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AN EXPERIMENTAL STUDY OF THE BUCKLING OF COMPLETE SPHERICAL SHELLS

by R. L. Carlson, R. L. Sendelbeck, and N. J. Hoff

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STANFORD UNIVERSITY
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SUMMARY

Complete spherical shells with radius to thickness ratios of from 1570 to 2120 were produced by the electroforming process. For specimens of good quality and for optimum testing conditions, buckling pressures up to 86 per cent of the classical buckling pressure were obtained.

The effect of loading system characteristics was examined by pressurizing spherical shells in both rigid and soft systems. Contrary to expectations based on the Tsien hypothesis, each shell buckled at the same pressure in both systems. This indicates that the Tsien hypothesis cannot be used as an explanation for the occurrence of low buckling pressures. The test results obtained suggest that test performance is related to the nature and severity of flaws or imperfections; i.e., low buckling pressures can be correlated with the presence of severe flaws or nonuniformities.

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INTRODUCTION

During the past decade, there has been an increasing amount of activity devoted to the development of shell theory and to experimentation with shells. Of the research that has been done a substantial part has been directed toward problems involving buckling of shell structures, and a number of interesting and important results—both analytical and experimental—have been obtained. For the studies described here data dealing with cylindrical shells loaded by an axial compression and spherical shells loaded by uniform, external pressure are of particular interest. No attempt, however, will be made to review in detail here the background which provided the motivation for research on these problems in the recent past. This has been done several times recently (see, for example, papers by Fung and Sechler [1]* and by Hoff [2]). A brief statement of the current status is, however, appropriate.

Much of the recent activity dealing with compressively loaded, thin circular cylinders and spherical shells under uniform external pressure has been directed toward resolving the large discrepancy between the classical theoretical buckling pressures and the experimentally observed ones. Until rather recently, the experimental values were much lower than the classical values. Speculation about the reason for the observed discrepancies produced several possible explanations, but a thorough evaluation of the possibilities appears to have only begun within the

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*Numbers in brackets designate References at the end of the report.

last five years.[*] The results of this recent activity, which is continuing, have provided for a substantial improvement in our understanding of the buckling process, particularly for the circular cylinder. It has been shown, for example, that boundary conditions different from those of the classical solution can yield a buckling load which is approximately one half of the classical value [5]. These results provide an explanation for at least some of the low buckling values that have been obtained, particularly those in which edge buckling occurred. In addition, it has been observed [6,7,8,9] that if special care in specimen preparation and testing is exercised, values very close to the classical can be achieved. This strongly suggests that specimen imperfection and improper testing technique are responsible for many of the low values that have been obtained. The experimental results of Almroth et al [7] are also important in one other respect. They indicate that the load at which buckling occurs in normal testing environments is not influenced by the rigidity of the test system; i.e., specimens buckle at the same value of load in both a rigid system and a soft system. This result is contrary to what would be expected if the "jump" phenomenon associated with the energy hypothesis of Tsien [10] were operative. In summary, all of these results for compressively loaded, circular cylindrical shells indicate that values approaching the classical can be obtained, and that when values substantially lower are observed, testing technique, particularly boundary conditions and specimen imperfection are responsible.

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[*] An exception to this statement is a remarkable Ph.D. thesis written by Koiter [3] in 1945. However, this work remained practically unknown until 1963, when a short English version [4] of the original Dutch text was published.

The problem of the axisymmetric buckling of thin spherical shells subjected to a uniform external pressure was solved by Zoelly [11] in 1915 and by Schwerin [12] in 1922. A solution considering unsymmetrical buckling modes was presented by van der Neut [13] in 1932. Since the work of van der Neut, however, most investigations—both experimental and analytical—have dealt with spherical caps. Data available on complete spheres are quite meager. What is available indicates that the experimental pressures are generally substantially less than the classical values. Fung and Sechler [1], for example, report data obtained by Sechler and Bollay on a hemispherical brass shell which buckled at about 25 per cent of the classical value. Klöppel and Jungbluth [14] obtained similar results from tests on deep, spherical steel shells which were rigidly supported along a circular boundary.

More recently, Thompson [15] presented results obtained from two experiments on polyvinyl chloride spheres. Buckling occurred at about three-quarters of the classical value. The radius to thickness ratio was 20, however, so the results provide little indication of the behavior that might be expected at the much larger radius to thickness ratios that are of interest in a number of applications.

Tests were also conducted recently by Krenzke [16] on aluminum alloy hemispheres which were integrally connected to circular cylinders. Buckling pressures ranged from 0.53 to 0.69 of the classical value. The radius to thickness ratio for these specimens was between about 80 and 100. Krenzke also reported results from tests conducted on small plastic spheres tested in rigid and soft loading systems. The results obtained for the two systems were reported to be essentially the same. No measurements of specimen dimensions or of the mechanical properties were obtained, however, so it is not possible to compare the experimental and classical buckling pressures.

Sabir [17] has presented results of tests on copper hemispherical specimens which were prepared in a unique manner. Small circular regions on the specimen surface were chemically polished to provide thin, shallow caps which were connected to the unthinned wall of the hemispheres. The unthinned, original thicknesses of the hemispheres were from three to four times thicker than the thinned "gage" sections. The specimens were thus, in essence, elastically supported, shallow caps. In most tests the specimens buckled at about six-tenths of the classical value for a complete sphere. A few buckled at values slightly less than nine-tenths of the classical value. The data obtained by Sabir are interesting, but because of the uncertainty of the conditions at the boundary between the thinned and the unthinned regions, the extent to which the results are representative of the response of complete spheres is not known.

From the preceding discussion it can be concluded that data on complete spheres, as contrasted with hemispheres and spherical caps, are very meager. In fact it appears that data on complete spheres with large radius to thickness ratios are non-existent. In most studies the spherical cap has been used with the hope that the results obtained are the same as would be obtained from a complete sphere. In practice, however, the presence of a boundary would appear to introduce problems which are not easily evaluated.

Experience with the compressively loaded, circular cylinder has shown that substantial contributions can be made to our understanding of thin shell buckling by carefully executed experiments. These experimental results, obtained in the recent past, and the knowledge that experimental data on complete spherical shells are almost non-existent, served as motivation for planning and executing the program described in this report. The program had two primary objectives. The first objective was to develop procedures for making good, complete spherical shells with large radius to thickness ratios. The goal for specimen quality was to provide specimens which gave buckling pressures substantially closer to the classical pressure than those obtained in previous experiments. The second objective of the program was to perform the buckling experiments in both "soft" and "rigid" testing systems to examine the Tsien energy hypothesis. According to the Tsien hypothesis, buckling should occur at different pressures in the two systems.

The plan of this report is to give detailed descriptions of the specimen fabrication, the equipment and test procedures, and finally, to present the experimental results. This plan is developed in the sections that follow.

EXPERIMENTAL PROGRAM

Selection of Material

Background

Elements of shells are frequently used in the construction of aircraft and missiles. As a consequence, the response of shells to different types of loading conditions has been of particular interest to the aircraft engineer, and a number of analytical and experimental studies have been undertaken. For experimental studies, investigators usually formed commercially available flat sheets to the desired shape and then, when necessary, connected edges by seams. This procedure has a number of obvious disadvantages, and often it has been concluded that test results have been influenced by some property introduced by the fabrication process.

Several years ago Thompson [18] recognized that the principles of electroplating could be used to form specimens which were free of many of the disadvantages common to the more customary methods of fabrication. Interest in making shell bodies by electroforming has become well established and several investigators [7,19,20] have used the process to make specimens for shell investigations. The results of these studies indicate that the apparent advantages of the process can be realized and that specimens of very good quality can be obtained.

The electroforming process was used in the program described by this report to make complete spherical shells. Three basic operations were necessary for the production of spherical shells. The first step involved the manufacture of a spherical wax mandrel upon which the shell specimen could be plated. The second step involved the plating of the mandrel, and the final step consisted of removing the wax mandrel from the shell specimen. Each of these steps is described in the following sections.

Production of Spheres

The plating process requires an electrically conducting mandrel for a cathode. To satisfy this requirement a special conducting wax* was obtained. Its surface was relatively hard and it could be machined without galling, flaking, or crumbling. The surface conductivity was greatly affected by the cooling rate from the liquid state, and it ranged from 40 ohms to infinity. The shrinkage was considerable and also was greatly affected by the cooling rate.

The several ingredients which made up the wax combined as a mixture and the constituents tended to separate upon cooling from the liquid state. When cooling the wax from the liquid state, it was necessary to prevent segregation. The wax began to soften at about 150°F and was in the liquid state at 300°F.

A shell casting procedure was developed since the amount of wax necessary is then only a small fraction of that of a solid mandrel. This simplifies the subsequent removal of the wax from the electroformed shell specimen, and it also reduces some of the solidification problems during the casting process.

The wax was placed inside two hemispherical molds which were bolted together along flanges. This assembly was then mounted on a rotating suspension device which was positioned inside an oven. The suspension mechanism, which can provide for continuous rotation about two axes,

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*Stevenson Bros. Wax Co., Philadelphia, Pa.

is shown in Figure 1.

The process consists of heating the wax in the mold until the liquid state is achieved. The power to the oven is then disconnected, and the rotation mechanism is turned on. As the mold cooled, wax solidified on the inside of the mold surface and a casting of continuous and uniform thickness was produced. The rate of cooling was carefully controlled.

Although the wax mandrel had good dimensional tolerances in the as-cast condition, there was a mold parting line which had to be blended into the mandrel surface. Also, the surface did not have the desired degree of smoothness (an as-cast mandrel is shown at the right in Figure 2). Finishing operations designed to obtain the desired surface condition were, therefore, introduced.

A shaving operation was performed by use of a hand tool. The tool was made from a short pipe section 4 inches in diameter which had been machined to give it an inside circular cutting edge. The tool was passed over the surface of the casting. The operation was executed in such a manner as to shave uniformly over the entire surface. The technique worked well and left the surface smooth and free of visible distortions. To minimize fine scratch lines left by the shaving tool, a solvent was rubbed over the surface. This disolved a thin surface layer, and continued polishing brought out a high luster. A typical, processed mandrel is shown in Figure 2.

For the studies described here nickel was chosen as the shell specimen material because deposits of relatively high strength and, more importantly, high elastic limit stress can be readily obtained. A proprietary nickel sulfamate plate solution* which contains nickel

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*Harstan Chem. Corp., Brooklyn, N. Y.

bromide was used because it produces deposits with low internal stresses. The composition of the bath was as follows:

Nickel sulfamate	42 oz./gal.
Nickel bromide	8 oz./gal.
Boric acid	4 oz./gal.
Anti-pitting agent	As Required

It was found that with the addition of 3.7 grams per liter of naphthalene 1-3-6 trisulfonic acid sodium salt [21] the internal stresses were further reduced. Also, the additional benefit of a harder plate and a bright reflective surface was obtained.

The mechanical properties of deposits in a given bath are influenced by the bath temperature, the current density, the pH of the bath, and impurities. Temperature and current density were monitored and controlled throughout the plating operation. A pH of from 3 to 4.5 was maintained by appropriate additions [22]. Impurities were minimized by maintaining the bath volume by additions of distilled water and by continuous filtration. All equipment and fixtures immersed in the bath were made of non-reactive materials and the entire system was covered by polyethylene sheeting to prevent airborne particles from getting into the solution. A photograph of the plating facility is shown in Figure 3.

The thickness of a deposit on the wax mandrel depends not only on the operating conditions of the bath, but also on its location in the bath. It follows that the method used to suspend the wax mandrel in the plating bath has a distinct influence on the type of thickness gradient produced.

Several different suspension methods were used for producing spherical shells. Each of the different methods could, however, be classified as modifications of one of two basic methods. The two basic methods used will be described.

The most uniform plate thickness was obtained by continuously rotating the wax mandrel about two axes. The suspension system used to make shells by this method is shown in Figure 4. This system produces continuous rotation about a vertical axis and a horizontal axis. By this method of suspension, shells which had thickness variations of about ± 5 per cent from the nominal thickness could be obtained. It was found, however, that the benefits of this thickness control could not be realized because the electrode holes were sources of weakness which made it impossible to obtain good test performance. This is discussed in detail in the section "Discussion of Results".

Exploratory tests which are described in a subsequent section indicated that the effect of the holes produced by the electrode contacts could be neutralized if the regions adjacent to the two holes were made somewhat thicker than the balance of the shell surface. The suspension system devised to produce a shell satisfying this type of requirement is shown in Figure 5. The assembly shown was positioned in the bath at a depth such that the uppermost surface of the mandrel received the same amount of deposit as the lowermost surface. During plating, the mandrel was rotated about the axis of the vertical rod at the top of the assembly.

Before testing the shell it was desirable to measure its thickness. This was done by the use of the nondestructive magnetic thickness gage which is shown in Figure 6. The gage is held, but spring loaded, so that it presses lightly against the shell. As readings are taken the shell is rotated to scan the surface.

A small bar magnet inside of the glass tube housing is attracted to the nickel surface. Wound around the housing is a small coil which, when energized, tends to pull the magnet away from the shell surface.

At some current level, this force just overcomes the magnet's attraction to the shell, and the magnet jumps away from the surface and towards the coil.

A current versus shell thickness curve was plotted and subsequently used to convert amperage readings directly to thickness measurements. The instrument was calibrated within the range of from 2.5 - 3.8 mils, and within this range, 0.0002 inch thickness variations were easily discerned. Measurements were taken at 15 degree intervals along each of three circles drawn with a felt pen on the surface. The planes of the circles were mutually perpendicular.

It should be emphasized that the magnetic gage was used to obtain thickness surveys prior to testing. It was, thus, useful in evaluating variations in plating technique quickly and prior to the destruction of specimens. Thickness values used in the computations presented later were obtained by the use of a hand micrometer with a hemispherical anvil.

Sphericity was measured along the same inked circles used to obtain thickness surveys. The variation in the diameters of each sphere was measured by the use of a Bausch and Lomb shadowgraph. Each specimen was positioned as shown in Figure 7 and rotated to measure the variation in diameter along each of the three inked circles. The shadowgraph magnified the shadow of the sphere by a factor of 62.5 and readings to within 0.001

inch were possible.

For the best shell produced the maximum difference in the measured diameter was 0.006 inch. For most of the shells the difference was 0.010 inch or less.

The buckling tests on the spherical shells revealed that surface irregularities or flaws could substantially reduce the buckling pressure. The surface condition was, therefore, extremely important. It may be noted also that the fact that the shells were of the order of 0.002 inch thick made intolerable what might be minor flaws in thicker shells.

Flaws were introduced primarily in two ways; i.e., they could be traced either to improper plating conditions or to the surface condition of the wax mandrel.

Flaws associated with the surface condition of the wax mandrel might be characterized as scratches, nicks or gouges. These created localized changes in the curvature of the plated shell, and if they were sufficiently severe, they reduced the resistance to buckling drastically. The response caused by flaws of this origin is described in the section "Discussion of Results". The elimination of flaws of this type was entirely dependent on the development of improved methods of processing the wax mandrels. The methods described previously in this section proved to be adequate.

The flaws introduced by improper plating conditions appeared as pitting, blisters and surface roughness. A variety of techniques were used to overcome these difficulties. Addition of an anti-pitting agent to the bath and plating at reduced current densities eliminated the pitting and also reduced the blistering. Roughness or nodular growth was effectively controlled by minimizing contamination of the bath by dust and foreign particles. This was controlled by constantly filtering the solution and by completely covering the entire system.

Since the shell specimens were very thin, they were quite sensitive to damage during processing for testing. Handling procedures were developed to minimize the possibility of introducing damage, but in spite of the precautions taken, specimens were occasionally damaged inadvertently. In some instances the flaws introduced in handling were apparent. In other instances damage could only be inferred from test results.

The two holes created by the electrical contacts during plating were used to remove the wax from the electroformed shell. The procedure involved placing the plated shell in an oven with one of the holes down. As the wax melted, it ran out of the bottom hole. Residue left on the inside surface was removed by submerging the shell in a container filled with solvent and heating for a prolonged period. The solvent was then drained out leaving a clean inside surface.

To prepare the shell specimens for testing in the rigid (water) system, one electrode hole had to be sealed and a fitting had to be added to the other hole. The method of sealing holes is shown in Figure 8. The patch shown was cut from a tested sphere and it was soldered, as shown, onto the test sphere to cap the hole.

The fitting, which provided the means for obtaining pressure differences in the testing cell, was applied as shown in Figure 9. It consisted of a small copper tube soldered to a washer which, in turn, was soldered to the specimen over one of the electrode holes.

The fitting and the closure introduced localized non-uniformities in the shell response to pressurization, and were responsible for causing premature buckling in initial tests. The effects introduced were, however, successfully neutralized. This behavior is discussed in the section "Discussion of Results".

The microstructure of the deposits produced during electroforming is complex and depends on the bath composition and the operating conditions. Also, the structure of the initial deposits can, if a metal mandrel is used, depend on the base metal and may, in some instances, be a continuation of the structure of the base metal [23]. Wax mandrels were used to produce the spherical shells, of course, so continuation growth effects should not be present.

When deposits are not constrained to adopt a particular texture, they assume orientations such that a crystal axis is perpendicular to the surface or parallel to the direction of current flow [23]. The orientation with respect to normals to the surface is random. Thus, for nickel, which has a face centered cubic lattice, the mechanical properties in elements tangent to the surface should be isotropic. Properties in a direction normal to the plated surface would be different from those of elements tangent to the surface.

The primary stresses in the spherical shells tested in this program act in directions tangent to the mid-surface of the shell wall. From the previous discussion regarding the growth characteristics of deposits, it follows that the shells can be considered to be isotropic; i.e., properties in the direction normal to the shell mid-surface need not be considered.

An examination of the microstructure of samples of the electroformed nickel was made and it was found that the material was sound and free of voids. Fibrous grains normal to the surface of the plate and in the direction of growth during deposition were observed. The structure obtained is typical of that for electrodeposited nickel [24].

Equipment and Procedures

Rigid Test System

An experimental evaluation of the Tsien hypothesis requires the use of testing systems representing extremes in rigidity. Two extremes, a rigid or hard system and a soft or dead-weight system, can provide the capability required. In this section the rigid system used will be described.

In a rigid system the water pressurizing the sphere from the outside is enclosed in a rigid container and the volume of the water remains unchanged during buckling. In practice it is not possible to obtain a rigid system. It can, however, be approximated. A simplified, section view of the system constructed for this purpose is shown in Figure 10. The system consists of a cylindrical pressure vessel with a two inch wall capped by three inch thick lids. The cylinder and lids are made of the aluminum alloy 7075-T6. The outside diameter of the cylinder is 14 inches and the overall length is 18.5 inches. The numbered parts are identified in the list given below:

Part number	Part
1	shell specimen
2	specimen connection
3	threaded pressure piston

4	pressure transducer
5	lead to pressure indicator
6	top cover
7	valve
8	valve
9	reservoir

The use of the system can be explained by outlining the procedure used to test the spherical shells.

With the top cover 6 removed and the lower valve 8 closed, the cylinder is partially filled with water which is the pressure medium. The specimen 1 is then placed in the tank and filled with water at 2. During this operation, the connection indicated at 2 has not yet been made. When the specimen is filled, the connection at 2 is made. The specimen is now completely submerged in the water of the tank. The cover 6 is then bolted down and additional water added at 9 and through 8 with the upper valve 7 open. When the water levels at 7 and 9 are even, the system is ready for testing. The valves 7 and 8 are then closed and external pressure loading is applied by advancing the piston 3 into the chamber. The value of pressure is obtained by feeding the output from the pressure transducer 4 to an indicator 5.

The piston 3 has been machined with a fine thread—56 threads per inch on a 1 inch diameter—and it is advanced into the chamber by rotation. The use of a fine thread eliminates leakage problems and also provides a sensitive measurement of volume change.

The pressure transducer used is a Consolidated Electrodynamics Type 4-312 with range of ± 10 psid.[*] For the tests described here

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[*] Pounds per square inch differential.

only the positive range is used because the transducer is vented to the atmosphere. In this gage a diaphragm which is exposed to the pressure medium actuates an unbonded strain gage. The strain sensitive element is part of a Wheatstone bridge which is enclosed in the transducer chamber. The output at 5 provides a measure of the unbalance of the bridge. The output is calibrated in terms of pressure so that readings can be converted to pressure values. The output of the bridge in the transducer is fed at 5 into the external bridge connections of a Baldwin Lima Hamilton SR-4 Strain Indicator, Type N.

Soft Test System

The characteristics of a soft system are such that as buckling proceeds, the external pressure remains constant. On a pressure versus volume change plot, the behavior after the initiation of buckling is depicted by a line parallel to the volume change axis. A system of this type can be approximated by the use of air as a pressure medium. The basic elements of the soft system are shown in Figure 11.

The operation of the system begins with the valves 2 and 6 closed and valve 3 open. The vacuum pump 4 is turned on and a large tank reservoir is evacuated. The valve 3 is then closed and the vacuum pump is turned off to eliminate spurious disturbances during testing. The needle throttling valve is then slowly opened and the specimen 1 is gradually evacuated. The manometer 5, which is vented to the atmosphere, is used to record the pressure difference acting on the specimen. After buckling occurs, valve 2 is closed and valve 6 is opened to restore atmospheric pressure inside of the specimen 1.

Two types of tests were conducted on the soft system and the method of specimen support was different for the two tests. These details will be described in the section on test results.

Tensile Tests

The computation of the classical buckling pressure makes use of two elastic constants—Young's modulus and Poisson's ratio. Young's modulus was determined from tensile tests on strips cut from spherical shell specimens after they had been buckled. Regions away from the buckle were selected in order to provide specimens free of kinks. It would, of course, have been possible to obtain specimens plated on more convenient mandrel shapes. The conditions of plating would have differed then, however. Using the spheres as a source eliminates the uncertainty associated with the more convenient procedure.

Specimen blanks were placed in templates, cut to the desired size, and drilled at the ends to receive pins for clamp-type loading shackles. The width of the specimens was 0.30 inch and the length 2.5 inches. The width dimension was small enough to effectively eliminate the curvature problem across the specimen width.

The specimen cross-sections were so small that direct, dead-weight loading was possible. Load increments of approximately two pounds were used. A Type AF-71-wire resistance strain gage was applied to each face of a tensile specimen. The two gages wired in series to average the strain readings which were obtained from an SR-4 Strain Indicator. This procedure compensated for the straightening effect which occurred at the initiation of loading, and good straight line plots of stress versus strain were obtained.

Both the variation of Young's modulus from specimen to specimen and the variation within a single shell specimen was checked. This was accomplished by obtaining blanks from specimens plated at different times (Specimens 9, 25, and 33) and from three locations in one specimen (33). The selection of the three locations was as follows: one specimen blank was adjacent to each of the poles and the remaining blank was halfway between the poles. The data from the tests are summarized in the section on test results.

Torsional Pendulum Test

As noted in the previous section, the value of Poisson's ratio for the shell material is needed. Since the shell material is isotropic in directions tangent to the mid-surface, it should be possible to compute Poisson's ratio from a knowledge of the modulus of rigidity. This latter elastic constant can be obtained from a simple torsional pendulum test in which a mass is suspended from a thin strip of the material of the specimen. The stiffness of the strip, which has a rectangular cross-section, must be computed from a result available from the theory of elasticity. The modulus of rigidity G, can then be computed from the equation

$$G = \frac{12\pi^2 LJf^2}{t^3 b(1-0.63\ t/b)} ,$$

where

L is the strip length,

J is the mass moment of inertia of the pendulum bob,

f is the frequency of oscillation,

t is the thickness of the strip,

and b is the width of the strip.

Specimen blanks for these tests were selected in the same manner as the blanks for the tensile tests. The values of L and b were:

L = 4.00 inches,

and b = 0.190 inch.

The quantity determined from the tests was the period of oscillation. This was obtained by measuring the time—by stop watch—required 70 cycles. Small amplitudes were used and the extremes of displacement were easily noted by observing a reflection from a small mirror fixed to the pendulum bob. The time for 70 cycles was repeatedly measured with a maximum deviation of less than one half of one per cent. The value of Poisson's ratio, ν, can be computed from the well known formula:

$$\nu = \frac{E}{2G} - 1 \quad ,$$

where E is Young's modulus. It should be emphasized that the values of E, G and ν in this equation apply to properties of the material in directions tangent to the mid-surface. They do not apply to the direction normal to the mid-surface.

Presentation of Experimental Results

Mechanical Properties

The tensile tests described in the previous section provided the value of Young's modulus and an approximate value of the proportional limit stress for the electroplated nickel used in the spherical shells. One specimen from each of two spheres (9, 25) and three specimens from one sphere (33) were tested in tension. The values of the modulus for all five specimens were within one half of one million of 28.0×10^6 psi. that is, Young's modulus was determined to be $28.0 \times 10^6 \pm 0.5 \times 10^6$ psi.

A typical plot of stress versus strain is presented in Figure 12. It appears that the proportional limit is about 50,000 psi. Due to the removal of curvature that occurs upon initiation of loading, the true proportional limit is somewhat greater than the apparent value. In any event, the value is relatively large and this is an important advantage in studies of elastic buckling.

Because the specimens were cut from blanks with double curvature, it was not convenient to have reduced gage sections. Thus, although it was possible to obtain Young's modulus and an approximate value of the proportional limit, it was not possible to obtain the tensile strength. Since the tensile strength has not bearing on the present studies, no special attempt was made to obtain it. It may be noted, however, that from Figure 12, the tensile strength can be deduced to be greater than 106,000 psi. The tensile strength of nickel plated from a sulfamate bath is reported [22] to range from 55,000 spi. to 155,000 psi. so that the strength of the nickel produced for this program exceeds the lower limit given in the Metals Handbook.

The modulus of rigidity was determined from tests on samples cut from the same sphere specimens as the blanks for the tensile tests. The computed values of the modulus of rigidity (see Equipment and Procedures) exhibited substantially greater variation from specimen to specimen than the values of Young's modulus. The computed values ranged from 13.0×10^6 psi. to 16.0×10^6 psi. In reviewing the test and the computation it was concluded that the measurement of thickness is the probable source of the variation observed. An error in thickness measurement of 1×10^{-4} inch is quite possible. The thickness value is cubed in computing the modulus of rigidity; so for a thickness of 3×10^{-3} inch, an error in G of 10 per cent is easily conceivable. The observed range is described, however, by taking the rigidity modulus to be $14.5 \times 10^6 \pm 1.5 \times 10^6$ psi., and thus the observed variation is of the order to be expected from the accuracy of the thickness measurement.

The equation for computing Poisson's ratio,

$$\nu = \frac{E}{2G} - 1 \ ,$$

can produce large variations in ν for relatively small errors in E and G . For the given values of E and G , the extremes can be computed as

$$\nu_{max} = \frac{28.5}{26} - 1 = 0.10 \quad ,$$

and

$$\nu_{min.} = \frac{27.5}{32.0} - 1 = -0.14 \quad .$$

Since it is not likely that Poisson's ratio would be negative[*], the above results indicate that the value can be described as

$$0 < \nu \leq 0.10 \quad .$$

In the computation of the classical buckling pressure the values of Young's modulus, E , and Poisson's ratio, ν , were taken as:

$E = 28 \times 10^6$ psi.

$\nu = 0.10$

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[*] Poisson's ratio is probably different in the thickness direction. It is, for example, well know that for deep drawing, low carbon steels, which are isotropic in the plane but anistropic normal to the plane, the plastic contraction accompanying tension can be substantially different in the plane of the sheet and the thickness directions.

Sphere Test Results

Three different types of tests were conducted on the spherical shell specimens. The first type of test was conducted before the wax mandrel was removed. The specimen was gradually evacuated as in the soft system test (see Equipment and Procedures), but because the mandrel was still inside, the inward displacement of the shell wall during buckling was limited to the clearance existing between the sphere specimen and the mandrel*. A photograph of a shell specimen prepared for this test is shown in Figure 13. A section view of a specimen and mandrel is shown in Figure 14. The clearance at the top was approximately 0.01 inch. Note that with the support used, most of the weight of the mandrel (2.3 lbs.) is transferred through the specimen wall to the plastic horn.

The results of the air tests with the mandrel inside are presented in Table 1 in the column for values of p_{am}. In a number of instances more than one test was conducted. Note that the second value is often slightly lower, indicating that some damage was incurred in the first test. Specimen no. 11 was the first specimen exposed to this type of test. A detailed discussion of the reasons for introducing the test is presented in the section "Discussion of Results".

Testing in the rigid test system has been discussed previously. The results of these experiments are presented in Table 1 in the columns identified as p_w and p_p. The values of p_w represent the buckling pressures for the tests. The values of p_p are the post-buckling pressures measured. More than one set of values of p_w and p_p was

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*When the plated mandrel was removed from the heated plating bath, the mandrel contracted more upon cooling than the nickel shell.

Table 1. Spherical Shell Test Data

Spec. No.	Surface Quality	p_{am} (psi)	p_w (psi)	p_p (psi)	p_a (psi)	Failure Location	Comments
5	poor		1.3		1.5	side	1
6	poor		4.7		4.8	side	2
7	fair		4.7 4.4	1.3 1.3	4.8	bottom	
8	fair		5.4 5.4	1.3 1.1	5.4	bottom	
9	fair		3.8 3.6 3.5	1.4 1.4	4.0	top	
11	poor	2.7 2.7	2.8 2.6 2.6	0.5 0.6 0.6	2.8	side	2
12	fair	4.1	2.7 2.7 2.7	0.8 0.8 0.8	2.8	side	3
13	fair		2.4 2.2	0.2 0.3	2.3	side	
15	fair	7.4	4.7 4.2	0.8 0.8	4.4	side	4,5
16	fair	9.0				side	3,4
17	fair	6.7				side	3
20	fair	2.6 2.6 2.6	2.5 2.2	0.2 0.2	2.2	side	4
21	fair	3.8 3.9 3.2	2.2 2.1	0.6 0.6	2.1	side	3,4,5
22	fair	4.3 4.2	3.6 3.5	0.7 0.7	3.5	side	2

Table 1. (Continued)

Spec. No.	Surface Quality	p_{am} (psi)	p_w (psi)	p_p (psi)	p_a (psi)	Failure Location	Comments
23	fair	5.9				side	2,6
24	fair	3.3	3.2 3.1	0.5 0.5	3.1	side	2
25	fair	10.0				side	5
26	fair	7.6 7.7 7.7	4.4 4.3	1.6 1.6	4.1		7
27	good	7.0 6.6	5.2 5.2	0.5	5.3	side	3,5
28	good	6.9 6.4	6.3 6.3	0.4 0.4	6.4	side	5
29	good	6.6 3.3				side	5
30	fair	4.6 4.5	4.2 4.1	0.4 0.4	4.3	side	
31	good	6.5 6.0	5.8 6.0	0.6 0.9	5.7	side	5
32	good	5.8 5.8	4.5 4.5	0.4 0.4	4.5	side	3
33	good	6.8 6.6	6.1 6.1	0.6 0.5	6.2	side	3,5
34	good	6.5 6.5	2.7 2.8 2.7	0.5 0.5 0.5	2.7	side	3
36	fair	9.3 8.7 8.8	5.2 5.2	0.8 0.8	5.0	side	3,5

Table 1. (Continued)

Spec. No.	Surface Quality	p_{am} (psi)	p_w (psi)	p_p (psi)	p_a (psi)	Failure Location	Comments
39	good	8.1 7.3	3.6 3.5	0.73 0.72	3.6	side	3,5
40	fair	8.8 8.2	8.4 8.3	0.67 0.72	8.0	side	5
41	good	9.5 9.3	7.3 6.8	1.4 1.4	6.8	side	3,5
43	good	7.5 7.2	6.1 6.1	0.45 0.44	5.9	side	3,5
45	good	7.5 7.1	5.9 5.7 5.7	0.47 0.47 0.49	5.8	side	3,5

Footnotes for Table 1

1. Specimen accidentally dented on side prior to testing.
2. Failed at small region of curvature change.
3. Specimen damaged during preparation for testing in the rigid test system.
4. Specimen chemically polished (see section - Discussion of Results)
5. Specimen damaged during air test with mandrel inside.
6. Mandrel collapsed during first test.
7. Buckle at side in first tests (P_{am}) and at top in subsequent tests. Specimen evidentally damaged while processing for final tests.

usually obtained, and all of the values determined are recorded in Table 1. The second value of p_w is sometimes slightly lower than the first.

As can be seen from the table, some of the sphere specimens were tested more than once in air with the mandrel inside and then tested a few times in the rigid, water system. The final test for most specimens was in the soft system which has been described in the section "Equipment and Procedures". In this test the specimen was destroyed since the pressure difference at buckling was not reduced during buckling, and no restriction against inward displacement existed. The buckling pressures for these tests are entered in Table 1 in the column entitled p_a .

Table 1 also includes the columns Surface Quality, Failure Location and Comments. Three disignations are used for the quality of the surface: good, fair, poor. The surface condition produced by the electroforming process has been discussed previously and its influence on the response of the spherical shells to uniform, external pressure is discussed in the section "Discussion of Results". Generally, however, the classification system of Table 1 refers to surface condition in terms of the following scheme:

poor - many flaws, surface repair necessary.

fair - few flaws, some surface repair necessary.

good - only minor flaws, no surface repair.

The location of failure designates the region in which buckling occurred. The classification used in Table 1 is based on a specimen orientation in which the axis of the points through which electrical contact to the mandrel was made during plating is vertical. Buckling adjacent to one of these points is indicated by the words top or bottom.

Buckling away from these sites is indicated by the word side. The implications of the failure location are discussed later.

The numbers entered in the column entitled Comments refer to the footnotes at the bottom of Table 1.

DISCUSSION OF RESULTS

Two aspects of the investigation will be emphasized in the discussion of the results that were presented in the previous section. The first aspect is the relation of the various forms of shell imperfection to the value of the buckling pressure. The second aspect is the influence of loading system characteristics—rigid or soft—on the value of the buckling pressure. A discussion of several subsidiary features of the results will also be included. These, however, are of secondary importance.

Relation of Specimen Quality to Test Performance

The fabrication of good quality, complete spherical shells by any process is a difficult task. The production of such shells by electrodeposition would appear—at least at this time—to be one of the more promising methods available. It is apparent, however, from the contents of the preceeding sections of this report that the use of electroforming is by no means a simple solution to the production problem. This conclusion will be further amplified in the discussion which follows.

Test results for 32 complete spherical shells are presented in this report. During the production of these shells, it was found that good control of the shell thickness and radius could be achieved. A quality generally comparable with that obtained by Almroth et al [7] for circular, cylindrical shells was attainable. Before the benefits of this quality could be realized in terms of high buckling pressures, however, it was found that the disturbance effects arising from the shell attachments and closures (see Figures 8 and 9) had to be alleviated.

The results of the first five tests indicated that when the surface of the shell was free of serious flaws such as dents or regions of reduced curvature (see Table 1), buckling started adjacent to either the attachment or the closure. The abrupt change in thickness near these points, which were functionally necessary, evidently prevented the remaining shell surface from realizing its potential resistance to buckling. A comparison of the experimental and the classical buckling pressures for the first five tests is presented in Table 2. The classical buckling pressure, p_c , was computed from the formula

$$p_c = \frac{2E}{\sqrt{3(1-\nu^2)}} \left(\frac{h}{R} \right)^2 ,$$

where h is the nominal thickness,

$R = 4.25$ inches,

$E = 28.0 \times 10^6$ psi.,

and $\nu = 0.10$.

The five specimens of Table 2 were tested first in the rigid system and then in the soft system. With this procedure, it was not possible to observe the buckling process until the second test was completed. In the second test the specimens were destroyed. Rapid inward displacement continued until the shells were crushed or fractured.

It was apparent from the results of Table 1 that a modification of the production process was required. The method of testing that was being used, however, did not provide for an efficient evaluation of variations in processing. A significant time delay and considerable handling were required before process modifications could be evaluated.

Table 2. Evaluation of Sphere Test Data

Spec. No.	Nominal Thickness* (10^{-3} inch)	Per Cent of Theoretical Buckling Pressure**
5	4.0	4.5
6	4.5	13
7	4.5	13
8	4.0	19
9	4.5	10

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*For these specimens the variation of thickness from the nominal values was about ± 15 per cent.

**Based on experimental results of Table 1 and theoretical pressures computed for values of the nominal shell thickness.

A change in the testing procedure was, therefore, introduced to remedy this situation.

The new testing procedure took advantage of the fact that when the plated mandrel was removed from the heated plating bath, the mandrel contracted more upon cooling than the nickel shell. This difference in contraction produced a gap between the mandrel and the shell (see Figure 14). In this condition the specimen could be air tested without being destroyed. The effect of modifications in specimen preparation could be quickly evaluated by the introduction of this test, and the location and the extent of buckling could be observed. A specimen prepared for air testing with the mandrel inside is shown in Figure 13. A small rubber suction cup attached to the line of the evacuation system was placed over one of the electrode holes to provide vacuum loading. The second electrode hole was covered with a small piece of masking tape. This procedure eliminated the need for the attachment and closure which were used in the previous tests.

Most of the specimens produced after No. 9 were tested first in air with the wax mandrel inside. When possible, they were then tested in the rigid system and finally in the soft system. Before reviewing the results of the additional tests in detail, it will prove instructive to describe the progression of testing in terms of observations made during the air tests with the mandrel inside.

It was generally found that buckling at a low pressure (less than 50 per cent of theoretical) appeared as a single dimple whose size depended, of course, on a gap dimension. The dimple was approximately circular and about 0.8 inch in diameter. Examination of the specimen revealed that the dimple was at a flaw or near one of the electrode holes. If the test was continued, additional dimples were observed to "pop in", usually singly, and at higher pressures*. In most instances, the subsequent dimples formed at flaws which appeared less severe than the flaw at the first dimple. In some instances, however, no apparent flaw could be detected.

At the higher buckling pressures, a different type of response was observed (initial buckling at pressures greater than 50 per cent of the theoretical value). In these instances the shell was more highly stressed, and the transition to the buckled state occured with the formation of a large number of dimples. The transition was very rapid and it was not possible to detect a dimpling sequence by visual observation, i.e., the dimples appeared to form simultaneously. An example of the type of pattern developed is shown in Figure 15. This is a photograph of Specimen No. 34.

The implications of the behavior observed are important. To illustrate this, consider a spherical shell which, when tested, will buckle at a low pressure, p_1, with the formation of a single dimple. If the flaw or surface nonuniformity responsible for the formation of the dimple could be eliminated prior to testing, it is to be expected that testing would then result in the formation of a dimple at a pressure $p_2 > p_1$. If, however, the flaw corresponding to p_2 were also eliminated, a buckling pressure $p_3 > p_2$ could be achieved. This hypothetical process can be continued to a level such that a pressure $p_n > p_{n-1}$

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*Horton and Durham [25] have presented results of interesting statistical studies in which the dimple formation process is followed in a mandrel restricted, circular cylindrical shell under an increasing axial compression.

might be achieved.

Suppose now, however, that the imperfections discussed in the preceeding paragraph were not eliminated, but only alleviated to the extent that they were equivalent in severity to the flaw corresponding to the pressure p_n . Excluding interaction effects, there should now be n sites subject to dimple formation at pressure p_n . If n is large, it might be concluded that the process outlined has produced a "balanced" structure; i.e., there are no large differences in the local resistances to buckling. By this terminology the spherical shell pictured in Figure 15 has a high degree of balance. Specimens in which dimples occur singly do not possess this balance.

Ideally, perfect balance would be achieved in a spherical shell which is without imperfections and has a constant thickness and a constant radius. Though this perfection cannot be achieved in the laboratory, it can be approached. The results of Table 2 and of Table 3, which provides an evaluation of the additional test data, illustrate the effect of continuing improvement in the quality and performance of the sphere specimens. The same trend is shown graphically in Figure 16.

The improvement in specimen performance depicted in Figure 16 was attained by developments in two areas of sphere production. First, effort was directed toward reducing the severity of surface flaws or irregularities. As noted in the section on the production of spheres, most of the surface flaws were traceable to irregularities of various types on the wax mandrel surface. Some flaws were already present on the mandrel surface in the as-cast condition, and some were inadvertently introduced in the surface conditioning process. To improve the surface quality, it was, therefore,

Table 3. Evaluation of Sphere Test Data

Specimen No.	Thickness* (10^{-3} inch)	Per Cent of Theoretical Buckling Pressure**
11	3.0	17
12	3.0	25
13	2.0	34
15	2.5	66
16	2.7	68
17	2.6	55
20	1.5	64
21	2.0	54
22	2.3	45
23	3.0	36
24	2.4	32
25	2.7	77
26	2.7	59
27	2.2	80
28	2.1	86
29	2.1	83
30	2.1	58
31	2.1	82
32	2.0	80
33	2.2	78
34	2.2	74
36	3.2	62
39	2.3	85

(Continued)

Table 3. (Continued)

Specimen No.	Thickness[*] (10^{-3} inch)	Per Cent of Theoretical Buckling Pressure[**]
40	2.6	72
41	2.5	84
43	2.2	86
45	2.2	86

Footnotes for Table 3

[*] The values of thickness listed are minimum values, but they represent the thickness in a region. Single, isolated minimum values were discarded from consideration. The variation in the shell thickness is discussed in detail in a subsequent section.

[**] The thickness values listed were used to compute the theoretical buckling pressure. The highest experimental buckling pressure obtained for each specimen was used in the computation.

necessary to modify both the mandrel casting process and the surface conditioning of the mandrel. The modifications which were introduced resulted in substantial improvements in surface quality. The details of the processing operations which were ultimately used are described in the section "Production of Spheres".

The second factor influencing the performance of the spherical shells was the disturbance arising from the presence of the electrode holes in the shell. It is apparent that if the sphere has a constant thickness and constant radius, and no reinforcements about the holes, the holes will be flaw sites at which buckling will occur at low pressure. If the holes are reinforced, but the reinforcements are too rigid and not surrounded by transition zones, bending will be introduced when the shell is pressurized, and again buckling will occur at a low pressure. Evidence of this type of response was obtained in some of the tests reported in Table 2. Buckling then occurred adjacent to the attachment or the closure sites.

The preceding comments can lead to the conclusion that the shell thickness should be gradually increased as the holes, which are diametrically opposite one another, are approached. As appropriate thickening adjacent to the holes may effectively neutralize the effect of the holes. In such a case a region of smaller, but uniform thickness would then lie in a circumferential belt which is axisymmetric with respect to an axis passing through the two holes.

A second approach might be to produce a region of a slightly reduced thickness in the neighborhood of the apex of a hemisphere whose base circle passes through the two holes. Ideally, this specimen should have a uniform thickness in a large central region surrounded by a transition zone, and a slightly thicker circular band which includes the holes. If the overall variation in thickness is not too great, the edge conditions of the elastically supported, deep cap formed should approximate the conditions which exist for a cap of a uniform, complete sphere.

Although thickness gradients can be produced in electroforming by various techniques, it was initially felt that the development work required to produce specimens with appropriate thickness variations might be too time consuming to be of value. It was discovered, however, that the second kind of specimen could be manufactured by the process of chemical-polishing [26]. By this process, uniform removal of metal layers is relatively simple. At the same time, the quality of the surface obtained is good as the process was originally developed for polishing. For the nickel shells a solution of the following composition was used:

50 ml	glacial acetic acid
30 ml	nitric acid
10 ml	sulphuric acid
10 ml	orthophosphoric acid

The process consisted of partial immersion in the polishing solution at a temperature of 85-95°C for 0.5 to 1 minute.

Specimen No. 15 was the first shell chemically polished. For this specimen there was an as-plated variation in thickness of ± 14 per cent from a nominal thickness of 0.0030 inch. The size of the chemically polished region can be described in terms of the angle of a cone whose apex is at the center of the sphere. A region within a cone angle of about 100 degrees was chemically polished. The tabulation given below for Specimen No. 15 gives the thickness variation for caps cut by cones centered in the chemically polished region.

Cone Angle (degrees)	Thickness Variation* (per cent)
30	4
60	8
90	8

*Variation from the average value for the given cap.

An evaluation of the efficiency of chemical polishing was made by conducting a sphere test on Specimen No. 15 in the soft system with the mandrel still inside. In contrast to previous tests in which a single dimple appeared upon buckling, a region within a cone angle of about 90 degrees was suddenly filled with dimples when buckling occurred. Also, the value of the buckling pressure (see Table 3) increased substantially.

Tests on several additional, chemically polished specimens (see Table 1 and 3) produced results which were essentially similar to those obtained for Specimen No. 15. These results led to attempts to develop a plating procedure which would provide the thickness variation achieved by chemical polishing. The method of suspending the mandrel shown in Figure 5 provided the desired results, and the subsequent specimens were plated in this manner (see section Production of Spheres). Note that for this process, two zones comparable to the chemically polished zone were produced. The two zones were diametrically opposite.

With all additional specimens, thickness measurements were also taken within cone angles centered about the region in which buckling occurred (see Figure 17). For a 30 degree cone, maximum variations in thickness were less than 5 per cent. For a 60 degree cone, the thickness variations were less than 15 per cent. The circumferential zones which included the pole holes were from 20 to 30 per cent thicker than the zones in which buckling was centered.

The measures introduced to improve the surface of the wax mandrels, and consequently the surface of the specimens, and the plating procedure adopted to neutralize the effect of the holes provided for substantial improvements in the performance of the shell specimens. It is seen from Table 3 and Figure 16 that buckling pressures up to 86 per cent of the classical buckling pressure were obtained. At the pressures attained in these tests, most of the surface, just prior to the onset of instability, was close to a state capable of sustaining a buckled configuration. This is vividly illustrated by the photograph of Figure 15. The response of this specimen is typical of those which buckled at high pressures.

It may be noted that often some damage occurred in the first air test with the mandrel inside. This occurred when the second test value was lower than the first under the same testing condition (see Table 1). No doubt the elastic limit was exceeded in some regions at high curvature on the dimpled surface. In still other instances specimens being prepared for additional testing were inadvertently damaged during the removal of wax or the application of the fittings. In several instances, however, the amount of degradation in proceeding to the rigid, water test was relatively small. Data for specimens of this latter group are given in Table 4. Of the quantities given, p_c represents the classical buckling pressure, p_{am} the buckling pressure obtained in the first air test with the mandrel inside, and p_w the buckling pressure obtained in the first water test (rigid system).

Table 4. Comparison of Selected Data

Specimen Number	$\frac{p_{am}}{p_c} \times 100$ (per cent)	$\frac{p_w}{p_c} \times 100$ (per cent)
20	64	62
22	45	38
24	32	31
28	86	78
30	58	53
31	82	76
33	78	70
40	72	69

A review of the data in Table 4 indicates that the amount of damage incurred in the first test or in preparing the specimen for the rigid system test was not severe, i.e., the decrease was of the order of ten per cent or less. For Specimens No. 28, 31, and 40, the damage can be traced to the first air tests with the mandrel inside. The result for the second air test with the mandrel inside and the result for the rigid system test are within 0.1 psi. of one another (see Table 1).

The Tsien Hypothesis

One of the primary objectives of this investigation was to examine the Tsien hypothesis that in test systems with different characteristics (rigid to soft) spherical shell specimens would buckle at different pressures. Specifically, the Tsien hypothesis suggests that a spherical shell loaded by a uniform external pressure can jump from an unbuckled to a buckled state at a substantially lower pressure in a soft system than in a rigid system. Tsien suggested that external disturbances, which were sufficient to actuate the jumps or transitions were usually present under normal testing conditions. Thus, although shells in soft and rigid systems might theoretically have the same buckling load, probability should favor lower buckling loads for shells tested in soft systems.

The rigid and soft systems used to examine the Tsien hypothesis in the investigation were described in the section "Equipment and Procedures". The data obtained from tests in these systems are presented in Table 1. Buckling pressures in the rigid system (water) are denoted as p_w . Buckling pressures in the soft system (air) are denoted as p_a . A photograph of a specimen buckled in the soft test system is shown in Figure 17.

A review of the data indicates that the buckling pressures obtained in the two systems are, in nearly every instance, the same to within 0.1 psi. This difference is about equal to the accuracy of the pressure measurements. It should be noted that each specimen was tested in both systems. Thus, problems which might arise from comparisons based on tests on similar but separate specimens are avoided.

It should also be noted that the same buckling pressures were obtained in both systems for specimens for a broad range of specimen quality. Indeed, the comparison covers a range of specimens which buckled at between 5 and 86 per cent of the classical buckling pressure. Regardless of the degree of imperfection, however, the performance of the specimens was the same in both systems. There is, thus, no support for the belief that the Tsien hypothesis can be used to explain the occurrence of low buckling values. These conclusions are in agreement with those which Almroth et al [7] reached on the basis of tests on circular, cylindrical shells.

Additional Features

The preceding discussion includes the features of the investigation which are considered to be of primary interest. There were, however, several other aspects of the test results which are of interest.

The thicknesses of the spherical shells tested were within a relatively narrow range of values. No effect of thickness could, therefore, be assessed. As noted previously, of course, flaws originating from scratches in the wax mandrel were observed to be more likely sites for buckling in very thin shells than in thicker shells. The resulting irregularity, for example, might easily have a depth greater than the shell thickness in the thinner specimens. Shells thinner than 0.002 inch were not produced simply because they imposed more rigid requirements on the mandrel surface preparation. For the range of thickness produced, the level of performance achieved was essentially uniform. This is illustrated in Figure 18, which includes the optimum results obtained for a range of radius to thickness ratios. The specimens may be identified by reference to Table 3.

The form of the pressure versus internal volume change curves obtained in the rigid (water) test system was essentially similar to that obtained by Thompson [15]. In all instances an abrupt, finite decrease in pressure signalled the onset of instability. This jump behavior, it should be noted, occurred even for the shells which buckled at the lowest pressures. This indicates that even for shells of large imperfection, the pressure versus displacement equilibrium curve can rise monotonically to a maximum. Beyond this point, it would follow that both the pressure and displacement values on the equilibrium curve decrease. Ultimately, the displacement begins to increase to provide the branch of the curve to which the pressure jumps in the rigid test. The post-buckling pressures are recorded as p_p in Table 1.

The relationship of the buckling pressure and the post-buckling pressure to specimen quality is shown in Figure 19. Here, the ratio of the post-buckling pressure to the buckling pressure is plotted against the ratio p_w/p_c . It can readily be seen that the ratio tends to decrease with improving specimen quality. There is, it may be noted, too much scatter to represent the trend by a curve. This scatter probably stems from differences in the nature and form of the flaws. It is interesting to note, however, that there is a decrease in scatter with improving specimen quality.

It may be concluded, that for tests in which the specimen quality was good, the post-buckling pressure tends to be ten per cent or less of the buckling pressure. These values are, no doubt, somewhat dependent on the characteristics of the total system; i.e., the pressure cell and the specimen.

CONCLUSIONS

The results of the investigation described in this report demonstrate that the process of electroforming can be used to produce spherical shell specimens of good quality. Surface flaws which are the result of either improper plating conditions or irregularities on the mandrel surface must first, however, be eliminated.

The holes in the shell made by the electrical contacts during deposition are sites of local nonuniformity and can cause a shell to have a poor test performance. If the regions adjacent to the holes are made slightly thicker than the remainder of the shell, the effects of the holes can be effectively neutralized.

Spherical shells with radius to thickness ratios ranging from about 1570 to 2120 have been produced, and by appropriate processing and testing techniques, good test results have been obtained. For specimens of good quality and for optimum testing conditions, buckling pressures up to 86 per cent of the classical value have been obtained.

The results of tests conducted in rigid and in soft loading systems indicate that pressurized spherical shells have no predisposition to buckle at different pressures in the two systems. There is, thus, no support for the belief that the Tsien hypothesis can be used to explain the occurrence of low buckling pressures. The results obtained do, however, provide strong support for the belief that the nature and severity of flaws and nonuniformities strongly influence test performance.

The role of imperfections in causing local instability can be vividly demonstrated if tests are conducted with a displacement restricting mandrel inside the specimen. For a specimen with severe flaws single dimples appear at the individual flaws. The dimples form at different pressures and the level of the pressure is a measure of the severity of the flaw. For spheres of good quality, on the other hand, most of the surface becomes unstable at the same pressure and dimples which cover much of the surface develop simultaneously.

Tests conducted in the rigid system indicate that when instability occurs, there is a finite drop in pressure. The abrupt drop can be observed for a broad range of test performance from poor to good. As the buckling pressure increases, however, the ratio of the post-buckling pressure to the buckling pressure decreases.

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26. Tegart, W. J. McG., <u>The Electrolytic and Chemical Polishing of Metals</u>, Pergamon Press, 1959, p. 102.

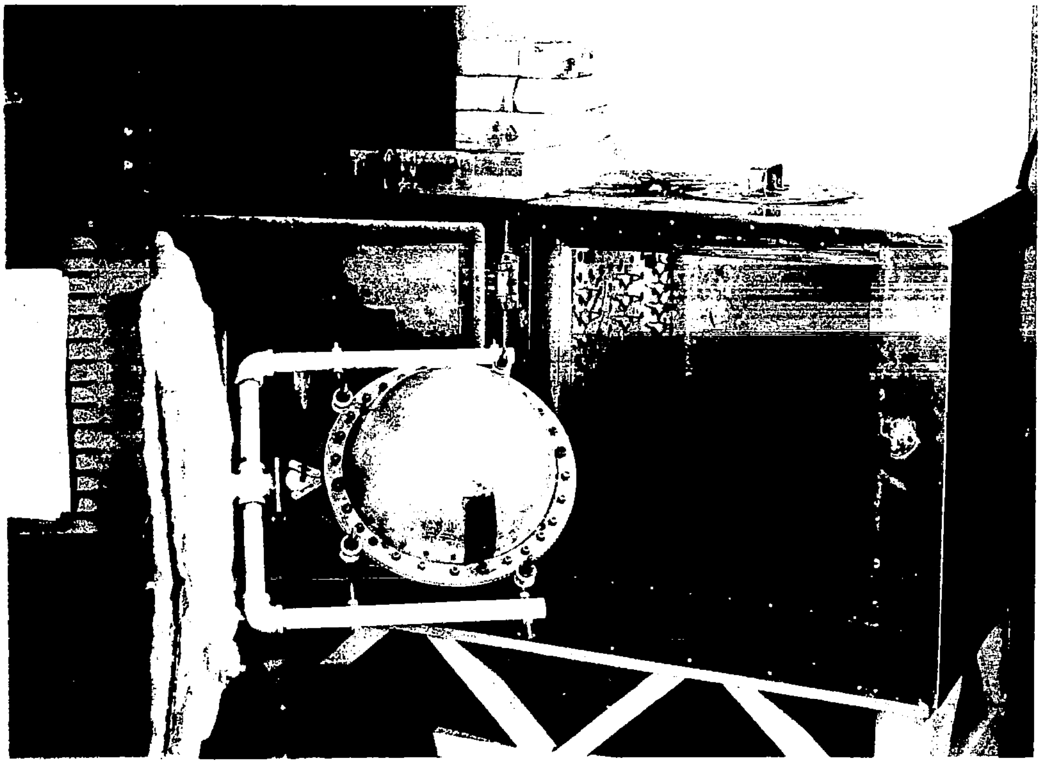


Figure 1.- Mold mechanism for production of wax mandrels.

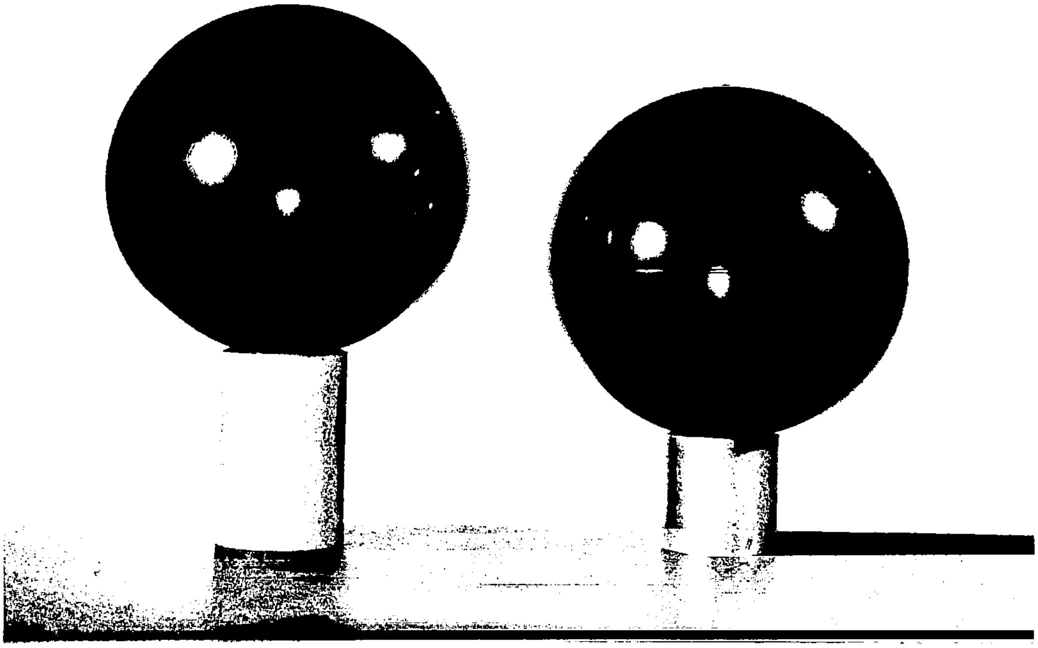


Figure 2.- Wax mandrels as-cast on the right and processed on the left.

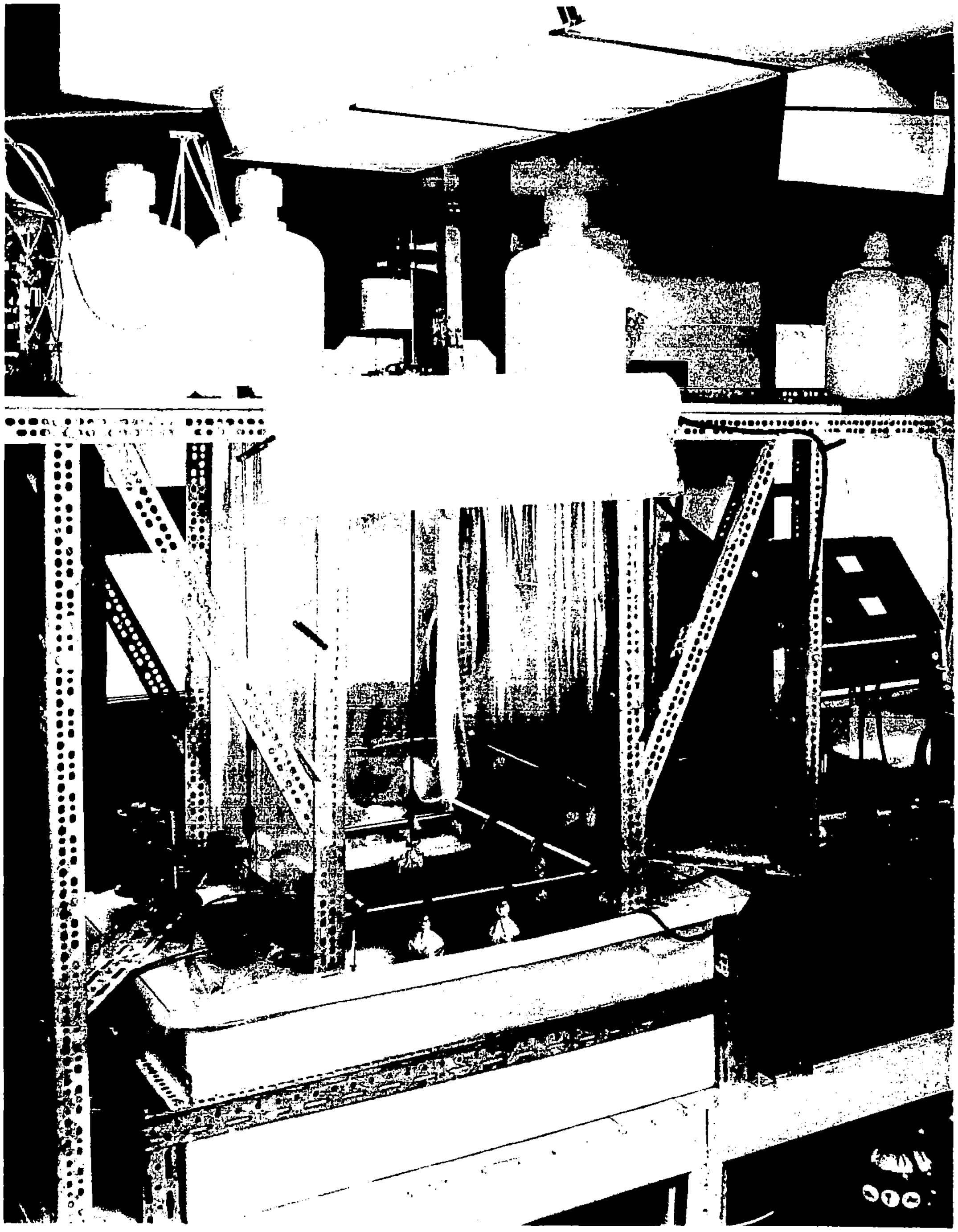


Figure 3.- Electroplating facility.

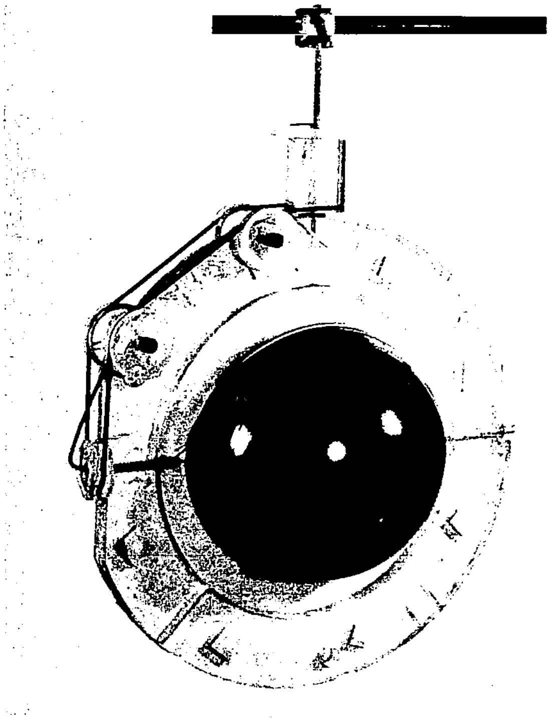


Figure 4.- Suspension system with two axes of rotation.

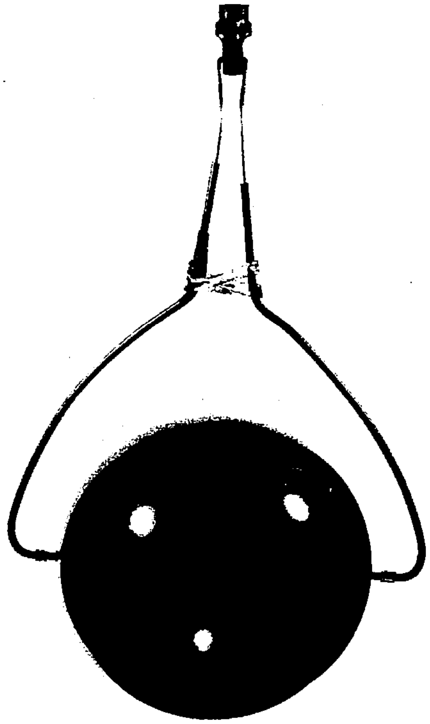


Figure 5.- Suspension system with one axis of rotation.

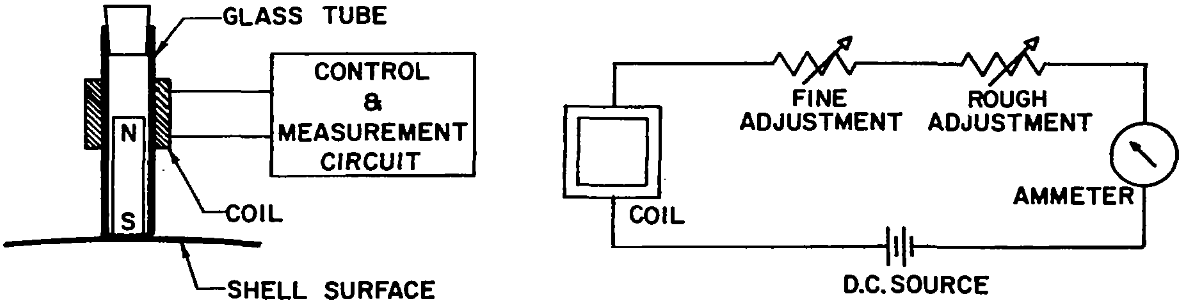


Figure 6.- Non-destructive magnetic thickness gage.

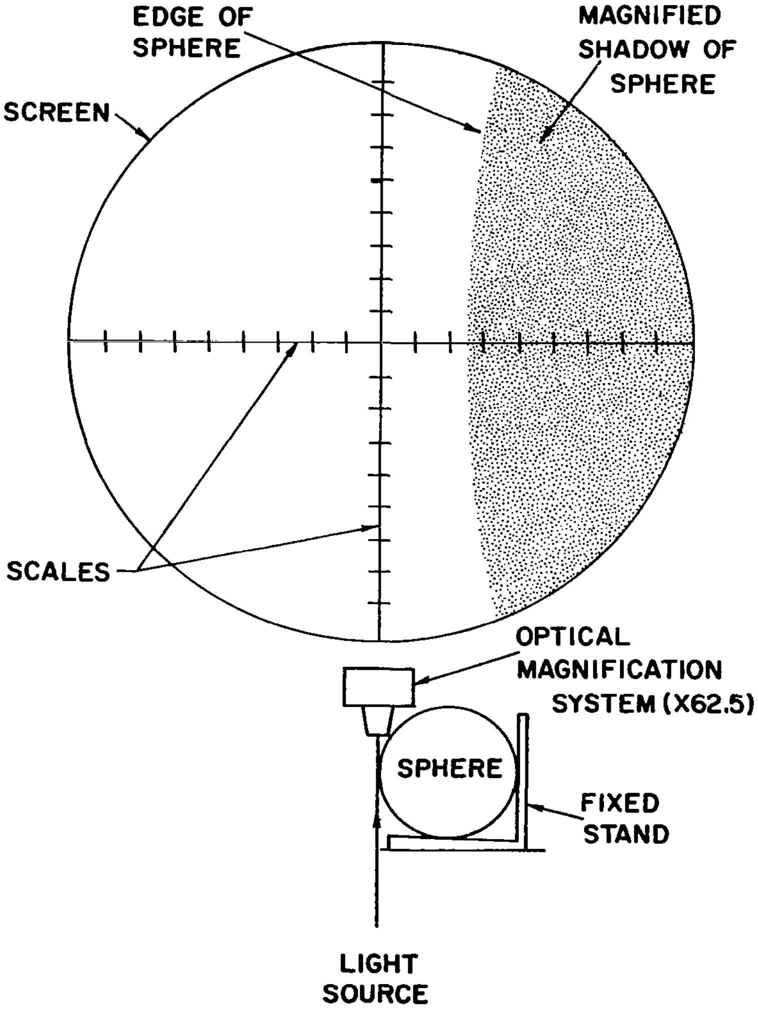


Figure 7.- Measurement of sphericity variation.

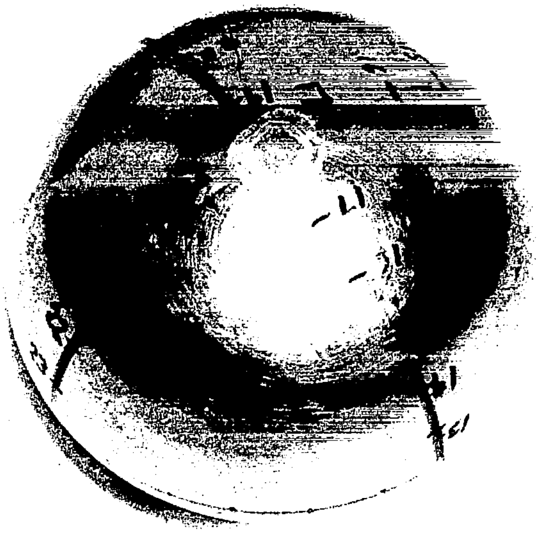


Figure 8.- Closure for spherical shell.

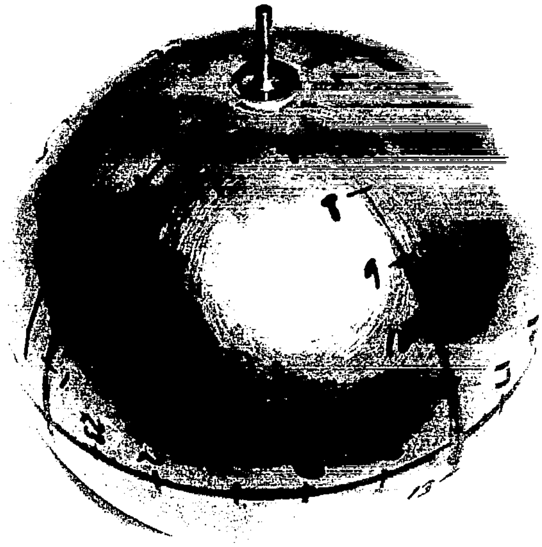


Figure 9.- Pressure fitting for spherical shell.

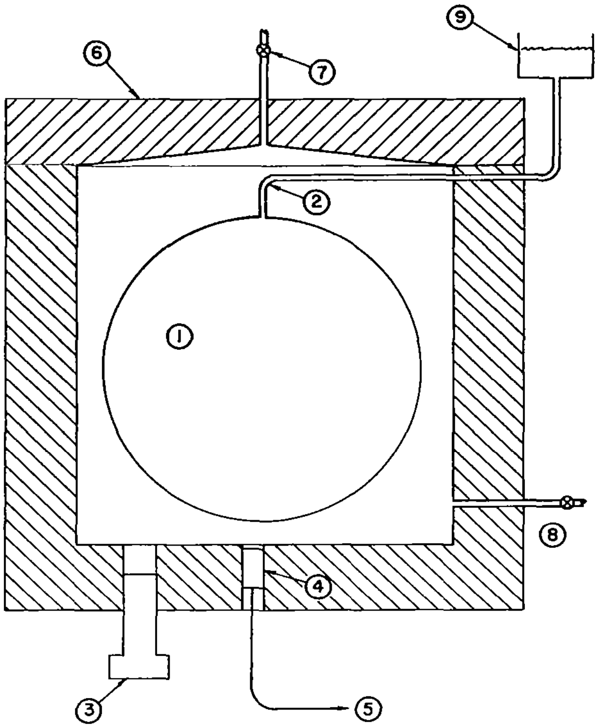
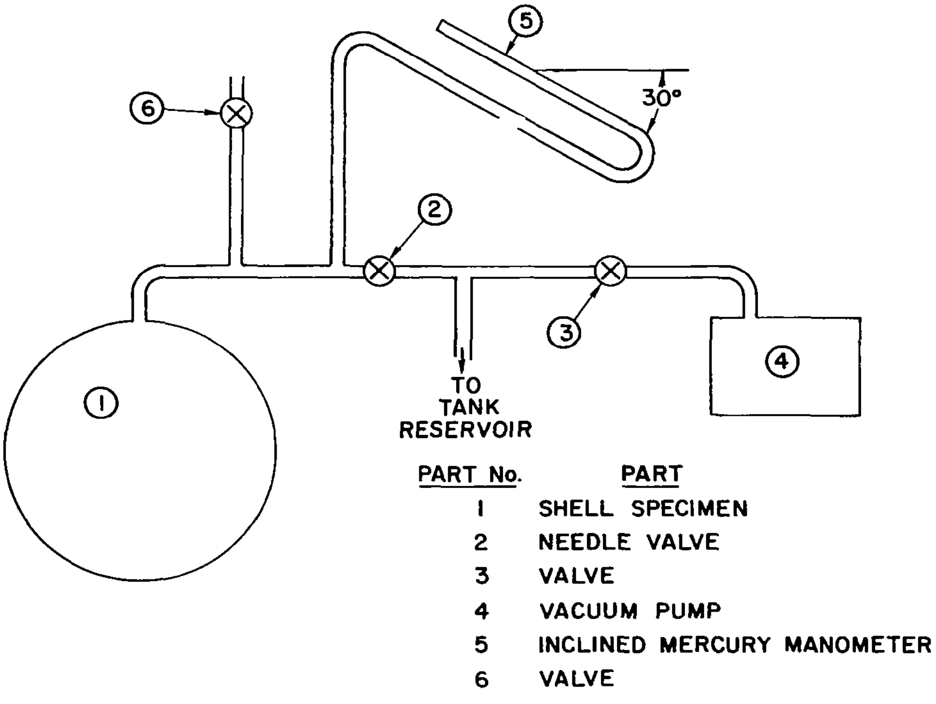


Figure 10.- Rigid test system.



PART No.	PART
1	SHELL SPECIMEN
2	NEEDLE VALVE
3	VALVE
4	VACUUM PUMP
5	INCLINED MERCURY MANOMETER
6	VALVE

Figure 11.- Soft test system.

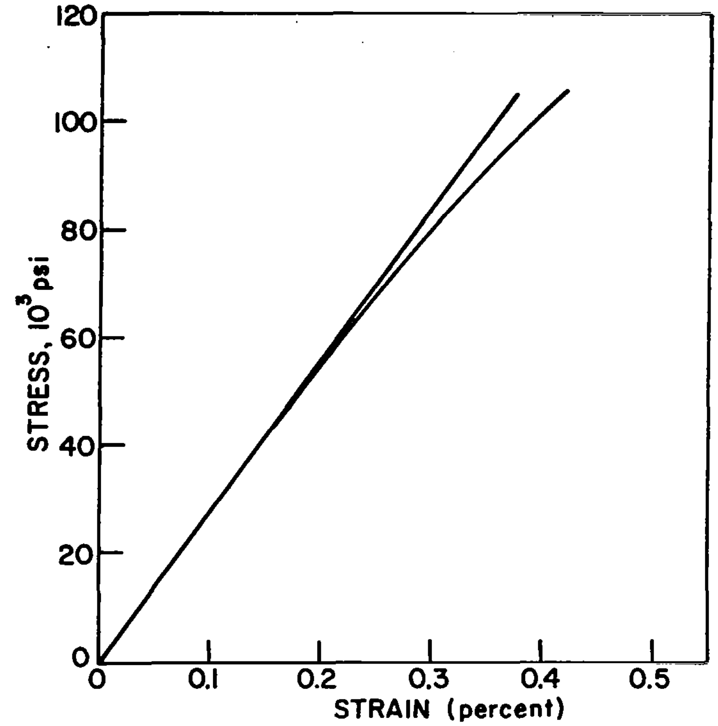


Figure 12.- Tensile stress versus strain for electrodeposited nickel.

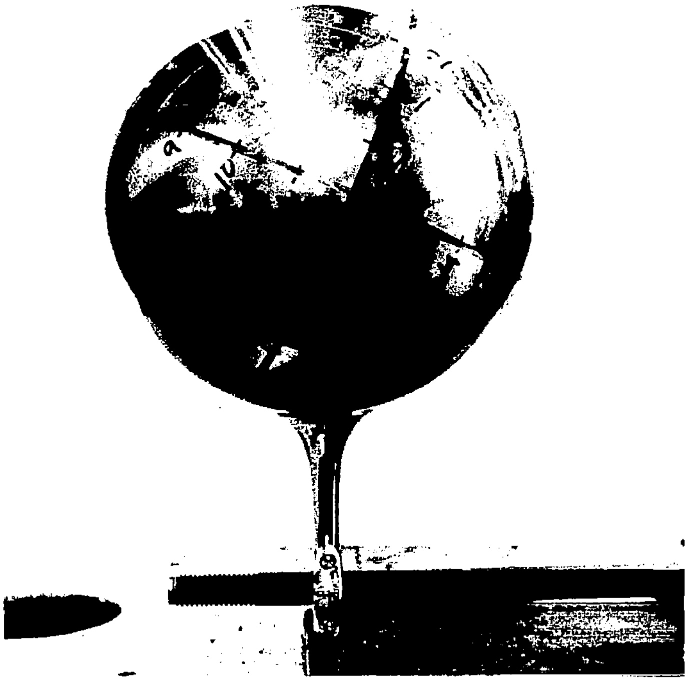


Figure 13.- Spherical shell with mandrel inside.

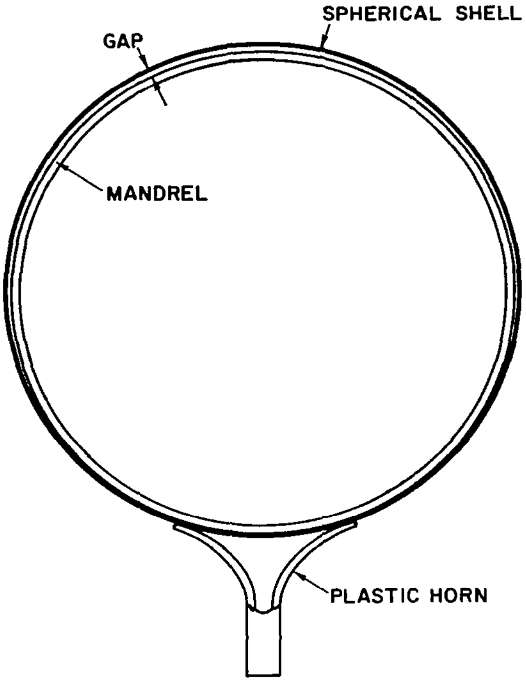


Figure 14.- View of section through specimen with mandrel.

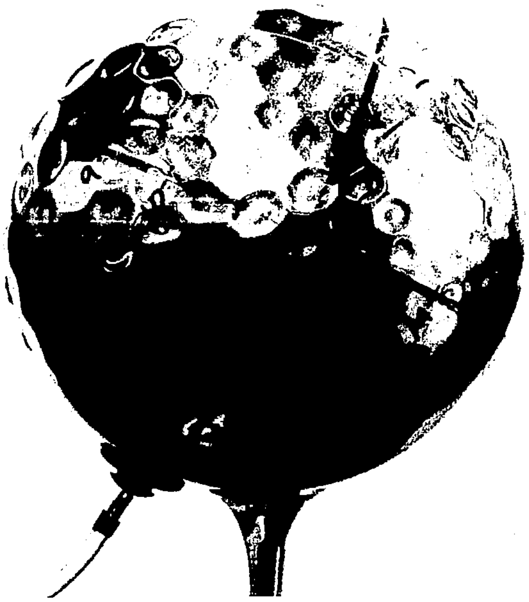


Figure 15.- Air system test with wax mandrel inside specimen.

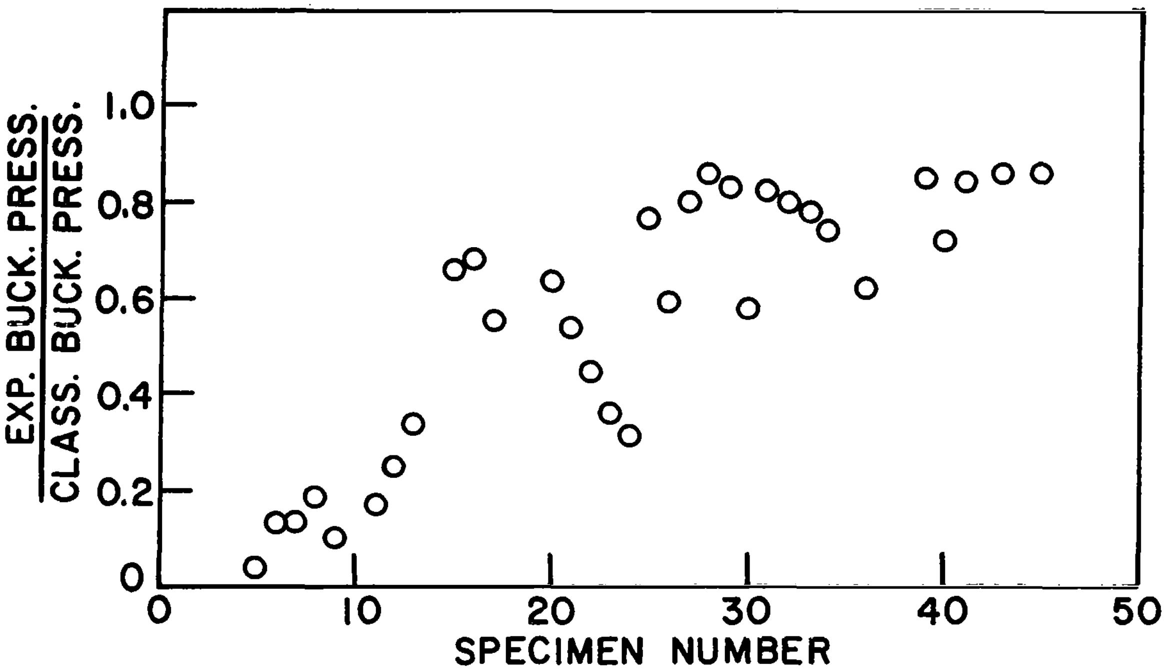


Figure 16.- Summary of sphere test performance.

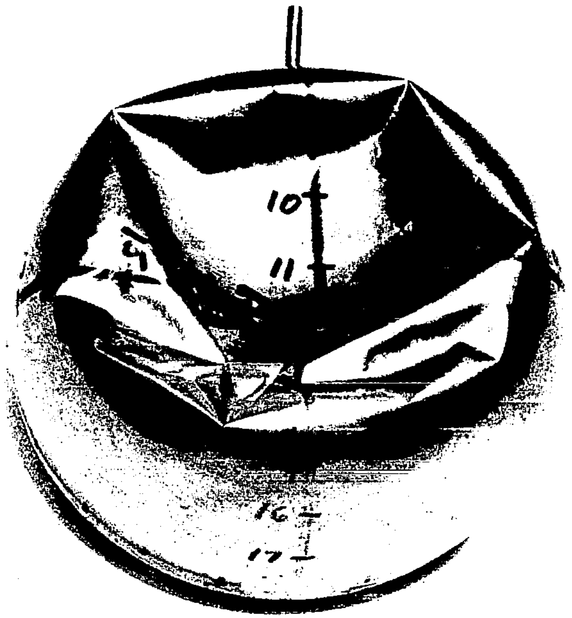


Figure 17.- Specimen buckled in the soft test system.

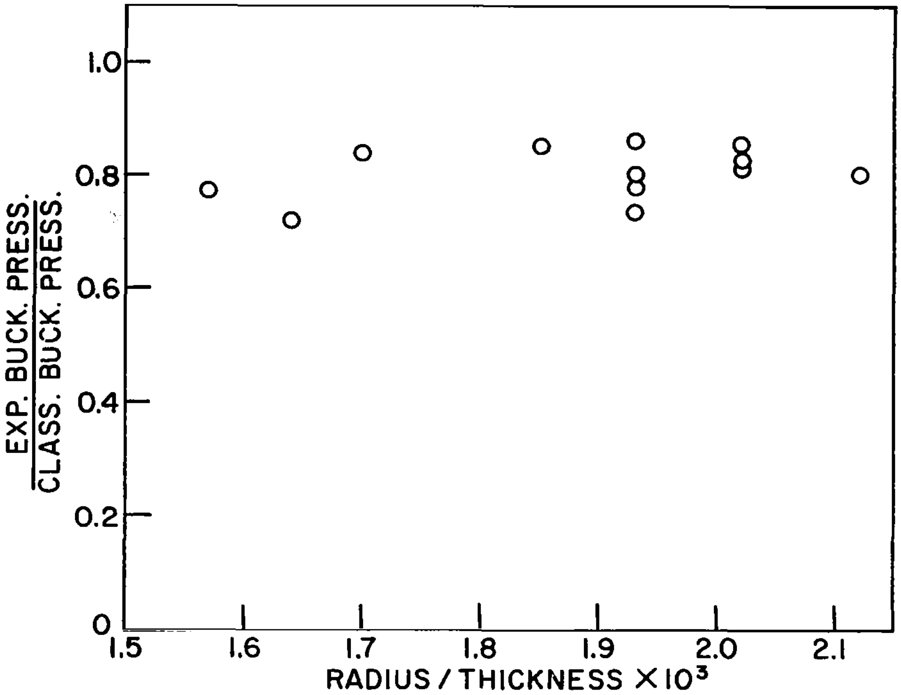


Figure 18.- Sphere test performance versus radius to thickness ratio.

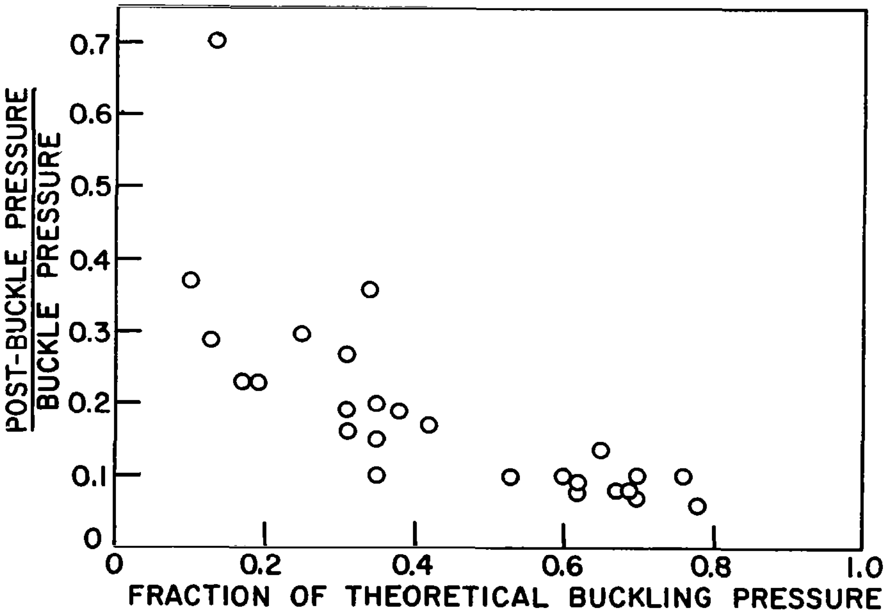


Figure 19.- Summary of post-buckling behavior.

NASA-Langley, 1966 CR-550

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> —NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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