$+ E_1 \sin\theta - E_2 \cos\phi \cos\theta - E_3 \sin\phi \cos\theta \quad , \tag{27}$$

$$B_\phi = \sum_{N=2}^{\infty} \sum_{M=1}^{N} \left(\frac{a}{r}\right)^{N+1} \frac{M-1}{\sin\theta} \Big((-g_{N,M,0} - g_{N,M,t}t - g_{N,M,tt}t^2) \sin(M-1)\phi$$

$$+ (h_{N,M,0} + h_{N,M,t}t + h_{N,M,tt}t^2) \cos(M-1)\phi \quad P_{N,M}(\theta)$$

$$+ E_2 \sin\phi - E_3 \cos\phi \quad , \tag{28}$$

$$B_r = - \sum_{N=2}^{\infty} \sum_{M=1}^{N} \left(\frac{a}{r}\right)^{N+1} N \Big((g_{N,M,0} + g_{N,M,t}t + g_{N,M,tt}t^2) \cos(M-1)\phi$$

$$+ (h_{N,M,0} + h_{N,M,t}t + h_{N,M,tt}t^2) \sin(M-1)\phi \Big) P_{N,M}(\theta)$$

$$- E_1 \cos\theta - E_2 \cos\phi \sin\theta - E_3 \sin\phi \sin\theta \quad . \tag{29}$$

The above formulation is rigorously correct only for a spherical earth. It is obvious that as the accuracy of evaluation of the geomagnetic field increases, it will eventually be necessary to take the earth's true shape into account. So long as the evaluation of the harmonic coefficients is done in spherical coordinates, r, θ, and ϕ, the resulting fields B_r, B_θ, and B_ϕ will be in strict geocentric directions. The only constant in the potential function pertaining to the earth is the radius $\underline{a}$ here chosen to be the mean radius or 6371.2 km . The only problem is that of converting positions in geodetic coordinates to geocentric coordinates. Both the Fit and

Error programs perform coordinate conversion for the oblateness of the earth only. Referring to Fig. 2, we can write:

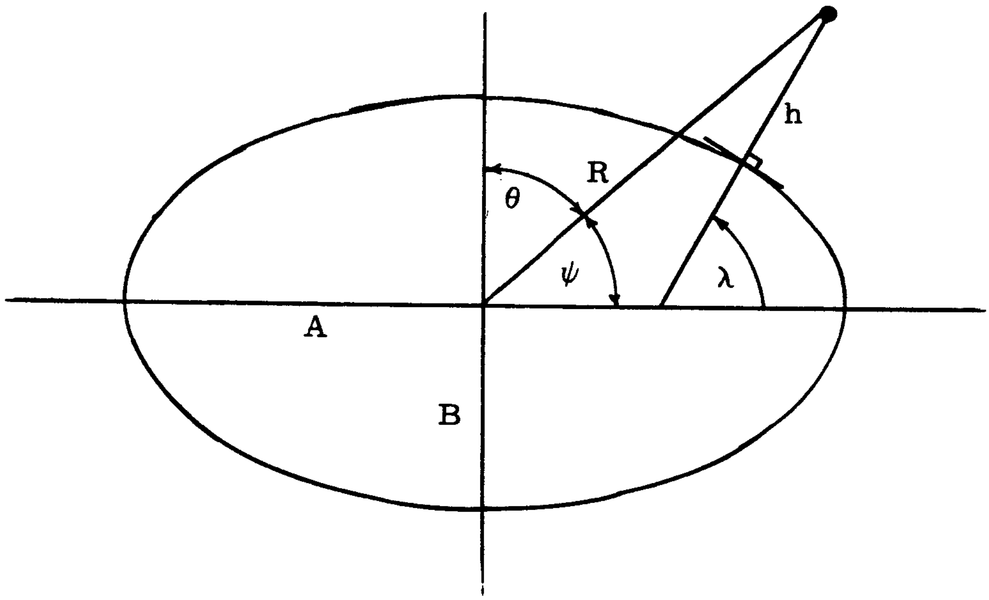


Fig. 2

$$\tan\psi = \frac{h\sqrt{A^2\cos^2\lambda + B^2\sin^2\lambda} + B^2}{h\sqrt{A^2\cos^2\lambda + A^2\sin^2\lambda} + A^2}\quad \tan\lambda \tag{30}$$

and

$$R^2 = h^2 + 2h\sqrt{A^2\cos^2\lambda + B^2\sin^2\lambda} + \frac{A^4\cos^2\lambda + B^4\sin^2\lambda}{A^2\cos^2\lambda + B^2\sin^2\lambda} \tag{31}$$

where:

h = height above the geoid

ψ = geocentric latitude $(90° - \theta)$

λ = geodetic latitude

R = geocentric distance

A = mean equatorial radius of 6378.165 km

B = polar radius of 6356.783 .

Using λ and h, the geocentric quantities $\theta = 90 - \psi$ and R can then be calculated. The conversion from B_r and B_θ to X and Z can then be done by the rotation:

$$X = -B_\theta \cos(\lambda - \psi) - B_r \sin(\lambda - \psi) \tag{32}$$

$$Z = B_\theta \sin(\lambda - \psi) - B_r \cos(\lambda - \psi) \tag{33}$$

To complete the coordinate systems, note that

$$Y = B_\phi \quad .$$

Geomagnetic data have been assembled from many sources. Hence the reliability of the information varies. Since instrument accuracy and some other factors are known for the different data sources, quantitative estimates of reliability can be made. These reliability estimates are the basis for a system of geomagnetic data weights. Table 1 below lists the standard error associated with the different data sources.

The quantities measured in Gamma were weighted inversely as the standard error listed. Thus values of H measured at an observatory would have a weighting factor of 1/5 and values of H measured in Canada would have a weighting factor of 1/60, so that the observatory data would count 12 times as much as the Canadian data. For those quantities measured in in degrees, the effect of the error is greater for points where the field is stronger. Thus, these data were weighted by the factor $\frac{1}{\delta D \cdot H}$ for D and $\frac{1}{\delta I \cdot F}$ for I .

Table 1

Estimated Standard Errors

	δD^{o}	δI^{o}	δH^{γ}	δZ^{γ}	δF^{γ}
Observatory	0.0033	0.006	5	15	15
Land Survey	0.1	0.1	30	50	-
Air Survey	0.3	0.1	-	-	30
Shipboard	0.083	0.083	25	-	-
Satellite	-	-	-	-	10
Towed magnetometer (proton)	-	-	-	-	10
Towed magnetometer (fluxgate)	-	-	-	-	40
Repeat observations	0.033	0.083	5	-	2
Canadian data	0.3	-	60	60	-

3.0 JENSEN'S FIT PROGRAM

Jensen's Fit Program is simply a computer program for determining small corrections to an already good set of parameters for the spherical-harmonic expansion of the geomagnetic function.

The general scheme for the program is a common least squares approach. The best available estimate of the coefficients of the potential function are used to estimate the magnetic field for observation locations.

Then the actual observations are compared with these estimates and corrections for the potential function coefficients are computed from all available data, both old and new, via the method of least squares. Since the corrections are not optimum, the procedure must be repeated until the corrections are no longer significant. Hence much computer time is consumed each time the coefficients are up-dated with new data.

The procedure is to express the geomagnetic measurements in terms of the field components X, Y, and Z developed in Section 2. 0 of this report. As functions of the g's and h's, these expressions are expanded into Taylor series that include only linear terms. Then, via the method of least squares, corrections for the improvement of the g's and h's are estimated. These corrections are applied and the procedure repeated until the g and h parameters converge.

As an example of a Taylor's expansion, consider the measurement declination,

$$D = \tan^{-1} \frac{Y}{X} = \tan^{-1} \frac{B_\phi}{-B_\theta \cos(\lambda-\psi) - B_r \sin(\lambda-\psi)} \tag{34}$$

As a function of X and Y, we can write an algebraic expression for declination first in terms of B_r, B_θ, and B_ϕ as is done above and then in terms of the g's and h's of the potential function. The Taylor's expansion for D as a function of a single g and h is as follows:

$$D(g+\Delta g, h+\Delta h) \approx D(g,h) + \Delta g \frac{\partial D(g,h)}{\partial g} + \Delta h \frac{\partial D(g,h)}{\partial h} \tag{35}$$

Now the term on the left is the observation while D(g,h) is a computed or expected measurement based on the best available set of g and h parameters. On rearranging this equation, we see that the error (or residue),

$$D(g+\Delta g, h+\Delta h) - D(g,h) = \Delta g \frac{\partial D(g,h)}{\partial g} + \Delta h \frac{\partial D(g,h)}{\partial h} \tag{36}$$

is linear in the correction terms Δg and Δh. Hence, a standard least-squares procedure can be used to find values for these corrections. These corrections are then applied to each of the coefficients, and the procedure repeated until the desired accuracy is obtained. It should be recognized at this point that g and h as here used can and do represent a large group of parameters. The number of parameters is determined by the limits on the summations in the potential function.

Since this linear expression resulted from a Taylor series that was truncated after the linear terms, it is accurate only as long as the summation term is small compared with the expected value. This requirement is not difficult to meet since there are several sets of coefficients available that represent the geomagnetic potential with very small error.

Similar expressions for dip,

$$\text{dip} = \tan^{-1} \frac{Z}{H} \quad , \tag{37}$$

horizontal field,

$$H = (X^2 + Y^2)^{1/2} \quad , \tag{38}$$

total field,

$$T = (X^2 + Y^2 + Z^2)^{1/2} \quad , \tag{39}$$

and the X, Y, and Z field components can be used to estimate the corrections for the g's and h's. It is also possible to mix the use of geomagnetic measurements so long as the corresponding expression is used for the g and h corrections.

The coefficients of the set of simultaneous equations form a symmettic matrix. To conserve computer storage, and hence to permit the estimation of a larger number of g's and h's, only half of these coefficients are stored in the computer program. If one thinks of the complete array required for the normal equations as consisting of the square matrix of the coefficients of the unknowns plus two additional columns, one for the observation terms and the other for the computation check sums, then the program stores the upper half of the square matrix including the diagonal terms and the two additional columns. This array, which is illustrated in Fig. 3 for a system of five unknowns, is stored by rows in a one dimensional array named D in the computer program. In the illustration, the usual two dimensional Fortran subscript is equated with the corresponding position in the one dimensional D array. Given the subscript (I, J) for any element in the complete rectangular array, its position in the D array is determined by

$$K = I*(NOR + 2) - I*(I - 1) / 2 + J - NOR - 2 \tag{41}$$

where NOR is the number of rows in the complete matrix. To illustrate how this formula determines the position in the D array, it is rewritten as follows:

$$K = I*(NOR+2) - I*(I-1)/2+J - NOR - 2 \tag{41}$$

					Observation Column	Check Sum
(1,1)=D(1)	(1,2)=D(2)	(1,3)=D(3)	(1,4)=D(4)	(1,5)=D(5)	D(6)	D(7)
	(2,2)=D(8)	(2,3)=D(9)	(2,4)=D(10)	(2,5)=D(11)	D(12)	D(13)
		(3,3)=D(14)	(3,4)=D(15)	(3,5)=D(16)	D(17)	D(18)
			(4,4)=D(19)	(4,5)=D(20)	D(21)	D(22)
				(5,5)=D(23)	D(24)	D(25)

Fig. 3

Upper Triangular Matrix Storage
(illustrated with a 5 x 5 matrix)

or

$$K = (I-1)*(NOR+2) - I*(I-1)/2+J \quad . \tag{42}$$

Now consider two positional notations. The first is the position in the complete array from which the array in Fig. 3 was taken. Number the elements in this complete array from left to right by rows. The second notation is the one used for the subscripts of D in Fig. 3. Then the interpretation of each of the three terms in the K equation is as follows:

(I-1)*(NOR+2) = position preceding the first position in row I of the complete rectangular array, i.e., each row has NOR+2 elements and this is multiplied by the row number less one. For row 1, this position is the position numbered zero or one less than position one.

I*(I-1) / 2 = the sum of the digits preceding the I^{th} digit. In the D array, one notes that in row 2, one position is not used; in row 3, two positions are not used, and so forth. Hence, the number of elements not used by row I is the sum of the positive digits less than I. Hence, the first two terms of K compute the position preceding the first position in a row of the D array.

J = column position which is added to the position preceding the first position in a row of the D array.

Each entry in the computation check column is the sum of the coefficients of the parameters in the respective row. For example, in Fig. 3, the third term in this column will be stored in D(18) and is the sum

$$D(3) + D(9) + D(14) + D(15) + D(16) \quad .$$

The computation check column is a pseudo-observation column in which all of the unknowns are assumed to be 1.0. Hence on solution of the system with this pseudo-observation column, estimates of the unknowns should approximate 1.0. Their failure to do so indicates the precision of the true estimates made from the observation terms themselves.

The procedure employed for the solution of the least squares equations is a modification of the common Gauss elimination method. The modification consists of computing the "back solution" at the same time that the "forward solution" is computed, i. e., the matrix of coefficients (here assumed to be the complete square array) is diagonalized at the same time as it is made into a triangle.

In the modified Gauss elimination procedure, the following sequence of operations is performed for each row:

(1) Each element in the row (beginning with the diagonal element) is divided by the diagonal element. Let the diagonal row and diagonal column be defined as the matrix row and column respectively that contain this element.

(2) Then beginning with the first column to the right of the diagonal column, each element in every row other than the diagonal row has subtracted from it the product of the corresponding element in the diagonal row and the corresponding element in the diagonal column.

From the procedure above, one can see that as each row is considered at step (1), the corresponding complete column is required for step (2). This column is contained in the triangle array D.

The solution of the set of simultaneous least squares equations yields adjustments or corrections for the parameters based on the average observation time. The parameters for starting the program are based on the year 1960. Hence, the corrections must be computed for 1960 instead of the average observation time. To do this, we note that $g_{N,M,0}$ for time t is estimated by

$$g_{N,M,0} + g_{N,M,t}t + g_{N,M,tt}t^2 \tag{43}$$

for time zero. Adding another subscript to indicate epoch, this relation may be put into an equation as follows:

$$g_{N,M,0,t} = g_{N,M,0,0} + g_{N,M,t,0}t + g_{N,M,tt,0}t^2 \quad . \tag{44}$$

Similarly,

$$g_{N,M,0,0} = g_{N,M,0,t} + g_{N,M,t,t}t + g_{N,M,tt,t}t^2 \quad . \tag{45}$$

Taking differences with respect to the g's ,

$$\Delta g_{N,M,0,0} = \Delta g_{N,M,0,t} + \Delta g_{N,M,t,t}t + \Delta g_{N,M,tt,t}t^2 \tag{46}$$

we obtain the correction for $g_{N,M,0}$ at time zero from the corrections at time t. In a similar manner the correction for $g_{N,M,t}$ at time zero from the corrections at time t can be shown to be

$$\Delta g_{N,M,t,0} = \Delta g_{N,M,t,t} + 2\,\Delta g_{N,M,tt,t}t \quad . \tag{47}$$

Similar expressions apply to the h's .

In Appendix A, Jensen's complete Fit program is listed. Appendix B is a brief description and cross reference of the program. Appendix C is a flow chart of Jensen's Fit program.

4.0 WALL'S ERROR

The following derivation establishes the relationship between errors in the given data and errors in the coefficients determined by a least-squares procedure. Assume that the values y_i are measured at points having coordinates $X_i = (x_{1i}, x_{2i}, x_{3i}, x_{4i})$ and that it is desired to fit a curve of the form $y = \sum_n a_n f_n(X)$ to the data. In this equation, a_n corresponds to the g's and h's in the spherical-harmonic expansion of the geomagnetic potential function and $f_n(x)$ corresponds to the known coefficients which are, incidentally, all functions of the position X in time and space. The least squares procedure involves the calculation of a vector $\underline{v}$ whose elements are

$$v_n = \sum_i f_n(X_i)\, y_i$$

and a matrix $\underline{A}$ whose elements are

$$A_{mn} = \sum_i f_m(X_i)\, f_n(X_i) \quad .$$

The coefficients a_n of the fitted curve are then found by solving the matrix equations

$$\underline{\underline{A}} \cdot \underline{a} = \underline{v}$$

for the coefficients a_n, the elements of vector $\underline{a}$. The solution is

$$\underline{a} = \underline{\underline{A}}^{-1} \cdot \underline{v}$$

where $\underline{\underline{A}}^{-1}$ is the inverse of $\underline{\underline{A}}$, and the coefficients are

$$a_n = \sum_j A^{nj} v_j = \sum_j \sum_i A^{nj} f_j(X_i)\, y_i \quad . \tag{48}$$

Now, since the y_i are independent measurements, the standard error σ_{a_n} of coefficient a_n can be found from the expression

$$\sigma_{a_n}^2 = \sum_i \left(\frac{\partial a_n}{\partial y_i}\right)^2 \sigma_{y_i}^2$$

Assuming that all σ_{y_i}'s have the same value $\sigma_{y'}$,* and using values for the partial derivatives obtained from Eq. (48),

$$\sigma_{a_n}^2 = \sigma_{y'}^2 \sum_i \left(\frac{\partial a_n}{\partial y_i}\right)^2 = \sigma_{y'}^2 \sum_i \left[\sum_j A^{nj} f_j(X_i)\right]^2$$

$$= \sigma_{y'}^2 \sum_i \sum_j \sum_k A^{nj} A^{nk} f_j(X_i)\, f_k(X_i) \quad .$$

---

* The assumption that all σ_{y_i} are the same can easily be satisfied if weighting factors are used when the a_n are determined. The value of $\sigma_{y'}$ should be thought of as $W_i \sigma_{y_i}$ where W_i is the normalized weight chosen to make the value a constant for all i.

Reordering this finite sum yields

$$\sigma^2_{a_n} = \sigma^2_{y'} \sum_j \sum_k A^{nj} A^{nk} \sum_i f_j(X_i) f_k(X_i)$$

$$= \sigma^2_{y'} \sum_j \sum_k A^{nj} A^{nk} A_{jk} \quad .$$

Note that

$$\sum_j A^{nj} A_{jk} = \delta_{nk}$$

and therefore

$$\sigma^2_{a_n} = \sigma^2_{y'} A^{nn} \quad . \tag{49}$$

Eq. (49) is the relation allowing the accuracy of the coefficients to be determined when the accuracy of the given data is known.

To calculate the accuracy of the field determined by the curve-fitting process, assume again that a function of the form $y = \sum_n a_n f_n(X)$ is to be fitted to the given data. The functional form itself gives

$$\frac{\partial y}{\partial y_i} = \sum_n \left(\frac{\partial a_n}{\partial y_i} \right) f_n(X) \quad , \tag{50}$$

and the fact that the values y_i are independently measured allows the expression

$$\sigma_y^2 = \sum_i \left(\frac{\partial y}{\partial y_i}\right)^2 \sigma_{y_i}^2 \quad .$$

Assuming all $\sigma_{y_i} = \sigma_{y'}$ as before, and using Eq. (50),

$$\sigma_y^2 = \sigma_{y'}^2 \sum_i \left(\frac{\partial y}{\partial y_i}\right)^2 = \sigma_{y'}^2 \sum_i \left[\sum_n \left(\frac{\partial a_n}{\partial y_i}\right) f_n(X)\right]^2$$

$$= \sigma_{y'}^2 \sum_i \sum_n \sum_m \frac{\partial a_n}{\partial y_i} \frac{\partial a_m}{\partial y_i} f_n(X) f_m(X) \quad .$$

Evaluating the partial derivatives from Eq. (48),

$$\sigma_y^2 = \sigma_{y'}^2 \sum_i \sum_n \sum_m \left\{\left[\sum_j A^{nj} f_j(X_i)\right] \left[\sum_k A^{mk} f_k(X_i)\right] f_n(X) f_m(X)\right\}$$

$$= \sigma_{y'}^2 \sum_i \sum_n \sum_m \sum_j \sum_k A^{nj} A^{mk} f_j(X_i) f_k(X_i) f_n(X) f_m(X) \quad .$$

Reordering terms yields

$$\sigma_y^2 = \sigma_{y'}^2 \sum_n \sum_m \sum_j \sum_k \left[A^{nj} A^{mk} f_n(X) f_m(X) \sum_i f_j(X_i) f_k(X_i)\right]$$

$$= \sigma_{y'}^2 \sum_n \sum_m \sum_j \sum_k \left[A^{nj} A^{mk} f_n(X) f_m(X) A_{jk}\right]$$

$$= \sigma_{y'}^2 \sum_n \sum_m \sum_k A^{mk} f_n(X) f_m(X) \delta_{nk}$$

since $\sum_j A^{nj} A_{jk} = \delta_{nk}$. Summing the remaining terms over k yields

$$\sigma_y^2 = \sigma_{y'}^2 \sum_n \sum_m a^{mn} f_n(X) f_m(X) \quad . \tag{51}$$

Wall's Error program is a computer implementation of Eq. (51). To provide data for the Error program, a minor change was made in Daniels' Matrix subroutine. Daniels' Matrix inverts the least squares matrix that is set up by Jensen's Fit program. However, Matrix records only the diagonal elements of this inverse. To estimate the error of the geomagnetic field computed from the parameters calculated by Jensen's Fit, the complete inverse is needed. (Reference Eq. (51) above. A^{mn} are the elements of this inverse.) Hence, the Matrix subroutine was modified so that the complete inverse plus the parameters of the potential function and certain other miscellaneous constants are recorded on magnetic tape 1. Then, whenever the Error program is used, the required data will be available from the last run of Daniels' Matrix subroutine.

On comparing the listing of Jensen's Fit in Appendix A and Wall's Error in Appendix D, one sees much similarity. The principal differences include: (1) the initial input data are different; (2) the Fit program receives all data from the RDATA subroutine while the Error program generates its own data to form a grid over the earth's surface; and (3) instead of computing the D matrix as in the Fit program, the Error program computes the standard error of estimate according to Eq. (51) above.

The errors computed are all in gammas. For dip and declination, points and errors expressed in degrees are more meaningful. Hence, immediately before printout for a grid point, the generated declination and dip and the respective standard errors of estimate are converted to degrees. The formulae for converting these errors are developed below.

For declination, let

D_γ = declination in gammas

D_o = declination in degrees

Note that H = horizontal component of field intensity.

Then

$$D_\gamma = D_o H$$

$$D_o = \frac{D_\gamma}{H}$$

$$\sigma^2_{D_o} = \frac{1}{H^2} \sigma^2_{D_\gamma} + \frac{D^2_\gamma}{H^4} \sigma^2_H$$

$$\sigma^2_{D_o} = \frac{1}{H^2} \sigma^2_{D_\gamma} + \frac{D^2_o}{H^2} \sigma^2_H$$

$$\sigma^2_{D_o} = \frac{\sigma^2_{D_\gamma} + D_o^2 \sigma_h^2}{H^2} \quad (52)$$

For dip, let

I_γ = dip in gammas

I_o = dip in degrees .

Note that F = total field intensity.

Then

$$I_\gamma = I_o F$$

$$I_o = \frac{I_\gamma}{F}$$

$$\sigma^2_{I_o} = \frac{1}{F^2} \sigma^2_{I_\gamma} + \frac{I_\gamma^2}{F^4} \sigma^2_F$$

$$\sigma^2_{I_o} = \frac{1}{F^2} \sigma^2_{I_\gamma} + \frac{I_o^2}{F^2} \sigma^2_F$$

$$\sigma^2_{I_o} = \frac{\sigma^2_{I_\gamma} + I_o^2 \sigma^2_F}{F^2}$$

In Appendix D, Wall's complete Error program is listed. Appendix E is a brief description and cross reference of the program. Appendix F is a flow chart of Wall's Error program.

5.0 CONCLUSIONS

While Jensen's Fit program including the external field provisions and Wall's Error program are in a sense complete, there are three minor tasks related to these projects that should be done. The first of these tasks concerns adding secular change coefficients for the external field. If an estimate of the external field based on a large data sample is as significant as Dikewood's initial estimate, then further calculations to determine if this field is time-dependent seem necessary.

The second task concerns the addition of the external field variables to Daniels' Matrix subroutine. This, of course, is absolutely essential if one wishes to include the external field parameters when using Wall's Error. The third task is the addition of the secular change coefficients for the external field to the Error program.

Future programs in geomagnetism will require new analytical techniques. The large volume of satellite data will necessitate a data reduction procedure preceding any data analysis. At two points per second, each orbit will produce about 10,800 data points. Some means of reducing these data to a more manageable quantity is imperative. One method recommended would fit a Fourier series to each orbit and then select points from this equation at equal distances along the orbit. Another recommendation is to try the same approach with elliptic functions.

Since this type of data reduction scheme preserves information on individual orbits, in a very short time the quantity of data will still become massive. A scheme to preserve orbital data in terms of secular variables would provide for even more reduction of the data, and hence would have some merit. If one views the potential function as a changing surface in three dimensional space, then, at a specific time in a very small area, it will appear as a simple plane. (For this discussion, the size of such a small area is not delineated.)

Now assuming that a simple plane can be used to approximate the potential function at a specific time in a small area, one might use all data in that area without regard to orbit to determine this plane as a function of time. Then in a manner similar to that which would be used for orbital data, one or more points could be chosen on the plane to represent the small area. One would hope that the plane would be simple enough in form to permit direct as opposed to iterative estimation of the parameters required to describe it. If this is possible, the reduction of satellite data and the estimation of the potential function parameters from the reduced data will be a manageable problem. In fact, such reduced data can be used with existing programs to estimate the potential function parameters.

By generating pseudo-satellite data, any or all of these proposed data reduction procedures can be studied. The utility of the results of these studies will be limited only by our ability to generate the pseudo-data.

In an appendix to our December, 1964 report, a new technique for handling geomagnetic data was outlined. Several questions exist concerning the general use of this method. For example, how are areas where no data exists handled? Most if not all of such questions do not apply where satellite data is concerned. Hence, the method as outlined can be developed into a system for processing satellite data only.

The problem of the westward drift of the geomagnetic field has been brought to the attention of Dikewood personnel. A cursory examination of the problem suggests that some form of correlation analysis may be useful in establishing the significance and hence validity of the drift.

The above problems were recognized by The Dikewood Corporation during previous work for the Goddard Space Flight Center. We look forward to future contractual effort in the field of geomagnetism.

APPENDIX A

PROGRAM LISTING FOR JENSEN'S FIT PROGRAM

```
C                  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C
C                  JENSEN FIT PROGRAM    HENDRICKS VERSION    TRIANGULAR
C
C                  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C
      COMMON /DD/D(7500)
      COMMON /DATAR/ISKIP,FLATT,ELONG,ALT,TIME,DECL,DECLWT,DIP,DIPWT,HOR
     $,HORWT,B,BWT,X,XWT,Y,YWT,Z,ZWT
      COMMON /COEFS/G(9,9),H(9,9),GT(9,9),HT(9,9),GTT(9,9),HTT(9,9),MAXD
      DIMENSION ERR(18,36),FNO2(18,36),JERR(18),F(127),SIDE(126)
      DIMENSION SHMIDT(9,9)
      DIMENSION DXDH(9,9),DYDH(9,9),DZDH(9,9)
      DIMENSION DXDG(9,9),DYDG(9,9),DZDG(9,9)
      DIMENSION CP(9),SP(9),P(9,9),DP(9,9),CONST(9,9)
      DIMENSION IERR(200),TYPE(8),SIG1(8),FNO1(8),SWT1(8),WD(7)
      INTEGER EXTFLD
      DATA RAD,A,FLAT,(TYPE(I),I=1,8),PI,PI2,LINE/57.2957795,6378.165,29
     18.3,1HD,1HI,1HH,1HB,1HZ,1HX,1HY,1H*,3.14159265,6.28318530,0/
C
C                  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C
      FLAT=1.-1./FLAT
C                  COMPUTATION WITH SPHERICAL EARTH
      FLAT=1.
      A=6371.2
      DO 1 I=1,18
      DO 1 J=1,36
      FNO2(I,J)=0.
      ERR(I,J)=0.
1     CONTINUE
      MAXD=7500
      A2=A**2
      A4=A**4
      B2=(A*FLAT)**2
      A2B2=A2*(1.-FLAT**2)
      A4B4=A4*(1.-FLAT**4)
      READ (5,2) XID1,XID2
2     FORMAT (2A6,24X,I2)
      READ (5,3) NMAX,NMAXT,NMAXTT,NSKIP,ITER
3     FORMAT (5I5)
      READ (5,4) ERRLIM,AVETIM
4     FORMAT (2F10.0)
      READ (5,5) EXTFLD
5     FORMAT (I5)
      WRITE (6,6) NMAX,NMAXT,NMAXTT,NSKIP,ITER,ERRLIM
6     FORMAT (6H1NMAX=,I5,3X,6HNMAXT=,I5,3X,7HNMAXTT=,I5,3X,6HNSKIP=,I5,
     $3X,5HITER=,I5,3X,7HERRLIM=,F10.0)
      WRITE (6,7) XID1,XID2
7     FORMAT (1X,2A6)
```

```
C                   COMPUTE CONSTANTS REQUIRED FOR GENERATING
C                   LEGENDRE POLYNOMIALS
      DO 8 N=2,NMAX
      FN=N
      DO 8 M=1,N
      FM=M
      CONST(N,M)=((FN-2.0)**2-(FM-1.0)**2)/(FN+FN-3.0)/(FN+FN-5.0)
8     CONTINUE
C                   COMPUTE CONSTANTS TO CONVERT FROM GAUSS TO
C                   SCHMIDT NORMALIZATION
      SHMIDT(1,1)=-1.0
      DO 9 N=2,NMAX
      FN=N
      SHMIDT(N,1)=SHMIDT(N-1,1)*(FN+FN-3.0)/(FN-1.0)
      FACT=2.0
      DO 9 M=2,N
      FM=M
      SHMIDT(N,M)=SHMIDT(N,M-1)*SQRT((FN-FM+1.0)*FACT/(FN+FM-2.0))
9     FACT=1.0
C                   SET VALUE OF FIRST LEGENDRE POLYNOMIALS
      P(1,1)=1.0
      DP(1,1)=0.0
C                   SET VALUE OF SIN(M-1)PHI AND COS(M-1)PHI WHEN M=1
      SP(1)=0.0
      CP(1)=1.0
C                   READ BEST SET OF PARAMETERS AS FIRST APPROXIMATION
10    READ (5,11) N,M,GNM,HNM,GTNM,HTNM,GTTNM,HTTNM
11    FORMAT (2I3,6F11.4)
      IF (N) 12,13,12
12    G(N,M)=GNM
      H(N,M)=HNM
      GT(N,M)=GTNM
      HT(N,M)=HTNM
      GTT(N,M)=GTTNM
      HTT(N,M)=HTTNM
      GO TO 10
C                   READ BEST SET FOR EXTERNAL FIELD
13    READ (5,14) E1,E2,E3
14    FORMAT (6X,3F11.4)
C                   RECORD STARTING PARAMETERS
      WRITE (6,11) ((N,M,G(N,M),H(N,M),GT(N,M),HT(N,M),GTT(N,M),HTT(N,M)
     $,M=1,N),N=2,NMAX)
      WRITE (6,15) E1,E2,E3
15    FORMAT (4HOE1=,F13.4/4H E2=,F13.4/4H E3=,F13.4)
C                   * * * * * * * * * * * * * * * * * * * *
C                   END INITIALIZATION, BEGIN DATA PROCESSING
C                   * * * * * * * * * * * * * * * * * * * *
      DO 120 ITNO=1,ITER
      REWIND 2
      DO 16 J=1,8
```

```
      SIG1(J)=0.
      FNO1(J)=0.
      SWT1(J)=0.
16    CONTINUE
      DO 17 I=1,200
      IERR(I)=0
17    CONTINUE
      DO 18 I=1,MAXD
      D(I)=0.0
18    CONTINUE
      LINE=0
      SUMTM=0.0
      ISKIP=NSKIP
C                  READ ONE DATA LINE
19    CALL RDATA
      WD(1)=DECL
      WD(2)=DIP
      WD(3)=HOR
      WD(4)=B
      WD(5)=Z
      WD(6)=X
      WD(7)=Y
      IF (ISKIP) 20,79,20
C                  COMPUTE GEOCENTRIC THETA FROM
C                  GEODETIC COORDINATES
20    FLATR=FLATT/RAD
      SINLA=SIN(FLATR)
      SINLA2=SINLA**2
      DEN2=A2-A2B2*SINLA2
      DEN=SQRT(DEN2)
      FAC=((ALT*DEN)+B2)/((ALT*DEN)+A2)
      THETA=ATAN(FAC*SINLA/(1.E-30+SQRT(1.-SINLA2)))
C                  COMPUTE GEOCENTRIC R FROM GEODETIC COORDINATES
      R=SQRT(ALT*(ALT+2.*DEN)+(A4-A4B4*SINLA2)/DEN2)
C                  COMPUTE SINE AND COSINE OF DIFFERENCE BETWEEN
C                  GEODETIC AND GEOCENTRIC LATITUDINAL COORDINATES
      SIND=SIN(FLATR-THETA)
      COSD=SQRT(1.0-SIND*SIND)
      AOR=6371.2/R
C                  COS THETA MEASURED FROM POLAR AXIS
      CT=SIN(THETA)
C                  SIN THETA MEASURED FROM POLAR AXIS
      ST=SQRT(1.0-CT*CT)
C                  LONGITUDE INDEX
      LON=AMIN(AMAX(ELONG/10.0+18.0,1.0),36.0)
C                  LATITUDE INDEX
      LAT=AMIN(AMAX(FLATT/10.0+9.0,1.0),18.0)
      SP(2)=SIN(ELONG/RAD)
      CP(2)=COS(ELONG/RAD)
      DO 21 M=3,NMAX
```

```
C                   SIN(M-1)PHI, EQ.(5)
      SP(M)=SP(2)*CP(M-1)+CP(2)*SP(M-1)
C                   COS(M-1)PHI, EQ.(6)
      CP(M)=CP(2)*CP(M-1)-SP(2)*SP(M-1)
21    CONTINUE
C                   GENERATE ASSOCIATED LEGENDRE POLYNOMIALS
      DO 24 N=2,NMAX
      DO 24 M=1,N
      IF (N-M) 23,22,23
22    P(N,N)=ST*P(N-1,N-1)
      DP(N,N)=ST*DP(N-1,N-1)+CT*P(N-1,N-1)
      GO TO 24
23    P(N,M)=CT*P(N-1,M)-CONST(N,M)*P(N-2,M)
      DP(N,M)=CT*DP(N-1,M)-ST*P(N-1,M)-CONST(N,M)*DP(N-2,M)
24    CONTINUE
C                   INITIALIZE TO COMPUTE X,Y,Z
      CX=0.0
      CY=0.0
      CZ=0.0
      AR=AOR*AOR
      TM=TIME-60.0
C                   COMPUTE X,Y,Z USING BEST AVAILABLE
C                   PARAMETERS
      DO 25 N=2,NMAX
      FN=N
      AR=AR*AOR
      DO 25 M=1,N
      FM=M-1
C                   APPLY SCHMIDT NORMALIZATION CONSTANTS
C                   AND MULTIPLY BY (A/R)**(N+1)
      P(N,M)=P(N,M)*AR*SHMIDT(N,M)
      DP(N,M)=DP(N,M)*AR*SHMIDT(N,M)
      TEMP=FN*P(N,M)*SIND-DP(N,M)*COSD
      DXDG(N,M)=TEMP*CP(M)
      DXDH(N,M)=TEMP*SP(M)
      TEMP=FM*P(N,M)/ST
      DYDG(N,M)=-TEMP*SP(M)
      DYDH(N,M)=TEMP*CP(M)
      TEMP=FN*P(N,M)*COSD+DP(N,M)*SIND
      DZDG(N,M)=TEMP*CP(M)
      DZDH(N,M)=TEMP*SP(M)
C                   ADD TIME TERMS
      GNM=(TM*GTT(N,M)+GT(N,M))*TM+G(N,M)
      HNM=(TM*HTT(N,M)+HT(N,M))*TM+H(N,M)
      CX=CX+GNM*DXDG(N,M)+HNM*DXDH(N,M)
      CY=CY+GNM*DYDG(N,M)+HNM*DYDH(N,M)
      CZ=CZ+GNM*DZDG(N,M)+HNM*DZDH(N,M)
25    CONTINUE
      IF (EXTFLD) 26,27,26
26    T1=E2*CP(2)+E3*SP(2)
```

```
      T2=E1*ST-T1*CT
      T1=E1*CT+T1*ST
      CX=CX-T2*COSD+T1*SIND
      CY=CY+E2*SP(2)-E3*CP(2)
      CZ=CZ+T2*SIND+T1*COSD
C                 COMPUTE HORIZONTAL, TOTAL FIELD, DIP, AND
C                 DECLINATION
27    CH=SQRT(CX*CX+CY*CY)
      CF=SQRT(CH*CH+CZ*CZ)
      CI=2.0*ATAN(CZ/(CF+CH))
      CD=2.0*ATAN(CY/(CH+CX))
C                 COMPUTE COEFFICIENTS OF G(N,M) AND
C                 H(N,M) FOR THE NEXT APPROXIMATION
28    I=1
      IF (DECL) 35,29,35
29    IF (DIP) 42,30,42
30    IF (HOR) 45,31,45
31    IF (B) 48,32,48
32    IF (Z) 51,33,51
33    IF (X) 53,34,53
34    IF (Y) 55,19,55
C                 COMPUTE COEFFICIENTS WHEN DECLINATION
C                 (DECL) IS GIVEN
C                 DECL=ARCTAN(Y/X)
C                 T1=DDECL/DY
C                 T2=DDECL/DX
35    T1=CX/CH
      T2=CY/CH
      ITYPE=1
      DO 37 N=2,NMAX
      DO 37 M=1,N
C                 DDECL/DG(N,M)
      F(I)=(T1*DYDG(N,M)-T2*DXDG(N,M))
      IF (M-1) 36,37,36
36    I=I+1
C                 DDECL/DH(N,M)
      F(I)=(T1*DYDH(N,M)-T2*DXDH(N,M))
37    I=I+1
      FI=DECL/RAD-CD
      IF (FI-PI) 39,39,38
38    FI=FI-PI2
      GO TO 41
39    IF (FI+PI) 40,41,41
40    FI=FI+PI2
41    FI=FI*CH
      WT=RAD/DECLWT/CH
      DECL=0.0
      GO TO 57
C                 COMPUTE COEFFICIENTS WHEN DIP IS GIVEN
C                 DIP=ARCTAN(Z/H)
```

```
C                   T1=DDIP/DZ
C                   T2=DDIP/DX
C                   T3=DDIP/DY
42    T1=CH/CF
      T2=CZ*CX/CH/CF
      T3=CZ*CY/CH/CF
      ITYPE=2
      DO 44 N=2,NMAX
      DO 44 M=1,N
C                   F(I)=DDIP/DG(N,M)
      F(I)=(T1*DZDG(N,M)-T2*DXDG(N,M)-T3*DYDG(N,M))
      IF (M-1) 43,44,43
43    I=I+1
C                   F(I)=DDIP/DH(N,M)
      F(I)=(T1*DZDH(N,M)-T2*DXDH(N,M)-T3*DYDH(N,M))
44    I=I+1
      FI=(DIP/RAD-CI)*CF
      WT=RAD/DIPWT/CF
      DIP=0.0
      GO TO 57
C                   COMPUTE COEFFICIENTS WHEN THE HORIZONTAL
C                   COMPONENT (HOR) IS GIVEN
C                   HOR=SQRT(X*X+Y*Y)
C                   T1=DHOR/DX
C                   T2=DHOR/DY
45    T1=CX/HOR
      T2=CY/HOR
      ITYPE=3
      DO 47 N=2,NMAX
      DO 47 M=1,N
C                   F(I)=DHOR/DG(N,M)
      F(I)=(T1*DXDG(N,M)+T2*DYDG(N,M))
      IF (M-1) 46,47,46
46    I=I+1
C                   F(I)=DHOR/DH(N,M)
      F(I)=(T1*DXDH(N,M)+T2*DYDH(N,M))
47    I=I+1
      FI=HOR-CH
      WT=1.0/HORWT
      HOR=0.0
      GO TO 57
C                   COMPUTE COEFFICIENTS WHEN TOTAL
C                   FIELD B IS GIVEN
C                   B=SQRT(X*X+Y*Y+Z*Z)
C                   T1=DB/DX
C                   T2=DB/DY
C                   T3=DB/DZ
48    T1=CX/B
      T2=CY/B
      T3=CZ/B
```

```
      ITYPE=4
      DO 50 N=2,NMAX
      DO 50 M=1,N
C                       F(I)=DB/DG(N,M)
      F(I)=(T1*DXDG(N,M)+T2*DYDG(N,M)+T3*DZDG(N,M))
      IF (M-1) 49,50,49
49    I=I+1
C                       F(I)=DB/DH(N,M)
      F(I)=(T1*DXDH(N,M)+T2*DYDH(N,M)+T3*DZDH(N,M))
50    I=I+1
      FI=B-CF
      WT=1.0/BWT
      B=0.0
      GO TO 57
C                       COMPUTE COEFFICIENTS WHEN THE Z COMPONENT IS GIVEN
51    DO 52 N=2,NMAX
      F(I)=DZDG(N,1)
      I=I+1
      DO 52 M=2,N
      F(I)=DZDG(N,M)
      F(I+1)=DZDH(N,M)
52    I=I+2
      ITYPE=5
      FI=Z-CZ
      WT=1.0/ZWT
      Z=0.0
      GO TO 57
C                       COMPUTE COEFFICIENTS WHEN THE X COMPONENT IS GIVEN
53    DO 54 N=2,NMAX
      F(I)=DXDG(N,1)
      I=I+1
      DO 54 M=2,N
      F(I)=DXDG(N,M)
      F(I+1)=DXDH(N,M)
54    I=I+2
      ITYPE=6
      FI=X-CX
      WT=1.0/XWT
      X=0.0
      GO TO 57
C                       COMPUTE COEFFICIENTS WHEN THE Y COMPONENT IS GIVEN
55    DO 56 N=2,NMAX
      F(I)=DYDG(N,1)
      I=I+1
      DO 56 M=2,N
      F(I)=DYDG(N,M)
      F(I+1)=DYDH(N,M)
56    I=I+2
      ITYPE=7
      FI=Y-CY
```

```
      WT=1.0/YWT
      Y=0.0
C                    ADD TIME**1 TERMS
57    NO=I-1
      TFACT=TIME-AVETIM
      IF (NMAXT) 58,60,58
58    J=1
      DO 59 N=2,NMAXT
      F(I)=F(J)*TFACT
      I=I+1
      J=J+1
      DO 59 M=2,N
      F(I)=F(J)*TFACT
      F(I+1)=F(J+1)*TFACT
      I=I+2
      J=J+2
59    CONTINUE
C                    ADD TIME**2 TERMS
60    NONOT=I-1
      IF (NMAXTT) 61,63,61
61    J=1
      TFACT=TFACT*TFACT
      DO 62 N=2,NMAXTT
      F(I)=F(J)*TFACT
      I=I+1
      J=J+1
      DO 62 M=2,N
      F(I)=F(J)*TFACT
      F(I+1)=F(J+1)*TFACT
      I=I+2
      J=J+2
62    CONTINUE
63    IF (EXTFLD) 64,72,64
C                    ADD EXTERNAL FIELD TERMS
64    DXDE1=CT*SIND-ST*COSD
      DZDE1=ST*SIND+CT*COSD
      DXDE2=CP(2)*DZDE1
      DXDE3=SP(2)*DZDE1
      DZDE2=-CP(2)*DXDE1
      DZDE3=-SP(2)*DXDE1
      GO TO (65,66,67,68,69,70,71),ITYPE
C                    COEFFICIENTS WHEN DECLINATION (D) IS GIVEN
C                    F(I)=DD/DE1
65    F(I)=-T2*DXDE1
      I=I+1
C                    F(I)=DD/DE2
      F(I)=T1*SP(2)-T2*DXDE2
      I=I+1
C                    F(I)=DD/DE3
      F(I)=-T1*CP(2)-T2*DXDE3
```

```
      I=I+1
      GO TO 72
C                     COEFFICIENTS WHEN DIP (I) IS GIVEN
C                     F(I)=DI/DE1
66    F(I)=T1*DZDE1-T2*DXDE1
      I=I+1
C                     F(I)=DI/DE2
      F(I)=T1*DZDE2-T2*DXDE2-T3*SP(2)
      I=I+1
C                     F(I)=DI/DE3
      F(I)=T1*DZDE3-T2*DXDE3+T3*CP(2)
      I=I+1
      GO TO 72
C                     COEFFICIENTS WHEN THE HORIZONTAL
C                     COMPONENT (HOR) IS GIVEN
C                     F(I)=DHOR/DE1
67    F(I)=T1*DXDE1
      I=I+1
C                     F(I)=DHOR/DE2
      F(I)=T1*DXDE2+T2*SP(2)
      I=I+1
C                     F(I)=DHOR/DE3
      F(I)=T1*DXDE3-T2*CP(2)
      I=I+1
      GO TO 72
C                     COEFFICIENTS WHEN TOTAL FIELD IS GIVEN
C                     F(I)=DF/DE1
68    F(I)=T1*DXDE1+T3*DZDE1
      I=I+1
C                     F(I)=DF/DE2
      F(I)=T1*DXDE2+T2*SP(2)+T3*DZDE2
      I=I+1
C                     F(I)=DF/DE3
      F(I)=T1*DXDE3-T2*CP(2)+T3*DZDE3
      I=I+1
      GO TO 72
C                     COEFFICIENTS WHEN THE Z COMPONENT IS GIVEN
69    F(I)=DZDE1
      I=I+1
      F(I)=DZDE2
      I=I+1
      F(I)=DZDE3
      I=I+1
      GO TO 72
C                     COEFFICIENTS WHEN THE X COMPONENT IS GIVEN
70    F(I)=DXDE1
      I=I+1
      F(I)=DXDE2
      I=I+1
      F(I)=DXDE3
```

```
      I=I+1
      GO TO 72
C                      COEFFICIENTS WHEN THE Y COMPONENT IS GIVEN
71    F(I)=0.0
      I=I+1
      F(I)=SP(2)
      I=I+1
      F(I)=-CP(2)
      I=I+1
      GO TO 72
C                      ADD OBSERVATION TERM
72    F(I)=FI
      NOR=I-1
      NOP=I
      NOPP=I+1
      CIDEG=CI*RAD
      CDDEG=CD*RAD
      IF (MOD(LINE,57)) 75,73,75
73    WRITE (6,74)
74    FORMAT (85H1  LAT   LONG  ALT TIME   DECL    DIP     H      B
     $ Z       X       Y    F(NOP)    /1HO)
75    WRITE (6,76) FLATT,ELONG,ALT,TIME,(WD(KK),KK=1,7),TYPE(ITYPE),CDDE
     $G,CIDEG,CH,CF,CZ,CX,CY,F(NOP)
76    FORMAT (1X2F6.1,F5.0,F5.1,2F7.2,5F7.0,5XA2/23X2F7.2,5F7.0,F7.0)
      LINE=LINE+1
C                      SUM DEVIATIONS FROM COMPUTED VALUES, WEIGHTS, DATE
C                      TYPE COUNTS, ETC. FOR STANDARD ERROR ESTIMATES
      K=AMAX(AMIN(F(I)/10.0+101.0,200.0),1.0)
      IERR(K)=IERR(K)+1
      SIG1(ITYPE)=SIG1(ITYPE)+F(NOP)*F(NOP)*WT
      FNO1(ITYPE)=FNO1(ITYPE)+1.
      SWT1(ITYPE)=SWT1(ITYPE)+WT
      SIG1(8)=SIG1(8)+F(NOP)**2*WT
      FNO1(8)=FNO1(8)+1.
      SWT1(8)=SWT1(8)+WT
      SUMTM=SUMTM+TM
      IF (ITNO-ITER) 78,77,78
77    FNO2(LAT,LON)=FNO2(LAT,LON)+WT
      ERR(LAT,LON)=ERR(LAT,LON)+F(NOP)*WT
C                      COMPUTE SQUARES AND CROSS-PRODUCTS AND
C                      SUM INTO TRIANGULAR MATRIX D
78    K=1
      CALL DLOOP (NOR,NOP,F,WT)
      GO TO 28
C                      *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C                      END DATA PROCESSING, BEGIN SOLUTION
C                      OF THE LEAST SQUARES EQUATIONS
C                      *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
79    DO 80 IJ=1,8
      SIG1(IJ)=SQRT(SIG1(IJ)/SWT1(IJ))
```

```
80    CONTINUE
C                 RECORD DATA FOR MATRIX SUBROUTINE
      WRITE (2) NMAX,NMAXT,NMAXTT,SWT1(8),FNO1(8),SIG1(8)
      WRITE (2) (D(I),I=1,MAXD)
C                 COMPUTE SUMS FOR CHECK COLUMN
      DO 82 I=1,NOR
      SUMD=0.0
      DO 81 J=1,NOR
      NROW=MINO(I,J)
      NCOL=I+J-NROW
      K=(NROW*(NOR+NOR+5-NROW))/2+NCOL-NOR-2
      SUMD=SUMD+D(K)
81    CONTINUE
      K=(I*(NOR+NOR+5-I))/2
      D(K)=SUMD
82    CONTINUE
C                 INVERT TRIANGULAR MATRIX
      DO 88 L=1,NOR
C                 SET UP ONE COMPLETE COLUMN
      DO 83 I=1,NOR
      NROW=MINO(I,L)
      NCOL=I+L-NROW
      K=(NROW*(NOR+NOR+5-NROW))/2+NCOL-NOR-2
      SIDE(I)=D(K)
83    CONTINUE
      K=(L*(NOR+NOR+7-L))/2-NOR-2
      RDKK=1.0/D(K)
      DO 84 J=L,NOP
      D(K+1)=D(K+1)*RDKK
      K=K+1
84    CONTINUE
      DO 88 I=1,NOR
      IF (I-L) 85,88,85
85    DO 87 J=L,NOP
      IF (J+1-I) 87,86,86
86    K=(I*(NOR+NOR+5-I))/2+J-NOR-1
      KJ=(L*(NOR+NOR+5-L))/2+J-NOR-1
      D(K)=D(K)-SIDE(I)*D(KJ)
87    CONTINUE
88    CONTINUE
C                 *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C                 END SOLUTION OF THE LEAST SQUARES EQUATIONS,
C                 BEGIN ESTIMATION OF THE PARAMETER CORRECTIONS
C                 *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      WRITE (6,89) (TYPE(IJ),SIG1(IJ),FNO1(IJ),IJ=1,8)
89    FORMAT (6H1SIGMA,5X,6HPOINTS/1X/(1X,A2,F5.0,5X,F6.0))
      WRITE (6,90)
90    FORMAT (5X,1HN,2X,1HM,15X,1HP,14X,2HDP,13X,3H1.0)
      TFACT=60.0-AVETIM
      I=1
```

```
      DO 100 N=2,NMAX
      DO 100 M=1,N
C                   COMPUTE TIME ADJUSTMENT FOR THE G(N,M) CORRECTIONS
      K=(I*(NOR+NOR+5-I))/2-1
      KP=I+NO
      KP=(KP*(NOR+NOR+5-KP))/2-1
      KPP=I+NONOT
      KPP=(KPP*(NOR+NOR+5-KPP))/2-1
      IF (N-NMAXT) 91,91,93
91    D(K)=D(K)+D(KP)*TFACT
      IF (N-NMAXTT) 92,92,93
92    D(K)=D(K)+D(KPP)*TFACT*TFACT
      D(KP)=D(KP)+2.0*D(KPP)*TFACT
C                   CORRECT G(N,M)
93    G(N,M)=G(N,M)+D(K)
C                   RECORD NEW G(N,M), CORRECTION, AND THE
C                   CORRESPONDING ITEM IN THE CHECK COLUMN
      WRITE (6,94) N,M,G(N,M),D(K),D(K+1)
94    FORMAT (3H G 2I3,4E20.8,F20.2)
      IF (M-1) 95,100,95
95    I=I+1
C                   COMPUTE TIME ADJUSTMENT FOR THE H(N,M) CORRECTIONS
      K=(I*(NOR+NOR+5-I))/2-1
      KP=I+NO
      KP=(KP*(NOR+NOR+5-KP))/2-1
      KPP=I+NONOT
      KPP=(KPP*(NOR+NOR+5-KPP))/2-1
      IF (N-NMAXT) 96,96,98
96    D(K)=D(K)+D(KP)*TFACT
      IF (N-NMAXTT) 97,97,98
97    D(K)=D(K)+D(KPP)*TFACT*TFACT
      D(KP)=D(KP)+2.0*D(KPP)*TFACT
C                   CORRECT H(N,M)
98    H(N,M)=H(N,M)+D(K)
C                   RECORD NEW H(N,M), CORRECTION, AND THE
C                   CORRESPONDING ITEM IN THE CHECK COLUMN
      WRITE (6,99) N,M,H(N,M),D(K),D(K+1)
99    FORMAT (3H H 2I3,4E20.8,F20.2)
100   I=I+1
      IF (NMAXT) 101,111,101
101   DO 105 N=2,NMAXT
      DO 105 M=1,N
C                   CORRECT GT(N,M)
      K=(I*(NOR+NOR+5-I))/2-1
      GT(N,M)=GT(N,M)+D(K)
C                   RECORD NEW GT(N,M), CORRECTION, AND THE
C                   CORRESPONDING ITEM IN THE CHECK COLUMN
      WRITE (6,102) N,M,GT(N,M),D(K),D(K+1)
102   FORMAT (3H GT2I3,4E20.8,F20.2)
      IF (M-1) 103,105,103
```

```
103   I=I+1
C                   CORRECT HT(N,M)
      K=(I*(NOR+NOR+5-I))/2-1
      HT(N,M)=HT(N,M)+D(K)
C                   RECORD NEW HT(N,M), CORRECTION, AND THE
C                   CORRESPONDING ITEM IN THE CHECK COLUMN
      WRITE (6,104) N,M,HT(N,M),D(K),D(K+1)
104   FORMAT (3H HT2I3,4E20.8,F20.2)
105   I=I+1
      IF (NMAXTT) 106,111,106
106   DO 110 N=2,NMAXTT
      DO 110 M=1,N
C                   CORRECT GTT (N,M)
      K=(I*(NOR+NOR+5-I))/2-1
      GTT(N,M)=GTT(N,M)+D(K)
C                   RECORD NEW GTT (N,M), CORRECTION, AND THE
C                   CORRESPONDING ITEM IN THE CHECK COLUMN
      WRITE (6,107) N,M,GTT(N,M),D(K),D(K+1)
107   FORMAT (4H GTT,I2,I3,3E20.8)
      IF (M-1) 108,110,108
108   I=I+1
C                   CORRECT HTT(N,M)
      K=(I*(NOR+NOR+5-I))/2-1
      HTT(N,M)=HTT(N,M)+D(K)
C                   RECORD NEW HTT(N,M), CORRECTION, AND THE
C                   CORRESPONDING ITEM IN THE CHECK COLUMN
      WRITE (6,109) N,M,HTT(N,M),D(K),D(K+1)
109   FORMAT (4H HTT,I2,I3,3E20.8)
110   I=I+1
C                   RECORD NEW E1,E2,E3 AND THE
C                   CORRESPONDING ITEM IN THE CHECK COLUMN
111   IF (EXTFLD) 112,116,112
112   K=(I*(NOR+NOR+5-I))/2-1
      E1=E1+D(K)
      WRITE (6,113) E1,D(K),D(K+1)
113   FORMAT (3H E1,6X,3E20.8)
      I=I+1
      K=(I*(NOR+NOR+5-I))/2-1
      E2=E2+D(K)
      WRITE (6,114) E2,D(K),D(K+1)
114   FORMAT (3H E2,6X,3E20.8)
      I=I+1
      K=(I*(NOR+NOR+5-I))/2-1
      E3=E3+D(K)
      WRITE (6,115) E3,D(K),D(K+1)
115   FORMAT (3H E3,6X,3E20.8)
      I=I+1
116   AVETIM=SUMTM/FNO1(8)+60.0
C                   RECORD ENTIRE ARRAY OF G AND H PARAMETERS
      WRITE (6,117) ITNO
```

```
117   FORMAT (20H1OUTPUT COEFFICIENTS,39X,5HITNO=,I4/1H0)
      WRITE (6,11) ((N,M,G(N,M),H(N,M),GT(N,M),HT(N,M),GTT(N,M),HTT(N,M)
     $,M=1,N),N=2,NMAX)
      WRITE (6,15) E1,E2,E3
      WRITE (6,118) AVETIM
118   FORMAT (10H0AVETIM=   ,F10.2)
C                   PUNCH CARDS FOR STARTING NEXT APPROXIMATION
      PUNCH 119,XID1,XID2,NMAX,NSKIP,SIG1(8)
      PUNCH 3,NMAX,NMAXT,NMAXTT,NSKIP,ITER
      PUNCH 4,ERRLIM,AVETIM
      PUNCH 5,EXTFLD
      PUNCH 11,((N,M,G(N,M),H(N,M),GT(N,M),HT(N,M),GTT(N,M),HTT(N,M),M=1
     $,N),N=2,NMAX)
      PUNCH 14
      PUNCH 14,E1,E2,E3
119   FORMAT (2A6,6H NMAX ,I1,7H NSKIP ,I3,5HSIG ,F6.0)
120   CONTINUE
C                   RECORD ERROR DISTRIBUTIONS
      WRITE (6,121) TYPE(IJ)
121   FORMAT (23H1ERROR DISTRIBUTION FOR,3X,A2)
      DO 124 JK=1,200,10
      JL=JK+9
      IF (JK-101) 122,123,123
122   JM=JK-101
      GO TO 124
123   JM=JK-100
124   WRITE (6,125) JM,(IERR(IK),IK=JK,JL)
125   FORMAT (I5,3X10I6)
C                   RECORD MEAN DEVIATION FOR LAT-LONG BLOCKS
      WRITE (6,126) (L,L=10,90,10)
126   FORMAT (38H1MEAN DEVIATION FOR LAT-LONG BLOCK     /1H0,58X,9I6)
      DO 128 K=1,36
      DO 127 J=1,18
127   JERR(J)=(ERR(J,K)/FNO2(J,K))
128   WRITE (6,129) K,(JERR(M),M=1,18)
129   FORMAT (1X12,3X,18I6)
      CALL MATRIX
      RETURN
      END
```

```
      SUBROUTINE MATRIX
      COMMON /DD/D(3400)
      COMMON /COEFS/G(9,9),H(9,9),GT(9,9),HT(9,9),GTT(9,9),HTT(9,9),MAXD
      DIMENSION FONE(150),DIAG(150)
      DIMENSION ROW(150),SROW(150)
      REWIND 2
      REWIND 1
      READ (2) NMAX,NMAXT,NMAXTT,FWNP,FNP,SIGMA
      READ (2) (D(I),I=1,MAXD)
      NOR=NMAX*NMAX-1
      IF (NMAXT) 1,2,1
1     NOR=NOR+NMAXT*NMAXT-1
2     IF (NMAXTT) 3,4,3
3     NOR=NOR+NMAXTT*NMAXTT-1
4     NOP=NOR+1
      NOPP=NOR+2
      DO 6 I=1,NOR
      DO 5 J=1,NOR
      II=MINO(I,J)
      JJ=I+J-II
      K=((NOR+NOR+5-II)*II)/2+JJ-NOR-2
5     ROW(J)=D(K)
6     WRITE (1) (ROW(J),J=1,NOP)
      REWIND 2
      REWIND 1
      DO 27 K=1,NOR
      IF (MOD(K,2)) 7,8,7
7     READ (1) (SROW(L),L=1,NOP)
      GO TO 9
8     READ (2) (SROW(L),L=1,NOP)
9     IF (K-1) 12,10,12
10    SROW(NOP)=0.0
      DO 11 II=1,NOR
11    SROW(NOP)=SROW(NOP)+SROW(II)
12    RDKK=1.0/SROW(K)
      SROW(K)=1.0
      DO 13 J=1,NOP
13    SROW(J)=SROW(J)*RDKK
      DO 23 I=2,NOR
      IF (MOD(K,2)) 14,15,14
14    READ (1) (ROW(L),L=1,NOP)
      GO TO 16
15    READ (2) (ROW(L),L=1,NOP)
16    IF (K-1) 19,17,19
17    ROW(NOP)=0.0
      DO 18 II=1,NOR
18    ROW(NOP)=ROW(NOP)+ROW(II)
19    T=ROW(K)
      ROW(K)=0.0
      DO 20 J=1,NOP
```

```
20    ROW(J)=ROW(J)-T*SROW(J)
      IF (MOD(K,2)) 21,22,21
21    WRITE (2) (ROW(L),L=1,NOP)
      GO TO 23
22    WRITE (1) (ROW(L),L=1,NOP)
23    CONTINUE
      IF (MOD(K,2)) 24,25,24
24    WRITE (2) (SROW(L),L=1,NOP)
      GO TO 26
25    WRITE (1) (SROW(L),L=1,NOP)
26    REWIND 2
27    REWIND 1
      DO 31 I=1,NOR
      IF (MOD(NOR,2)) 28,29,28
28    READ (2) (ROW(L),L=1,NOP)
      GO TO 30
29    READ (1) (ROW(L),L=1,NOP)
      WRITE (2) (ROW(L),L=1,NOP)
30    DIAG(I)=ROW(I)
31    FONE(I)=ROW(NOP)
      WRITE (6,32) SIGMA,FWNP,FNP
32    FORMAT (19H1STATISTICS FOR FIT/1X,5HSIGMA,F5.0,3X15HWEIGHTED POINT
     $S,F6.1,3X,6HPOINTS,F6.0)
      WRITE (6,33)
33    FORMAT (5X1HN2X1HM8X1HP20X4HSIGP16X3H1.019X2HTC)
      I=0
      DO 37 N=2,NMAX
      DO 37 M=1,N
      I=I+1
      SIGP=SQRT(ABS(DIAG(I)))*SIGMA
      TC=ABS(G(N,M)/SIGP)
      WRITE (6,34) N,M,G(N,M),SIGP,FONE(I),TC
34    FORMAT (4H G  2I3,2E20.8,2F20.2)
      IF (M-1) 35,37,35
35    I=I+1
      SIGP=SQRT(ABS(DIAG(I)))*SIGMA
      TC=ABS(H(N,M)/SIGP)
      WRITE (6,36) N,M,H(N,M),SIGP,FONE(I),TC
36    FORMAT (4H H  2I3,2E20.8,2F20.2)
37    CONTINUE
      IF (NMAXT) 38,43,38
38    DO 42 N=2,NMAXT
      DO 42 M=1,N
      I=I+1
      SIGP=SQRT(ABS(DIAG(I)))*SIGMA
      TC=ABS(GT(N,M)/SIGP)
      WRITE (6,39) N,M,GT(N,M),SIGP,FONE(I),TC
39    FORMAT (4H GT 2I3,2E20.8,2F20.2)
      IF (M-1) 40,42,40
40    I=I+1
```

```
      SIGP=SQRT(ABS(DIAG(I)))*SIGMA
      TC=ABS(HT(N,M)/SIGP)
      WRITE (6,41) N,M,HT(N,M),SIGP,FONE(I),TC
41    FORMAT (4H HT 2I3,2E20.8,2F20.2)
42    CONTINUE
43    IF (NMAXTT) 44,49,44
44    DO 48 N=2,NMAXTT
      DO 48 M=1,N
      I=I+1
      SIGP=SQRT(ABS(DIAG(I)))*SIGMA
      TC=ABS(GTT(N,M)/SIGP)
      WRITE (6,45) N,M,GTT(N,M),SIGP,FONE(I),TC
45    FORMAT (4H GTT2I3,2E20.8,2F20.2)
      IF (M-1) 46,48,46
46    I=I+1
      SIGP=SQRT(ABS(DIAG(I)))*SIGMA
      TC=ABS(HTT(N,M)/SIGP)
      WRITE (6,47) N,M,HTT(N,M),SIGP,FONE(I),TC
47    FORMAT (4H HTT2I3,2E20.8,2F20.2)
48    CONTINUE
49    I=-1
      DO 53 N=2,NMAX
      DO 53 M=1,N
      I=I+2
      R=SQRT(G(N,M)**2+H(N,M)**2)
      SIGP=SQRT(G(N,M)**2*ABS(DIAG(I))+H(N,M)**2*ABS(DIAG(I+1)))/R*SIGMA
      IF (M-1) 51,50,51
50    I=I-1
51    TC=R/SIGP
      WRITE (6,52) N,M,R,SIGP,TC
52    FORMAT (4H R  2I3,2E20.8,20X,F20.2)
53    CONTINUE
      IF (NMAXT) 54,59,54
54    DO 58 N=2,NMAXT
      DO 58 M=1,N
      I=I+2
      R=SQRT(GT(N,M)**2+HT(N,M)**2)
      SIGP=SQRT(GT(N,M)**2*ABS(DIAG(I))+HT(N,M)**2*ABS(DIAG(I+1)))/R*SIG
     $MA
      IF (M-1) 56,55,56
55    I=I-1
56    TC=R/SIGP
      WRITE (6,57) N,M,R,SIGP,TC
      PRINT 57,N,M,R,SIGP,TC
57    FORMAT (4H RT 2I3,2E20.8,20X,F20.2)
58    CONTINUE
59    IF (NMAXTT) 60,65,60
60    DO 64 N=2,NMAXTT
      DO 64 M=1,N
      I=I+2
```

```
      R=SQRT(GTT(N,M)**2+HTT(N,M)**2)
      SIGP=SQRT(GTT(N,M)**2*ABS(DIAG(I))+HTT(N,M)**2*ABS(DIAG(I+1)))/R*S
     $IGMA
      IF (M-1) 62,61,62
61    I=I-1
62    TC=R/SIGP
      WRITE (6,63) N,M,R,SIGP,TC
      PRINT 63,N,M,R,SIGP,TC
63    FORMAT (4H RTT2I3,2E20.8,20X,F20.2)
64    CONTINUE
65    IF (FNP-100.) 66,66,67
66    TCT95=0.0
      TCT50=0.0
      GO TO 68
67    TCT95=1.96
      TCT50=.674
68    WRITE (6,69) TCT95,TCT50
69    FORMAT (29H TC ABOVE SHOULD BE GREATER  F10.3,26H FOR 95 PERCENT C
     $ONFIDENCE/29X,F10.3,27H FOR 50 PERCENT CONFIDENCE )
      REWIND 1
      REWIND 2
      WRITE (1) NMAX,NMAXT,NMAXTT,FWNP,FNP,SIGMA,NOR,NOP,NOPP
      WRITE (1) ((G(N,M),H(N,M),M=1,N),N=2,NMAX)
      WRITE (1) ((GT(N,M),HT(N,M),M=1,N),N=2,NMAX)
      WRITE (1) ((GTT(N,M),HTT(N,M),M=1,N),N=2,NMAX)
      DO 70 I=1,NOR
      READ (2) (ROW(L),L=1,NOP)
      WRITE (1) (ROW(L),L=1,NOP)
70    CONTINUE
      END FILE 1
      REWIND 1
      REWIND 2
      RETURN
      END
```

APPENDIX B

JENSEN'S FIT

Introduction

The purpose of this appendix is to document the sequence of operations and to discuss various programming aspects of Jensen's Fit program. This program has been written to find time-dependent coefficients for a spherical-harmonic expansion of the geomagnetic potential function.

The mathematical formulas which form the basis of the computer program are not restated in this appendix. Each time that a formula is required to explain a Fortran variable, a reference is made to an equation in Sections 2.0 or 3.0 of this report or to one of the reports listed in the bibliography. When referencing this report, it should be noted that the Fortran variable N is equal to n+1. Similarly, M = m+1.

The computer program is relatively linear, i.e., there are few alternate calculation sequences, as can be seen from the flow-charts in Appendix C. Hence, the calculation sequence will be described in a linear manner.

The program may be roughly divided into five phases as follows: (1) initializing; (2) data processing for the coefficients in the least squares equations; (3) solution of the least squares equations; (4) estimation of the corrections for the coefficients of the spherical-harmonic expansion of the geomagnetic potential function; and (5) recording. This appendix will likewise be divided into five principal sections to describe respectively these five phases. Within each section, the Fortran name for variables will be used whenever possible. A glossary identifying these variables is included at the end of this appendix.

Initialization

As with all computer programs, initialization consists of doing the things that must be done once at the beginning of the execution of the program. (Similarly, parts of a program, i. e., subprograms, may require initialization. While such initialization may subsequently be discussed, it is not the subject of this section of the appendix.) Initialization for this program includes setting or computing the value of certain constants that will be used throughout the other phases of the program. Among these are FLAT, A2, A4, B2, A2B2, A4B4, CONST(N, M), SHMIDT(N,M), P(1, 1), DP(1, 1), SP(1) and CP(1) all of which are identified in the glossary.

The equation for computing CONST(N, M) is found in Eq. (19) of Section 2.0. Note that N = n+1 and M = m+1. The equations for computing SHMIDT(N, M) are found in Eqs. (20) of Section 2.0. Again note that N = n+1 and M = m+1.

The value of the first associated Legendre polynomial, P(1, 1), and its derivative, DP(1, 1), are constants and may be found in Eqs. (19) of Section 2.0. Similarly the value of $\sin(M-1)\phi$ and $\cos(M-1)\phi$, i. e., SP(M) and CP(M) respectively, are constants for M = 1, i. e., SP(1) = 0.0 and CP(1) = 1.0.

Another common function of initialization is the clearing of tables. This program requires that the tables FNO2(I, J) and ERR(I, J) be cleared, i. e., all entries be made equal to zero. A third function of initialization is the input of variable control and starting data. Among the variables that must be input for this program are XID1, XID2, NMAX, NMAXT, NMAXTT, NSKIP, ITER, ERRLIM, AVETIM, EXTFLD, G(N, M), H(N, M), GT(N, M), HT(N, M), GTT(N, M), HTT(N, M), E1, E2, and E3. These variables are all identified in the glossary.

A final function of initialization is often the recording of initial values of pertinent variables. This program records NMAX, NMAXT, NMAXTT, NSKIP, ITER, ERRLIM, XID1, XID2, G(N, M), H(N, M), GT(N, M), HT(N, M), GTT(N, M), HTT(N, M), E1, E2, and E3 as a permanent record of the starting data used by the program. These variables are all identified in the glossary.

Except for setting or computing the values of certain other variables required by the initialization functions enumerated above, this completes the initialization phase of the program.

Data Processing

In Section 2. 0 of this report, it was noted that the procedure must be repeated until the corrections estimated for the coefficients of the spherical-harmonic expansion of the potential function are no longer significant. The first Fortran statement in the data processing phase (card 98) is the DO statement that controls the number of repeats, or iterations, that will be made for any particular run of the computer program.

Certain tables, SIG1(J), FNO1(J), SWT(J), IERR(I), and D(I) must be cleared, i. e., all entries made equal to zero, at the beginning of each iteration. In addition to these tables, the value of LINE and SUMTM must be set equal to zero and ISKIP must be set equal to NSKIP which was input during the initialization phase. With the setting of ISKIP, the initialization of the data processing phase of the program is completed.

Beginning with the Fortran statement, CALL RDATA (card 115), the remainder of the data processing phase of the program is repeated for each observation that is to be used in the calculation. An observation input by the subroutine RDATA may consist of any combination of the following field measurements: DECL, DIP, HOR, B, Z, X, and Y. These are all identified in the glossary. In addition to these measurements, the location of the observation in time and space is recorded. This location is specified by the Fortran variables FLATT, ELONG, ALT, and TIME. All location data as well as measurement data are transmitted to the main program via the common field DATAR. RDATA signals the end of the data by setting the value of ISKIP to zero. To recognize this signal, the main program, following each CALL RDATA, examines ISKIP and terminates the data processing phase of the program upon sensing this signal.

Beginning with card 126, geocentric coordinates are computed from the geodetic measurements made for each observation. The equation for computing THETA is Eq. (30) of Section 2.0. The equation for computing geocentric R is Eq. (31) of Section 2.0.

The five variables SIND, COSD, AOR, CT, and ST which are all identified in the glossary are computed next. SIND and COSD are required for converting from geocentric to geodetic coordinates. CT and ST are required for the generation of the associated Legendre polynomials and AOR is a term that appears in the equations for estimating X, Y, Z, etc. from the best available set of parameters. One should note that the Fortran statement for computing CT and ST redefines θ to be measured from the polar axis instead of from the equatorial plane, i.e., colatitude.

The variables LON and LAT are computed next. These constants are required later for weight and error tabulations. Next, SP(2) and CP(2) are computed, followed by the computation of SP(M) and CP(M) for $M > 2$. The equations

$$\sin(M-1)\phi = \sin\phi \cdot \cos(M-2)\phi + \cos\phi \cdot \sin(M-2) \quad \text{(B1)}$$

and

$$\cos(M-1)\phi = \cos\phi \cdot \cos(M-2)\phi - \sin\phi \cdot \sin(M-2) \quad \text{(B2)}$$

used for computing SP(M) and CP(M) are available in standard texts on trigonometry under the subject, "functions of sums of angles."

Next the program evaluates the necessary associated Legendre polynomials, P(N, M), and their derivatives, DP(N, M), employing the recurrence relationships which are found in Section 2.0, Eqs. (19). $K_{N,M}$ corresponds to the Fortran variable CONST(N,M) and was discussed above in the section on initialization.

Beginning with card 167, the Fortran variables CX, CY, and CZ are set equal to zero in preparation for the estimation of X, Y, and Z. AR and TM, two Fortran variables identified in the glossary, must also be initialized in preparation for the estimation of X, Y, and Z.

The Fortran statements through card 198 are required to estimate X, Y, and Z, i.e., the Fortran variables CX, CY, and CZ. The rotation formulas required for computing X and Z are given in Section 2.0, Eqs. (32) and (33). Equations for B_θ, B_r, and B_ϕ are given in several different forms in Section 2.0 of this report.

Near the beginning of the group of Fortran statements required to estimate X, Y, and Z (specifically cards 181 and 182), the Gauss normalized polynomials are Schmidt normalized and multiplied by the appropriate power of $\frac{6371.2}{r}$, i.e., AR.

Depending on when it is calculated, the Fortran variable TEMP is the common factor in the coefficients of the two parameters $g_{N,M,0}$ and $h_{N,M,0}$ in the formulas for computing X, Y, or Z. DXDG, DXDH, DYDG, DYDH, DZDG and DZDH complete the calculation of the coefficients of $g_{N,M,0}$ and $h_{N,M,0}$ in the formulas for computing X, Y, and Z. Next, the Fortran variables GNM and HNM are computed using Eqs. (21) of Section

2.0 and finally all terms are summed for the respective estimates of X, Y, and Z.

The X, Y, and Z components of the external field are estimated next and added to the respective estimates of X, Y, and Z (cards 200 through 205). This step is completely skipped if EXTFLD is zero.

From the estimates of X, Y, and Z, estimates of the horizontal field, total field, dip, and declination (Fortran variables CH, CF, CI, and CD respectively) are made using Eqs. (34) and (37-39) of Section 3.0.

The program now calculates the coefficients of the unknowns in the system of simultaneous least squares equations. Beginning with card 214, the first non-zero measurement is processed and then its value set equal to zero. The program returns to statement number 28 and since the value of the previous measurement was set equal to zero, processes the second non-zero measurement. This continues until all measurements have been processed and the value of the respective Fortran variables have all been set equal to zero.

Formulas for the coefficients of the unknowns in the system of simultaneous least squares equations may be derived easily from simple theorems in differential calculus and the various equations of Section 3.0. The calculus theorem results in the following:

If $x = f(u)$ and $u = g(w)$

then $$\frac{dx}{dw} = \frac{df}{du} \cdot \frac{dg}{dw} \quad . \qquad \text{(B3)}$$

Applying this theorem to Eq. (34) of Section 3. 0, the following formulas can be derived for the declination of the total field strength:

$$\frac{d\,DECL}{d\,g_{N,M,0}} = \frac{d\,DECL}{dX} \cdot \frac{dX}{d\,g_{N,M,0}} + \frac{d\,DECL}{dY} \cdot \frac{dY}{d\,g_{N,M,0}} \tag{B4}$$

$$\frac{d\,DECL}{d\,g_{N,M,0}} = -\frac{Y}{HOR}\,\frac{dX}{d\,g_{N,M,0}} + \frac{X}{HOR}\,\frac{dY}{d\,g_{N,M,0}} \tag{B5}$$

$$\frac{d\,DECL}{d\,h_{N,M,0}} = \frac{d\,DECL}{dX} \cdot \frac{dX}{d\,h_{N,M,0}} + \frac{d\,DECL}{dY} \cdot \frac{dY}{d\,h_{N,M,0}} \tag{B6}$$

$$\frac{d\,DECL}{d\,h_{N,M,0}} = -\frac{Y}{HOR} \cdot \frac{dX}{d\,h_{N,M,0}} + \frac{X}{HOR}\,\frac{dY}{d\,h_{N,M,0}} \tag{B7}$$

Similarly, from Eqs. (37-39) of Section 3. 0, the following can be derived for field dip:

$$\frac{d\,DIP}{d\,g_{N,M,0}} = -\frac{X \cdot Z}{H \cdot F}\,\frac{dX}{d\,g_{N,M,0}} - \frac{Y \cdot Z}{H \cdot F}\,\frac{dY}{d\,g_{N,M,0}} + \frac{H}{F}\,\frac{dZ}{d\,g_{N,M,0}} \tag{B8}$$

$$\frac{d\,DIP}{d\,h_{N,M,0}} = -\frac{X \cdot Z}{H \cdot F}\,\frac{dX}{d\,h_{N,M,0}} - \frac{Y \cdot Z}{H \cdot F}\,\frac{dY}{d\,h_{N,M,0}} + \frac{H}{F}\,\frac{dZ}{d\,h_{N,M,0}} \tag{B9}$$

for the horizontal component:

$$\frac{d\,HOR}{d\,g_{N,M,0}} = \frac{X}{HOR}\,\frac{dX}{d\,g_{N,M,0}} + \frac{Y}{HOR}\,\frac{dY}{d\,g_{N,M,0}} \tag{B10}$$

$$\frac{d\,HOR}{d\,h_{N,M,0}} = \frac{X}{HOR}\,\frac{dX}{d\,h_{N,M,0}} + \frac{Y}{HOR}\,\frac{dY}{d\,h_{N,M,0}} \tag{B11}$$

and for total field:

$$\frac{dB}{dg_{N,M,0}} = \frac{X}{B}\frac{dX}{dg_{N,M,0}} + \frac{Y}{B}\frac{dY}{dg_{N,M,0}} + \frac{Z}{B}\frac{dZ}{dg_{N,M,0}} \tag{B12}$$

$$\frac{dB}{dh_{N,M,0}} = \frac{X}{B}\frac{dX}{dh_{N,M,0}} + \frac{Y}{B}\frac{dY}{dh_{N,M,0}} + \frac{Z}{B}\frac{dZ}{dh_{N,M,0}} \tag{B13}$$

Similar expressions for derivatives with respect to $g_{N,M,t}$, $h_{N,M,t}$, $g_{N,M,tt}$, and $h_{N,M,tt}$ can be derived from the equations cited.

Beginning at card 227, the program employs Eqs. (B4-B7) to compute the coefficients of the $g_{N,M,0}$ and $h_{N,M,0}$ when declination is observed. The observation term (Fortran variable FI) is then computed followed by the weight assigned to the observation. Finally, the value of DECL is set equal to zero so that when program control is returned to statement number 28, the next data type will be processed.

In a similar manner the program processes field dip beginning with card 254, the horizontal field strength beginning with card 276, the total field strength beginning with card 298, the Z component beginning with card 316, the X component beginning with card 329, and the Y component beginning with card 342.

As each observation is processed, the program, beginning at card 354, adds the time terms. Then beginning at card 369, the squared time terms are added.

If external field terms are to be used and corrected, the program, beginning with card 385, computes the coefficients of the unknowns E_1, E_2, and E_3 in the system of simultaneous least squares equations. The equations employed are similar to equations (B4) through (B13) with $\frac{d}{dg_{N,M,0}}$ or $\frac{d}{dh_{N,M,0}}$ being replaced with $\frac{d}{dE_1}$, $\frac{d}{dE_2}$, or $\frac{d}{dE_3}$. Beginning with card 385, $\frac{dX}{dE_1}$, $\frac{dX}{dE_2}$, $\frac{dX}{dE_3}$, $\frac{dZ}{dE_1}$, $\frac{dZ}{dE_2}$, and $\frac{dZ}{dE_3}$ are computed. The quantities $\frac{dY}{dE_1} = 0$, $\frac{dY}{dE_2} = \sin\phi$, and $\frac{dY}{dE_3} = -\cos\phi$ are not set up explicitly. The program processes field declination beginning with card 394, field dip beginning with card 405, the horizontal field strength beginning with card 417, the total field strength beginning with card 428, the Z component beginning with card 438, the X component beginning with card 446, and finally the Y component beginning with card 454. Then at card 462, the observation term is added to the Fortran vector F(I).

Beginning with card 463, the three Fortran constants NOR, NOP, and NOPP (identified in the glossary) required to specify the matrix size, etc. are computed.

Observations are recorded by cards 466 through 475. After the observation is recorded, various counts, weights, and errors (i. e., IERR, SIG1, FNO1, SWT1, and SUMTM which are all identified in the glossary) are computed and summed. On the last iteration, the Fortran variables FNO2 and ERR are summed.

Card 498 calls subroutine DLOOP. This subroutine computes the sums of squares and cross-products required for the coefficients of the unknown parameters in the system of simultaneous least squares equations.

A description of D as well as a description of the intricate manipulations required of D for the solution of the set of least squares equations is given in some detail in Section 3. 0 of this report.

Solution of the Least Squares Equations

This phase of the program accomplishes four functions. First, calculations for standard errors are completed and a record made of the results. Second, the D array is recorded for the MATRIX subroutine calculations. Third, the computation check column is computed and finally, the least squares equations are solved.

Details of the D matrix storage and manipulation are presented in Section 3. 0 of this report and will not be repeated here. However, note that cards 506 through 516 compute the computation check column.

From the procedure described in Section 3. 0, one can see that as each row is considered at step (1), the corresponding complete column is required for step (2). This column is contained in the triangle matrix. As a column is needed, it is transferred to the vector SIDE by the following Fortran statements:

```
          DO 83  I=1,NOR
          NROW=MIN(I, J)
          NCOL=I+L-NROW
          K=(NROW*(NOR+NOR+5-NROW))/2+NCOL-NOR-2
          SIDE(I)=D(K)
83        CONTINUE
```

where NOR = number of rows in the complete matrix
NROW = row number
NCOL = column number
MIN = function subprogram to choose minimum of the arguments
D = the triangle matrix stored.

(These variables are identified in the glossary.) The cards beginning with card 520 and ending with card 525 transfer the required column to the vector SIDE. Then beginning with card 526 and ending with card 531, step (1) above is accomplished. Step (2) follows ending with card 540. This completes the third phase of the program.

Corrections and Output

The last two phases of the program, viz., the estimation of the corrections for the parameters and the recording of all results, are intermingled so that while the functions are distinct, the Fortran statements are not. As is customary with Fortran programs, final results are not stored but are written as soon as available.

The output begins with a record of the Fortran variables TYPE (IJ), SIG1 (IJ), and FNO1 (IJ) which are all identified in the glossary.

The solution of the set of simultaneous least squares equations yields adjustments or corrections for the parameters based on the average observation time (AVETIM). The G, H, GT, HT, GTT, HTT input during the initialization phase are based on the year 1960. Hence, the corrections must be computed for 1960 instead of the average observation time. This time adjustment in the corrections begins with card 549. Card 560 computes and adds the time adjustment while card 562 adds the squared time adjustment to the $g_{N,M,0}$ correction. Card 563 computes and adds the time adjustment to the $g_{N,M,t}$ correction. Card 565 adds the correction to the $g_{N,M,0}$ and the next two Fortran statements record the new $g_{N,M,0}$, the total correction, and the computer check column. In a similar manner, the cards from 573 through 589 compute, apply, and record the same information for the $h_{N,M,0}$ and $h_{N,M,t}$.

If time terms were used and are to be corrected, the cards beginning with 590 and going through 609 apply and record the corrections for $g_{N,M,t}$ and $h_{N,M,t}$. Finally, if squared time terms were used and are to be corrected, the cards beginning with 610 and going through 629 apply and record the corrections for $g_{N,M,tt}$ and $h_{N,M,tt}$.

If external field terms were used, and are to be corrected, the program beginning with card 632 and continuing through card 647 records the new values for E_1, E_2, and E_3 and the corrections applied.

Final Output

The final output consists of a permanent record of the results of the calculation plus the inputs required for the next updating of the parameters in the spherical-harmonic expansion of the geomagnetic potential function. First, a printed record is made of all the G, H, GT, HT, GTT, and HTT and the E1, E2, and E3. Next, a punched record is made of XID1, XID2,

NMAX, NSKIP, SIG1(8), TYPE(I), NMAX, NMAXT, NMAXTT, NSKIP, ITER, ERRLIM, AVETIM, all of the G, H, GT, HT, GTT and HTT, and the E1, E2, and E3. All of these Fortran variables are identified in the glossary.

With the punching of the new starting data, one iteration has been completed. The program now transfers control to the beginning of the data processing phase for the next iteration.

When all iterations have been completed, a printed record is made of the Fortran variables IERR(IK) and ERR(J,K)/FNO2(J,K). ERR(J,K)/FNO2(J,K) is the mean deviation for latitude-longitude blocks. This concludes the program.

GLOSSARY

Jensen's Fit

A	a, the mean equatorial radius of the earth in kilometers
A2	a^2
A4	a^4
A2B2	(mean equatorial radius of the earth)^2 - (polar radius of the earth)^2
A4B4	(mean equatorial radius of the earth)^4 - (polar radius of the earth)^4
ALT	altitude of the observation
AOR	radius of the sphere having volume equal to the earth's volume/R
AR	(radius of the sphere having volume equal to the earth's volume/R)$^{N+1}$
AVETIM	average time for all observations
B	total observed field strength
B2	(polar radius of the earth)^2
BWT	standard error of total field strength
CD	estimated declination in radians
CDDEG	estimated declination in degrees
CF	estimated total field strength
CH	estimated horizontal field strength
CI	estimated field dip in radians
CIDEG	estimated field dip in degrees
CONST	a set of constants required for the generation of the associated Legendre polynomials, P_n^m

$$\frac{(n-1)^2 - m^2}{(2n-1)(2n-3)} = \frac{(N-2)^2 - (M-1)^2}{(2N-3)(2N-5)}$$

n and m are common formula notation while N and M are used in the computer program

COSD	cosine of difference between geodetic coordinate λ and geocentric coordinate θ
CP(M)	cosine of the product of (M-1) and the longitudinal coordinate ϕ
CT	cosine of $\pi/2$ minus the geocentric coordinate θ, i. e., coaltitude
CX	estimated X component of the field strength
CY	estimated Y component of the field strength
CZ	estimated Z component of the field strength
D	triangular matrix of sums of squares and cross-products of the coefficient of $g_{N,M,0}$, $h_{N,M,0}$, $g_{N,M,t}$, $h_{N,M,t}$, $g_{N,M,tt}$, and $h_{N,M,tt}$.
DECL	angle of declination D
DECLWT	standard error of the angle of declination
DEN	$(a^2 \cos^2\lambda + b^2 \sin^2\lambda)^{1/2}$ where a = mean equatorial radius of the earth b = polar radius of the earth λ = geodetic coordinate of latitude
DEN2	$(DEN)^2$
DIP	angle of dip, I
DIPWT	standard error of the angle of dip
DP	derivative of an associated Legendre polynomial
DXDE1	$\frac{dX}{dE_1}$ where X is the X component of the field strength

DXDE2	$\frac{dX}{dE_2}$	where X is the X component of the field strength
DXDE3	$\frac{dX}{dE_3}$	where X is the X component of the field strength
DXDG	$\frac{dX}{dg}$	where X is the X component of the field strength
DXDH	$\frac{dX}{dh}$	where X is the X component of the field strength
DYDG	$\frac{dY}{dg}$	where Y is the Y component of the field strength
DYDH	$\frac{dY}{dh}$	where Y is the Y component of the field strength
DZDE1	$\frac{dZ}{dE_1}$	where Z is the Z component of the field strength
DZDE2	$\frac{dZ}{dE_2}$	where Z is the Z component of the field strength
DZDE3	$\frac{dZ}{dE_3}$	where Z is the Z component of the field strength
DZDG	$\frac{dZ}{dg}$	where Z is the Z component of the field strength
DZDH	$\frac{dZ}{dh}$	where Z is the Z component of the field strength

E1 — external field term along the polar axis

E2 — external field term in the equatorial and prime meridian planes

E3 — external field term in the equatorial plane but perpendicular to the plane of the prime meridian

DLONG — longitudinal coordinate ϕ in degrees

ERR — observations times weights summed for global grid-points

ERRLIM — error limit for detecting measurement errors

EXTFLD — a code to identify when external field terms are to be used (EXTFLD $\neq$ 0) and when they are not to be used (EXTFLD = 0)

F — one of the numbers used to form the sums of squares and cross-products of the triangular matrix

FAC — $\tan\theta / \tan\lambda$ where θ and λ are the geocentric and geodetic latitudinal coordinates

FACT — a factor used in generating the factors for converting from Gauss normalization to Schmidt normalization. When M=2, FACT= 2.0; when M > 2, FACT = 1.0. Also used in forming squares and cross products. Hence a temporary storage.

FI — the "Y" or observation term used in forming the least squares matrix

FLAT — polar radius of the earth / mean equatorial radius of the earth

FLATR — λ in radians

FLATT — latitudinal coordinate λ

FM — the index M or M-1 in floating point notation

FN — the index N in floating point notation

FNO1 — a set of eight storages to contain counts of the seven types of data and a count of the total number of data

FNO2 — a set of (18, 36) storages to contain the sum of the data weights at points regularly spaced over the surface of the earth at 10° intervals

FWT — standard error of the field strength

G — the coefficient $g_{N,M,0}$ in the spherical-harmonic expansion of the geomagnetic potential function

GNM — G(N, M), i.e., a specific G

GT	the coefficient $g_{N,M,t}$ in the spherical-harmonic expansion of the geomagnetic potential function
GTNM	GT(N, M), i. e., a specific GT
GTT	the coefficient $g_{N,M,tt}$ in the spherical harmonic-expansion of the geomagnetic potential function
GTTNM	GTT(N, M), i. e., a specific GTT
H	the coefficient $h_{N,M,0}$ in the spherical-harmonic expansion of the geomagnetic potential function
HNM	H(N, M), i. e., a specific H
HOR	observed horizontal component of field strength
HORWT	standard error of the observed horizontal component of field strength
HT	the coefficient $h_{N,M,t}$ in the spherical-harmonic expansion of the geomagnetic potential function
HTNM	HT(N, M), i. e., a specific HT
HTT	the coefficient $h_{N,M,tt}$ in the spherical-harmonic expansion of the geomagnetic potential function
HTTNM	HTT(N, M), i. e., a specific HTT
I	an index
IERR	distribution of FI
IJ	an index
IK	an index
ISKIP	determines frequency of the data selected for a test calculation, e. g., ISKIP = 3 selects every third observation, ISKIP = 10 selects every tenth, etc.

ITER	iteration limit, i. e., maximum number of iterations to be performed
ITNO	iteration counter
ITYPE	identifies data type, e. g., DECL = 1 DIP = 2 HOR = 3 B = 4 Z = 5 X = 6 Y = 7
J	an index
JERR	average observation times weights for global grid-points
JK	an index
JL	an index
JM	an index
K	a computed subscript
KJ	a subscript computed from J and L
KK	an index
KP	an index
KPP	an index
L	an index
LINE	an index for counting the number of lines output for a page
LON	a longitude code
M	an index and subscript
MAXD	maximum size of the triangular matrix C
N	an index and subscript

NCOL column number in triangular matrix C

NMAX maximum N in the terms of the form $g_{N,M,0} \cos(M-1)\phi$ or $h_{N,M,0} \sin(M-1)\phi$ in the spherical-harmonic expansion of the geomagnetic potential function

NMAXT maximum N in the terms of the form $g_{N,M,t} t \cos(M-1)\phi$ or $h_{N,M,t} t \sin(M-1)\phi$ in the spherical-harmonic expansion of the geomagnetic potential function

NMAXTT maximum N in the terms of the form $g_{N,M,tt} t^2 \cos(M-1)\phi$ or $h_{N,M,tt} t^2 \sin(M-1)\phi$ in the spherical-harmonic expansion of the geomagnetic potential function

NO first row or column in the least squares equations for terms of the form $g_{N,M,t} t \cos(M-1)\phi$ or $h_{N,M,t} t \sin(M-1)\phi$

NONOT first row or column in the least squares equations for terms of the form $g_{N,M,tt} t^2 \cos(M-1)\phi$ or $h_{N,M,tt} t^2 \sin(M-1)\phi$

NOP number of parameters in the spherical-harmonic expansion of the geomagnetic potential function

NOPP number of parameters plus one

NOR number of rows in the triangular matrix D

NROW row number in the triangular matrix D

NSKIP constant used to set the value of ISKIP

P an associated Legendre polynomial

PI $\pi = 3.14159265$

PI2 $2\pi = 6.28318530$

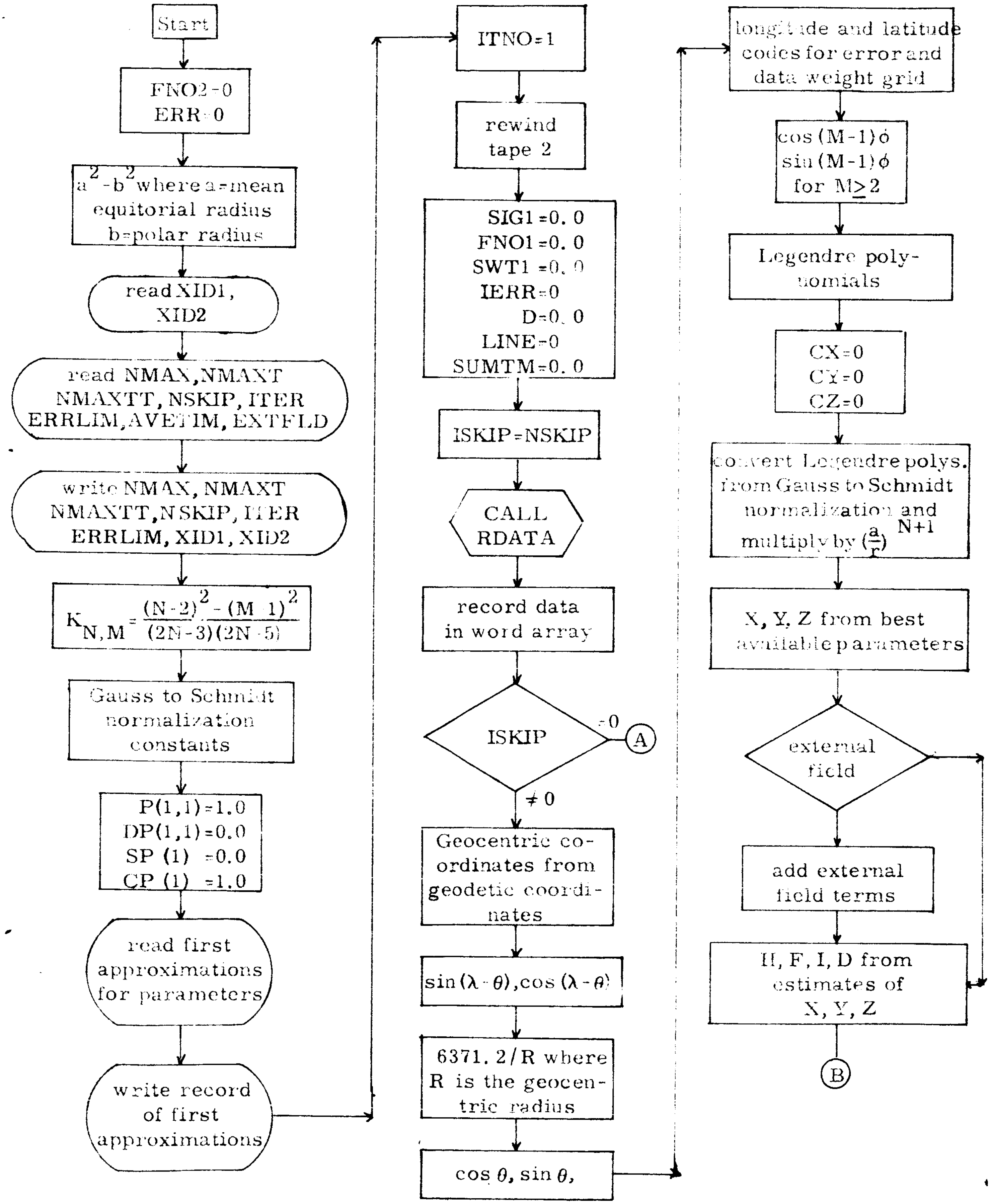
R geocentric coordinate of radius

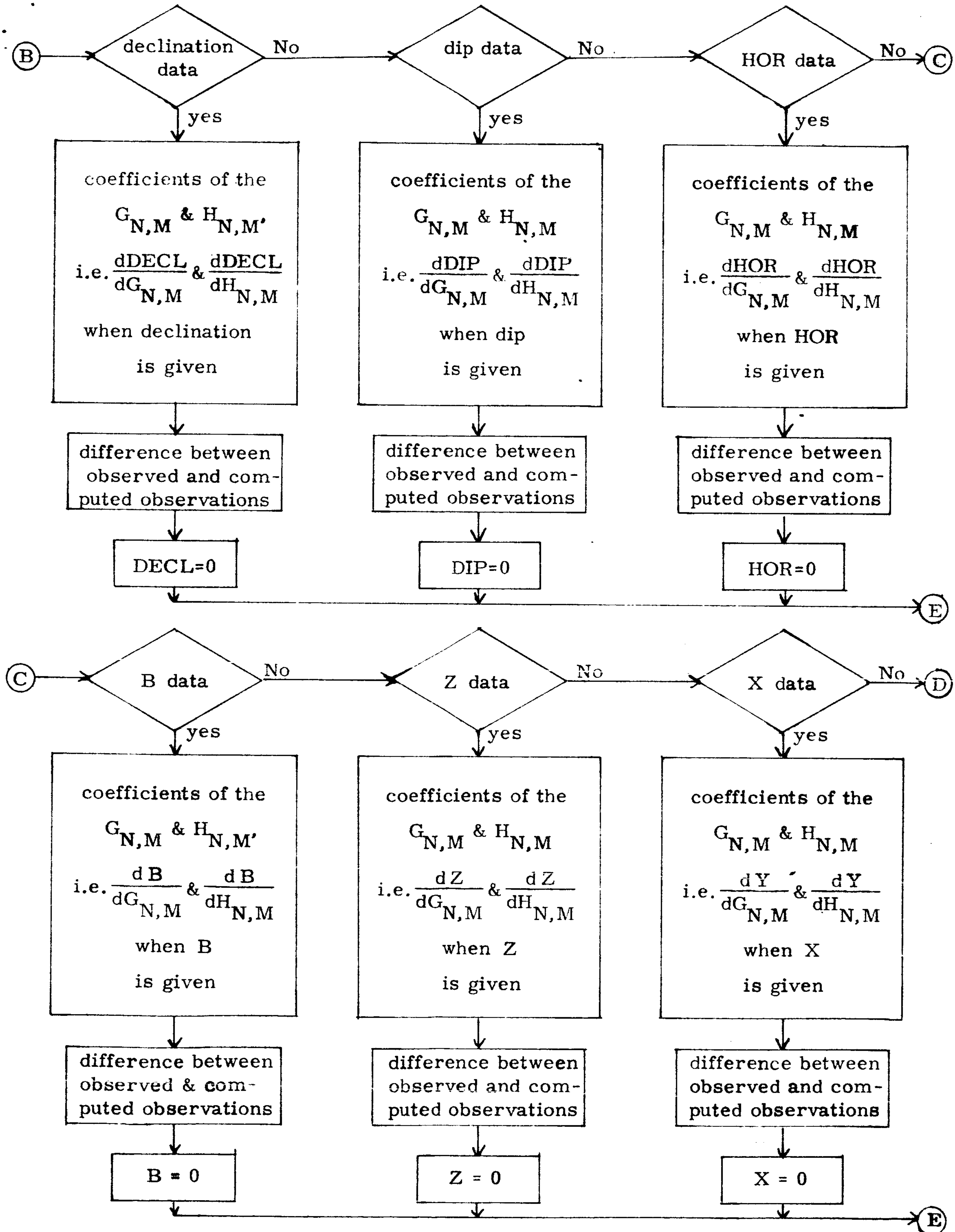
RAD	degrees in one radian = 57.2957795
RDATA	name of subroutine for reading data from magnetic tapes
RDKK	reciprocal of the (K, K) element in the triangular matrix D
SHMIDT	constants to convert from Gauss normalization to Schmidt normalization
SIDE	a "complete" column in the matrix D required for inversion
SIG1	a set of eight storages to contain the sums of the squared observations times the assigned weight types and the total for all data
SIND	sine of difference between geodetic coordinate λ and geocentric coordinate θ
SINLA	sine of the geodetic latitudinal coordinate λ
SINLA2	$(SINLA)^2$
SP(M)	sine of the product of (M-1) and the longitudinal coordinate ϕ
ST	sine of $\pi/2$ minus the geocentric coordinate θ, i. e., coaltitude
SUMD	sum storage for forming check sum column of D matrix
SUMTM	sum of (time - 60.0)
SWT1	a set of eight storages to contain the sum of the weights of the seven types of data and a sum of the total weights of all data
T1	a temporary storage
T2	a temporary storage
T3	a temporary storage
TEMP	a temporary storage
TFACT	time factor
THETA	the geocentric coordinate of latitude

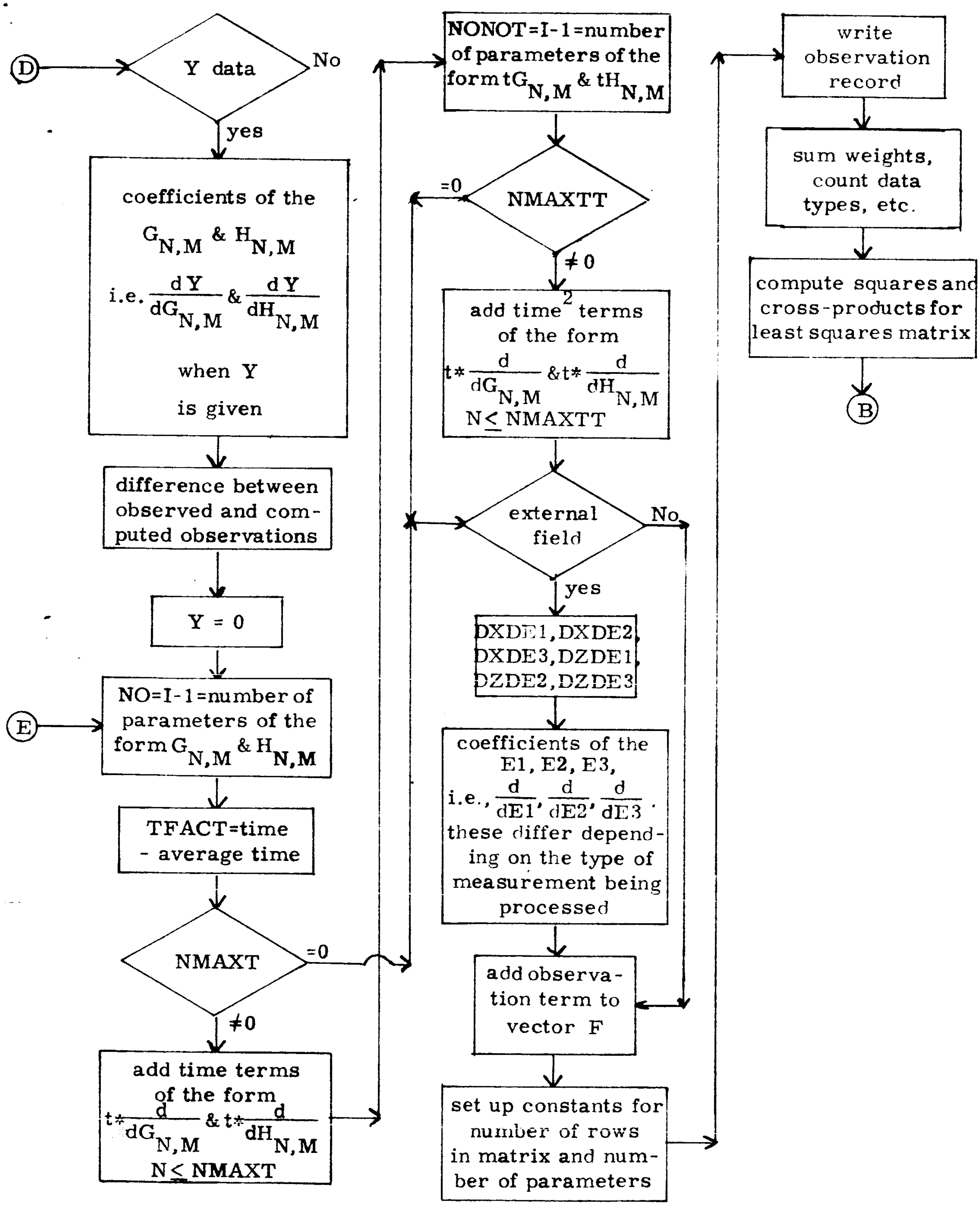
TIME	time of observation
TM	time - 60.0
TYPE	alphabetic code to identify ITYPE's on output
WD	seven storages for input data in the following order--DECL, DIP, HOR, B, Z, X, and Y
WT	an assigned data weight
X	observed X component of the field strength
XID1,XID2	storages to identify computer run
XWT	standard error of observed X component of the field strength
Y	observed Y component of the field strength
YWT	standard error of observed Y component of the field strength
Z	observed Z component of the field strength
ZWT	standard error of observed Z component of the field strength

APPENDIX C

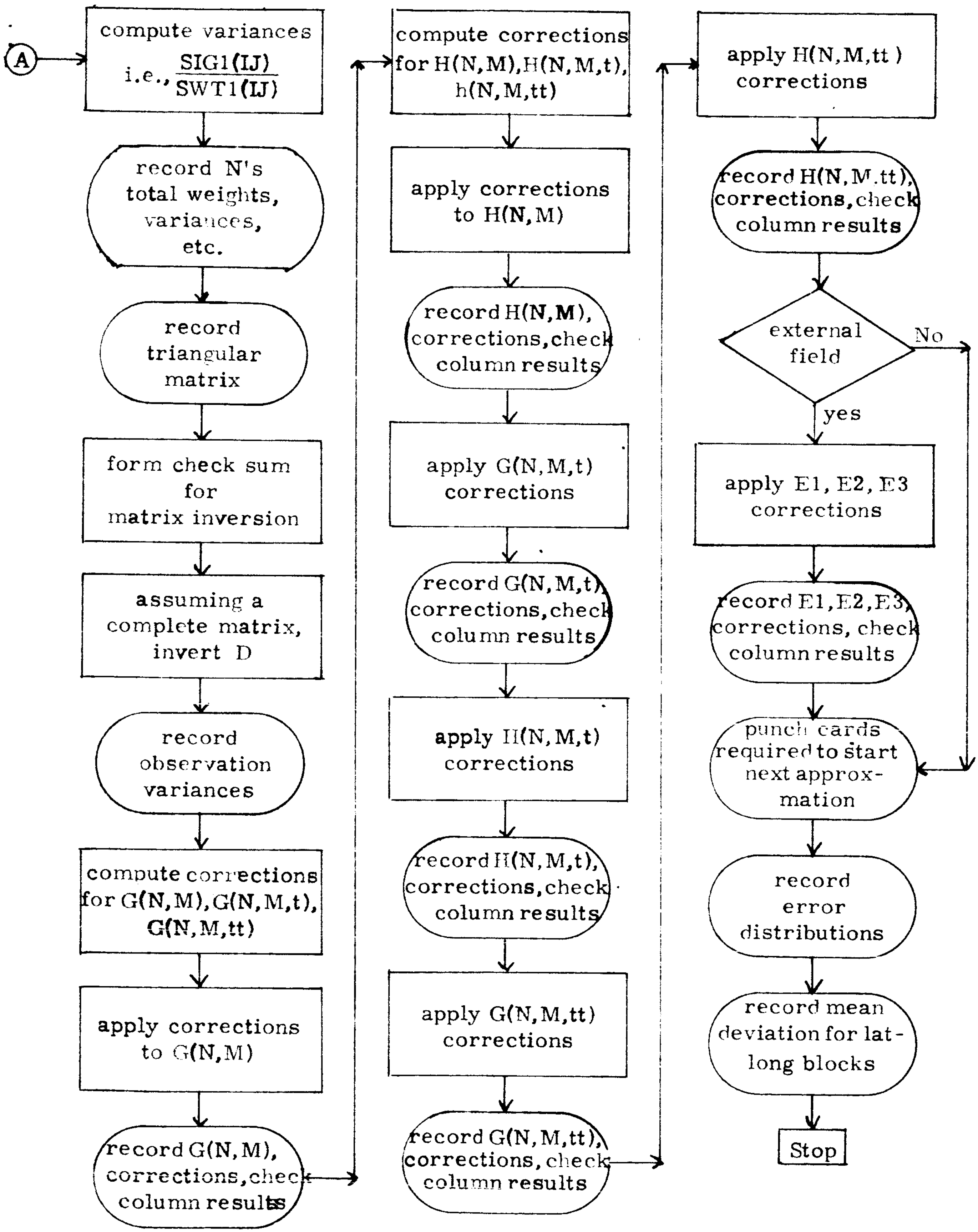
FLOW CHARTS FOR JENSEN'S FIT PROGRAM


Start
FNO2=0
ERR=0
a² - b² where a=mean equitorial radius b=polar radius
read XID1, XID2
read NMAX, NMAXT NMAXTT, NSKIP, ITER ERRLIM, AVETIM, EXTFLD
write NMAX, NMAXT NMAXTT, NSKIP, ITER ERRLIM, XID1, XID2
$K_{N,M} = \frac{(N-2)^2 - (M-1)^2}{(2N-3)(2N-5)}$
Gauss to Schmidt normalization constants
P(1,1)=1.0
DP(1,1)=0.0
SP (1) =0.0
CP (1) =1.0
read first approximations for parameters
write record of first approximations
ITNO=1
rewind tape 2
SIG1=0.0
FNO1=0.0
SWT1=0.0
IERR=0
D=0.0
LINE=0
SUMTM=0.0
ISKIP=NSKIP
CALL RDATA
record data in word array
ISKIP
=0
A
≠0
Geocentric co-ordinates from geodetic coordi-nates
sin (λ-θ), cos (λ-θ)
6371.2/R where R is the geocentric radius
cos θ, sin θ,
longitude and latitude codes for error and data weight grid
cos (M-1)φ
sin (M-1)φ
for M≥2
Legendre poly-nomials
CX=0
CY=0
CZ=0
convert Legendre polys. from Gauss to Schmidt normalization and multiply by $(\frac{a}{r})^{N+1}$
X, Y, Z from best available parameters
external field
add external field terms
H, F, I, D from estimates of X, Y, Z
B


B
declination data
No
dip data
No
HOR data
No
C
yes
yes
yes
coefficients of the $G_{N,M}$ & $H_{N,M}$, i.e. $\frac{dDECL}{dG_{N,M}}$ & $\frac{dDECL}{dH_{N,M}}$ when declination is given
coefficients of the $G_{N,M}$ & $H_{N,M}$ i.e. $\frac{dDIP}{dG_{N,M}}$ & $\frac{dDIP}{dH_{N,M}}$ when dip is given
coefficients of the $G_{N,M}$ & $H_{N,M}$ i.e. $\frac{dHOR}{dG_{N,M}}$ & $\frac{dHOR}{dH_{N,M}}$ when HOR is given
difference between observed and computed observations
difference between observed and computed observations
difference between observed and computed observations
DECL=0
DIP=0
HOR=0
E
C
B data
No
Z data
No
X data
No
D
yes
yes
yes
coefficients of the $G_{N,M}$ & $H_{N,M}$, i.e. $\frac{dB}{dG_{N,M}}$ & $\frac{dB}{dH_{N,M}}$ when B is given
coefficients of the $G_{N,M}$ & $H_{N,M}$ i.e. $\frac{dZ}{dG_{N,M}}$ & $\frac{dZ}{dH_{N,M}}$ when Z is given
coefficients of the $G_{N,M}$ & $H_{N,M}$ i.e. $\frac{dY}{dG_{N,M}}$ & $\frac{dY}{dH_{N,M}}$ when X is given
difference between observed & computed observations
difference between observed and computed observations
difference between observed and computed observations
B = 0
Z = 0
X = 0
E



D
Y data
No
yes
coefficients of the
G N,M & H N,M
i.e. dY/dG N,M & dY/dH N,M
when Y
is given
difference between observed and computed observations
Y = 0
E
NO=I-1=number of parameters of the form G N,M & H N,M
TFACT=time - average time
NMAXT
=0
≠0
add time terms of the form
t* d/dG N,M & t* d/dH N,M
N ≤ NMAXT
NONOT=I-1=number of parameters of the form tG N,M & tH N,M
NMAXTT
=0
≠0
add time² terms of the form
t* d/dG N,M & t* d/dH N,M
N ≤ NMAXTT
external field
No
yes
DXDE1, DXDE2, DXDE3, DZDE1, DZDE2, DZDE3
coefficients of the E1, E2, E3,
i.e., d/dE1, d/dE2, d/dE3.
these differ depending on the type of measurement being processed
add observation term to vector F
set up constants for number of rows in matrix and number of parameters
write observation record
sum weights, count data types, etc.
compute squares and cross-products for least squares matrix
B


A
compute variances i.e., SIG1(IJ) / SWT1(IJ)
record N's total weights, variances, etc.
record triangular matrix
form check sum for matrix inversion
assuming a complete matrix, invert D
record observation variances
compute corrections for G(N,M), G(N,M,t), G(N,M,tt)
apply corrections to G(N,M)
record G(N,M), corrections, check column results
compute corrections for H(N,M), H(N,M,t), h(N,M,tt)
apply corrections to H(N,M)
record H(N,M), corrections, check column results
apply G(N,M,t) corrections
record G(N,M,t) corrections, check column results
apply H(N,M,t) corrections
record H(N,M,t), corrections, check column results
apply G(N,M,tt) corrections
record G(N,M,tt), corrections, check column results
apply H(N,M,tt) corrections
record H(N,M,tt), corrections, check column results
external field
No
yes
apply E1, E2, E3 corrections
record E1, E2, E3, corrections, check column results
punch cards required to start next approx-mation
record error distributions
record mean deviation for lat-long blocks
Stop

APPENDIX D

PROGRAM LISTING FOR WALL'S ERROR PROGRAM

```
C                    *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C
C                    WALL@S ERROR
C
C                    *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C
C                    D MUST EXCEED (NMAX**4+9*NMAX**2+14)/2
C                                 +(NMAXT**4+9*NMAXT**2+14)/2
C                                 +(NMAXTT**4+9*NMAXTT**2+14)/2
C                    F AND SIDE MUST EXCEED NMAX**2+NMAXT**2+NMAXTT**2+6
      DIMENSION G(9,9),H(9,9),GT(9,9),HT(9,9),GTT(9,9),HTT(9,9)
      DIMENSION F(30),DINV(30,30),ERR(7)
      DIMENSION SHMIDT(9,9)
      DIMENSION DXDH(9,9),DYDH(9,9),DZDH(9,9)
      DIMENSION DXDG(9,9),DYDG(9,9),DZDG(9,9)
      DIMENSION CP(9),SP(9),P(9,9),DP(9,9),CONST(9,9)
      INTEGER EXTFLD
      DATA (RAD=57.2957795),(A=6378.165),(FLAT=298.3),(LINE=0)
      DATA (PI=3.14159265),(PI2=6.28318530)
      DATA (EXTFLD=0)
C
C                    *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C
      FLAT=1.-1./FLAT
C                    COMPUTATION WITH SPHERICAL EARTH
      FLAT=1.
      A=6371.2
      MAXD=3400
      A2=A**2
      A4=A**4
      B2=(A*FLAT)**2
      A2B2=A2*(1.-FLAT**2)
      A4B4=A4*(1.-FLAT**4)
      REWIND 1
C                    READ DATA FROM MATRIX SUBROUTINE
      READ (1) NMAX,NMAXT,NMAXTT,FWNP,FNP,SIGMA,NOR,NOP,NOPP
      READ (1) ((G(N,M),H(N,M),M=1,N),N=2,NMAX)
      READ (1) ((GT(N,M),HT(N,M),M=1,N),N=2,NMAX)
      READ (1) ((GTT(N,M),HTT(N,M),M=1,N),N=2,NMAX)
      DO 1 I=1,NOR
      READ (1) (DINV(I,J),J=1,NOP)
1     CONTINUE
C                    READ DELTA LONGITUDE, DELTA LATITUDE, AND BASE TIME
      READ 2,DLONG,DLATT,TIME
2     FORMAT (3F10.0)
      VAR=SIGMA*SIGMA
      TFACT=TIME-60.0
      TFACT2=TFACT*TFACT
C                    FIND STARTING POINT FOR GRID
      SLONG=0.0
```

```
      SLATT=0.0
3     IF (SLONG-DLONG+180.0) 5,4,4
4     SLONG=SLONG-DLONG
      GO TO 3
5     IF (SLATT-DLATT+90.0) 7,6,6
6     SLATT=SLATT-DLATT
      GO TO 5
7     SLATT=SLATT-DLATT
      ELONG=SLONG
      FLATT=SLATT
C                   COMPUTE CONSTANTS REQUIRED FOR GENERATING
C                   LEGENDRE POLYNOMIALS
      DO 8 N=2,NMAX
      FN=N
      DO 8 M=1,N
      FM=M
      CONST(N,M)=((FN-2.0)**2-(FM-1.0)**2)/(FN+FN-3.0)/(FN+FN-5.0)
8     CONTINUE
C                   COMPUTE CONSTANTS TO CONVERT FROM GAUSS TO
C                   SCHMIDT NORMALIZATION
      SHMIDT(1,1)=-1.0
      DO 9 N=2,NMAX
      FN=N
      SHMIDT(N,1)=SHMIDT(N-1,1)*(FN+FN-3.0)/(FN-1.0)
      FACT=2.0
      DO 9 M=2,N
      FM=M
      SHMIDT(N,M)=SHMIDT(N,M-1)*SQRT((FN-FM+1.0)*FACT/(FN+FM-2.0))
9     FACT=1.0
C                   SET VALUE OF FIRST LEGENDRE POLYNOMIALS
      P(1,1)=1.0
      DP(1,1)=0.0
C                   SET VALUE OF SIN(M-1)PHI AND COS(M-1)PHI WHEN M=1
      SP(1)=0.0
      CP(1)=1.0
C                   RECORD PARAMETERS TO BE USED FOR THE GRID
      WRITE (6,10) NMAX,NMAXT,NMAXTT,SIGMA,FNP,FWNP,NOR,NOP,NOPP
10    FORMAT (6H1NMAX=,I5,3X,6HNMAXT=,I5,3X,7HNMAXTT=,I5,3X,8HSIGMA *=,F
     $5.0,3X,7HPOINTS=,F6.0,3X,11HWEIGHT SUM=,E16.8/3X,4HNOR=,I5,3X,4HNO
     $P=,I5,3X,5HNOPP=,I5/1X)
      WRITE (6,11) DLONG,DLATT,TIME
11    FORMAT (12H DELTA LONG=,F8.2,3X,10HDELTA LAT=,F8.2,3X,10HBASE TIME
     $=,F8.2/1X)
      WRITE (6,12) ((N,M,G(N,M),H(N,M),GT(N,M),HT(N,M),GTT(N,M),HTT(N,M)
     $,M=1,N),N=2,NMAX)
12    FORMAT (2I3,6F11.4)
C                   *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C                   END INITIALIZATION, BEGIN FIELD GENERATION
C                   *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
13    IF (FLATT+DLATT-90.0) 18,14,14
```

```
14      IF (ELONG-180.0) 17,15,15
15      WRITE (6,16)
16      FORMAT (1H1/1H1/1H1)
        STOP END
17      ELONG=ELONG+DLONG
        FLATT=SLATT
        LINE=1
        WRITE (6,64)
18      FLATT=FLATT+DLATT
C                      COMPUTE GEOCENTRIC THETA FROM
C                      GEODETIC COORDINATES
        FLATR=FLATT/RAD
        SINLA=SIN(FLATR)
        SINLA2=SINLA**2
        DEN2=A2-A2B2*SINLA2
        DEN=SQRT(DEN2)
        FAC=((ALT*DEN)+B2)/((ALT*DEN)+A2)
        THETA=ATAN(FAC*SINLA/(1.E-30+SQRT(1.-SINLA2)))
C                      COMPUTE GEOCENTRIC R FROM GEODETIC COORDINATES
        R=SQRT(ALT*(ALT+2.*DEN)+(A4-A4B4*SINLA2)/DEN2)
C                      COMPUTE SINE AND COSINE OF DIFFERENCE BETWEEN
C                      GEODETIC AND GEOCENTRIC LATITUDINAL COORDINATES
        SIND=SIN(FLATR-THETA)
        COSD=SQRT(1.0-SIND*SIND)
        AOR=6371.2/R
C                      COS THETA MEASURED FROM POLAR AXIS
        CT=SIN(THETA)
C                      SIN THETA MEASURED FROM POLAR AXIS
        ST=SQRT(1.0-CT*CT)
        SP(2)=SIN(ELONG/RAD)
        CP(2)=COS(ELONG/RAD)
        DO 19 M=3,NMAX
C                      SIN(M-1)PHI, EQ.(5)
        SP(M)=SP(2)*CP(M-1)+CP(2)*SP(M-1)
C                      COS(M-1)PHI, EQ.(6)
        CP(M)=CP(2)*CP(M-1)-SP(2)*SP(M-1)
19      CONTINUE
C                      GENERATE ASSOCIATED LEGENDRE POLYNOMIALS
        DO 22 N=2,NMAX
        DO 22 M=1,N
        IF (N-M) 21,20,21
20      P(N,N)=ST*P(N-1,N-1)
        DP(N,N)=ST*DP(N-1,N-1)+CT*P(N-1,N-1)
        GO TO 22
21      P(N,M)=CT*P(N-1,M)-CONST(N,M)*P(N-2,M)
        DP(N,M)=CT*DP(N-1,M)-ST*P(N-1,M)-CONST(N,M)*DP(N-2,M)
22      CONTINUE
C                      INITIALIZE TO COMPUTE X,Y,Z
        CX=0.0
        CY=0.0
```

```
      CZ=0.0
      AR=AOR*AOR
C                    COMPUTE X,Y,Z USING BEST AVAILABLE
C                    PARAMETERS
      DO 23 N=2,NMAX
      FN=N
      AR=AR*AOR
      DO 23 M=1,N
      FM=M-1
C                    APPLY SCHMIDT NORMALIZATION CONSTANTS
C                    AND MULTIPLY BY (A/R)**(N+1)
      P(N,M)=P(N,M)*AR*SHMIDT(N,M)
      DP(N,M)=DP(N,M)*AR*SHMIDT(N,M)
      TEMP=FN*P(N,M)*SIND-DP(N,M)*COSD
      DXDG(N,M)=TEMP*CP(M)
      DXDH(N,M)=TEMP*SP(M)
      TEMP=FM*P(N,M)/ST
      DYDG(N,M)=-TEMP*SP(M)
      DYDH(N,M)=TEMP*CP(M)
      TEMP=FN*P(N,M)*COSD+DP(N,M)*SIND
      DZDG(N,M)=TEMP*CP(M)
      DZDH(N,M)=TEMP*SP(M)
C                    ADD TIME TERMS
      GNM=(TM*GTT(N,M)+GT(N,M))*TM+G(N,M)
      HNM=(TM*HTT(N,M)+HT(N,M))*TM+H(N,M)
      CX=CX+GNM*DXDG(N,M)+HNM*DXDH(N,M)
      CY=CY+GNM*DYDG(N,M)+HNM*DYDH(N,M)
      CZ=CZ+GNM*DZDG(N,M)+HNM*DZDH(N,M)
23    CONTINUE
      IF (EXTFLD) 24,25,24
24    T1=E2*CP(2)+E3*SP(2)
      T2=E1*ST-T1*CT
      T1=E1*CT+T1*ST
      CX=CX-T2*COSD+T1*SIND
      CY=CY+E2*SP(2)-E3*CP(2)
      CZ=CZ+T2*SIND+T1*COSD
C                    COMPUTE HORIZONTAL, TOTAL FIELD, DIP, AND
C                    DECLINATION
25    CH=SQRT(CX*CX+CY*CY)
      CF=SQRT(CH*CH+CZ*CZ)
      CI=2.0*ATAN(CZ/(CF+CH))
      CD=2.0*ATAN(CY/(CH+CX))
C                    *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C                    END FIELD GENERATION, BEGIN STANDARD ERROR ESTIMATION
C                    *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C                    COMPUTE COEFFICIENTS OF G(N,M) AND H(N,M)
      ITYPE=1
26    I=1
      GO TO (27,30,33,36,39,41,43,62),ITYPE
C                    COMPUTE COEFFICIENTS FOR DECLINATION
```

```
C                   DECL=ARCTAN(Y/X)
C                   T1=DDECL/DY
C                   T2=DDECL/DX
27    T1=CX/CH
      T2=CY/CH
      DO 29 N=2,NMAX
      DO 29 M=1,N
C                   DDECL/DG(N,M)
      F(I)=(T1*DYDG(N,M)-T2*DXDG(N,M))
      IF (M-1) 28,29,28
28    I=I+1
C                   DDECL/DH(N,M)
      F(I)=(T1*DYDH(N,M)-T2*DXDH(N,M))
29    I=I+1
      GO TO 45
C                   COMPUTE COEFFICIENTS FOR DIP
C                   DIP=ARCTAN(Z/H)
C                   T1=DDIP/DZ
C                   T2=DDIP/DX
C                   T3=DDIP/DY
30    T1=CH/CF
      T2=CZ*CX/CH/CF
      T3=CZ*CY/CH/CF
      DO 32 N=2,NMAX
      DO 32 M=1,N
C                   F(I)=DDIP/DG(N,M)
      F(I)=(T1*DZDG(N,M)-T2*DXDG(N,M)-T3*DYDG(N,M))
      IF (M-1) 31,32,31
31    I=I+1
C                   F(I)=DDIP/DH(N,M)
      F(I)=(T1*DZDH(N,M)-T2*DXDH(N,M)-T3*DYDH(N,M))
32    I=I+1
      GO TO 45
C                   COMPUTE COEFFICIENTS FOR HORIZONTAL
C                   FIELD
C                   HOR=SQRT(X*X+Y*Y)
C                   T1=DHOR/DX
C                   T2=DHOR/DY
33    T1=CX/CH
      T2=CY/CH
      DO 35 N=2,NMAX
      DO 35 M=1,N
C                   F(I)=DHOR/DG(N,M)
      F(I)=(T1*DXDG(N,M)+T2*DYDG(N,M))
      IF (M-1) 34,35,34
34    I=I+1
C                   F(I)=DHOR/DH(N,M)
      F(I)=(T1*DXDH(N,M)+T2*DYDH(N,M))
35    I=I+1
      GO TO 45
```

```
C                   COMPUTE COEFFICIENTS FOR TOTAL FIELD
C                   B=SQRT(X*X+Y*Y+Z*Z)
C                   T1=DB/DX
C                   T2=DB/DY
C                   T3=DB/DZ
36    T1=CX/CF
      T2=CY/CF
      T3=CZ/CF
      DO 38 N=2,NMAX
      DO 38 M=1,N
C                   F(I)=DB/DG(N,M)
      F(I)=(T1*DXDG(N,M)+T2*DYDG(N,M)+T3*DZDG(N,M))
      IF (M-1) 37,38,37
37    I=I+1
C                   F(I)=DB/DH(N,M)
      F(I)=(T1*DXDH(N,M)+T2*DYDH(N,M)+T3*DZDH(N,M))
38    I=I+1
      GO TO 45
C                   COMPUTE COEFFICIENTS WHEN THE Z COMPONENT IS GIVEN
39    DO 40 N=2,NMAX
      F(I)=DZDG(N,1)
      I=I+1
      DO 40 M=2,N
      F(I)=DZDG(N,M)
      F(I+1)=DZDH(N,M)
40    I=I+2
      GO TO 45
C                   COMPUTE COEFFICIENTS WHEN THE X COMPONENT IS GIVEN
41    DO 42 N=2,NMAX
      F(I)=DXDG(N,1)
      I=I+1
      DO 42 M=2,N
      F(I)=DXDG(N,M)
      F(I+1)=DXDH(N,M)
42    I=I+2
      GO TO 45
C                   COMPUTE COEFFICIENTS WHEN THE Y COMPONENT IS GIVEN
43    DO 44 N=2,NMAX
      F(I)=DYDG(N,1)
      I=I+1
      DO 44 M=2,N
      F(I)=DYDG(N,M)
      F(I+1)=DYDH(N,M)
44    I=I+2
C                   ADD TIME**1 TERMS
45    NO=I-1
      IF (NMAXT) 46,48,46
46    J=1
      DO 47 N=2,NMAXT
      F(I)=F(J)*TFACT
```

```
      I=I+1
      J=J+1
      DO 47 M=2,N
      F(I)=F(J)*TFACT
      F(I+1)=F(J+1)*TFACT
      I=I+2
      J=J+2
47    CONTINUE
C                       ADD TIME**2 TERMS
48    NONOT=I-1
      IF (NMAXTT) 49,51,49
49    J=1
      DO 50 N=2,NMAXTT
      F(I)=F(J)*TFACT2
      I=I+1
      J=J+1
      DO 50 M=2,N
      F(I)=F(J)*TFACT2
      F(I+1)=F(J+1)*TFACT2
      I=I+2
      J=J+2
50    CONTINUE
51    IF (EXTFLD) 52,60,52
C                       ADD EXTERNAL FIELD TERMS
52    DXDE1=CT*SIND-ST*COSD
      DZDE1=ST*SIND+CT*COSD
      DXDE2=CP(2)*DZDE1
      DXDE3=SP(2)*DZDE1
      DZDE2=-CP(2)*DXDE1
      DZDE3=-SP(2)*DXDE1
      GO TO (53,54,55,56,57,58,59),ITYPE
C                       COEFFICIENTS WHEN DECLINATION (D) IS GIVEN
C                       F(I)=DD/DE1
53    F(I)=-T2*DXDE1
      I=I+1
C                       F(I)=DD/DE2
      F(I)=T1*SP(2)-T2*DXDE2
      I=I+1
C                       F(I)=DD/DE3
      F(I)=-T1*CP(2)-T2*DXDE3
      I=I+1
      GO TO 60
C                       COEFFICIENTS WHEN DIP (I) IS GIVEN
C                       F(I)=DI/DE1
54    F(I)=T1*DZDE1-T2*DXDE1
      I=I+1
C                       F(I)=DI/DE2
      F(I)=T1*DZDE2-T2*DXDE2-T3*SP(2)
      I=I+1
C                       F(I)=DI/DE3
```

```
      F(I)=T1*DZDE3-T2*DXDE3+T3*CP(2)
      I=I+1
      GO TO 60
C                       COEFFICIENTS WHEN THE HORIZONTAL
C                       COMPONENT (HOR) IS GIVEN
C                       F(I)=DHOR/DE1
55    F(I)=T1*DXDE1
      I=I+1
C                       F(I)=DHOR/DE2
      F(I)=T1*DXDE2+T2*SP(2)
      I=I+1
C                       F(I)=DHOR/DE3
      F(I)=T1*DXDE3-T2*CP(2)
      I=I+1
      GO TO 60
C                       COEFFICIENTS WHEN TOTAL FIELD IS GIVEN
C                       F(I)=DF/DE1
56    F(I)=T1*DXDE1+T3*DZDE1
      I=I+1
C                       F(I)=DF/DE2
      F(I)=T1*DXDE2+T2*SP(2)+T3*DZDE2
      I=I+1
C                       F(I)=DF/DE3
      F(I)=T1*DXDE3-T2*CP(2)+T3*DZDE3
      I=I+1
      GO TO 60
C                       COEFFICIENTS WHEN THE Z COMPONENT IS GIVEN
57    F(I)=DZDE1
      I=I+1
      F(I)=DZDE2
      I=I+1
      F(I)=DZDE3
      I=I+1
      GO TO 60
C                       COEFFICIENTS WHEN THE X COMPONENT IS GIVEN
58    F(I)=DXDE1
      I=I+1
      F(I)=DXDE2
      I=I+1
      F(I)=DXDE3
      I=I+1
      GO TO 60
C                       COEFFICIENTS WHEN THE Y COMPONENT IS GIVEN
59    F(I)=0.0
      I=I+1
      F(I)=SP(2)
      I=I+1
      F(I)=-CP(2)
      I=I+1
      GO TO 60
```

```
C                    COMPUTE STANDARD ERROR OF ESTIMATE
60    SUMD=0.0
      DO 61 I=1,NOR
      TEMP=F(I)
      DO 61 J=1,NOR
      SUMD=SUMD+TEMP*F(J)*DINV(I,J)
61    CONTINUE
      ERR(ITYPE)=SQRT(SUMD*VAR)
      ITYPE=ITYPE+1
      GO TO 26
C                    * * * * * * * * * * * * * * * * * *
C                    END ERROR ESTIMATION, BEGIN ONE POINT OUTPUT
C                    * * * * * * * * * * * * * * * * * *
62    CD=CD*RAD
      CI=CI*RAD
      ERR(1)=SQRT((ERR(1)*ERR(1)+CD*CD*ERR(3)*ERR(3))/(CH*CH))*RAD
      ERR(2)=SQRT((ERR(2)*ERR(2)+CI*CI*ERR(4)*ERR(4))/(CF*CF))*RAD
      IF (MOD(LINE,51)) 65,63,65
63    WRITE (6,64)
64    FORMAT (118H1  LAT    LONG     DECLINATION      FIELD DIP      HORIZON
     $TAL      TOTAL FIELD     Z COMPONENT     X COMPONENT      Y COMPONENT)
65    WRITE (6,66) FLATT,ELONG,CD,ERR(1),CI,ERR(2),CH,ERR(3),CF,ERR(4),C
     $Z,ERR(5),CX,ERR(6),CY,ERR(7)
66    FORMAT (1X,F6.1,F7.1,2(F8.2,F7.3),5(F8.0,F7.2))
      LINE=LINE+1
      GO TO 13
      END
```

APPENDIX E

WALL'S ERROR

Introduction

The purpose of this appendix is to document the sequence of operations and to discuss various programming aspects of Wall's Error program. This program has been written to find standard errors of estimation for a grid of the geomagnetic potential over the earth's surface. The grid is based on the time-dependent coefficients estimated by Jensen's Fit program for the spherical-harmonic expansion of the geomagnetic potential function.

The mathematical formulas which form the basis of the computer program are not restated in this appendix. Each time that a formula is required to explain a Fortran variable, a reference is made to an equation in Sections 2. 0, 3. 0, or 4. 0 of this report or to one of the reports listed in the bibliography. When referencing this report, it should be noted that the Fortran variable N is equal to $n+1$. Similarly, $M = m+1$.

The computer program is relatively linear, i. e., there are few alternate calculation sequences. Hence, the calculation sequence will be described in a linear manner.

The program can be roughly divided into four phases as follows: (1) initializing; (2) field generation; (3) standard error estimation; and (4) one point output or recording. This appendix will likewise be divided into four principal sections to describe respectively these four phases. Within each section, the Fortran name for variables will be used whenever possible. A glossary identifying these variables is included at the end of this appendix.

Initialization

Initialization consists of doing the things that must be done once at the start of the execution of the program. For this program, these things include setting or computing the value of certain constants that will be used throughout the other phases of the program. Among these are FLAT, A2, A4, B2, A2B2, A4B4, VAR, TFACT, TFACT2, ELONG, FLATT, CONST(N, M), SHMIDT(N, M), P(1, 1) DP(1, 1), SP(1), and CP(1) all of which are identified in the glossary at the end of this appendix.

The equation for computing CONST(N, M) is found in Eqs. (19) of Section 2.0. Note that N = n+1 and M = m+1. The equations for computing SHMIDT(N, M) are found in Eqs. (20) of Section 2.0.

The value of the first associated Legendre polynomial, P(1, 1), and its derivative, DP(1, 1), are constants and may be found in Eqs. (19) of Section 2.0. Similarly, the value of SP(M) and CP(M) are constants for M=1, i.e., SP(1) = 0.0 and CP(1) = 1.0.

Another common function of initialization is the reading of variable control and starting data. Among the variables that must be read for this program are NMAX, NMAXT, NMAXTT, FWNP, FNP, SIGMA, NOR, NOP, NOPP, G(N, M), H(N, M), GT(N, M), HT(N, M), GTT(N, M), HTT(N, M), and DINV(I, J). These variables, which are all identified in the glossary, are recorded by the modification of Daniels' Matrix subroutine specifically for this program. In addition to these, DLONG, DLATT, and TIME must also be supplied, these are identified in the glossary.

A final function of initialization is often the recording of initial values of pertinent variables. This program records all input data listed above except DINV.

Except for setting or computing the values of certain other minor variables required by the initialization functions enumerated above, this completes the initialization phase of the program.

Field Generation

Output page control and the incrementing of longitude and latitude is accomplished by cards 100 through 109.

Beginning with card 112, geocentric coordinates are computed from the geodetic grid point assignments made for each observation. The equation for computing THETA is Eq. (30) of Section 2.0. The equation for computing geocentric R is Eq. (31) of Section 2.0.

The five variables SIND, COSD, AOR, CT and ST which are all identified in the glossary are computed next. SIND and COSD are required for converting from geocentric to geodetic coordinates. CT and ST are required for the generation of the associated Legendre polynomials and AOR is a term that appears in the equations for estimating X, Y, Z, etc. from the best available set of parameters. One should note that the Fortran statement for computing CT and ST redefines θ to be measured from the polar axis instead of from the equatorial plane, i.e., colatitude.

SP(2) and CP(2) are computed, followed by the computation of SP(M) and CP(M) for $M > 2$. The equations

$$\sin(M-1)\phi = \sin\phi \cdot \cos(M-2)\phi + \cos\phi \cdot \sin(M-2)\phi \qquad \text{(E1)}$$

and

$$\cos(M-1)\phi = \cos\phi \cdot \cos(M-2)\phi - \sin\phi \cdot \sin(M-2)\phi \qquad \text{(E2)}$$

used for computing SP(M) and CP(M) are available in standard texts on trigonometry under the subject, "functions of sums of angles."

Next the program evaluates the necessary associated Legendre polynomials, P(N, M), and their derivatives, DP(N, M), employing the recurrence relationships which are found in Section 2.0, Eqs. (19). $K_{N,M}$ corresponds to the Fortran variable CONST(N, M) and was discussed above in the section on initialization.

Beginning with card 149, the Fortran variables CX, CY, and CZ are set equal to zero in preparation for the estimation of X, Y, and Z. AR and TM, two Fortran variables identified in the glossary, must also be initialized in preparation for the estimation of X, Y, and Z.

The Fortran statements through card 179 are required to estimate X, Y, and Z, i.e., the Fortran variables CX, CY, and CZ. The rotation formulas required for computing X and Z are found in Section 2.0, Eqs. (32)

and (33). Equations for B_θ, B_r, and B_ϕ are given in several different forms in Section 2.0 of this report.

Near the beginning of the group of Fortran statements required to estimate X, Y, and Z (specifically cards 162 and 163), the Gauss normalized polynomials are Schmidt normalized and multiplied by the appropriate power of $\frac{6371.2}{r}$, i.e., AR.

Depending on when it is calculated, the Fortran variable TEMP is the common factor in the coefficients of the two parameters $g_{N,M,0}$ and $h_{N,M,0}$ in the formulas for computing X, Y, or Z. DXDG, DXDH, DYDG, DYDH, DZDG and DZDH complete the calculation of the coefficients of $g_{N,M,0}$ and $h_{N,M,0}$ in the formulas for computing X, Y, and Z. Next, the Fortran variables GNM and HNM are computed using Eqs. (21) of Section 2.0 and finally all terms are summed for the respective estimates of X, Y, and Z.

The X, Y, and Z components of the external field are next estimated and added to the respective estimates of X, Y, and Z (cards 180 through 186). This step is completely skipped if EXTFLD is zero.

From the estimates of X, Y, and Z, estimates of the horizontal field, total field, dip, and declination (Fortran variables CH, CF, CI, and CD respectively) are made using Eqs. (34) and (37-39) of Section 3.0.

This concludes the field generation phase of the Error program.

Standard Error Estimation

The program now computes the coefficients of the g's and h's in the spherical-harmonic expansion of the geomagnetic potential function. These coefficients are the f(x) in Eq. (51) of Section 4.0.

Formulas for the coefficients of the g's and h's in the system of simultaneous least squares equations may be derived easily from simple theorems in differential calculus and the various equations of Section 3.0. The calculus theorem results in the following:

$$\text{If} \qquad x = f(u) \qquad \text{and} \qquad u = g(w)$$

$$\text{then} \qquad \frac{dx}{dw} = \frac{df}{du} \cdot \frac{dg}{dw} \quad . \tag{E3}$$

Applying this theorem to Eq. (34) of Section 3.0, the following formulas can be derived for the declination of the total field strength:

$$\frac{d\,DECL}{d\,g_{N,M,0}} = \frac{d\,DECL}{dX} \cdot \frac{dX}{d\,g_{N,M,0}} + \frac{d\,DECL}{dY} \cdot \frac{dY}{d\,g_{N,M,0}} \tag{E4}$$

$$\frac{d\,DECL}{d\,g_{N,M,0}} = -\frac{Y}{HOR}\,\frac{dX}{d\,g_{N,M,0}} + \frac{X}{HOR}\,\frac{dY}{d\,g_{N,M,0}} \tag{E5}$$

$$\frac{d\,DECL}{d\,h_{N,M,0}} = \frac{d\,DECL}{dX} \cdot \frac{dX}{d\,h_{N,M,0}} + \frac{d\,DECL}{dY} \cdot \frac{dY}{d\,h_{N,M,0}} \tag{E6}$$

$$\frac{d\,DECL}{d\,h_{N,M,0}} = -\frac{Y}{HOR} \cdot \frac{dX}{d\,h_{N,M,0}} + \frac{X}{HOR}\,\frac{dY}{d\,h_{N,M,0}} \tag{E7}$$

Similarly, from Eqs. (37-39) of Section 3. 0, the following can be derived for field dip:

$$\frac{d\,DIP}{dg_{N,M,0}} = -\frac{X\cdot Z}{H\cdot F}\frac{dX}{dg_{N,M,0}} - \frac{Y\cdot Z}{H\cdot F}\frac{dY}{dg_{N,M,0}} + \frac{H}{F}\frac{dZ}{dg_{N,M,0}} \tag{E8}$$

$$\frac{d\,DIP}{dh_{N,M,0}} = -\frac{X\cdot Z}{H\cdot F}\frac{dX}{dh_{N,M,0}} - \frac{Y\cdot Z}{H\cdot F}\frac{dY}{dh_{N,M,0}} + \frac{H}{F}\frac{dZ}{dh_{N,M,0}} \tag{E9}$$

for the horizontal component:

$$\frac{d\,HOR}{dg_{N,M,0}} = \frac{X}{HOR}\frac{dX}{dg_{N,M,0}} + \frac{Y}{HOR}\frac{dY}{dg_{N,M,0}} \tag{E10}$$

$$\frac{d\,HOR}{dh_{N,M,0}} = \frac{X}{HOR}\frac{dX}{dh_{N,M,0}} + \frac{Y}{HOR}\frac{dY}{dh_{N,M,0}} \tag{E11}$$

and for total field:

$$\frac{dB}{dg_{N,M,0}} = \frac{X}{B}\frac{dX}{dg_{N,M,0}} + \frac{Y}{B}\frac{dY}{dg_{N,M,0}} + \frac{Z}{B}\frac{dZ}{dg_{N,M,0}} \tag{E12}$$

$$\frac{dB}{dh_{N,M,0}} = \frac{X}{B}\frac{dX}{dh_{N,M,0}} + \frac{Y}{B}\frac{dY}{dh_{N,M,0}} + \frac{Z}{B}\frac{dZ}{dh_{N,M,0}} \tag{E13}$$

Similar expressions for derivatives with respect to $g_{N,M,t}$, $h_{N,M,t}$, $g_{N,M,tt}$, and $h_{N,M,tt}$ can be derived from the equations cited.

Beginning with card 204, the Error program employs Eqs. (E4-E7) to compute the coefficients of the $g_{N,M,0}$ and $h_{N,M,0}$ in the equation for estimating declination. At card 221, the same coefficients in the equation for dip are computed. Then at cards 239 and 256, these coefficients are computed for horizontal and total field respectively. Finally, at cards 270, 279, and 288, the coefficients for Z, X, and Y respectively are processed. Finally, coefficients for $g_{N,M,t}$ and $h_{N,M,t}$ are computed at 296 while coefficients for $g_{N,M,tt}$ and $h_{N,M,tt}$ are computed at card 310.

If external field terms are to be used and corrected, the program, beginning with card 325, computes the coefficients of the unknowns E_1, E_2, and E_3 in the system of simultaneous least squares equations. The equations employed are similar to Eqs. (E4) through (E13) with $\frac{d}{dg_{N,M,0}}$ or $\frac{d}{dh_{N,M,0}}$ being replaced with $\frac{d}{dE_1}$, $\frac{d}{dE_2}$, or $\frac{d}{dE_3}$. Beginning with card 325, $\frac{dX}{dE_1}$, $\frac{dX}{dE_2}$, $\frac{dX}{dE_3}$, $\frac{dZ}{dE_1}$, $\frac{dZ}{dE_2}$, and $\frac{dZ}{dE_3}$ are computed. The quantities $\frac{dY}{dE_1} = 0$, $\frac{dY}{dE_2} = \sin\phi$, and $\frac{dY}{dE_3} = -\cos\phi$ are not set up explicitly. Then beginning at cards 334, 345, 357, and 368 these derivatives are used to compute the coefficients of E_1, E_2, and E_3 for declination, dip, horizontal field and total field, respectively. At cards 378, 386, and 394, the E coefficients for Z, X, and Y respectively are processed.

Cards 402 through 408 compute the sum specified in Eq. (51) of Section 4.0. This concludes the Standard Error Estimation phase of the Error program.

One Point Output

This last phase of Wall's Error is very short, consisting only of cards 414 through 425. The first four of these cards convert declination and dip and their estimated errors from gammas to degrees. This conversion is discussed in Section 4. 0 and specific equations for the conversion are (52) and (53). The remaining eight cards control paging and produce the output of one line.

This concludes Wall's Error program.

GLOSSARY

Wall's Error

A	a, the mean equatorial radius of the earth in kilometers
A2	a^2
A4	a^4
A2B2	$(\text{mean equatorial radius of the earth})^2 - (\text{polar radius of the earth})^2$
A4B4	$(\text{mean equatorial radius of the earth})^4 - (\text{polar radius of the earth})^4$
ALT	altitude of the observation
AOR	radius of the sphere having volume equal to the earth's volume/R
AR	$(\text{radius of the sphere having volume equal to the earth's volume}/R)^{N+1}$
B2	$(\text{polar radius of the earth})^2$
CD	estimated declination in radians
CF	estimated total field strength
CH	estimated horizontal field strength
CI	estimated field dip in radians
CONST	a set of constants required for the generation of the associated Legendre polynomials, P_n^m

$$\frac{(n-1)^2 - m^2}{(2n-1)(2n-3)} = \frac{(N-2)^2 - (M-1)^2}{(2N-3)(2N-5)}$$

n and m are common formula notation while N and M are used in the computer program

COSD	cosine of difference between geodetic coordinate λ and geocentric coordinate θ

CP(M)	cosine of the product of (M-1) and the longitudinal coordinate ϕ
CT	cosine of $\pi/2$ minus the geocentric coordinate θ i. e., colatitude
CX	estimated X component of the field strength
CY	estimated Y component of the field strength
CZ	estimated Z component of the field strength
DEN	$(a^2 \cos^2\lambda + b^2 \sin^2\lambda)^{1/2}$ where a = mean equatorial radius of the earth; b = polar radius of the earth; λ = geodetic coordinate of latitude
DEN2	$(DEN)^2$
DINV	inverse of the D matrix of Jensen's Fit program
DLATT	latitude interval for the potential function grid
DLONG	longitude interval for the potential function grid
DP	derivative of an associated Legendre polynomial
DXDE1	$\frac{dX}{dE_1}$ where X is the X component of the field strength
DXDE2	$\frac{dX}{dE_2}$ where X is the X component of the field strength
DXDE3	$\frac{dX}{dE_3}$ where X is the X component of the field strength
DXDG	$\frac{dX}{dg}$ where X is the X component of the field strength
DXDH	$\frac{dX}{dh}$ where X is the X component of the field strength
DYDG	$\frac{dY}{dg}$ where Y is the Y component of the field strength
DYDH	$\frac{dY}{dh}$ where Y is the Y component of the field strength

DZDE1	$\frac{dZ}{dE_1}$ where Z is the Z component of the field strength
DZDE2	$\frac{dZ}{dE_2}$ where Z is the Z component of the field strength
DZDE3	$\frac{dZ}{dE_3}$ where Z is the Z component of the field strength
DZDG	$\frac{dZ}{dg}$ where Z is the Z component of the field strength
DZDH	$\frac{dZ}{dh}$ where Z is the Z component of the field strength
E1	external field term along the polar axis
E2	external field term in the equatorial and prime meridian planes
E3	external field term in the equatorial plane but perpendicular to the plane of the prime meridian
ELONG	longitudinal coordinate ϕ in degrees
ERR	observations times weights summed for global grid-points
EXTFLD	a code to identify when external field terms are to be used (EXTFLD $\neq 0$) and when they are not to be used (EXTFLD = 0)
F	one of the numbers used to form the sums of squares and cross-products of the triangular matrix
FAC	$\tan\theta / \tan\lambda$ where θ and λ are the geocentric and geodetic latitudinal coordinates
FACT	a factor used in generating the factors for converting from Gauss normalization to Schmidt normalization. When M=2, FACT =2.0; when M>2, FACT = 1.0. Also used in forming squares and cross products. Hence a temporary storage.
FLAT	polar radius of the earth/mean equatorial radius of the earth
FLATR	λ in radians
FLATT	latitudinal coordinate λ
FM	the index M or M-1 in floating point notation

FN	the index N in floating point notation
FNP	FNO1(8) in Jensen's Fit program
FWNP	SWT1(8) in Jensen's Fit program
G	the coefficient $g_{N,M,0}$ in the spherical-harmonic expansion of the geomagnetic potential function
GNM	G(N, M), i. e., a specific G
GT	the coefficient $g_{N,M,t}$ in the spherical-harmonic expansion of the geomagnetic potential function
GTT	the coefficient $g_{N,M,tt}$ in the spherical-harmonic expansion of the geomagnetic potential function
H	the coefficient $h_{N,M,0}$ in the spherical-harmonic expansion of the geomagnetical potential function
HNM	H(N, M), i. e., a specific H
HT	the coefficient $h_{N,M,t}$ in the spherical-harmonic expansion of the geomagnetical potential function
HTT	the coefficient $h_{N,M,tt}$ in the spherical-harmonic expansion of the geomagnetical potential function
I	an index
ITYPE	identifies data type, e. g., DECL = 1 DIP = 2 HOR = 3 B = 4 Z = 5 X = 6 Y = 7
J	an index
LINE	an index for counting the number of lines output for a page

M — an index and subscript

MAXD — maximum size of the triangular matrix D

N — an index and subscript

NMAX — maximum N in the terms of the form $g_{N,M,0} \cos(M-1)\phi$ or $h_{N,M,0} \sin(M-1)\phi$ in the spherical harmonic expansion of the geomagnetic potential function

NMAXT — maximum N in the terms of the form $g_{N,M,t}\, t \cos(M-1)\phi$ or $h_{N,M,t}\, t \sin(M-1)\phi$ in the spherical-harmonic expansion of the geomagnetic potential function

NMAXTT — maximum N in the terms of the form $g_{N,M,tt}\, t^2 \cos(M-1)\phi$ or $h_{N,M,tt}\, t^2 \sin(M-1)\phi$ in the spherical-harmonic expansion of the geomagnetic potential function

NO — first row or column in the least squares equations for terms of the form $g_{N,M,t}\, t \cos(M-1)\phi$ or $h_{N,M,t}\, t \sin(M-1)\phi$

NONOT — first row or column in the least squares equations for terms of the form $g_{N,M,tt}\, t^2 \cos(M-1)\phi$ or $h_{N,M,tt}\, t^2 \sin(M-1)\phi$

NOP — number of parameters in the spherical-harmonic expansion of the geomagnetic potential function

NOPP — number of parameters plus one

NOR — number of rows in the triangular matrix D

P — an associated Legendre polynomial

PI — $\pi = 3.14159265$

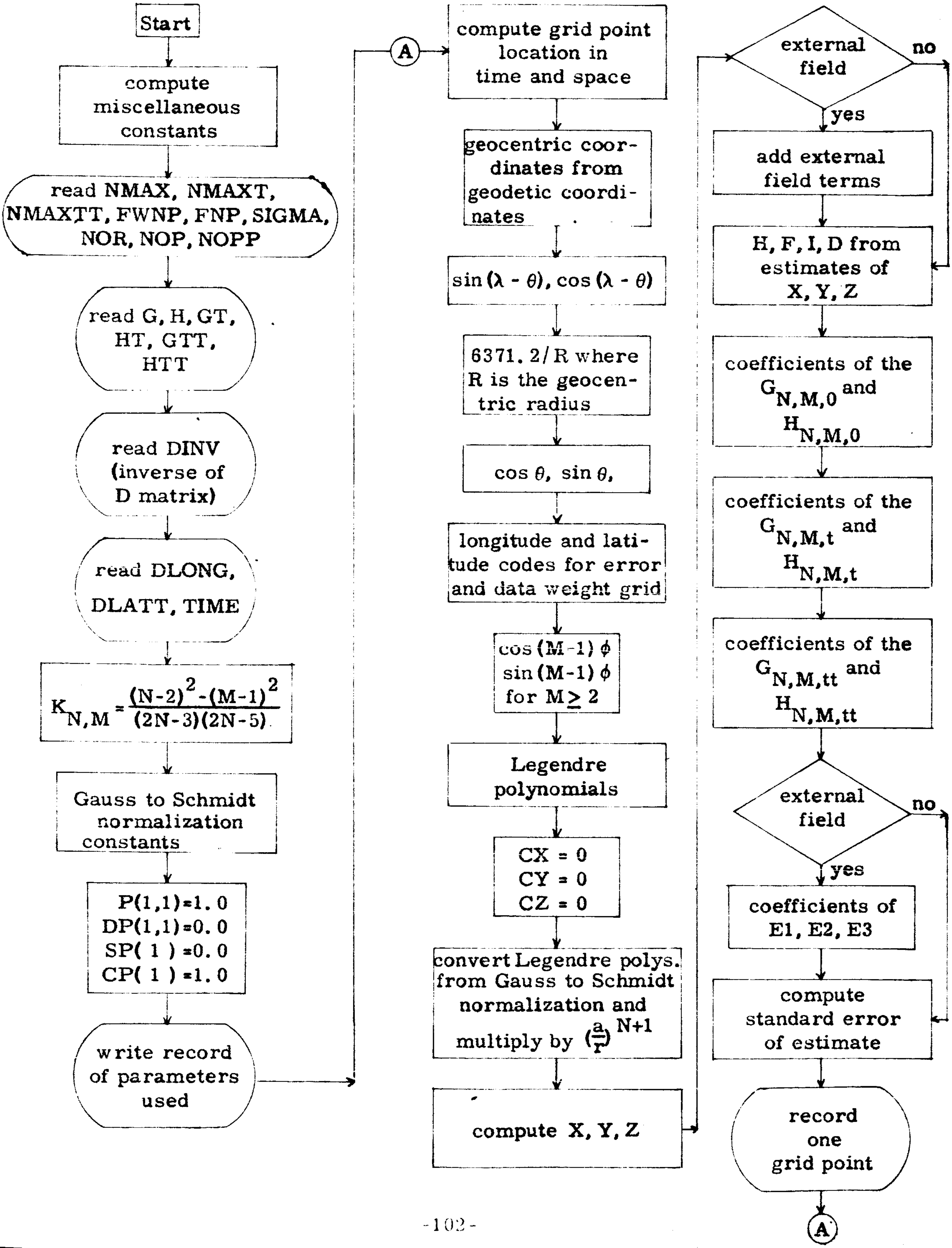
PI2 — $2\pi = 6.28318530$

R — geocentric coordinate of radius

RAD	degrees in one radian = 57.2957795
SHMIDT	constants to convert from Gauss normalization to Schmidt normalization
SIGMA	SIG1(8) of Jensen's Fit program
SIND	sine of difference between geodetic coordinate λ and geocentric coordinate θ
SINLA	sine of the geodetic latitudinal coordinate λ
SINLA2	$(SINLA)^2$
SLATT	starting latitude for grid
SLONG	starting longitude for grid
SP(M)	sine of the product of (M-1) and the longitudinal coordinate ϕ
ST	sine of $\pi/2$ minus the geocentric coordinate θ , i.e., colatitude
SUMD	sum storage for forming check sum column of D matrix
T1	a temporary storage
T2	a temporary storage
T3	a temporary storage
TEMP	a temporary storage
TFACT	time factor
TFACT2	TFACT * TFACT
THETA	the geocentric coordinate of latitude
TIME	time of computed grid
TM	time - 60.0

APPENDIX F

FLOW CHART FOR WALL'S ERROR PROGRAM


Start
compute miscellaneous constants
read NMAX, NMAXT, NMAXTT, FWNP, FNP, SIGMA, NOR, NOP, NOPP
read G, H, GT, HT, GTT, HTT
read DINV (inverse of D matrix)
read DLONG, DLATT, TIME
$K_{N,M} = \frac{(N-2)^2-(M-1)^2}{(2N-3)(2N-5)}$
Gauss to Schmidt normalization constants
P(1,1)=1.0
DP(1,1)=0.0
SP(1)=0.0
CP(1)=1.0
write record of parameters used
A
compute grid point location in time and space
geocentric coordinates from geodetic coordinates
sin (λ - θ), cos (λ - θ)
6371.2/R where R is the geocentric radius
cos θ, sin θ,
longitude and latitude codes for error and data weight grid
cos (M-1) φ
sin (M-1) φ
for M ≥ 2
Legendre polynomials
CX = 0
CY = 0
CZ = 0
convert Legendre polys. from Gauss to Schmidt normalization and multiply by $(\frac{a}{r})^{N+1}$
compute X, Y, Z
external field
no
yes
add external field terms
H, F, I, D from estimates of X, Y, Z
coefficients of the $G_{N,M,0}$ and $H_{N,M,0}$
coefficients of the $G_{N,M,t}$ and $H_{N,M,t}$
coefficients of the $G_{N,M,tt}$ and $H_{N,M,tt}$
external field
no
yes
coefficients of E1, E2, E3
compute standard error of estimate
record one grid point
A

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