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A General Method for the Calculation of Axis-Crossing Moments

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Summary. A formula for the moments of the number of crossings of a level by a stationary normal process in a given time was obtained by Cramér and Leadbetter [3]. Ylvisaker [14] has weakened the conditions for the validity of this result to the minimal possible, by a proof which includes non stationary normal cases. In this report we give an alternative derivation applicable to normal processes. Non normal processes are also discussed in a manner analogous to that given in [7] for the mean.

1. Introduction. There is a long history of interest in the problem of obtaining the mean number of crossings of a level by normal, and other types of stochastic processes in a given time. References [6] [10] [5] [2] [4] [13] [7] contain much of the development of this theory. Until recently there has been less interest in moments of higher order than the first. This is due in part to the fact that the simple results available for the mean are undoubtedly the most useful and are of paramount importance in applications, whereas the difficulty of making practical use of the higher moments increases greatly with the order of these moments. Further, the difficulty of obtaining rigorous results is certainly somewhat greater for moments of higher order than the first, in view of singularities of the probability densities involved "at diagonal points", as will be seen below.

However there are excellent reasons for studying moments of higher order than the first. Certainly the second moment plays an important role in applications of various sorts - for example in deciding on the length of "counting periods" to give very accurate time measurements by means of "noisy" standard frequency transmissions. The problem of obtaining the variance of the number of crossings of a level by a (particular type of) stationary normal process was first considered by Steinberg, Schultheiss, Worgrin, and Zweig [12], using somewhat heuristic methods. Rozanov and Volkonski [11] point out precise sufficient (though by no means necessary) conditions for the validity of this formula, and Leadbetter and Cryer [8] weaken these restrictions to give a result under conditions which are close to being minimal. This latter result includes also a rather sharp condition for finiteness of the variance.

There are also important applications for results concerning moments of higher order, as, for example, in connection with one method of discussing the asymptotic distribution of the maximum of a stationary normal process in a given time (cf. [1]). A partial result concerning moments of higher order than the second was indicated by Ivanov at the end of his paper [5]. For stationary normal processes the moments of all orders were obtained by Cramér and Leadbetter [3] under mild restrictions. One of the conditions there assumed is that $\xi(t)$ should possess a continuous sample derivative, with probability one. Belayev [1] derives this result under slightly more restrictive conditions, but including non stationary (normal) cases also. It is also shown there that the k-th moment is finite if $\xi(t)$ possesses k (q.m.) derivatives. However, as can be seen from [8] (and as noted by Belayev) this sufficient condition for finiteness is not by any means necessary. Finally, Ylvisaker [14] has shown that it is possible to slightly further weaken the conditions for the validity of the formula for the moments, to avoid explicit reference to the existence of the sample derivative. Ylvisaker's derivation hinges on an interesting application of the martingale convergence theorem. It refers to stationary normal processes, but may also be adapted to cover non stationary cases.

It will be our purpose here to give an alternative derivation of the moments for the normal case under the (minimal) conditions assumed by Ylvisaker, but without appealing to martingale theory. We shall then give explicit results (in terms of convergence of certain densities) for processes which may be non normal as well as non stationary. These results are analagous to those of [7] for the mean. The method of proof to be given uses, in part, ideas from each of the available works on this subject.

2. A general result. We shall consider a process $\xi(t)$ possessing, a.s., continuous sample functions and, for a given integer k, absolutely continuous 2k-dimensional distributions with corresponding densities of the form $f_{t_1 \dots t_{2k}}(x_1 \dots x_{2k})$. Our discussion will be in terms of the k-th factorial moment of the number N of upcrossings of zero by $\xi(t)$ in $0 \leq t \leq 1$. Certainly (as in [7])N is a well defined random variable. The modifications required to deal with downcrossings, the total number of crossings, crossings of other levels and curves, more general time intervals, and ordinary moments will be clear.

For $\underline{t} = (t_1 \dots t_k)$ lying in the k-dimensional unit cube, let m_r denote the unique integer such that $m_r/2^n \leq t_r < (m_r+1)/2^n$. Write $E_n(\underline{t})$ for the k-dimensional cube whose sides are the intervals $[m_r/2^n, (m_r+1)/2^n)$. We shall refer to $E_n(\underline{t})$ as "the cube of side 2^{-n} containing $\underline{t}$". For $\epsilon > 0$, let $A_{n\epsilon}$ denote the set of all points $\underline{t}$ in the unit cube such that for all $\underline{s} = (s_1 \dots s_k) \in E_n(\underline{t})$, we have $|s_i - s_j| > \epsilon$ whenever $i \neq j$, and write $\lambda_{n\epsilon}(\underline{t})$ for the characteristic function of the set $A_{n\epsilon}$. Finally let the random variable $\chi_{i,n} = 1$ if $\xi(i/2^n) < 0 < \xi[(i+1)/2^n]$, $\chi_{i,n} = 0$ otherwise.

The main results concerning the factorial moments of N will be relatively straightforward consequences of the following lemma.

LEMMA. Let $M_{n\epsilon} = \Sigma' \chi_{i_1,n} \dots \chi_{i_k,n} \lambda_{n\epsilon}(\frac{i_1}{2^n} \dots \frac{i_k}{2^n})$, where the summation is extended over all ordered sets of distinct integers $i_1 \dots i_k$, $0 \leq i_r \leq 2^n - 1$. Then

(i) $M_{n\epsilon}$ is non decreasing as n increases for fixed ϵ, and as ϵ decreases for fixed n.

(ii) $\lim\limits_{n \to \infty} \lim\limits_{\epsilon \to 0} M_{n\epsilon} = N(N-1)\dots(N-k+1)$, with probability one.

To prove the first statement of (i) we note that each term of the sum for $M_{n\epsilon}$ corresponds to a cube of side 2^{-n} (viz. $E_n(\frac{i_1}{2^n} \dots \frac{i_k}{2^n}])$). For fixed $\epsilon > 0$, the typical term is unity if (a) every point $\underline{s} = (s_1 \dots s_k)$ in the cube is such that $|s_i - s_j| > \epsilon$ for $i \neq j$, and (b) $\chi_{i_1,n} = \chi_{i_2,n} = \dots = \chi_{i_k,n} = 1$. When n is increased by unity, the cube divides into 2^k subcubes, in each of which property (a) still holds. Correspondingly, the typical term of the sum divides into 2^k terms formed by replacing n by $(n+1)$, and each i_j by either $2i_j$ or $2i_j+1$. Since if $\chi_{i_j,n} = 1$ we must have either $\chi_{2i_j,n+1} = 1$ or $\chi_{2i_j+1,n+1} = 1$ (with probability one), it follows that at least one of these 2^k terms is unity, and hence the first statement of (i) follows. The second statement of (i) is obvious.

To prove (ii) we note first that if the typical term in the sum of $M_{n\epsilon}$ is non zero it follows that $|i_r - i_s| > 1$ for $r \neq s$, since it is impossible to have $\chi_{i,n} = \chi_{i+1,n} = 1$ for any i. It is easy to see from this that, for such a term, $\lim\limits_{\epsilon \to 0} \lambda_{n\epsilon}([\frac{i_1}{2^n} \dots \frac{i_k}{2^n}]) = 1$. Further if $N_n = \sum\limits_{i=0}^{2^n-1} \chi_{i,n}$ it follows easily as in [3] that $\Sigma' \chi_{i_1,n} \ldots \chi_{i_k,n} = N_n(N_n-1) \ldots (N_n-k+1)$. But $N_n \longrightarrow N$ with probability one (cf. [7]). From these facts (ii) follows at once, and the proof of the lemma is complete.

From the monotonicity properties proved in (i) of the lemma it follows that the order of the ϵ and n-limits in (ii) may be interchanged. Hence, writing $M_k = \mathcal{E}\, N(N-1)\ldots(N-k+1)$, it follows by two applications of monotone convergence that $M_k = \lim_{\epsilon \to 0} \lim_{n \to \infty} \mathcal{E}\, M_{n\epsilon}$. From the definition of $M_{n\epsilon}$ and a simple transformation of variables it now follows at once that

$$(1) \qquad M_k = \lim_{\epsilon \to 0} \lim_{n \to \infty} \Sigma' \lambda_{n\epsilon}\left(\frac{i_1}{2^n} \ldots \frac{i_k}{2^n}\right) P\{\xi_{i_r} < 0 < \xi_{i_r} + 2^{-n}\eta_{i_r},\ r = 1,2\ldots k\}$$

in which $\xi_r = \xi(r/2^n)$ and $\eta_r = 2^n(\xi_{r+1}-\xi_r)$. As noted earlier, the only non zero terms in the sum on the right correspond to integers $i_1 \ldots i_k$ satisfying $|i_r - i_s| > 1$ for $r \neq s$. Thus we can (and do now) regard Σ' as indicating summation only over such sets of integers. For such integer sets, the random variables $\xi_{i_1} \ldots \xi_{i_k}$, $\eta_{i_1} \ldots \eta_{i_k}$ clearly possess a joint density. Let us write $\Psi_{n\underline{t}\epsilon}(x_1 \ldots x_k, y_1 \ldots y_k)$ to be equal to this joint density for all $\underline{t} = (t_1 \ldots t_k)$ in $A_{n\epsilon}$ such that $i_r/2^n \leq t_r < (i_r+1)/2^n$, $r = 1 \ldots k$, i.e. for all $\underline{t}$ of $A_{n\epsilon}$ lying in the cube $E_n\left(\frac{i_1}{2^n} \ldots \frac{i_k}{2^n}\right)$, and $\Psi_{n\underline{t}\epsilon} = 0$ otherwise. Then from (1),

$$M_k = \lim_{\epsilon \to 0} \lim_{n \to \infty} 2^{kn} \int_0^1 \!\ldots\! \int d\underline{t} \int_0^\infty \!\ldots\! \int d\underline{y} \int_{-2^{-n}y_1}^{0} \ldots \int_{-2^{-n}y_k}^{0} \Psi_{n\underline{t}\epsilon}(\underline{x},\underline{y})\, d\underline{x}$$

By a simple change of variable we may now obtain the following general result.

THEOREM 1. _For the process_ $\xi(t)$ _defined at the beginning of this section,_

(2) $$M_k = \lim_{\epsilon \to 0} \lim_{n \to \infty} \int_0^1 \cdots \int d\underline{t} \int_0^\infty \cdots \int d\underline{y} \int_{-y_1}^0 \cdots \int_{-y_k}^0 \Psi_{n\underline{t}\epsilon}(2^{-n}x_1 \ldots 2^{-n}x_k, y_1 \ldots y_k)d\underline{x}.$$

3. _Normal processes._ In this section $\xi(t)$ will denote a (separable) stationary normal process. It will be shown how the result given by Cramér and Leadbetter for the k-th factorial moment $M_k(u)$ of the number of upcrossings of the level u by $\xi(t)$ in $0 \leq t \leq 1$, can be obtained from Theorem 1. The less restrictive (in fact minimal) conditions assumed by Ylvisaker [14] will be used for this calculation. Specifically we shall assume that $\xi(t)$ has zero mean, and spectrum $F(\lambda)$ which is not purely discrete and possesses a finite second moment $\lambda_2 = \int_0^\infty \lambda^2 dF(\lambda)$. _Under these conditions it follows that_

(3) $$M_k(u) = \int_0^1 \cdots \int d\underline{t} \int_0^\infty \cdots \int y_1 \ldots y_k \, p_{\underline{t}}(u,\underline{y})d\underline{y}$$

where $\underline{t} = (t_1 \ldots t_k)$ and $p_{\underline{t}}(u,\underline{y})$ is obtained from the joint density $p_{\underline{t}}(\underline{x},\underline{y})$ for $\xi(t_1) \ldots \xi(t_k)$ _together with the (q.m.) derivatives_ $\xi'(t_1) \ldots \xi'(t_k)$, _by putting all the components of_ $\underline{x}$ _equal to_ u.

To obtain this result from Theorem 1 we note first that the conditions of that theorem are clearly satisfied by $\xi(t)$ (cf. [7]). It follows by considering the process $\xi(t) - u$ that

(4) $$M_k(u) = \lim_{\epsilon \to 0} \lim_{n \to \infty} \int_0^1 \cdots \int d\underline{t} \int_0^\infty \cdots \int d\underline{y} \int_{-y_1}^0 \cdots \int_{-y_k}^0 \Psi_{n\underline{t}\epsilon}(u+2^{-n}x_1 \ldots u+2^{-n}x_k, y_1 \ldots y_k)d\underline{x}.$$

Let $D(\epsilon) = \{\underline{t} = (t_1 \ldots t_k): \; |t_i - t_j| > \epsilon \text{ for all } i \neq j\} = \lim_{n \to \infty} A_{n\epsilon}$. Then it follows simply (cf. [9]) that for any given $\underline{t} \in D(\epsilon)$, the covariances occurring in the normal distribution described by $\Psi_{n\underline{t}\epsilon}$ converge to those of $p_{\underline{t}}(\underline{x},\underline{y})$; in fact this occurs uniformly for $\underline{t} \in D(\epsilon)$. In particular this implies that the integrand in (4) converges to $p_{\underline{t}}(u,\underline{y})$ in $D(\epsilon)$, as $n \to \infty$. If we can show that the n limit may be taken inside all the integral signs it would then follow that

$$M_k = \lim_{\epsilon \to 0} \int \ldots \int_{D(\epsilon)} d\underline{t} \int_0^{\infty} \ldots \int y_1 \ldots y_k \; p_{\underline{t}}(u,\underline{y}) d\underline{y}$$

from which the final result follows by monotone convergence.

To demonstrate that the order of the n limit and the integral signs may be interchanged, it is sufficient to show that the integrand in (4) is, for each fixed $\epsilon > 0$, dominated by a function which is independent of n and integrable over the indicated $\underline{x}$, $\underline{y}$, $\underline{t}$ region.

Let $\Lambda_{n\underline{t}}$ denote the covariance matrix appearing in the density $\Psi_{n\underline{t}\epsilon}$, and $B_{n\underline{t}}$ its lower right hand k x k submatrix. That is $B_{n\underline{t}}$ is the covariance matrix for $\eta_{i_1} \ldots \eta_{i_k}$, with the above notation. A simple minimization over the first k variables shows that $[\underline{x}', \underline{y}'] \, \Lambda_{n\underline{t}}^{-1} \begin{bmatrix} \underline{x} \\ \underline{y} \end{bmatrix} \geq \underline{y}' \, B_{n\underline{t}}^{-1} \, \underline{y}$. Now the elements of $B_{n\underline{t}}$ converge (uniformly in $D(\epsilon)$) to those of $B_{\underline{t}}$, the covariance matrix for $\xi'(t_1) \ldots \xi'(t_k)$. But $\min_{\underline{y}} \underline{y}' B_{n\underline{t}}^{-1} \underline{y} \, / \, \underline{y}' B_{\underline{t}}^{-1} \underline{y}$ is simply the smallest characteristic root of $B_{n\underline{t}}^{-1} B_{\underline{t}}$, and this tends to unity as $n \to \infty$ (uniformly for $\underline{t} \in D(\epsilon)$). Hence we may take n_o independent of $\underline{t}$ and such that $\underline{y}' B_{n\underline{t}} \underline{y} \geq \frac{1}{2} \underline{y}' B_{\underline{t}}^{-1} \underline{y}$ for all $\underline{y}$ when $n \geq n_o$.

Similarly by using the continuity of the smallest characteristic root of $B_{\underline{t}}^{-1} B_{\underline{t}_o}$, (for any fixed $\underline{t}_o \in D(\epsilon)$), it can be seen that $\underline{y}' B_{\underline{t}}^{-1} \underline{y} \geq \alpha \, \underline{y}' B_{\underline{t}_o}^{-1} \underline{y}$ for some

$\alpha > 0$, all $\underline{t} \in D(\epsilon)$ and all $\underline{y}$. Further since $\Lambda_{n\underline{t}}$ is non singular it follows by continuity and uniform convergence that $|\Lambda_{n\underline{t}}|$ is uniformly bounded away from zero in this region. Hence, for some constant K,

$$\Psi_{n\underline{t}\epsilon}(\underline{x},\underline{y}) \leq K \exp(-\alpha\, \underline{y}' B_{\underline{t}_o}^{-1} \underline{y}/4)$$

from which the desired conclusion follows.

4. Generalizations. The above proof carries through for a non stationary normal process subject to a non degeneracy assumption (cf. [7]). The condition $\lambda_2 < \infty$ is replaced by the requirement that the covariance function r(t,s) should possess a continuous mixed second partial derivative $\partial^2 r/\partial t \partial s$.

For non normal processes we may obtain a result for higher moments, similar to that for the mean given in [7]. Specifically suppose $\xi(t)$ satisfies the conditions stated early in Section 2 and write for $\underline{t} = (t_1 \ldots t_k)$

$$g_{\underline{t},\tau}(\underline{x},\underline{y}) = \tau^k f_{t_1 \ldots t_k,\ t_1+\tau \ldots t_k+\tau}(x_1 \ldots x_k,\ x_1+\tau y_1 \ldots x_k+\tau y_k).$$

That is $g_{\underline{t},\tau}$ is the joint density for the random variables $\xi_{t_1} \ldots \xi_{t_k}$ and the incrementary ratios $(\xi_{t_i+\tau} - \xi_{t_i})/\tau$. It then follows that

$$\Psi_{n,\underline{t},\epsilon}(\underline{x},\underline{y}) = g_{(\frac{m_1}{2^n} \ldots \frac{m_k}{2^n}),2^{-n}}(\underline{x},\underline{y})$$

at points $\underline{t}$ for which the left hand side is non zero, when $m_r/2^n \leq t_r < (m_r+1)/2^n$.)
The following result holds for M_k as defined in Section 2.

THEOREM 2. Consider points $\underline{t} = (t_1 \ldots t_k)$ such that $t_i \neq t_j$ for $i \neq j$ and suppose that, with the above notation,

(i) $g_{\underline{t}\tau}(\underline{x},\underline{y})$ is continuous in $(\underline{t},\underline{x})$ for each $\underline{y},\tau$

(ii) For each $\epsilon > 0$, $g_{\underline{t}\tau}(\underline{x},\underline{y}) \longrightarrow p_{\underline{t}}(\underline{x},\underline{y})$ as $\tau \longrightarrow 0$ uniformly in $(\underline{t},\underline{x})$ for $\underline{t} \in D(\epsilon)$ and each $\underline{y}$.

(iii) For each $\epsilon > 0$, there is a function $h_\epsilon(\underline{y})$ such that for $\underline{t} \in D(\epsilon)$,

$$g_{\underline{t},\tau}(\underline{x},\underline{y}) \leq h_\epsilon(\underline{y}) \text{ and } \int_0^\infty \!\!\ldots\!\int y_1 \ldots y_k\, h_\epsilon(\underline{y})dy < \infty .$$

Then

$$M_k = \int_0^1 \!\!\ldots\!\int d\underline{t} \int_0^\infty \!\!\ldots\!\int y_1 \ldots y_k\, p_{\underline{t}}(0,\underline{y})d\underline{y} \leq \infty .$$

To prove this result we note that by (i) and (ii) it follows that $\Psi_{n,\underline{t},\epsilon}(2^{-n}x_1 \ldots 2^{-n}x_k, y_1 \ldots y_k) \longrightarrow p_{\underline{t}}(0,\underline{y})$ as $n \longrightarrow \infty$. Hence as for the normal case the result follows once it is established that the n-limit in (2) may be taken inside the integral signs, for each fixed $\epsilon > 0$. But this latter property follows at once from (iii) by dominated convergence.

Finally we repeat that these results can be modified to refer to downcrossings, arbitrary time intervals, crossings of curves, and so on. The appropriate modifications are completely analogous to those given in [7] for the mean and hence will not be considered further here.

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In this report the moments of the number of crossings of a level or curve by a stochastic process $\xi(t)$ are obtained, for a wide class of such processes. This extends results of Cramér and Leadbetter [3], Ylvisaker [14] to non normal cases and provides an alternative derivation to that of Ylvisaker (loc. cit.) in the normal case.

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