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OPTICAL MASER PHOTON RATE GYROSCOPE

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ABSTRACT

The basic equations for the resonant frequencies of an electromagnetic cavity in an accelerated system of reference are considered. The removal of the degeneracy between clockwise and counter-clockwise resonant modes in vacuum is discussed in terms of the angular momentum of the photon. If matter with an index of refraction is placed in the cavity the shift in resonant frequency is shown to depend upon the moment of the energy flux. A maser media in the cavity permits oscillation and the beat frequency which is caused by rotation is discussed for a Fabry-Perot square cavity with all flat mirrors and which is oscillating at 3.39 microns. Saturation problems are considered.

INTRODUCTION

Historical references and references to recent research on the measurement of absolute rotation with electromagnetic radiation is given in references 1 through 10. In the accelerated frame of reference of a system which rotates with angular velocity Ω about the z-axis and for which the source and detector are attached to the rotating system, Maxwell's equations have their convential appearance for Cartesian ordering of the coordinates,[5,6]

$$\operatorname{curl} \underset{\sim}{E} + \partial \underset{\sim}{B}/\partial t = 0 \qquad \operatorname{div} \underset{\sim}{B} = 0$$
$$\operatorname{curl} \underset{\sim}{H} - \partial \underset{\sim}{D}/\partial t = \underset{\sim}{J} \qquad \operatorname{div} \underset{\sim}{D} = \rho \tag{1}$$

The constitutive equations are modified by rotation and are given to first order in Ω by

$$\underset{\sim}{B} = K_m \mu_o \underset{\sim}{H} + c^{-1}\left[\left(\frac{\underset{\sim}{\Omega} \times \underset{\sim}{r}}{c}\right) \times \underset{\sim}{E}\right] + \mathcal{O}(\Omega^2) \tag{2a}$$

$$\underset{\sim}{D} = K_e \epsilon_o \underset{\sim}{E} - c^{-1}\left[\left(\frac{\underset{\sim}{\Omega} \times \underset{\sim}{r}}{c}\right) \times \underset{\sim}{H}\right] + \mathcal{O}(\Omega^2) \tag{2b}$$

In the short wavelength limit these equations are equivalent to an index of refraction

$$\mu = (K_e K_m)^{\frac{1}{2}} + c^{-1}(\underset{\sim}{\Omega} \times \underset{\sim}{r}) \cdot \hat{k}_o + \mathcal{O}(\Omega^2) \tag{3}$$

where $\hat{k}_o$ is an unit vector in the direction of propagation. Rotation

[redacted]

has the effect of making even the vacuum appear anisotropic. The current $\underset{\sim}{J}$ or $\frac{\partial \underset{\sim}{P}}{\partial t}$ the polarization current of electric dipoles remain the same to first order in Ω. If the fields are expanded in orthonormal functions for the electromagnetic cavity for $\Omega = 0$, the frequency splitting between the degeneate cw and ccw modes introduced by rotation is

$$\frac{\Delta \nu}{\nu_a} = (K_e K_m c^{+2})^{-\frac{1}{2}} \underset{\sim}{\Omega} \cdot \int dv \left[\underset{\sim}{r} \times (\underset{\sim}{E}_a \times \underset{\sim}{H}_a^* + \underset{\sim}{E}_a^* \times \underset{\sim}{H}_a) \right] + \mathcal{O}(\Omega^2) \tag{4}$$

If E_a and H_a are the modes of a simple cylindrical cavity rotating about its symmetry axis, the observed frequency of the photon is $h\nu \simeq h\nu_0 + \underset{\sim}{L} \cdot \underset{\sim}{\Omega}$ where $\underset{\sim}{L}$ is the orbital angular momentum of the photon and was referred to as the "Coriolis-Zeeman" effect for the photon. This simple analogy is true only for vacuum and in the presence of matter it becomes apparent that the effect is due to the moment of the energy flux. For the convential optical maser photon rate gyroscopes cavities such as those shown in Figure (1), the modes have approximately a gaussian cross-section[11,12] and integration of the approximate modes over the beam cross-section yields a rotation beat frequency of

$$\Delta \nu \simeq 2R \cos \theta \; / \lambda \tag{5}$$

where R is the radius of the inscribed circle, λ the wavelength in meters, θ the angle between the perpendicular to the plane of the cavity and the axis of rotation. This is in accord with the angular momentum interpretation since the angular momentum of the photon about the axis of rotation is $L_z = (h/\lambda)R \cos\theta$ and $\Delta\nu = 2L_z \Omega$.

With the advent of microwave maser it was evident that the Q of the system could be increased by the introduction of maser medium and a beat frequency proportional to rotation would occur[7]. The discovery of the optical maser made possible systems with photons with much larger photon orbital angular momentum and lead to the suggestion of the optical maser photon rate gyroscope[5,8]. Such rotation beats were first reported by Macek and Davis[9] and subsequently by Cheo and Heer[10].

THEORY OF OPERATION

The general case is much too complex for analysis and in the following discussion it is assumed that only one almost degenerate mode is excited and this mode is linearly polarized. Also in order to simplify the discussion a waveguide closed in a circle is used to guide the waves as shown in Figure (2). The approximate modes are

$$E_{mnq} \rightarrow (2\pi)^{-\frac{1}{2}} \underset{\sim}{u} \, e^{\pm iq\varphi} \tag{6}$$

where $\underset{\sim}{u}$ is the linear polarization of the transverse mode and q is the number of modes around the periphery of the circle. The effective field in the cavity is approximately

$$\underset{\sim}{E}(\underset{\sim}{r},t) = E_{+q}(t)\, \underset{\sim}{u}\, \frac{e^{+iq\varphi}}{(2\pi)^{\frac{1}{2}}} + E^{*}_{+q}(t)\, \underset{\sim}{u}\, \frac{e^{-iq\varphi}}{(2\pi)^{\frac{1}{2}}} \tag{7}$$

and from equation (1) and (2), the equation of motion for a single mode is

$$\ddot{E}_{+q} + \omega_o^2 E_{+q} - i2\Omega q\dot{E}_{+q} = -\epsilon_o^{-1}\ddot{P}_{+q} - \frac{\omega_o}{Q}\dot{E}_{+q} \tag{8}$$

where ω_o is the resonant frequency of the cavity, Q the cavity and load loss term, and $\ddot{P}_{+q}$ the contribution of the polarization of the maser medium. Since ω_o^2 is large, equation (8) is examined for fields

$$E_{+q}(t) = A(t)\, e^{-i\omega_o t} + B(t)\, e^{+i\omega_o t} \tag{9}$$

where A(t) and B(t) are slowly varying functions of time. The A coefficient corresponds to ccw traveling waves and the B coefficient to cw.

Polarization of the Optical Maser Media

Following the procedure introduced by Lamb[13] a development for the polarization of traveling wave modes was made by the author[14]. The polarization including first and third order terms may be written as

$$\epsilon_o^{-1} P_{+q} \simeq (C_1 + C_2|A|^2 + C_3|B|^2)A\, e^{-i\omega_o t} + (C_1^* + C_3^*|A|^2 + C_2^*|B|^2)B\, e^{+i\omega_o t} \tag{10}$$

where C_1 is the first order coefficient to the polarization and C_2 and C_3 the third order coefficients. For large doppler broadening

$$C_1'' \propto i \text{ const} \exp -(\omega_{ab} - \omega_o)^2/D^2 \tag{11a}$$

and C_2 and C_3 are of the form for atoms at rest

$$C_j = C_j' + iC_j'' = \text{const} \frac{(\omega_{ab} - \omega_o) - i\gamma_{ab}}{(\omega_{ab} - \omega_o)^2 + \gamma_{ab}^2} \tag{11b}$$

and, $C_2' \simeq 0$, $C_2'' \simeq 1/\gamma_{ab}$ for large doppler motion where $\omega_{ab} = 2\pi\nu_{ab}$ is the frequency of the optical maser line. If the following notation is used for A and B,

$$A(t) = U\, e^{iu} \quad ; \quad B(t) = V\, e^{iv}$$
$$C_1 = C_1' + iC_1'' \quad ; \quad C_2 = C_2' + iC_2'' \quad ; \quad C_3 = C_3' + iC_3'' \tag{12}$$

then in the quasi-linear approximation equation (8) becomes

$$\dot{u} = \Omega q + \tfrac{1}{2}\,\omega_o(C_1' + C_2'U^2 + C_3'V^2) \qquad (13a)$$

$$\dot{v} = \Omega q - \tfrac{1}{2}\,\omega'_o(C_1' + C_3'U^2 + C_2'V^2) \qquad (13b)$$

$$2\dot{U} = -\,\omega_o(Q^{-1} + C_1'' + C_2'U^2 + C_3''V^2)U \qquad (13c)$$

$$2\dot{V} = -\,\omega_o(Q^{-1} + C_1'' + C_3''U^2 + C_2''V^2)V \qquad (13d)$$

Steady state requires $\dot{U} = \dot{V} = 0$ and the solution for the amplitude is

$$U^2 = V^2 = -(Q^{-1} + C_1'')/(C_2'' + C_3'') \qquad (14)$$

and the beat frequency between modes by

$$\omega_{cw} - \omega_{ccw} = 2\,\Omega q + \frac{\omega_o}{2}\,(C_2'-C_3'')(U^{\,2}-V^2) \qquad (15)$$

To this degree of approximation the c.w. and c.c.w. modes are equally excited; the frequency of each mode is shifted an equal amount in the same direction by the frequency sensitive C' terms and the amplitude terms. <u>The difference in frequency between the two modes is $2\Omega q$ and is not sensitive to the effects of ~~saturation~~ for the ideal system</u>.

Equation (42a) gives the stored energy energy and the power output of each mode is

$$P_{cw} = P_{ccw} = \omega W/Q = \omega\,\epsilon_o U^2/Q = -\,\epsilon_o(\omega/Q)(Q^{-1}+C_1'')/(C_3''+C_2'') \qquad (16)$$

For extreme doppler motion C_1'' has the doppler frequency response given by equation (38) and for $(Q^{-1} + C_1'') < 0$ the system oscillates. C_3'' has the response given by equation (39) and the frequency response of the power ouput is of the form

$$\left(-Q^{-1} + \text{const}\; e^{-(\omega_{ab} - \omega_o)^2/D^2}\right)\left(\frac{(\omega_{ab} - \omega_o)^2 + \gamma_{ab}^2}{(\omega_{ab} - \omega_o)^2 + 2\,\gamma_{ab}^2}\right) \qquad (17)$$

and shows the dip in power output discussed by Lamb[13]. Frequency pulling is given by

$$\nu_{ccw} = \nu_{cw} = \nu_o + \nu_o\,\tfrac{1}{2}C_3'V^2 \simeq \nu_o + (\nu_o - \nu_{ab})\;\Delta\nu_L/\,\Delta\nu_{ab} \qquad (18)$$

The approximate expression is in agreement with the expression introduced by Townes[15].

Stability

Equations 13c and 13d lead to a stable solution, that is $\dot{U} = \dot{V} = 0$ and $U = V$, for values of $C_2'' > C_3''$. Phase trajectories are shown in Figure (8). For atoms at rest C_3'' is larger. For atoms with large doppler motion C_2'' becomes larger than C_3'' and is insensitive to frequency. This solution to the equations indicates that large doppler broadening is necessary for the observation of rotation beats. Other types of broadening have not been investigated as yet. This solution also indicates that at the atomic line frequency, $C_2'' \simeq C_3''$ and the system is near a point of instability.

Bias Beats

If a unidirectional coupler as shown in Figure 5 is inserted in the beam or scattering occurs the equation of motion is modified. Equations 13a and 13b remain the same in the absence of a phase shift, and equations 13c and 13d become of the form

$$2\dot{U} = -\omega_o(Q^{-1}+C_1'' + C_2'' U^2 + C_3'' V^2)\, U + \omega_o V/Q_B \qquad (13c')$$

$$2\dot{V} = -\omega_o(Q^{-1}+ C_1'' + C_3'' U^2 + C_2'' V^2)\, V + \omega_o U/Q_A \qquad (13d')$$

Q_A and Q_B are a masure of the power coupled from one traveling wave into the other. The simplest case to consider is the unidirectional coupler shown in Figure (5), $Q_B \to \infty$ and then $U^2-V^2 = Q_A^{-1}/(C_3''-C_2'')\, U/V$. For weak coupling, $Q_A >> Q$, $U/V \simeq 1$ and a phenomena known as bias beats occurs,

$$\omega_{ccw} - \omega_{cw} = 2\,\Omega q + \Delta\omega_{bias} \;;\Delta\omega_{bias} \simeq \tfrac{1}{2}\,\omega_o\,\gamma_{ab}/(\omega_{ab} - \omega_o)QA \qquad (14)$$

Bias beats as a function of frequency are shown in Figure (9). Mr. Little, a graduate student in our laboratory, has observed the effect shown in Figure(9) as the cavity is tuned across the doppler broadened line. Q_A may be estimated from the coupled power by $Q_A/Q = P_L/P_A$, where P_L is the oscillator power and P_A the coupled power.

Frequency Locking

Equations 13c' and 13d' are similar to the equations which are considered in the entrainment of an oscillator by an external source[16].

In this problem the coupling of the linear term of the electric field of the cw wave to the ccw wave is similar to the external source term. Frequency entrainment or locking becomes a problem when the two frequencies are within $\Delta\nu$ of each other and,

$$\Delta\nu_{\text{locking}} < \Delta\nu_L \, (P_A/P_L)^{\frac{1}{2}} \tag{15}$$

where $\Delta\nu_L$ is the cavity bandwidth. For simplicity this criterion is used, and the more detailed examination of equations 13 is not used, to estimate the region of entrainment. This coupling could be due to a unidirectional coupler or to scattering of energy of one beam into the other. If in the latter case f P_L is the scattered power and $\Delta\Omega$ the angular spread of the maser beam ($\Delta\Omega \simeq$ spot size/$4\pi L^2$ and the spot sized is of the order of $L\lambda$) the coupled energy is of the order of

$$P_A/P_L \simeq f\,\Delta\Omega \simeq f\lambda /4\pi L \tag{16}$$

For f = 0.01, $\lambda \simeq 10^{-6}$, $L \simeq 1$, $\Delta\nu_L \simeq 10^{6}$, entrainment or frequency locking problems are of importance in the hundreds of cycles per second frequency separation region. A more exact expression indicates that entrainment may be reduced by reducing the power level of the maser.

Optimum Design Considerations

From the previous discussion it is apparent that a maser media with a large doppler broadening is necessary of stable oscillation at two frequencies, that is $C_2'' > C_3''$ is necessary. In the ideal system and to the approximation to which equations 13 are correct, these two frequencies do not entrain or lock and rotation beats are expected over the entire doppler line. Operation near the atomic frequency is near an unstable point, but the frequency response of the coefficients must be examined in greater detail to discuss this in more detail. In the presence of coupling between the two waves, the bias beat frequency is shown in Figure (9). Operation at a frequency γ_{ab} away from either side of the atomic line ω_{ab}, yields a minimum bias beat frequency. Since the sense of the bias is fixed by the choice of the right or left minimum, the sense of rotation may be determined.

The bandwidth for frequency entrainment may be reduced by selecting a maser media with a small value for the C'' saturation coefficient and a large value for the linear gain C_1'' and also selecting a cavity with a high Q or low losses. Further-more any properties of the mirrors, brewster angle windows, or other objects in the maser beam path which cause unidirectional coupling or scattering of power between the two beams must be minimized. In order to avoid objects in the path the design shown in Figure 7 does not have brewster angle windows, etc. but is entirely enclosed. The maser beam is free from foreign objects, dust, moisture, etc. and the only surfaces are the mirrors and the quartz maser tubes. Some investigations of Mr. Little indicate that even the maser tubes may be important. In a system similar to that shown in Figure (6) and oscillating, the insertion of a quartz tube in the beam path could enhance the power level of oscillation, attenuate the level, or have almost no effect depending on the diameter of the tube. For very small angles of reflection the interior of the tube is almost a perfect reflector and acts as a guide. Rough surfaces attenuated the beam.

Thermal vibrations of the mounting platform and spontaneous emission noise are not as yet a limitation on the photon rate gyroscope and should not be of interest until beats in the region of 1 cps are of interest. Mechanical vibrations or mechanical noise could be as effective as thermal noise and increase the effective noise temperature of the system well above that of the temperature of the surroundings. Resonat frequencies of the mounting platform can be troublesome, but since each platform is a special case these are not discussed.

EXPERIMENTAL APPARATUS

Slides of the experimental apparatus being used, mechanical details, of the apparatus, and the performance of the apparatus will be shown.

REFERENCES

*This work supported in part by NASA Grant No. NsG-552

1. A. A. Michelson, Phil. Mag 8 716 (1904)
2. G. Saynac, Compt. Rend. 157 708 and 1410 (1913); F. Harress, dissertation, Jena, 1911(unpublished); and B. Pogany, Ann. Physik 85 244 (1928)
3. A. A. Michelson and H. G. Gale, J. Astrophys. 61 1401 (1925)
4. C. V. Heer, Bull. Am. Phys. Soc. 6 58 (1961)
5. C. V. Heer, "Preceedings of the Third International Conference on Quantum Electronics" (Dunod Cie, Paris, and Columbia University Press, New York (1963)) p 1305
6. C. V. Heer, Phys. Rev. 134 799 (1964)
7. C. V. Heer - proposal submitted to NASA, AFOSR, and ONR on 16 March 1961
   C. V. Heer - Annual progress report to NSF - 6 January 1960
8. C. V. Heer - supplementary material supplied upon request to NASA 29 January 1962 and copies to AFOSR and ONR
9. W. Macek and D. Davis, Appl. Phys. Letters 2 (1963)
10. P. K. Cheo - Ph.D. dissertation, Ohio State University 1964; P. K. Cheo and C. V. Heer, Applied Optics 3 788 (1964)
11. A. G. Fox and Tingye Li, Bell System Tech. J. 40 453 (1961)
12. S. A. Collins, Jr. - Applied Optics - (to be published); G. D. Boyd and J. P. Gordon, B.S.T.J. 40 487 (1963), 41 1347 (1962); A. G. Fox and T. Li, Proc. I.E.E.E. 51 80 (1963); P.P. Clark, Proc. I.E.E.E. 51 70 (149) (1963).
13. W. E. Lamb, Jr. Phys. Rev. 134 1429 (1964)
14. C. V. Heer, "An Evaluation of the Optical Maser Photon Rate Gyroscope" Colloque sur les Gyroscopes Avances, Centre National d'Etudes Spatiales Nov. 1964(copies available upon request)
15. C. H. Townes, "Advances in Quantum Electronics," (Columbia University Press, New York, (1961)) p3
16. A. Andronow and A. Witt, Archiv für Elektrotechnik, 24 (1930); N. Minorsky, "Non-Linear Mechanics", (J. W. Edwards, Ann Arbor, 1947, p. 341)

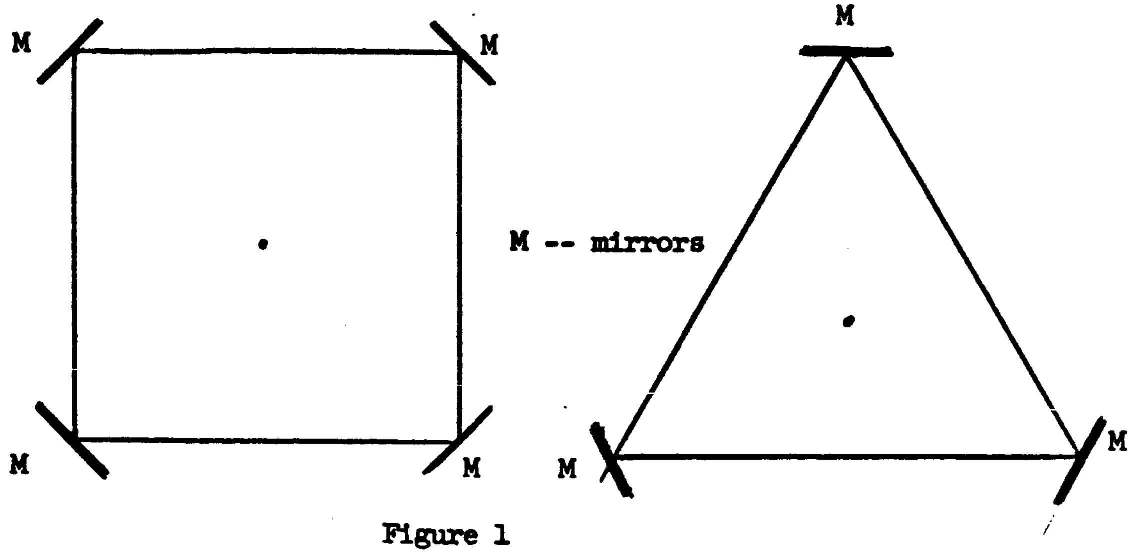


Figure 1
Typical Cavity Designs

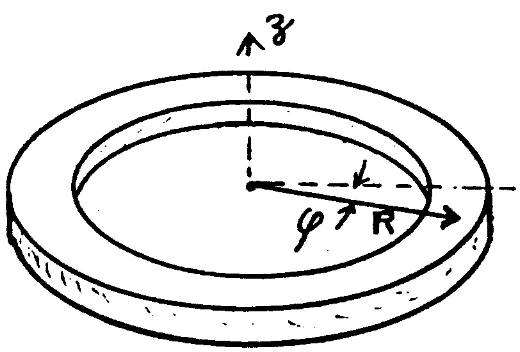


Figure 2
Ideal Cavity

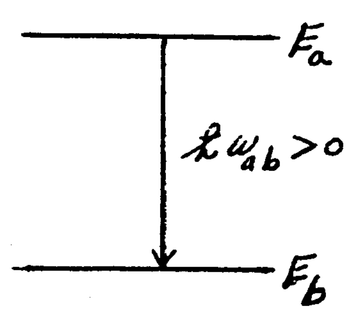


Figure 3
Energy Levels

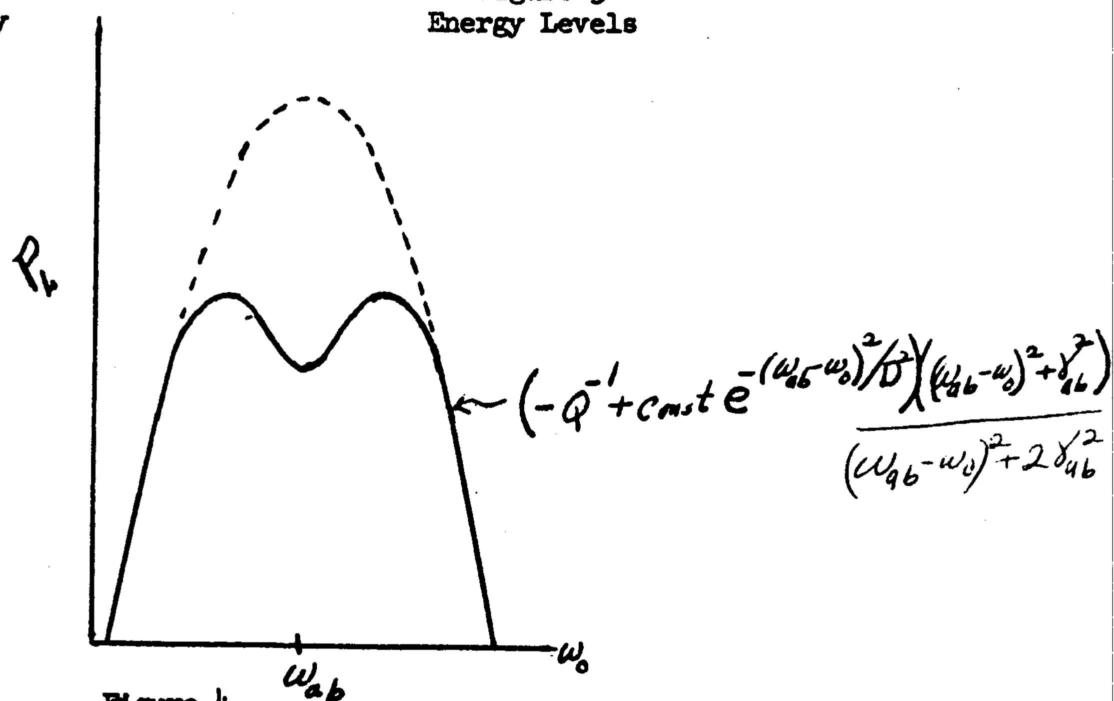


Figure 4
Power Dip at ω_{ab} by Tuning

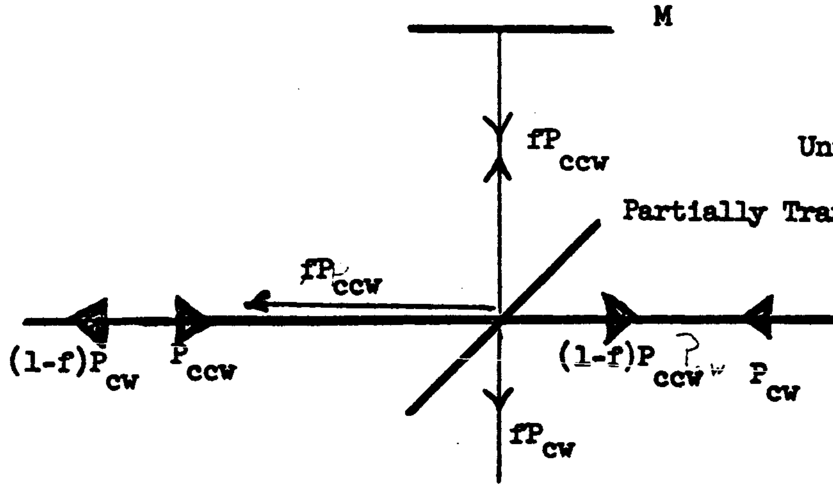


Figure 5

Unidirectional Coupler

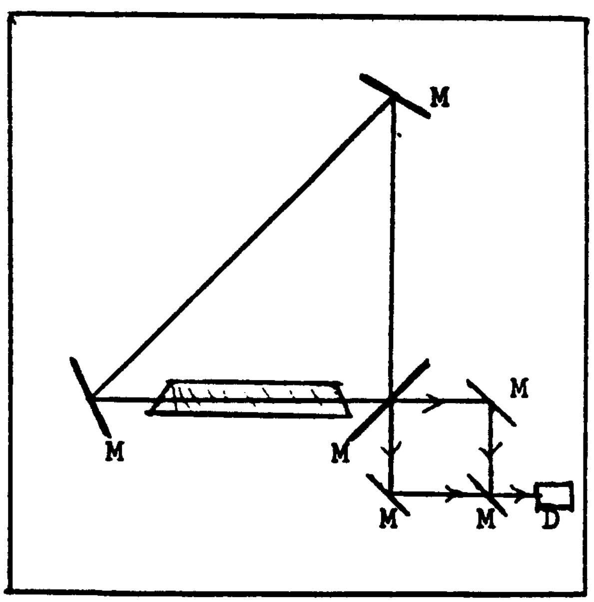
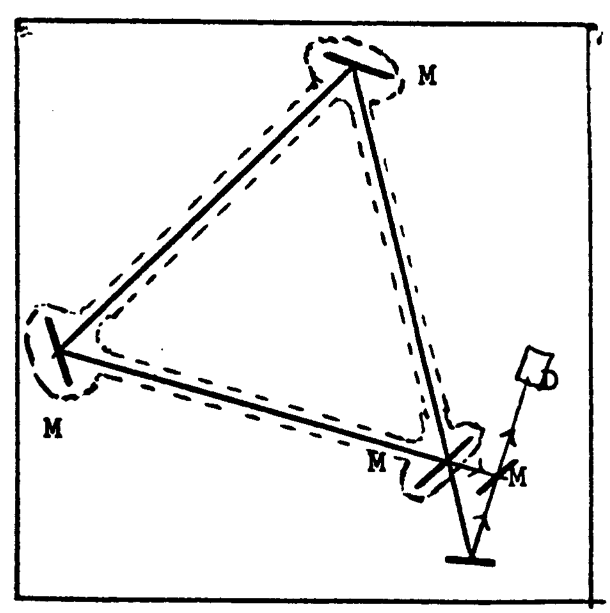


Figure 6

Photon Rate Gyroscope

Figure 7

Photon Rate Gyroscope



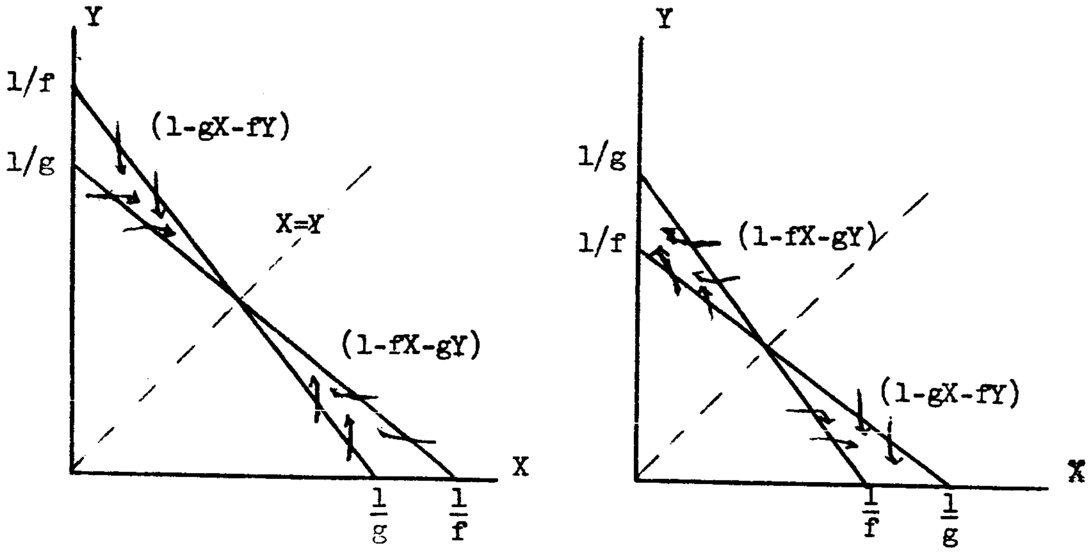


Figure 8
Stability

$$\frac{dY}{dX} = \frac{(1 - f\,X - g\,Y)Y}{(1 - g\,X - f\,Y)X} \quad \text{where } X = U^2 \text{ and } Y = V^2$$

$$g = -C_2''/(Q^{-1} + C_1'') \qquad f = -C_3''/(Q^{-1} + C_1'')$$

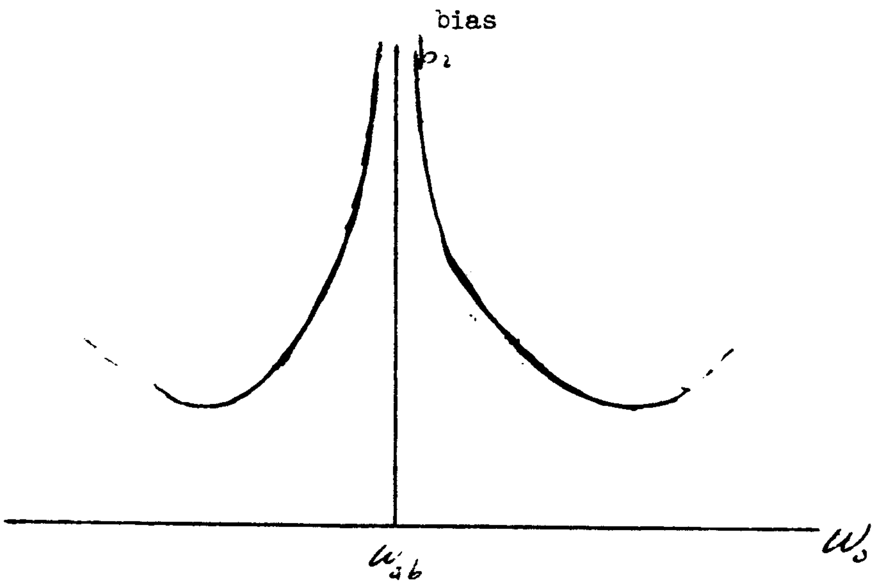


Figure 9
Bias Beats