## SPECLAL-PURPOSE INTERPLANETARY TRAJECTORY

 COMPUTATION PROGRAM FOR GUIDANCE AND NAVIGATION STUDIES
## User's Manual

## by

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1

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# SPECIAL-PURPOSE INTERPLANETARY TRAJECTORY COMPUTATION PROGRAM FOR GUIDANCE AND NAVIGATION STUDIES User's Manual 


#### Abstract

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This manual provides the necessary information for use and modification of the interplanetary trajectory computation program. It has been prepared as a supplement to Ref. (1), which gives a technocal description of the program. This manual describes input and output data formats and contains detailed functional flow charts for the program. An example of a typical trajectory computation is given also.


William T. McDonald July 1965

## 1. Introduction

This manual has been prepared as a supplementary document to Ref.(1). The reference is a technical description of the program, while this manual provides basic information necessary for use and modification of the program. Sections 2 and 3 describe the inpuc and output data formats, respectively, and Section 4 contains detailed functional flow charts for the program.

Reference is made throughout this manual to a set of numbers whichidentify the Sun and planets in the program. They are listed below for convenient reference, and they also appear in each output data listing (see, example, Fig. 3-1):

| Number | Body |
| :---: | :--- |
| 1 | Earth |
| 2 | Mars |
| 3 | Venus |
| 4 | Jupiter |
| 5 | Saturin |
| 6 | Sun |

## 2. Description of Input Data Card Format and Use

Data input to the program is by means of punched cards. The total number of input data cards varies from 22 to 42 , with allowance for from 0 to 20 cards specifying times of required data printouts. The data are punched according to FORTRAN standard word formats.

Cols. 1 through 72 contain the data words. Cols. 73 through 80 are not used (except that Cols. 79 and 80 may conveniently be used for card numbering). The first data word of each card begins in Col. 1, and all data words are right-justified. The card formats and data words are described below. Refer to Fig. 2-1, which is a sample coding form for the input data.
SPACE TRAJEC ICRY INTEGRATION I RCGRAM
Input Data Card Format


Julian dates of start time (TSTART) and end time (TEND) of the trajectory. The difference of these two times is the duration of the trip. If the trajectory integration is forward in time, TSTART is the injection time and TEND is the arrival time. If the integration is backward in time, TSTART is the arrival time and TEND is the injection time. Julian date measured from an arbitrary epoch should be used, and it is convenient to use the last three or four digits of the whole Julian day number.
Day numbers larger than $131,172\left(2^{17}\right)$ cannot be read in the integer word format used, so whole number Julian data cannot be used.

Word Format

| TSTART (DAY) | - | 19 |
| :--- | :--- | :--- |
| TSTART (HR) | - | 19 |
| TSTART (MIN) | - | 19 |
| TSTART (SEC) | - | F9.6 |
| TEND (DAY) | - | 19 |
| TEND (HR) | - | 19 |
| TEND (MIN) | - | 19 |
| TEND (SEC) | - | F9.6 |

The number of the primary body from which the integration starts (NSTART) and the position vector (USTART) of the trajectory starting point. If the integration is forward, NSTART is the number of the primary body at injection and USTART is the position at injection. If the integration is backward, NSTART is the number of the target body and

Card No.
(cont.) 2

3

## Description

USTART is the position at arrival. Position components are in starting body-centered coordinates (usually ecliptic of 1950.0 ) and units are kilometers. Components 1, 2, 3 correspond to $\mathrm{x}, \mathrm{y}, \mathrm{z}$.

NSTART may be either positive or negative. If it is positive, Card No. 3 (VSTART) will be read in decimal format. If NSTART is negative, Card No. 3 will read in octal format.

## Format

NSTART - I3
USTART (1) - El5.8
USTART (2) - E15.8
USTART (3) - E15.8
Estimated velocity at the starting point.
(VSTART) required to cause the trajectory to ready the end point. VSTART will usually be simply an initial guess, and the program will search to find the correct starting velocity. The velocity components are in starting bodycentered coordinates (usually ecliptic of 1950.0 ) and the units are $\mathrm{km} / \mathrm{sec}$. Components $1,2,3$ correspond to $x, y, z$. If NSTART is positive, VSTART will be read by the program in decimal format; if NSTART is negative, VSTART will be read in octal format. There are thus two different formats for this card. The octal readin option is to allow precise duplication of initial conditions in separate runs.


## Format:

| NEND | - | I3 |
| :--- | :--- | :--- |
| UEND (1) | - | E15.8 |
| UEND (2) | - | E15.8 |
| UEND (3) | - | E15.8 |
| TOL | - | El5.8 |

Card No.
(cont.) 5

## Description

Program control constants and the calendar date of launch.

Format and Function:
NPRINT - I3:
Specifies the number of time points along the trajectory at which special data printouts are required. The upper limit on NPRINT is 20 ; no more than 20 time points can be specified for any one run. $0 \leq$ NPRINT $\leq 20$.

MiODE - I 3
Program operating mode, 1, 2, or 3.
NCSC - 13
Flag which causes the osculating conic routine (OSCON), to compute and print out osculating conic data at each printout point.
NOSC $=0$ for no osculating conic data,
NOSC $=1$ for osculating conic data.
MAX - I3
Maximum number of trajectory search iterations allowed. $M A X \geq 1$.

NCYCLE - 13
Flag which causes printout at all specified times and at the beginning, end, and phase change points during every cycle of the trajectory search. NCYCLE $=0$ : print out only during last cycle (after search convergence); NCYCLE = 1: print out every cycle during search.

KPHINV - I3
Flag which causes the product of the state transition matrix and its inverse to be

Card No.
(cont.) 5

6 through 25

## Description

computed and printed out in all normal printout records in which the state transition matrix is printed. KPHINV $=0$ : product is not computed and not printed; KPHINV = 1: product is computed and printed.
LNCHMO - I5 Launch date - month.
LNCHDY - I5 Launch date - day.
LNCHYR - I5 Launch date - year.
Calendar date of launch, month, day, and year only.

HMULT - F5. 3
Scaling factor for the time step computation. Value is usually about . 100.

Time points along trajectory at which data printouts are required. The times are always positive and measured from TSTART regardless of the direction of the integration (forward from injection to target, or backward from target to injection). Each card specifies one time point in days, hours, minutes, and seconds from TSTART. A maximum of 20 time points are allowed; any number from 0 to 20 is permitted, the number of time points (cards) must correspond to NPRINT on Card No. 5.

Format (each card)
DAYS - I9
HRS - 19
MIN - I9
SEC - F9.6

Description
Epochs (Julian dates) of the reference plant ephemerides in the following data cards 31 through 40. TSTART, TEND, and these epochs must all be measured from the same zero time reference. Crdinarily, the reference ephemerides are chosen as follows:

Launch planet - on or near launch date
Target planet - on or near arrival date
Other planets - date abouve midway between launch and arrival dates.

These choices provide the best approximation to actual planet ephemerides in all phases of the flight.

Format:
EPCCH DAY - 19
EPOCH HR - 19
EPCCH MIN - I9
EPOCH SEC - F9.6
Reference ephemerides of the 5 planets, Earth, Mars, Venus, Jupiter, and Saturn, in order (planet numbers 1 through 5, respectively, in the program). Corresponding to the 5 epochs in cards 26 through 31). Cards 31 through 35 contain components of the planet positions ( $3 \mathrm{x}, \mathrm{y}, \mathrm{x}$ components per card, 1 card per planet). Cards 36 through 40 contain components of the planet velocities ( $3 x, y, z$ components per card, l card per planet). The ephemerides are in heliocentric coordinates (usually ecliptic of 1950.0 ) and units are a.u. and a.u. / day. These units are convenient

|  | Card No. |  | Description |
| :---: | :---: | :---: | :---: |
| (cont.) | $\begin{gathered} 31 \\ \text { through } \\ 40 \end{gathered}$ |  | because they are used in the sources of the reference ephemerides; conversion to km and $\mathrm{km} / \mathrm{sec}$ takes place in the program. |
|  |  |  | Format: |
|  |  | each position card | UPO(1,N) (x position component, planet N)-El 5. <br> $\mathrm{UPO}(2, \mathrm{~N})$ (y position component, planet $N$ )-E15. <br> $\operatorname{UPC}(3, N)(z$ position component, planet $N$ )-E1 5. |
|  |  | each velocity card | VPO(1, N) (x velocity component, planet N)-E15. <br> $\mathrm{VPO}(2, \mathrm{~N})$ (y velocity component, planet $N$ )-E15. <br> $\operatorname{VFO}(3, N)$ (z velocity component, planet $N$ )-E15. |
|  | 41 <br> and <br> 42 |  | Gravitational constants ( $\mathrm{mu}=\mathrm{G} \times \mathrm{mass}$ ) of the |
|  |  |  | 5 planets, Earth, Mars, Venus, Jupiter, and |
|  |  |  | Saturn, and the Sun, in order. Units are $\mathrm{km}^{3} / \mathrm{sec}^{2}$. Suggested values have been computer using $1.49599 \times 10^{8} \mathrm{~km}$ per astronomica' unit and the Sun/planet mass ratios given in Ref. (2): |
|  |  |  | SMU (1) (Earth) $3.986032 \times 10^{5} \mathrm{~km}^{3} / \mathrm{sec}^{2}$ <br> SMU (2) (Mars) 4, $296455 \times 10^{4}$ <br> SMU (3) (Venus) 3, $2576185 \times 10^{5}$ <br> SMU (4) (Jupiter) $1.2690142 \times 10^{8}$ <br> SMU (5) (Saturn) $3.7957172 \times 10^{7}$ <br> SMU (6) (Sun) $1.3291083 \times 10^{11}$ |
|  |  |  | The influence of any or all of the solar bodies may be deleted by setting the appropriate mu's to zero. For conic analysis, for example, 5 of the mu's are set to zero, and only the one for the primary body of conic problem is nonzero. |
|  |  |  | Format: |

3 words per card, each word is El5.8.

## 3. Description of Cutput Data Formats

The program puts out printed data only, and this section describes the formats of the printed records. There are five basic types of printed records to be described:

1. A record of the input data as read from the input daca cards.
2. An identification record.
3. The basic data record.
4. An end of cycle record.
5. Csculating conic data record.

The program operations and options which determine the sequence of these records are explained in Ref. (1). The subsections below describe these basic record types and indicate the modifications caused by different program modes and options.

In addition to the normal data records, the program prints special messages, most of which are intended to indicate an error condition in the computations. Most of these special messages are self-exaplanatory, but some of the more commonly appearing ones are illustrated in the last subsection.
3.1 Record of Input Data as Read From Input Data Cards

This record begins the printed listing for each execution of the program. The purpose is to list the input data cards in an unmodified format for convenient reference if the need arises. Fig. 3-1 shows a typical illustration of this record together with other records to be described below. The card identification is noted in the figure, and the formats may be compared directly with the card formats shown in Fig. 2-1.

### 3.2 Identification Record

Fig. 3-1 shows a typical identification record which appears with each execution of the program. The information in the record is self-explanatory. Only a few comments are necessary about interpretation of some quantities.
どこし

The body number identification table is shown for convenient reference. "Launch" and "target" designations generally refer to the primary bodies at the injection and arrival points on the trajectory. Thus the "launch planet", for example, might very well be the Sun if the program is used to determine the effect of a midcourse correction.

The time conversions done in the program suffer two types of errors. The first is the error in binary-to-decimal conversion of the numbers in the computer during the printing. The second error arises from the limited 8 -decimal figure word length in the computer. Time quantities in which the number of significant digits exceed this limitation suffer round-off errors. Generally speaking, dates and durations exceeding 100 days should be rounded to the nearest whole second for both input and interpretation of output data. For smaller times, fractional seconds are handled accurately in the program to within the 8 decimal figure precision limitation.

Reference to ecliptic coordinates appears in the Identification Record because these are the coordinates most often used. In the event the equatorial or other coordinates are used, this reference will of course be in error.

### 3.3 Data Record

Fig. 3-2 shows the basic data for Mode 1 of the program. This record is modified for different time points in the trajectory integration and print options. The basic record shown is explained in the following paragraphs and the modifications are pointed out.

At key time points in the trajectory integration the record will have an identifying title at the top. The title and time point correspondences are listed below:

| Starting point | $* * *$ TRAJECTORY STARTING POINT |
| :---: | :---: |
| Phase change |  |
| point | *** PHASE CHANGE POINT-DATA |
| Before phase | REFERENCED TC OLD PRIMAR |
| change | BCDY |

 $10380283 E 02$
204514126440
$10380283 E 02$
$83469652 E-08$ -. $35970882 E 101$

 Matrix
$-.86451766 E 00$
-.17386288 E 00
.13928973 E 03
$-.18314472 \mathrm{E}-01$
$-.46000486 \mathrm{E}-02$
.10011728 E 01
.21821308 E 00
-.29098029 E 00
$-.15258789 \mathrm{E}-04$
$.16969051 \mathrm{E}-02$
$-.27302583 \mathrm{E}-02$
.10049741 E 01
$10-387059618^{\circ}$
$40-326999026^{\circ}-$
$50-3882950 ヶ \varepsilon^{\circ}-$
$80-329866998^{\circ}-$
$40-315565921^{\circ}-$
$50-382 \angle 9591 \varepsilon^{\circ}$
$-.17996569 E-05$
$-.64111590 \mathrm{E}-06$
$.18138734 \mathrm{E}-06$
$-.63254680 E-17$
$-.72776254 E-17$ -. $41018878 \mathrm{E}-13$
$\begin{array}{r}.77240524 E-06 \\ -.10372091 E-05 \\ \hline .64111590 E-06\end{array}$

$.60924681 E-13$ $.609106217 \mathrm{E}-13$
$-.72776254 \mathrm{E}-17$ $-.63254680 \mathrm{E}-17 \quad-.72776254 \mathrm{E}-17$

## $.85582175 E-06$ $.77240524 F-06$

- 

$.11912662 E-13$
$.60924681 E-13$
$.13978776 E 03$


45013428 E-03

$.86309219 \mathrm{E}-08$
$.75643438 \mathrm{E}-04$ $.26555524 \mathrm{E}-02$
$.95630141 \mathrm{E}-04$ $.95630141 \mathrm{E-04}$
$.10789688 \mathrm{E}-04$
$11165005 E 09$


STATE TRANSITION MATRIX

## g(PERTURBATION) MATRIX


-. 387
 $.21455288 E-02$

## 

 $. .75307544 \mathrm{E}-04$


| After phase change | *** | PHASE CHANGE POINT-DATA REFERENCED TO NEW PRIMARY BODY |
| :---: | :---: | :---: |
| Endpoint | *** | TRAJECTORY ENDPOINT |
| Special printout point | *** | REQUIRED PRINTCUT POINT N |

In Modes 2 and 3 each data record will have one of the titles listed above followed by the information appearing in Fig. 3-2. In Mode 1, the majority of data records have no title.

TIME FROM START appears both as whole number of seconds and days, hours, minutes, and seconds with starting time as the zero reference. For integration backward along a trajectory, the whole number of seconds will be preceded by a negative sign, but the days, hours, minutes, and seconds quantity does not have a negative sign within the parentheses.

The primary body may be identified from PRIMARY BODY NUMBER $N$ by reference to the identification table in the Identification Record.

The positions and velocities are given in primary-body centered coordinates (usually ecliptic) and in units of km and $\mathrm{km} / \mathrm{sec}$. The true velocity is given in octal as well as decimal format to facilitate initialization of other trajectory computations without suffering a decimal-tobinary conversion error in reading in this sensitive parameter. ENCKE POS and ENCKE VEL are the conic state variables in the Encke method (Ref. 1). DELTA POS and DELTA VEL as the numerically integrated corrections to the conic state in the Encke method. In all cases the listed components are in $x, y, z$ order.

GRAV FORCE is the many-body perturbation gravitational force per unit mass (acceleration) used in the Encke method (see Eq. 3.4, Ref. (1) ). Units are $\mathrm{km} / \mathrm{sec}^{2}$ and components are in $x, y, z$ order.

G(PRIMARY) and G(PERTURBATION) are the gravity gradient matrices of the primary body attraction and the vector sum of the
perturbing body effects, respectively. Units are $\mathrm{km} / \mathrm{sec}^{3}$ (See Section 3. 2, Ref. (1)).

STATE TRANSITION MATRIX is defined in Section 3.2 of Ref. (1). The matrix relates state perturbations at the present time point to small variations in the initial state. The units are as follows:

$$
\begin{aligned}
& \text { upper left } 3 \times 3 \text { submatrix }-\mathrm{km} / \mathrm{km} \\
& \text { upper right } 3 \times 3 \text { submatrix }-\mathrm{km} / \mathrm{km} / \mathrm{sec} \\
& \text { lower left } 3 \times 3 \text { submatrix }-\mathrm{km} / \mathrm{sec} / \mathrm{km} \\
& \text { lower right } 3 \times 3 \text { submatrix }-\mathrm{km} / \mathrm{sec} / \mathrm{km} / \mathrm{sec}
\end{aligned}
$$

The two matrices following the state transition matrix are described in some detail in Section 2.2.2. 2 of Ref. (1). They are accuracy checks on the state transition matrix computations, and are optional, depending on the value of KPHINV in the program input data (see... Section 2, and Card No. 5, of this manual).

The planet position data are printed only in Mode 1. The units are km, and the components are in $x, y, z$ order. The reference to ecliptic coordinates appears because those are most often used. If equatorial or other coordinates are used, the position data will be in the correct coordinates, and the ecliptic reference will be in error.

The modifications of this record format which normally occur are listed below:

Option or Mode
Modes 2,3
KPHINV $=0$

Mode 3

Phase change point

## Modification

Planet position data do not appear.
PRODUCT CF STATE TRANSITION MATRIX AND ITS INVERSE and RMS MATRIX do not appear (both Modes 1 and 2).

STATE TRANSITION MATRIX and all following data do not appear.

Normal data record occurs before phase is changed, after phase is changed a shortened record is printed with state transition matrix and subsequent data deleted.

### 3.4 End of Cycle Record

The end of cycle record appears after the end point of a full. trajectory computation. It appears in Mode 1 automatically, and in Modes 2 and 3 it appears either at the end of each search cycle or only at the end of the last cycle, depending on the input data control NCYCLE (see Section 2, Card No. 5) .

The basic record format as it appears in Mode 1 is shown in Fig. 3-3. The updated starting velocity appears in both decimal and octal formats; units are $\mathrm{km} / \mathrm{sec}$. The matrix product is an error check on the inversion of the upper right $3 \times 3$ submatrix $N$ of the state transition matrix. The inversion is done in the computation of the updated velocity. The miss vector is the computed endpoint position vector minus the target position vector. The $x, y, z$ components and the magnitude of the miss are listed; units are km .

In Modes 2 and 3 only the miss vector appears in the end of the cycle record. The other two printed quantitites are deleted in these search modes.

### 3.5 Osculating Conic Data Record

Osculating conic data are computed and printed only when the NOSC input data control is specifically set (see Section 2, Card No. 5). Then the osculating conic data record follows each normal data record and lists parameters of the osculating conic of the actual orbit at that time point.

Fig. 3-4 shows an osculating conic data record. The type of conic is identified initially, and the data following are referred to the primary coordinates of the program, that is, planetocentric or heliocentric coordinates, depending on the primary body. UXI, UETA, and UZETA are unit vectors locating the principle axes of the oscula-. ting conic.

## RECORD

.sist4t37E C?
$-.26 C 18 C 18 E O$
$6 C E \subseteq 15 C 17534$
$-.13694 C 71 E 02$
$60466615252 \epsilon$


.9 ScSSSSSE CC
$.59 \in 62852 E-09$
$-582076 \in 1 E-1 C$
MISS VECTCR -.43252832E 03
UPDATED STARTIVG VELCCITY
MATRIX PRCDUCT N*N-INVERSE






FIGURE 3－4


$$
\begin{aligned}
& .065136496 \\
& .994874999 \\
& .077336164 \\
& .0 c 000018 \\
& .997005068 \\
& .98554233 \\
& .150671719 \\
& .468215436 \\
& .880223520 \\
& 6 \text { ANGLE } \\
& \text { UMENT OF PE }
\end{aligned}
$$



UXI - in plane of conic in the direction of periapse.
UZETA - perpendicular to the plane of the orbit in the direction of the angular momentum vector.
UETA - completes right-handed coordinate set XI, ETA, ZETA ULN; UM, and UZETA are unit vectors, locating another set of osculating conic coordinates:
ULN - along the line of ascending nodes.
UZETA - as above.
UM - completes right hand coordinate set LN, M, ZETA UR, US, and UZETA are unit vectors locating the $R, S, Z$ coordinate system:
UR - along the radius vector from primary body to the current point on the orbit at which the osculating conic is defined.
UZETA - as above.
US - completes right-handed coordinate set $R, S$, ZETA UP, UQ, and UZETA are unit vectors locating the $P, Q, Z$ coordinates (Ref. (3)):
UQ - along the velocity vector at the current point on the orbit at which the osculating conic is defined.
UZETA as above.
UP - completes right-handed coordinate set $P, Q$, ZETA FLIGHT PATH ANGLE GAMMA AND ANGLE $G$ have the special definitions givin in Ref. (3) and are associated with the $P, Q, Z$ coordinate system. Units are radians.
The three angle parameters of the osculating conic all have standard definitions with respect to the primary $x, y, z$ coordinate system. Units of these angle are radians.

The following osculating conic parameters listed depend upon whether the conic is an ellipse or hyperbola. For an hyperbola the semi-major axis (km), eccentricity, time of pericenter passage
(seconds), true anomaly (radians), hyperbolic anomaly, and "hyperbolic mean anomaly" are listed. The latter three quantities are all referenced to pericenter, and the last quantity is defined as

$$
\text { hyperbolic mean anornaly }=\sqrt{\frac{d}{f a f^{3}}}(t-T) \text {. }
$$

For an ellipse the semi-major axis ( km ), eccentricity, time of pericenter passage (seconds), true anomaly (radians), eccentric anomaly (radians), and mean anomaly are listed. All anomalies are referenced to pericenter.

### 3.6 Special Messages

At a number of points in the program special messages are printed to indicate either error conditions in the computations or changes in the operating sequence. An example of the latter type appears in Fig. 3-1. This message is specifically chosen for illustration because it appears regularly in data listings. It indicates rectification of the Encke conic in the ENCON subroutine, the number of the primary body for the new conic, and the time of rectification referenced to start time.

Error messages which appear are always self-explanatory. They emanate from a number of points in the program, and reference to the flow charts in the next section shows these error message sources.

## 4. Program Flow Charts

The following pages show the flow charts of the main program and each of the subroutines and function subprograms. Section 4 of Ref. (1) briefly explains the purpose of each subprogram. The charts show detailed functional flow; detailed computational procedures are shown. They are intended to make the program listings easily understandable.

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NOTE: Equation numbers refer to
Section 3.3 of refenence 1
INTEG Subroutine Flow Chart






目国
7xevo motit uresioxdqns uotpouns DVW


$$
\begin{aligned}
& \text { Call } A \times B(A, B, \text { VRKN) } \\
& \text { Compute } \\
& \text { YPRDD }=A \times B
\end{aligned}
$$

ADOTB Function Subprogram Flow Chart

Call TIME TARGG, SDAY, THR, ,MIN, XSEC
TIME Subroutine Flow Chart

