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SATELLITE TRACKING WITH A LASER

C. G. Lehr

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ABSTRACT

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For satellite tracking, laser systems have characteristics that supplement the capabilities of Smithsonian's Baker-Nunn cameras. A laser system can measure range. It can operate when a satellite is in the earth's shadow. It can range on a satellite when the sky is too bright for the Baker-Nunn cameras.

The range equation and the statistics of the background noise are used to analyze a laser system. The results are applied to the experimental system now in operation at the Smithsonian astrophysical observing station in New Mexico. Ranging at night and during the day is considered for satellites with retroreflecting mirrors. The laser energy needed to photograph such a satellite in the earth's shadow is computed. Calculations show that the present system can only range on satellites that incorporate retroreflectors; however, commercially available components could be used to build a system that should obtain a return signal from a large noncooperative satellite like Echo 2.

SATELLITE TRACKING WITH A LASER¹

C. G. Lehr²

1. INTRODUCTION

1.1 Scope

This report considers laser systems for satellite tracking. They are examined particularly as supplements to the Baker-Nunn cameras of the Smithsonian Astrophysical Observatory (SAO). This specific application involves considerations that differ somewhat from other uses of lasers for satellite tracking. For example, the scientific program of the Observatory requires that satellite orbits be obtained to the greatest accuracy possible. There is little interest, however, in communicating with a satellite or in identifying the satellite from characteristics of the laser return. Real-time processing of the information from the laser system is not necessary since the precisely reduced data from the cameras are not obtained for several months. Fairly modest requirements for the reliability of a laser system can be tolerated because of the redundancy involved if equipment is installed in all the observing stations. These latter considerations should lead to ways of reducing the complexity and cost of the laser system.

In order to lend substance to the material covered, the experimental laser system now in use at the Smithsonian astrophysical observing station, Organ Pass, New Mexico, will be used as a basis for discussion. This system employs a laser and a photoelectric detector built by the Re-entry Systems Department, Missile and Space Division, General Electric Company (GE). The present system is similar to the one used by GE in obtaining what was probably the first laser return from a satellite. The experimental laser equipment was not designed specifically for the purpose at hand.

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The positioning systems for the laser and photoelectric detector are relatively unsophisticated ones that require observers to track the satellites visually. The full capability of the Observatory in predicting a satellite's coordinates at a given time is not fully utilized by such a tracking system. The ways in which the present laser system can be modified for improved performance will be indicated in the report.

1.2 Characteristics of a Laser System

It is interesting to observe how well a laser system supplements, but does not duplicate, the functions of the Baker-Nunn camera. For instance, the laser can be used to illuminate a satellite when it is in the earth's shadow. If the satellite has a retroreflecting mirror on it, the reflected laser energy can be photographed with the Baker-Nunn camera. More importantly, the laser determines range, something that the cameras cannot obtain directly. A range measurement obtained simultaneously with a Baker-Nunn photograph defines the position vector of the satellite at the time of the observation. Laser range measurements should also be obtainable when the sky is too bright to use the camera. These returns could be distinguished from the noise of the sky background by use of a narrow-band photoelectric detector whose filter is centered at the laser's wavelength. Range measurements can, of course, be obtained with UHF or microwave radar systems. For the present application, however, lasers have the following advantages:

a. Relatively simple and lightweight retroreflectors can be attached to a satellite to increase the directivity,³ and consequently the strength, of the returned signal by a factor of about 10^8 . These retroreflectors

³The divergence of the retroreflector on the BE-B satellite is 2.8×10^{-9} sterad, whereas the divergence from a specularly reflecting sphere is 4π sterad.

are passive devices, requiring no electrical energy from the satellite. They are much simpler than the beacons or transponders used with such electronic tracking systems as Minitrack, Secor, Tranet, or the Range and Range-Rate System. The precise point on the satellite to which range is measured is also more accurately known at the shorter optical wavelengths.

b. Lasers can generate hundreds of megawatts (Mw) of peak power, whereas the most powerful radars, such as the Millstone or the AN/FPQ-6, produce less than 10-Mw peak power even though they generate much more average power than a laser.

c. The generation of a very short pulse of energy is an inherent characteristic of the laser. A pulse-forming circuit is not required. The pulse length of a laser is typically tens of nanoseconds (nsec) when the laser is in the Q-switched mode of operation. Such a pulse introduces an error of a meter or two into the range measurement obtained from a single return; consequently, the correlation techniques that would have to be used with typical radar pulses of a microsecond (μ sec) or so are not required.

d. The laser needs a lens of only modest aperture to concentrate the transmitted energy within a beam of several minutes of arc. For example, a 3-min beam can be obtained with a 3-inch lens, whereas the FPQ-6 radar (Mason, 1965) requires a 30-ft antenna to produce a 24-min beam. The Haystack antenna (Weiss, 1965), 120 ft in diameter, has a beamwidth of 0.9 min when operating at a frequency of 39 Gc. A pulsed ruby laser typical of those now being manufactured could produce a 0.9-min beam with a 14-inch lens.

e. The ionosphere does not affect the propagation of the visible laser radiation, whereas it has a significant affect on propagation in the UHF range (but not at microwave wavelengths). The troposphere introduces an error of several meters into the range measurement of both

a laser and a radar system. This error can be reduced by introducing a correction based on a model of the atmosphere.

The laser also has several disadvantages with respect to radar. Because quanta of visible light are more energetic than microwave quanta, a larger amount of received energy is required to assure the reception of at least one quantum. Lasers in the band of visual wavelengths cannot operate under adverse weather conditions, since clouds and rain are not transparent to the laser radiation. The narrow beam of the laser requires either visual tracking or highly refined predictions for the acquisition of a satellite.

The accuracy with which a laser system determines range depends on: (1) the error in measuring the two-way transit time of the pulse to the satellite and back, (2) the error in correcting atmospheric effects, and (3) the error in the value used for the velocity of light in vacuum. The accuracy of the measurement of the transit time is essentially the accuracy with which the transmitted and received pulses can be superposed. This resolution depends on the rise time of the transmitted pulse and the amount of the distortion in the received pulse. An estimate of this resolution is difficult to make. For the present purpose, a standard deviation of 5 nsec, about one-half the minimum practical value of a Q-switched laser, will be used. The equivalent error in range is about 1 m. Since the velocity of light in vacuum (American Institute of Physics Handbook, 1963) is known to only about 1 part in 10^6 , the corresponding error for a satellite at a typical distance of 2 Mm is 2 m. This error is not significant when range measurements that are scaled to the velocity of light have a useful application. Appendix A shows that the correction for the earth's atmosphere will be less than 4 m for elevations above 40° . If the accuracy of this correction is assumed to be within 10%, the correction will have a standard deviation of about 0.4 m. If local meteorological data are used, the accuracy will be better. From

these estimates we see that the overall error in the range measurement might have a standard deviation of about

$$(1^2 + 2^2 + 0.4^2)^{1/2} \doteq 2 \text{ m} .$$

The laser may be used to illuminate a retroreflecting satellite when it is in the earth's shadow and not visible by reflected sunlight. Then the satellite can be photographed against a star background with the Baker-Nunn camera. For this purpose the laser is not Q-switched, since maximum energy rather than minimum pulse length is desired. Calculations show that enough energy is returned to expose the Kodak 2475 film used in the camera. Experiments (Anderson et al., 1966), using a red filter over the lens to remove reflected sunlight confirmed this possibility when a satellite with a retroreflector was tracked visually and illuminated with a laser pulse.

Several additional applications of the laser become attractive if satellite positions can be predicted to a few minutes of arc and the necessity for visual tracking can be eliminated. Such accuracy of predictions is not presently needed for the SAO network because of the wide field-of-view of the Baker-Nunn cameras; however, this accuracy is consistent with the accuracy of the field-reduced data from the Baker-Nunn cameras.

With predictions whose accuracy is several minutes of arc, satellites can be acquired with the laser beam when the satellite is in the earth's shadow. It will be pointed out later that ranging when the sky is too bright for photography should also be feasible.

Ranging on a noncooperative satellite (i. e. , one without a retro-reflector) has a marginal chance of success if the satellite is a very

large one, such as Echo 2. The main difficulty is that the returned signal is very weak. If the satellite is tracked visually, the strong background of sunlight reflected from the satellite may mask the returned signal. If the laser is directed toward the satellite when it is in shadow, the laser beamwidth may have to be increased to compensate for the less accurate predictions available on a balloon-type satellite. If the beamwidth is increased without increasing the transmitted power, the returned signal becomes still weaker.

The type of laser system best adapted to the Baker-Nunn network is probably one in which the laser and the photoelectric receiver can be accurately positioned to predicted angular coordinates. Such a system is predicated on the fact that the satellite's position can be predicted to within the same error as that associated with visual tracking. The laser should have high pulsed power and narrow pulse width. Such a laser favors range measurements over photography in shadow, since these measurements provide probably the most useful additional information needed in computing satellite orbits. The fixed positioning system permits fewer range measurements per satellite pass but simplicity and economy favor the installation of a number of laser systems, one at each observing station. Probably at least one of these stations should have a mount suitable for visual tracking and one should have a laser capable of producing a long, high-energy pulse for photography.

1.3 A Summary of Early Work

Lasers became practical for satellite tracking when the first satellite incorporating a retroreflector (the Explorer 22 or BE-B) was launched on October 10, 1964. G. L. Snyder (Snyder et al., 1965) of GE reported the reception of a photoelectric return on October 18, 1964. H. H. Plotkin (Plotkin et al., 1965) reported a return on October 31,

1964. A photoelectric return was also obtained by Bivas and Blamont (1964) on January 24, 1965. R. L. Iliff (1965) made a laser photograph of a satellite on January 21, 1965. The Smithsonian Astrophysical Observatory made its first successful observation on June 19, 1965, obtaining both photographic and photoelectric data simultaneously.

2. LASERS AS TRANSMITTERS

Lasers (Birnbaum, 1964; Brinton, 1964) are of three types: crystal or glass lasers, gaseous lasers, and semiconductor lasers. A crystal or glass laser characteristically generates short, high-energy pulses. The two most important materials for these lasers are ruby and neodymium-doped glass. The ruby laser is probably more suitable for satellite tracking because its output is in the deep-red end of the visual band rather than in the infrared where photographic film and photo-emissive devices are less sensitive. The dimensions of the ruby rod used in a laser are large compared to the wavelength of the laser; thus, the rod can support a large number of modes. Although the Fabry-Perot reflecting structure limits their number, single-mode operation is not obtained when the energy output is high. The large number of modes in which the laser operates simultaneously makes the output beam significantly wider than the diffraction-limited value. For example, the diffraction-limited beamwidth for a ruby rod 3/8-inch in diameter is (Jenkins and White, 1957)

$$\theta = 1.22 \frac{\lambda}{D} = 0.89 \times 10^{-4} \text{ rad} \quad ,$$

where $\lambda = 0.694 \mu$, the wavelength of the ruby laser, and $D = 0.95 \text{ cm}$, the diameter of the rod, whereas the actual beamwidth of such a laser is about 0.01 rad. Even though laser beams are not presently diffraction limited, their widths can be decreased by collimating them with a lens system having an exit pupil larger than the cross section of the ruby rod. Since the brightness of the beam remains constant (Jenkins and White, 1957, pp. 111-112) within the collimator, its solid angle on emergence is inversely proportional to the area of

the exit pupil. Thus, for the 0.95-cm-diameter ruby considered above, a 13-inch lens is required for a beamwidth of 1 min of arc. A Galilean telescope is usually used for the optical system because it brings the beam to a virtual focus rather than a real focus. The high concentration of energy at a real focal point might ionize the air and cause the laser beam to defocus.

The gaseous lasers that are now available commercially probably do not have sufficient power for satellite tracking. Laboratory models of molecular lasers have higher powers and greater efficiencies. Gaseous lasers can be operated CW and can produce diffraction-limited beams. The use of these properties in satellite tracking is not as straightforward as the use of the high-pulsed power of a ruby laser. There is no reason, however, why a system using a CW gas laser might not eventually equal or exceed the performance of a system using a pulsed ruby laser.

At present, semiconductor lasers have neither the high power nor the low beam divergence needed for satellite tracking. Most of them need to be operated at low temperatures. This requirement may be inconvenient under field conditions.

3. THE RANGE EQUATION

If the laser transmits E joules how many photons S will be returned to expose the film of the Baker-Nunn camera or to activate a photoelectric detector? This question is answered by the range equation, which will be derived below.

We assume that the reflector on the satellite spreads the beam sufficiently to compensate for a velocity aberration (see Appendix B) of $2 v/c$ rad, where v is the component of the velocity of the satellite that is perpendicular to its position vector and c is the velocity of light.

The fraction of the transmitted light reaching the satellite's reflector equals the solid angle subtended by this reflector divided by the solid angle of the laser beam. The fraction of this light returned to the receiver (telescope or photoelectric detector) is the solid angle subtended at the satellite by the receiver divided by the solid angle of the beam that returns from the satellite's reflector. Thus,

$$S = E \cdot \frac{A_s/R^2}{\Omega_T} \cdot \frac{A_r/R^2}{\Omega_S} \cdot T^2 \cdot \frac{10^{19}}{2.86} \text{ photons ,} \quad (1)$$

where A_s is the effective area of the satellite's reflecting surface (see Appendix C), A_r is the effective area of the light-collecting aperture of the receiver (see Appendices D and E), Ω_T is the solid angle of the transmitted laser beam (see Appendix F), Ω_S is the solid angle of the laser beam reflected from the satellite, R is the slant range between the laser and the satellite, and T is the atmospheric extinction along a path between the laser and the satellite (see Appendix G).

The numerical factor in equation (1) converts joules to photons at a wavelength of 6943 Å. The experimental verification of equation (1) is complicated by atmospheric effects and by the properties of the retroreflectors on the satellites. The energy of the outgoing beam acquires a definitely nonuniform distribution as it goes through the atmosphere (Whitten et al., 1965; Lehr et al., 1966a). For this reason the energy reaching the satellite's reflector may be more or less than the proportional amount indicated in equation (1). The returning beam also appears to suffer some atmospheric effect. A returning pulse seems to break into several pulses separated by time intervals so short that they are not resolved by the oscilloscope at the sweep rate usually employed for the experiment (Anderson et al., 1966; Lehr et al., 1966b).

The low eccentricity of the BE-B and BE-C orbits adds to the difficulty of verifying equation (1), because large values of R correspond to low elevation angles of the satellite. When the elevation angle is low, the arriving laser beam makes a large angle with the axis of the satellite's reflector. The reflectance of the corner-cube assembly deteriorates under this condition. Consequently, a decrease in S may be caused either by an increase in R or a decrease in the reflectance. The higher eccentricity of GEOS-I makes its orbit more suitable for verifying the range equation, but a variation in the effective area of the reflector with elevation introduces a complication. Limited data from range measurements on the three satellites show (Lehr et al., 1966b) that the returned signal was always more than 16 db below the value of S obtained from equation (1).

We obtain the range by multiplying the measured time interval between the transmitted and received pulses by one-half the velocity of light in vacuum, and making a correction for the effect of the atmosphere. If the satellite is at a slant range of 1.5 Mm, the time interval is 10 msec. The present system has a timing resolution of ± 10 nsec, which corresponds to ± 1.5 m.

4. DETECTION AT NIGHT

The range equation gives S , the number of photons received for a transmitted pulse of E_j . In order to use the range equations to determine E , we must know the smallest value of S that gives us the information we require.

Suppose that we are transmitting a 20-nsec pulse and that we have no background noise. In order to detect the corresponding received pulse we must receive a signal strong enough to generate at least 1 photoelectron in the detector. Let us say that for practical purposes the incoming signal is strong enough if it generates at least 1 photoelectron with a probability of 99%. We assume that $p(n; \lambda)$, the probability that the signal generates n photoelectrons, is given by the Poisson⁴ distribution:

$$p(n; \lambda) = \frac{\lambda^n}{n!} e^{-\lambda} \quad , \quad (2)$$

where λ is the average number of photoelectrons that would be generated if the detection were performed many times with the same signal strength. For a 99% probability of generating at least 1 photoelectron we have

⁴We neglect photon clumping (Purcell, 1956) because the intensity is low. We also neglect the fact that we have an assembly of corner-cube reflectors and that this assembly gives a negative binomial distribution rather than a Poisson distribution (Goodman, 1965).

$$\sum_{\ell=1}^{\infty} p(n; \lambda) = 1 - p(0; \lambda)$$

$$= 1 - e^{-\lambda} = 0.99 \quad . \quad (3)$$

We obtain $\lambda = \ln 100 = 4.6$ by solving equation (3). If the quantum efficiency of the detector is 3%, the returned signal under these conditions must be at least

$$\frac{4.6}{0.03} = 153 \text{ photons} \quad .$$

If we wish to detect a returned laser pulse in the presence of noise, we must discriminate against the photoelectrons generated by the noise. This discrimination can be accomplished if we predict the range of the satellite and gate the receiver "on" only over a time interval that brackets the uncertainty in the prediction. Or we can increase the detection threshold of the receiver and consequently the minimum S needed to assure that the receiver will, with a sufficiently high probability, respond to the signal but not to noise. The second method leads to simpler operation but requires a higher powered laser. The increased power for typical operation probably does not exceed that of available commercial lasers.

The receiver collects noise from the sky background. This noise passes through the interference filter and then on to the phototube. We assume that its total value⁵ at the phototube is 15 photons/ μ sec. This noise is 15 times that of the dark-night sky. We use it as an approximation to actual conditions. For a quantum efficiency of 0.03, it is equivalent

⁵ See Appendix D.

to an average of 9×10^{-3} photoelectrons in a 20-nsec interval. The probability of the emission of k or more photoelectrons in a 20-nsec interval is

$$P(k) = \sum_{n=k}^{\infty} p(n; 0.009) \quad , \quad (4)$$

from which Table 1 is obtained (General Electric Co., 1962).

TABLE 1. Emission probabilities of k or more photoelectrons

k	$P(k)$
0	1.0
1	9.0×10^{-3}
2	4.0×10^{-5}
3	1.2×10^{-7}
4	2.7×10^{-10}

For a satellite at 1.5 Mm, the propagation time of the pulse is 10 msec. Within this time are 5×10^5 20-nsec intervals in which a signal could be received. The counter that measures the propagation time is triggered first by the transmitted pulse and then by the received pulse. No range gating is assumed. In order to be reasonably sure that a noise pulse does not interfere with this measurement, we must set the threshold of the counter high enough to exclude noise pulses that occur often enough to cause significant numbers of false measurements. We must then transmit sufficient laser power to be sure that

the returning signal exceeds this threshold. If we set the threshold just below k photoelectrons, we are interested in the probability P_k that 1 or more of the 5×10^5 intervals contains a noise pulse of k or more photoelectrons. This probability is

$$P_k = 1 - p(0; \lambda_k) = 1 - e^{-\lambda_k} \quad , \quad (5)$$

where $\lambda_k = 5 \times 10^5 P(k)$, the average (over the 10-msec propagation time) of the number of noise pulses containing k or more photoelectrons. From equation (5) and Table 1 we tabulate the following result.

TABLE 2. The probability that one or more intervals contains a noise pulse

k	P_k
0	1.0
1	1.0
2	1.0
3	6.2×10^{-2}
4	1.2×10^{-4}

Table 2 shows that the threshold level of the counter should be set at 4 photoelectrons for a probability of less than 0.01 for a false range measurement. To make sure that the incoming signal will activate the counter, we need a signal strong enough to provide 4 or more photoelectrons with a probability of say, 0.99. If we let λ be the strength of the returned signal in terms of photoelectrons produced in the phototube, we have

$$0.99 = \sum_{n=4}^{\infty} p(4; \lambda) = 1 - \sum_{n=0}^3 p(4; \lambda) \quad . \quad (6)$$

The solution of equation (6) gives 10 photoelectrons⁶ for the value of λ . For a quantum efficiency of 0.03, this value is equivalent to a received signal of 330 photons. Three hundred and thirty photons in 20 nsec corresponds to 9.4×10^{-17} j or 4.7×10^{-9} w. This is the minimum value of the received energy needed to attain at 0.99 probability that a return is received from a satellite and that this return is not a noise pulse. To produce 330 photons the range equation (1) shows that for the BE-B or BE-C satellite and the laser transmitter and receiver described in the appendices, we need transmit only 1.5×10^{-3} j for a slant range of 1.5 Mm. For 0.5-j transmitted energy the maximum range would be 6.4 Mm if the same reflector were used on the satellite.

⁶We note in comparison that Flint (1964) obtains 18 photoelectrons in 16.7 nsec for a detection probability of 10^{-3} , when he analyzes a system that is similar, but not identical, to the present one.

5. DETECTION IN DAYLIGHT

Let us now examine the possibility of ranging on a satellite in daylight. From Appendix D we see that 1 photon/ μsec reaches the phototube when the sky is dark. Consequently 0.03 photoelectron/ μsec is generated. Appendix H shows that the daylight sky is about 10^7 times as bright as the night sky; thus an average of 6000 photoelectrons is emitted in a 20-nsec interval. This large average value permits a simplification in the calculations. We may use the normal distribution function $\Phi[(x - m)/\sigma]$, where

$$\Phi(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^u e^{-\omega^2/2} d\omega \quad , \quad (7)$$

with $m = 6000$ and $\sigma = 6000^{1/2} = 77.5$, instead of the Poisson distribution with $\lambda = 6000$, in determining the probability that x photoelectrons are emitted in a 20-nsec interval. The large average value also gives us an advantage in our receiving equipment. We can utilize the integration in our measuring circuit to obtain the average (DC) value of the background noise. The circuit eliminates this average value, leaving a fluctuating (AC) background against which the returned signal is to be measured. In deciding how strong a signal is needed to overcome the noise background, we first determine a value $y = x - 6000$, such that $P(y)$, the probability of the emission of y electrons above the average in any 20-nsec interval, is 0.01. Using equation (7), we obtain the following expression:

$$1 - \Phi\left(\frac{y}{77.5}\right) = 0.01 \quad \text{or} \quad \Phi\left(\frac{y}{77.5}\right) = 0.99 \quad . \quad (8)$$

From a table of the normal distribution (see Table 1 from Parzen (1960)), we obtain the solution to equation (8): $y = 181$ photoelectrons. This is the signal needed if the range were already known and the receiver could be gated "on" only during the proper 20-nsec interval. If the receiver is not gated, we need a higher signal strength to assure that the noise exceeds the signal with a low probability over all the 5×10^5 intervals that might contain a return.

We now calculate the value of $P(y)$ that satisfies the condition that the probability is only 0.01 of one or more intervals containing more than y photoelectrons. On the average, λ intervals will contain more than y photoelectrons, where

$$\lambda = 5 \times 10^5 P(y) \quad . \quad (9)$$

If we use this expression for λ , the condition above may be

$$\sum_1^{\infty} p(n;\lambda) = 1 - p(0;\lambda) = 1 - e^{-\lambda} \approx \lambda = 0.01. \quad (10)$$

From equations (9) and (10) we see that

$$P(y) = \frac{0.01}{5 \times 10^5} = 2 \times 10^{-8} \quad .$$

We now determine what value of y corresponds to this value of $P(y)$. Again we use the normal distribution, and we now obtain the following equation:

$$1 - \Phi\left(\frac{y}{77.5}\right) = 2 \times 10^{-8} \quad . \quad (11)$$

For such a low value of $P(y)$, the following inequality (see p. 192 of Parzen (1960)) is useful:

$$1 - \Phi(u) \leq \frac{1}{u\sqrt{2\pi}} e^{-u^2/2} \quad . \quad (12)$$

From equations (11) and (12), $y = 430$ photoelectrons when the equality sign in equation (12) is used. If we let $S = 430$, we have a signal-to-noise ratio of

$$\frac{S}{N} = \frac{S}{\sigma} = \frac{430}{77.5} = 5.5 \quad .$$

It is of interest that this is about the signal-to-noise ratio (Rose, 1948) necessary to distinguish a point image from the background on a photographic film. For a quantum efficiency of 0.03, the received signal must be $430/0.03 = 14,000$ photons. For the BE-B satellite at a range of 1.5 Mm, the range equation gives

$$\frac{S}{E} = 2.2 \times 10^5 \frac{\text{photons}}{\text{j}}$$

for the experimental laser and photodetector being considered in this report. This means that the transmitted energy must be at least 0.065 j for detection in daylight. Since actual returns have been 16 to 20 db below theoretical values, a minimum energy of between 2.6 and 6.5 j

may actually be required. For a 20-nsec pulse this corresponds to a transmitted power of between 130 Mw and 320 Mw. This amount of transmitted power should be obtainable from lasers now available commercially. The required transmitted power could be reduced by use of a receiver that integrates successive pulses. This integration, however, introduces the following difficulties. The pulse repetition frequency and average power of the laser are increased. The integration introduces an additional error into either the range measurement itself or the exact time at which it is made.

6. NONCOOPERATIVE SATELLITES

Another interesting possibility is ranging on a noncooperative satellite (i. e. , one without a retroreflector) such as a large reflecting sphere like Echo 2 (see Appendix C). The solid angle of the laser beam reflected from a sphere is 4π . This is $4\pi/(2.8 \times 10^{-9}) = 4.5 \times 10^9$ times that for the BE-B and BE-C satellites. We have seen that we should be able to detect a retroreflecting satellite at 6.4 Mm. The range of Echo 2 is 1.15 Mm at perigee. For detection with a 0.5-j laser, the range equation gives the following requirement for the cross-sectional area of the sphere

$$\left(\frac{1.15}{6.4}\right)^4 \times (4.5 \times 10^9) \times (8 \times 10^{-3}) = 3.8 \times 10^4 \text{ m}^2 .$$

This cross-sectional area is about 30 times that of Echo 2. Consequently, a signal from Echo 2 cannot be obtained with a strength sufficient to result in a precise range measurement with the laser system and night-time noise background that have been assumed in this report. And the background noise might be greater than that assumed if Echo 2 were ranged on when it was sunlit. In order to range on Echo 2 at perigee, 14 j or 710 Mw would be required for the probability of detection assumed in Section 4. The minimum value of energy that could possibly produce a useful return is that amount needed to produce 1 photoelectron. This value is one-tenth that considered above. Consequently, the transmitted power would have to be only 71 Mw. Of course, a fairly accurate prediction of the range would be needed to distinguish this return from one of the photoelectrons in the noise background. Possibly, two receivers and a coincidence circuit could be used to distinguish signal

from noise. In practice, the additional 16- to 20-db attenuation of unknown origin that seems to be present in the present system makes the detection of a return from Echo 2 more difficult.

7. A COMPARISON OF LASER AND RADAR SYSTEMS

It is interesting to compare lasers and microwave radars for satellite tracking. To make this comparison definite we shall compare the Millstone radar with our present experimental laser system. The Millstone radar was chosen because it can range on passive satellites at distances greater than 1 Mm. It is, of course, a large, highly developed, general-purpose instrument, not specifically designed to perform the function for which it is being compared. Nor is our present experimental laser system representative of what might ultimately be achieved. However, both systems now exist, and the comparison is a realistic one for the present if not for the future.

We note immediately that the radiance (transmitted power/solid angle) for a laser is larger than that for a radar. For example, our laser, operating in the Q-switched mode, has a radiance of about 1.3×10^{13} w/sterad, whereas the radiance for the Millstone radar⁷ is 4.3×10^{10} w/sterad. The laser obtains its advantage from the short wavelength that leads to good collimation and the short pulse that is inherent in the physical mechanism of the stimulated emission. On the other hand, the laser receiver is less sensitive than the microwave receiver. Assume the equivalent noise of the laser system is the fluctuation in the background of 15 photons/ μ sec. For a Poisson distribution this fluctuation is $15^{1/2} = 3.9$ photons/ μ sec or 1.1×10^{-12} w. If the receiver responds to a 20-nsec pulse, its bandwidth is at least

⁷For the characteristics of the Millstone radar, see Appendix I.

50 Mc. Consequently, the noise per unit bandwidth is

$$N_0 = 2 \times 10^{-20} \text{ w/cps}$$

For the Millstone radar we have

$$N_0 = kT = 2 \times 10^{-21} \text{ w/cps} \quad ,$$

where k is Boltzmann's constant and T , the temperature of the receiver, is taken to be 150° K .

The area of the receiving antenna also favors the radar. The area of the Millstone antenna is 2900 times the effective area of the present laser receiver.

The pulse-repetition frequency of the Millstone radar is 900 times that of our experimental laser system. Since the signal-to-noise ratio is proportional to the number of pulses that can be integrated (see p. 36, eq. 2.31 of Skolnik (1962)), the higher repetition rate of the radar might be expected to increase its sensitivity appreciably. The integration required for this improvement, however, is not easily realized when precise ranging is required. It can only be carried out by approximating the rate of change of range from a knowledge of the satellite's orbit. When integration is used, the problem of determining the instant corresponding to the range measurement is compounded.

The laser has a particular advantage when a retroreflector is placed on a satellite. For example, an assembly of corner cubes like the one used on the BE-B satellite returns 3.6×10^7 times more power

than a specularly reflecting sphere 1 m^2 in cross section. This increase in returned power corresponds to an increase of 77 times in range. For an object 1 m^2 in cross section with a retroreflector similar to that on BE-B, both the Millstone radar and our laser system have ranges of several megameters on a single-pulse basis. The accuracy of the radar is 1 km, whereas that of the laser should be several meters. We see that a large radar has a far greater range capability on a 1-m^2 target than does a laser system, but the laser competes when a retroreflector is used and precise range measurements are required. The radar, of course, has the advantage of all-day and all-weather tracking. Its more rapid pulse-repetition frequency permits some searching for the satellite if the predictions are not sufficiently accurate.

8. PHOTOGRAPHY OF A SATELLITE BY REFLECTED LASER LIGHT

Appendices E, J, and K show that an exposure of 10^5 photons should produce an image on the film used in the Baker-Nunn camera. If we put this value into the range equation, we see that about 1 j must be transmitted if the satellite is at a range of 1.5 Mm. For a transmitted pulse of 36 j, the calculated maximum range is 3.7 Mm. The limited experimental data (Lehr et al., 1966b) now available indicate that the returned signal is about 16 db below the calculated value. If these data are representative, the maximum range is only 1.5 Mm.

A laser provides useful illumination of a satellite only when the satellite is in the earth's shadow. In this case the satellite cannot be tracked visually; its position at a given instant must be predicted accurately. Predictions of sufficient accuracy are not made routinely by SAO. Consequently, a particular experiment was carried out that did not yield any useful data on satellite location but that demonstrated the feasibility of photography with a laser. For this experiment the BE-B satellite was tracked when it was visible and a Wratten No. 70 filter was used over the lens of the Baker-Nunn camera to reduce the reflected sunlight by 2.2 mag. This filter removed the star background and the sunlit image of the BE-B satellite. but the laser image was visible (Anderson et al., 1966).

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APPENDIX A

ATMOSPHERIC REFRACTION

Freeman (1964) considers the quantity ΔR , the difference between the optical path through the earth's atmosphere and the optical path through vacuum. The path is a straight line between a point on the earth's surface and a point above the atmosphere. He assumes that the refractivity of the earth's atmosphere has the following exponential variation:

$$N = (n - 1) 10^6 = N_s e^{-d(h - h_s)} \quad , \quad (A-1)$$

where n is the refractive index at height h , N_s is the refractivity at the earth's surface, h_s , and d is a constant. Using

$$g = \left(\frac{dr_o}{2} \right)^{1/2} \tan \beta_0 \quad , \quad (A-2)$$

where r_o is the radius of the earth and β_0 is the elevation angle, he obtains the following series:

$$\Delta R = \frac{10^{-6} N_s}{d \sin \beta_0} \left(1 - \frac{1}{2g^2} + \frac{3}{4g^4} - \dots \right) \quad . \quad (A-3)$$

For $r_o = 6378$ km, $N_s = 292$, $d = 0.1385/\text{km}$, and $\beta_0 = 40^\circ$, equation (A-3) becomes

$$\Delta R = 3.3(1 - 0.002 + \dots) \text{ m} \quad , \quad (\text{A-4})$$

which shows that the atmospheric correction should be about 3.3 m (since laser returns are not usually obtained at elevations below 40°) and the neglect of the earth's curvature introduces an error of only 2 parts in 1000.

The atmosphere may also introduce an error in the range measurement by bending the ray from ground to satellite away from a straight line. The maximum error this bending could introduce may be estimated from the following approximation. The atmosphere is assumed to be concentrated in a uniform layer extending t meters above the earth. The index of refraction within this layer is given the value at the earth's surface. The layer extends up to the point where the refractivity is 1/100 the value of the earth's surface. From equation (A-1) and the values of N_s and d used above, this criterion gives the value $t = 33.2$ km. Figure A-1 shows a ray traversing this layer. From Snell's law we have

$$n \sin \theta = \sin (\theta + \delta\theta)$$

Consequently,

$$(n - 1) \sin \theta = \sin(\theta + \delta\theta) - \sin \theta = \delta(\sin \theta) \approx \delta\theta \cos \theta \quad ,$$

or

$$\delta\theta \approx (n - 1) \tan \theta \quad (\text{A-5})$$

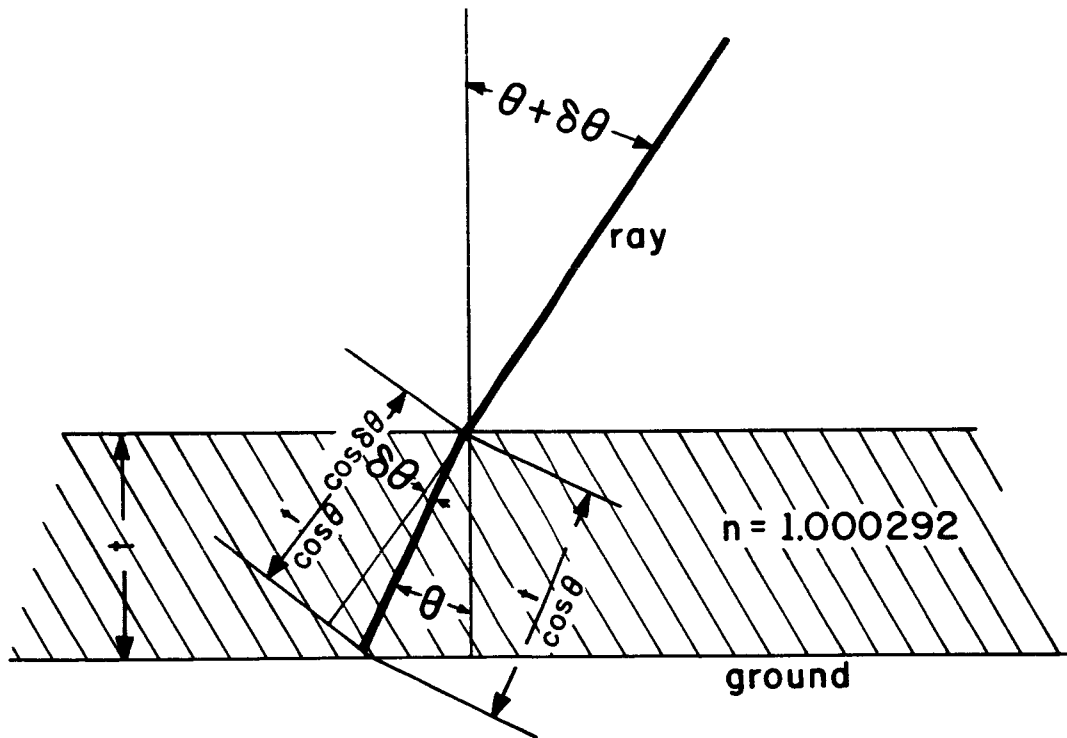


Figure A-1. Ray path through a medium that roughly approximates the earth's atmosphere.

If Δ is the increase in path length resulting from the atmospheric bending,

$$\Delta = \frac{t}{\cos \theta} - \frac{t}{\cos \theta} \cos \delta\theta = \frac{t}{\cos \theta} (1 - \cos \delta\theta)$$

$$\approx \frac{t}{\cos \theta} \cdot \frac{(\delta\theta)^2}{2} \quad . \quad (A-6)$$

From equations (A-5) and (A-6) we have

$$\Delta \approx \frac{t(n-1)^2}{2} \cdot \frac{\tan^2 \theta}{\cos \theta} = 1.41 \frac{\tan^2 \theta}{\cos \theta} \text{ mm} \quad . \quad (A-7)$$

For an elevation of 40° we have $\theta = 60^\circ$ and $\Delta \approx 8.5$ mm. This value is negligible compared to the correction of 3.3 m that is due to the fact that the atmosphere lowers the velocity of light.

The use of local meteorological data will allow a more accurate atmospheric correction to be made when it is needed.

APPENDIX B
VELOCITY ABERRATION

Plotkin (1964) has shown that a laser beam directed toward a retro-reflecting satellite at the zenith will return at an angle $\Delta = 2v/c$ from zenith if the satellite is moving with velocity v .

We obtain the expression for Δ by using the fact that in the moving coordinate system of the satellite the laser beam is returned in the same direction that it arrives. The transformation from the earth's coordinate system and back gives the angle Δ .

For a satellite at culmination and at a slant range R from the earth the beam returns to the ground a distance $d = R\Delta = 2Rv/c$ from the laser. If the satellite is not at culmination, d is smaller for the same value of R because only the component of satellite velocity perpendicular to R causes the aberration.

The velocity v may be obtained by using the following expression (Mueller, 1964) for the angular velocity of an earth satellite:

$$\frac{2\pi}{P} = \left(\frac{k^2 M}{a^3} \right)^{1/2},$$

where P is the period, k^2 is the gravitational constant, M is the mass of the earth, and a is the semimajor axis of the orbit. If the satellite is in a circular orbit, D Mm from the earth, the velocity is the angular velocity times the radius of the orbit. Thus the velocity of the satellite can be written as follows:

$$v = \frac{2\pi}{P} (D + R_o) = \left(\frac{k^2 M}{D + R_o} \right)^{1/2} ,$$

where R_o is the radius of the earth. When the values of the physical constants are used, this equation takes the following form:

$$v = \left(\frac{399}{D + 6.37} \right)^{1/2} \text{ km/sec} ,$$

where D is in megameters. The orbit of the BE-B satellite is nearly circular. For D we use 0.99 Mm, the mean of the perigee and the apogee, and obtain $v = 7.4$ km/sec. Consequently,

$$\Delta = 4.9 \times 10^{-5} \text{ rad} = 10'' \text{ arc} ,$$

and

$$d = 74 \text{ m} ,$$

when $R = 1.5$ Mm and the satellite is at culmination. Since the half beamwidth of the retroreflector on the BE-B satellite is 3×10^{-5} rad, satisfactory performance possibly may not always be obtained when the laser and the searchlight receiver are collocated.

APPENDIX C

CHARACTERISTICS OF PERTINENT SATELLITES*

Satellites with retroreflectors

BE-B (6406401)

apogee	1.09 Mm
perigee	0.89 Mm
inclination	80°
period	105 min
average magnitude	8.5
retroreflector (360 corner cubes)	
effective area	$8 \times 10^{-3} \text{ m}^2$
divergence	$12'' = 6 \times 10^{-5} \text{ rad}; 2.8 \times 10^{-9} \text{ sterad}$

BE-C (6503201)

apogee	1.32 Mm
perigee	0.94 Mm
inclination	41°
period	108 min
average magnitude	approximately that of BE-B
retroreflector	same as BE-B

* See SAO (1965a), Systems Science Corporation (1965), Pilkington (1965), Snyder et al. (1965), and GE (1965).

GEOS I (6508901)

apogee	2.27 Mm
perigee	1.12 Mm
inclination	59°
period	120 min
retroreflector (334 corner cubes)	
area	290 inch ² = 0.187 m ²
effective area	(all energy incident on this area reflected within 20" divergence angle) 9.35 × 10 ⁻² m ²
divergence	20" = 10 ⁻⁴ rad; 7.3 × 10 ⁻⁹ sterad
magnitude	5-8

Large satellites without retroreflectors

Echo 2 (6400401)

apogee	1.1 Mm
perigee	1.1 Mm
inclination	82°
period	108 min
shape	sphere: 41 m diameter
cross-sectional area	1.32 × 10 ³ m ²
average magnitude	1.0

Saturn 5 (6400501)

apogee	0.42 Mm
perigee	0.23 Mm
inclination	31°
period	91 min
shape	cylinder: 6.5 m diameter, 25.6 m long
maximum cross-sectional area	166 m ²

Pegasus 2 (6503901)

apogee	0.73 Mm
perigee	0.51 Mm
inclination	32°
period	97 min
shape	wings: 29.3 m × 4.3 m
maximum cross-sectional area	126 m ²

APPENDIX D
CHARACTERISTICS OF PHOTOELECTRIC RECEIVER

<u>Effective aperture*</u>	0.177 m ²
<u>Bandwidth</u>	70 Å
<u>Beamwidth</u>	20' arc; 2.66×10^{-5} sterad
<u>Focal length</u>	less than 26.5 inches
<u>Secondary mirror</u> (transmits a parallel beam to the photomultiplier)	4-inch diameter
<u>Photomultiplier</u> (RCA 7265, S-20 surface)	
effective diameter	2 inches
gain	10^7
load resistance	50 ohms
quantum efficiency at 7000 Å	0.03
dark current	10^{-14} amp
half-power points (approx)	3200 Å, 6300 Å
<u>Oscilloscope</u> (Tektronix model 545A with type H preamplifier)	
vertical scale	typically 5 mv/cm
horizontal scale	typically 100 μsec/cm
resolution	30 nsec
<u>Approximate attenuation</u> of white light by the 70 Å filters:	

$$\frac{70}{6300 - 3200} = 0.023 \doteq 4 \text{ mag}$$

* About 1/10 the actual area of the modified 60-inch searchlight.

Various noise sources in terms of the rates of arrival of photons:

	At the aperture of the receiver	At the phototube, after passing through the filter
	<u>(photons/μsec)</u>	<u>(photons/μsec)</u>
<u>Noise of the night sky</u>	50	1
<u>Equivalent noise of the dark current (without amplitude gating of dynode noise)</u>	100	2
<u>Estimated total noise under operating conditions</u>	750	15

APPENDIX E

CHARACTERISTICS OF BAKER-NUNN CAMERA*

Aperture area (less film bridge)	0.151 m^2
Optical efficiency	0.6
Effective area of aperture	$9.04 \times 10^{-2} \text{ m}^2$
Image diameter	less than $1.26 \times 10^3 \mu^2$
Focal length	0.508 m
Solid angle subtended by image (image diameter/focal length) ²	6.20×10^{-9} sterad
Bandwidth	4000-7000 Å

*See Henize (1957) and SAO (1965b).

APPENDIX F
CHARACTERISTICS OF GE LASER

Energy output

normal mode:

36 j in 1 msec (a superposition of random spikes, each having a duration of about 1 μ sec or less)

passive Q-switched mode:

a series of approximately 20 pulses, each of length approximately 30 nsec. A single pulse has an energy of about 1/3 j and is separated from a neighboring pulse by a continually increasing interval averaging about 50 μ sec.

rotating Q-switched mode:

0.5 j in less than 100 nsec

Efficiency Less than 1%

Characteristics of transmitted beam

0.375" diameter, 34' arc (without collimating lens)

3.75" diameter, 3.4' arc (with collimating lens)

Solid angle of transmitted beam, $\Omega_T = 7.8 \times 10^{-7}$ sterad

Wavelength 6943 Å at room temperature

Estimated bandwidth less than 1 Å

Dimensions of ruby 3/8-inch diameter, 8 inches long

Collimating lens 5-inch diameter (only 3.75-inch diameter central portion is used)

APPENDIX G
ATMOSPHERIC EXTINCTION

Kaula (1962) gives the following expression for the atmospheric extinction:

$$T = e^{-\tau \sec Z} \quad , \quad (G-1)$$

where $\tau = 0.0090 \lambda^{-4} + 0.223$, Z is the zenith angle, and λ is the wavelength in microns. T is the fraction of light at wavelength λ transmitted in a one-way path through the atmosphere from an incoherent source outside the atmosphere to an observer at sea level. The first term in τ represents the scattering by gas molecules; the second, the scattering by water droplets. We assume that the two-way path of a laser beam between the ground and the satellite has a transmission coefficient of T^2 , where $\lambda = 0.6943 \mu$. If θ is the elevation angle, equation (G-1) may be used to construct the following table:

<u>θ (degrees)</u>	<u>T^2</u>
10	0.074
20	0.27
30	0.40
40	0.49
50	0.56
60	0.59
70	0.62
80	0.63
90	0.64

Since the satellites now equipped with retroreflectors reflect poorly when θ is less than 40° , $T^2 = 0.49$ has been used for the calculations in this report.

APPENDIX H
BRIGHTNESS OF THE SKY

Baum (1962) gives the following values for the brightness of the sky:

Dark-night sky	4 th mag/square degree,
Moonlit sky (quarter phase)	twice brightness of dark-night sky
Moonlit sky (full)	greater than 10 times brightness of night sky
Sunlit sky	4 th mag/sec ²
(1.3 × 10 ⁷ times the brightness of the dark-night sky)	

Using the conversion factors in Appendix G, we can express these values as follows:

	<u>Brightness (photons-sec m² sterad)</u>
Dark-night sky	2.6 × 10 ¹²
Moonlit sky (quarter phase)	5.2 × 10 ¹²
Moonlit sky (full)	2.6 × 10 ¹³
Sunlit sky	3.4 × 10 ¹⁹

In addition to the light from the dark-night sky and the moon we have the average light of the star field. This light corresponds to about six stars (each of 9 mag) per square degree (Staff SAO, 1964), or

$$1.5 \times 10^{11} \frac{\text{photons}}{\text{sec m}^2 \text{ sterad}} .$$

Compared to the dark-night sky, this amount of light is negligible.

For an average value of the night sky we use

$$10^{13} \frac{\text{photons}}{\text{sec m}^2 \text{ sterad}} .$$

APPENDIX I

CHARACTERISTICS OF THE MILLSTONE RADAR^{*†}

Frequency	1.295 gc
Transmitted power	5 Mw
Pulse length	2 msec
Diameter of antenna	25.6 m
Resolution (corresponding to 2 msec pulse)	300 km
Accuracy of range measurement (6-sec integration)	± 1.5 km
Beamwidth	42' arc
Receiver temperature (with liquid N ₂ cooling)	150° K
Pulse repetition frequency	15/sec
Maximum range	9.5 Mm

* Millstone Long Range Tracking Radar, Millstone Hill Field Station,
MIT Lincoln Laboratory.

† See Pineo, 1965.

APPENDIX J
PHOTOMETRIC CONVERSION FACTORS

1 lumen	= $\frac{1}{206} w \doteq 1.35 \times 10^{16}$ photons/sec (for sunlight over the wavelength range of 3500 Å to 6500 Å) (Courtney-Pratt, 1961).
1 lux	= 1 lumen/m ² = 1 m-candle = 0.0929 ft-candles
1 candle	= 1 lumen/sterad
1 candle/cm ²	$\doteq 1.35 \times 10^{20}$ photons/sterad m ² sec (3500 Å - 6500 Å)
m	= -2.5 log L - 14.1 (Veis, 1963), where m is in magnitudes and L is in lumens/m ²
1 m candle-sec	= $\frac{1}{206} j/m^2 = 4.85$ erg/cm ²
1 photon (6943 Å)	= 2.86×10^{-19} j
1 photon (5500 Å)	= 3.60×10^{-19} j

APPENDIX K
CHARACTERISTICS OF KODAK 2475 FILM

For our experiments with a laser, film characteristics such as "sensitivity" and "exposure" are more conveniently expressed in photons incident on the Baker-Nunn camera than in conventional photographic units. The scales on the curves of the film characteristics are converted in the following manner.

If N_p photons strike the Baker-Nunn camera and form an image of $1.26 \times 10^{-5} \text{ cm}^2$, the energy density at the film is

$$N_p \frac{2.86 \times 10^{-12}}{1.26 \times 10^{-5}} = 2.27 \times 10^{-7} N_p \text{ ergs/cm}^2 ,$$

since $h\nu$ at 6943 \AA is 2.86×10^{-12} ergs.

Thus $\log S$ where S is the sensitivity of the film can be written as follows:

$$\begin{aligned} \log S &= \log (2.27 \times 10^{-7} N_p)^{-1} \\ &= 6.64 - \log N_p , \end{aligned}$$

since the unit of sensitivity is the reciprocal of the energy density in ergs/cm^2 . This expression was used to change the vertical scale of the manufacturer's curve. The result is shown in Figure K-1. The curve shows that the sensitivity at 6943 \AA is about 10^5 photons, a value that

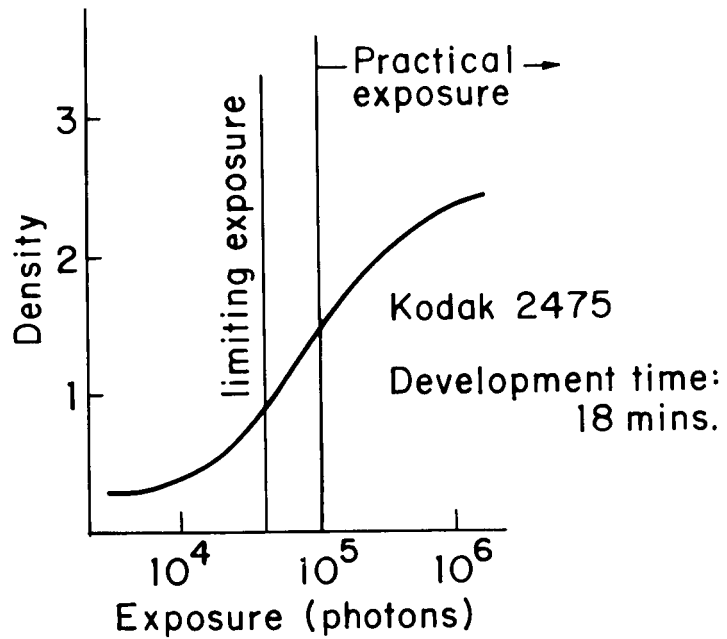
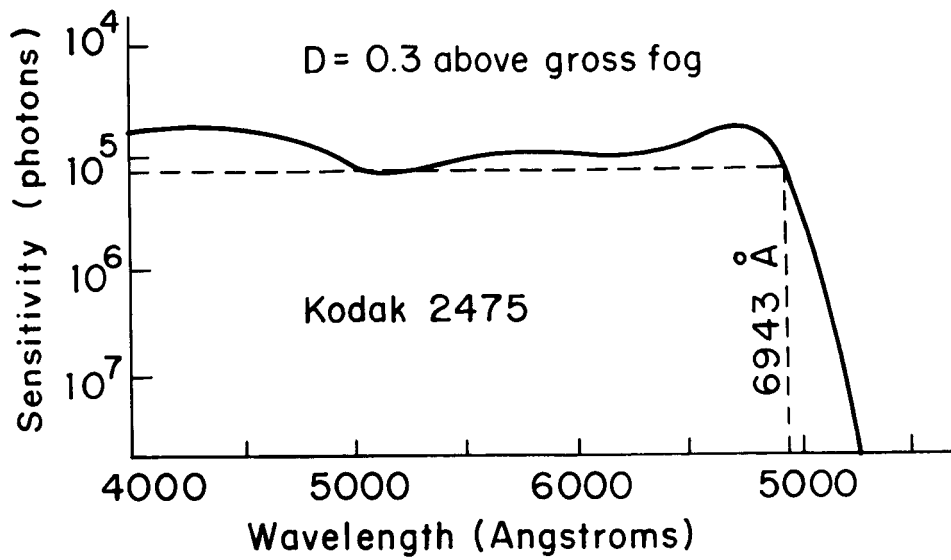


Figure K-1. Film characteristics in terms of photons incident on the lens of the Baker-Nunn camera.

may be considered to be the smallest practical value of the exposure. If we use the fact that 1/206 w of visible light is equivalent to 1 lumen and the fact that 1 lumen-sec/m² = 1 m-candle-sec, we find that exposures can be written as follows:

$$E = 4.69 \times 10^{-8} N_p \text{ m-candle-sec} \quad ,$$

where we have used the energy density of $2.27 \times 10^{-7} N_p \text{ ergs/cm}^2$ that was given above.

If we use the above conversions, the exposures indicated in the Kodak 2475 film characteristics are changed to a form that is useful when the Baker-Nunn camera and a ruby laser are used in combination. These changes have been made in Figure K-1.

In comparing exposures from reflected sunlight to exposures from laser light, a formula converting magnitudes to photons is useful. We assume that the Baker-Nunn camera is operated with a 0.4-sec exposure, a minimum practical value. The relation between magnitude and illumination is the following (Veis, 1963)

$$m = -2.5 \log L - 14.1 \quad ,$$

where m is the magnitude and L is illumination in lux (lumens/m²). For visible light, 1 lumen gives a 0.4-sec exposure of

$$0.4 \times 1.35 \times 10^{16} = 5.40 \times 10^{15} \text{ photons.}$$

Since the effective area of the Baker-Nunn camera is $9.04 \times 10^{-2} \text{ m}^2$, the illuminance L of the exposure is related to the exposure in photons N_p as follows:

$$L = \frac{N_p}{5.40 \times 10^{15} \times 9.04 \times 10^{-2}} = 2.05 \times 10^{-15} N_p \text{ lux} .$$

Thus,

$$m = -2.5 \log (2.05 \times 10^{-15} N_p) - 14.1 ,$$

or

$$\log N_p = 9.0 - 0.40 m .$$

When the shutter is open for 0.4 sec, the light from an 11th-mag object is known to give a barely detectable image. From this information we obtain a limiting exposure of $N_p = 4 \times 10^4$ photons.

NOTICE

This series of Special Reports was instituted under the supervision of Dr. F. L. Whipple, Director of the Astrophysical Observatory of the Smithsonian Institution, shortly after the launching of the first artificial earth satellite on October 4, 1957. Contributions come from the Staff of the Observatory.

First issued to ensure the immediate dissemination of data for satellite tracking, the reports have continued to provide a rapid distribution of catalogs of satellite observations, orbital information, and preliminary results of data analyses prior to formal publication in the appropriate journals. The Reports are also used extensively for the rapid publication of preliminary or special results in other fields of astrophysics.

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