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AN EXPERIMENTAL STUDY OF LIQUID SURFACE OSCILLATIONS IN LONGITUDINALLY EXCITED COMPARTMENTED CYLINDRICAL AND SPHERICAL TANKS

by Daniel D. Kana

Prepared by
SOUTHWEST RESEARCH INSTITUTE
San Antonio, Texas
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ABSTRACT

An experimental investigation of liquid surface oscillations has been conducted for liquid propellants in longitudinally excited, rigid, 90° sector cylindrical, and spherical tanks. It is determined that for longitudinal excitation, propellant motions in these tanks are basically a nonlinear 1/2 subharmonic surface motion in some predominant mode, similar to the general liquid behavior that has earlier been found to occur in a rigid cylindrical tank. However, in the case of the 90° sector tank it is found that the surface shape of the predominant mode is different from that of the corresponding mode for lateral translational excitation. Stability boundaries, as well as liquid response curves are presented for two modes under various excitation conditions in each tank.

INTRODUCTION

The importance of liquid propellant dynamics on the overall stability of space vehicles has long been recognized, and much data has been accumulated from studies of liquid propellant behavior under various dynamic conditions. Most of these studies have concentrated on liquid surface responses to translational or pitching inputs to rigid tanks of various geometries. More recently, attention has focused on liquid surface responses resulting from longitudinal inputs, owing to the occurrence of strong longitudinal transients at liftoff, and longitudinal oscillations (POGO) that have occurred on some vehicles during flight.

Studies to date have indicated that liquid surface responses resulting from longitudinal oscillational inputs are considerably more complicated than those for either translational or pitching inputs, since in this case the liquid surface first experiences an instability in small motions, and the resulting response is a nonlinear periodic motion whose dominant component is a 1/2 subharmonic wave in some corresponding mode. Many investigators have studied the behavior of liquid surfaces under vertical excitation, the more recent of these being Dodge, et al, (1) who have studied the problem more from the point of view of space vehicle applications. Liquid surface subharmonic responses were studied both analytically and experimentally for the circular cylindrical tank, and the complexity of the nonlinear analysis required to predict such responses was amply demonstrated.

Since tanks of various geometries are used in different vehicles, it is apparent that liquid surface responses for longitudinal inputs must also be considered for tanks of other types. This report presents the results of current experimental work that has been conducted to study the corresponding liquid surface responses in 90° sector cylindrical and spherical tanks, in order to extend the knowledge of liquid behavior under longitudinal vibration to tanks of these geometries. Concurrently, a theoretical analysis of this problem, applicable to tanks of arbitrary geometry, has been conducted, the results being presented in a separate report (2)*.

Experiments of the present study were chosen so that a good general description of the overall liquid behavior in each of the tanks could be presented with a minimum amount of data; hence, only the first two modes considered to be most important in each tank were investigated. The effects of liquid depth were considered in the spherical tank only, since in the 90° sector tank, as in a cylindrical tank, depth effects become significant only at depths below one-quarter full. For each mode investigated, both stability boundaries for small motion, as well as the resulting nonlinear responses are presented, since both are required for an overall description of this complicated type of liquid response.

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* Unfortunately, it was not possible to carry out computations for either the compartmented cylindrical or spherical tanks, so that no comparisons can be made with the experimental data given here. The numerical example given in (2) was the rectangular tank, chosen for simplicity.

EXPERIMENTAL APPARATUS

The experimental observations of the present study were quite similar to those conducted in the cylindrical tanks in the earlier study (1); therefore, the apparatus used here is essentially the same as that described for that work, except for modifications required by the different geometry of the tanks used.

Basically, the apparatus consists of the model fuel tank, mounted on an electrodynamic shaker so that it can be excited vertically, perpendicular to the liquid free surface. The excitational motion is guided by a fixture designed to minimize lateral perturbations, and is monitored by means of a transducer located on the shaker armature, the output of which is recorded on an oscilloscope and frequency counter.

Liquid surface motion is monitored by a specially designed liquid displacement transducer, whose output is also observed on the oscilloscope and frequency counter. The details of this transducer have been reported in (3). It can be described briefly as a device whose signal is proportional to the varying electrical conductance path that is generated by the liquid surface, as it oscillates up and down, over two closely spaced sensor wires that extend into the liquid. The sensors are, of course, positioned so that they will monitor the motion of some desired single point of the liquid surface. In the present study these sensors had to be secured in the desired position by a different method than was used in the earlier study, because of the difference in the geometry of the tanks used.

Observations were made of liquid responses in two different 90° sector cylindrical tanks, and a spherical tank. One sector tank was made from a 14.5 cm inner diameter, 3.18 mm thick, 30.5 cm long, lucite plastic cylinder, having a rigid, flat, aluminum bottom. The solid sector walls were made of 3.18 mm thick lucite plastic, and were completely sealed at the tank bottom and walls so that no interchange of liquid between the sectors was possible. This small sectored tank was used primarily for qualitative observations of the liquid surface behavior. Water was used as the model liquid in this as well as all of the tanks studied. It was found that surface tension effects were significant, particularly in this small tank. Natural frequencies of a given mode tended to shift slightly as water conditions changed, and were as much as 20% greater than theoretically predicted values if ordinary tap water was used. This effect was virtually eliminated by using distilled water having a very small amount of Triton X-100 wetting agent added to reduce the surface tension.

Observations of the liquid motion in the smaller rigid sectored tank indicated, as might be surmised, that the liquid behaved as if it were contained in four separate tanks, each of quarter-section geometry. Therefore, a second, larger tank was used for the detailed experimental observations, in order to minimize further the surface tension effects. This tank was made in the form of a single quarter sector of a 24.6 cm inner diameter, 48.3 cm high, lucite plastic tank, having 3.18 mm thick walls and a rigid, flat lucite bottom. One liquid displacement sensor (3), was located at one joint of the sector and tank wall, and another was located at the joint of the two sector walls, the positions for antinodes of motion in the first and second modes respectively for liquid surface motion in a 90° sectored cylindrical tank.

The spherical tank used for the present study was made of 19.3 cm diameter, 3.18 mm thick lucite plastic. Liquid displacement sensors were located at the center of the sphere for observations of the first symmetric liquid surface mode; however, observations of the first antisymmetric mode required a more elaborate positioning device for the sensors. The plane of the dominant surface motion in this mode tended to shift to different positions for different input conditions, similar to the behavior reported in (1) for this mode, as it occurred in the cylindrical tank. Therefore, it was necessary to secure the liquid sensors to a small fixture inside the tank, so that they could be rotated into any vertical diametral plane of the sphere. Further, they were positioned vertically at one-half the <u>tank</u> radius. This position was chosen as one that would give the best amplitude description at all depths, since the wall curvature complicates the description of the liquid amplitude at the tank wall. It must then be pointed out, that the maximum liquid amplitudes at the tank wall are greater than the measured values given herein, by a factor of approximately 2 b/r, where b is the radius of the quiescent liquid surface at a given depth between h/d = 1/4 and h/d = 3/4.

EXPERIMENTAL RESULTS

The data obtained in the present study is presented here in nondimensional form for convenience in comparing these results to possible behavior in similar tanks of more practical dimensions. It is recognized that various nondimensional forms for the parameters can be used; however, the ones selected were considered to be those that give the best overall description of the data, and still retain a good physical picture of the relationship of one mode relative to another in a given tank. Further, it must be emphasized that the sloshing motions, as they occur for longitudinal excitation, are nonlinear periodic responses whose periods are twice that of the excitational frequency. The dominant component of the responses appear in the form of one of the natural modes of the liquid surface, although it is a 1/2 subharmonic motion. Therefore, the responses will be identified in terms of this dominant mode of the motion, but it must be kept in mind that the basic response frequency is one-half that of the excitation frequency for the longitudinal inputs.

90° Sector Cylindrical Tank

Figures 1a and 1b, respectively show the first and second 1/2 subharmonic modes for the 90° sectored cylindrical tank. It was determined that these basic mode shapes are somewhat different from their corresponding modes as they occur for lateral translational excitation of a sectored tank, (this contrasts with the shapes being essentially the same for a complete cylindrical or a spherical tank). Hence, it appeared desirable to describe the differences for the two cases of excitation. For comparison, a photograph of the corresponding modes for translational inputs are shown in Figure 2a and 2b, respectively.* Further, a forced response curve for lateral translation (4, 5) is given in Figure 3, along with a drawing indicating the direction of lateral excitation considered. Reference here will be made to the small amplitude linear free response frequency, as indicated in Figure 3, although it is recognized, as has been previously reported (5), that even for lateral translational excitation, liquid surface responses are nonlinear at appreciable amplitudes, while the dominant motion remains at the same frequency as the excitation in that case.

Figure 1a shows the basic form of the first 1/2 subharmonic mode in the 90° sectored tank. It may be noted that this mode corresponds to the first mode for lateral translational excitation shown in Figure 2a. However, several basic differences can occur in the shape of the mode for the two cases, in addition to its being excited as a 1/2 subharmonic for the case of longitudinal excitation. For translational excitation, as shown in Figure 2a, this mode always appears as a sector 2-4 (see Fig. 3) resonance, and causes the lower

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* It should be noted that the crazing that appears highly illuminated from the electronic flash photography in Figures 1 and 2, is within the tank wall, and should not be confused with the liquid motion.

frequency peak in the total force response. The motion occurs such that the liquid surfaces in sectors 2 and 4 respond with large motions in phase with each other, having a common nodal line along a diametral line that bisects sectors 2 and 4 of the tank, while the liquid surfaces in sectors 1 and 3 are relatively quiescent. A glance at Figure 1a shows that this is no longer the case for the corresponding response for longitudinal excitation. Here all four sectors respond simultaneously with a large motion that corresponds to that experienced by sectors 2 and 4 for the lateral translation input, and the nodal lines of the motion in each opposite sector pair are now perpendicular to each other. Further, the motion in one sector may be either in or out of phase with that in another sector, at a given excitational amplitude and frequency. In Figure 1a the surfaces in the near and far sectors are in phase with each other, and the surfaces in the left and right sectors are also in phase with each other. This final phase relationship for a given set of input parameters appeared to depend on the initial conditions; that is, any one of the other several possible phase relationships could result, if the same final excitational conditions were approached from different initial conditions.

The second 1/2 subharmonic mode for the 90° sectored tank is shown in Figure 1b and corresponds to the second mode for lateral translation shown in Figure 2b. Here also, differences in the two may be noted. This mode occurs principally as a sector 1-3 resonance, as shown in Figure 2b, and causes the second peak in the total force response shown in Figure 3. For lateral translation the antinode of the sloshing occurs at the tank center for sectors 1 and 3 only, and the two peaks are always out of phase with each other, (in Figure 2b the peak at the tank center is up in the left sector and down in the right sector), while it can be seen from Figure 1b that the antinode of the motion in all four sectors occurs at the center of the tank, and no set phase relationship always results. Here the peaks are up at the center in the front, rear, and left sectors, while the peak at the center in the right sector is down. Any of the other combinations of phase relationships appeared possible, depending on the initial conditions.

Stability boundaries for the first two 1/2 subharmonic liquid modes in the 90° sector tank experiencing longitudinal excitation are shown in Figure 4, while the corresponding liquid response curves for two different fixed excitation amplitudes are shown in Figure 5. The stability boundary data were obtained by setting excitation frequency near twice the natural frequency of the desired mode, and slowly increasing excitation amplitude. Below amplitude values on the boundary, only high order, small amplitude liquid surface motion could be observed, but at the boundary, or above it, the liquid surface would grow into a 1/2 subharmonic motion. Once the motion appeared, it could remain at a relatively small amplitude, or could grow to a very large steady state value. depending on the input conditions, as indicated by the upward jumps in Figure 5. It may be observed that the minima in the stability boundaries occur at exactly twice the linear free natural frequency for a corresponding mode, and agree very well with theoretical values (4, 5, 6) indicated in Figure 3. However, these linear natural frequencies are somewhat higher than those of the corresponding peaks in the total force response for lateral inputs shown in Figure 3. It has been shown in (5) that this difference, as well as the jumps that appear, result from nonlinearities in the response for lateral translational inputs.

The response curves of Figure 5 indicate the final steady state amplitude that is achieved by the 1/2 subharmonic modes after the initial instability of the liquid surface. The amplitude factor plotted is the mean amplitude of the measured peak to peak values. It must be pointed out that up amplitudes are always somewhat greater than down amplitudes, measured from the quiescent liquid level position. The amplitude values were measured at the positions indicated in the previous section. In every case the responses are nonlinear softening; hence, large upward jumps could occur, as indicated, for frequencies below twice the natural frequency for a given mode and, likewise, sudden jumps back down could be experienced at even lower excitation frequency values for the same fixed excitation amplitude. At the jump down, the 1/2 subharmonic motion was lost completely. The dashed lines at large wave amplitudes indicate that the wave was very unstable and complicated, and breaking waves were readily formed. It was observed from the 14.5 cm complete 90° sector tank that both the jumps up and the jumps down did not always occur at exactly the same input conditions for all four sectors; the stability boundaries for each individual sector did not exactly coincide. It was surmised that this behavior resulted from imperfections in tank geometry and input motion. The indication is, as was pointed out in the previous section, that each sector behaved as a separate tank; hence, the data presented in Figures 4 and 5 were taken in the 24.6 cm diameter, single sector tank.

Spherical Tank

The first antisymmetric and first symmetric 1/2 subharmonic modes for the longitudinally excited spherical tank are shown in Figures 6a and 6b, respectively. The amplitudes at which the motion was photographed was relatively large, for it can be seen that a breaking wave has formed in Figure 6a. It might be pointed out that these are not the first two modes occurring for a spherical tank since another mode occurs in between these two, similarly to the cylindrical tank (1); however, these modes were considered to be the most significant from the point of view of space vehicle applications. The basic surface motion for the first antisymmetric mode is the same as that observed in laterally excited tanks except that here, of course, the motion is a 1/2 subharmonic, while the symmetric mode cannot be excited at any appreciable amplitude with lateral excitation.

Figure 7 has been included for convenient reference in comparing minima points for the stability boundaries, shown in Figures 8 and 9 for the two liquid modes considered for three different liquid depths.

The solid line of Figure 7 gives the theoretical natural frequency for the first antisymmetric liquid mode at various depths in a spherical tank (7). It may be seen that frequency parameter values based on twice this natural frequency compare very well with the values at which the minima occur in Figures 8 and 9. To the author's knowledge, there is no similar theoretical curve available for the natural frequencies of the symmetric mode in a spherical tank;* hence, a dashed curve has been faired in through the three points that correspond to the minima in Figure 9, simply for the sake of comparing these values with those for the antisymmetric mode. One further interesting feature of the stability curves can be pointed out: It can be seen in Figure 8 that the excitation amplitude value for the minimum point of the stability boundary for the antisymmetric mode gets progressively larger for lower liquid levels, and similarly the width of the unstable region becomes smaller. This effect is a result of the increase in damping which is known to occur at lower liquid levels. Only the same trend in excitation amplitude at the minimum point holds true for the symmetric mode, as can be seen in Figure 9, as the width of the region appears to be influenced more by geometrical factors in this case.

Liquid responses for the first antisymmetric and first symmetric 1/2 subharmonic liquid modes for the spherical tank, at three different

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* Although one could be computed from the analysis presented by Chu (2).

liquid depths, are shown in Figures 10 and 11. Mean liquid amplitudes are again plotted, but were measured at the tank center for the symmetric mode, and at the one-half radius point for the antisymmetric mode, as indicated in the previous section. Thus, for the antisymmetric mode, the average liquid amplitude at the wall would be approximately 2b/r times greater than the values given in Figure 10. Both responses can be seen to be nonlinear softening, although it would appear that the symmetric response bends more for a given amplitude, than does the antisymmetric mode. This may also reflect the fact that the antisymmetric amplitudes were not measured at the wall where liquid tends to pile up. Here again the thick dashed lines indicate breaking waves at the larger amplitude, and also interference with the top of the tank for the h/d = 3/4 case. The dashed line at lower liquid amplitudes for the symmetric mode at h/d = 1/2 depth, and $X_o/d = 0.00428$, indicates where irregular motion occurred as a result of coupling with a higher mode. Jumps up and jumps down occurred for these modes as indicated, similar to their occurrence for the 90° sector tank.

CONCLUSIONS

In general, liquid surface sloshing in 90° sector tanks and spherical tanks, resulting from longitudinal excitation, has been observed to be essentially similar to its behavior in cylindrical tanks under the same type of forced oscillation. That is, the basic surface response is a large amplitude, nonlinear, 1/2 subharmonic motion that displays a softening characteristic. Responses in the spherical tank appear to be quite similar to their corresponding cases in the cylindrical tank, except, of course, for the greater dependence of the modes on depth in the spherical tank.

Compared to the responses for similar modes resulting from lateral translational excitation, it has been observed that the basic mode shapes of the 1/2 subharmonic responses resulting from longitudinal excitation are different in the case of the 90° sectored tank. The most significant implication of this behavior is that the total lateral force exerted by the liquid on the tank, during the sloshing in one of these modes, can have several different values for the same input conditions, depending on the phase pattern that is set up in the liquid motion in the different sectors. The total force felt by the tank, being the vector sum of the total forces of each individual sector, can have a different value, at the same input conditions, for each possible different phase combination. The total lateral force exerted by the liquid, of course, is of major concern in the design of the guidance system. Further, the fact that both jumps up and down in the motion can occur at slightly different input conditions, because of imperfections in the tank geometry, indicates that a further complicated total force response can be anticipated in any practical tank. Finally, it can be anticipated that motion of the liquid in the first mode of the 90^{o} sector tank, can exert a net torque on the vehicle about its longitudinal axis, if the proper phase relationships exist in the motion. Such a torque cannot occur in a non-sectored tank, or even in a sectored tank under lateral translational inputs.

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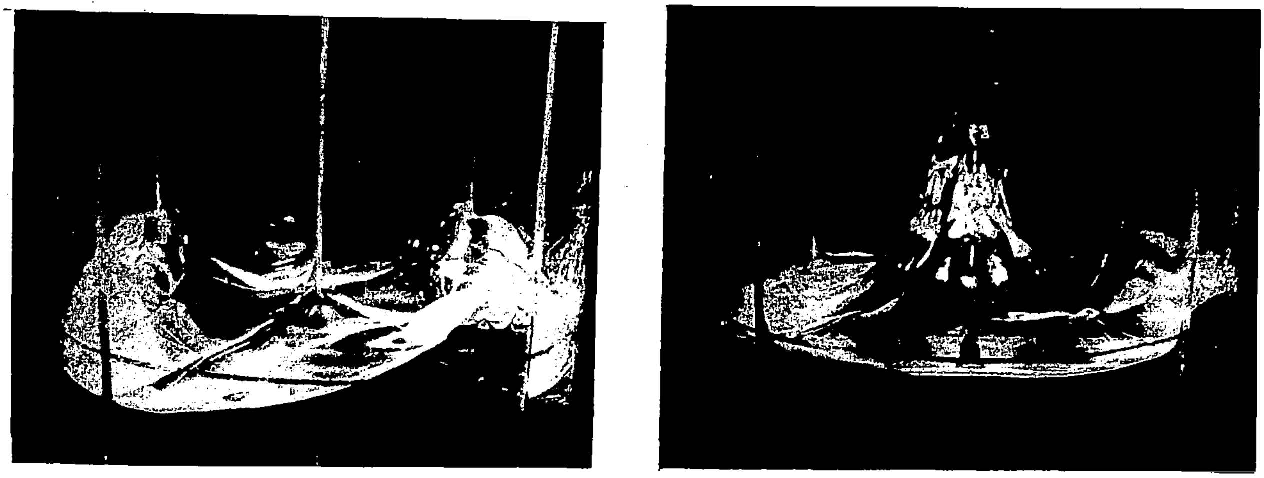
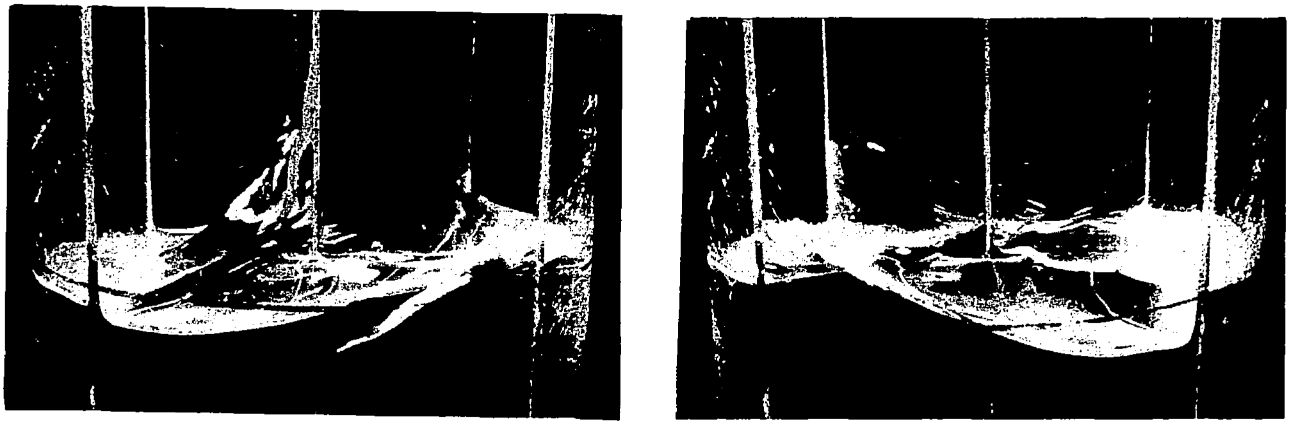


(a) First mode

(b) Second mode

Figure 1. Liquid 1/2-subharmonic motion in a longitudinally excited 90° sector cylindrical tank



(b) Second mode

(a) First mode

Figure 2. Liquid harmonic motion in a laterally excited 90° sector cylindrical tank

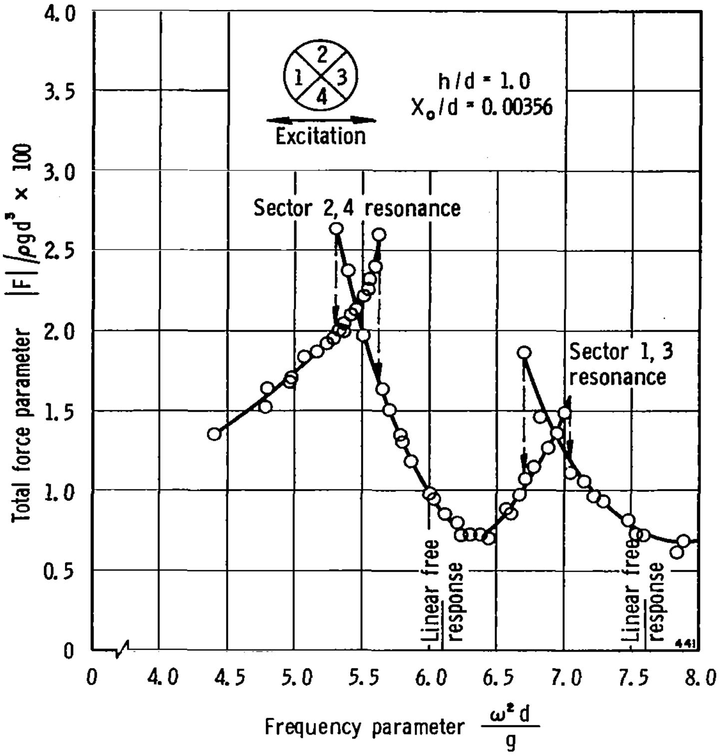


Figure 3. Total force response for laterally excited 90° sector cylindrical tank

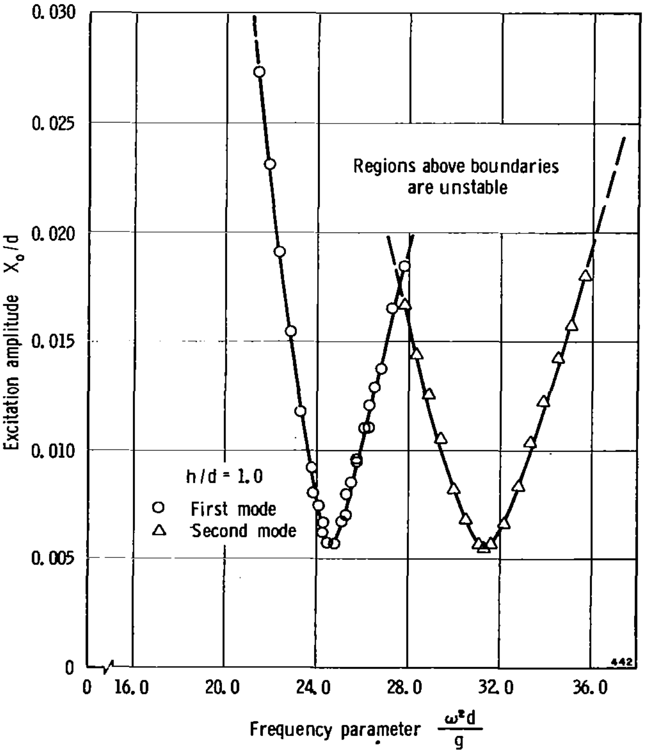


Figure 4. Stability boundaries for liquid in a 90° sector cylindrical tank

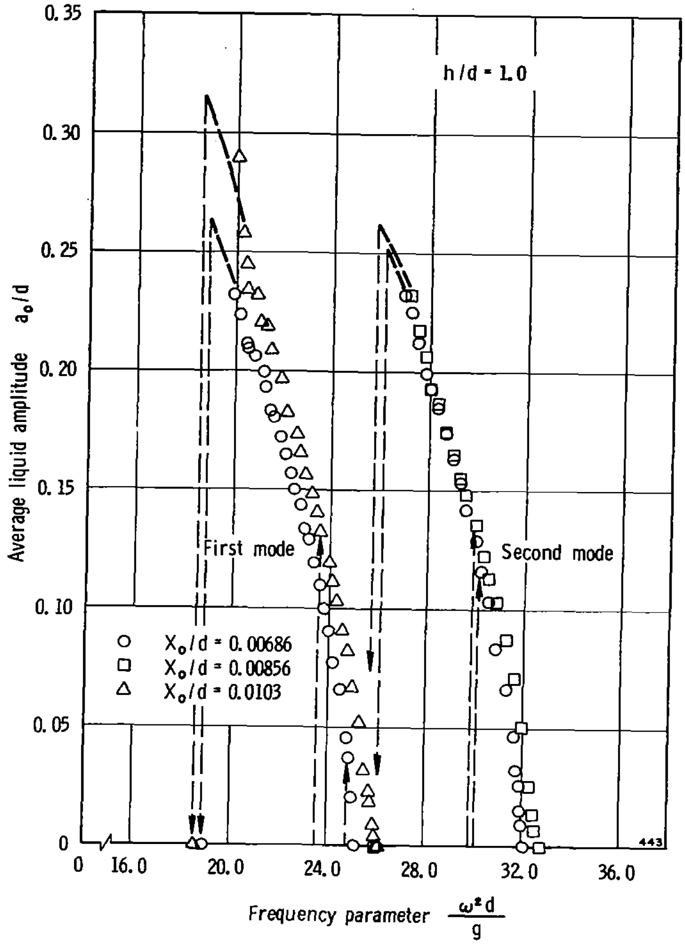
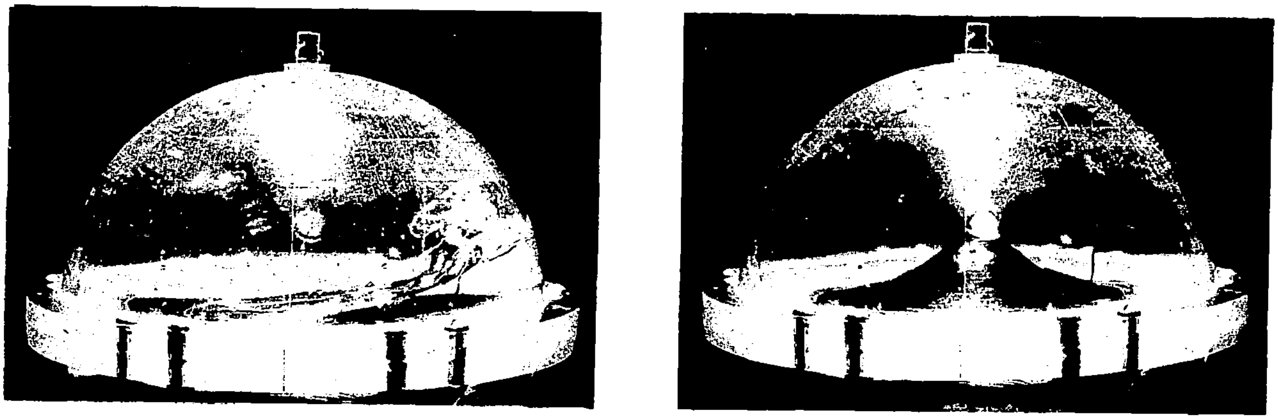


Figure 5. Liquid 1/2-subharmonic response in a 90° sector cylindrical tank



(a) First antisymmetric mode

(b) First symmetric mode

Figure 6. Liquid 1/2-subharmonic motion in a longitudinally excited spherical tank

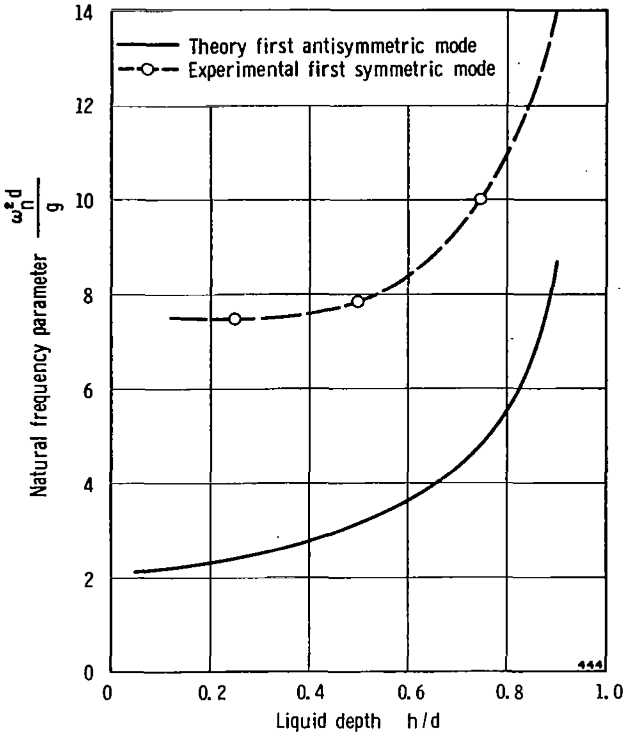


Figure 7. Natural frequencies for liquid motion in a spherical tank

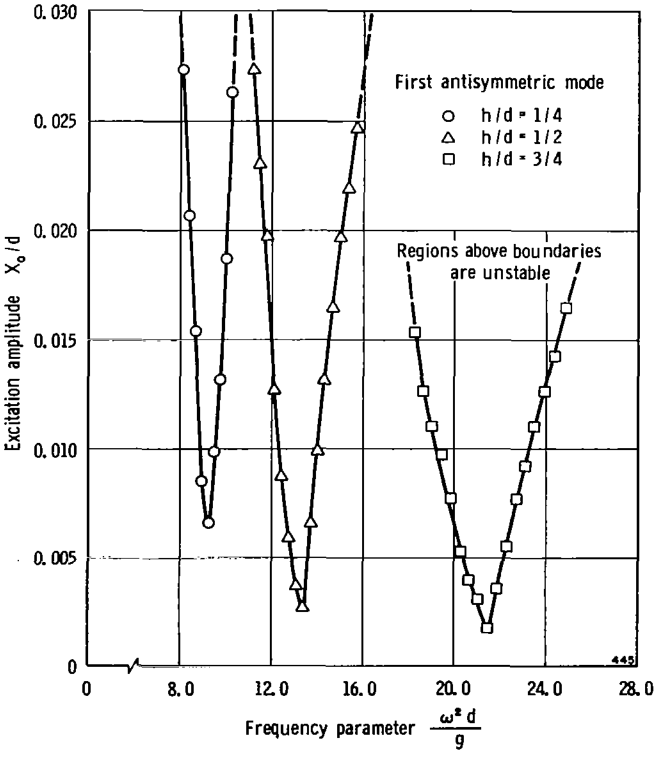


Figure 8. Stability boundaries for liquid in a spherical tank

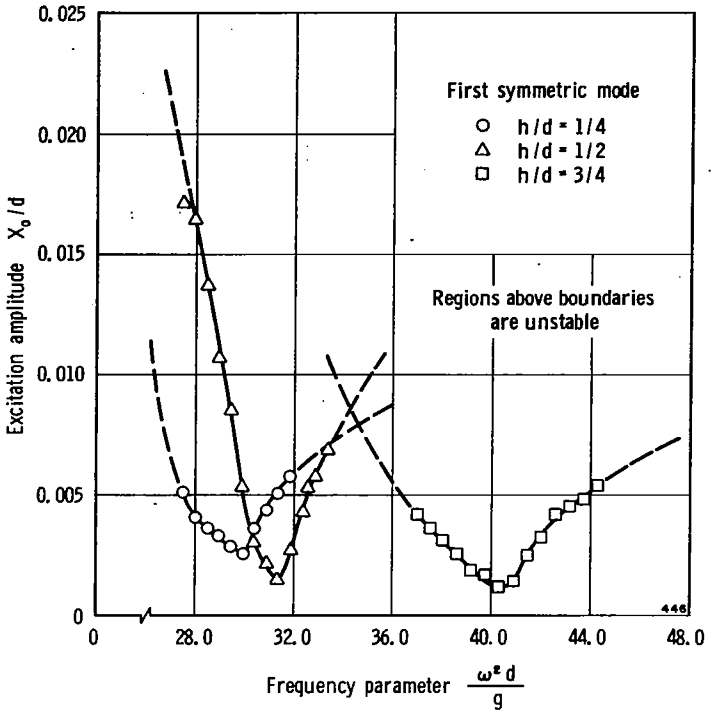


Figure 9. Stability boundaries for liquid in a spherical tank

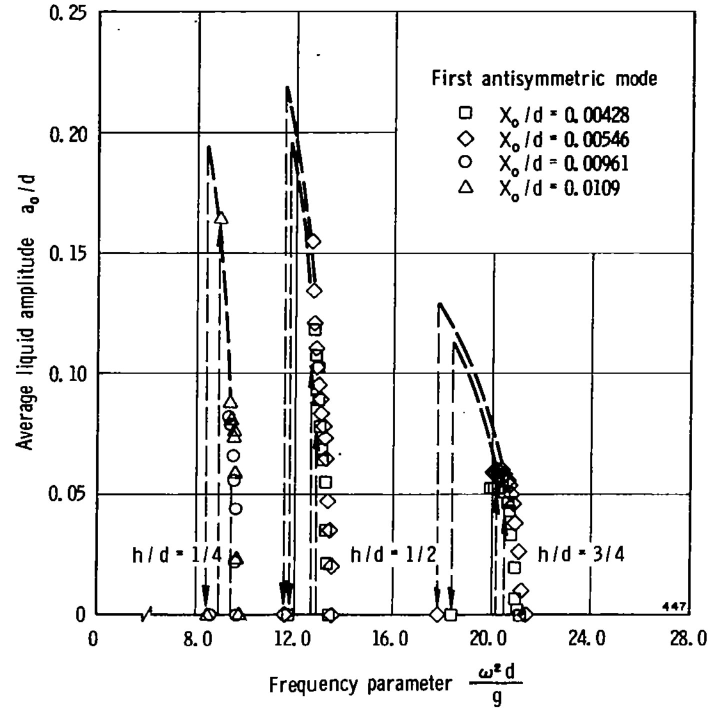


Figure 10. Liquid 1/2-subharmonic response in a spherical tank

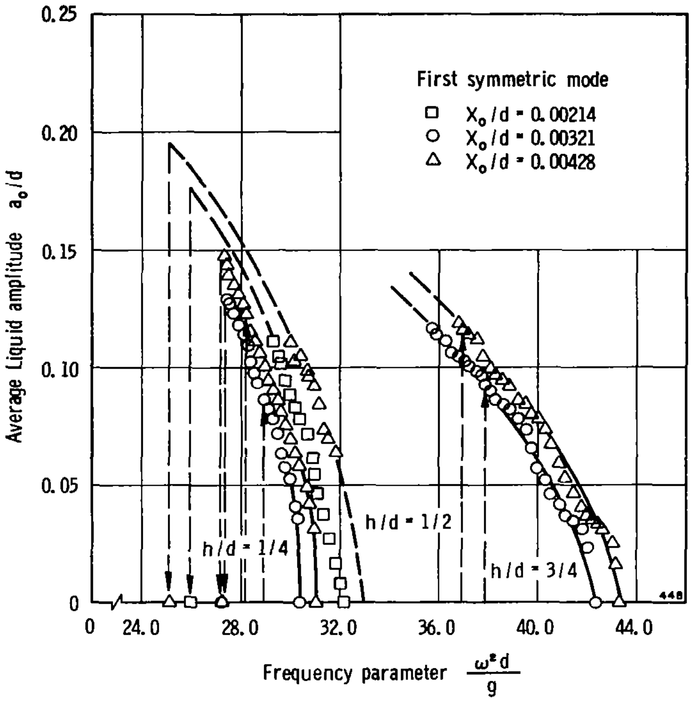


Figure 11. Liquid 1/2 subharmonic response in a spherical tank

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