

# TECHNOLOGY UTILIZATION REPORT

## Technology Utilization Division

# VIBRATING DIAPHRAGM PRESSURE TRANSDUCER

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VIBRATING DIAPHRAGM PRESSURE  
TRANSDUCER

Prepared under contract for NASA  
by Southwest Research Institute

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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August 1966

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## Foreword

Although a number of transducers, working on a variety of principles, have been developed for measuring pressures between  $10^{-4}$  and  $10^3$  mm Hg, all have limitations for many applications. The most commonly encountered limitations are large size, slow response, dependence of response on gas composition, fragile construction, sensitivity to differential rather than absolute pressure, poor accuracy, and intermittent indication. Some of the more important previous transducers are discussed in chapter 4.

To overcome the many limitations of previous sensors when applied to the problem of monitoring pressures encountered in high-velocity wind tunnels, Dimeff et al. (ref. 1) at NASA Ames Research Center developed a new transducer based on the principle of gas damping of a vibrating diaphragm. This device has been used for several years in the high-velocity wind tunnels at Ames Research Center and has given good results in measuring pressures from atmospheric down to about  $10^{-4}$  mm Hg.

Improved accuracy, speed, and reliability in measuring pressure in this range would be worthwhile in many industrial process and laboratory applications. The Ames vibrating diaphragm transducer could make these improvements possible.

This report was prepared by J. Derwin King of Southwest Research Institute, under contract to the Technology Utilization Division of the National Aeronautics and Space Administration.

GEORGE J. HOWICK, DIRECTOR,  
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## Chapter 1. General Description

The vibrating diaphragm pressure transducer is an instrument for measuring absolute gas pressure between  $10^{-5}$  and  $10^3$  mm Hg. When used with appropriate electronic circuitry, continuous pressure indication is possible with an accuracy of 1 percent over most of the range.

The transducer measures pressure by sensing the damping effect of the gas on the vibration of a thin metal diaphragm. The diaphragm is under radial tension and is electrically maintained in continuous vibration at its mechanical resonant frequency. Thus, the transducer basically is similar to Langmuir's quartz fiber gage (ref. 2) and its modified versions by Haber and Kerschbaum (ref. 3) and Wetterer (ref. 4), all of which measure pressure through the damping effect of the gas on a vibrating structure. However, the vibrating diaphragm gage is a great improvement over the earlier instruments which were delicate, difficult to use, and limited in their useful range. They also required a large gage volume, had long response times, and did not provide continuous indication. In contrast, the Ames device is small, relatively rugged, and reliable. It also has fast response, is not subject to damage by overpressure, does not require a vacuum reference, and lends itself readily to automatic operation.

The vibrating diaphragm transducer is shown assembled in figure 1. Figure 2 is a simplified sketch of the transducer which is composed of a thin, metallic diaphragm under radial tension, between two closely spaced, insulated metal plates. The two metal plates provide a means of electrically forcing and detecting the diaphragm's vibration. The space between the plates and the diaphragm is filled with gas at the pressure to be measured. Gas damping causes a loss of energy from the diaphragm, and this loss is sensed by its effect on the diaphragm damping factor. The electrical power required for a given amplitude of vibration is therefore a measure of the gas pressure.

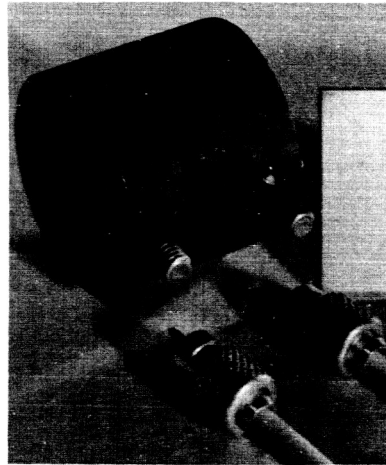


FIGURE 1.—Assembled vibrating diaphragm pressure transducer, shown approximately one and one-half times actual size.

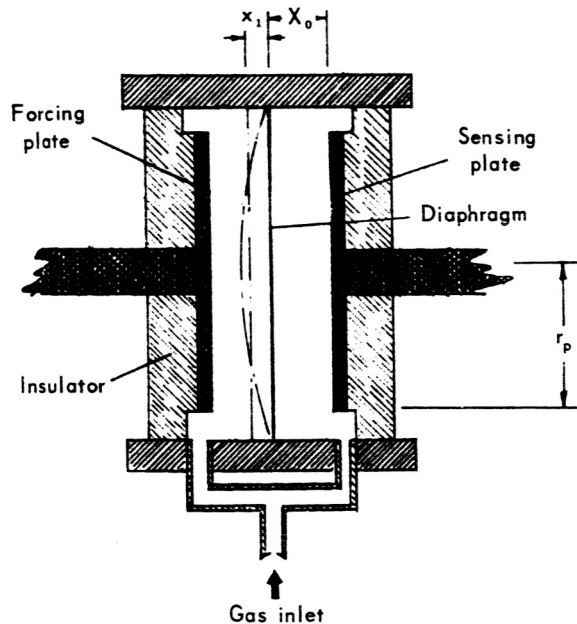


FIGURE 2.—Simplified sketch of the transducer.

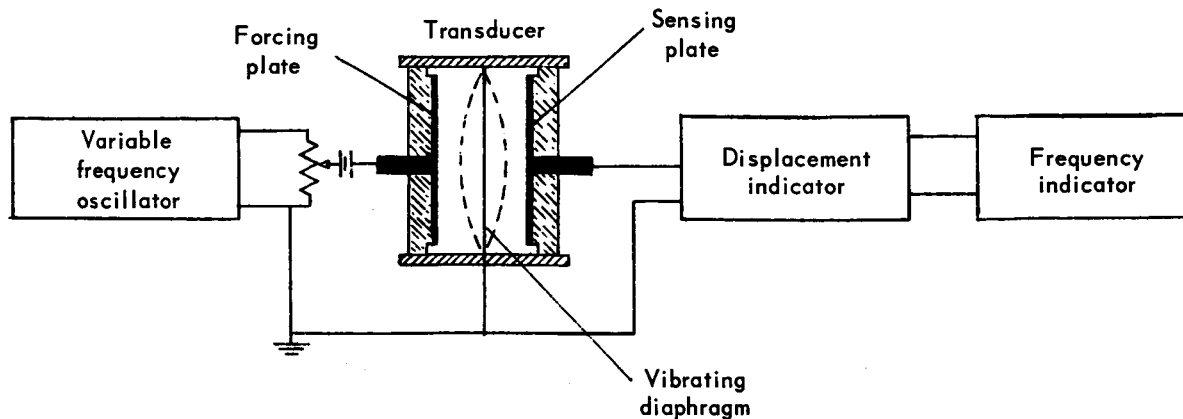


FIGURE 3.—Basic pressure-measuring system, showing simplest mode of use.

Figure 3 shows a simplified arrangement of the transducer and associated electronics. The forcing plate of the transducer is supplied with a dc potential and an ac voltage from an electronic oscillator set at the mechanical resonant frequency of the tensioned diaphragm. The electrostatic forcing function thus generated causes the diaphragm to vibrate synchronously with the ac driving voltage and at an amplitude which is a function of the driving signal and the diaphragm damping factor  $Q$ . The diaphragm's vibration causes a corresponding periodic variation in the capacitance between the diaphragm and the sensing plate that may be detected and used to indicate the vibration's amplitude. Since the power required for a given amplitude of vibration is a function of the damping on the diaphragm, which in turn is a function of the gas pressure, the power can be monitored as an indication of the pressure.

The basic circuit in figure 3 would not be suitable for actual use because of the difficulty in maintaining the oscillator on the exact mechanical resonant frequency of the diaphragm which can have a  $Q$  of about 20 000 at low pressures. In addition, the circuit does not take advantage of the inherent transducer characteristics that permit automatic operation. The electronic system shown in figure 4 was developed to overcome these problems. With this arrangement the vibrating diaphragm transducer itself is used as the frequency-selective element of the electronic oscillator, so that the ac driving signal is automatically maintained at the diaphragm mechani-

cal resonant frequency. In addition, a servo system automatically forces the diaphragm vibration level to follow a selected curve by controlling both the ac and dc signals applied to the fixed plates of the transducer. The servo control is connected to an indicator dial which may be calibrated in units of pressure.

A dc potential  $V_0$  is applied to the transducer sensing plate through a very high resistance  $R_1$ . The variation in capacitance between the diaphragm and the sensing plate produces an ac output voltage  $V_2$ . If the change in capacitance occurs over a time interval that is short compared with the  $R_1C_0$  time constant, then

$$V_2 = V_0 \frac{C_1}{C_0} \quad (1)$$

where  $C_1$  is the change in capacitance and  $C_0$  is the static capacitance. Since the variation in capacitance is sinusoidal,  $V_2$  is sinusoidal, with a peak value dependent on the peak value of the capacitance change.  $V_2$  is amplified by the displacement-sensing amplifier and the output connected to an ac-to-dc amplitude detector and through a phase shifter to a peak-to-peak clipper circuit that produces an output with a fixed peak-to-peak amplitude. A portion of the clipper output is selected by a servo-controlled potentiometer and fed back to the forcing plate of the transducer. If the amplifier gain is sufficient to overcome the losses, the circuit will oscillate at the resonant frequency of the transducer diaphragm with an amplitude dependent on the setting of the clipper-output potentiometer.

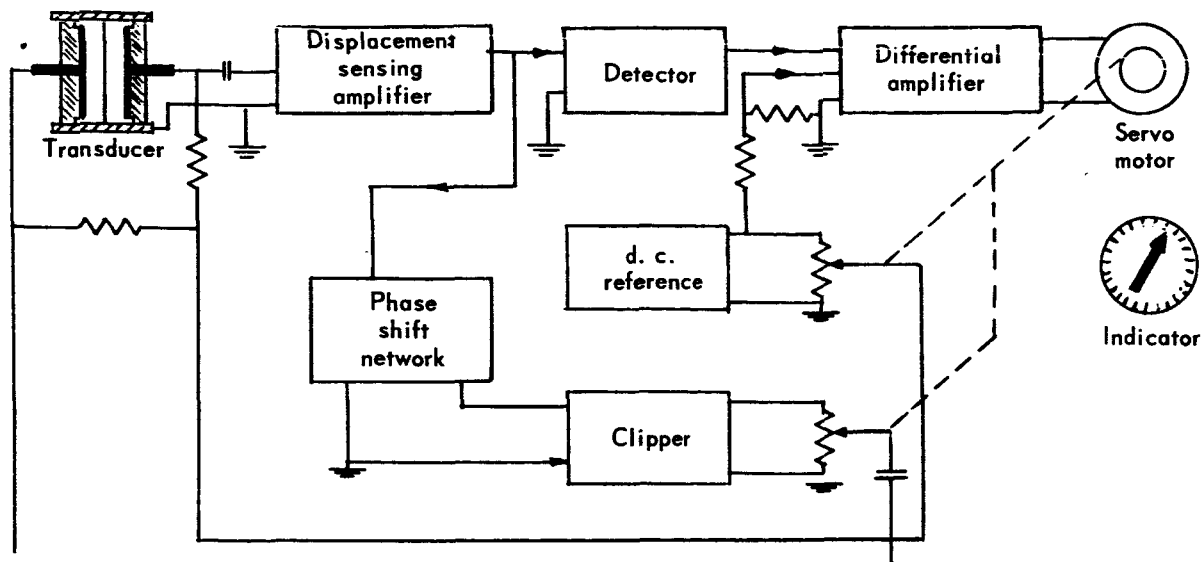


FIGURE 4.—Automatic pressure-measuring system, showing electromechanical servo system.

To control automatically the feedback amplitude, the output of the ac-to-dc amplitude detector is compared with a fixed dc reference potential in a differential amplifier. Any difference between the detector level and the dc reference level causes the servomotor to rotate so that the difference is reduced by altering both the dc applied to the transducer plates and the portion of the clipper output applied to the forcing plate. The control action is such that as the gas pressure is decreased, a smaller portion of the clipper output is used, because of the decreased loss of energy from the diaphragm to the gas. To increase the pressure range, the dc voltage applied to the fixed transducer plates is also reduced as the damping is decreased. Therefore, higher dia-

phragm vibration levels are necessary to maintain the detector output voltage equal to the fixed reference at the input to the differential amplifier. The result of controlling both the dc bias and the ac driving voltage is that the servomotor shaft rotation is approximately proportional to the cube root of the pressure.

With the system shown in figure 4, the mechanical servo limits the time response to transient pressure changes. A transistorized electronic servo system has been designed using similar concepts, and provides a dc output voltage proportional to pressure. This system has a frequency response of up to 100 cycles per second and has provisions for an expanded scale over selected portions of the pressure range.



## Chapter 2. Theoretical Considerations

### ENERGY LOSSES

Energy lost through diaphragm vibration must be replaced for the diaphragm to vibrate at a required amplitude. The total energy loss is the sum of the electrical energy losses, the mechanical energy losses, and the energy losses caused by the gas. Electrical energy losses result from resistance and dielectric losses in the transducer driving and sensing circuits as well as in the amplifier input circuit. Mechanical losses are caused by the presence of viscous foreign materials on the diaphragm, the vibration of the diaphragm-edge support, and the transfer of energy from the transducer housing to the transducer support. For good transducer sensitivity and accuracy, the electrical and mechanical losses must be very constant and preferably very small compared with the energy loss from gas damping. The low-pressure end of the range is determined by the smallest electrical and mechanical losses possible in a practical instrument as compared with the loss from damping.

#### Electrical Power

The electrical power required to maintain diaphragm vibration can be determined by the method of Dimeff et al. (ref. 1). The electrostatic force between the diaphragm and the forcing plate is given by

$$F = \frac{eV^2}{2x_0^2} \quad (2)$$

where

$F$  force per unit area

$e$  dielectric constant

$V$  voltage applied to the forcing plate

$x_0$  distance between the forcing plate and the diaphragm

If  $V = V_0 + V_1 \sin \omega t$ , equation (2) becomes

$$F = \frac{e}{2x_0^2} \left[ V_0^2 + \frac{V_1^2}{2} (1 - \cos 2\omega t) + 2V_0V_1 \sin \omega t \right] \quad (3)$$

Since only the fundamental frequency term is transferred through the transducer to any appreciable extent, all except the last term within brackets in equation (3) may be ignored. The force function is then simplified to

$$F = \frac{eV_0V_1}{x_0^2} \sin \omega t \quad (4)$$

By integrating the product of the electrostatic force times the diaphragm velocity over the area of the stationary forcing plate for a complete cycle of displacement, the following expression is obtained for the electrical work per cycle:

$$W_e = \frac{\pi^2 e V_0 V_1 r_p^2 \Delta C}{x_0 C} \quad (5)$$

where

$r_p$  effective radius of forcing plate

$\Delta C$  peak capacitance change

$C$  capacitance between forcing plate and undisturbed diaphragm

For small displacements occurring at the first vibrational mode of the diaphragm and for assumed spherical deformation

$$\frac{\Delta C}{C} = \frac{x_1}{x_0} \quad (6)$$

where  $x_1$  is the peak displacement of the diaphragm averaged over the area of the fixed plate. The

equation for average electrical power is then

$$P_e = \frac{\pi r_p^2 e V_0 V_1 x_1 \omega}{2x_0^2} = \frac{A e V_0 V_1 x_1 \omega}{2x_0^2} \quad (7)$$

where  $\omega$  is the resonant frequency of the diaphragm in radians per second and  $A$  is the area of the diaphragm.

#### Gas Losses

Dimeff et al. (ref. 1), following the methods of Crandall (ref. 5), Dushman (ref. 6), and Hurlbut (ref. 7), derived an equation for the power loss  $P_g$  from the vibrating diaphragm to the gas. For frequencies and dimensions used in the transducer and for pressures where the mean free path is large compared with  $x_0$ , this equation is

$$P_g = 16\pi x_1^2 \beta \bar{c} p_0 \quad (8)$$

For higher pressures

$$P_g = \frac{8\pi x_0}{9\mu} x_1^2 p_0^2 \quad (9)$$

where

$p_0$  equilibrium pressure being measured by the transducer

$\beta$  momentum transfer accommodation coefficient, characteristic of the gas and surface involved

$\bar{c}$  molecular velocity

$\mu$  viscosity coefficient

Other investigators (ref. 8) have also analyzed and evaluated the vibrating diaphragm transducer and arrived at expressions for the power loss in the gas in both the continuum and free molecule pressure regions. For the continuum region

$$P_g = \left( \frac{\pi \gamma^2}{6\mu} \right) x_0 x_2^2 p_0^2 \quad (10)$$

For the free molecule region

$$P_g = \left( \frac{\pi R T \gamma^2}{2\bar{\alpha}_1 \bar{c}} \right) x_2^2 p_0 = \frac{\pi^2 \bar{c} m \gamma^2}{16\bar{\alpha}_1} x_2^2 p_0 \quad (11)$$

where

$\gamma$  ratio of specific heats

$R$  gas constant

$T$  temperature

$\bar{\alpha}_1$  analogous shear stress coefficient

$m$  molecular weight of the gas

$x_2$  peak diaphragm displacement in center  
( $\approx (\pi/2)x_1$ )

Equations (9) and (10) are equivalent if  $\gamma^2 = 192/9\pi^2$ , which is approximately correct for air. Equations (8) and (11) are equivalent if  $\beta = \pi^2 m \gamma^2 / 1024 \bar{\alpha}_1$ , which is a reasonable approximation based on the definition of the parameters.

Equating the electrical power in equation (7) and the power loss to the gas for continuum conditions as given in equation (10)

$$\frac{V_0 V_1}{x_1} = \frac{\pi^2 \gamma^2}{12 A e \mu \omega} x_0^3 p_0^2 \quad (12)$$

Similarly, equating (7) and (11) for the free molecule region

$$\frac{V_0 V_1}{x_1} = \frac{\pi^2 \bar{c} m \gamma^2}{32 A e \omega \bar{\alpha}_1} x_0^2 p_0 \quad (13a)$$

$$= \frac{\pi^{7/2} (2RTm)^{1/2} \gamma^2}{16 A e \omega \bar{\alpha}_1} x_0^2 p_0 \quad (13b)$$

These relations are in good agreement with experimental measurements; however, some factors are not usually sufficiently well known to permit absolute dependence on the equations, and the equation does not consider the mechanical and electrical losses. In practice the following relation is used and the exact relationship between  $V_0 V_1 / x_1$  and pressure is determined experimentally.

$$\frac{V_0 V_1}{x_1} = f(p_0, \text{gas properties}) + P_m + P_r \quad (14)$$

where

$P_m$  mechanical losses

$P_r$  electrical losses

The two voltages can be measured and the value of  $x_1$  determined from the variation in capacity between the diaphragm and the sensing plate.

## Mechanical and Electrical Losses

Not only is power dissipated in the gas, it is also lost in the mechanical structure and in the electrical circuit used to read out the transducer signal. The mechanical loss is difficult to completely describe analytically, since it is primarily controlled by minor factors in assembly; however, such factors as the diaphragm material and the overall design are important. The diaphragm vibrates at its lowest natural resonant frequency with a figure of merit  $Q$  inversely proportional to the power loss, which may be described by

$$Q = \frac{\text{energy stored} \times \omega}{\text{power dissipated}} \quad (15)$$

as well as by the relation

$$Q = \frac{f}{\Delta f} \quad (16)$$

where

$f$  resonant frequency

$\Delta f$  bandwidth at 0.707 amplitude

The mechanical loss is indicated by the transducer  $Q$  at pressures so low that gas damping is insignificant.

The major factors affecting the value of the structure  $Q$  are the purity of resonance in the diaphragm as determined by equal radial tension, the dimensional accuracy of the diaphragm effective diameter in all directions, the presence of viscous foreign material deposits on the diaphragm, and the variation of the diaphragm-mount contact points during the vibration cycle. Design of the diaphragm mount to minimize energy loss to the transducer structure is also important. A high  $Q$  structure is desirable, since the lower limit of the range is determined by the level at which the internal losses become large compared with the gas damping losses.

The electrical loss is primarily the energy dissipated in the input impedance of the amplifier (displacement indicator in fig. 3) which follows the transducer. To determine the effect of this power loss on the system, the capacity between the diaphragm and the sensing plate of the transducer has the following form (ref. 8):

$$C = C_0 + C_1 \sin \omega t \quad (17)$$

If the potential across the capacitor is  $V$ , then the charge  $q$  is

$$q = V(C_0 + C_1 \sin \omega t) \quad (18)$$

and the current flow in the output circuit is

$$i = \dot{q} = \dot{V}(C_0 + C_1 \sin \omega t) + VC_1\omega \cos \omega t \quad (19)$$

If a dc potential  $V_0$  is applied to the transducer through a resistance  $R$  that represents the input impedance of the amplifier, then

$$i = \frac{V_0 - V}{R} \quad (20)$$

Considering the solution for  $V$  to be sinusoidal and of the form

$$V = V_0 + V_2 \sin(\omega t + \theta) \quad (21)$$

then

$$V = V_{2\omega} \cos(\omega t + \theta) \quad (22)$$

Combining equations (19), (20), (21), and (22) results in

$$V_2 \sin(\omega t + \theta) = -R [V_{2\omega} C_0 \cos(\omega t + \theta) + V_{0\omega} C_1 \cos \omega t + V_{2\omega} C_1 \sin(2\omega t + \theta)] \quad (23)$$

Considering only the fundamental frequency terms

$$V_2 \sin(\omega t + \theta) = -R [V_{2\omega} C_0 \cos(\omega t + \theta) + V_{0\omega} C_1 \cos \omega t] \quad (24)$$

By properly choosing the magnitude of  $\theta$

$$\begin{aligned} V_2 [\sin(\omega t + \theta) + RC_{0\omega} \cos(\omega t + \theta)] \\ = -V_2 \sqrt{1 + (RC_{0\omega})^2} \cos \omega t \\ = -RV_{0\omega} C_1 \cos \omega t \end{aligned} \quad (25)$$

Then

$$V_2 = \frac{RV_{0\omega} C_1 \cos \omega t}{\sqrt{1 + (RC_{0\omega})^2}} = \frac{RV_{0\omega} C_0}{\sqrt{1 + (RC_{0\omega})^2}} \frac{x_1}{x_0} \quad (26)$$

The power lost in the resistance  $R$  is then

$$P_r = \frac{(V_2)^2}{2R} = \frac{RV_0^2 \omega^2 C_1^2}{2[1 + (RC_{0\omega})^2]} \quad (27)$$

Since  $C_1$  is proportional to the diaphragm displacement, the electrical power loss can be considered in terms of the reduction of the effective transducer  $Q$ . For this reason the input impedance of the transducer amplifier and any resistance furnishing a path for the dc potential to the transducer plates must be high to prevent lowering of the transducer  $Q$ .

#### RESPONSE SPEED

The transducer response speed is limited by two factors—the time required for the diaphragm to assume a new equilibrium energy level after a change in pressure, and the time for the change in pressure to become the effective pressure within the transducer sensing region. The first factor is dependent on the effective  $Q$ . A lightly damped, freely vibrating first-order system decays to an amplitude of  $1/e$  of its initial value after completing  $n$  cycles, where

$$n = \frac{Q}{\pi} \quad (28)$$

Since the time  $t$  for  $n$  cycles is

$$t = \frac{n}{f_0} \quad (29)$$

where  $f_0$  is the resonant frequency. The time constant for the system is

$$t = \frac{Q}{\pi f_0} = \frac{2Q}{\omega_0} \quad (30)$$

The effective time constant of the transducer at low pressures has been found to be of the form of equation (30), but with an empirically determined constant of 3.4 instead of 2.0.

Because of the transducer's large  $Q$  in the free molecule region, the response time for low-pressure measurements is controlled primarily by this factor. At high pressures the transducer  $Q$  becomes small, and the response time is limited by the airflow through the inlet tubes.

$$t = \frac{32\bar{\alpha}_1 L^2}{\pi D \bar{c}} \quad (31)$$

where

$\bar{\alpha}_1$  accommodation coefficient

$L$  inlet tube length

$D$  inlet tube diameter

For a typical transducer this time constant is about 0.005 second.

By using circuitry that maintains a constant amplitude of diaphragm vibration, the effective time constant of the transducer at low pressures can be greatly reduced and made to approach that at higher pressures. Since the stored energy in the diaphragm remains constant with this mode of operation, the time to restore equilibrium energy does not directly affect the response time.

#### GAS-COMPOSITION AND TEMPERATURE EFFECTS

Equation (12) shows that in the higher pressure ranges the ratio  $V_0 V_1 / x_1$  is dependent on the ratio  $\gamma^2 / \mu$ . Not only are the specific heat ratio and viscosity different for different gases, but the viscosity varies with temperature much more than the specific heat ratio. Therefore, both gas composition and temperature are expected to affect the calibration. Through the choice of materials and transducer design, the temperature effect can be so minimized that either or both  $x_0$  and  $\omega$  vary to compensate for the change in the ratio  $\gamma^2 / \mu$  as a function of temperature. Similarly, the effect of gas composition can be somewhat reduced by causing  $1/\omega$  to vary as a function of  $\gamma^2 / \mu$ .

In the free molecule region the ratio  $V_0 V_1 / x_1$  is dependent on the gas properties as given in equation (13b) by the factors  $[(Tm)^{1/2} \gamma^2] / \bar{\alpha}_1$ . Here again the transducer sensitivity is generally expected to vary with both gas composition and temperature. Very possibly, the transducer design and materials selection can minimize this effect by changes in  $x_0$  or  $\omega$ , or both.

## Chapter 3. Practical Experience

### TESTS AT AMES RESEARCH CENTER

Figure 5 is a plot (ref. 1) of  $V_0V_1/x_1$  as a function of pressure for the theory developed in chapter 2 and for an experimental gage in both air and helium. The experimental values shown have been reduced from the actual instrument readings by subtracting a value of  $5 \times 10^6$  volts<sup>2</sup>/meter. This has been done to account for the electrical and mechanical losses, which are independent of the gas, so that agreement with the theory developed for the gas damping can be

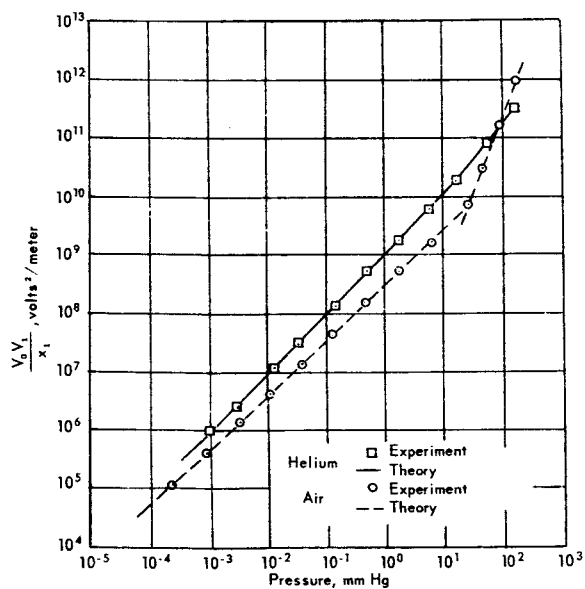


FIGURE 5.—Performance curves obtained for air and helium, corrected for mechanical and electrical losses.

checked. The experimental curves of  $V_0V_1/x_1$  as a function of pressure exhibit a considerable leveling off at pressures below  $10^{-3}$  mm Hg for that particular gage, as shown in figure 6. The readings shown were obtained at a temperature of  $70^\circ$  F and are the results obtained by the

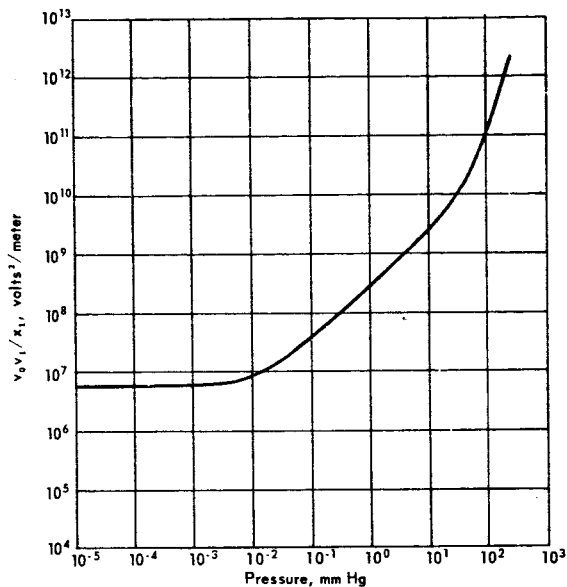


FIGURE 6.—Performance curve obtained for air, uncorrected.

innovators who also report that measurements at  $135^\circ$  F in air over the range of  $10^{-3}$  to 100 mm Hg showed insignificant differences from those at the lower temperatures.

Measurements made over a period of several months indicate good stability. These measurements were compared with those made with an ion gage from  $10^{-3}$  to  $10^{-2}$  mm Hg, an oil manometer (cathetometer) from  $10^{-2}$  to 10 mm Hg, and a standard oil manometer from 10 to 60 mm Hg. The readings obtained were within 1 percent of comparison instrument readings for pressures greater than 1 mm Hg, within 2 percent between 1 and  $10^{-1}$  mm Hg, within  $3\frac{1}{2}$  percent between  $10^{-1}$  and  $10^{-2}$  mm Hg, and within 5 percent between  $10^{-2}$  and  $10^{-3}$  mm Hg. All these readings are within the expected accuracy of the comparison instruments so that the absolute accuracy of the vibrating diaphragm transducer

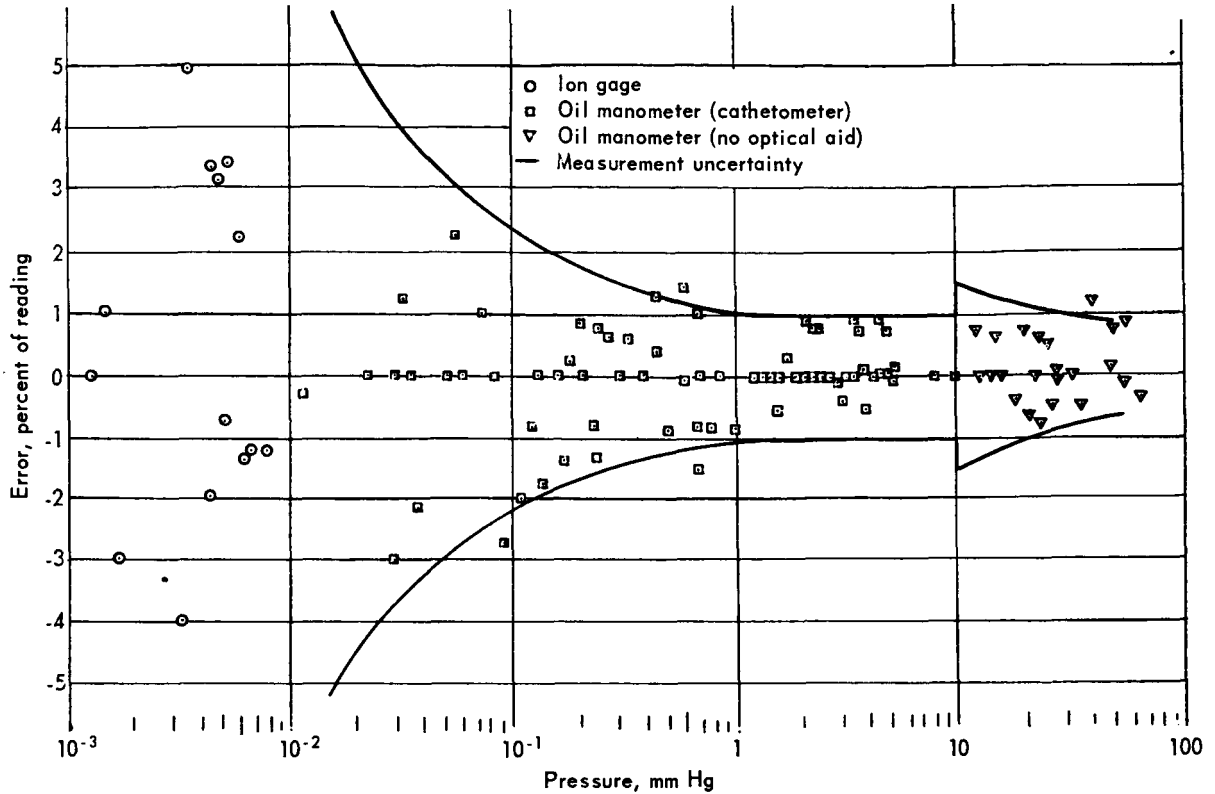


FIGURE 7.—Transducer stability.

may be much better than these figures, which are summarized in figure 7.

Differences in performance were obtained for one transducer from tests made in both air and helium (fig. 5). The experimental measurements are in good agreement with the theory.

The innovators (ref. 1) conducted several experiments to determine the effects of varying frequency and electrode spacing and shape. Information is available on the optimum parameters for particular pressure ranges. Figure 8 shows the effect of changes in the diaphragm frequency while maintaining the spacing constant, and figure 9 shows the effect of changing the spacing while maintaining the frequency constant.

#### EVALUATION AT NORTRONICS

The Nortronics Division of Northrup Corp. has conducted tests (ref. 8) to evaluate the vibrating diaphragm transducer. Of the two units made available for these tests, performance data were determined on either one or both.

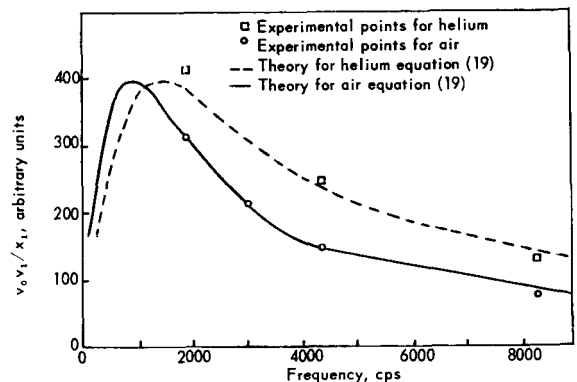


FIGURE 8.—Effect of varying the frequency while maintaining constant spacing. Pressure = 1 mm Hg; temperature = 70° F.

#### Test Conditions and Apparatus

In the derivation of equations (10) and (11) it was assumed that for 1 percent accuracy the amplitude of the diaphragm peak displacement  $x_1$  must be less than 10 percent of the nominal spacing  $x_0$  between the diaphragm and the fixed

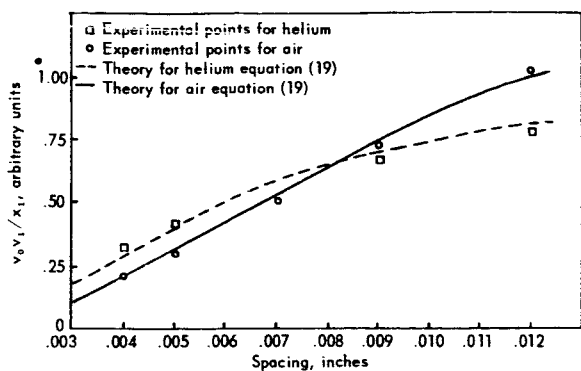


FIGURE 9.—Effect of changing the spacing while maintaining a constant frequency. Pressure = 1 mm Hg; temperature = 70° F.

plate, and the ac voltage  $V_1$  must be less than 10 percent of the dc voltage  $V_0$ . That the ratio  $V_0V_1/x_1$  is constant within these limitations was shown by experimental measurements at constant pressures ranging from  $3.6 \times 10^{-2}$  to 9.4 mm Hg. The ratio remained constant within 1 percent for all values of  $x_1/x_0$  between  $2.7 \times 10^{-5}$  and 0.1. For  $x_1/x_0 = 0.23$ , the ratio is reduced by 16 percent. The lower  $x_1$  limit corresponds to the limit of detecting diaphragm motion with 1 percent accuracy. All subsequent tests were made with  $x_1/x_0 = 0.01$ . The ac-to-dc drive voltage ratio was also varied, and readings taken at several constant pressures confirmed that the output

ratio ( $V_0V_1/x_1$ ) remained constant within 1 percent for all values of  $V_1/V_0 < 0.1$ . All tests were subsequently made with  $V_1/V_0 = 0.0707$ .

The test circuitry is shown in figure 10. To set the oscillator to the peak of the resonance to within 1 percent accuracy, the oscillator frequency must be within  $\pm 0.0707 f_0/Q$  of the diaphragm resonant frequency  $f_0$ . A 36:1 gear train was added for tuning the oscillator with such precision, and a five-digit counter was used for measuring the oscillator frequency to within 0.1 cps. This resolution at a transducer resonant frequency of 5500 cps meets the 1-percent peak setting criterion for a  $Q$  of 3890 or less. Since the transducer  $Q$  can be higher than this at pressures below  $10^{-2}$  mm, there may be some measurement error at the lower pressures. Some error may also result from the use of a 1-megohm resistor in series with the dc potential on the sensing plate. A higher resistance should have been used to avoid any such error.

A McLeod gage (CVC Type GM-100A) was used as the standard of comparison for pressure measurements. The absolute accuracy of this gage, as quoted by the manufacturer, is  $\pm 1.2$  percent of the indicated reading. The repeatability of measurements with the gage is determined by the accuracy of reading the meniscus levels, which was experimentally determined as 0.0345 millimeter. The effect of such a reading

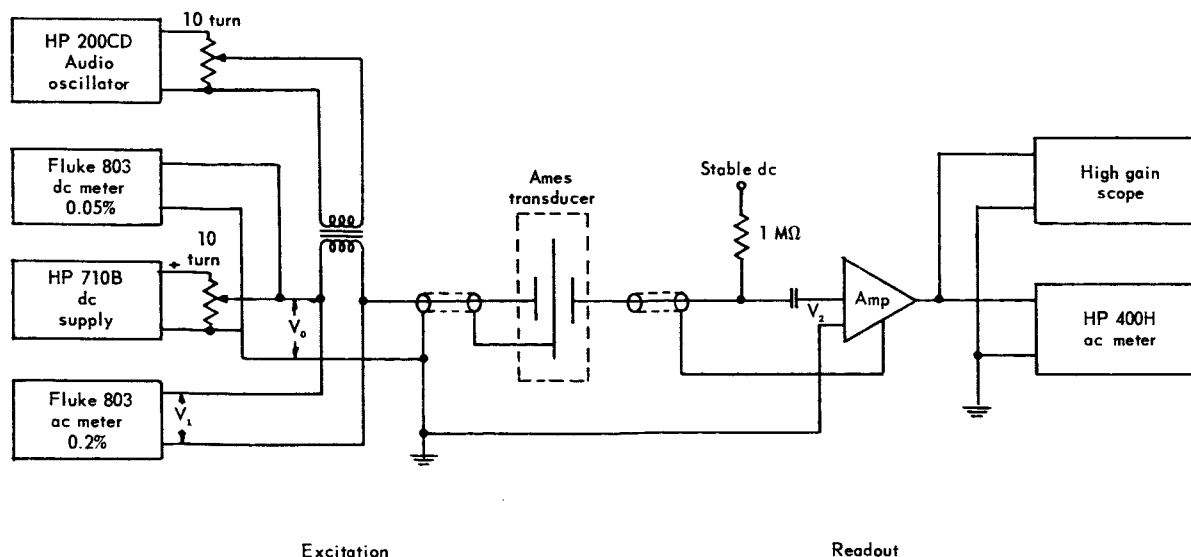


FIGURE 10.—Test circuitry used by Nortronics.

error is shown in figure 11. The reading error of the McLeod gage is less than that of the Ames transducer by an order of magnitude over most of the range. The pressure system had provision for controlling the static pressure on the transducer as well as modulating the pressure by  $\pm 10$  percent at frequencies up to 100 cps.

and pressure, the sensitivity would have been exactly 1.0. At low pressures the sensitivity approaches zero because of significant power lost to the structure instead of the gas. When the pressure equals  $2.2 \times 10^{-3}$  mm Hg, the sensitivity is 0.5, and essentially half the power is lost to the structure.

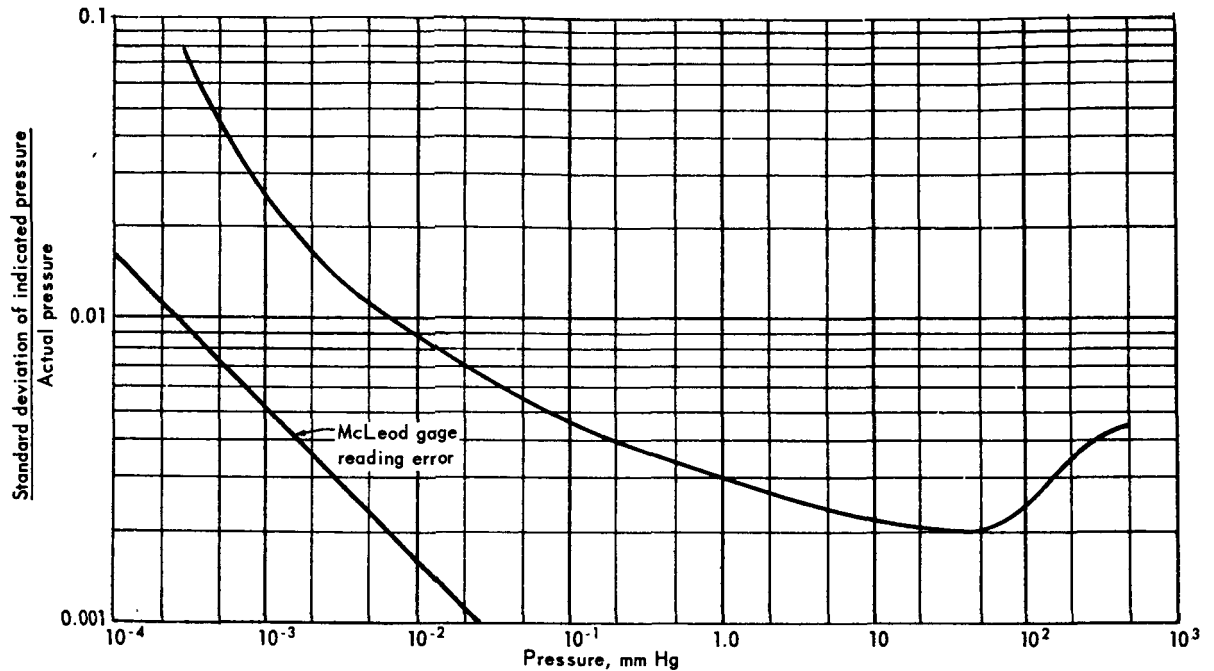


FIGURE 11.—Pressure indication stability shown as a function of pressure.

### SENSITIVITY AND RANGE

At atmospheric pressure the drive voltages were set at  $V_0 = 200$  volts and  $V_1 = 14.14$  volts. The resultant  $x_1/x_0$  was approximately  $5.7 \times 10^{-5}$ , which is about twice the lower limit for the measurability cited above. Thus, the transducer can operate at pressures as great as 1 atm. Accuracy and response decrease with pressure. Although there is no distinct low-pressure limit, the low-pressure limit for a specific application depends on the accuracy and response requirements.

Figure 12 shows the experimentally measured sensitivity of the Ames transducer. In the middle-pressure regions the sensitivity is about 1.0. Had the solution of equations (10) and (11) been the exact relation between the output ratio

### Stability

Figure 13 shows the short-time stability of the transducer output and the standard deviation of the observed transducer output at constant temperature over a period of several weeks. The sample of experimental data is sufficient to conclude that there is a 79-percent probability that the standard deviation of the output of a randomly chosen transducer is less than the value indicated in figure 13.

The stability of the indicated pressure is shown in figure 11. It is the ratio of the sensitivity given in figure 12, to the output stability, given in figure 13. The probability that the indicated pressure will be in error by less than the value shown is 68.3 percent.



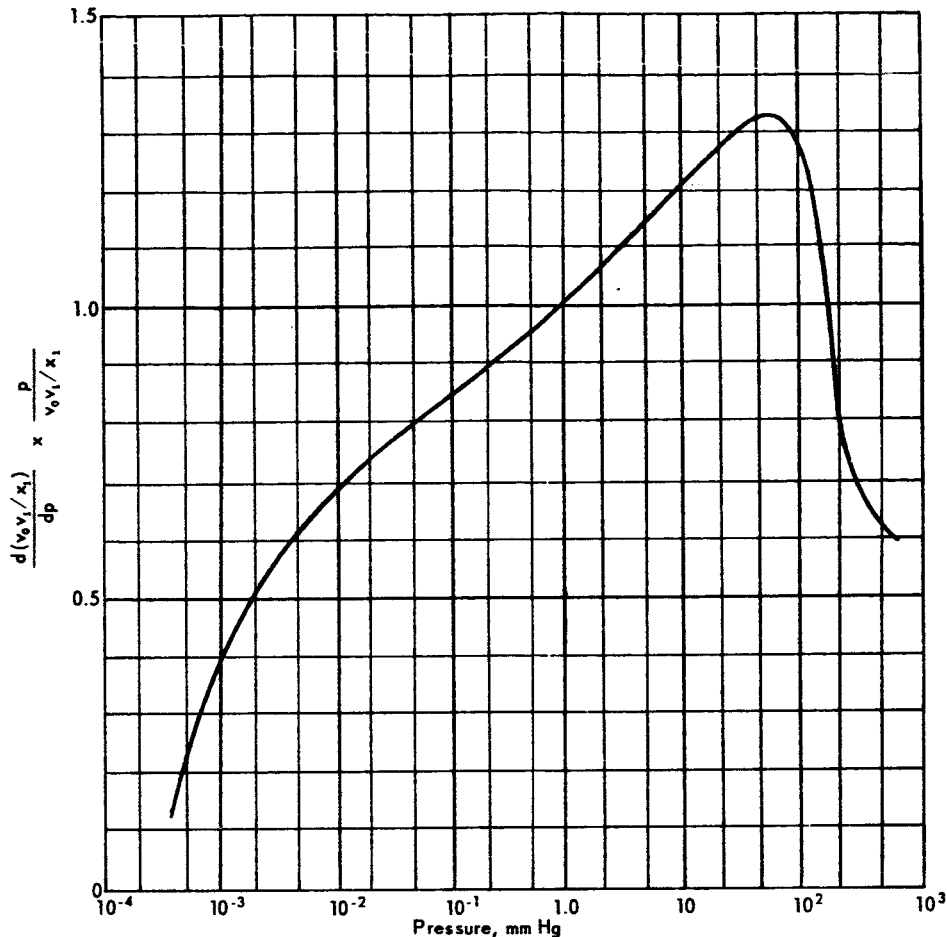


FIGURE 12.—Normalized sensitivity plotted against pressure.

#### Resonant Frequency, $Q$ , and Response

Figure 14 shows the variation in the diaphragm resonant frequency as a function of pressure, normalized with respect to the low-pressure limit. Figure 15 shows the experimental values of the total  $Q$  for the transducer as a function of pressure. In the middle-pressure regions the curve is approximately proportional to  $p_0^{-1}$ . At lower pressures  $Q$  approaches the value corresponding to the structure alone, about 20,000. If this  $Q$  corresponding to the structure could be increased, the low-pressure sensitivity would be improved. The same advantage could be obtained by reducing the  $Q$  resulting from the gas and thus changing the point at which the structure loss becomes significant.

Figure 16 shows the transducer time constant as a function of pressure. The time constant can

be reduced by making the effective  $Q$  smaller by any means; however, if the structure  $Q$  is reduced, the low-pressure sensitivity and accuracy will suffer. It is advantageous, for both sensitivity and response, to lower the  $Q$  resulting from the gas.

#### Temperature

Figure 17 shows the temperature sensitivity of the transducer output as a function of pressure. At pressures less than  $3 \times 10^{-2}$  mm Hg, tests were conducted on only one transducer and no conclusion was drawn concerning the probability that the temperature sensitivity shown represents a randomly chosen transducer. At pressures greater than  $3 \times 10^{-2}$  mm Hg, the probability is 79 percent that the temperature sensitivity of a random transducer will not exceed the value shown in figure 17. Temperature-change rates of less than

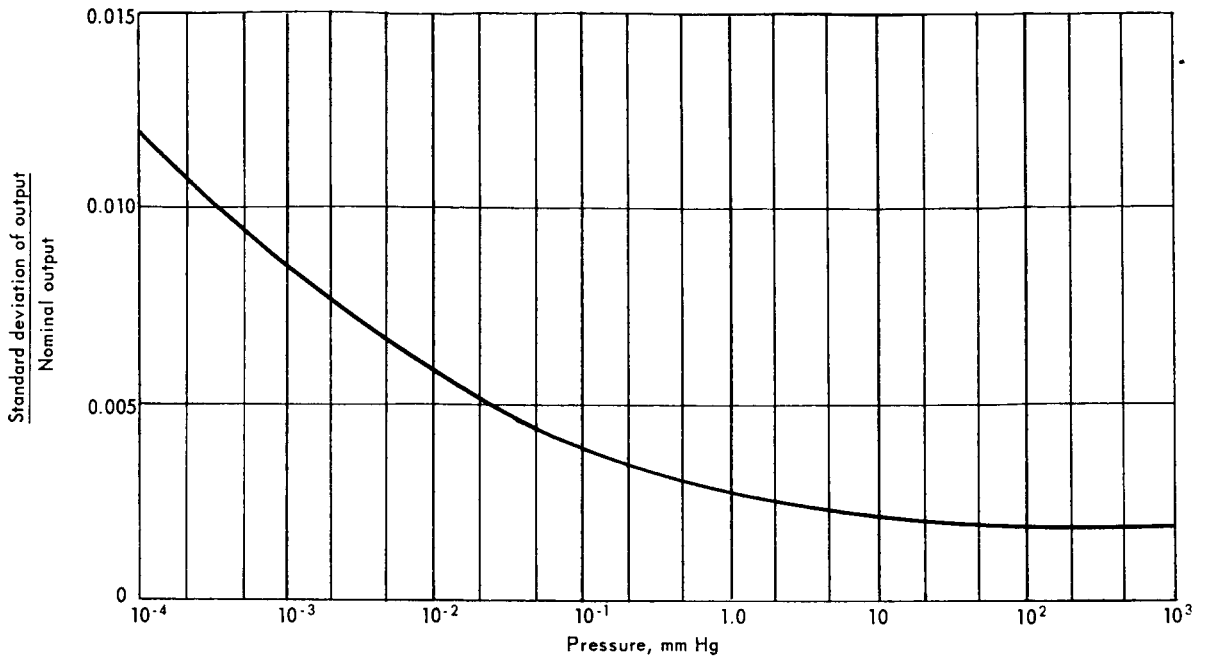


FIGURE 13.—Output stability shown as a function of pressure.

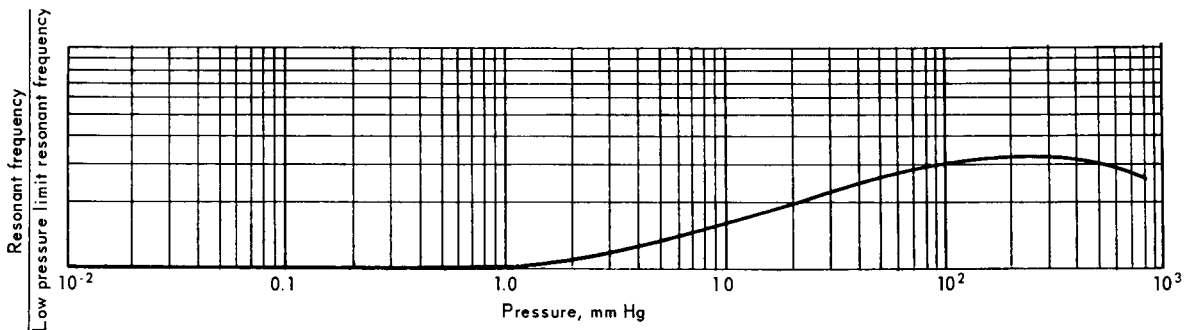


FIGURE 14.—Resonant frequency variation with pressure, normalized to low pressure limit frequency.

0.625° F per second do not contribute an output deviation other than that caused by the integrated temperature change.

#### Vibration and Acceleration

Vibration at the diaphragm mechanical resonant frequency contributes energy to the system. It was experimentally determined that the resulting diaphragm motion relates to the acceleration  $a$ , applied to the transducer housing by

$$\frac{x_1}{a} = 36 \frac{\text{volts}^2}{g} \times \frac{1}{\left(\frac{V_0 V_1}{x_1}\right)} \quad (33)$$

where  $(V_0 V_1 / x_1)$  was determined as a function of pressure by the calibration curve. Figure 18 gives  $x_1/a$  for pressures between  $10^{-3}$  and 1.0 mm Hg. Since the transducer can be operated with  $x_1 = 0.1x_0$ ,  $x_1$  can be as great as 0.0003 inch ( $x_0$  is 0.003 inch). If the contribution from vibration is to be insignificant, the  $x_1$  from vibration must be less than 1 percent of that, or  $x_1 = 3 \times 10^{-6}$  inch.

Because of the diaphragm's high  $Q$ , the transducer is sensitive only to vibration frequency components within a narrow band centered on the diaphragm resonance. As the pressure is

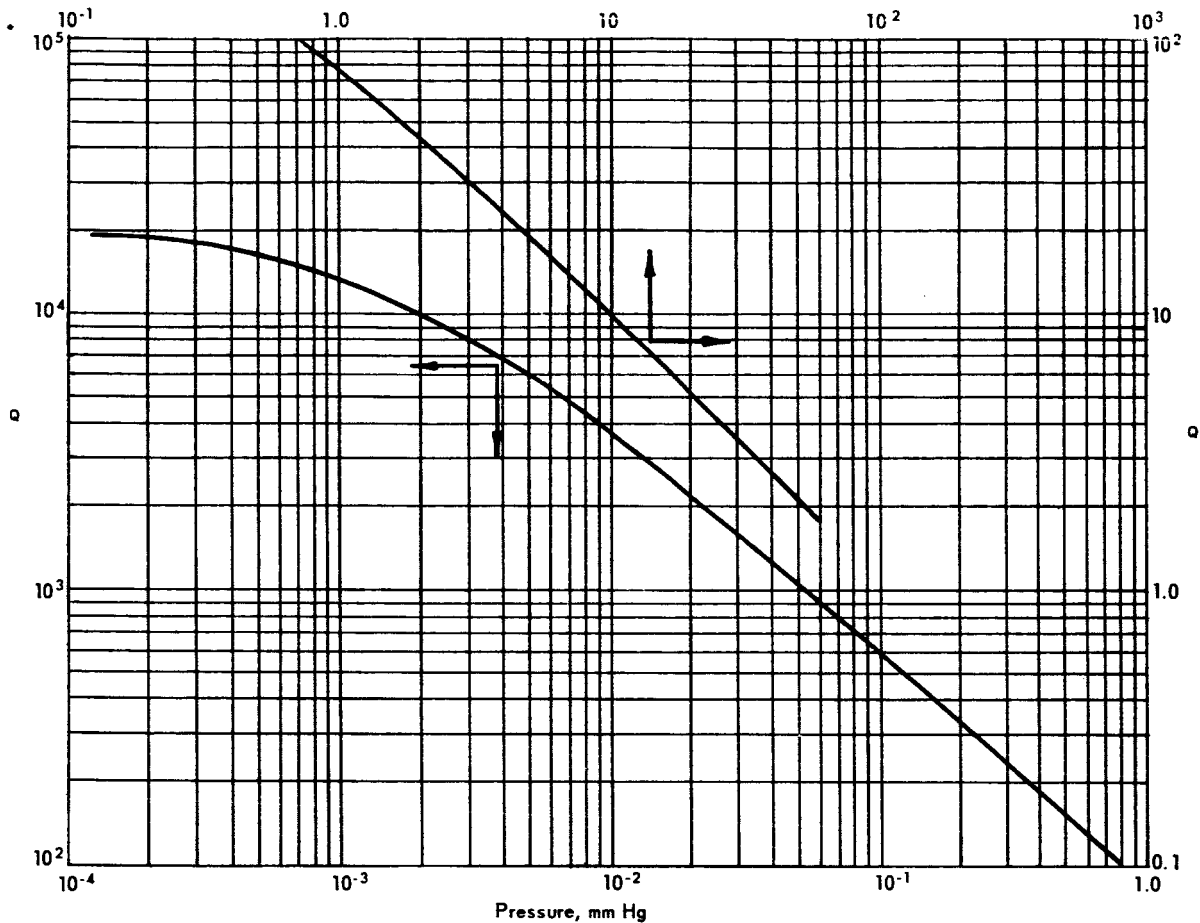


FIGURE 15.—Transducer  $Q$  plotted against pressure.

decreased,  $Q$  increases and the bandwidth over which the vibration energy is detrimental is reduced. In applications involving random vibration, the two effects tend to cancel, and the increased sensitivity to vibration as pressure decreases is not significant. Except in adverse environments, the sensitivity to vibration is adequately low.

Isolation may be required when the transducer is used in some high-vibration environments. For a specific vibration level and lower pressure measurement limit, figure 18 can be used to determine the isolation required for vibration components of the diaphragm resonant frequency. The transducer exhibits no lower harmonics, and although higher harmonics exist, they cause no output in operation because of the electronic oscillator system. Figure 19 shows the effect on the output

of a static acceleration in the direction normal to the diaphragm as a function of pressure. The data correspond to  $x_1 = 3 \times 10^{-5}$  inch. The effect is so low that a 40-g acceleration in the direction normal to the diaphragm would be required to alter the output by 1 percent at  $10^{-2}$  mm Hg.

#### Moisture

As discussed previously, the output of the transducer will vary with the gas composition, depending on the viscosity coefficient and the ratio of specific heats. One-hundred percent water vapor would give an output equal to 1.612 times that of pure air. Assuming that the composite effect for moist air is in proportion to the fraction of water vapor, the moist air output would be 1.021 times the dry air output for 3.43 percent

water vapor. The output for air with this level of water vapor was experimentally determined to be between 1.03 and 1.07 times that for dry air at pressures between  $10^{-2}$  mm Hg and atmospheric. This sensitivity is computed on the assumption

that the moisture in the transducer is in the vapor state. For most transducers (including the Ames transducer) the output is more significantly affected by vapor adsorbed on the transducer walls than by the free vapor.

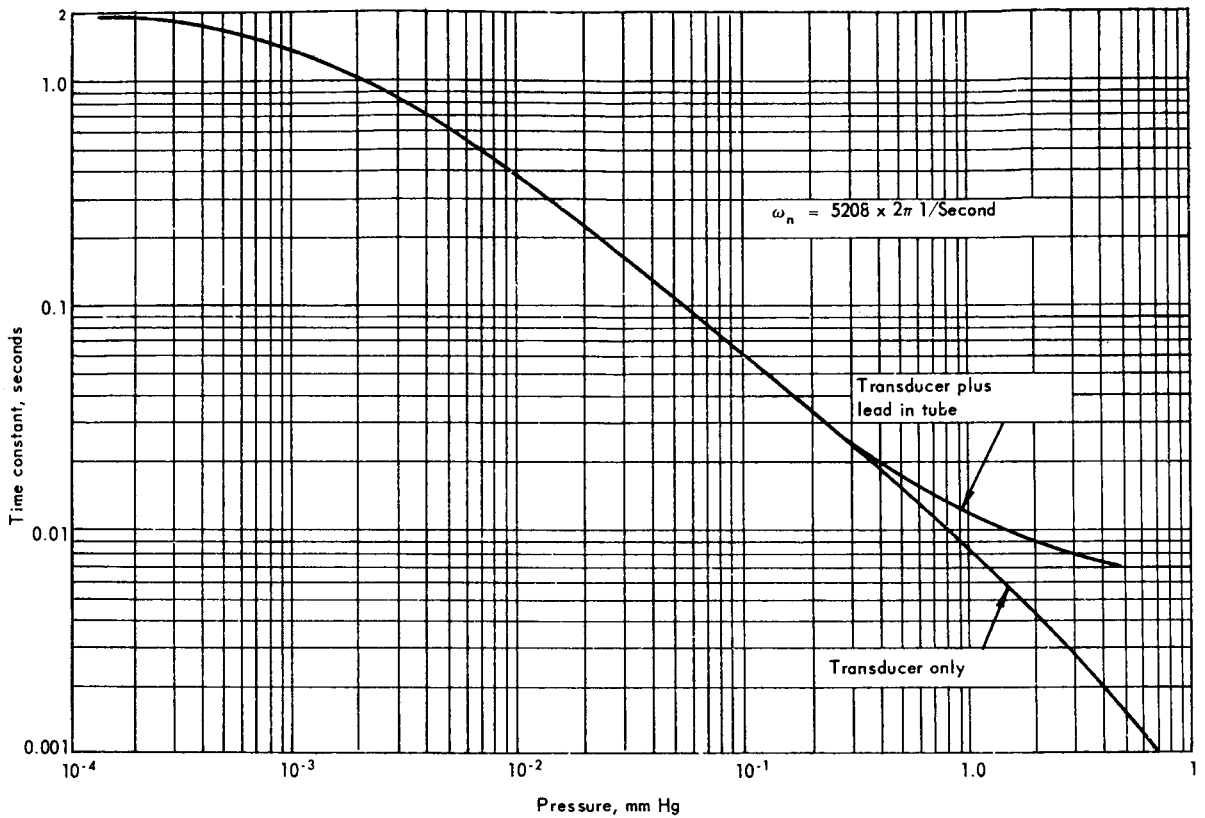


FIGURE 16.—Time constant plotted against pressure.

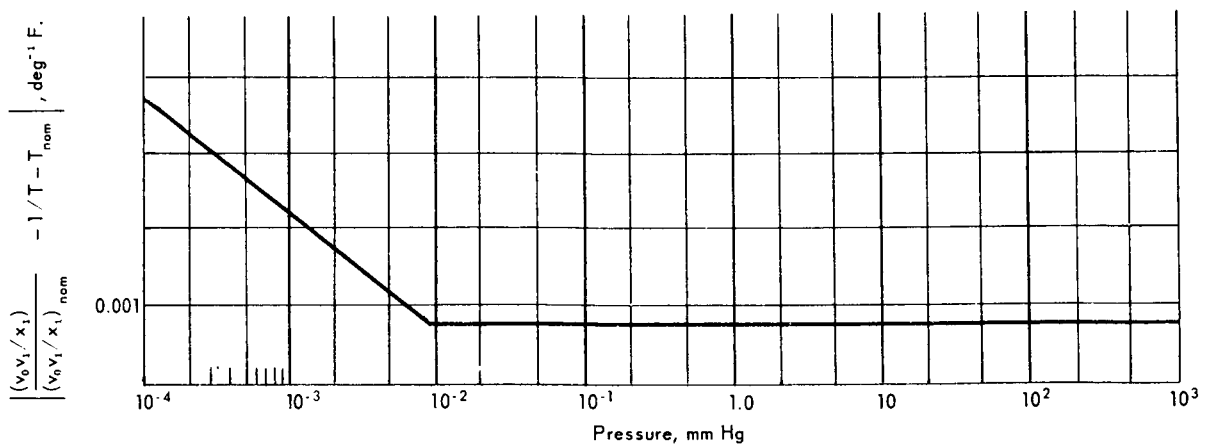


FIGURE 17.—Temperature sensitivity.

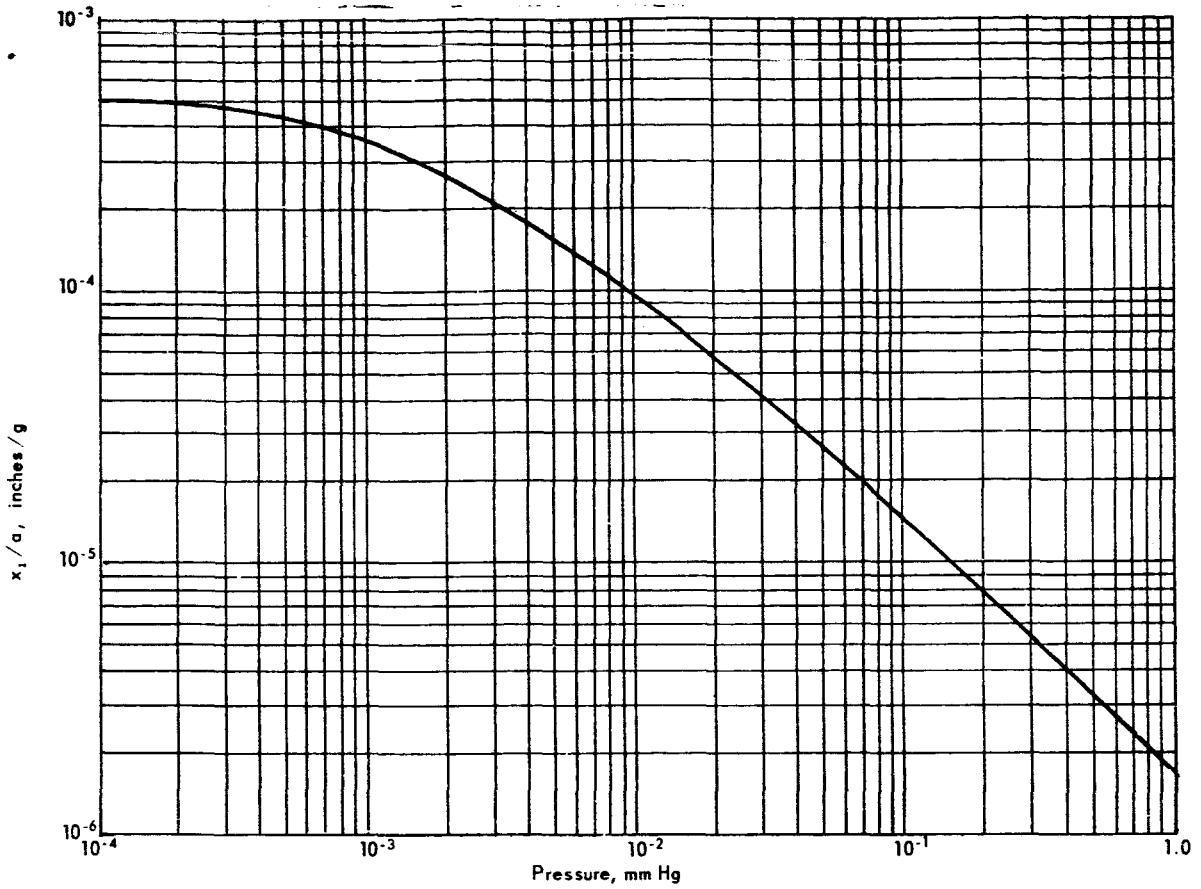


FIGURE 18.—Sensitivity to vibration at diaphragm resonance frequency.

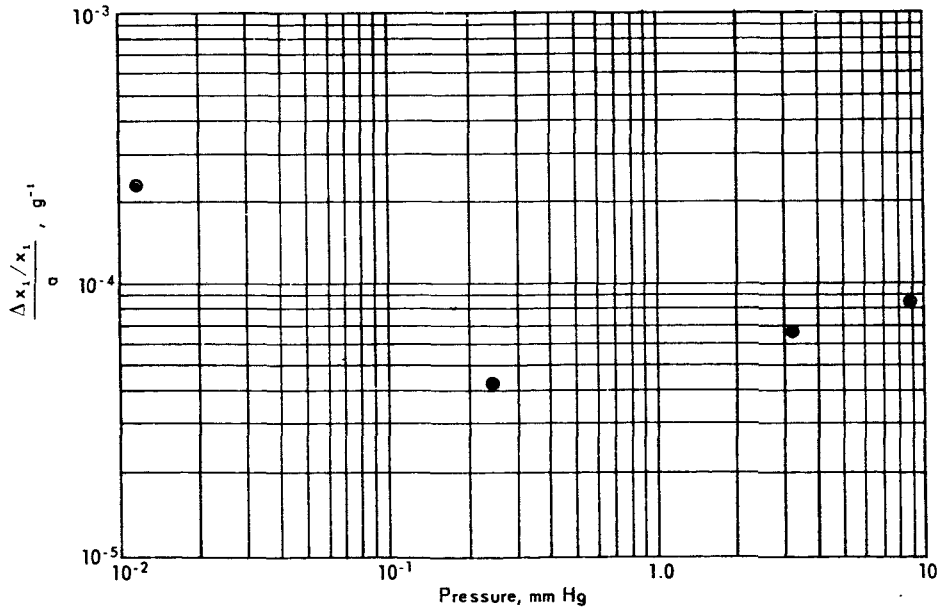


FIGURE 19.—Acceleration sensitivity at various pressures.

### DETAILS OF TRANSDUCER CONSTRUCTION

The selection of materials and the care taken in fabrication determine, to a large extent, the ultimate performance of the transducer. The diaphragm material must have low internal losses to insure a high mechanical  $Q$ . The thermal expansion coefficients must be well matched for all materials, and the internal losses in the diaphragm should remain constant to reduce temperature effects. The diaphragm must be under equal radial tension in all directions. Lint, dust, viscous materials, and other foreign matter on the diaphragm or at the diaphragm attachment point will cause a reduction in the mechanical  $Q$ .

Various materials have been investigated at Ames Research Center, and the most dependable results have been achieved using Kovar cell bodies, No. 7052 Corning glass insulator, and 42 percent nickel-iron 0.0001-inch-thick diaphragms. Reasonably good results have been achieved with diaphragms made from Therlo and 0.00025-inch-thick Dynavar combined with the above body and insulator materials. Another combination which has been used is 446 stainless-steel bodies with No. 00120 Corning glass and a heat-hardened, 17-7PH, 0.00025-inch-thick stainless-steel diaphragm. Recently, transducers have had  $Al_2O_3$  ceramic insulators, with the body, diaphragm, and all metal parts made of 42 percent nickel-iron. Results with this combination seem comparable with the best built of other materials.

Figure 20 shows a detailed cross section of an actual transducer. With this design, the overall outside diameter is 0.786 inch, the vibrating diaphragm is 0.728 inch in diameter, the fixed plates 0.562 inch in diameter, and the overall thickness is 0.619 inch. The diaphragm, which is made of 0.0001-inch-thick 42 percent nickel-iron alloy, is spaced 0.003 inch from the fixed plates and tensioned to achieve a resonant frequency of 2500 cps.

Electrical connections to the fixed plates are made through miniature (Microdot) coaxial connectors. The glass or ceramic insulator is

mounted on the outer metal ring by fusing at high temperatures, and the diaphragm is attached by multiple overlapping spot welds.

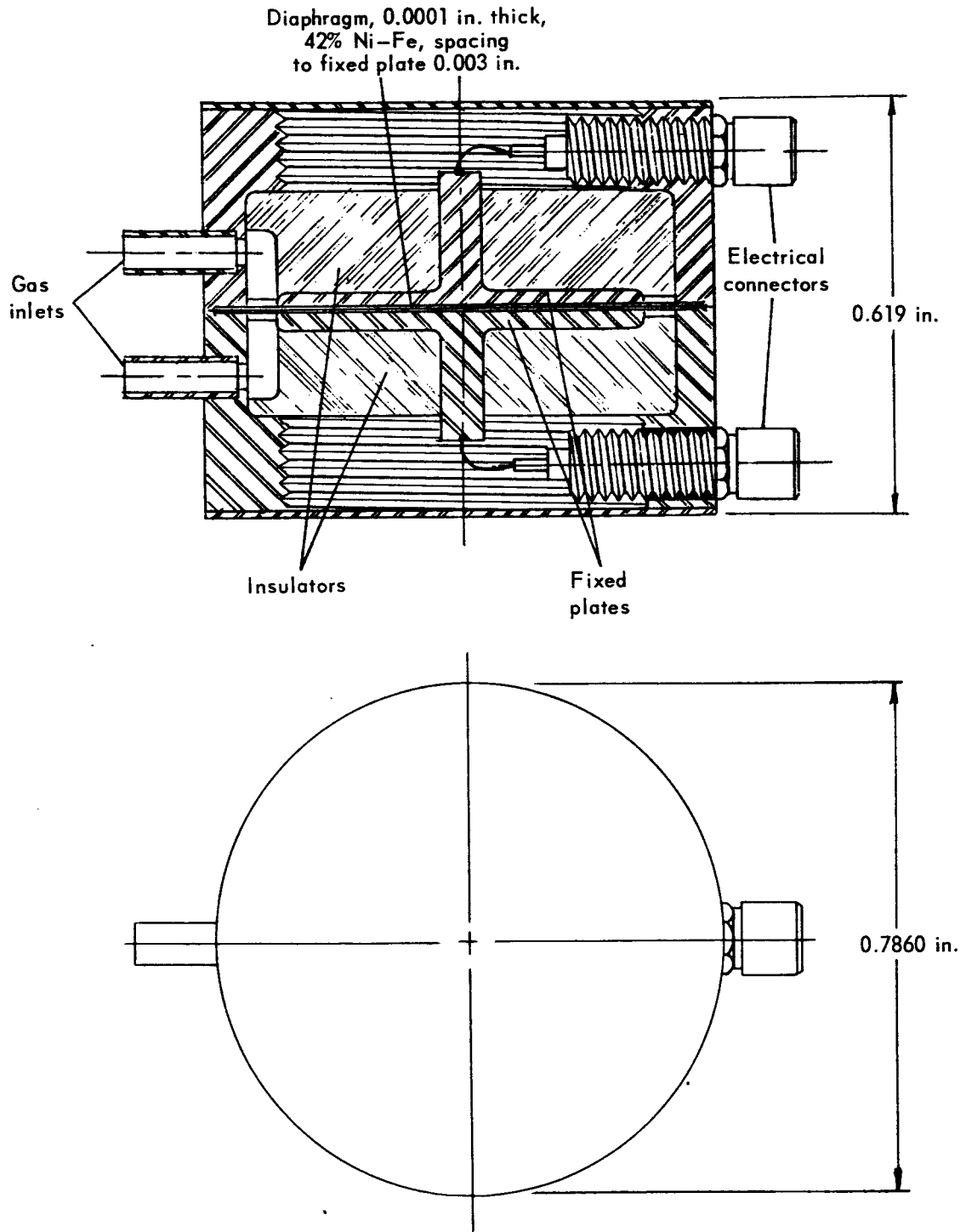
Numerous transducers of this size have been fabricated at Ames, and transducers that operate on a similar principle, but only have a 0.28-inch outside diameter, have also been built. These smaller transducers performed well between  $10^{-2}$  and 10 mm Hg.

### PRESENT STATE OF DEVELOPMENT

Approximately 50 vibrating diaphragm pressure transducers have been built at Ames Research Center for use in NASA wind tunnels. Much information has been accumulated on the characteristics of these devices and on the effects of certain design variations. They have been used for several years and have been useful for the intended application; however, existing problems and areas of insufficient information about the transducers and the electronic system indicate that further development and investigation are desirable.

Transducers having all the desired characteristics are not easily fabricated. Much of the difficulty involves tensioning and welding the diaphragm to obtain equal radial tension in all directions. The small diaphragm-to-fixed-plate spacing requires close tolerances on several parts as presently designed and close control of cleaning and environmental conditions for assembly. Problems are also met in producing leak-free seals and in achieving the same characteristics in each transducer, so that individual calibration is still required.

The original electromechanical servo system was too slow for some applications, and Ames Research Center has developed a transistorized, all-electronic control system which works quite well. However, it is not as accurate as should be possible and does not work over as wide a pressure range as is desirable. Some individual tailoring of the electronic system to each transducer is required.



Scale: 4:1

FIGURE 20.—Transducer assembly details.

## Chapter 4. Previous Techniques

Many vacuum gages have been designed and used, based on a variety of principles and having either mechanical or electrical readout. Some disadvantages include slow response, with varying gas composition, differential variation rather than absolute nature, large size, and fragility.

Many vacuum gages are useful mainly in the laboratory where manual reading, fragility, or large size are not major disadvantages. In this class are such devices as liquid manometers, McLeod gages, mechanical manometers, and forms of the Knudsen gage. These devices will not be considered in detail as previous art because they would not be usable in applications where the vibrating diaphragm pressure transducer is most advantageous.

### GAS DAMPING GAGES

The principle that gas damping varies as a function of pressure is the basis for several devices designed for the measurement of low pressures (ref. 9). The first of these, suggested by Sutherland (ref. 10) and Hogg (ref. 11), had a thin glass disk suspended on a wire between two horizontal plates. The disk is set in rotation, and torsion in the support wire produces a restoring force that causes the disk to oscillate. The rate of decrease of the oscillation is observed optically. Shaw (ref. 12) has found this type of gage useful down to about  $0.35 \times 10^{-3}$  mm Hg.

Quartz fiber gages were first suggested by Langmuir (ref. 2), and improved by Haber and Kerschbaum (ref. 3), and Wetterer (ref. 4). A thin quartz fiber is supported at one end and set in oscillation by plucking, tapping the support, or other means. The rate of vibration decay is observed optically. This type of gage is useful down to as low as  $5 \times 10^{-6}$  mm Hg. Variations of the fiber gage have been described by Andrews (ref. 13) and King (ref. 14). Bruche (refs. 15 and

16) and Wetterer (ref. 4) have used membranes attached to the fiber to extend the useful range down to  $10^{-7}$  mm Hg.

Langmuir (ref. 17), Dushman (ref. 18), and Timiriazeff (ref. 19) have reported gages that used a continuously rotating disk or cylinder to cause a force on a restrained disk or cylinder in close proximity. The torque on the restrained disk or cylinder depends on viscous drag and is a measure of the pressure. Gages of this type have been found useful down to about  $10^{-7}$  mm Hg.

All these gages are mechanical, most are very slow in response, and none are suitable for continuous pressure indication except the continuously rotating type described last. Brooks and Reis (ref. 20) investigated the gaseous damping effect on a vibrating wire in a magnetic field. Their device had a tensioned wire suspended in a transverse magnetic field and set in vibration by a pulse of electrical current through the wire. The oscillation decay time was determined by counting the vibrations with an electronic counter. The input to the counter was the voltage generated by the wire vibrating in the magnetic field. Wires of nickel, phosphor bronze, and tungsten of  $5 \times 10^{-4}$  inches in diameter and 0.4 inch long were used. Measurements were made down to  $10^{-1}$  mm Hg.

King (ref. 21) developed a vibrating wire pressure transducer based on the variation of gas damping on the wire as a function of pressure. This transducer used a tensioned wire suspended in a transverse magnetic field as the sensing element. The wire was used as a frequency selective filter in the feedback loop of an electronic amplifier and was thus maintained in continuous, constant-amplitude vibration at the transverse mechanical resonant frequency of the tensioned wire. A continuous electrical output that was a function of the pressure was developed. The device was useful for pressures down to  $10^{-3}$  mm



Hg and had a response time of a fraction of a second. Variations in internal damping of the wire as a function of temperature and time were limitations in its usefulness.

A pressure transducer system based on the gas damping of a vibrating metal ribbon has been developed and marketed under the trade name "Reva" by Arthur Pfeiffer GmbH, Wetzlar, Germany (ref. 22). The corrugated ribbon is suspended in a magnetic field and is electronically maintained in continuous vibration. A continuous electrical indication of pressure is provided. The device is larger and more fragile than the vibrating diaphragm gage.

### ELECTRICAL GAGES

The vibrating wire and vibrating ribbon gages are based on principles similar to the vibrating diaphragm pressure transducer and provide a continuous electrical output. Several other gages based on different principles provide continuous electrical output and are useful over at least a portion of the pressure range covered by the vibrating diaphragm gage. These must be considered competitive devices for some applications and evaluated in relation to the vibrating diaphragm device.

For pressure measurements in the higher ranges, transducers operating on the principle of the deflection of a diaphragm as a measure of the pressure difference are often used with various methods of registering the deflection. Some common ways to provide an electrical output are strain gages, vibrating wire, variable capacitance effects, variable reluctance effects, and linear variable differential transformers. For sensing small pressures, the diaphragm must be large and of very thin material, and the sensing element must not require appreciable force. Because of this restriction, deflecting diaphragm transducers that have been most successful for measuring in the range of a few mm Hg have used either the variable capacitance or variable reluctance effects. Because this is a differential method, a high-vacuum reference pressure is needed on one side of the diaphragm for accurate low-pressure measurements. To achieve sufficient sensitivity with this type of gage, the diaphragm frequently is too thin to withstand a differential pressure of 1-atm pressure and is thus subject to damage from over-

pressure. The same considerations apply to variable-reluctance, differential pressure gages in which the diaphragm forms a portion of a magnetic circuit. Commercial variable-capacitance differential pressure sensors and associated electronics for use in the range of a few mm Hg have been marketed by several companies. Sensitivity to pressure changes of  $2 \times 10^{-4}$  mm Hg and long-term stabilities on the order of  $10^{-2}$  mm have been reported. Variable-reluctance differential pressure gages useful in the mm Hg range also have been marketed by several companies. Sensitivity and accuracy of a few microns Hg have been reported with this type of device. Response speeds of the transducer and associated electronics of several milliseconds for both types have been achieved. Compared to the vibrating diaphragm gage, these devices have a smaller range, which does not extend to as low a pressure; they are generally larger; and they have larger internal volume, which limits response speed at low pressures. An advantage of these units is that the reading is independent of the gas composition and temperature (except for general temperature effects on the sensor).

Thermocouple and Pirani gages are widely used in the range of  $10^{-3}$  to  $10^2$  mm Hg. Both gages provide electrical output and depend on the variation in heat conductivity of a gas with pressure. The thermocouple gage generates an electrical signal proportional to the temperature of a heated conductor. The gages do not cover as wide a range as the vibrating diaphragm device, have relatively slow response especially at lower pressures, and are sensitive to gas composition and heating current; however, they are relatively inexpensive and of simple construction.

Several common forms of vacuum gage depend on gas ionization by electrons or a radioactive source. Of these gages, only the radioactive type covers an appreciable portion of the range of the vibrating diaphragm gage. One such device is the Alphasatron developed by Downing and Mellen (ref. 23) and marketed by National Research Corp. This device uses a sealed radium source and ionizes gas by alpha radiation. As a result an electric current that is a linear function of the pressure is generated between electrodes. The range of the Alphasatron is  $10^{-3}$  to  $10^3$  mm Hg. A similar device having a tritium source was de-

veloped by Spencer and Boggess (ref. 24) and covers a range of  $10^{-3}$  to about 100 mm Hg. These devices are very sensitive to gas composition and their response is slow because of the electrometer required to measure the small

currents and the statistical nature of the ionization process. To cover the complete pressure range, two sets of electrodes and chambers are necessary. The internal volume is much larger than that of the vibrating diaphragm gage.

## Chapter 5. Applications and Conclusion

### NASA APPLICATIONS

The vibrating diaphragm transducer was developed for pressure measurements in wind tunnels. The requirement in this application for wide range, small size and internal volume, accuracy, rugged construction, sensitivity to absolute pressure, freedom from overpressure damage, and fast response could not be met by available types of transducers. Devices of this type have been used since 1959, with approximately 50 built to date. Development of the transducer and associated electronics is continuing.

### OTHER APPLICATIONS

#### Pressure Measurements

The transducer should be useful in many applications where pressure in the range of  $10^{-4}$  to  $10^3$  mm Hg must be measured. It should be applicable for electronic control systems and automatic data recording, as a general-purpose vacuum gage.

Because of its related ruggedness and quick response, the transducer could be useful in many manufacturing processes using vacuum.

Static and dynamic pressure measurements in other wind tunnels, aircraft, and missiles require transducers with the characteristics of the vibrating diaphragm device. The transducer could be useful as an altimeter with a range extending from sea level to 400,000 feet.

#### Other Possible Uses

The structure of the device is the same as a differential capacitor pressure transducer. It could be used as such by altering its electronic and pressure connections.

The transducer structure could serve as the basis for an electrometer with a high input imped-

ance and a current sensitivity possibly greater than that available in other devices, or for an accurate multiplier for use in electronic analog computers.

### ESTIMATED COST AND SAVINGS

Based on the experience at Ames Research Center, the price of the transducers is estimated at \$200 to \$400 each, depending upon quantity. The price of the associated electronics would be in the same range. Thus, the total cost would be approximately the same as for similar gages. The instrument's principal advantage is a combination of characteristics not presently available in a single device.

### CONCLUSION

The vibrating diaphragm pressure transducer has a combination of advantageous characteristics that cannot be fully matched by any other type of transducer now used in the range of  $10^{-5}$  to  $10^3$  mm Hg. It covers a wider range than most other types, is sensitive to absolute pressure, has small size and internal volume, has fast response, is relatively rugged, provides an electrical output, and provides a continuous indication of pressure.

With little or no modification, the transducer can be used as a differential capacitor pressure transducer, an electrometer input device of very high sensitivity, a magnetic damping measuring device, and an accurate multiplier for use with electronic analog computers.

Although the transducer system has been found useful, obtaining the desired characteristics and uniformity still remains a fabrication problem. Further development of the electronic circuitry is desirable for greater stability, accuracy, and range.

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