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Technical Report No. 32-893

Spectral Analysis Applied to a Digitally Coded PM RF Carrier

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CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

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Spectral Analysis Applied to a Digitally Coded PM RF Carrier

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ABSTRACT

To ensure a high degree of confidence in calibrating a digitally coded phase-modulated (PM) system, it is desirable to calibrate directly in terms of the digital modulation. The complexity of the spectra resulting from the single-tone case suggests that the spectra from a digitally modulated PM system must be extremely complex and are, therefore, not usable for direct calibration.

This Report demonstrates that the contrary is true, by investigating the spectrum of a square-wave PM carrier. It is shown that the spectra obey simple relationships which permit accurate calibration of a PM system.

The geometric tolerances of the square- and trapezoidal-wave modulation functions are investigated in terms of harmonic content. It is shown that the symmetry and the rise-time of these functions provide a sensitive indication of the performance of a PM system.

I. INTRODUCTION

Calibrating and establishing the performance criteria of a digitally coded PM communication system is intimately related to a knowledge of the spectral characteristics defining this class of modulation.

This Report derives the spectral characteristics of a square-wave phase-modulated (PM) RF carrier and also demonstrates the geometric tolerances for the modulation function in terms of harmonic content.

It is of particular interest to note the general simplicity of the square-wave PM spectra in contrast to the relative complexity of the single-tone PM case. The unique properties of the square-wave PM spectra provide a simple means of accurately calibrating a digitally coded PM system in terms of the clocking frequency of the digital system.

II. FOURIER ANALYSIS OF SQUARE-WAVE PHASE MODULATION

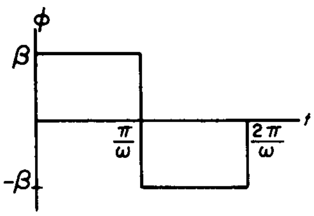
A PM function can be expanded into a Fourier series such that (Ref. 1)

$$i = I \sum_{-\infty}^{\infty} C_n e^{j(\omega_0 + n\omega)t} \tag{1}$$

where

$$C_n = \frac{\omega}{2\pi} \int_{-\pi/\omega}^{\pi/\omega} e^{[j\phi(t)]} e^{-jn\omega t}\, dt \tag{2}$$

The function $\phi(t)$ is a square wave and may be expressed analytically as



$$\phi(t) = \beta \qquad 0 < t < \frac{\pi}{\omega}$$

$$\phi(t) = -\beta \qquad \frac{\pi}{\omega} < t < \frac{2\pi}{\omega}$$

$$\phi(t + 2\pi) = \phi(t)$$

To avoid integrating through a finite discontinuity, C_n must be evaluated as the sum of two integrals:

$$C_n = K \int_0^{\pi/\omega} P(t)\, dt + K \int_{\pi/\omega}^{2\pi/\omega} Q(t)\, dt = A + B$$

Evaluating Part A by substituting for $\phi(t)$ in Eq. (2) its value of $+\beta$, we have

$$A = \frac{\omega}{2\pi} \int_0^{\pi/\omega} e^{j(\beta - n\omega t)}\, dt = -\frac{\omega}{2\pi} \left[\frac{1}{jn\omega} e^{j(\beta - n\omega t)} \right]_0^{\pi/\omega}$$

By Euler's formula

$$e^{-jn\pi} = \cos n\pi - j \sin n\pi = (-1)^n \qquad \text{for all } n$$

Therefore

$$A = \frac{e^{j\beta}}{j2n\pi} [1 - (-1)^n]$$

Part B may be evaluated by substituting for $\phi(t)$ in Eq. (2) the value $-\beta$:

$$B = \frac{\omega}{2\pi} \int_{\pi/\omega}^{2\pi/\omega} e^{-j(\beta + n\omega t)}\, dt = \frac{-\omega}{2\pi} \left[\frac{1}{jn\omega} e^{-j(\beta + n\omega t)} \right]_{\pi/\omega}^{2\pi/\omega}$$

$$B = \frac{e^{-j\beta}}{j2n\pi} [e^{-jn\pi} - e^{-j2n\pi}]$$

By Euler's formula

$$C^{-jn\pi} = \cos n\pi - j \sin n\pi = (-1)^n \qquad \text{for all } n$$

$$C^{-j2n\pi} = \cos 2n\pi - j \sin 2n\pi \equiv 1$$

Therefore

$$B = -\frac{e^{-j\beta}}{j2n\pi} [1 - (-1)^n]$$

Since C_n is the algebraic sum of the corresponding values for A and B,

$$C_n = \frac{e^{j\beta}}{j2n\pi} [1 - (-1)^n] - \frac{e^{-j\beta}}{j2n\pi} [1 - (-1)^n]$$

$$C_n = \left[\frac{1 - (-1)^n}{\pi n} \right] \left[\frac{e^{j\beta} - e^{-j\beta}}{2j} \right] = \left[\frac{1 - (-1)^n}{\pi n} \right] [\sin \beta]$$

The spectral distribution of the square-wave PM wave can now be obtained by substituting in Eq. (1) the derived value of C_n. Thus

$$i = \frac{I \sin \beta}{\pi} \sum_{n=-\infty}^{\infty} \frac{1 - (-1)^n}{n} e^{j(\omega_0 + n\omega)t} \tag{3}$$

Equation (3) can be evaluated for $\pm n$, ultimately leading to

$$i|_n = \frac{I \sin \beta}{\pi} \left[\frac{1 - (-1)^n}{n} \cdot e^{j(\omega_0 + n\omega)t} + \frac{1 - (-1)^n}{n(-1)^n} \cdot e^{j(\omega_0 - n\omega)t} \right]$$

$$i|_n = \frac{I \sin \beta}{\pi} \sum_{n=1}^{\infty} \frac{1 - (-1)^n}{n} \left[e^{j(\omega_0 + n\omega)t} + \frac{1}{(-1)^n} e^{j(\omega_0 - n\omega)t} \right] \tag{4}$$

When n is even,

$$\frac{1 - (-1)^n}{n} = 0 \qquad n \neq 0$$

When n is odd,

$$\frac{1 - (-1)^n}{n} = \frac{2}{n} \qquad n \neq 0$$

and

$$(-1)^n = -1$$

When n is zero,

$$\frac{1-(-1)^n}{n} = \frac{0}{0}$$

Equation (4) cannot be evaluated for $C_n = 0$, the RF carrier, because the indeterminate $0/0$ occurs. Also, L'Hospital's rule fails because of the appearance of the $\ln(-1)$ when $1 - (-1)^n$ is differentiated. Therefore, it becomes necessary to revert to the exponential form of C_n; applying L'Hospital's rule we find

$$C_0 = \lim_{n \to 0} \frac{d\left[e^{j\beta} - e^{j(\beta - n\pi)} - e^{-j(\beta + 2n\pi)} + e^{-j(\beta + n\pi)}\right]}{d(j2\pi n)}$$

$$C_0 = \lim_{n \to 0} \frac{j\pi\left[e^{j(\beta - n\pi)} + 2e^{-j(\beta + 2n\pi)} - e^{-j(\beta + n\pi)}\right]}{j2\pi}$$

$$C_0 = \frac{e^{j\beta}}{2} + e^{-j\beta} - \frac{e^{-j\beta}}{2} = \frac{e^{j\beta} + e^{-j\beta}}{2} = \cos\beta$$

The complete square-wave PM wave can be precisely stated as follows:

$$i = \frac{2I}{\pi} \sin\beta \sum_{n=1,3,5}^{\infty} \frac{1}{n}\left[e^{j(\omega_0 + n\omega)t} - e^{j(\omega_0 - n\omega)t}\right] + I\cos\beta\, e^{j\omega_0 t} \tag{5}$$

If Eq. (5) is viewed on a spectrum analyzer, the absolute magnitude of the nth term will appear as:

$$|i_n| = I\cos\beta + \frac{2I\sin\beta}{\pi n} \qquad (n \text{ is odd})$$

Equations (4) and (5) establish several facts, each of which has been substantiated experimentally:

1. When n is an even integer, all even sidebands disappear because of the term $1 - (-1)^n$
2. When β is an odd multiple of $\pi/2$, the carrier amplitude is zero. The spectrum envelope of the sidebands describes the hyperbola K/n
3. When β is an even multiple of $\pi/2$, all sidebands are zero and the carrier is a maximum
4. Energy in the carrier is transferred to the sidebands as a cosinusoidal function of β and the total energy remains constant

It is interesting to note that there is a unique correspondence between the spectrum of a square-wave balanced modulated carrier and a square-wave PM carrier for values of the modulation index β equal to odd multiples of $\pi/2$.

The surprising simplicity of the square-wave PM carrier can be appreciated by comparing Eq. (5) to the expression for single-tone PM:

$$i = I \sum_{-\infty}^{\infty} J_n(\beta)\, e^{j(\omega_0 + n\omega)t} \tag{6}$$

where $J_n(\beta)$ is an nth-order Bessel function. It is immediately evident from Eq. (6) that to calibrate a PM system using sinusoids involves the uncertainty inherent in equating sidebands and determining zeros in accordance with $J_n(\beta)$. Also, the bandwidth required for a constant β depends upon the modulation frequency and, if inadequate, may introduce subtle calibration errors. Figure 1 is a typical sinusoidal PM system calibration curve which illustrates the technique.

In calibrating a digitally coded PM system, it is desirable to calibrate in terms of a square-wave. Only in this manner can the system performance be established with a high degree of certainty.

Equation (5) indicates that the modulus of the unit carrier vector, $e^{j\omega_0 t}$, varies cosinusoidally with the modulation index. Thus, β can be accurately calculated from trigonometric tables by simply noting the ratio of the unmodulated carrier to its modulated value. A cross-check can be obtained by multiplying the value of the unmodulated carrier by $2/\pi$ and the sine of the modulation index, and comparing this number with the amplitude of the first sideband.

Figure 2 is a plot of Eq. (5) for modulation indices $\pi/2$, $\pi/6$, and $\pi/18$. Although the sideband spectra are discrete, the envelope contour has been shown for clarity. It should be noted that for symmetrical square-wave modulation, the sidebands are composed of odd harmonics.

The two essential conditions that a communication system must satisfy in order that the modulation shall be geometrically similar to the demodulated product are that the system must be linear and nonselective with regard to frequency (Ref. 2). Equation (5) uniquely describes a square-wave PM waveform. This uniqueness is predicated on the preservation of amplitude and phase-angle components. Square-wave modulation is a particularly good method of investigating the characteristics of

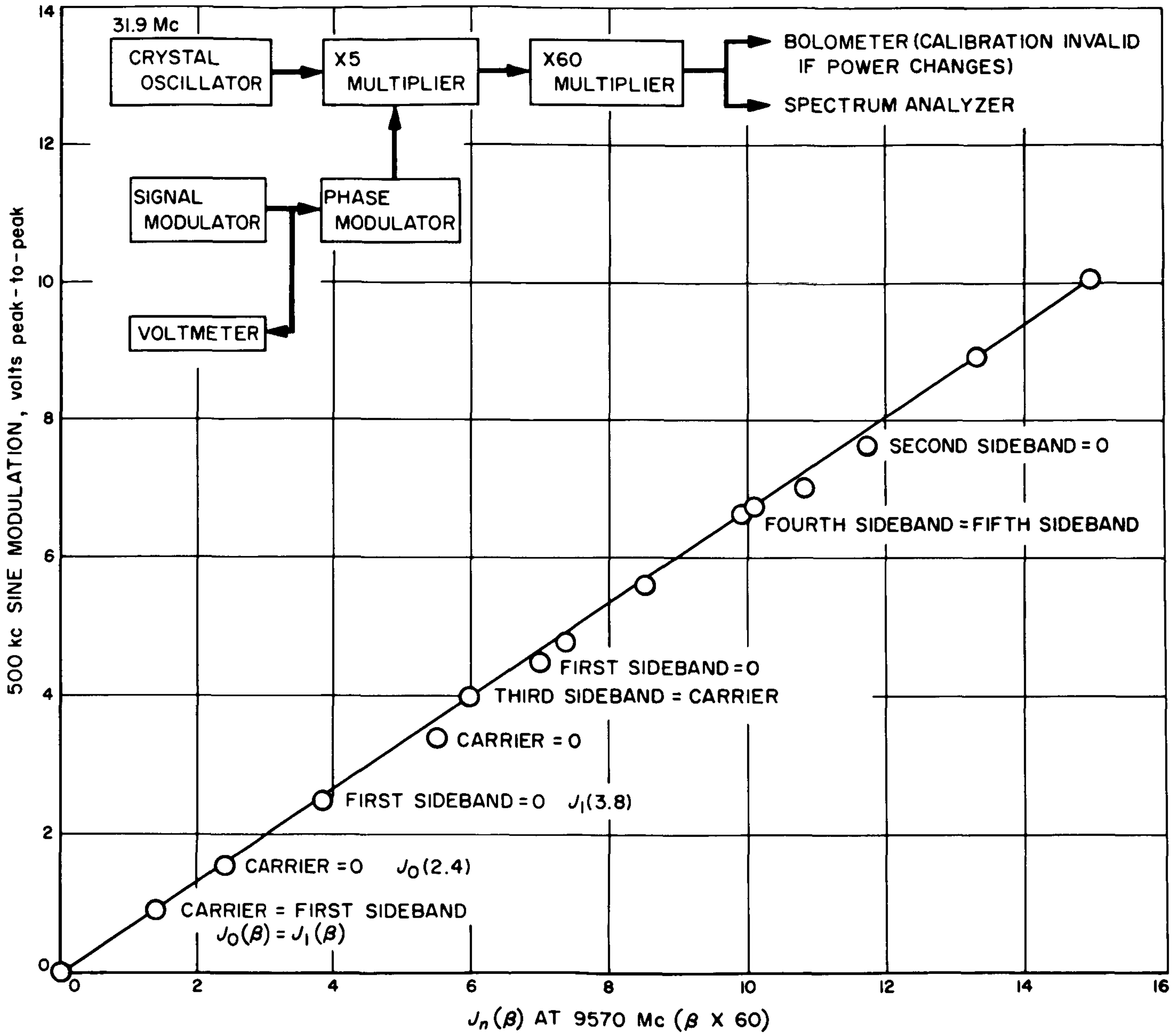
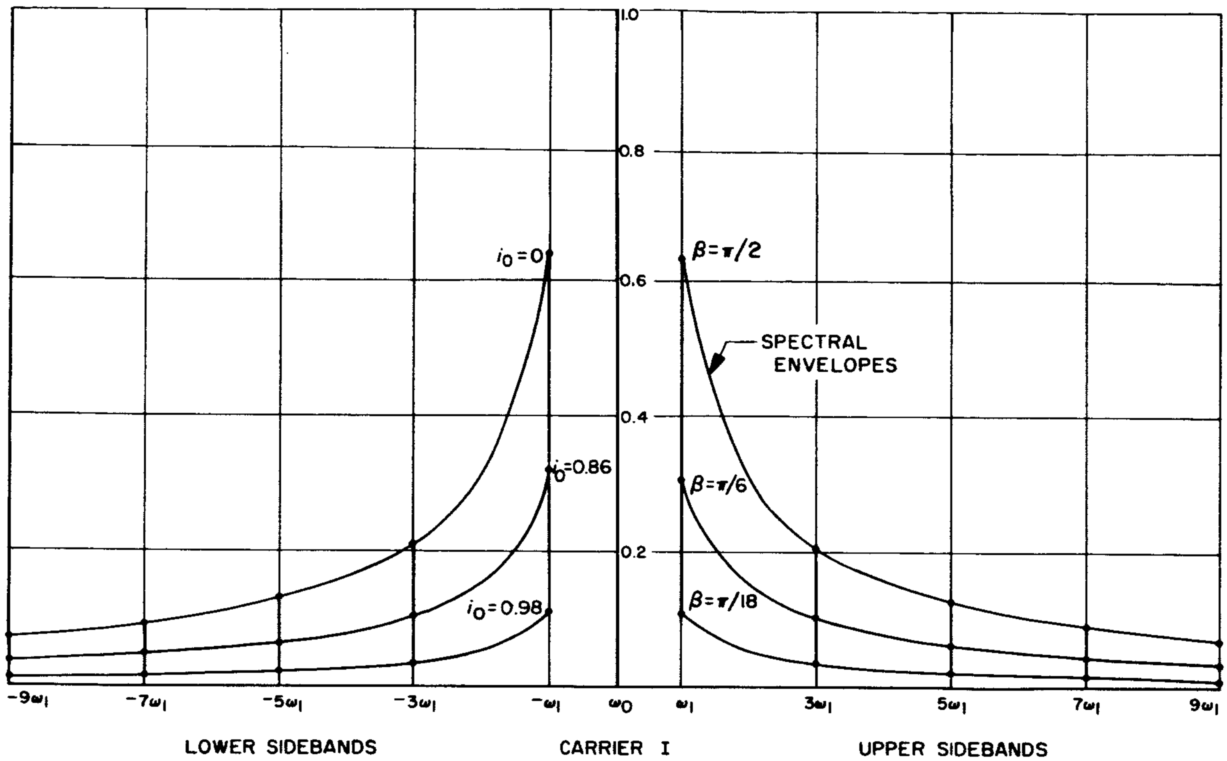


Fig. 1. Typical PM calibration curve, sinusoidal modulation

a PM channel. For example, any asymmetry in the system will cause the appearance of even harmonics. If this asymmetry is caused by nonuniform phase-shift, then the total RF power will not change because the effect of phase distortion is to remove part of the energy from the beginning of the signal and replace it later in the signal. Thus phase distortion cannot modify the energy in the signal. If the RF power changes with an increase of the modulation index without introducing appreciable even-harmonic distortion, the bandwidth of the system is symmetrical yet inadequate. In this case the demodulated waveform will be trapezoidal.

In summary, the use of a square-wave to calibrate a PM system provides an accurate method of calibration through the linear measurement of line spectra which behave according to simple relationships. A high degree of confidence can be achieved by observing the constancy of the RF power level and the absence of even-harmonic or sideband components.



CARRIER VECTOR = $I(\cos\beta)e^{j\omega_0 t}$

$$i = 0.86\, e^{j\omega_0 t} + 0.31\left[e^{j(\omega_0+\omega_1)t} - e^{j(\omega_0-\omega_1)t}\right] + 0.10\left[e^{j(\omega_0+3\omega_1)t} - e^{j(\omega_0-3\omega_1)t}\right]$$

$$+ \frac{2\sin\frac{\pi}{6}}{n\pi}\left[e^{j(\omega_0+n\omega_1)t} - e^{j(\omega_0-n\omega_1)t}\right] + \cdots$$

Fig. 2. Square-wave PM line spectra (parameter: modulation index β)

III. EVEN-HARMONIC ENERGY VERSUS SYMMETRY OF A SQUARE-WAVE

In order to calibrate a PM system using a square-wave, it is necessary to estimate the effect of waveform symmetry in terms of even-harmonic components. The following derivation provides this information.

A rectangular wave of unit amplitude can be defined as (see Fig. 3):

$$f(t) = \frac{4}{\pi}\sum_{n=1}^{\infty}\left[\frac{1}{n}\sin\frac{\pi n t_0}{T}\right]\sin(n\omega t + \phi)$$

where the absolute magnitude of the nth harmonic as displayed by a spectrum analyzer is

$$C_n = \left|\frac{4}{\pi n}\cdot\sin\frac{\pi n t_0}{T}\right|$$

The voltage ratio of the nth to the $(n+1)$th harmonic, expressed in db, is

$$\mathrm{db} = 20\log_{10}\left|\frac{C_n}{C_{n+1}}\right|$$

where

$$C_n = \left|\frac{4}{\pi n}\sin\frac{\pi n t_0}{T}\right|$$

and

$$C_{n+1} = \left|\frac{4}{\pi(n+1)}\sin\frac{\pi(n+1)t_0}{T}\right|$$

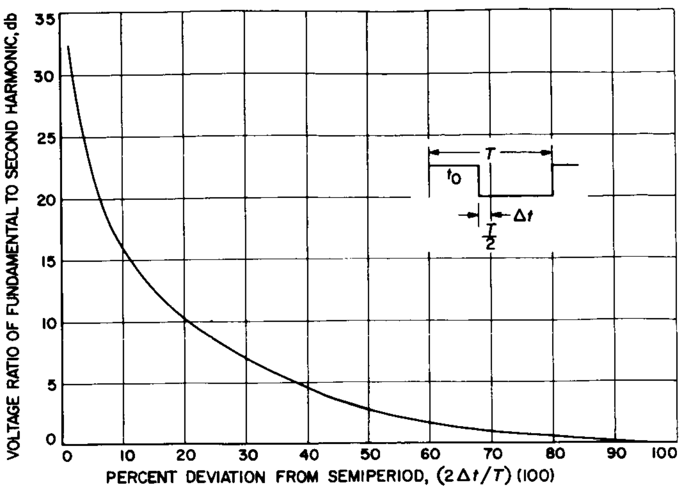


Fig. 3. Voltage ratio of fundamental to second harmonic in db as a function of percentage deviation from semiperiod

Therefore

$$\text{db} = 20\log_{10}\left[\frac{n+1}{n}\right]\left|\frac{\sin\frac{\pi n t_0}{T}}{\sin\frac{\pi(n+1)t_0}{T}}\right|$$

Of particular interest is the voltage ratio of the fundamental to the second harmonic ($n = 1$):

$$\frac{C_n}{C_{n+1}} = \frac{C_1}{C_2} = \frac{2\sin\frac{\pi t_0}{T}}{\sin\frac{2\pi t_0}{T}}$$

$$= \frac{2\sin\frac{\pi t_0}{T}}{2\sin\frac{\pi t_0}{T}\cos\frac{\pi t_0}{T}} = \sec\frac{\pi t_0}{T}$$

$$\text{db} = 20\log_{10}\left|\sec\frac{\pi t_0}{T}\right| \tag{7}$$

Formula (7) was evaluated for a unit period T within the interval $0 \leqq t_0 \leqq T/2$. Tabulated values may be found in Appendix A.

Figure 3 is a graph of the voltage ratio, in db, of the fundamental to the second harmonic as a function of the percentage deviation from the semiperiod $T/2$. It should be noted that the fundamental frequency in this case will appear as the first sideband of the PM carrier. Figure 3 indicates that 2% asymmetry will result in a second-harmonic component which is 30 db down from the fundamental.

IV. EVEN HARMONIC ENERGY VERSUS SYMMETRY AND RISE TIME OF A TRAPEZOIDAL WAVE

Although the rectangular wave represents the design goal of a digital system, it very often happens that the best approximation that can be achieved is in the form of a trapezoid. The following analysis investigates the effect of symmetry and rise time of a trapezoidal wave in terms of harmonic content.

The nth harmonic of a trapezoidal wave is given by (see Fig. 4)

$$C_n = \left[\frac{2T}{(\pi n)^2 t_1}\right]\left[\sin\frac{\pi n t_1}{T}\right]\left[\sin\frac{\pi n(t_0+t_1)}{T}\right]$$

The ratio of the nth to the $(n+1)$th harmonic is

$$\frac{C_n}{C_{n+1}} = \left(\frac{n+1}{n}\right)^2\left[\frac{\left(\sin\frac{\pi n t_1}{T}\right)\left(\sin\frac{\pi n(t_0+t_1)}{T}\right)}{\left(\sin\frac{\pi(n+1)t_1}{T}\right)\left(\sin\frac{\pi(n+1)(t_0+t_1)}{T}\right)}\right]$$

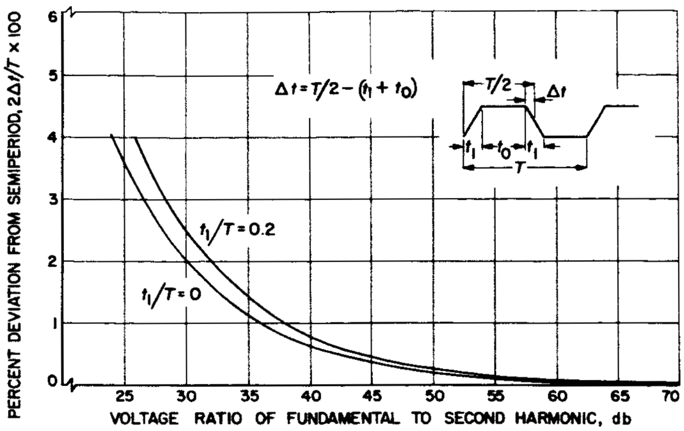


Fig. 4. Voltage ratio of fundamental to second harmonic in db

As with the rectangular wave, the ratio of the fundamental to the second harmonic ($n = 1$) is of primary value. Thus

$$\frac{C_n}{C_{n+1}} = \frac{C_1}{C_2} = \frac{4\left[\sin\frac{\pi t_1}{T}\right]\left[\sin\frac{\pi(t_0+t_1)}{T}\right]}{\left[\sin\frac{2\pi t_1}{T}\right]\left[\sin\frac{2\pi(t_0+t_1)}{T}\right]}$$

$$\frac{C_1}{C_2} = \left[\sec\frac{\pi t_1}{T}\right]\left[\sec\frac{\pi(t_0+t_1)}{T}\right]$$

where

$$0 \leqq (t_0 + t_1) \leqq T/2$$

The voltage ratio of the fundamental to the second harmonic, expressed in db, is

$$\mathrm{db} = 20\log_{10}\left[\sec\frac{\pi t_1}{T}\right]\left[\sec\frac{\pi(t_0+t_1)}{T}\right] \tag{8}$$

A plot of $100\,[(T/2) - (t_0 + t_1)]/(T/2)$ versus Eq. (8), with T taken as unity, is given by Fig. 4. The parameter t_1/T is a measure of the rise-and-fall time of the trapezoid. Figure 4 indicates that less than 2% asymmetry is permitted before the ratio of the fundamental to the second harmonic exceeds 30 db when the rise time is zero (the rectangular case). When the rise-time is 20% of the period ($t_1/T = 0.2$), asymmetry of 2.5% is permitted before the above ratio is exceeded.

Appendix B contains tabulated values giving the ratio of the fundamental to the second harmonic in db as a function of the percentage deviation from the semiperiod $100\,[(T/2) - (t_0 + t_1)]/(T/2)$ in terms of the parameters t_1, and t_0. It should be noted that $(t_1 + t_0)$ are restricted to the range $0 \leqq (t_1 + t_0) \leqq T/2$. Figure B-1 illustrates the table; t_1 was not extended beyond 0.2 because, in general, digital systems will seldom exhibit values of t_1/T exceeding 0.06.

V. CONCLUSION

The Fourier analysis of a square-wave PM RF carrier leads to a remarkably simple expression in which the carrier and sideband energy vary mutually as the cosine and sine of the modulation index. The resultant line spectra are indistinguishable from that of a square-wave function. This is in contrast to the multiplicity of line spectra resulting from single-tone modulation.

The fact that the spectra of the square-wave PM carrier are the same as that of the square-wave modulation modified by the sine of the modulation index provides an easy method of calibrating and confirming the operation of the PM system.

The existence of even-harmonic components indicates that the modulation is asymmetrical. A tolerance for symmetry can be established in terms of the ratio of the fundamental to the second harmonic. If 30 db is arbitrarily established, 2% asymmetry can be tolerated for the square-wave. The tolerance for the trapezoidal case appears to be less critical because 2.5% asymmetry is possible for a trapezoid whose rise-time is 20% of its period.

NOMENCLATURE

C_n Amplitude of nth Fourier term (harmonic)

db Decibel

e Base to napierian logarithms, 2.71828

i Instantaneous value of RF spectra

I Maximum amplitude of RF carrier

j Complex operator: $j^2 = -1$

$J_n(\beta)$ nth-order Bessel function

n An integer

t Time, sec

t_0 Time duration of flat-top region of a waveform

t_1 Rise-and-fall time of a trapezoidal wave

T Repetition period

β Maximum phase excursion expressed in radians (commonly referred to as modulation index: $\beta = \Delta\theta = \Delta\omega/\omega_1$

π 3.14

ϕ A phase constant

ω $2\pi f$: angular frequency of fundamental component of a square-wave modulating function, rad/sec

ω_0 Angular frequency of RF carrier, rad/sec

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2. Cherry, C., *Pulses and Transients in Communication Circuits*, New York: Dover Publications, Inc., 1950, p. 147.

APPENDIX A

Square-Wave

t_0	db = 20 $\log_{10}$ [sec $\pi t_0/T$]	$(2\Delta t/T)$ (100), %
0.500	infinity	0.0
0.499	49.897	0.2
0.490	30.056	2.0
0.480	24.038	4.0
0.460	18.037	8.0
0.440	14.544	12.0
0.420	12.086	16.0
0.400	10.200	20.0
0.300	04.614	40.0
0.200	01.840	60.0
0.100	00.432	80.0
0.000	00.000	100.0

$\Delta t = T/2 - t_0$
$T = 1.00$

APPENDIX B

I. Trapezoidal-Wave

t_0	t_1													
	0.00		0.04		0.06		0.08		0.10		0.12		0.20	
	db	%	db	%	db	%	db	%	db	%	db	%	db	%
0.00	00.00	100	00.13	92	00.31	88	00.55	84	00.87	80	01.26	76	03.68	60
0.02	00.01	96	00.22	88	00.43	84	00.71	80	01.06	76	01.50	72	04.10	56
0.04	00.06	92	00.34	84	00.59	80	00.91	76	01.30	72	01.78	68	04.58	52
0.06	00.15	88	00.50	80	00.78	76	01.14	72	01.58	68	02.10	64	05.13	48
0.08	00.27	84	00.70	76	01.02	72	01.42	68	01.90	64	02.47	60	05.75	44
0.10	00.43	80	00.93	72	01.30	68	01.74	64	02.27	60	02.89	56	06.45	40
0.12	00.63	76	01.21	68	01.62	64	02.11	60	02.70	56	03.37	52	07.26	36
0.14	00.86	72	01.53	64	01.99	60	02.54	56	03.18	52	03.92	48	08.18	32
0.16	01.14	68	01.91	60	02.42	56	03.02	52	03.72	48	04.54	44	09.25	28
0.18	01.46	64	02.33	56	02.90	52	03.57	48	04.34	44	05.24	40	10.52	24
0.20	01.84	60	02.81	52	03.44	48	04.18	44	05.05	40	06.05	36	12.04	20
0.22	02.26	56	03.36	48	04.06	44	04.89	40	05.85	36	06.97	32	13.92	16
0.24	02.74	52	03.98	44	04.77	40	05.69	36	06.77	32	08.04	28	16.38	12
0.26	03.29	48	04.68	40	05.57	36	06.62	32	07.85	28	09.31	24	19.88	08
0.28	03.91	44	05.48	36	06.49	32	07.69	28	09.11	24	10.83	20	25.88	04
0.30	04.61	40	06.41	32	07.57	28	08.95	24	10.63	20	12.72	16	infin	00

Parameter: t_1
Independent variable: t_0
% = 100 $[(T/2) - (t_0 + t_1)]/(T/2)$
$T = 1.00$
db = 20 $\log_{10}$ [sec $\pi t_1/T$] [sec $\pi (t_0 + t_1)/T$]

APPENDIX B (Cont'd)

I. Trapezoidal-Wave (Cont'd)

t_0	t_1 = 0.00		0.04		0.06		0.08		0.10		0.12		0.20	
	db	%	db	%	db	%	db	%	db	%	db	%	db	%
0.32	05.41	36	07.48	28	08.83	24	10.47	20	12.52	16	15.17	12		
0.34	06.34	32	08.74	24	10.35	20	12.36	16	14.98	12	18.67	08		
0.36	07.41	28	10.27	20	12.24	16	14.82	12	18.47	08	24.67	04		
0.38	08.68	24	12.15	16	14.70	12	18.31	08	24.47	04	infin	00		
0.40	10.20	20	14.61	12	18.19	08	24.32	04	infin	00				
0.42	12.08	16	18.10	08	24.19	04	infin	00						
0.44	14.54	12	24.11	04	infin	00								
0.46	18.03	08	infin	00										
0.48	24.04	04												
0.49	30.05	02												
0.50	infin	00												

Parameter: t_1
Independent variable: t_0
$\% = 100\,[(T/2) - (t_0 + t_1)]/(T/2)$
$T = 1.00$
$db = 20 \log_{10} [\sec \pi t_1/T]\,[\sec \pi (t_0 + t_1)/T]$

II. Trapezoidal-Wave Intermediate Values

$t_1 = 0.00$			$t_1 = 0.04$			$t_1 = 0.06$		
t_0	db	%	t_0	db	%	t_0	db	%
0.4825	25.20	3.5	0.4425	25.27	3.5	0.4225	25.35	3.5
0.4850	26.53	3.0	0.4450	26.60	3.0	0.4250	26.69	3.0
0.4855	26.83	2.9	0.4455	26.90	2.9	0.4255	26.99	2.9
0.4875	28.12	2.5	0.4475	28.19	2.5	0.4275	28.27	2.5
0.4900	30.05	2.0	0.4500	30.13	2.0	0.4300	30.21	2.0
0.4925	32.55	1.5	0.4525	32.62	1.5	0.4325	32.71	1.5
0.4950	36.07	1.0	0.4550	36.14	1.0	0.4350	36.23	1.0
0.4975	42.09	0.5	0.4575	42.16	0.5	0.4375	52.25	0.5
0.4985	46.54	0.3						
0.4995	56.10	0.1						

$t_1 = 0.08$			$t_1 = 0.10$			$t_1 = 0.12$		
t_0	db	%	t_0	db	%	t_0	db	%
0.4025	25.48	3.5	0.3825	25.63	3.5	0.3625	25.83	3.5
0.4050	26.81	3.0	0.3850	26.97	3.0	0.3650	27.17	3.0
0.4055	27.11	2.9	0.3855	27.27	2.9	0.3655	27.46	2.9
0.4075	28.39	2.5	0.3875	28.55	2.5	0.3675	28.75	2.5
0.4100	30.33	2.0	0.3900	30.49	2.0	0.3700	30.69	2.0
0.4125	32.83	1.5	0.3925	32.99	1.5	0.3725	33.19	1.5
0.4150	36.35	1.0	0.3950	36.51	1.0	0.3750	36.71	1.0
0.4175	42.37	0.5	0.3975	42.53	0.5	0.3775	42.73	0.5

APPENDIX B (Cont'd)

II. Trapezoidal-Wave Intermediate Values (Cont'd)

$t_1 = 0.20$		
t_0	db	%
0.2825	27.04	3.5
0.2850	28.38	3.0
0.2855	28.67	2.9
0.2875	29.94	2.5
0.2900	31.89	2.0
0.2925	34.39	1.5
0.2950	37.92	1.0
0.2975	43.93	0.5
0.2985	48.38	0.3
0.2995	57.92	0.1
0.2999	71.87	0.02

$$\% = \frac{100\left[T/2 - (t_0 + t_1)\right]}{T/2} \qquad \Delta t = T/2 - (t_0 + t_1)$$

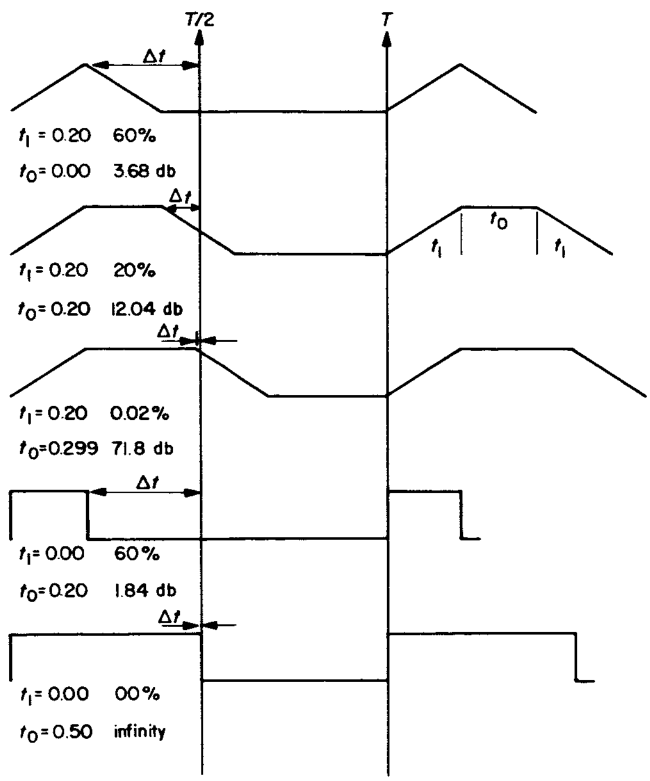

Fig. B-1. Waveform geometry vs second-harmonic distortion