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TECHNICAL REPORT WR 64-8

Contract Number NAS8-11312

THE HOT WIRE ANEMOMETER

by

R. C. Potter

GPO PRICE $ ____________

CFSTI PRICE(S) $ ____________

Hard copy (HC) 3.00

Microfiche (MF) .75

ff 653 July 65

WYLE LABORATORIES
TESTING DIVISION, HUNTSVILLE FACILITY

December, 1964

FACILITY FORM 602

N66 30303 (ACCESSION NUMBER)	(THRU)
75 (PAGES)	1 (CODE)
CR-76135 (NASA CR OR TMX OR AD NUMBER)	14 (CATEGORY)

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THE HOT WIRE ANEMOMETER

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Date December, 1964

SUMMARY

A description of the hot wire anemometer and its application to measurements in turbulent flows is given. The method of operation and the basic cooling equations are presented. Actual techniques for construction of the necessary equipment are reported and new techniques for determining the mean flow direction in three dimensional flow are included. An appendix is concerned with the operation of yawed wires and includes experimental results. These are used, in comparison with theoretical values, to determine the accuracy that may be expected when measurements of transverse velocity components are made.

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1.0 INTRODUCTION

The noise produced by a rocket is formed directly by the turbulent mixing of the exhaust gas with the stationary atmosphere. The prediction of the sound field produced is normally found from comparison with the results of similar flows. However it could be obtained completely by a solution of the basic flow pattern and the turbulence produced. This latter method is obviously complicated, and because of the present lack of a general theory of turbulence is in fact insoluble. Wyle Laboratories Research Staff is initiating a program of turbulence measurements in high speed flows which is designed to produce results which will help in determining a basic understanding of turbulent mixing in a high speed shear region, and which will be directly applicable to rocket noise production.

This report discusses the hot wire anemometer as an aerodynamic tool in the study of turbulent flows, and as such serves two purposes. Firstly it forms an introduction to the technique of construction and operation of hot wires, and gives details of the various measurements that can be made. Secondly it presents certain new results. These include the operation of a single wire as a flow direction measuring device in a three dimensional gas flow. Also the case of pairs of wires to measure cross-components of the velocity fluctuations is analyzed, and a technique to produce a linear system is developed including details of the expected accuracy.

The discriptions and analyses given here form the basis of the program using hot wire anemometers to be completed at Wyle Laboratories. This will initially be concerned with developing techniques for accurate calibration of the wires in fluctuating flows for absolute measurements of the turbulence, and will be extended to cover the use of the wires in supersonic flows. This program will include studies of the turbulent mixing regions of supersonic jet and rocket flows, and details of the correlation patterns of the turbulence. The use of the Wyle CDC 3200 computer will allow more complex correlation studies than have previously been attempted.

Of special interest is the turbulence and noise produced by the exhaust flow when it is altered, both by clustering nozzles and deflecting the exhaust stream. This latter case is important since the deflected flow of a rocket on the launch stand is a prime acoustic noise source of loading on the rocket structure. A study of the turbulence formed in such a deflected flow will help in determining the noise generating mechanism of the flow, and in predicting the sound field formed for the large rockets presently being designed.

The hot wire anemometer is probably the most useful tool available for the study of turbulent air flows. Because of its small size the anemometer is capable of following the most rapid velocity fluctuations without greatly disturbing the flow. It will enable measurements of the mean values and also the intensity of the turbulence, as the r.m.s. values of the fluctuating components of velocity. Hot wire anemometers are especially suitable for correlation measurements in turbulent flows, both in the longitudinal and the transverse directions, and in determining the convection and the propagation of the turbulence through the flow. Note that in this report, longitudinal refers to the direction of the mean flow and transverse means normal to this mean flow direction. A complete hot wire anemometer is composed of several components; the wire, the probe, the Wheatstone bridge, the amplifier to drive the bridge, power supplies, compensating and linearizing circuits, and read-out meters and recorders. Each individual part is discussed in detail in this report.

Basically, the system consists of a very small wire forming one arm of a Wheatstone bridge circuit. This wire is heated by passing a current through it so its resistance increases to balance the Wheatstone bridge. When the wire is placed in an airflow, it is cooled by convection and its temperature, and hence resistance, drops. The bridge becomes unbalanced, and this unbalance is a measure of the resistance change, and hence of the velocity.

Consider Figure 1, where E_b is the voltage drop across the bridge, $R_1 = R_2$ and R_w equals the new resistance of the wire. Then $R_w = R_b - \Delta R$ where ΔR is the resistance change, since $R_w = R_b$ when the bridge is balanced. The bridge out of balance voltage e_o, across BD, is then given by

$$e_o = \frac{E_b}{2} - \frac{R_w}{R_w + R_b} \cdot E_b$$

$$= E_b \left[\frac{1}{2} - \frac{(R_b - \Delta R)}{(2R_b - \Delta R)} \right] = E_b \frac{\Delta R}{4R_b - 2\Delta R}$$

and since $\Delta R << R_b$

$$e_o = E_b \frac{\Delta R}{4R_b}$$

and if E_b and R_b are made constant values, then

$$e_o = K\Delta R \tag{1}$$

Hence the out of balance voltage is directly proportional to the resistance change, and the resistance change is a function of the velocity.

The earliest hot-wire anemometer systems worked exactly this way, with a constant current supplied at all times to the bridge, see Figure 2. Then the velocity fluctuations caused the bridge out of balance to vary and the out of balance voltage gave a direct electric signal which was a function of velocity. Such constant current systems were almost universal, but they suffer from frequency limitations. Because of the finite size of the wire itself, high frequency fluctuations in velocity cannot be followed as effectively as lower frequency fluctuations, and this results in a fall-off of the signal as the frequency of the velocity fluctuations, increases. It is accordingly necessary to construct a compensating circuit to allow for this effect. It was realized long ago that a system where the wire resistance is kept constant would be much more efficient. Such a system operates by varying the heating current in accordance with the out of balance of the bridge, and keeps the wire at a constant resistance, which means constant temperature.

The basic lay-out of such a constant temperature system is also shown in Figure 2. The out of balance voltage is fed back to a high gain amplifier which supplies the heating current to the bridge. Because of the very high gain used, such systems are very likely to become unstable and to cause self-oscillations, and for this reason the constant current method was generally preferred. However advances in electronic technique have allowed the constant temperature system to become practicable, and such systems are now, in their turn, becoming more popular (and necessary) for measurements involving frequencies of over 1,000 cycles per second. The system works in the following way. The bridge is balanced by the feed-back system, the wire temperature is raised to that point at which its resistance equals its opposite resistance in the bridge. Then any change in velocity causes the cooling on the wire to change and the bridge to start to go out of balance. The feed-back circuit recognizes this tendency to unbalance and changes the heating current correspondingly to keep the bridge balanced. The output voltage from the amplifier is a function of this change in cooling; that is a function of the velocity change.

The advantages of the constant temperature to the constant current system are as follows:

1) It can be used even when high frequencies occur. In effect the frequency limitation is taken from the hot-wire itself, with a time constant determined by its physical size, to the electronic amplifier circuit. This can then be designed for the highest frequencies expected and so the use of a frequency compensating circuit is not necessary.

2) The signal output of the hot-wire anemometer may be linearized. The voltage output of the set, both for the constant current and the constant temperature systems, will be a function of the velocity. The relationship will be complicated and dependent on many factors, such as the wire size, material, operating temperature and thermal properties. For the constant current system the picture is further complicated because the resistance and the temperature of the wire change, and it is also necessary to add the frequency compensating circuit. Thus, for the constant current system, a simple function generator cannot be used to form a linearized output of the voltage with velocity. However such a system is suitable with the constant temperature method and will be necessary if the intensity of the signals, i.e. the r.m.s. value of the fluctuating components, and the correlation coefficients are needed.

The main disadvantage of the constant temperature system is that it is not so sensitive as the constant current method, and the signal to noise ratio will be less. However, for the turbulent flow in the mixing regions of jets and boundary layers, the values of turbulent intensities are usually very high so that this problem is not at all critical.

The hot wire anemometer is usually described as a velocity measuring device, as its name implies. But in fact it measures mass flow or density times velocity, and only this if the temperature of the flow is kept constant. The temperature and density of cold subsonic flows can normally be assumed constant, and since the cooling of the wire depends also on the diameter of the wire and the viscosity of the fluid, the wire actually measures the Reynolds number of the flow as explained in Section 2.

The wire is calibrated by placing it in a steady airflow, with its axis normal to the flow direction. It is assumed that the calibrations for the fluctuating velocities is identical to the steady velocity result. Since the cooling process could alter with the fluctuating airstream, this could be wrong, but the major difficulty of calibrating for turbulent flows is the creation of a known turbulent stream. Wyle Laboratories is developing a system of calibration which consists of sinusoidally shaking the wire and its probe in a steady airflow. It is hoped that this technique can be extended into supersonic flow regions where the moving shock wave in turbulent supersonic flow may cause other complicating effects.

In operation, a wire positioned normal to the mean airstream gives a good measurement of velocity (times density) in the stream direction and the fluctuating part of the signal is limited to the longitudinal velocity component, provided the intensity of the fluctuating part is less than about 0.20 of the mean value. The mean flow velocity causes the transverse velocity components to have little effect on the resultant velocity over the normal wire; this is explained more fully in Appendix A where actual figures are quoted. In order to measure the transverse velocity fluctuations, it is necessary to use pairs of wires, yawed at equal but opposite angles to the mean flow. The use of sum and difference amplifiers allows the two parts of the resultant signal, due to the longitudinal and transverse velocities, to be separated.

In the next section the analysis of the cooling of the wires is presented and later sections give measuring techniques and some practical examples of manufacturing and operating hot wire anemometers.

2.0 HOT WIRE OPERATION

2.1 Subsonic Flow

The initial analysis of the cooling of cylinders in an airstream is due to King (Reference 1). The heat transferred depends upon the velocity and the physical properties of the fluid, the temperature difference between the fluid and the cylinder or wire, and the physical properties of the wire.

King's relationship was derived theoretically on the assumption of potential flow around the wire and a heat flow distribution along the wire that does not approach the practical case. His relationship can be reduced to:

$$H = (T_w - T_e)\,(k + \sqrt{2\pi k C_p \,\rho\, U\, d}) \qquad (2)$$

where
- H is the heat lost
- T_w is the wire temperature
- T_e is the wire equilibrium temperature, the value it attains in the flow when not heated
- k is the heat conductivity of the fluid
- C_p is the specific heat of the fluid at constant pressure
- ρ is the density of the fluid
- d is the wire diameter

and U is the fluid velocity

This can be non-dimensionalized to produce:

$$Nu = \sqrt{(2/\pi) Pr \; Re} + (1/\pi) \qquad (3)$$

Nu is the Nusselt Number $\quad Nu = \dfrac{\alpha\, d}{k}$ (4)

Pr is the Prandtl Number $\quad Pr = \dfrac{C_p \mu}{k}$ (5)

Re is the Reynolds Number $\quad Re = \dfrac{\rho U d}{\mu}$ (6)

where α is the heat transfer coefficient,
and μ is the viscosity of the fluid.

This equation formed the basis of further analysis and the most generally accepted form now is that due to Kramers (Reference 2). This expression for Heat Lost was derived empirically from a study of a wide range of cylinders at various temperatures and for a number of gases.

$$Nu = 0.42\, Pr^{0.2} + 0.57\, Pr^{0.33}\, Re^{0.50} \tag{7}$$

This relationship has been accepted as applicable over a wide range of Reynolds numbers of the flow. The gas properties must be given at some known temperature, and the use of the 'film' temperature is usually advocated. The film temperature is defined as the mean between the wire temperature T_w, and the gas temperature T_g.

$$T_f = \frac{T_w + T_g}{2} \tag{8}$$

The relationship (7) is derived for long wires, that is when the length is great compared to the diameter. Real wires will always have end effects and the temperature distribution along the wires will not be uniform. Since the hot wire is supported by probes which must necessarily be much greater in mass than the wire for structural reasons, these probes act as heat sinks. Thus a temperature gradient is formed at the ends of the wire and some heat is lost in conduction to the probes. As the velocity increases, and so the cooling, this temperature gradient will become steeper and the heat flow to the probes will increase. This means the expressions for heat lost must be further complicated to allow for this end loss.

Since Prandtl number, for a given gas, has a negligible variation with pressure and temperature, particularly at atmospheric temperatures (Reference 3), equation 7 can be written:

$$Nu = A_1 + B_1 \sqrt{Re} \tag{9}$$

Various authors have attempted to allow for the end correction and provide for the operation of real wires by altering the power of the Reynolds Number, Reference (4).

$$Nu = A_2 + B_2 R_e^x \tag{10}$$

where new values for the constants A and B are chosen.

Values for x as low as 0.3 have been suggested for very short wires; however the actual value of x will depend on the wire physical characteristics and the operating conditions. Therefore when using actual wires, it is essential that all the wires used should have identical properties so it can be assumed that they all have the same response. Then a detailed calibration of only one wire is required and the others can be quickly calibrated by checking out a few points.

Taking equation 7 and equating the heat lost to the power generated in the wire,

$$I^2 R_w = \pi k \ell (T_w - T_g) \left[0.42\, Pr^{0.2} + 0.57\, Pr^{0.33} Re^{0.50} \right] \tag{11}$$

where I is the current through the wire,
R_w is the wire resistance,
and ℓ is the wire length.

It will be necessary to include a conversion coefficient, since the two sides of Equation 11 will be obtained in different units; the LHS will be in watts and the RHS in BTU per second or calories per second depending on the units used.

It is more suitable to keep the wire resistance at a constant value which allows a simple measurement of the voltage to give a direct measure of the heat supplied to the wire. If the gas temperature is constant the film temperature will be constant and the values of the gas properties to be used in the non-dimensional numbers will be constant through a wide range of densities. If the Prandtl number is constant, the heat loss is solely a function of the Reynolds number, and in a varying pressure (density) flow no way of differentiating between density and velocity fluctuations in the measured results is available.

Now

$$I^2 R_w = A_3 + B_3\, Re^{0.5} \text{ for a given wire}$$

$$= A_3 + B_3 \sqrt{\frac{\rho U d}{\mu}} \tag{12}$$

d is fixed and μ will not vary significantly over a wide range of pressures (Reference 5), so that:

$$I^2 R_w = A_3 + B_4 \sqrt{\rho U} \qquad (13)$$

The density and velocity are intimately linked in the Reynolds number term and no manipulation of the basic cooling equation will allow one parameter to be eliminated. Therefore if the density is unknown or fluctuates, the resultant measurement is a direct function of the multiple, of density and velocity, which represents the mass flow.

Because the hot wire anemometer is sensitive to the temperature difference between the wire and the fluid, in a separate way from the sensitivity to the mass flow (velocity and density), it is possible to use the wire to measure the flow temperature. Equation 11 shows that the temperature of the fluid occurs twice in the expression. Once in the temperature difference term and also indirectly in the Reynolds Number. The mass flow term only occurs in the Reynolds number. Simply, if the resultant output of the hot-wire anemometer is E, then it is a function of mass flow and temperature difference.

$$E = S_1 (\rho U) + S_2 T \qquad (14)$$

where S_1 is the sensitivity due to mass flow

S_2 is the sensitivity due to temperature

The use of two wires, or the operation of one wire at two different temperatures, will allow the temperature and mass flow effects to be separated, allowing a measurement of the mean flow temperature. This is more fully explained in Section 3.

When the wire is yawed to the flow the velocity component across the wire will change. Figure 3 represents a yawed wire where θ is the angle between the wire and the flow direction. Then the effective velocity for the wire cooling can be represented as:

$$U_{eff} = U \sin \theta \qquad (15)$$

The normalized heat loss equation can be written as,

$$Nu = A\sqrt{Re \sin\theta} + B \tag{16}$$

In fact an actual hot wire suffers additional effects since the component of velocity along the wire also effects the cooling. When the wire is yawed, the end effects change and the temperature distribution along the wire will become asymmetrical. Hinze in Reference 6 suggests that equation 15 can be modified to,

$$U_{eff}^2 = U^2(\sin^2\theta + a^2\cos^2\theta) \tag{17}$$

where a is some small number

Webster in Reference 7 has made a series of low speed measurements of the response of yawed hot wires and has found that "a" is nearly always equal to 0.2. Sandborne and Laurence, (Reference 8), have made a series of measurements on yawed wires and have expressed their results in the form of a modified heat loss equation.

$$Nu = (A\sqrt{Re \sin\theta} + B)\sin\theta + (C\sqrt{Re\cos\theta} + D)\cos\theta \tag{18}$$

where A, B, C and D are constants.

In the initial analysis, in Section 3.4, to determine the flow direction by rotating the probes, the simple expression of equation 15 is used to determine the way the effective velocity alters as the wire is rotated. Appendix A presents some experimental results of yawing a wire in an air stream and these indicate that for wires with a large length to diameter ratio, equation 15 is a fair representation at angles of θ greater than 30 degrees.

2.2 Supersonic Flow

The whole basis of the hot wire cooling is changed once the flow becomes supersonic, since a shock wave is created in front of the wire, as shown in Figure 4. Kovasznay in Reference 9 and Kovasznay and Tormark in Reference 10 report on a series of hot wire measurements in high velocity air flows. They found some difficulty in calibrating their wires for absolute measurements because of the shock effects. Also because of the difficulty of obtaining theoretical allowances for the compressibility effects, they suggest that most analyses must depend upon experimental results. A major difficulty of supersonic flow analysis is the range of temperatures necessarily introduced by the high speed flow. The flow temperature will range from the stagnation temperature to the stream temperature both ahead and behind the shock wave. Additional effects caused by yawing the wire will further complicate the understanding of the flow problem.

Fortunately Kovasznay's results show that certain trends are apparent, and these allow measurements to be taken in supersonic flow. The results indicate that the heat balance equation can be simplified, and presented in a form of the Nusselt number depending on the Reynolds number and Mach number alone, if the temperature difference is small,

$$Nu = Nu\ (Re, M) \tag{19}$$

In this analysis the length to diameter ratio of the wire is assumed large although some overall end correction may be necessary. The Prandtl number is also taken as a constant. After trying several methods, Kovasznay and Tormark reduced their results to the standard form of the heat balance equation, by taking the properties of the flow parameters at the conditions behind the shock wave.

$$N_u' = A + B\sqrt{R_e'} \tag{20}$$

where suffix ' means the values of the parameters are taken after the shock wave.

$$N'_u = \frac{H}{\pi \Delta T k'} \quad (21)$$

$$R'_e = \frac{\rho' U' d}{\mu'} \quad (22)$$

Since $\rho' U' = \rho U$ the mass flow, where no suffix indicates the free stream conditions, and because the heat conduction coefficient and the viscosity only change slowly with temperature, then the total temperature can be used.

For air, Kovasznay and Tormark further reduce their results to apply at the total conditions of the flow.

$$Nu_o = 0.580 \sqrt{Re_o} - 0.795 \quad (23)$$

where the subscript o means the thermal parameters are taken at total temperature conditions.

This compares to their suggested heat loss formula for incompressible flow,

$$Nu = 0.690 \sqrt{Re} + 0.318 \quad (24)$$

Expressions 23 and 24 are shown in Figure 5, which is extracted from Reference 9.

These results show that hot wires can be used in supersonic flows, but very careful calibration will be necessary to obtain accurate results. The full effect of density variation will have to be particularly examined, as will the effect of yawing the wires.

2.3 Limitations of Density

When the wires are operated in a low density region, the operation can be effected if the mean free path of the gas molecules approaches the order of the wire diameter dimension. This would occur, for instance, in the simulation of altitude in flow models. For hot wires, the diameter of the wire must always be kept much larger than the mean free path, otherwise the cooling expressions, which are for a continuum, will not be applicable. The effects of radiation and heat conduction to the supports become more critical, see Reference 11, and have to be included in the analysis. Fixing an arbitary limit of 10 mean free paths as the smallest diameter that can be tolerated, then wires 0.002 inches in diameter cannot be used at simulated altitudes about 45,000 feet. This problem could be overcome by using larger diameter wires, but then their length must also be increased to keep the length to diameter ratio reasonable. This, in turn, will reduce the resolution which can be obtained in the flow. Therefore the choice of wire size in such models will have to be a compromise between the two requirements.

The Knudsen number for the wire is defined as,

$$Kn = \frac{\lambda}{d} \tag{25}$$

where λ is the mean free path of the gas molecules and d is the wire diameter.

Also $$\frac{M}{Re} = 0.606 \; Kn \tag{26}$$

where M is the Mach number.

This relationship, equation 26, is derived in Reference 12, hence if the Knudsen number has a maximum allowed value of 0.1, and the Mach number of the flow is of order 1, then the Reynolds number of the wire can have a minimum value of order 10. Examination of Figure 5 shows that only a limited range can be covered for the small wire diameter used in References 9 and 10. Therefore the choice of wire dimensions will become critical in these low density flows and it may prove necessary to use different probes in different flow regions.

3.0 MEASURING TECHNIQUES

3.1 Measurement of the Mean Velocity and the Longitudinal Velocity Fluctuations

When the wire is set with its axis perpendicular to the mean flow, the cooling is controlled by the mean flow and the longitudinal velocity fluctuations. If the flow is cold and the Mach Number is sufficiently low for density variations in the flow to be neglected, then a curve of voltage against velocity can be constructed. For a constant temperature system, it will resemble the curve shown in Figure 6. The output of voltage with velocity is non-linear and the voltage has a positive value at zero velocity. For mean velocity measurements, this non-linear result is usually satisfactory, since the values of velocity can be read off directly from the calibration curve. However if a measure of the intensity of the turbulence is required, this will not be a suitable arrangement. The non-linearity will be reflected in the fluctuating output and so the output cannot be fed directly to an r.m.s. voltmeter, to determine the intensity. The voltage output must be corrected to form a linear relationship with velocity. First a potentiometer is used to remove the zero velocity and then a function generator circuit is used to correct the voltage and produce the required linear relationship, as shown in Figure 6. Then r.m.s. voltage measurements can be made directly using a suitable meter. This technique of linearization is most suitable for use with the constant temperature method.

It may also be required to measure the mean velocity when the mean density varies through the flow. If the temperature effects can be assumed negligible or eliminated, then the output voltage is a measure of the mass flow. If other measurements will allow the density to be found, then the velocity can be obtained. The use of a pitot static combination to measure the total and static pressures is one method of finding the mean velocity in such a varying density flow.

From Bernoulli's equation, the relationship between velocity and the flow parameters can be written for an adiabatic flow as

$$1/2\ U^2 + \frac{\gamma}{\gamma - 1}\frac{p}{\rho} = 1/2\ U_1^2 + \frac{\gamma}{\gamma - 1}\frac{p_1}{\rho_1} \tag{27}$$

The values of p, ρ and T where the velocity is brought to zero are the stagnation values and denoted by subscript o. Hence for a given point in the flow.

$$U^2 = \frac{2\gamma}{\gamma - 1}\left(\frac{p_o}{\rho_o} - \frac{p}{\rho}\right) \qquad (28)$$

Using the isentropic relationship between pressure and density, ρ can be eliminated to give the Saint-Venant and Wantzel equation, relating pressure to velocity.

$$U^2 = \frac{2\gamma}{\gamma - 1}\ \frac{p_o}{\rho_o}\left[1 - \left(\frac{p}{p_o}\right)^{\frac{\gamma - 1}{\gamma}}\right] \qquad (29)$$

If however we chose to eliminate ρ_o, equation (28) becomes,

$$U^2 = \frac{2\gamma}{\gamma - 1}\ \frac{p}{\rho}\left[\left(\frac{p_o}{p}\right)^{\frac{\gamma - 1}{\gamma}} - 1\right] \qquad (30)$$

which gives a direct measure of the term ρU^2 if the local static and total pressure are known.

$$\rho U^2 = \frac{2\gamma}{\gamma - 1}\ p\left[\left(\frac{p_o}{p}\right)^{\frac{\gamma - 1}{\gamma}} - 1\right] \qquad (31)$$

This is the compressible version of the fundamental equation relating velocity to difference to total and static pressures.

A pitot static combination will allow p_o the total pressure and p the static pressure to be measured. Equation (31) thus gives a value for ρU^2 which when divided by the value for ρU obtained using the hot wire anemometer, allows U the mean velocity to be obtained. When a mixture of gases is used to create the flow field, for example rocket exhaust gas mixing with the atmosphere, or a substitute gas such as helium is used to represent a rocket exhaust, then the composition of the gas at each point is also required. This is so that a value of the specific heat ratio γ can assigned in Equation 31.

3.2 Measurement of Temperature

As derived in Section 2.1, the wire is sensitive to the mass flow and to the temperature difference.

Equation 14 is repeated here

$$E = S_1 \ (\rho U) \ + \ S_2 T \tag{14}$$

where S_1 is the sensitivity due to mass flow

and S_2 is the sensitivity due to temperature difference (or gas temperature if the wire is held at a constant temperature).

Squaring and taking the mean of the output gives,

$$\overline{E^2} = \overline{S_1^2 \ (\rho U)^2} \ + \ \overline{S_2^2 \ (T^2)} \ - \ 2 S_1 S_2 \ (\overline{\rho U T}) \tag{32}$$

Then if $S_1 << S_2$ the equation can be reduced to an expression which gives the mean temperature difference and hence the mean gas temperature directly. Similarly is $S_1 >> S_2$ the expression reduced to a relationship for $\rho \overline{U^2}$, independent of the temperature.

The sensitivity to velocity can be made small by making the wire temperature equal to the mean temperature of the gas. The mean temperature must be measured and the wire resistance set at its correct value for this temperature. Then the wire is extremely sensitive to any change in temperature and the r.m.s. output can be used to estimate the magnitude of the temperature fluctuations.

An alternative method of measuring temperature is based on the fact that the sensitivities of the wire depend in different ways on the ratio of the wire and gas temperature. Knowing the dependence of the sensitivities on the wire temperature ratio from previous calibrations, the wire can be operated at three different temperatures to obtain values for the velocity fluctuations, the temperature fluctuations and the correlation of the velocity and the temperature. It is often more useful to operate two identical wires at different temperatures and place them, in turn, at the point under consideration. Then the choice of suitable resistances and operating conditions will allow the temperature or the velocity to be obtained from the two outputs.

An example of the use of two wires to eliminate the temperature effect is given by Semenov in Reference 13. He used two elements as part of a bridge circuit, which was controlled by a feedback amplifier. The first wire was short and operated at a very high temperature, the second was longer and its temperature was just below that of the gas stream. This latter wire acted as a temperature correction device and enabled direct measurements of the velocity fluctuations in the cylinder head of an internal combustion engine.

3.3 Measurement of Transverse Velocity Fluctuations

It will be necessary to use pairs of wires to measure transverse velocity fluctuations. This is assuming that the density and the temperature are constant, otherwise the effects discussed above are introduced. The wires are placed in the plane of the mean flow and the desired transverse axis, and inclined at equal but opposite angles to the mean flow; see Figure 7. The instantaneous signal from each wire is then a function of both the instantaneous longitudinal velocity component U_x and the instantaneous transverse velocity component U_y. Addition and subtraction of the signals allows the two parts to be separated, if the two wires are identical in all operating respects. This involves the use of variable amplifiers to set the wire sensitivities equal and necessitates exact positioning in the flow, although this can be accomplished by studying the mean signals produced when the array is rotated. Carefully controlled manufacturing methods for the wires are necessary so that series of identical wires can be produced.

If E_1 is the instantaneous output of the wire 1, then,

$$E_1 = f(U_x) + g(U_y) \tag{33}$$

and if the two wires are equal in all respects except for the angles being opposite, then,

$$E_2 = f(U_x) - g(U_y) \tag{34}$$

Subtraction yields

$$E_{Resultant} = E_1 - E_2 = 2g(U_y) \tag{35}$$

Thus the angle that the probes are set to the mean flow and the function g must be known. Calibration is normally completed by rotating the probes in a steady airstream. Appendix A discusses the yawed wire and also the accuracy that can be expected from this method of measuring the transverse velocities.

3.4 Measurement of the Flow Direction in a Three Dimensional Flow*

An initial analysis to determine flow direction was based on the use of a cross wire probe similar to that used for transverse velocity measurements. The probe would be rotated and the mean signals from the two wires monitored. The two wires will show equal outputs, if they have previously been carefully matched, when the plane of the flow direction bisects the angle of the wires. To determine completely the flow direction, a second such probe will have to be set up in another axis. This method is obviously cumbersome and so the following analysis was completed to see if a single wire would be suitable to determine the flow direction. It should be noted that normally the flow direction is not completely unknown, since it can be estimated from previous measurements or just common sense viewing of the flow. This means that a flow direction determining technique need not cover completely every possible direction.

First consider a single wire as shown in Figure 8a, the probe is in the z axis and the wire is normal to this axis. Then when the probe rotates in the z axis, the wire rotates in the x y plane. The rotation of the probe is given by the angle γ, which is the angle the wire makes with the x axis in the x y plane. If the flow direction is given by the angles ϕ and β as shown in Figure 8a, then it is desired to obtain a value of these angles.

When the probe and wire are rotated, the angle between the flow and the wire, θ , changes. The effective velocity on the wire depends on this angle of yaw and so the output of the set changes. Since the wire is unable to differentiate

* The author is grateful to Dr. M. V. Lowson who suggested the basic analysis of this section.

between obtuse and acute angles, the output of the hot wire set reaches a maximum when θ equals 90 degrees and a minimum when the projected flow direction is in line with the wire. Therefore the angle of the flow in the x y plane, ϕ, can be obtained from the angle of the probe to give the minimum output, since here ϕ equals γ. Or it may be obtained from the position of the maximum outputs, since here ϕ differs from γ by 90 degrees. In all, four points are available to determine this value, although the sense of the flow must be obtained from either flow visuliza-tion or common sense. The value of the angle of elevation of the flow, β, can be found by comparison of the maximum and minimum outputs of the wire. If it can be assumed that the behavior of the wire, when it is yawed in a flow, is exactly like equations 15 or 16, then the angle β is quickly calculated. However in practice, it will be more suitable to obtain an actual calibration of the wire with yaw.

This method will be most sensitive when the ratio of the maximum and minimum values of voltage varies at the greatest rate with change of flow angle of elevation β. This means the system is most sensitive for smaller angles of β, and that the technique is not suitable when β exceeds 60 degrees. Assuming that equation 15 gives a typical wire response and that the set is linearized, then the effective velocity on the wire is given by the sine of θ. Figure 9 shows a typical response for various angles of β, in this case, as the probe is rotated. The obvious disadvantage of this method is that a calibration of the yawed wire is necessary. To avoid this a second possible method is suggested below.

Consider now a wire set at an angle $90 - \alpha$ to the z axis on the probe. The wire angle is now given by γ and α, and let the flow direction be given by ϕ and β as before, see Figure 8b.

The wire direction is ($\cos\alpha \cos\gamma$, $\cos\alpha \sin\gamma$, $\sin\alpha$) and the flow direction is ($\cos\beta\cos\phi$, $\cos\beta \sin\phi$, $\sin\beta$). Then the angle between the flow and the wire, θ, is given by the scalar product of the two dir-ections.

$$\cos\theta = \cos\alpha \cos\gamma \cos\beta \cos\phi + \cos\alpha \sin\gamma \cos\beta \sin\phi + \sin\alpha \sin\beta \qquad (36)$$

Equation 15 gives the effective velocity measured by a yawed wire,

$$U_{eff} = U \sin\theta \qquad (15)$$

Some small changes can be expected to occur in this relationship because of end effects and flow components along the wire, but the above expression will be used here to indicate the overall effects of the analysis. Also experimental results show that equation 15 holds for angles between 30 and 90 degrees, for example see Appendix A. The probe is rotated about the z axis, and the wire is at an angle (90 - α) to this axis; then α is constant but γ changes. The variation of the wire signal with γ is given by,

$$\frac{\partial(\sin\theta)}{\partial\gamma} = \frac{-\cos\theta}{\sqrt{1-\cos^2\theta}} \cdot \frac{\partial(\cos\theta)}{\partial\gamma}$$

Since $\sin\theta = \sqrt{1-\cos^2\theta}$

A maximum or minimum will occur when $\partial(\sin\theta)/\partial\gamma$ equals zero. This is when

$$\cos\theta = 0 \qquad \text{i.e.} \quad \theta = \pi/2 \text{ or } -\pi/2 \tag{37}$$

or

$$\frac{\partial(\cos\theta)}{\partial\gamma} = 0 \qquad \text{i.e.} \quad \gamma = \phi \text{ or } \phi + \pi \tag{38}$$

When $\sin\theta$, the representative wire response, is plotted against $\gamma - \phi$, two types of curve are possible, and these are shown in Figure 10. If the condition given in equation 37 occurs, that is an angle θ of 90 degrees occurs at some point in the rotation of the probe, then a curve like a in Figure 10 results. Here the curve has two maxima and two minima. The minima occur when the projected angle of the wire and the flow are the same, that is equation 38. If the condition of equation 37 does not occur then the result is as curve b in Figure 10. Both the single maximum and the single minimum are given by the condition in equation 38. The previous analysis of the wire normal to the z axis is a special case of this analysis where the angle α equals zero. The angle ϕ is obtained by comparing the angles at that point where the wire gives a minimum output, as before. The value of the angle of elevation of the flow, β, can be obtained by comparison of the position of the maxima with the minima, in the case when the double maxima curve is produced. These double maxima occur at that position where θ equals 90 degrees and hence $\sin\theta$ equals 1.0. The condition for the double maxima to occur for a given flow and wire is,

$$\alpha + \beta < 90$$

$$\text{or} \qquad 90 - \beta > \alpha \qquad (39)$$

To determine β, consider the point where the angle θ equals 90 degrees, then $\cos\theta$ equals zero here.

$$\cos\theta = 0 = \cos\alpha \cos\beta \cos(\gamma - \phi) + \sin\alpha \sin\beta \qquad (40)$$

Divide by $\cos\alpha \cos\beta$

$$\tan\beta = -\cot\alpha \cos(\gamma - \phi)_{\theta = 90} \qquad (41)$$

Then the value of $\gamma - \phi$ when θ equals 90 degrees can be read off the wire output plot by putting $\gamma - \phi$ equal to zero at one of the minimum values. The condition of equation 39 suggests that to cover the greatest range of angle β, the angle α should be made small. The limit would be the normal case as studied before but this would not allow a measure of angle β from the graph. The sensitivity of this method of determining β is decided by the $\cot\alpha$ term in equation 41 and the choice of α must be a compromise. Figure 11 shows a plot of the position of the maximum, as $\gamma - \phi$, where θ equals 90 degrees, against the flow angle β, for various wire angles α. This figure gives the sensitivity of determining β by this method. When α is near zero, a greater range of β is covered, but the sensitivity of the $(\gamma - \phi)$ change with β is small. The choice of wire angle must obviously be a compromise and the suggested values using this method are α equals 30 degrees. This will allow the angles of β up to 50 degrees to be measured with a good degree of accuracy, without any requirement for calibration.

This latter method is based purely on determining the position of the maximum and the minimum points of the hot wire set voltage output. The response of the wire to yaw is not critical and need not be that given in equation 15. If it is somewhat different, or perhaps the set is not linearized, then the output plots will not be like those of Figure 10. However the curve will still show the same basic shape and the maximum and minimum points will occur at the same angles of $(\gamma - \phi)$. Thus this method does not depend on the calibration of the wire.

The major difficulty will be in determining the exact position of the maxima and minima, but by rotating the probe through 360 degrees a completely symmetrical plot will be produced which will help in fixing these points. If double maxima do occur, the value of the voltage at these points will be that appropriate to an unyawed wire.

Some kind of pen recorder is necessary to allow the curve to be directly plotted. Otherwise a large number of results will have to be tabulated and plotted manually. The pen recorder will also allow a check as to whether the result is suitable or not. It may prove possible to fully automate the process, so that curves of β against $(\gamma - \phi)$ are directly plotted. Then a simple table of β against $(\gamma - \phi)$ where γ equals 90 for the appropriate wire angle will allow the flow direction to be read off.

3.5 Correlation Measurements in Turbulent Flow

Measurements, using two wires, with the probes separated to be in different parts of the flow, are used to construct the various correlation coeficients of the turbulence. Recordings of the signal from the hot wire probes, or the direct signals, are played into a correlator to construct the space correlations. If the mean flow is in the direction of the 1 axis, then normal wires in the 2 axis record the longitudinal velocity fluctuations. By placing the wires one behind the other in the flow at various separations, the longitudinal space correlation R_{11} can be constructed. It is usual to normalize the results by dividing by the root mean squares of the signals to present a correlation coefficient of 1 for maximum correlation. The integral of the longitudinal space correlation is used to obtain the longitudinal integral eddy scale which is defined as

$$L_{11} = \int_0^\infty R_{11} \, dx$$

and represents the energy bearing eddies of the turbulent flow.

By placing the probes at various separations across the flow, the radial correlation R_{12} is estimated, and used to determine a radial scale L_{12}. These values of scale are used to estimate the correlation volume.

By correlating the signal from a single probe with itself at various time delays, the *autocorrelation function* of the turbulence fluctuations, R_τ, is obtained. This is related to the spectral density of the turbulence fluctuations by a Fourier transform relationship.

$$R_\tau = \int_0^\infty w(f) \cdot \cos 2\pi f \tau \cdot df$$

and (42)

$$w(f) = 4 \int_0^\infty R_\tau \cdot \cos 2\pi f \tau \cdot d\tau$$

where $w(f)$ is the power spectral density function.

The fixed axis spectral density is thus easily obtained from the autocorrelation, or may be found directly by narrow band filter analysis of the hot wire signal.

A further important parameter in the study of turbulence in the presence of a mean shear is the convection speed of the turbulence. This is the speed at which an observer would see the least rate of change of the turbulent eddies. It can be obtained from a series of hot wire measurements using sets of probes spaced at various distances apart in the mean flow direction. These recordings are correlated, in turn, with the leading probe signal, as for space correlations but with a variable time delay, and the maximum peak of correlation plotted against time delay. All the results are plotted on one graph and the envelope of the various cross correlation curves is drawn. The points where this curve touches the correlation curves are then plotted for the various separations and time delays and a convection speed is obtained. The moving axis time scale is then defined as the time lag at which the correlation decreases to a given fraction of its maximum value. Figure 12 shows a typical example taken from Reference 14.

From Lighthill's hypothesis, where the flow is considered as being composed of a series of eddy volumes with unit correlation within each, it should be possible to estimate the acoustic power output of each volume, using measurements made at various points in the flow. These measurements are taken as typical of the values within each volume. In calculating the acoustic power output from the experimental results, care must be taken to choose an expression for which the required terms can easily be measured. The possibility of working in terms of a representative frequency is limited, since some kind of a scale must first be measured, and it is usually easier to use this scale value directly. The probes that take the measurements are operated at fixed points in the flow, since using them in a moving axis framework would naturally be rather complicated. However the use of two probes, separated in space, enables some measurements to be made in the moving axis frame, but these are usually limited to the longitudinal velocity fluctuations. To measure transverse components and correlations it is necessary to use an arrangement of two sets of multiple-wire probes, with all the corresponding complications.

4.0 MATERIALS AND CONSTRUCTION

4.1 The Wires

Anemometer wires are usually made from tungsten or platinum; while for hot flows an alloy of platinum and iridium is used. For turbulence work the wire diameter is restricted to the smallest practical size, and tungsten wires as small as 0.00015 inches in diameter have been used. Such small wires must be operated in a clean dry airflow since a small speck of dust or drop of water vapor will be capable of breaking the wire. In a recirculatory wind tunnel this air cleanliness is produced by fitting screens in the tunnel return circuit and letting the air be circulated many times before the wires are placed in the flow. The wire must have a suitable resistance at its operating condition to allow the bridge resistors to be chosen, and Figure 13 shows typical resistances for wires of 0.40 inches long at the given operating temperatures.

The wire is soldered or welded across two stiff high-conducting supports, which are often pointed at their ends, and designed to create the minimum disturbance to the flow. The wires are usually copper plated at the ends for two reasons. The first is to allow the wire to be soldered to the probe and the second is to precisely control the working length, which is the unplated section. If the soldered joint is considered to be too weak for cases of high aerodynamic loading, the wires can be spot welded to the probes for a firmer and more reliable joint. Two techniques of producing the required wire are available, the first is to take a completely plated wire and to etch the central working length clean, or the alternative is to plate the wire in such a manner as to leave the required working length bare.

In the first method the wire is either drawn with a high conducting outer coat, a technique that enables very fine wire to be drawn, or it is plated in a bath before hand. Then the wire is positioned under a buret from which the electrolyte falls in a steady stream into a beaker. The wire is placed in the stream and its width determines the working length to be etched away. The current is applied to the electroyte and the wire and the outer covering over the working part is removed. Figure 14 shows the experimental set up. This technique is most suitable when the wire is obtained already plated. The second technique, of plating the wires leaving the center portion bare affords greater control and is more suitable for producing a large number of wires all with identical characteristics.

One method of plating the wires consists of winding unplated tungsten wire around a frame and then positioning this frame so that the wires pass through the menisci of two parallel baths of copper sulfate solution. The width of the barrier between the two baths determines the unplated working length formed when a current is applied to the system. Copper is deposited on the wire in those regions that are immersed in the copper sulfate solution. A sketch of the bath is shown in Figure 15. By plating the wires in batches of 20 to 50, exactly equal wires should be produced. The electrical connections to the wires are made by squeezing the wires between two copper plates, and the connection to the electrolyte is through a piece of copper in the bottom of the baths. The two tanks are joined and the whole system is levelled to ensure an even meniscui in both baths. The wire is cleaned with carbon tetrachloride fumes, both before and after the plating process. Cleaning dust off wires after they have been used for sometime can be difficult. However once a technique of manufacturing large quantities of wires quickly has been attained, then cleaning is not necessary, since the wires can be discarded. The elimination of cleaning used wires should save time and help to avoid disruptions in the experimental program through wire breakages due to damage caused by cleaning.

4.2 The Probes and Techniques for Soldering the Wire to the Probe

The probe consists of a body containing two relatively heavy gage wires. These extend from the body a short distance and are normally tapered with a small flat at the end to take the hot wire element. They can be most suitably made from theomocouple tube with both wires made of the same highconducting stiff material. Phosphor bronze is a satisfactory material for these supporting wires. The key to the technique of soldering the wire onto the probes is the ability to place and hold the wire accurately in place across the prongs of the probe. The support wires are already tinned and a simple traversing apparatus is used to bring the wire up to the probe which is firmly held. The wire is positioned across the tips of the probe wires and soldering iron is applied to the latter wires, causing the solder to run and fix the hot wire in place. It obviously is essential that all parts involved, the hot wires and the tips, should be thoroughly cleaned before the operation is attempted. An alternative method has the probe fixed vertically in the work surface with the probe wires just jutting up out of the surface, at a predetermined height. Then the frame holding the hot wire is placed over the probe so the wire lies across the supports, just in tension. Application of the soldering iron to the probe wires as before completes the attachment. The wires are then broken from the frame and trimmed close to the probe, the small loose ends can be turned back, to cause the minimum disturbance, if required. The soldering technique needs a steady hand, but practice is the major requirement.

4.3 The Amplifier and Bridge

The Wheatstone bridge consists of 3 resistors and the wire as shown in Figure 16.

R_1 and R_2 are normally made equal and some 7 to 10 times greater than R_3 which is equal to the resistance of the wire at its operating temperature. R_3 can be made a variable resistor to allow the bridge to accept a range of wires, although if this wire resistance changes greatly it will be necessary to change R_1 and R_2 also. By placing a potentiometer across the bridge drive lines it is possible to alter the sensitivity of the circuit to enable two sets to be matched. This causes no change to the operating conditions and is a simple way of selecting a desired fraction of the output voltage.

The first successful constant temperature amplifier is due to Ossofsky (Reference 15) and many successful devices have since been built. NACA, at Lewis Research Center, have been responsible for the development of a whole series of hot wire anemometer equipment, (References 4 and 16). A fully transistorized design has recently been developed at the University of Southampton, England, and the circuit is given in Reference 17.

For intensity measurements a linearizer is required, and for a constant temperature set, this will comprise a simple function generator. For wires of similar size and operating conditions, the voltage output function will be the same in all cases and a single circuit should be sufficient to produce the required relationship. A variable amplifier can be included to allow slight differences to be corrected and also to give a standard signal of voltage with velocity.

5.0 CONCLUSION

This report has presented a brief description of hot wire anemometers, their method of operation, limitations and of some of the possible measurements that can be completed in turbulent flow. This will represent a foundation for further work at Wyle Laboratories concerned with producing calibration techniques for wires in supersonic flow, and applications of the wires to study of the turbulence occuring in jets and separated boundary layer flow.

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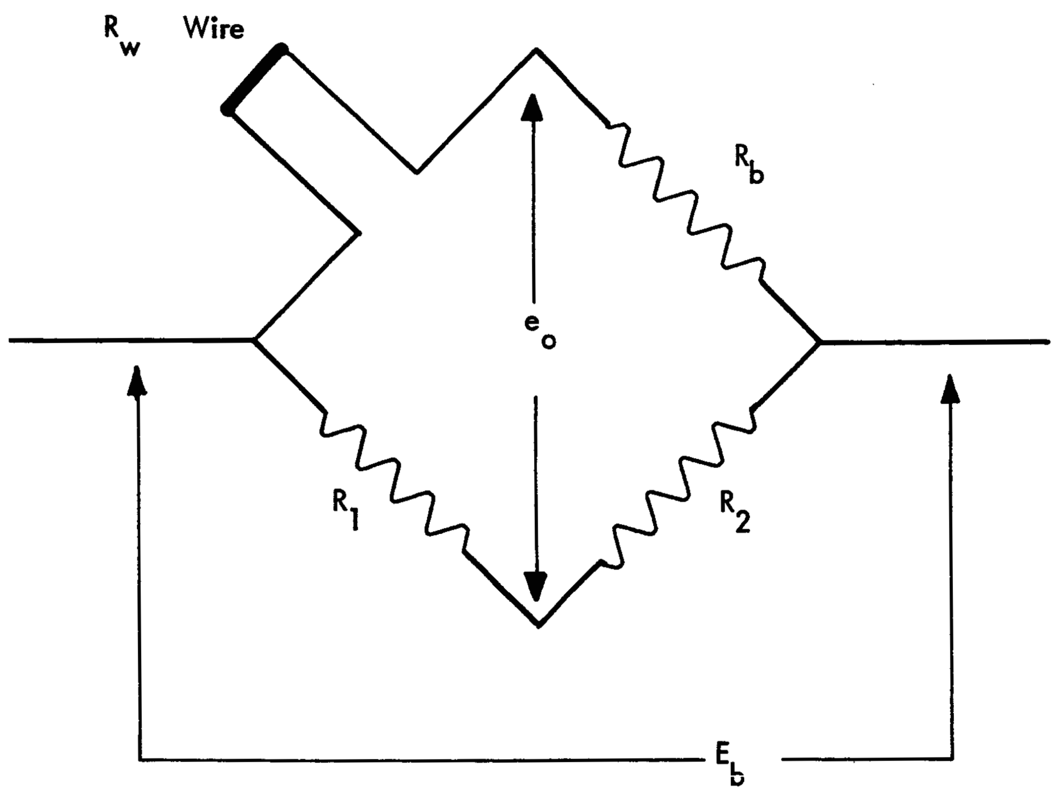
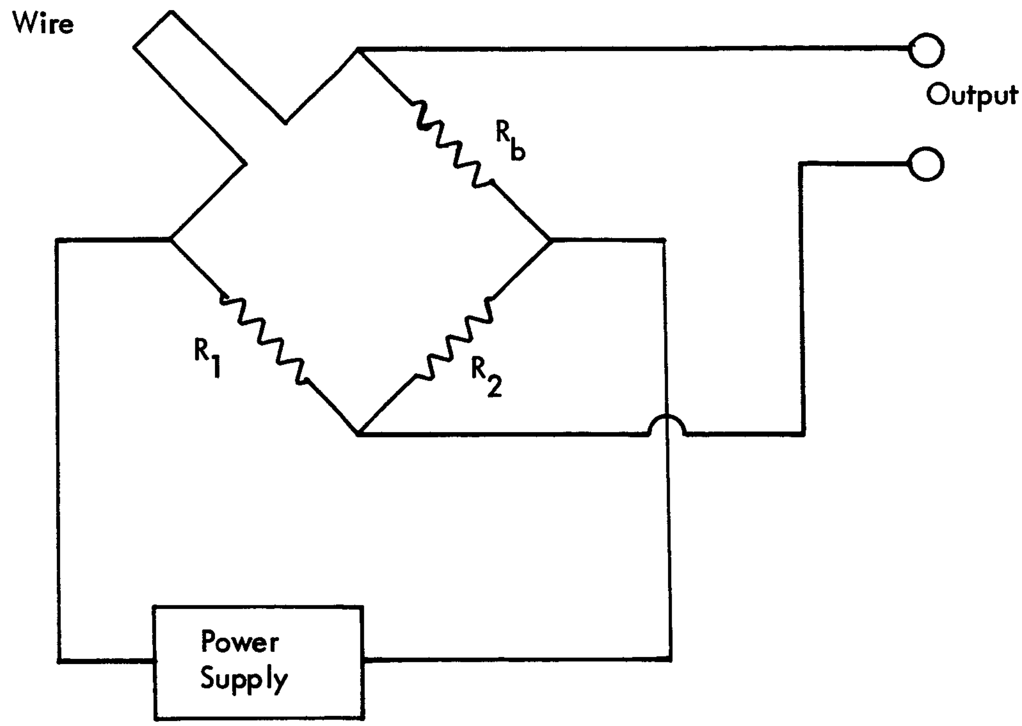
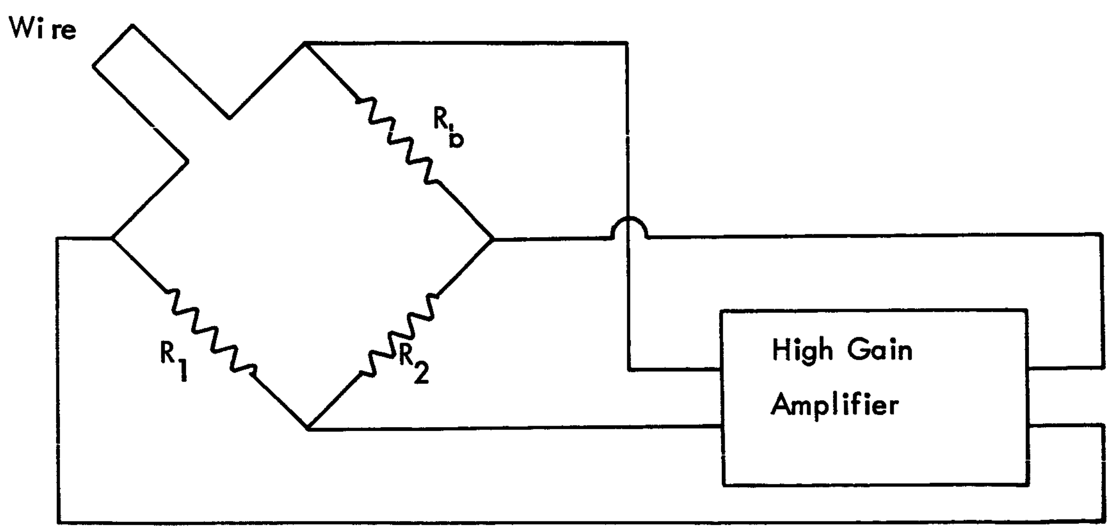


Figure 1 Hot Wire Bridge



Constant Current Anemometer



Constant Temperature Anemometer

Figure 2 Types of Hot Wire Anemometer

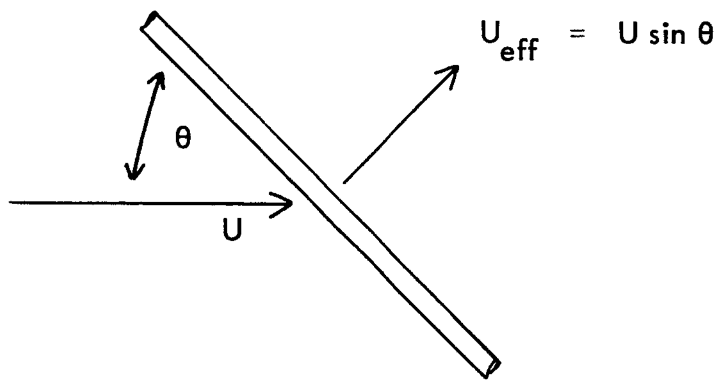


Figure 3 Yawed Hot Wire

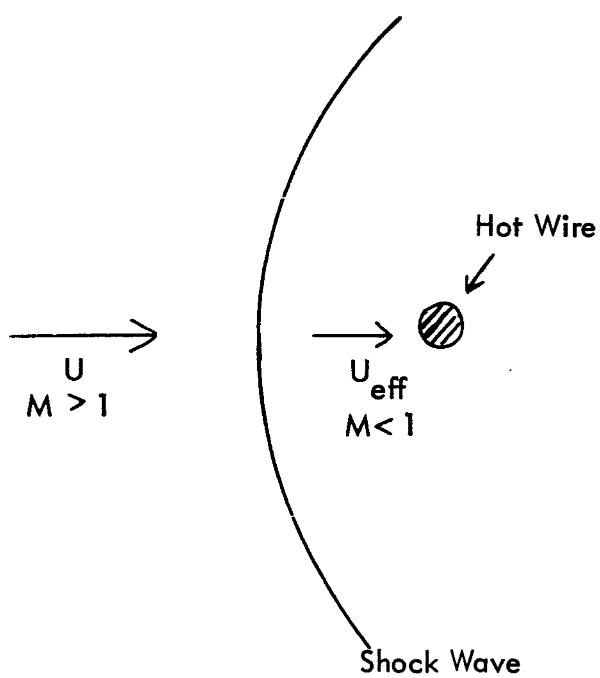


Figure 4 Hot Wire in Supersonic Flow

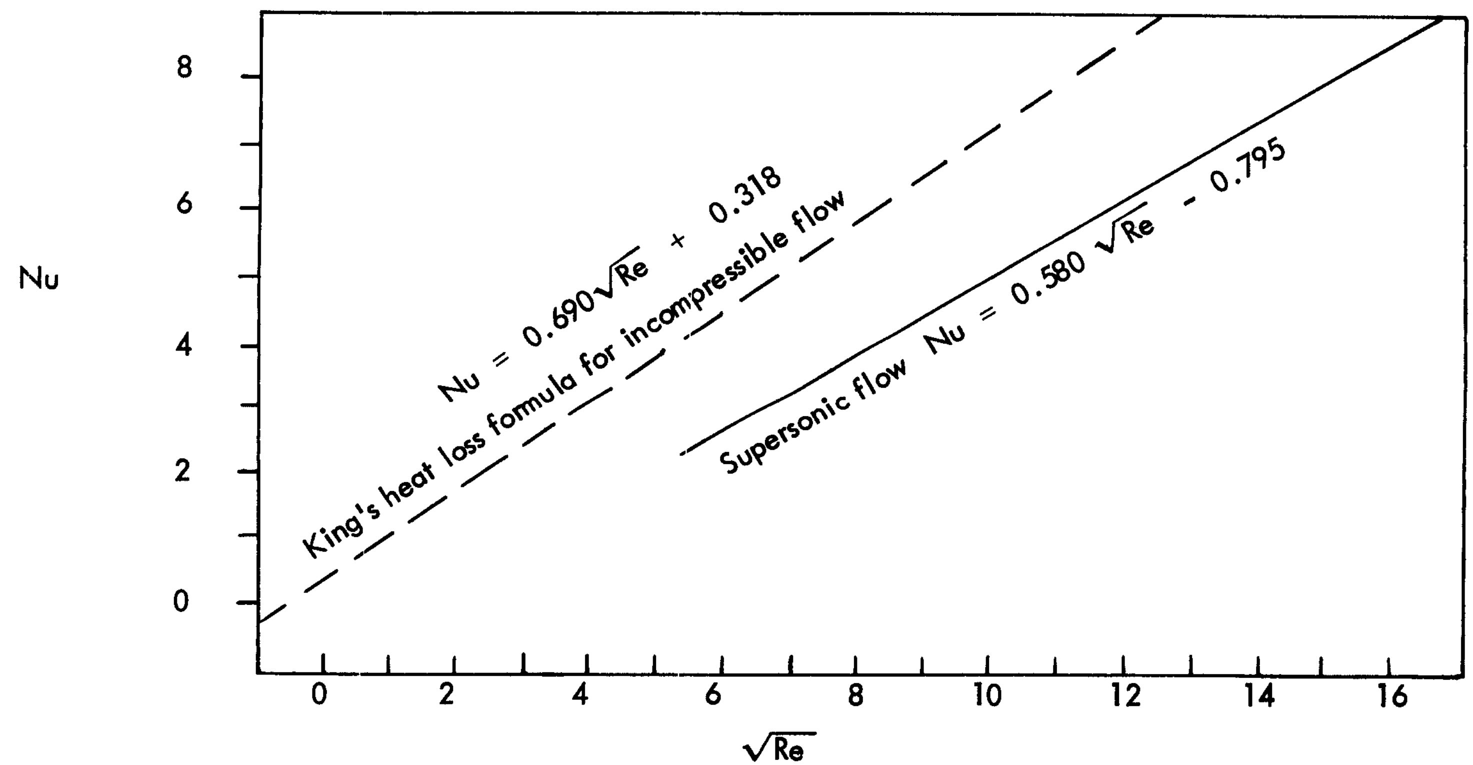


Figure 5 Heat Loss of Wire in Supersonic Flow (from Reference 9)

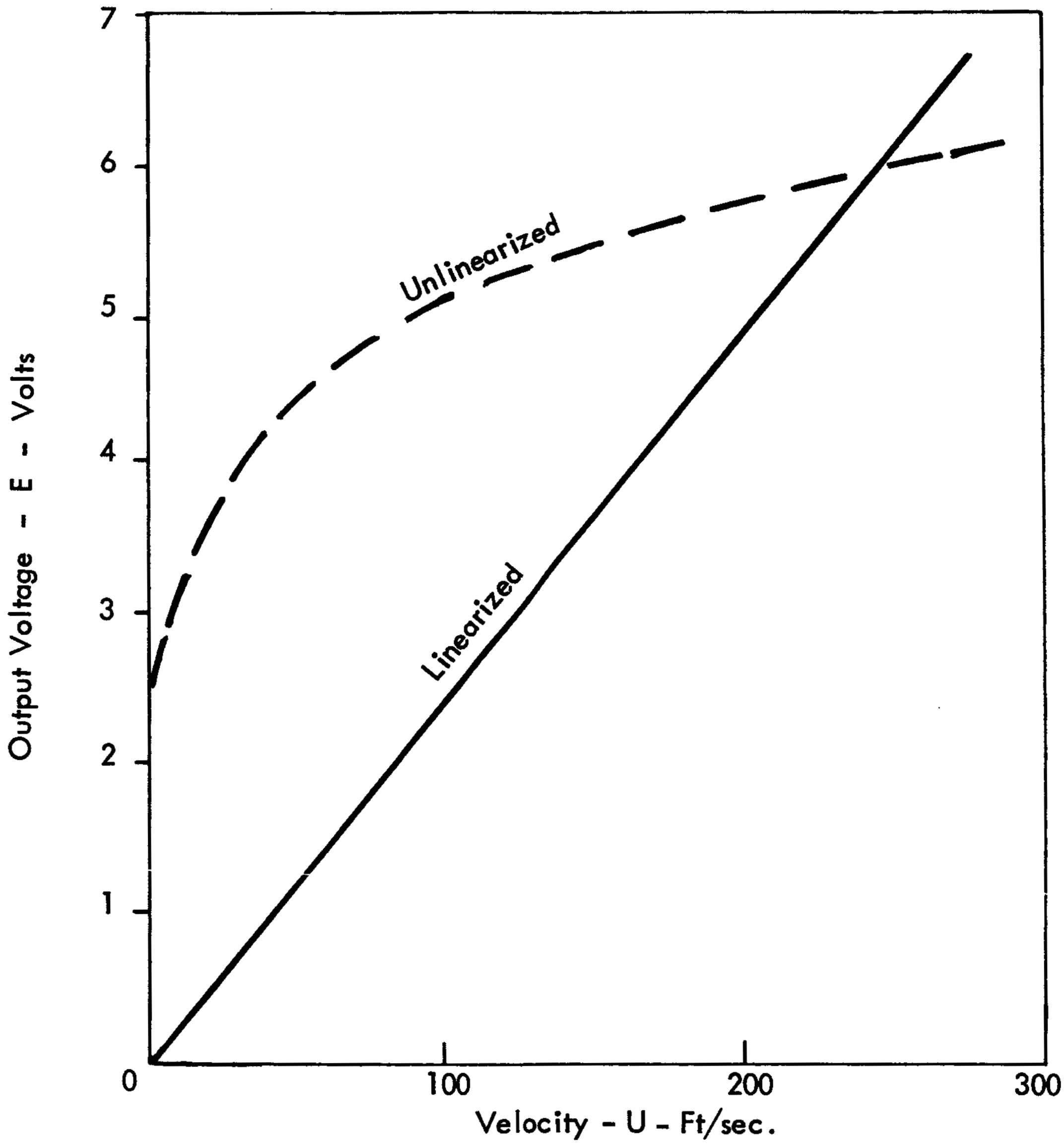
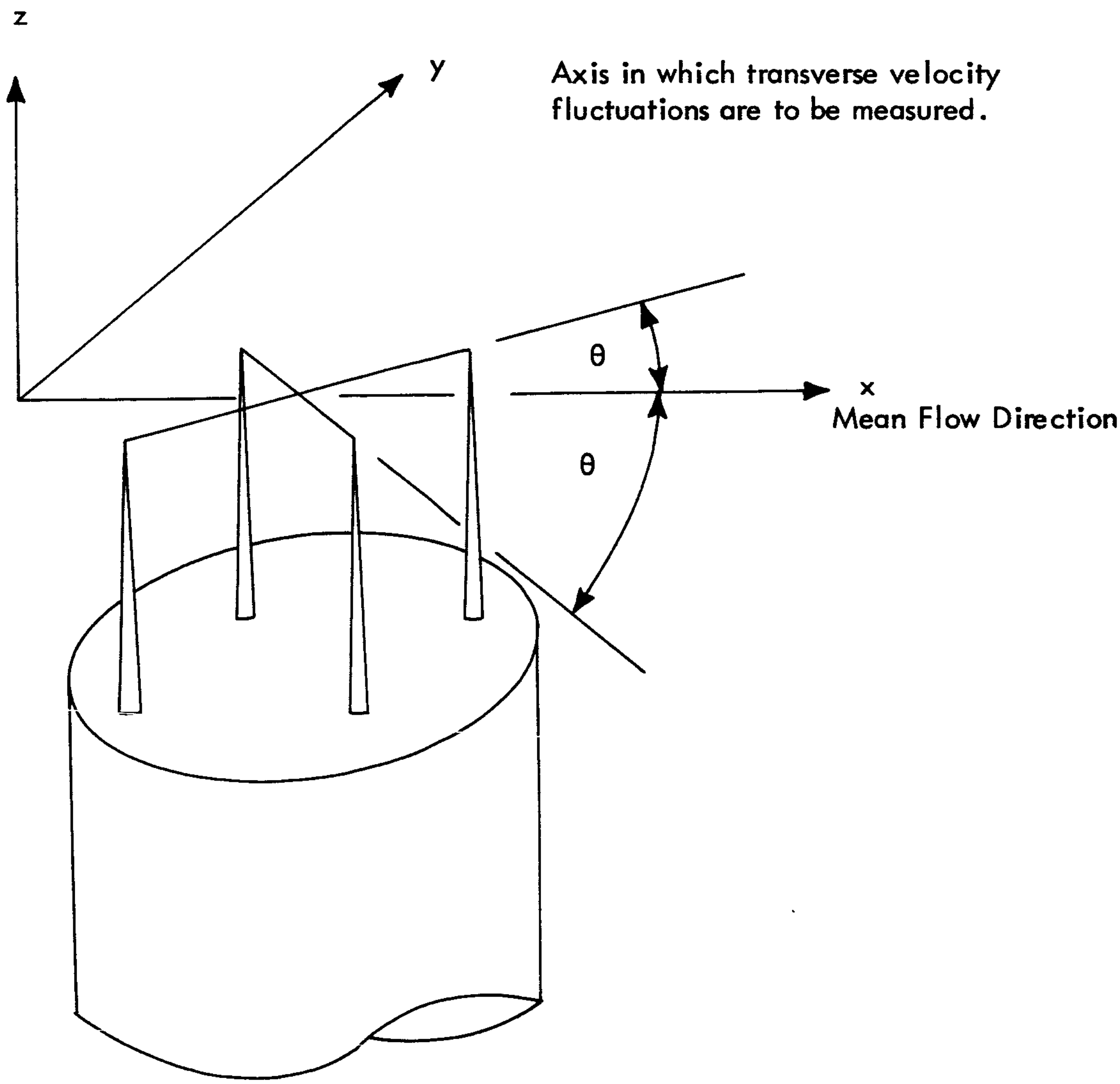


Figure 6 Typical Constant Temperature Anemometer Voltage Output with Velocity



The two wires are set at equal angles to the mean flow direction

Figure 7 Pair of Yawed Wires, Measurement of Transverse Velocity Fluctuations

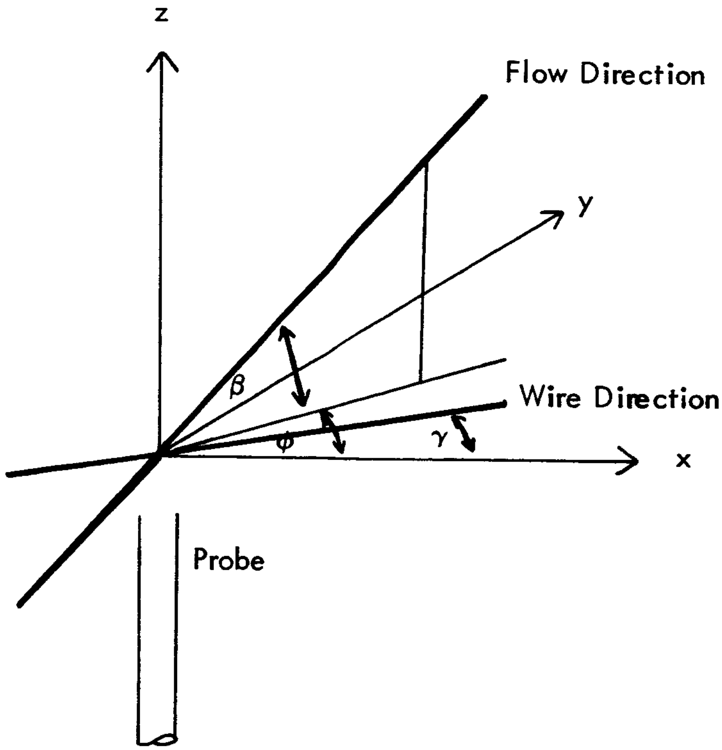


Figure 8a   Normal Wire Alignment

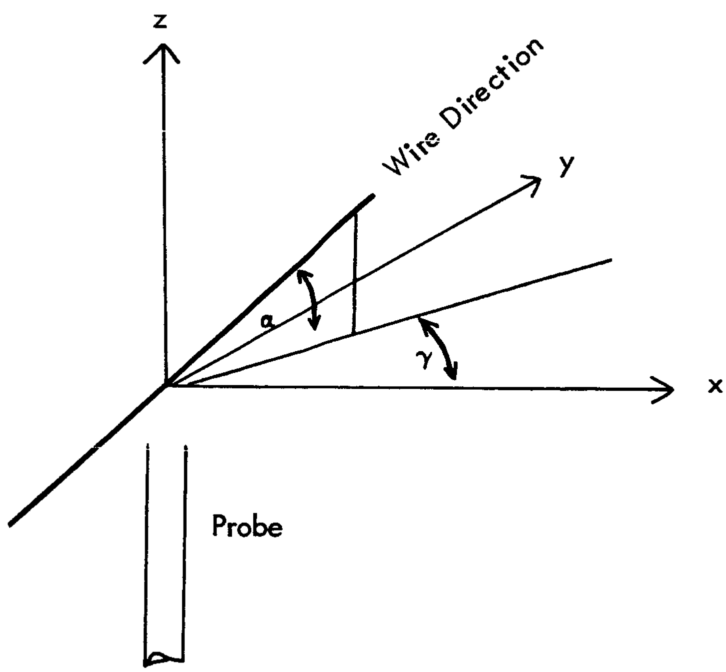


Figure 8b   Single Wire Alignment

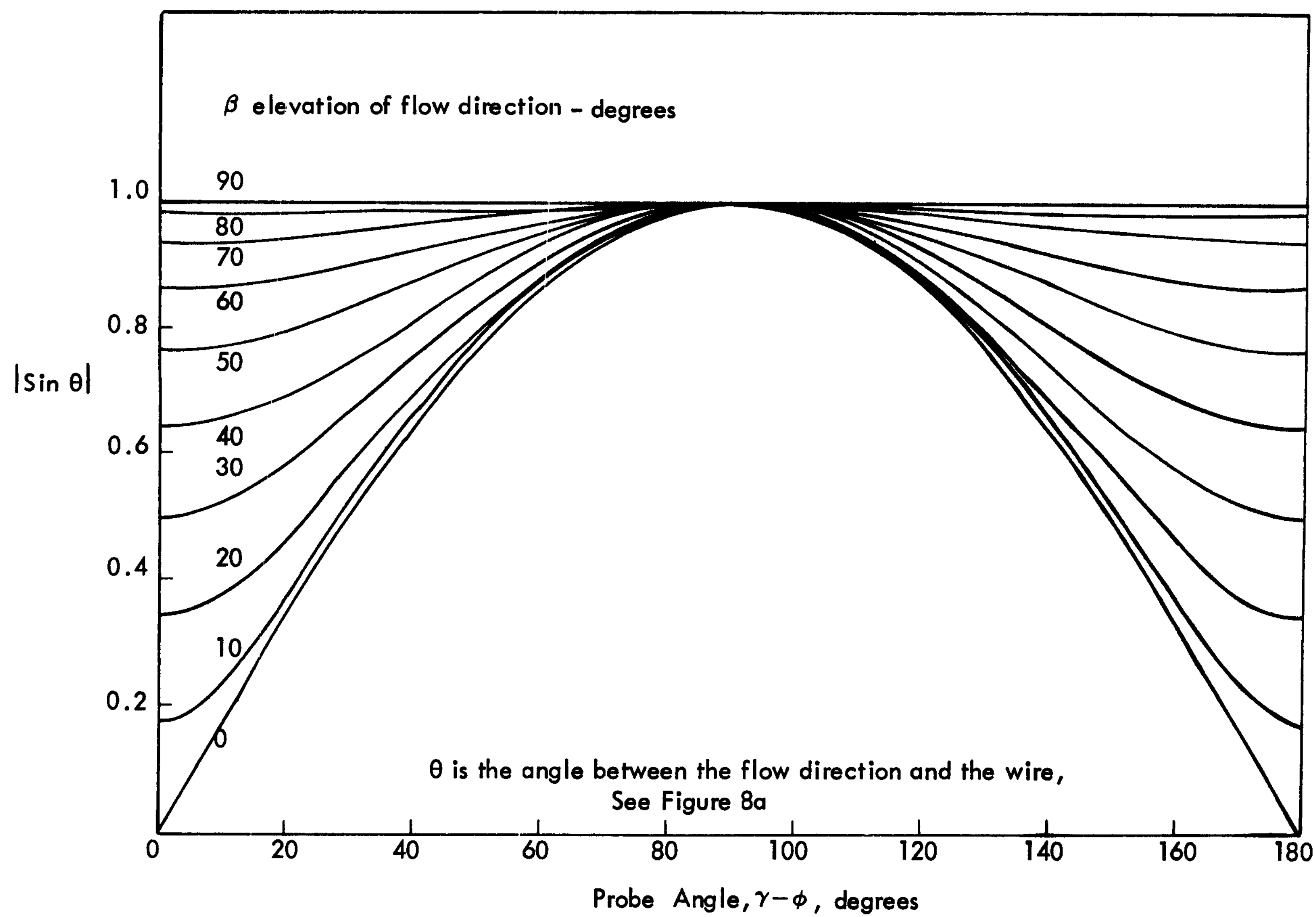


Figure 9 Response of Normal Wire, when Rotated in a Flow of Unknown Direction

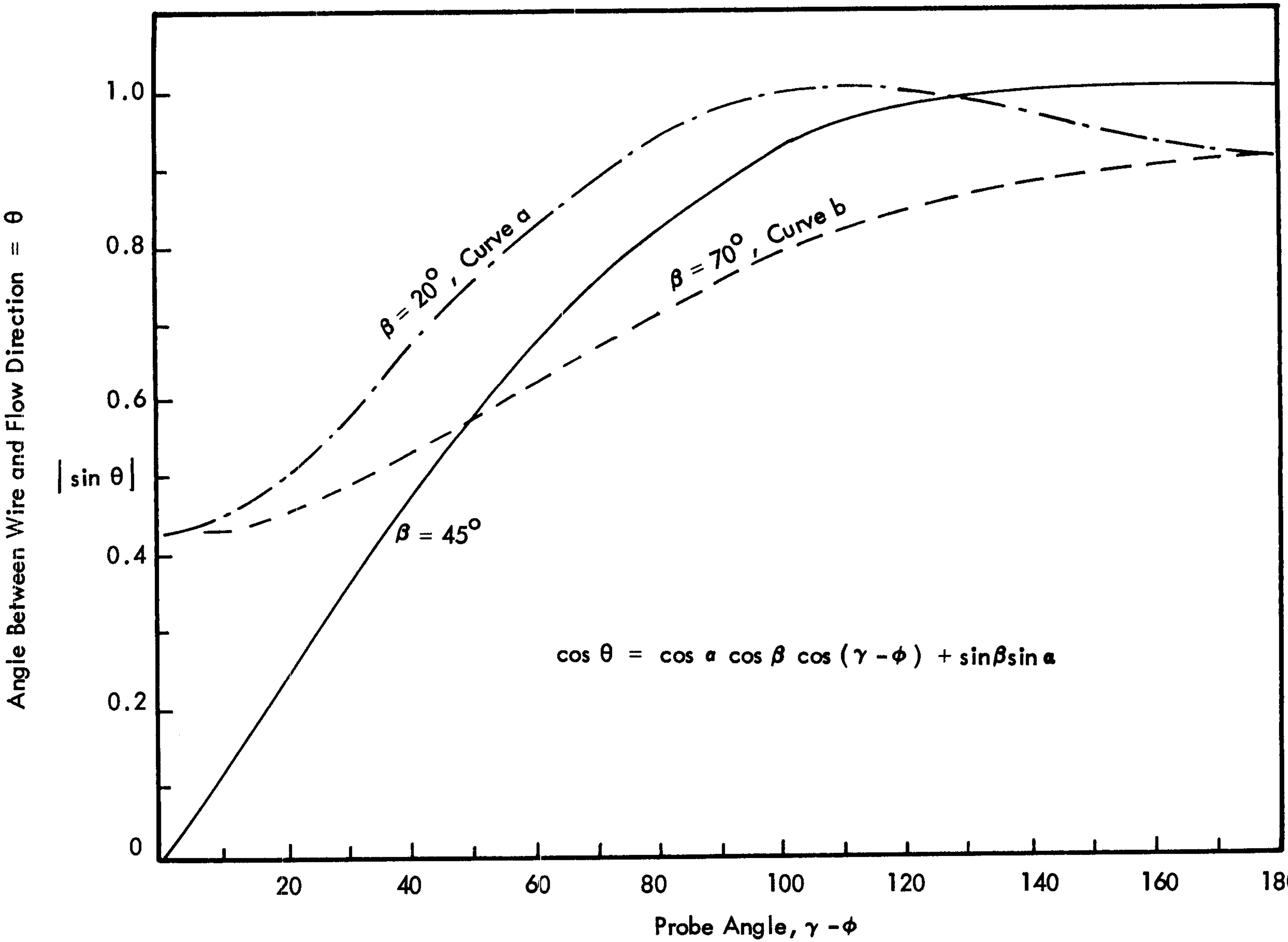


Figure 10 Response of Yawed Wire, $\alpha = 45^{\circ}$, when Rotated in a Flow of Unknown Direction

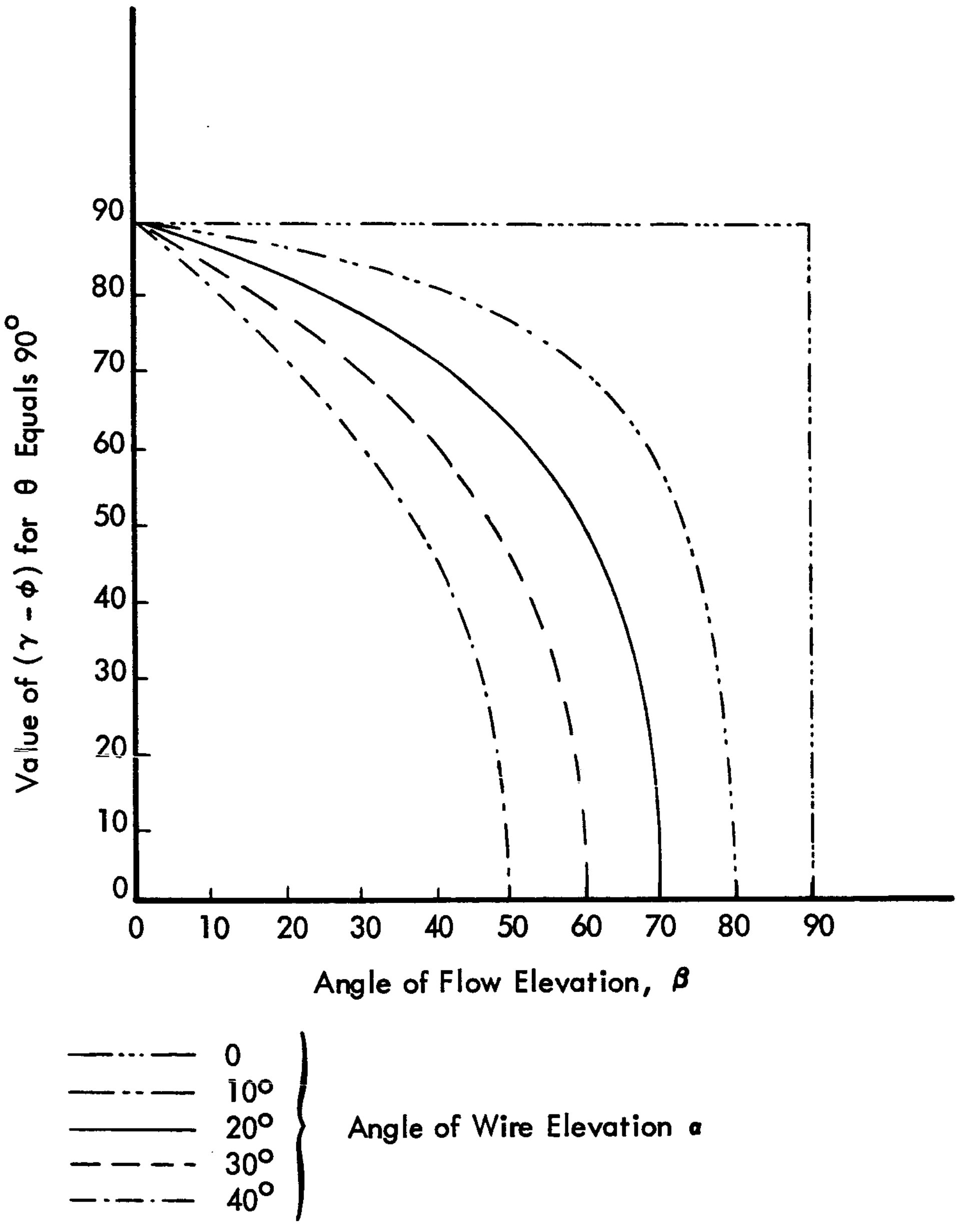
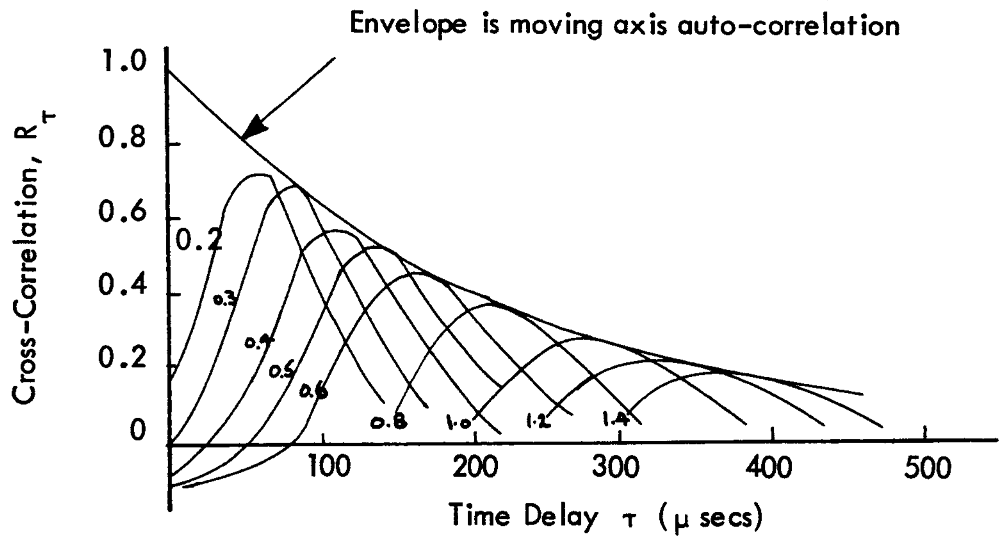
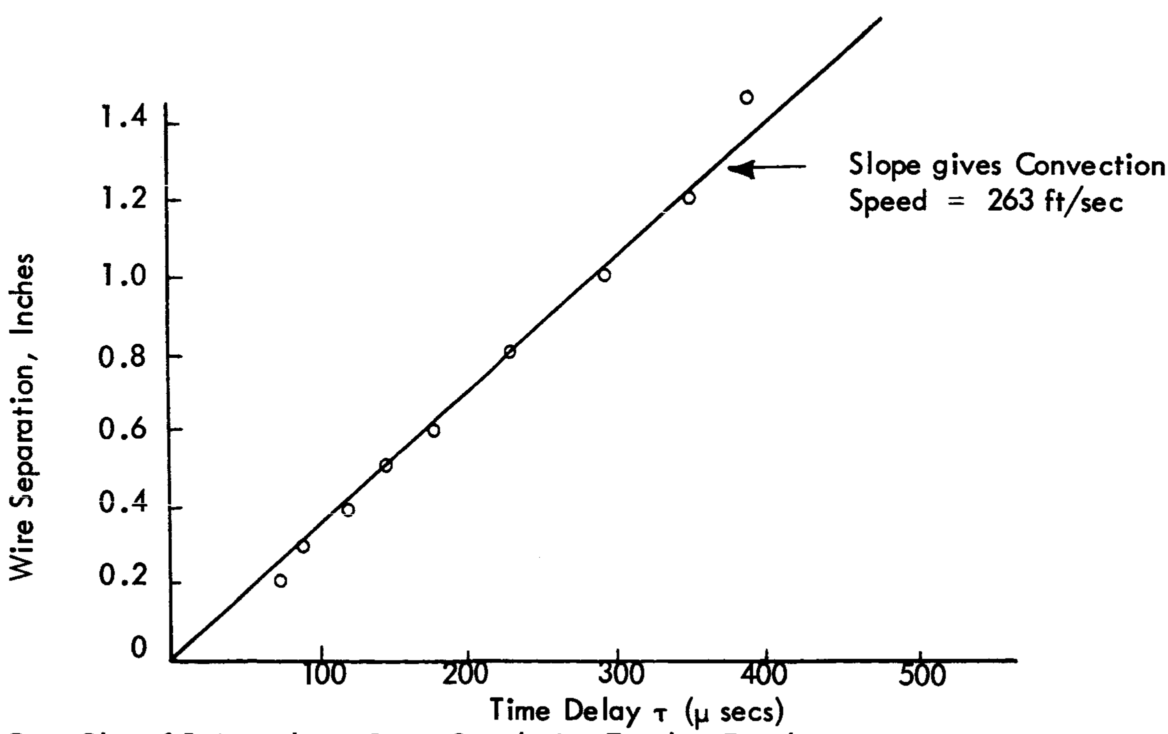


Figure 11 Sensitivity of Angle of Flow Elevation, β, Determination. $(\gamma - \phi)$ against β when $\theta = 90^{\circ}$

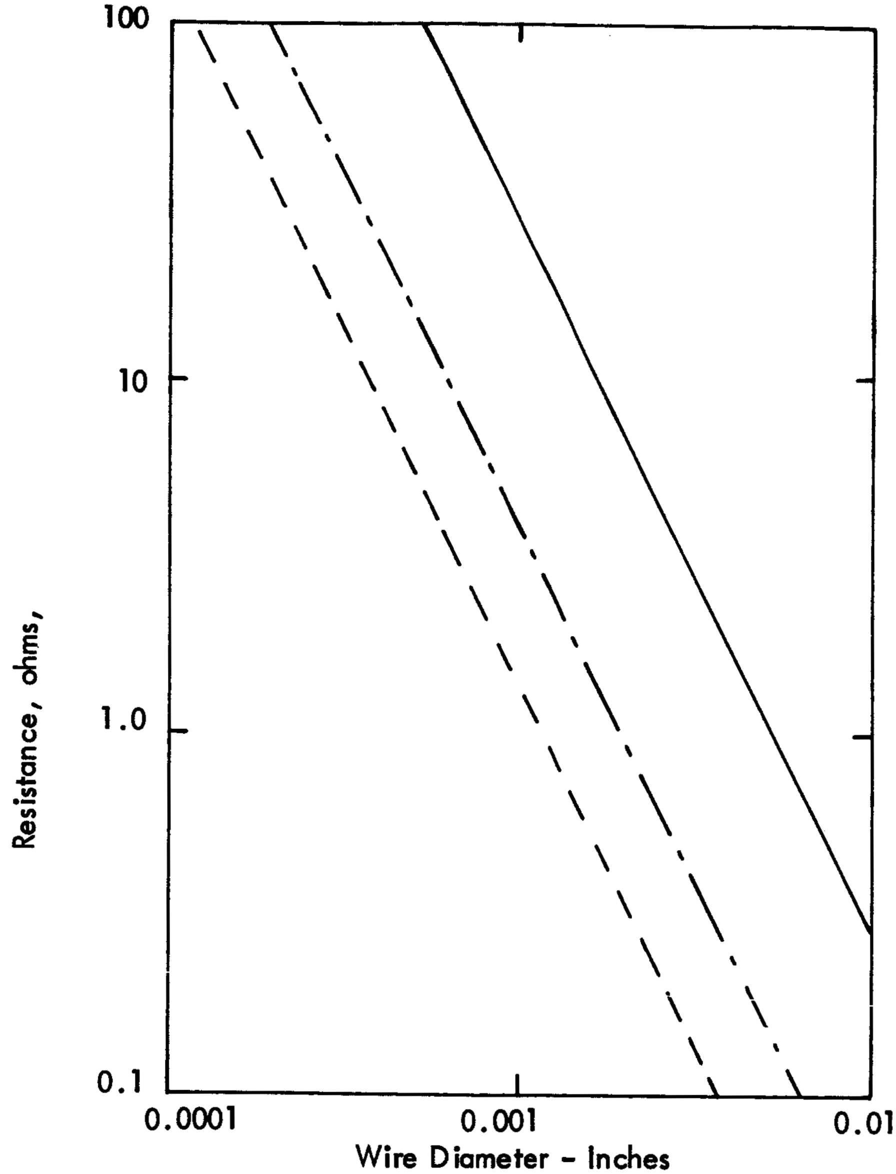


Cross- Correlation of Axial Velocity Fluctuations with Downstream Wire Separation in Mixing Region of 1 inch diameter Air Jet



Cross Plot of Points where Cross-Correlation Touches Envelope

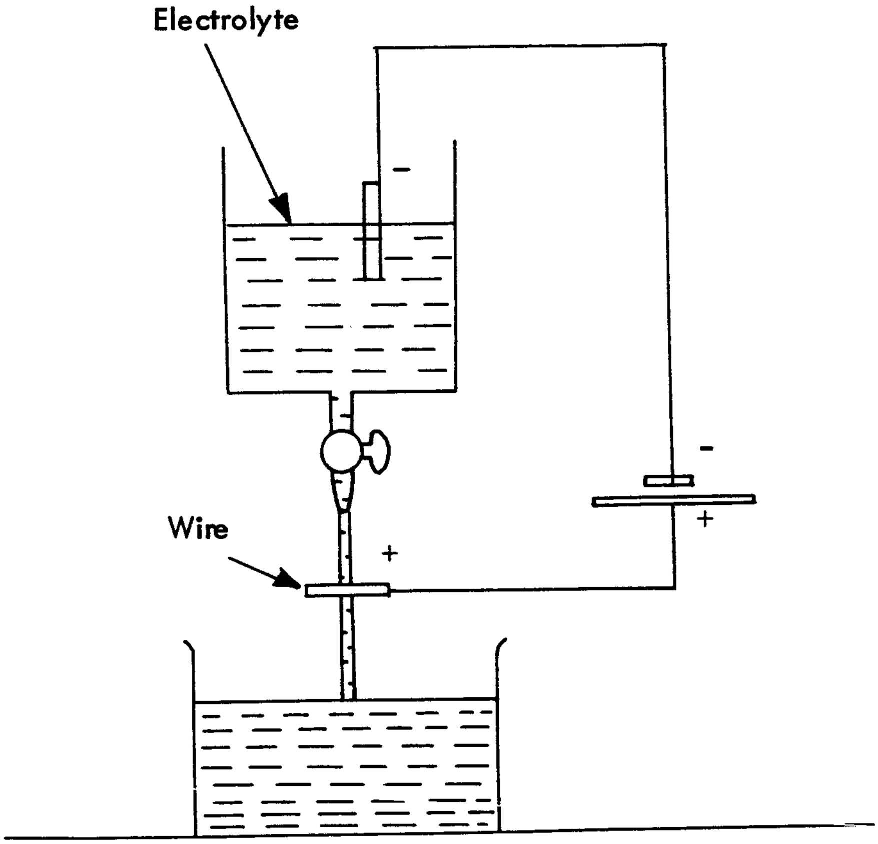
Figure 12 Determining Convection Speed of Turbulence (Results from Reference 14)



——— Platinum-Iridium at 1000° C
—·— Platinum at 300° C
– – – Tungsten at 300° C

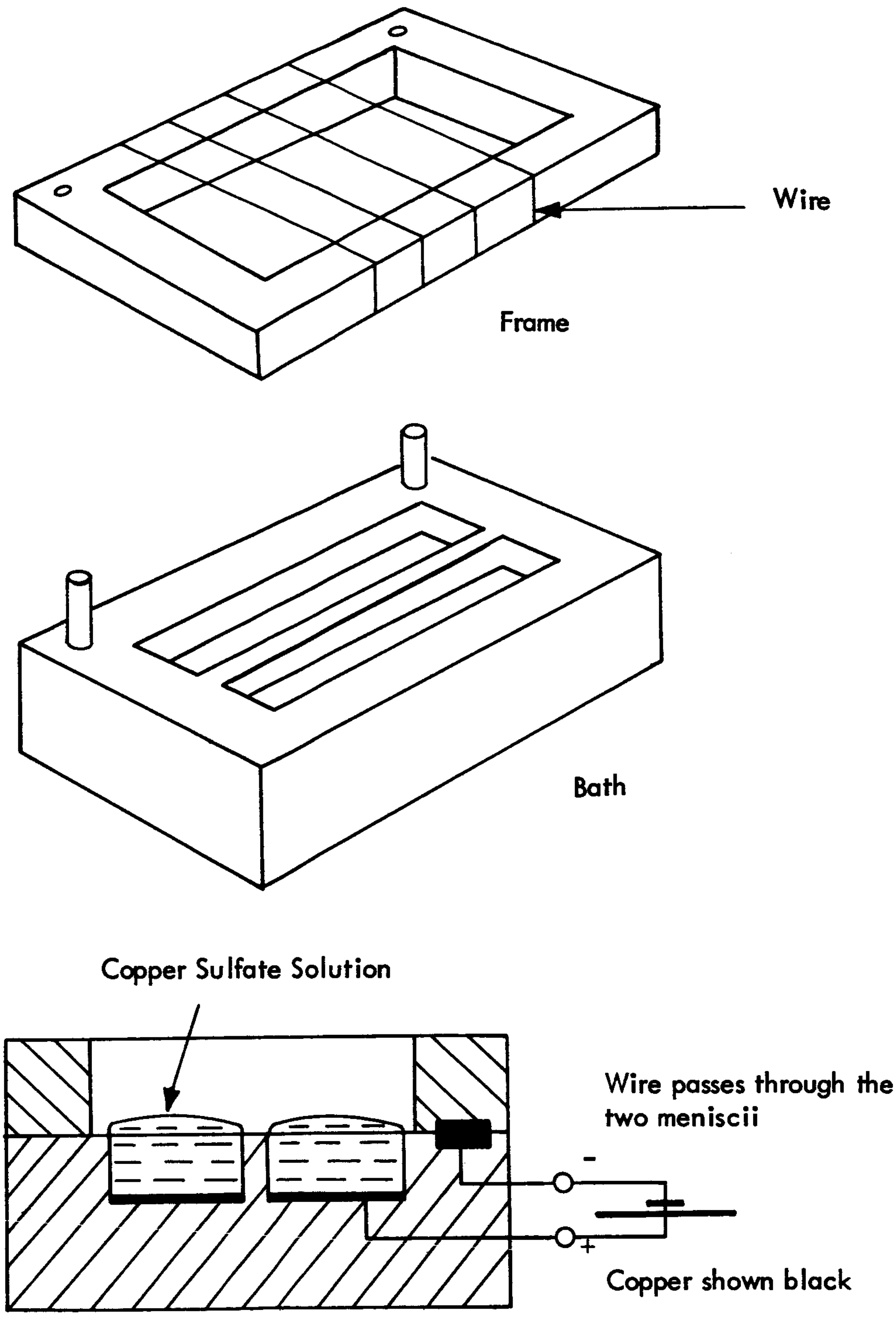
All wires are 0.40 inches long.

Figure 13 Resistance of Various Wire Materials



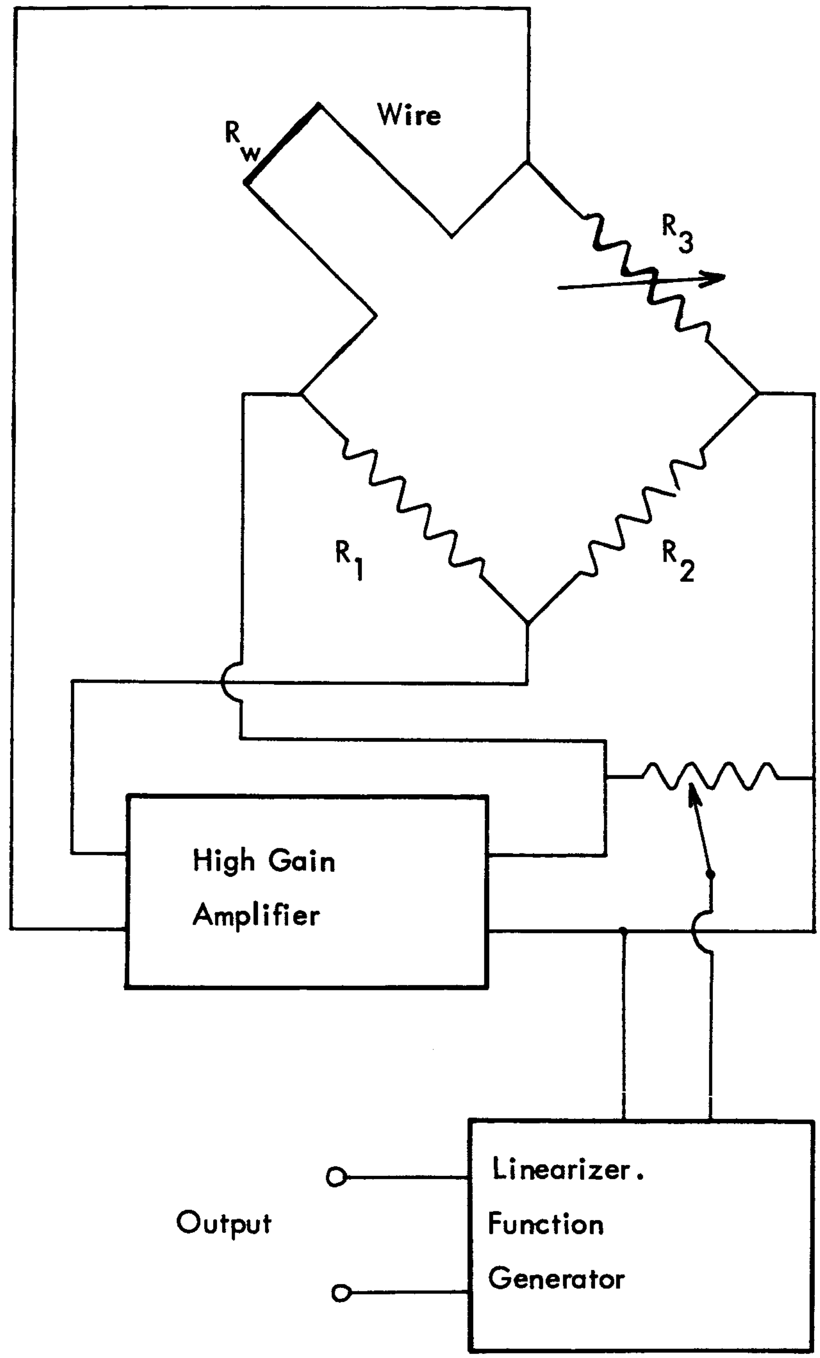
If the wire is Wollaston wire, platinum with a silver coat, then the electrolyte is normally diluted Nitric Acid.

Figure 14 Etching Technique



Section - In Operating Position

Figure 15 Hot Wire Plating Technique



$R_w = R_3$ at operating conditions; $R_1 = R_2$;

$R_1 = 7$ times R_w .

Figure 16: Wheatstone Bridge, Amplifier and Linearizer

APPENDIX A THE USE OF YAWED WIRES TO MEASURE THE CROSS COMPONENTS OF VELOCITY, AND THE EXPECTED ACCURACY.

A1 Introduction

As discussed in Section 3.3, it is necessary to use two wires yawed to the mean airstream to determine the transverse velocity fluctuations. This Appendix is concerned with a simple interpretation of the results produced by this method, based on an empirical argument and measurements of yawed wires.

Initially yawed wires were considered as being sensitive only to the flow normal to the wire, on the basis of classical aerofoil theory for infinite cylinders, and as the wires were usually made with a length-to-diameter ratio greater than 200:1, they were considered suitable for such a treatment. Early experiments showed that this assumption held quite well for all angles of yaw, and any deviation from it was accepted as experimental error. If the velocity is U and the incidence of the wire is θ , i.e. the angle between this velocity and the axis of the wire, then the effective velocity at the wire U_{eff} is given by

$$U_{eff} = U \sin \theta \tag{A1}$$

As a more exact expression of yawed wire performance, Hinze (Reference A1) suggested that the component of the flow along the wire could not be ignored and that the following expression would be more suitable.

$$U_{eff}^2 = U^2 (\sin^2 \theta + a^2 \cos^2 \theta) \tag{A2}$$

where "a" is some small number between 0.1 and 0.3. Webster (Reference A2) made measurements to check this relationship, using platinum wires of a nominal diameter of 2.5×10^{-3} mm. in a wind tunnel running at speeds up to 19 ft/sec. He tested a series of wires with length-to-diameter ratios of 80 to 1300 : 1, and calculated a value of "a" from the results for each case. Although there was some small amount of scatter, he obtained an almost constant value for "a" of 0.2 for all the wires tested.

Corrsin in Reference A3, studied theoretically the cooling effects of a finite yawed wire. He obtained expressions for the departure of the effective velocity from the sine law of equation A1. Two effects influenced his results, the heat sink effects of the supports and the unsymmetrical temperature profile formed along the wire when it is yawed. This effect is also considered by Davies and Fisher (Reference A4).

An expression for the heat convected from a hot cylinder into an airstream is best given in terms of the non-dimensional Nusselt, Reynolds and Prandtl numbers. Such expressions have been put forward for normal wires and variations for yawed wires have been suggested by Sandborne and Laurence (Reference A5). However, the whole picture of hot wire operation is complicated because a temperature variation exists along the wire. A heat flow to the larger mass of the wire supports must take place and this causes a temperature gradient to be formed at the ends of the wire. The equations of heat transfer then become very complicated and involve details of the way the thermal properties of the wire and the air vary with temperature. Changes in velocity will cause the temperature distribution along the wire to change, and yawing the wire will make this distribution unsymmetrical. In a turbulent flow, the various fluctuating velocity components cause the resultant velocity and the effective angle of yaw of the wire, and hence the temperature distribution, to change rapidly. So that even when the overall resistance of the wire is held constant in a constant temperature set-up, this change in temperature distribution will cause unsteady heat flows to occur in the wire. The way that this occurs has been examined in detail by Davies and Fisher (Reference A4).

A2 Resultant Velocities and Angles

Consider a flow system, where a velocity of value 1 exists in the 1 axis and other velocity components are added in the 1, 2 and 3 axes. Then consider two wires, one normal to the standard flow and positioned in the 2 axis, and the other at 45 degress to the standard flow in the 1, 2 plane.

Representing the flow by (U_1, U_2, U_3) and the two wires by,

(0, 1, 0) for the normal wire,
and (1, 1, 0) for the yawed wire.

Then for a given set of the velocity components, the resultant velocity is given by,

$$U_{res} = (U_1^2 + U_2^2 + U_3^2)^{1/2} \qquad \text{(A3)}$$

and the angle that this resultant velocity makes with the two wires is given by,

$$\cos\theta_n = \frac{U_2}{(U_1^2 + U_2^2 + U_3^2)^{1/2}} \tag{A4}$$

for the normal wire

and
$$\cos\theta_y = \frac{U_1 + U_2}{\sqrt{2}\,(U_1^2 + U_2^2 + U_3^2)^{1/2}} \tag{A5}$$

for the yawed wire.

Now consider these equations for a given range of velocity components. Let the mean flow be 1.0 in the 1 direction and additional velocity components of up to $\pm$ 0.2 be allowable due to the turbulence in the flow.

$$0.8 < U_1 < 1.2$$
$$-0.2 < U_2 < 0.2$$
$$-0.2 < U_3 < 0.2$$

The resultant velocity, given by equation A4, will be almost equal to the value of U_1, because the components are squared. When U_1 equals 1.2 and U_2 and U_3 equal 0.2, then the resultant velocity is 1.23. The extreme error of this assumption, within the limits given above, is when U_1 equals 0.8 and U_2 and U_3 equal 0.2. Then the resultant velocity is 0.848 and the error in assuming it equal to U_1 is 6 percent. Thus, it is fair to assume that a normal wire in a turbulent flow only sees the velocity fluctuations is the mean flow direction.

The value of the angle that this resultant velocity makes with the normal wire is given by equation A4, and shows that it is almost completely determined by the transverse velocity component U_2. If the resultant velocity is taken as U_1, then

$$\cos\theta_n \simeq \frac{U_2}{U_1} \tag{A6}$$

and within the limits suggested above, this will vary between -0.25 and +0.25, the extreme cases being given when U_1 equals 0.8 and U_2 is $\pm$ 0.2. The angle in this case is 75 degrees. Therefore, the resultant angle and the flow will vary over a range of 30 degrees. The effect of U_1 will be to change the value for $\cos \theta_n$ from 0.25 to 0.167 if it goes from 0.8 to 1.2 while U_2 stays at 0.2, this changes the angle from 75° to about 80°. Therefore it can be seen that the U_2 component of velocity will be the major factor in determining the angle of the resultant flow to the normal wire, and that the U_1 component acts to a lesser extent. The U_3 component will have very little effect at all.

A similar process will occur for the yawed wire, where the angle is given by equation A5. Taking the resultant velocity as U_1, as before, then

$$\cos \theta_y \simeq \frac{U_1 + U_2}{\sqrt{2}\, U_1} \tag{A7}$$

Therefore θ_y will vary about the 45 degree value as the U_2 and U_1 components of velocity change. Again it is seen that it is the U_2 component that chiefly determines the resultant angle, since a change of U_2 gives a greater change to the value of $\cos \theta_y$ than a corresponding change in U_1. In fact, the angle made between the resultant velocity and the 45 degree yawed wire, will be exactly 45 degrees different for that of the case of the normal wire; if the U_3 component is zero. In the case of the yawed wire, the U_3 component will have a slightly greater effect since the resultant velocity term is more important because of the $\sqrt{2}$ term, and the changes in value of the r.h.s. of equation A5 have a greater effect in the region around θ_y equals 45 degrees. However once again the change in U_2 is the major factor in determining the change produced in the angle the resultant velocity makes with the yawed wire. Figure A1, shows the effects of U_2 and U_1 velocity changes on the resultant angle when U_3 is zero. A further calculation taking U_3 equal to $\pm$ 0.2 showed a maximum change in angle of 1.7 degrees. This figure shows clearly that a change in U_2 causes a change in the angle of the resultant velocity to the flow and that the greater the value of U_1, the less the change.

A3 Experiments with Yawed Hot-Wire Anemometers

The results of Section A2 must be considered in conjunction with the operating characteristics of a yawed hot-wire anemometer. The wires used in the experiments were constructed from tungsten wire with a nominal diameter of 2×10^{-4} inches. They were copper-plated so as to leave an unplated working section and to allow the wires to be soldered onto the probes. The working length was fixed at 0.080 inches, giving a length-to-diameter ratio of 400:1. The wires were operated in a constant temperature system which was also linearized. The probes were then placed in the potential core of a jet whose velocity was measured using pitot-static combinations with water and mercury manometers.

The jet was run at speeds up to 500 ft/sec. with the wires fixed at a series of angles of yaw. The voltage outputs from the hot wire sets and the velocity of the jet were noted, and a typical set of results is shown in Figure A2. Because some trouble has been experienced from probe vibration effects causing extraneous signals when the wires were tight, the wires were normally soldered onto the probes in a slightly slack condition. This means that they bowed in the airflow, so that the angles quoted are in fact the angles of the wire supports, but they are typical for the wire. It will be seen that a stright line can be drawn through the results at each angle of yaw, showing that the linearization still holds at the various angles. These results are cross-plotted as voltage against angle of incidence with velocity as the parameter in Figure A3. All the results obtained are plotted in Figure A4 as effective velocity on the wire divided by the true velocity against the sine of the angle of incidence of the wire. Also plotted is the empirical relationship due to the results of Webster (Reference A2). This figure shows, that except at very small angles of incidence, when the wire axis is almost in the direction of the flow, the wire follows the sine law. For the large angles of incidence; the results fit the sine law, or if anything gives a function slightly less than the sine law would predict. Exact agreement with the empirical relationship suggested by Webster cannot be expected. The wires used here are made of tungsten and not platinum and these materials have very different thermal properties, notably for thermal conductivity. Also the measurements were made in flows at speeds up to 500 ft/sec., which is much higher than those used in Webster's experiments.

There are two regions of interest concerned with these results. The first is at positions where θ, the angle of incidence of the wire, is near 90 degrees, which is the operating conditions of the normal wire, and secondly at the position where θ is near 45 degrees, which is the operating condition of the yawed wire in cross-wire pairs. For the normal wire, a slight decrease in the voltage is observed as the angle of incidence of the wire varies from 90 degrees. This is only small for the small angle changes of about ± 15 degrees which can be expected due to the cross-components of the velocity. At the 45 degree region, the curves of Figure A3 can be approximated to a series of straight lines, that is the change in voltage is linear with the change in angle of incidence; the lines are at different slopes with the greater gradient for the greater mean velocity. Comparing Figure A3 with Figure A1, it can be seen that a higher velocity means a smaller angle change for a given U_2 component of velocity but that a higher voltage change is caused by a given angle change. This suggests that it should be possible to match these two curves so that a given U_2 component produces a constant voltage change regardless of the mean velocity.

The curves of Figure A3 can be approximated to a series of straight lines, over the working range (30 to 60 degrees). Also, since the line for zero velocity will be horizontal, the slope of all these lines and the points where they cut the voltage axis will be proportional to the mean or resultant velocity.

So if E is the voltage output,

$$E = a\,\theta + b \qquad \text{(A8)}$$

where a and b are particular constants for a given resultant velocity, and in terms of this resultant velocity,

$$E = A\,U_{res}\,\theta + U_{res}\,B \qquad \text{(A9)}$$

where A and B are now universal constants for this particular wire.

Next consider Figure A1.

If U_2 and U_3 are taken as small (< 0.2 of the mean velocity), then it can be assumed that,

(a) The resultant velocity is given by U_1,

and (b) That the effect of U_3 is negligible.

The relationship between θ and U_2 is again linear, for a given U_1 or resultant velocity.

$$\theta = c\,U_2 + d \qquad \text{(A10)}$$

where c and d are constants for a given U_1, but d has the same value for all resultant velocities and equals 45 degrees, and c is dependent of the resultant velocity and inversely so.

Therefore,

$$\theta = \frac{C}{U_{res}} U_2 + D \qquad \text{(A11)}$$

where C and D are constant for all values of the resultant velocity.

Substituting equation A11, in equation A9,

$$E = A\,U_{res}\left(\frac{C\,U_2}{U_{res}} + D\right) + U_{res}\,B$$

$$= A\,C\,U_2 + U_{res}\,(AD + B) \qquad \text{(A12)}$$

and writing $U_1 = U_{res}$

$$E = A\,C\,U_2 + U_1\,(AD + B) \qquad \text{(A13)}$$

Then the voltage produced by the hot-wire set is directly proportional to the two components of the velocity. Now if two wires are inclined at equal and opposite angles to the mean flow and if these two wires have equal responses, then, by subtracting one voltage from the other, an output is obtained which is proportional to the cross-component of the velocity. The voltage change produced by a change in transverse component will be independent of the resultant velocity, even though this may change considerably.

This result of separate linear response to the velocity components U_1 and U_2, can also be obtained from the basic equation A5 and the sine law of Figure A4. If the sine law holds over the range of interest, then all the curves in Figure A3 are sine laws.

Hence $$E = E_{90} \sin \theta \qquad \text{(A14)}$$

where E_{90} is the voltage at the angle of incidence θ equals 90 degree point on Figure A3, and since the output is linear with velocity at this point

$$E = G\, U_{res} \sin\theta \qquad (A15)$$

where G is a constant.

Equation A5 gives

$$\cos\theta = \frac{U_1 + U_2}{\sqrt{2}\; U_{res}}$$

and substituting in equation A15 gives

$$E = \frac{G}{\sqrt{2}}\left[2\, U_{res}^2 - (U_1 + U_2)^2\right]^{1/2}$$

and letting U_{res} equal U_1

$$E = \frac{G}{\sqrt{2}}(U_1 - U_2) \qquad (A16)$$

Thus the voltage output is directly proportional to the two velocity components and independently as in equation A13.

$$\frac{G}{\sqrt{2}} = AC = AD + B$$

If the wire is not inclined at exactly 45 degrees to the flow, but is still in the 1, 2 plane, then the independent linear result still holds. Let the direction of the wire be given by,

(g, h, o)

$$\cos\theta = \frac{U_1 + U_2}{(g^2 + h^2)^{1/2}\ U_{res}} \qquad \text{(A17)}$$

Substituting this value of θ into equation A15 gives

$$E = G\left[\frac{h^2 U_1^2 + g^2 U_2^2 + (g^2 + h^2)\ U_3^2 - 2 g h U_1 U_2}{(g^2 + h^2)}\right]^{1/2}$$

The two significant terms are $h^2 U_1^2$ and $2gh\ U_1\ U_2$, and the problem again relates to the effect of the U_3 term. If this term can be ignored then the resultant output is approximately

$$E \simeq \frac{G}{(g^2 + h^2)^{1/2}}\left[h\ U_1 - g\ U_2\right] \qquad \text{(A18)}$$

Thus a pair of wires not at 90 degrees to each other, but set up at equal angles to the flow, still gives a linear independent output with the velocity components. However, a different rate of output with velocity exists for the two components, and so some form of calibration is necessary.

By using Figures A1 and A3, the results of Figure A5 were produced. Here U_3 was taken as 400 ft/sec., U_1 was then varied through the range 320 to 480 ft/sec. and the voltage changes for various changes of the U_2 velocity component were obtained. It will be seen that a line can be drawn through all the points and the effect of the mean velocity, U_1, is lost. Because a linearized hot-wire system was used, the result is a straight line through the origin.

Next a series of experiments were perfomed using pairs of crossed wires. The probe was again placed in the core of the jet with the two wires at equal angles to the flow. The jet was run at various speeds and the probe rotated to various angles through $\pm$ 20 degrees, so as to put the equivalent of various cross-components of velocity onto the wires. The resultant voltages from the two hot-wire sets were measured and one subtracted from the other. The cross-components of velocity were calculated and plotted against these voltages in Figure A6. The wire was subjected to a range of speeds from 287 to 437 ft/sec., a ratio of 0.66:1, which gives cross-components of velocity up to 149 ft/sec. for the case when the probe was rotated through 20 degrees.

A4 Discussion

Study of the basic properties of the resultant velocity formed by various velocity components, and the angle that this makes with a yawed wire indicated the method of operation of cross-wire systems. When the maximum velocity components are limited to 0.2 of the mean flow velocity, then the analysis and the results of Figure A1 show the following properties exist. First the resultant velocity is nearly exactly equal to the velocity component in the direction of the flow, and the transverse components only slightly affect this value. Secondly, the transverse component in the plane of the wire, U_2, has the greatest effect on the angle this resultant velocity makes with the wire. The U_1 component changes the angle only slightly, and in a way that can be allowed in the operation of the yawed wire, whilst the U_3 component has only a very small effect. The angle change with U_2 component is an almost exactly linear relationship, and the results show that by matching the linearized output of the two hot-wires over the critical working range of 30 to 60 degrees of yaw, a system can be produced where a change in the transverse component of velocity produces a linear change in voltage output.

These conclusions are only applicable when the velocity components of the flow are small. If the intensity of the flow is such that instantaneous velocity components greater than 0.2 of the mean flow velocity occur, then the curves cannot be reasonably approximated to straight lines. Further to this, the resultant velocity formed at this moment will be considerably altered from the value of the velocity component in the flow direction. The third assumption, that the U_3 velocity components do not change the angle of incidence formed, will also fail. For example, consider the results produced by specific examples for the yawed wire case. Taking a flow where the instantaneous values of the velocity components are $U_1 = 1.0$ and $U_2 = 0.2$. Then when $U_3 = 0$ the resultant velocity formed is 1.02 and if this were to go to $U_3 = 0.2$, then the resultant velocity becomes 1.04. Thus the error in assuming the resultant velocity is the U_1 component, is 2 percent and 4 percent respectively for these two examples. Now consider a flow where the maximum variation of velocity component is increased from $\pm$ 0.2 to $\pm$ 0.6. First let $U_1 = 1.0$

and $U_2 = 0.6$, when $U_3 = 0$ the resultant velocity equals 1.17, so the resultant velocity assumption is now in error by 17 percent. When $U_3 = 0.6$, the resultant velocity is 1.32 and the error is increased to 32 percent. As the U_1 component varies, the error will again change, to give a maximum error of 80 percent when $U_1 = 0.4$, $U_2 = 0.6$ and $U_3 = 0.6$ in the extreme case.

Again consider a flow where instantaneously $U_1 = 1.0$ and U_2 goes from 0 to 0.2, while U_3 remains at 0. Then the angle of incidence formed by the wire with the resultant velocity change from 45 degrees to 33.8 degrees. Then if U_3 goes to 0.2 the incidence will become 35.6 degrees, a change of 1.6 degrees. This means the angle of incidence formed in this example can vary by up to 14.3 percent due to the U_3 velocity fluctuations. For the case of the larger velocity components, let $U_1 = 1.0$, $U_3 = 0$ and U_2 go from 0 to 0.6. Then the angle of incidence will change from 45 to 15 degrees, a 30 degree difference. If U_3 goes to 0.6 the angle of incidence changes to 32 degrees, a change of 17 degrees. Thus in this case the U_3 velocity fluctuations can cause the resultant angle of yaw of the wire to vary by 57 percent.

If the velocity components are limited to 0.2 of the mean velocity, then with a Gaussian distribution of the velocity fluctuations the intensity components must not exceed about 0.08 of the mean velocity value. In this case, the maximum error in any measurements made will be of the order of 15 percent. However, the error will increase rapidly if greater intensities exist, to reach possible values of over 50 percent when the intensity is about 0.24 of the mean velocity. In a jet the values of the local intensity of the velocity fluctuations can reach 0.2 in the outer edges of the mixing region, and similar values have been noted in boundary layers. Thus when hot-wire anemometers are used, the error possible in the measurements must not be neglected and care must be taken not to place too high a reliability on the results and on any small differences found. The figures also show that the yawed wire will be more critical than the normal wire, because in this case the third velocity component will have a greater effect.

The proposed operating procedure for cross-wire combinations based on this method is best given next. The two wires are placed, in turn, normal to a uniform non-turbulent flow. The response of the wires is checked to be linear with velocity and the outputs from the hot-wire sets are adjusted to be equal. The probe is then rotated to the position when the output from both sets is equal; the probe is then set up with each wire at an identical but opposite angle to the flow. This position is noted and if the wires are known to be at exactly 90 degrees angle to each other, then the calibration can be deduced from the normal calibration. The wires are more easily calibrated by rotating the probe through 20 degrees each side of the operating position, so as to put a known transverse component of velocity on the wires. The individual voltage outputs from the sets can then be subtracted to produce a graph like Figure A6 which is the transverse velocity calibration for the probe. Then the probe is returned to its operating position and is ready to be placed in the turbulent flow. In order to measure the intensities of the cross-components of the turbulence, the two signals are continuously subtracted and the resultant signal fed to a true r.m.s. voltmeter. If the two wires do not have exactly identical responses, then when the two voltages are subtracted an extra signal will be formed due to the longitudinal velocity components, i.e. in addition to the voltage due to the transverse components. This means that unless the two wires are perfectly matched the result obtained for the transverse velocity intensities will be greater than the true value.

The two wires do not have to be set up at exactly 45 degrees to the flow, but only at equal angles to the mean flow. Of course a calibration by rotation of the probe in a steady stream is then necessary. However, the whole process of setting up and calibration can be carried out without having to measure the wire angle exactly, only the probe position is required. When the wires are set normal to the airstream, for the initial check on linearization and equal response, the angle is not critical to within a few degrees and the wires may be positioned by eye.

The problem of calibrating the probes still remains, and the process is tedious and likely to be inaccurate since the exact behavior of the wire in a turbulent flow is unknown. When this effect is considered with the other limitations given by the approximations in the analysis, the whole approach must appear to become very unreliable. However, while the hot-wire anemometer remains the only suitable instrument for turbulent airflow measurements, it is necessary that certain assumptions be made, so that it may be used. The true explanation of its working will be very complicated, since the involved heat transfer process could hardly be described in a few simple terms. Davies and Fisher (Reference A4) explained how a temperature variation will occur along the wire because of the end losses, so this means the various thermal properties in any theoretical expression will have different values at different points. Very exact relationships are necessary to describe the variation of these quantities, such as conduction and convection coefficients, and simple linear laws with temperature are not accurate enough. A velocity change will cause the temperature distribution to change, so that the temperature at a given point on the wire is not constant. When the unsymmetrical temperature distribution of the yawed wire is also considered, the exact description of heat transfer in a turbulent airstream becomes intricate.

A theoretical approach to understand the working of yawed hot wires is obviously essential, otherwise a purely empirical approach must be evolved, but this should include measurements in a known turbulent flow. The main criticism of hot-wire calibration must be the use of a steady flow to calibrate a wire which is to be used in a fluctuating flow. It is recognized that serious drawbacks are involved in the method, especially the intensity limitations. Provided it is realized that these effects exist, the operation of cross-wire combinations in turbulent flows is feasible and some meaning can be given to the results obtained.

The measurements made with the yawed wires in a steady stream suggest that, except at very small angles of incidence, the sine law of effective velocity is followed. It is unlikely that a simple empirical expression can be found that will cover the whole range of angles and all types of wires. It must necessarily be complicated to account for the temperature distribution effect, and the different properties of the various materials used to form hot-wire anemometers. For the limited range of incidence of 30 - 60 degrees, the results show that the expression of linear response with angle is probably the simplest. This is as likely to be as accurate as a more complicated expression, which attempts to cover a greater range of angles.

A5 Conclusions

The analysis and experiments suggest a method whereby crossed hot-wire anemometer pairs may be used to produce a linear system for measuring the cross-components of velocities in a turbulent flow. The two wires must have identical linear responses and must be set up at equal but opposite angles to the mean flow direction.

The accuracy of the method will depend on the intensity of the turbulence and the system must be used with care in high intensity regions. If the total local intensity of the velocity fluctuations is limited to 0.1 of the mean flow velocity, then the possible errors in the measurements made will be no greater than 15 percent.

The problem of calibrating hot-wire anemometers will only be fully overcome when a complete understanding of the heat transfer process in a fluctuating flow is formulated.

The experimental work reported in this Appendix was completed at the University of Southampton, England, in the Department of Aeronautics and Astronautics. The author acknowledges the financial support of the Ministry of Aviation for this part of the program.

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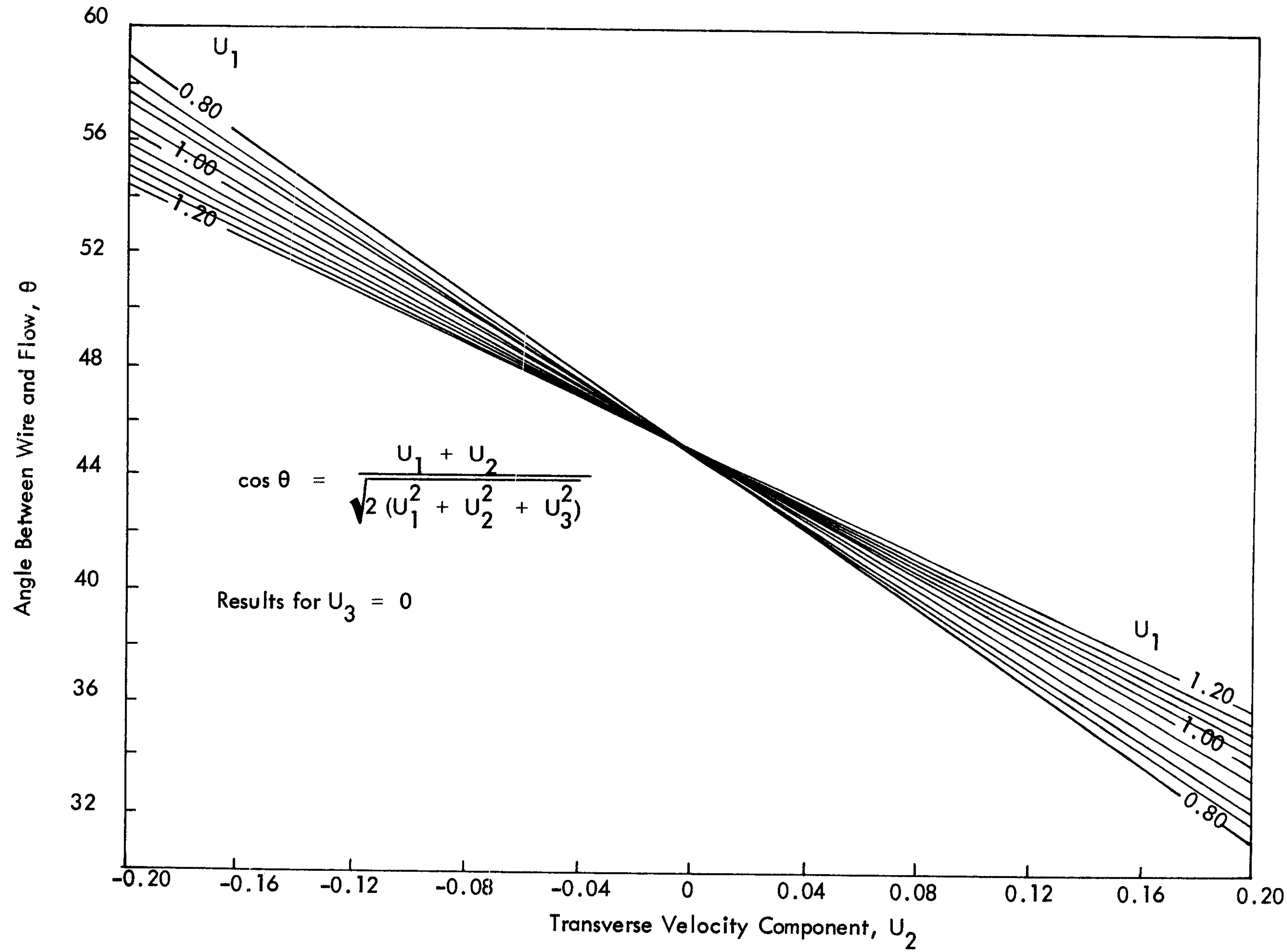


Figure A1 Resultant Angle with Yawed Wire

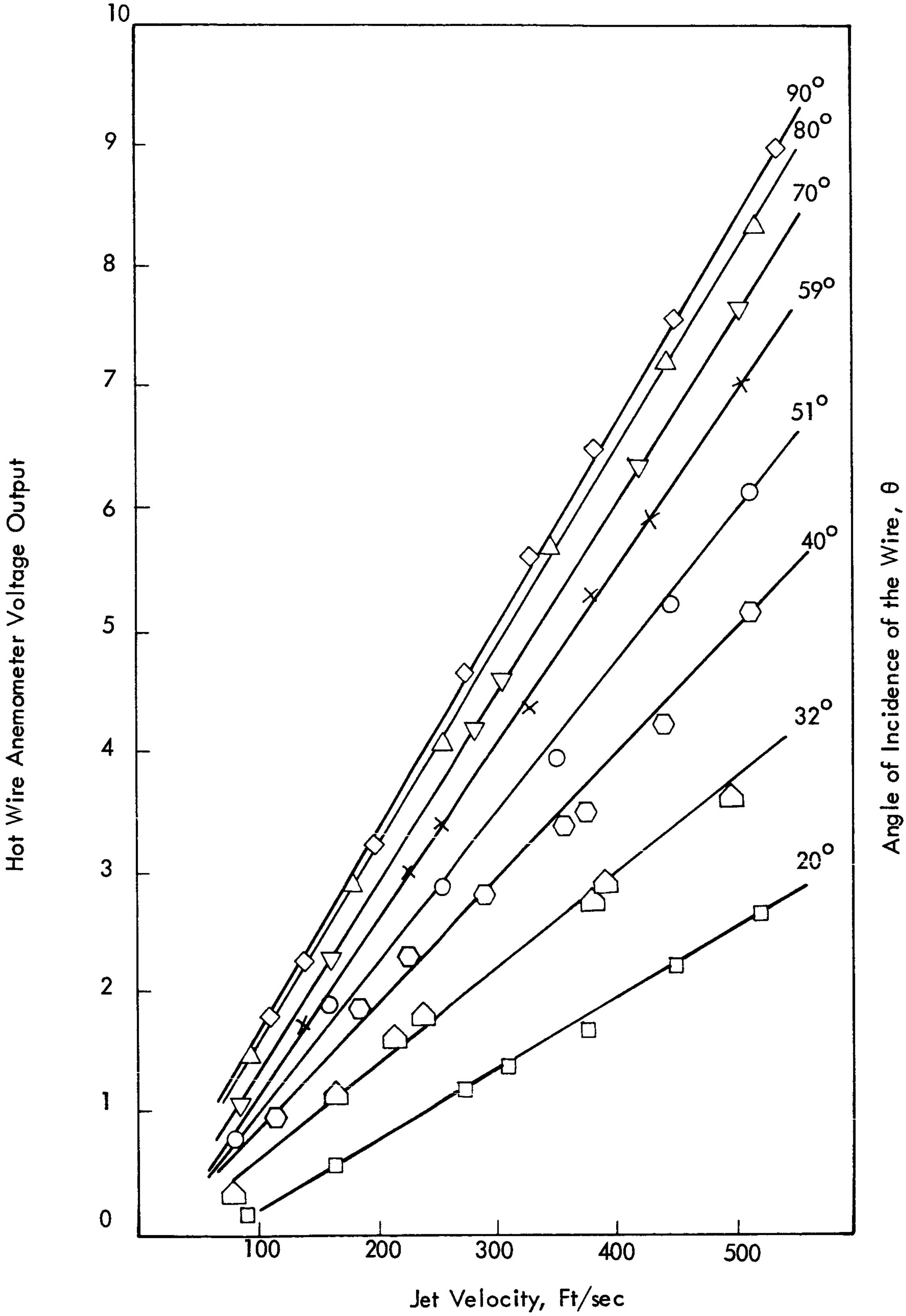


Figure A2 Yawed Hot Wire Anemometer Perfomance

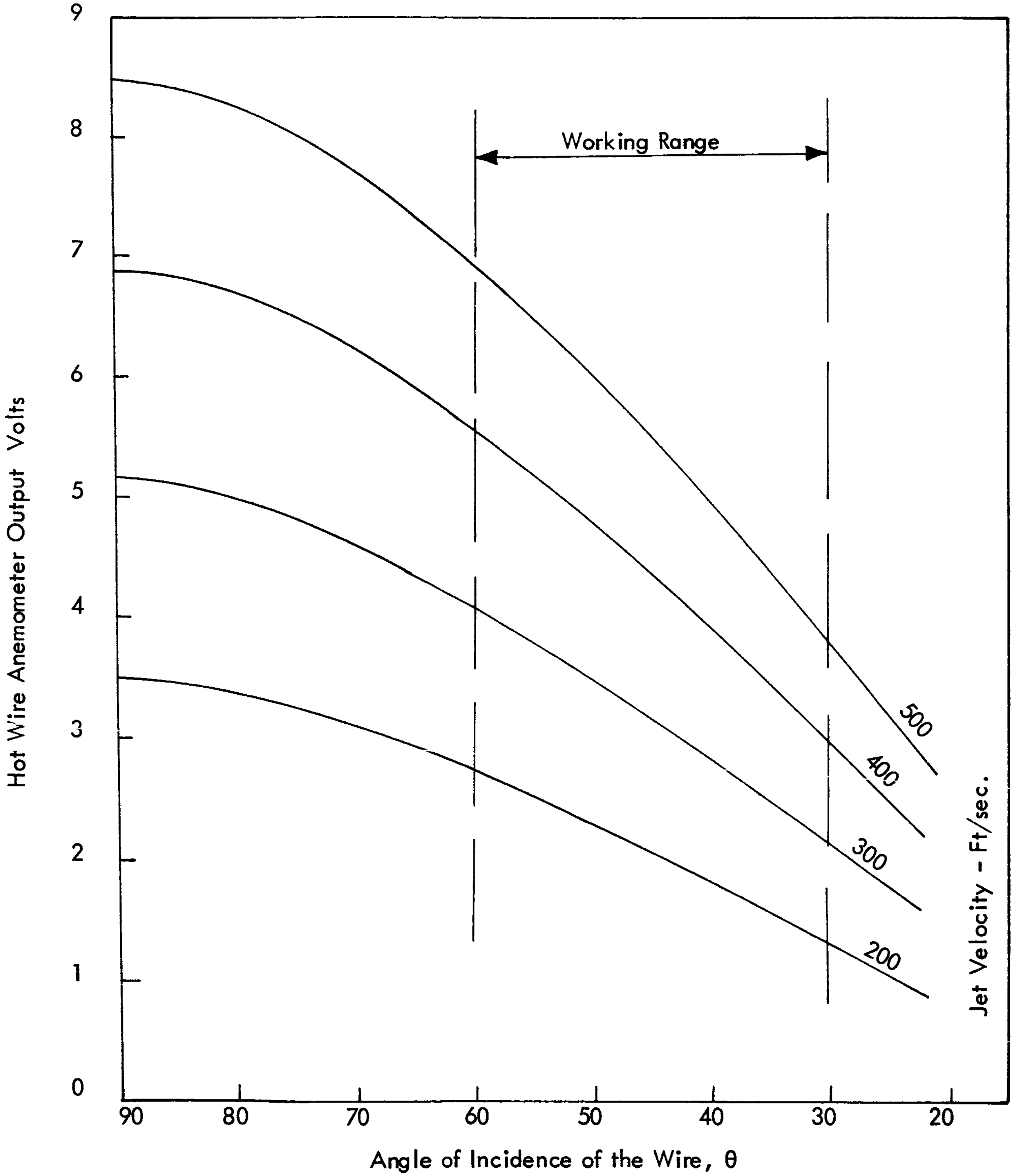


Figure A3 Yawed Hot Wire Anemometer Performance

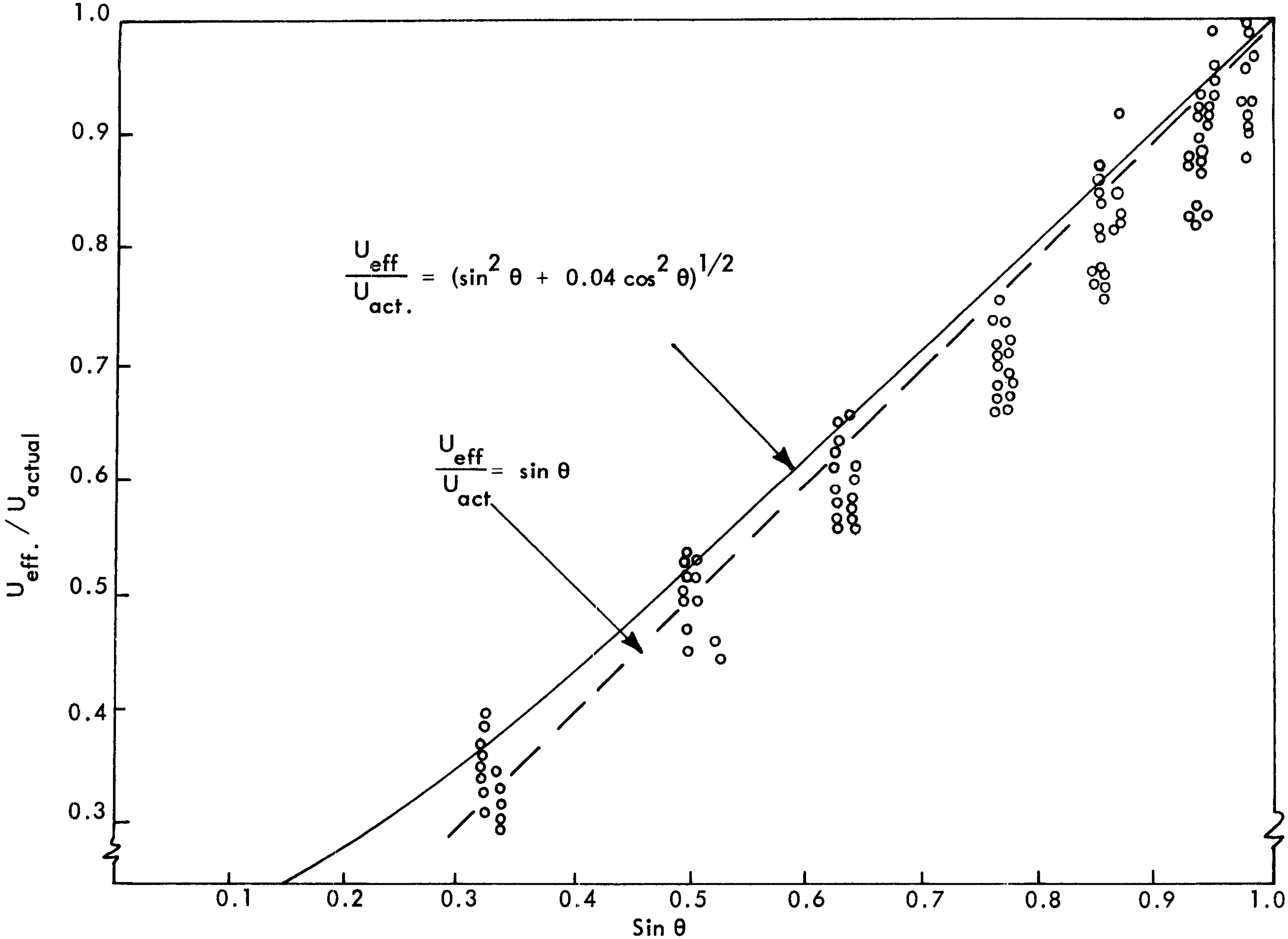
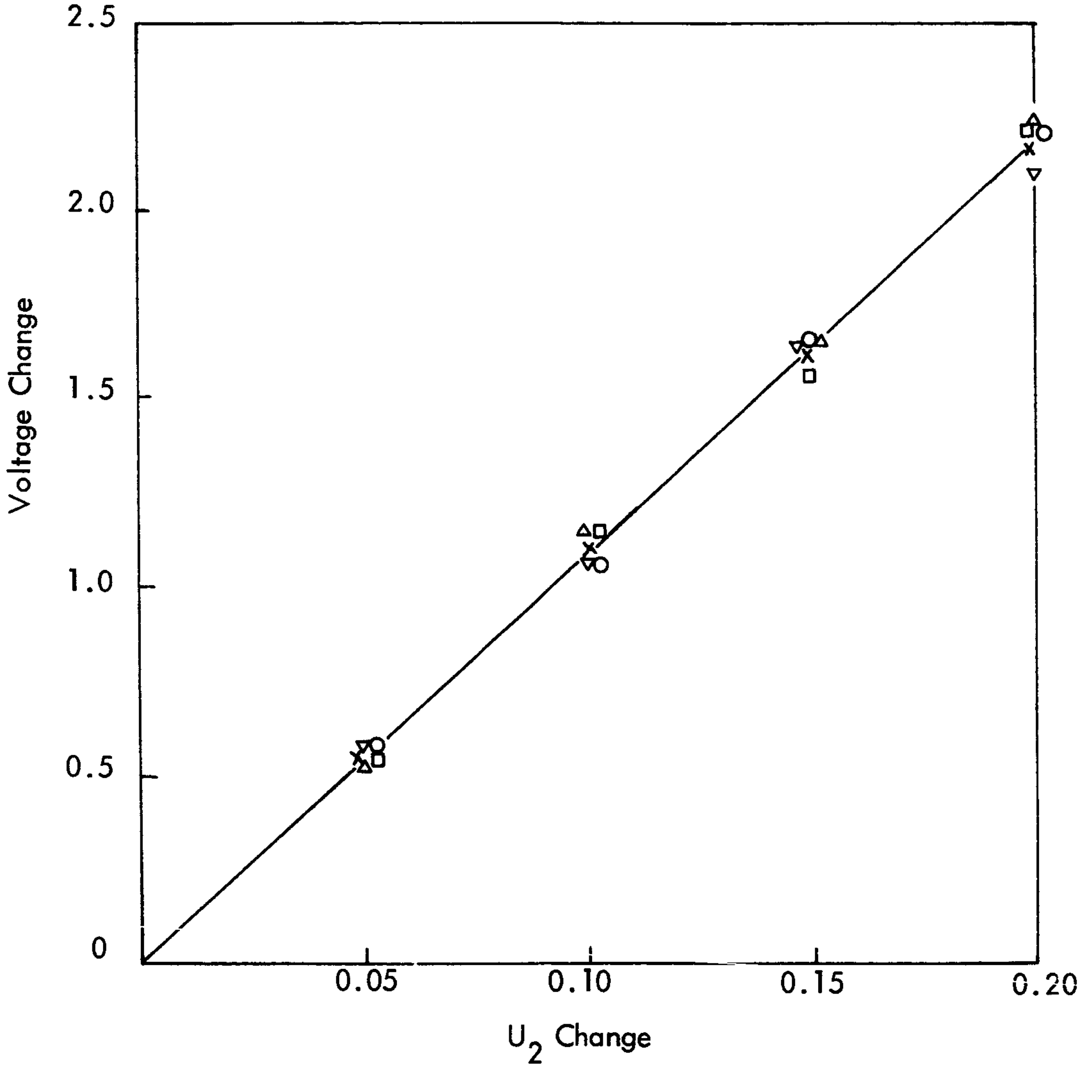


Figure A4 Effective Velocity as Measured by Wire



U_1 = 1.00 = 400 Ft/sec.

× U_1 = 0.80
△ U_1 = 0.92
○ U_1 = 1.00
▽ U_1 = 1.12
□ U_1 = 1.20

Figure A5 Effect of U_2 Change

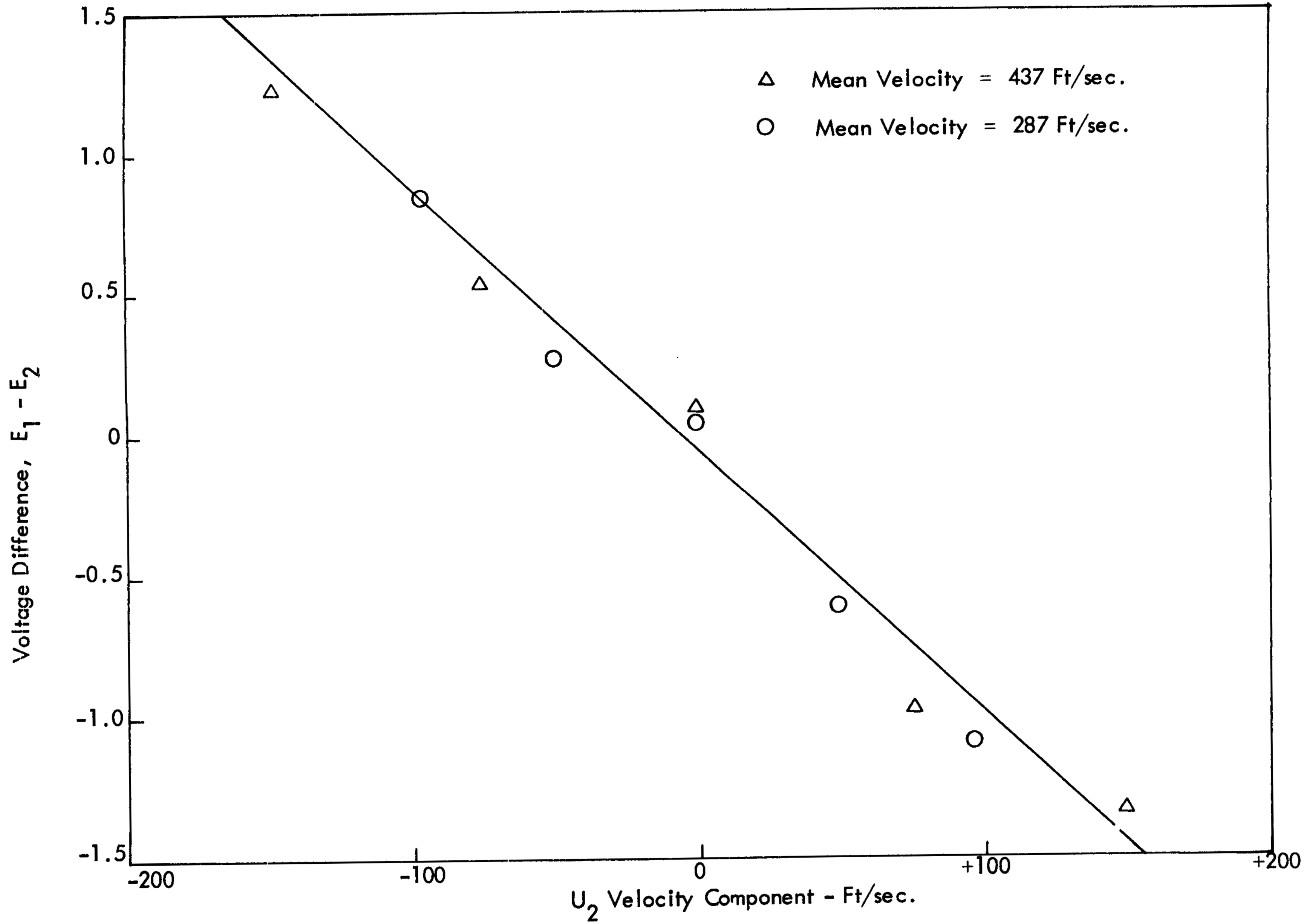


Figure A6 Cross Wire Combination, Voltage Difference Due to Transverse Velocity Component