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# Research in Space Science

## SPECIAL REPORT

Number 214

# THE DECAY OF SATELLITE 1965-79A

William P. Hirst

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# THE DECAY OF SATELLITE 1965-79A

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4-66-2

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# ABSTRACT

The decay of Satellite 1965-79A took place on 1965 October 29 and was observed across Europe. The observers' descriptions of the decay are quoted and position measures are tabulated.

The object was already self-luminous when first seen at a height computed as 92.6 km; a luminous trail began to form at a height of 91 km; and fragments were first seen breaking away at a height of about 82 km.

The decay trajectory as indicated by the measures is investigated with the help of an ephemeris supplied by NORAD. It is concluded that the observations fix the height with an uncertainty probably not greater than 0.5 km.

A rough indication of the probable impact point is given.

# THE DECAY OF SATELLITE 1965-79A[1]

William P. Hirst[2]

## 1. INTRODUCTION

The decay of Satellite 1965-79A took place over Europe on 1965 October 29. Predictions based on the Spiral Decay (S. D. )Program had been received from the Space Defense Center at ENT Airbase, Colorado Springs, and Moonwatch teams having a chance of seeing the entry into the atmosphere had been alerted.

The first intimation that the entry had been successfully observed was a telephone call from Berlin Moonwatch, and this was closely followed by a cable from Bochum Moonwatch. These reports were confirmed and amplified by letter. Written reports were also received from Zurich, Rodewisch, Eilenberg, and Copenhagen Moonwatch teams and from Dr. A. V. Nielsen of the Ole Roemer Observatory at Aarhus in Denmark. Later, a very useful photograph, which unfortunately has insufficient contrast for reproduction, was supplied by Bochum Moonwatch. Figure 1 is a drawing of this photograph.

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[1] This work was supported in part by Grant No. NsG 563 from the National Aeronautics and Space Administration.

[2] Chief, Moonwatch Division, Smithsonian Astrophysical Observatory.

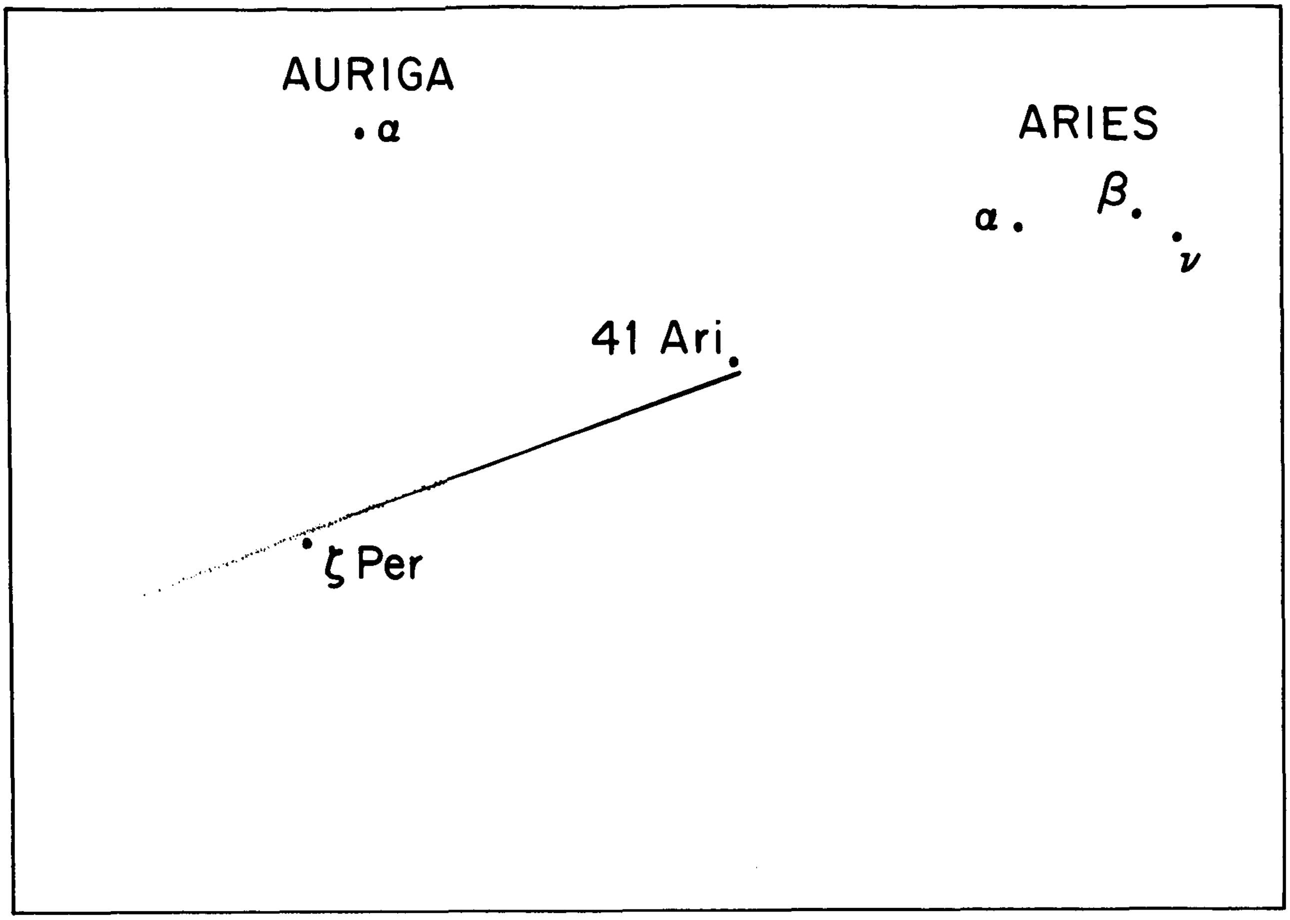


Figure 1. Drawing of the Bochum photograph.

All these observers gave detailed and graphic descriptions of what they had seen, and most of them made position measures or naked-eye estimates. All agreed that it was a most spectacular sight.

This decay appeared to have been carefully observed over a longer arc than any previous one. It therefore seemed desirable to make a detailed study of the observations to determine, if possible:

A. the height of the satellite at various points along the trajectory (it is also of interest to know the exact height at which the luminous trail reported by the observers first began to appear, and that at which pieces of the satellite were first seen to be breaking away);

B. how the observations compare with the Spiral Decay Ephemeris;

C. the approximate point of impact, supposing that part of the satellite survived passage through the atmosphere.

It was evident from the observations that the satellite was definitely lower than had been predicted, but it seemed probable that the track of the subsatellite point was pretty close to the predicted one.

## 2. DESCRIPTIONS

As some of the letters received dealt with other things besides the observations of the entry, they will not be quoted verbatim, summaries of only the relevant portions being given. Where necessary, we have translated from the original German. The sites are taken in order from South to North.

### Zurich Moonwatch (H. R. Epprecht)

The telegram giving the prediction arrived about 3 minutes before the satellite. The observations were therefore made in rather a hurry.

The sky had been overcast but cleared in places just before the satellite arrived. The satellite emerged from behind the clouds at the predicted time at an altitude of about 35°. The magnitude was estimated as -3. The color was almost white with a greenish-blue tint. No variation in brightness was detected. I saw no tail, but my father thought he saw a very short one.

The speed was so great that it was impossible to pick the satellite up with the theodolite. It touched $\eta$ Draconis, but this position was not timed, owing to some trouble with the stopwatch. A timed position was estimated when the satellite was in line between Polaris and $\alpha$ Ursae Majoris, the distance from $\alpha$ Ursae Majoris being about 1.3 times the distance between $\alpha$ and $\beta$ Ursae Majoris.

The satellite was in view for about half a minute.

Eilenburg Moonwatch (E. Otto)

When the object was first seen, the magnitude was estimated as -2. It was greenish in color at first and then changed to yellowish white and finally to yellow. It left behind a continuous vapor trail that was clearly visible despite the darkness; it was similar to the vapor trail left in the sky by a jet aircraft in the daytime. The trail had an average width of 20 to 25 min of arc and could still be distinguished in the telescope 2 or 3 minutes later.

Rodewisch Moonwatch (Prof. E. Penzel)

The sky had been overcast but cleared up in the West 2 minutes before the predicted time.

At $17^h54^m$ UT, object 1615 was seen with the naked eye as a bright, star-like object of magnitude -4. An exact measurement of time and position was made at $17^h54^m57\overset{s}{.}3$ UT.

At this instant, I noticed the beginning of the formation of a tail. The object increased in brightness by about half a magnitude on approaching culmination.

The object could be followed with the naked eye almost to the Northern horizon, where it vanished into haze.

The width of the tail was about half a degree, and toward the end of the period of observation, it was estimated to be 80° to 90° in length, becoming brighter all the time.

No sign of breakup into fragments was observed.

Bochum (H. Kaminski) (Telegram)

| | |
|---|---|
| Color: | yellow to red |
| Magnitude: | -3 to -4 |
| Tail: | about 120° long with strong afterglow |

Berlin (H. Zimmer)

A. Naked eye: One object. Magnitude -6 to -7. Color, almost white. The coma-like disk had a diameter of about 5 min of arc. The object had a long, reddish-yellow tail of estimated length 20°. A long, persistent, luminescent trail of light greenish-yellow color was observed along the satellite's path.

B. Telescopic: No definite nucleus was seen in the coma-like disk, but strong, turbulent motions. We had the impression that little flames erupted in the disk, and there was a shock wave in front of the object. The brightness was steady and increased slowly.

The object was first seen at about $17^h55^m30^s$ UT and last seen at $17^h56^m48^s$ (estimated).

Ole Roemer Observatory (Reported by Dr. A. V. Nielsen)

| | |
|---|---|
| Observer: | Svend Holm |
| Place of Observation: | Rungsted, Denmark |
| Color: | A small blue-white nucleus with red-orange border |
| Magnitude: | Brighter than Venus at its brightest |

The object disappeared behind a house; shortly before this, it was seen to pass about 1° West of β Aurigae.

A long train was seen from SSW to NNE (East of the zenith). The train was lying about half a degree or less West of 5 Lacertae.

<u>Copenhagen (Reported by Dr. N. Wieth-Knudsen)</u>

A. <u>Observer: B. Petersen</u>

| | |
|---|---|
| Place: | Tisvildeleje |
| Magnitude: | -1 |
| Color: | yellow |
| Size: | about 10' |
| Maximum altitude: | about 60° |
| Train | 20° long; color: yellow. About 10 bright objects broke away from the head and gradually faded out. |

B. <u>Observer: T. S. Pedersen</u>

| | |
|---|---|
| Place: | Nejlinge |
| Magnitude: | -10 |
| Color: | white |
| Size: | 15' |
| Maximum altitude: | about 80° |
| Train: | 15° long; color: reddish white. Ten or more bright objects broke away from the head and slowly faded. |

## 3. MEASURES

All the reported measures are listed in Tables 1 and 2. Those in Table 2 are rough estimates and were not used in the computation.

The numbers in the first column are for reference and are in order from South to North along the trajectory in each table:

(1) is the position of $\eta$ Draconis; (1a) is that of $\theta$ Draconis, which appears to be the more probable identification.

(6) and (7) were measured by the author on the photograph from Bochum. Backward extension of the trail in the photograph showed it to pass through the position reported by Bochum (5).

(10) is the position of 5 Lacertae, no adjustment being made for the "less than 1/2° West" reported.

(11) was computed by converting the position of $\beta$ Aurigae to altitude and azimuth, subtracting 1° from the azimuth and converting back to right ascension and declination.

Table 1. Measures

| Site | Position no. | Longitude (W) | Latitude | Height | Time UT | R. A. | Dec. | Epoch |
|---|---|---|---|---|---|---|---|---|
| | | | | m | h m s | h m | | |
| Zurich | (1) | 351° 29'25" | + 47° 23'48" | 476 | | 16 23.5 | + 61° 35' | Date |
| | (1a) | | | | | 16 01.0 | + 58 39 | " |
| | (2) | | | | 17 54 57.6 | 11 05 | + 69 00 | " |
| Rodewisch | (3) | 347 35 17 | + 50 31 51 | 467 | 17 54 57.3 | 17 07.8 | - 3 39 | 1950 |
| | (6) | | | | | 13 16.2 | + 46 07 | " |
| Bochum | (4) | 352 48 17 | + 51 25 44 | 127 | 17 55 32 | 1 51.5 | + 19 36 | " |
| | (5) | | | | | 2 47.7 | + 26 40 | " |
| | (7) | | | | | 3 50.6 | + 31 58 | " |
| Rungsted | (10) | 347 28 | + 55 53 | | | 22 28.1 | + 47 32 | Date |
| | (11) | | | | 17 57 | 6 02.0 | + 45 23 | " |
| | | | | | | Azimuth | Altitude | |
| Berlin | (8) | 346 38 49 | + 52 27 32 | 55 | 17 56 24.3 | 330° 06' | 20° 06' | |
| | (9) | | | | 17 56 40 | 355 06 | 7 42 | |

Table 2. Estimates

| Site | Position no. | Longitude (W) | Latitude | Height m | Time UT | Azimuth | Altitude |
|---|---|---|---|---|---|---|---|
| Rodewisch | (12) | 347° 35'17 | + 50° 31'51'' | 467 | $17^h 54^m$ | 237° | 7° |
| Berlin | (13) | 346 38 49 | + 52 27 32 | 55 | 17 55. 5 | 226 ± 5 | 4. 5 ± 0. 5 |
| | (14) | | | | 17 56. 8 | 8. 5 | 0. 5 |
| Tisvildeleje | (15) | 347 54 54 | + 56 03 23 | 30 | 17 59 ± 2 | 25 ± 1 | 6 ± 1 |
| Nejlinge | (16) | 347 46 36 | + 56 01 37 | 28 | 18 00 ± 3 | 23 ± 1 | 12 ± 1 |

## 4. THE HEIGHTS

If the orbit plane can be taken as known, the easiest way to determine the height of the satellite from an observation is to find the point of intersection of the direction of the satellite, as seen by the observer, with the orbit plane. This was one of the methods used by Solomon.[3]

The situation here is, however, rather different from that with which Solomon had to deal. In his case, the satellite was observed only over a short distance and was then more than 100 km high. It could therefore be assumed to be moving in a geocentric plane without significant error.

That may not be so in the present case, since the observed arc was some 600 miles long and the heights (as will shortly appear) were less.

To get some idea of the reliability of the observations and the order of magnitude of the heights, a simple graphical method was first employed.

Space Defense Center very kindly supplied a copy of the S. D. Ephemeris, computed at intervals of 1 minute. The relevant part of this, rounded off to about five significant figures, is given in Table 3. The rectangular

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[3] Solomon, L. H., Observation of the GT-5 rocket-body reentry-preliminary analysis, Smithsonian Astrophysical Observatory Special Report No. 191, 1965, page 4, ¶ b.

Table 3. S. D. Ephemeris

| UT $17^h_+$ | $\phi$ | $\lambda$(E) | h km | x km | y km | z km | $\dot{x}$ km/sec | $\dot{y}$ km/sec | $\dot{z}$ km/sec |
|---|---|---|---|---|---|---|---|---|---|
| $54^m 28^s.74$ | 46°.758 | 7°.008 | 103.21 | + 3060.9 | - 3227.7 | + 4698.3 | - 1.5068 | + 5.8603 | + 4.9438 |
| 55 28.74 | 50.576 | 9.206 | 101.69 | + 2962.6 | - 2868.0 | + 4982.1 | - 1.7706 | + 6.1236 | + 4.5113 |
| 56 28.74 | 54.330 | 11.829 | 99.80 | + 2848.7 | - 2493.6 | + 5239.2 | - 2.0243 | + 6.3525 | + 4.0536 |
| 57 28.74 | 57.997 | 15.019 | 97.43 | + 2719.9 | - 2106.5 | + 5468.1 | - 2.2661 | + 6.5444 | + 3.5724 |
| 58 28.74 | 61.544 | 18.973 | 94.47 | + 2577.0 | - 1709.1 | + 5667.4 | - 2.4929 | + 6.6943 | + 3.0688 |
| 59 28.74 | 64.917 | 23.694 | 90.73 | + 2421.1 | - 1304.2 | + 5835.9 | - 2.6996 | + 6.7922 | + 2.5426 |
| $18^h_+$ | | | | | | | | | |
| $00^m 28^s.74$ | 68.032 | 30.351 | 85.85 | + 2253.7 | - 895.6 | + 5972.0 | - 2.8724 | + 6.8097 | + 1.9899 |
| 01 28.74 | 70.752 | 38.491 | 79.08 | + 2078.0 | - 490.5 | + 6073.9 | - 2.9622 | + 6.6443 | + 1.3954 |
| 02 28.74 | 72.853 | 48.244 | 68.57 | + 1903.4 | - 109.0 | + 6137.9 | - 2.7790 | + 5.9099 | + 0.7248 |
| 03 28.74 | 74.051 | 57.182 | 51.03 | + 1762.8 | + 182.5 | + 6159.6 | - 1.6854 | + 3.3844 | + 0.0158 |
| 04 28.74 | 74.358 | 60.502 | 31.86 | + 1710.0 | + 285.6 | + 6150.5 | - 0.3038 | + 0.5546 | - 0.2135 |
| 05 28.74 | 74.382 | 60.803 | 19.52 | + 1701.3 | + 301.0 | + 6139.4 | - 0.0666 | + 0.1305 | - 0.1383 |
| 06 28.74 | 74.383 | 60.809 | 13.09 | + 1698.2 | + 308.3 | + 6133.2 | - 0.0444 | + 0.1199 | - 0.0795 |
| 07 00.41 | 74.383 | 60.809 | 10.71 | + 1696.9 | + 312.1 | + 6130.9 | - 0.0410 | + 0.1204 | - 0.0662 |

coordinates and velocity components in the last six columns are referred to the usual fixed geocentric equatorial axes: Z toward the North Pole, X toward the equinox of date, and Y toward right ascension 6 hours. The latitudes, longitudes, and heights are geodetic.

The geodetic positions of the observing sites were plotted on a large-scale conical projection chart (Figure 2) together with the track of the subsatellite point from the ephemeris. Observations (1) through (11) were converted to altitude and azimuth where necessary, by use of roughly interpolated times where no observed times were available. Lines were then drawn in these azimuths from the sites to intersect the track.

The great circle distance $\sigma$ from site to intersection, together with the altitude E, then gives the approximate height h by the formula:

$$\rho + h = \frac{\rho \cos E}{\cos (E + \sigma)} \quad , \tag{1}$$

where $\rho$ is the radius of the earth at the latitude of the site.

Measurement of the positions of the intersections along the track relative to the points plotted from the ephemeris gives the computed times of the observations, assuming the azimuths to be ideally accurate.

Table 4 gives the results.

The heights derived from observations (3) through (8) fall into a reasonable sequence, in view of the roughness of the method, although they are systematically lower than those predicted. The naked-eye observations, (1), (2), (10), and (11), are more erratic, as would be expected; (9) also fails to fall into line.

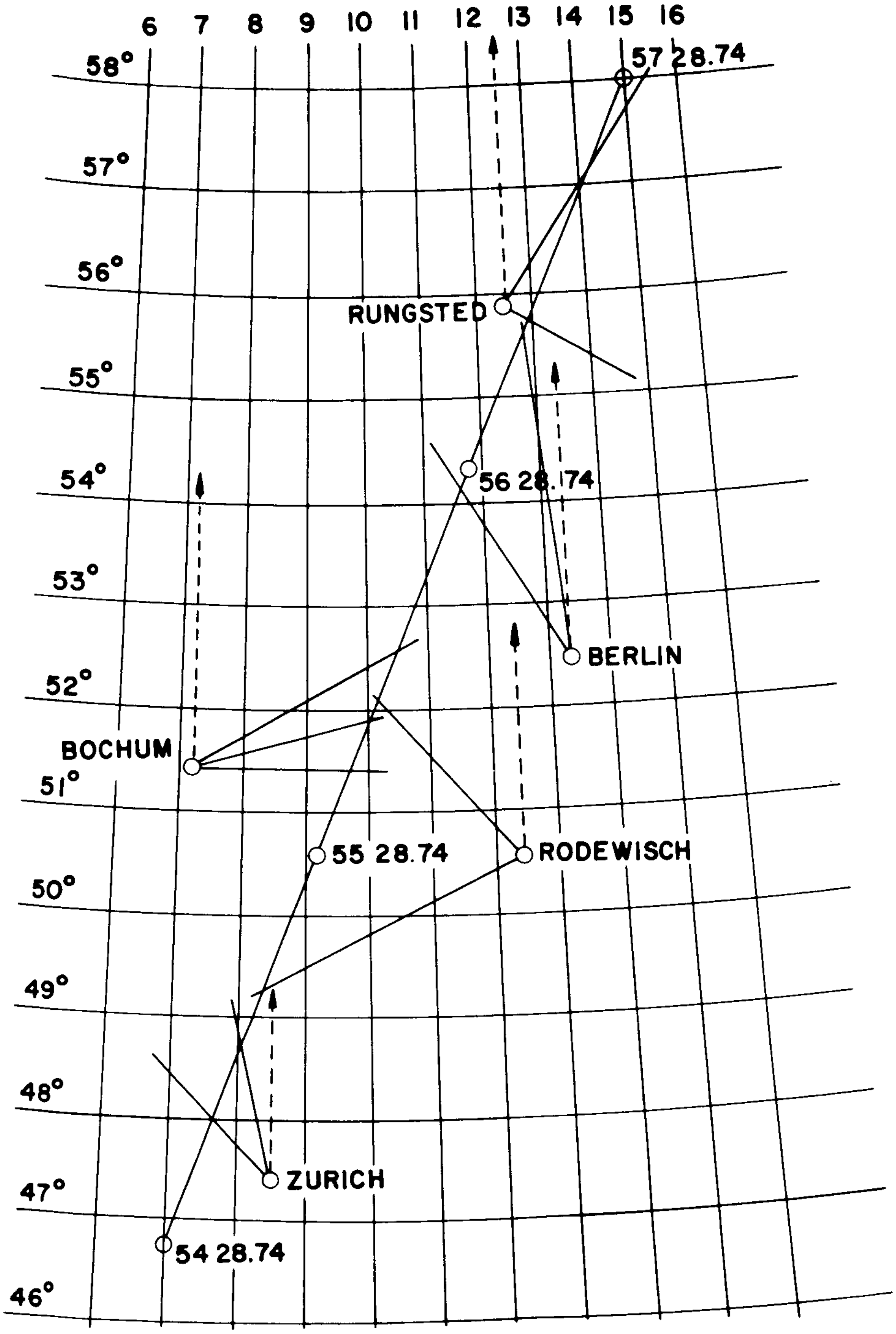


Figure 2. Subsatellite track.

Table 4. Heights and times

| Position no. | Site | Azimuth | Altitude | Time (obs.) UT | Time (calc.) UT | O - C | $h_O$ km | $h_C$ km |
|---|---|---|---|---|---|---|---|---|
| | | | | h m s | | s | | |
| (1) | Zurich | 316° 38' | 49° 38' | | $17^h 54^m 28^s.7$ | | 109 | 103 |
| (2) | | 347 48 | 28 39 | 17 54 57.6 | 17 54 59.9 | - 2.3 | 83 | 102 |
| (3) | Rodewisch | 245 20 | 14 28 | 17 54 57.3 | 17 55 07.7 | - 10.4 | 90 | 102 |
| (6) | | 320 11 | 19 41 | | 17 55 52.7 | | 87 | 101 |
| (4) | Bochum | 89 39 | 25 08 | 17 55 32 | 17 55 41.4 | - 9.4 | 83 | 102 |
| (5) | | 74 04 | 21 45 | | 17 55 49.3 | | 82 | 101 |
| (7) | | 59 26 | 16 48 | | 17 55 59.5 | | 81 | 101 |
| (8) | Berlin | 330 06 | 20 06 | 17 56 24.3 | 17 56 25.0 | + 0.7 | 84 | 100 |
| (9) | | 355 06 | 7 35* | 17 56 40 | 17 56 50.7 | - 10.7 | 58 | 99 |
| (10) | Rungsted | 120 31 | 76 24 | | 17 56 51.4 | | 117 | 99 |
| (11) | | 34 20 | 19 24 | 17 57 | 17 57 12.5 | | 57 | 98 |

*Corrected for refraction.

The times observed at positions (2) and (8) agree with the computed times, while those of (3), (4), and (9) all have residuals of about -10 sec.

One further point should be noticed. The two Rodewisch measures, made from East of the track, indicate rather greater heights than the three Bochum measures, made from the West. This suggests that the plotted track lies too far to the West, as would be the case if the ephemeris times were systematically late.

At this point, it must be frankly admitted that we have probably obtained all the information we could expect from observations of this standard of accuracy. However, as this was the first time a satellite had been visually observed and its positions measured over such a long arc so close to the decay point, and as the observers took so much trouble to obtain those measures, I thought that I should try to get as much information as possible from them.

Before we proceed to accurate numerical work, we must settle the question about the times.

Too much significance should not be attached to the agreement of positions (2) and (8) with the ephemeris times: the latter could easily be in error by much more than 10 sec. In fact, as we have just seen, there is some indication that they are in error. Next observe that the times of the two Berlin measures disagree with each other and cannot be reconciled by any reasonable assumption about the position of the track. Finally, the Zurich observations were made in a great hurry ( at 3 minutes' notice!).

On the other hand, there is no obvious reason to question the reliability of the Bochum and Rodewisch times. I therefore decided to accept these as approximately correct and to "anchor" the computed times to them. Times are, of course, needed to compute the coordinates of the observers relative to fixed axes.

## 5. THE ENTRY TRAJECTORY

If the trajectory of the entering satellite lies in a fixed geocentric plane, the computation of the heights indicated by the measures presents no difficulty.

In actual fact, as it descends through the atmosphere, the satellite departs from its almost fixed plane, finally moving in a plane defined by the small circle of the impact latitude. We have to decide whether the departure will be large enough to have a significant effect over the observed arc. This can best be seen in polar coordinates.

From the rectangular coordinates, we have:

$$\tan A = \frac{y}{x} \quad , \tag{2}$$

$$\tan D = \frac{z}{\sqrt{x^2 + y^2}} \quad , \tag{3}$$

where A and D are the geocentric R. A. and Dec. Writing A′ for the R. A. in a plane, we have

$$\sin (A' - \Omega) = \cot i \tan D \quad . \tag{4}$$

Let the plane be defined by

$$ax + by + z = 0 \quad . \tag{5}$$

Differentiating with respect to time and solving the resulting equation simultaneously with (5) for a and b, we get

$$a = \frac{y\dot{z} - \dot{y}z}{x\dot{y} - \dot{x}y} \quad , \tag{6}$$

$$b = \frac{z\dot{x} - \dot{z}x}{x\dot{y} - \dot{x}y} \quad . \tag{7}$$

Then

$$\tan \Omega = - \frac{a}{b} \quad , \tag{8}$$

$$\tan i = \sqrt{a^2 + b^2} \quad . \tag{9}$$

Taking the plane defined by the vector velocity and radius when the height (see Table 3) is 90.73 km (near the beginning of the observed arc), we find:

$$a = -3.3236 \quad ; \tag{10}$$

$$b = -1.6953 \quad ; \tag{11}$$

$$\Omega = 297\overset{\circ}{.}025 \quad ; \tag{12}$$

$$i = 74\overset{\circ}{.}996 \quad . \tag{13}$$

Table 5, computed from data in Table 3, shows that at a height of 79.08 km (near the end of the observed arc), A - A′ amounts to only a little over 1 min of arc.

Table 5. Departure from planar orbit

| Minute no. | Time | h | D | A′ | A | A - A′ | θ |
|---|---|---|---|---|---|---|---|
| 0 | $17^h 58^m 28\overset{s}{.}74$ | 94.47 | 61° 23′.0 | 326° 26′.8 | 326° 26.8 | 0′.0 | 65°.343 |
| 1 | 59 28.74 | 90.73 | 64 46.1 | 331 41.4 | 331 41.4 | 0.0 | 69.468 |
| 2 | 18 00 28.74 | 85.85 | 67 53.9 | 338 19.5 | 338 19.6 | + 0.1 | 73.579 |
| 3 | 01 28.74 | 79.08 | 70 37.8 | 346 41.9 | 346 43.1 | + 1.2 | 77.592 |
| 4 | 02 28.74 | 68.57 | 72 44.7 | 356 40.1 | 356 43.3 | + 3.2 | 81.375 |
| 5 | 03 28.74 | 51.03 | 73 57.0 | 5 42.9 | 5 54.7 | + 11.8 | 84.228 |
| 6 | 04 28.74 | 31.86 | 74 15.4 | 8 58.1 | 9 28.9 | + 30.8 | 85.175 |
| 7 | 05 28.74 | 19.52 | 74 17.0 | 9 17.1 | 10 02.0 | + 44.9 | 85.258 |
| 8 | 06 28.74 | 13.09 | 74 17.0 | 9 17.1 | 10 17.4 | + 60.3 | 85.258 |
| | 07 00.41 | 10.71 | 74 17.1 | 9 18.3 | 10 25.3 | + 67.0 | |

If the departure from the plane is about the same in the observed trajectory, where the above heights are reached at lower latitudes, we can ignore it. The simple calculation in the Appendix justifies that assumption.

The second and last columns in Table 5 will be explained later.

## 6. CALCULATION OF THE HEIGHTS

We define the following symbols:

$\alpha, \delta$: measured R. A. and Dec. corrected to equinox of date
$\ell$, m, n: direction cosines of observed position
X, Y, Z: geocentric coordinates of observer
x, y, z: geocentric coordinates of satellite
r: radius vector of satellite
$\rho$: radius of earth along radius vector
h: height, defined as $r - \rho$
R: range from observer to satellite
$\tau$: local sidereal time
$\phi$, H: geodetic latitude and height above sea level of observer.

Then

$$\left.\begin{aligned} X &= \rho \cos\phi' \cos\tau \\ Y &= \rho \cos\phi' \sin\tau \\ Z &= \rho \sin\phi' \end{aligned}\right\} , \tag{14}$$

and $\rho \cos\phi'$ and $\rho \sin\phi'$ are computed from Table VII in The American Ephemeris and Nautical Almanac with arguments $\phi$ and H. We have

$$\left.\begin{aligned} R\cos\delta\cos\alpha &= R\ell = x - X \\ R\cos\delta\sin\alpha &= Rm = y - Y \\ R\sin\delta &= Rn = z - Z \end{aligned}\right\} . \tag{15}$$

Substituting for x, y, z in equation (5) and solving for R, we have

$$R = - \frac{aX + bY + Z}{a\ell + bm + n} \quad . \tag{16}$$

Then

$$x = R\ell + X, \text{ etc.} \quad , \tag{17}$$

$$r^2 = x^2 + y^2 + z^2 \quad , \tag{18}$$

$$h = r - \rho \quad . \tag{19}$$

(Note: Substituting R from (16) in (17), we get the equivalent of Solomon's equations on page 15 of Special Report 191. He points out that there are misprints in the third of these equations. The plus signs inside the brackets should be minus signs.)

The above coordinates are referred to fixed axes.

We are now faced with a difficulty. To compute X, Y, Z, from (14), we need the times and, as explained in Section 4, we have to use computed times. These are obtained as follows:

The last column in Table 5 gives the geocentric angle $\theta$ between the satellite and the node at 1-minute intervals, which are numbered for reference in the second column. If we assume, as we probably can with sufficient accuracy, that $d\theta/dt$ is a function of height alone, then comparison of $\theta$ computed from the observations with $\theta$ from the table at corresponding heights will give us the time intervals. The required times then follow by "anchoring" these to the accepted time of observation (4).

The trouble is that, in order to compute $\theta$ from the observations by the formula

$$\sin \theta = \frac{z}{r \sin i} \quad , \tag{20}$$

we need z, which is a function of X, Y, Z. We therefore have to proceed by successive approximations. This is done graphically. We first plot $\theta$ and h from Table 5 against the minute numbers in column 2 (Figure 3).

We then add 9.4 sec to the computed times in Table 4 and use the preliminary times so obtained to compute X, Y, Z and hence $\theta$ and h for the observations. The values of h are plotted on a transparent overlay on Figure 3 against the preliminary time intervals, and the overlay is moved along the time axis until the best fit with the h curve is found.

This gives the time on the minute number scale corresponding to observation (4) and enables us to convert the computed intervals of $\theta$ into time intervals either by use of the graph or by interpolation in Table 5.

Further approximations were found to be unnecessary. The final results appear in Table 6 and the computed heights are plotted in Figure 3 against the final computed time scale marked below the time axis.

The times show much the same picture as before except that they have now been adjusted to make the residual of position (4) zero.

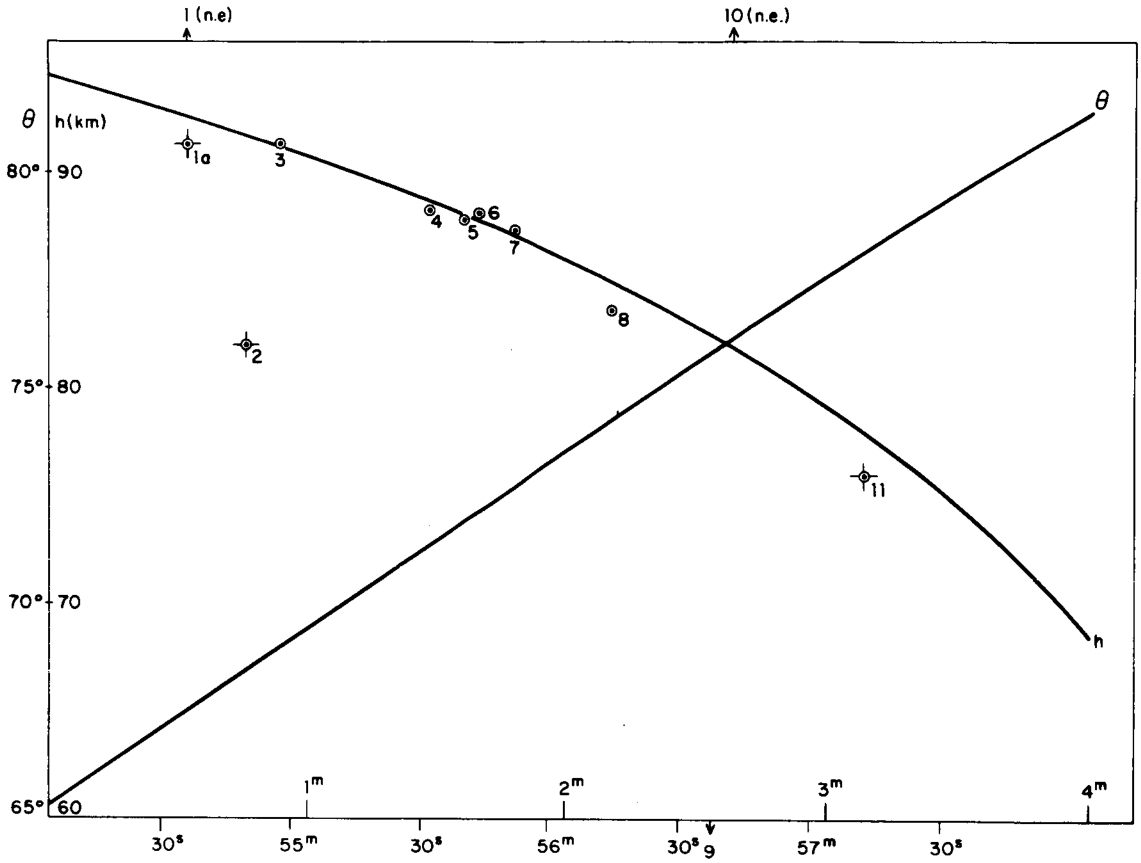


Figure 3. Naked-eye estimates: -⊙- or (n. e.); telescopic or photographic: ⊙ .

Table 6. Heights and times

| Site | Position no. | R. A. (date) | Dec. (Date) | t (obs.) | t (calc.) | O-C | $h_O$ km | $h_C$ km | O-C km |
|---|---|---|---|---|---|---|---|---|---|
| | | | | h m s | | s | | | |
| Zurich | (1) | $16^h 23^m.5$ | + 61° 35' | | $17^h 54^m 36^s.1$ | - | 105.5 | 92.6 | + 12.9 |
| | (1a) | 16 01. 0 | + 58 39 | | 17 54 33.6 | | 91.3 | 92.6 | - 1.3 |
| | (2) | 11 05. 0 | + 69 00 | 17 54 57.6 | 17 54 48.5 | + 9.1 | 82.0 | 91.8 | - 9.8 |
| Rodewisch | (3) | 17 08. 6 | - 3 40 | 17 54 57.3 | 17 54 57.3 | 0.0 | 91.3 | 91.2 | + 0.1 |
| | (6) | 13 16. 9 | + 46 02 | | 17 55 43.5 | | 88.1 | 87.7 | + 0.4 |
| Bochum | (4) | 1 52. 3 | + 19 44 | 17 55 32 | 17 55 32.0 | 0.0 | 88.2 | 88.6 | - 0.4 |
| | (5) | 2 48. 6 | + 26 44 | | 17 55 40.3 | | 87.8 | 87.9 | - 0.1 |
| | (7) | 3 51. 6 | + 32 01 | | 17 55 51.5 | | 87.3 | 87.0 | + 0.3 |
| Berlin | (8) | 12 29. 1 | + 50 12 | 17 56 24.3 | 17 56 14.3 | + 10.0 | 83.6 | 84.8 | - 1.2 |
| | (9) | 9 48. 9 | + 44 57 | 17 56 40.0 | 17 56 37.7 | + 2.3 | 56.9 | 82.2 | - 25.3 |
| Rungsted | (10) | 22 28. 1 | + 47 32 | | 17 56 43.8 | | 140.5 | 81.7 | + 58.8 |
| | (11) | 6 02. 0 | + 45 23 | 17 57 | 17 57 12.5 | | 76.0 | 77.9 | - 1.9 |

The fit of the heights computed from the telescopic measures is now very close except for position (9). Of the naked-eye estimates, (11) now fits quite well, as does (1a) ($\theta$ Draconis). Position (2) is a very rough estimate and could easily be in error by a couple of degrees.

Position (10) is not that of the satellite itself but of the trail left behind. Even so, there seems to be something wrong here. We shall discuss this and position (9) later.

## 7. COMPARISON WITH THE EPHEMERIS

The observations disagree with the ephemeris mainly by placing the satellite lower. If we consider the predicted trajectory as lying in a plane, it can be made to represent the heights computed from the more reliable observations by rotating it backward in that plane. Measures (1a) (if this is the correct identification of the star) and (11) do seem to suggest that the h graph ought to be more strongly curved, but not much significance can be attached to this.

The S. D. Ephemeris is computed by numerical integration using a method similar to Cowell's method for special perturbations. It starts from a given position and vector velocity, and the accelerations are functions of the position and velocity, depending on the gravitational field and the assumed density distribution of the atmosphere. Disagreement with observation can thus be due either to errors in the starting data or to incorrect acceleration functions. This differs from the ordinary astronomical case in which the latter are not in doubt.

Loosely speaking, a wrong atmospheric density distribution will change the shape of the trajectory, while wrong starting data will shift it bodily. It is a pity that we do not have accurate measures at the ends of the observed arc. In the absence of such measures, no positive conclusions can be drawn.

To allow a final comparison with the observations, the S. D. Ephemeris is adjusted as follows:

Starting with the plane defined by equations (12) and (13), we compute A and $\theta$ for a series of values of D. The times are then computed as before, and values of h from Figure 3 are used to compute the corresponding values of r. Table 5 is used to correct the values of A for the drag of the rotating atmosphere.

The ephemeris in Table 7 is constructed by interpolation and conversion of polar to rectangular coordinates.

The residuals in Table 8 are computed relative to the nearest point on the track as seen by the observer, thus throwing the maximum possible residual into the time. The resultant cross-track residual is represented by $\Delta N$.

Table 7. Ephemeris from observations

| Time UT $17^h$ + | h km | ΔA | A′ | A | D | Equatorial earth radii x | y | z |
|---|---|---|---|---|---|---|---|---|
| $54^m 35^s.0$ | 92.7 | 0!0 | 314° 07!9 | 314° 07!9 | + 47° 39!8 | + 0.47490 | - 0.48941 | + 0.74857 |
| 45.0 | 92.0 | 0.0 | 314 31.8 | 314 31.8 | + 48 17.9 | + 47238 | - 48019 | + 75598 |
| 55.0 | 91.4 | 0.0 | 314 56.3 | 314 56.3 | + 48 55.9 | + 46979 | - 47081 | + 76328 |
| 55 05.0 | 90.6 | 0.0 | 315 21.4 | 315 21.4 | + 49 33.8 | + 46714 | - 46137 | + 77045 |
| 15.0 | 89.9 | 0.0 | 315 47.1 | 315 47.1 | + 50 11.5 | + 46442 | - 45187 | + 77750 |
| 25.0 | 89.1 | 0.0 | 316 13.5 | 316 13.5 | + 50 49.1 | + 46163 | - 44229 | + 78441 |
| 35.0 | 88.3 | + 0.1 | 316 40.5 | 316 40.6 | + 51 26.6 | + 45877 | - 43267 | + 79120 |
| 45.0 | 87.4 | + 0.1 | 317 08.3 | 317 08.4 | + 52 03.9 | + 45586 | - 42302 | + 79787 |
| 55.0 | 86.6 | + 0.2 | 317 36.7 | 317 36.9 | + 52 41.0 | + 45290 | - 41334 | + 80440 |
| 56 05.0 | 85.8 | + 0.3 | 318 05.8 | 318 06.1 | + 53 17.9 | + 44987 | - 40361 | + 81079 |
| 15.0 | 84.8 | + 0.4 | 318 35.7 | 318 36.1 | + 53 54.5 | + 44678 | - 39387 | + 81703 |
| 25.0 | 83.9 | + 0.5 | 319 06.3 | 319 06.8 | + 54 30.8 | + 44365 | - 38413 | + 82312 |
| 35.0 | 82.7 | + 0.6 | 319 37.7 | 319 38.3 | + 55 06.7 | + 44049 | - 37438 | + 82905 |
| 45.0 | 81.5 | + 0.7 | 320 10.0 | 320 10.7 | + 55 42.2 | + 43729 | - 36463 | + 83481 |
| 55.0 | 80.3 | + 0.9 | 320 42.9 | 320 43.8 | + 56 17.5 | + 43405 | - 35489 | + 84040 |
| 57 05.0 | 79.0 | + 1.1 | 321 16.5 | 321 17.6 | + 56 52.3 | + 43076 | - 34517 | + 84583 |
| 15.0 | 77.5 | + 1.3 | 321 50.9 | 321 52.2 | + 57 26.7 | + 42742 | - 33549 | + 85110 |

Table 8. Residuals

| Position No. | Site | Time UT $17^h +$ | O-C | ΔN | R. A. (date) | O-C $\Delta\alpha \cos\delta$ | Dec. (date) | O-C |
|---|---|---|---|---|---|---|---|---|
| | | m s | s | | | | | |
| (1) | Zurich | | | 232' | 245° 52' | +181' | +61° 35' | +144 |
| (1a) | | | | 8 | 240 15 | -6 | +58 39 | -4 |
| (2) | | 54 57.6 | +8.9 | 129 | 166 15 | +32 | +69 00 | -125 |
| (3) | Rodewisch | 54 57.3 | -0.1 | 1 | 257 09 | +1 | -3 40 | 0 |
| (6) | | | | 5 | 199 13 | +2 | +46 02 | +4 |
| (4) | Bochum | 55 32.0 | +0.4 | 0 | 28 04 | 0 | +19 44 | 0 |
| (5) | | | | 4 | 42 09 | -2 | +26 44 | +3 |
| (7) | | | | 10 | 57 54 | -3 | +32 01 | +9 |
| (8) | Berlin | 56 24.3 | +8.7 | 24 | 187 16 | -9 | +50 12 | -22 |
| (9) | | 56 40.0 | -2.3 | 219 | 147 14 | +75 | +44 57 | -205 |
| (10) | Rungsted | | | 568 | 337 02 | -404 | +47 32 | +397 |
| (10a) | | | | 9 | 356 06 | +5 | +46 14 | -7 |
| (11) | | 57 | (12.1) | 26 | 89 15 | -16 | +44 57 | -20 |

Note: Position (1a) is θ Draconis
Position (10) is 5 Lacertae
Position (10a) is ψ Andromedae
Position (11) is β Aurigae

## 8. DISCUSSION OF THE RESIDUALS

### Zurich

These were naked-eye observations, made at very short notice, and there was a lot of cloud.

Position (1) was reported as a coincidence with $\eta$ Draconis, but the predicted track passed nearly 4° away from that star. It passed very close to $\theta$ Draconis, however, and in view of the conditions and the agreement with later observations, it seems probable that this is the star the observers saw.

As noted before, position (2) was a very rough estimate.

### Rodewisch and Bochum

All these observations agree very well with the adjusted ephemeris; remarkably well, in fact, for such a large, bright, and fast-moving object.

### Berlin

The observer estimated the probable error of position (8) as half a degree, which agrees with the residual. He thought position (9) was even more accurate, but it seems that a mistake of some sort must have been made, possibly in reading a circle.

Note that, in addition to making two measures with an altaz-mounted telescope in less than half a minute, he also took some time to study the appearance of the object, of which he gave a detailed description; and, according to his own estimate, the object was in view for little more than a minute all told. This is very fast working and it is easy to make slips under such conditions. Doubtless this also accounts for the evident time error.

Rungsted

The observer noted that the object passed "about 1° West of $\beta$ Aurigae." The residual makes it half a degree, which is satisfactory for a naked-eye observation at low altitude (position (11)).

The large residual (nearly 10°) of position (10) is not easy to account for. It was, of course, the trail and not the satellite that was observed, and some drift is possible, but hardly so much. If this was another misidentification, the only star that seems to fit is $\psi$ Andromedae, which is half a magnitude fainter than 5 Lacertae. If the object really did pass West of 5 Lacertae, position (11) would surely have had a much larger residual; and position (11), referred to a bright and easily recognized star, seems the more reliable of the two.

## 9. THE IMPACT POINT

The only reliable way to find the impact point - assuming that the satellite was not totally destroyed in the atmosphere - would be to use the S. D. Program, starting with coordinates selected from Table 7.

It is possible to get a rough position, however, without going to so much trouble. We assume that the length of the trajectory from some selected height to impact will be much the same as that predicted in the S. D. Ephemeris. If we now make the further assumption, which is, however, only very approximately true, that the displacement in R. A. due to atmospheric rotation is the same in both cases, the procedure is obvious.

Starting at a height of 79.08 km, we find the length of the subsatellite track to impact to be 7° 39' (geocentric). Table 3 shows that from a height of about 19 km, the satellite will fall almost vertically, and at this point, the departure from the plane is 45' of R. A. (Table 5). The time taken to drop from 79.08 km to 19.52 km is 4 minutes.

Using these figures to extrapolate the ephemeris in Table 7 and converting to geodetic latitude and longitude, we find that the object would be falling vertically at 19.52 km near:

| | |
|---|---|
| longitude | 21° 57' East |
| latitude | 63° 30' North |
| at | $18^h 01^m 04^s$ UT. |

This point is just off the coast of Finland, not far from the town of Vaasa.

Since the above was written, I have heard that NORAD, using preliminary coordinates I sent them, have computed the impact point as:

longitude 19°.9 East
latitude 62°.2 North .

This was not computed by the S. D. Program, which, it appears, cannot be started at a point so near to decay, but by another program specially designed for such cases.

This impact point is about 100 miles up range from the one I indicated. NORAD's figures are based on preliminary coordinates and mine on an approximate method, so perhaps the true impact point (if any) is somewhere between the two. If so, it is at sea, in the Gulf of Bothnia.

## 10. CONCLUSIONS

(A) The formation of a luminous trail commenced at a height of 91 km.

(B) Fragments were first seen breaking away at about 82 km.

(C) The S. D. Ephemeris represents the variation of height with time as closely as the observations were accurate enough to determine.

(D) While the S. D. Ephemeris gave the time over a given point accurate to about 10 sec, impact probably occurred some 4 1/2 minutes earlier than predicted.

(E) The more reliable measures were able to fix the height with an uncertainty probably not greater than 0. 5 km.


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## APPENDIX

## EFFECT OF ROTATING ATMOSPHERE

Let the components of the velocity v of the satellite be

$$\dot{r},\ r\dot{\delta},\ r\dot{\alpha}\cos\phi \quad . \tag{A-1}$$

Then

$$v^2 = \dot{r}^2 + r^2\dot{\delta}^2 + r^2\dot{\alpha}^2\cos^2\phi \quad . \tag{A-2}$$

In a nonrotating atmosphere, the acceleration due to air drag is

$$\dot{v} = -F v^2 \quad , \tag{A-3}$$

where F can be taken to be independent of v.

The eastward component of the acceleration is

$$\frac{r\,\dot{v}\,\dot{\alpha}\cos\phi}{v} = -F\,r\,v\,\dot{\alpha}\cos\phi \quad . \tag{A-4}$$

If we take the first two terms of Taylor's Series

$$r\,\Delta\alpha\cos\phi = r\,\dot{\alpha}\cos\phi\,\Delta t - \frac{1}{2}F\,r\,v\,\dot{\alpha}\cos\phi\,\Delta t^2 \quad ,$$

or

$$\Delta\alpha = \dot{\alpha}\,\Delta t - \frac{1}{2}F\,v\,\dot{\alpha}\,\Delta t^2 \quad . \tag{A-5}$$

In an atmosphere rotating with angular velocity $\omega$, the components of the velocity $V$ relative to the atmosphere are:

$$\dot{r}, \; r\dot{\delta}, \; r(\dot{\alpha} - \omega)\cos\phi \tag{A-6}$$

and

$$V^2 = \dot{r}^2 + r^2\,\dot{\delta}^2 + r^2(\dot{\alpha} - \omega)^2\cos^2\phi \quad . \tag{A-7}$$

We have

$$\Delta\alpha_r = \dot{\alpha}\;\Delta t - \frac{1}{2}\,F\;V(\dot{\alpha} - \omega)\,\Delta t^2 \quad . \tag{A-8}$$

Hence,

$$\Delta\alpha_r - \Delta\alpha = \frac{1}{2}\,F\left\{\dot{\alpha}(v - V) + V\omega\right\}\Delta t^2 \quad . \tag{A-9}$$

If $\dot{\alpha} >> \omega$, then $V \simeq v$ and

$$\Delta\alpha_r - \Delta\alpha \simeq \frac{1}{2}\,F\;v\,\omega\;\Delta t^2 \quad , \tag{A-10}$$

which is independent of $\phi$. Obviously, this approximation does not hold unless $\dot{\alpha} >> \omega$.

## NOTICE

This series of Special Reports was instituted under the supervision of Dr. F. L. Whipple, Director of the Astrophysical Observatory of the Smithsonian Institution, shortly after the launching of the first artificial earth satellite on October 4, 1957. Contributions come from the Staff of the Observatory.

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