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ABSTRACT

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Plasma ringing phenomena have been excited by Alouette I, and other ionosphere probes, at the local electron plasma frequency, upper hybrid frequency, and gyrofrequency harmonics. It has been suggested that observation of this effect is closely linked to the fact that, near to these frequencies, the warm plasma electrostatic waves, which could normally carry the energy away, have near-zero group velocity. The energy consequently remains in the vicinity of the antenna, and manifests itself in the ringing. This hypothesis may be verified by studies of the group delay of a wave-packet propagating between two antennas. In this paper, theoretical predictions of such delay are presented and have been found to agree very closely with laboratory plasma measurements. The success of these studies suggests a powerful new diagnostic technique based on group delay measurement. Its applicability to ionospheric studies is examined, and it is concluded that the method should be readily usable to obtain plasma parameters with high accuracy. The method has the particular advantage that it is influenced very little by local plasma perturbations produced by the vehicle and antennas.

CONTENTS

LIST OF FIGURES

1. INTRODUCTION

The Canadian satellite "Alouette I" was launched into a circular, 1000 km polar orbit on September 29, 1962. The vehicle carried a pulsed transmitter, to be used for topside ionosphere sounding, whose frequency was swept from 0.5 to 12.0 Mc in an 18 sec. period. Studies of the resulting ionograms made by Lockwood [1963] and by Calvert and Goe [1963] indicated the occurrence of a wider variety of plasma resonances than had been observed previously with rocket-borne fixed-frequency topside sounding transmitters [Knecht et al, 1961; Knecht and Russell, 1962]. Those with which we shall be concerned in this paper are the upper hybrid frequency, $\omega_T = (\omega_N^2 + \omega_H^2)^{1/2}$, and harmonics of the electron gyrofrequency, $n\omega_H$. The latter were identifiable up to $n \sim 10$, though the fundamental was not observed consistently for $\omega_H < \omega_N$. The electron plasma frequency, ω_N , also contributed a strong resonance trace to the ionograms, though an important distinction should be made related to the antenna orientation with respect to the local magnetic field: resonances at ω_T and $n\omega_H$ are strongest when the antenna and the magnetic field lines are parallel, while that at ω_N is strongest when they are perpendicular.

Calvert and Goe [1963] pointed out that the observations at ω_N and ω_T could be explained in terms of electrostatic plasma waves excited parallel and perpendicular to the magnetic field lines. Since the group velocity of these waves should tend to zero at ω_N and ω_T , respectively, the energy should remain in the vicinity of the antenna and result in the observed ringing. This still left unexplained the origin of the resonances at $n\omega_H$, however.

Although gyrofrequency harmonics were a newly-observed phenomenon in the ionosphere at this time, there are already a considerable volume of experimental work on laboratory plasmas in which noise emission and absorption at both electron and ion gyrofrequency harmonics had been observed [Crawford, 1965a]. There was also available a comprehensive theory of electrostatic wave propagation in warm magnetoplasmas [Bernstein, 1958; Stix, 1962] which showed that propagation perpendicular to the magnetic field should occur, free from collisionless damping, and should be characterized by transmission bands centered on the electron gyrofrequency harmonics (see Figure 1).

In 1964, Crawford, Kino and Weiss carried out a laboratory plasma experiment which has been explained in terms of the perpendicular plasma permittivity component [Harp, 1965], and proposed that the zero group velocity effect occurring at the gyrofrequency harmonics should help to explain the Alouette I observations. The analytic basis of this suggestion has since been developed in some detail by Fejer and Calvert [1964], Sturrock [1965], Wallis [1965], Dougherty and Monaghan [1965], and Deering and Fejer [1965]. Much of this work has been carried out with the object of predicting the decay time of the ringing. To make the mathematics tractable, numerous simplifying assumptions are employed involving infinitesimal antenna dimensions, neglect of the actual antenna-plasma coupling through inhomogeneities, and neglect of collisional effects. Even so, the consensus is that, despite other possibilities that have been advanced [Lockwood, 1963; Johnston and Nuttall, 1964; Nuttall, 1965], the foregoing explanation is adequate, and that the actual decay rate observed was principally due to the vehicle moving away from the energy pulsed into the plasma. This has led to further extension of the analysis to study those waves whose group velocity is matched to that of the vehicle. This has been carried out by Shkarofsky and Johnston [1965], taking into account electromagnetic corrections to the electrostatic wave dispersion at long wavelengths.

In parallel with the theoretical work just described, further experimental work on electrostatic plasma wave propagation and resonances has been proceeding. In the laboratory, very strong support for the warm magnetoplasma permittivity and wave dispersion theory for perpendicular propagation has been given by Harp [1965, 1966] who has been able to determine experimental dispersion characteristics for the propagation between two antennas separated radially in a discharge which agree closely with computations [Crawford, 1965 a,b], based on the Bernstein [1958] theory. Buchsbaum and Hasegawa [1964, 1966] have obtained very good agreement between computed eigenfrequencies for standing waves of

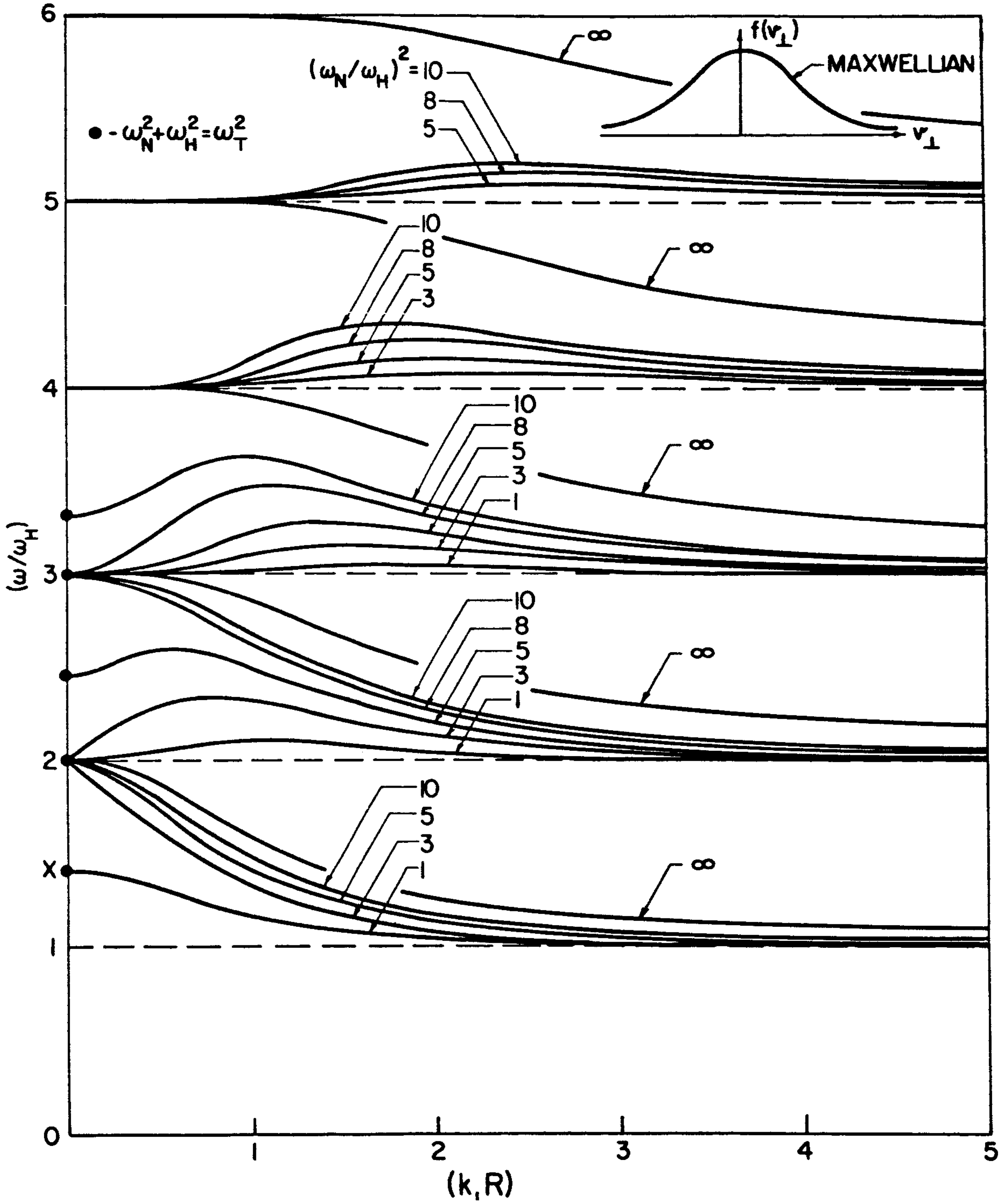


Fig. 1. Dispersion characteristics for perpendicular propagation of cyclotron harmonic waves. Maxwellian transverse velocity distribution.

this type in an inhomogeneous plasma, and experimental absorption resonances. In the ionosphere, the topside sounder, "Explorer 20" (=Ionosphere Explorer I) launched on August 25, 1964, has added new data on plasma resonances. Some preliminary results were discussed by Calvert, Knecht and Van Zandt [1964], and a more detailed description has been given recently by Calvert and Van Zandt [1966]. Observations made with six fixed-frequency transmitters carried by the vehicle, and pulsed in sequence, confirmed the previous rocket and satellite observations of the ionospheric plasma resonances, and allowed them to be studied for periods of up to several minutes.

The foregoing brief review serves to illustrate the two basic points motivating the work to be described in this paper. They are first that, although it is now generally accepted that ionospheric resonances at ω_T and ω_H are associated with warm plasma wave properties, no experimental laboratory demonstration of this has been reported, and second, that although there is now strong experimental confirmation of the propagation of such waves in the laboratory, their potentialities for application to ionospheric sounding do not seem to have been exploited, or even fully explored. The purpose of this paper is to present the results of theoretical and experimental studies of these points. Section 2 describes how group delay measurements of transmission between two antennas could be employed as a diagnostic technique, and presents appropriate computations. Section 3 reports experimental results on a laboratory plasma validating this theory and stimulating the ionospheric resonances. Section 4 deals with the practical realization and applicability of the method to ionospheric work, while Section 5 terminates the paper with a brief general discussion.

2. ELECTROSTATIC PLASMA WAVE THEORY

Dispersion Characteristics

The dispersion relation for electrostatic wave propagation perpendicular to a magnetic field is obtained by equating to zero the appropriate component, $\epsilon_\perp$, of the warm plasma relative permittivity tensor. This is given by Stix, [1962], as

$$\epsilon_\perp = 1 - \frac{\omega_N^2}{\omega_H^2} \sum_{n=1}^{\infty} \frac{\exp(-\lambda) I_n(\lambda)}{\left(\frac{\lambda}{2}\right)\left[\left(\frac{\omega}{n\omega_H}\right)^2 - 1\right]} = 0 \ , \qquad (1)$$

where $\lambda = (k_\perp R)^2$, $k_\perp$ is the perpendicular wave number, and $R \left[= (\kappa T_e/m)^{1/2}/\omega_H\right]$ is the gyroradius of a particle with thermal velocity, $v_t \left[= (\kappa T_e/m)^{1/2}\right]$. Here, κ is Boltzmann's constant and m is the electron mass. It has been assumed in the derivation of (1) that the transverse electron velocity distribution, $f(v_\perp)$, is Maxwellian with temperature T_e . Other distributions must be studied individually. If they are highly non-Maxwellian, the waves may become unstable [Crawford, 1965 b]. Although ion motions have been neglected, they may easily be taken into account by adding summation terms of similar form to that of (1) for each species.

A highly significant feature of (1) is that it does not contain any collisionless damping terms. Computations [Tataronis and Crawford, 1966] show that these can become very strong for propagation at angles only a few degrees off exact perpendicularity. The purely real solutions for perpendicular propagation are shown in Figure 1. The parameter range $1.0 \le \omega_N^2/\omega_H^2 \le 10.0$ has been chosen to span the range of Alouette I data reported by Calvert and Goe [1963]. It will be noted that as the upper hybrid frequency, ω_T (Point X) , moves up the frequency axis, the propagation windows in the frequency bands bounded below by the cyclotron harmonics for which $n\omega_H \ge \omega_T$ become wider. There is never any gap in the transmission for $\omega_H \le \omega \le \omega_T$.

Possible diagnostic techniques.

The theoretical expression (1) has potential application to diagnostic techniques in at least three distinctly different ways: First it may be used in the prediction of the admittance between two electrodes in the plasma. In this case, $\epsilon_\perp \neq o$ and in principle the analysis can only be carried out exactly for geometries in which the wave equation is separable. Even there, the practical effects of non-ideal electrode shapes, and the presence of space-charge sheaths, may prohibit accurate quantitative estimates. So far, only the admittance between two parallel wires has been studied. This has been done in the experiments of Crawford, Kino and Weiss [1964], Crawford and Weiss [1966], and Harp [1965,1966], with results that Harp has shown to agree well with qualitative predictions from (1). These and other aspects of the problem are currently under study by the authors.

The second way in which the expression (1) for $\epsilon_\perp$ may be used stems from the experimental observation of fine structure in the peaks obtained either in admittance measurements, or in noise emission from magnetoplasmas. These have been shown to be interpretable quantitatively in terms of propagating or standing cyclotron harmonic waves, for which $\epsilon_\perp = 0$, by Buchsbaum and Hasegawa [1964,1966] and by Harp [1965,1966]. In the latter's experiments, interference effects were observed as the separation between two antennas was varied. These were interpreted to give the wavelength, and hence the dispersion characteristics of the waves. Fitting such data to the universal curves of Figure 1 will yield ω_N , ω_H , and T_e .

The third use of (1) which we wish to discuss as the main subject of this paper will be termed the "group delay method". In this, a transmitting antenna will be pulsed and the resulting fields will be sampled by a receiving antenna some distance away. This will allow the group delay of a wave-packet travelling between the two antennas to be measured, and the plasma parameters to be deduced more simply than by either of the other two methods suggested. We shall now examine this technique in more detail:

The group delay method

For simplicity, consider first an infinite plasma system. A wavepacket transmitted from an infinite planar antenna will transverse the plasma at the group velocity, $v_g (=\partial\omega/\partial k_\perp)$, so that its time of arrival, t_g , at a second antenna will depend on v_g^{-1} . Figure 2 shows plots corresponding to the curves of Figure 1 in which the quantity $\alpha = (v_t/v_g) = (t_g/L)(\kappa T_e/m)^{1/2}$ expresses the ratio of the group delays for transit between antennas L apart of signals travelling at the group and thermal velocities. The frequency variation of α in each passband is reminiscent of "nose"-whistler transmission characteristics [Helliwell, 1965], and in what follows we shall adopt this description for the general shape of the curves.

It is interesting to note, that, for $\omega > \omega_T$, the humped nature of dispersion curves of Figure 1 gives rise to double-values for α , as shown in Figure 2. It is implied that under these conditions two signals with the same frequency may arrive at the second antenna with different delays. It will also be noted from Figure 2 that the ratio, α , always exceeds unity. Its absolute minimum value occurs for the parameters $(\omega/\omega_H) = 1.85$, $(\omega_N/\omega_H)^2 = 3.25$, $\alpha = 1.29$.

The group delay curves show clearly how $\alpha \rightarrow \infty$ at the gyrofrequency harmonics, and Alouette-type ringing occurs at the transmitting antenna. There is, however, just one special combination of parameters for which the group delay is not infinite at a gyrofrequency harmonic. This occurs at $(\omega_N/\omega_H)^2 = 3.0$, where the upper hybrid frequency equals the second harmonic. It will be seen that α passes smoothly through the point (4/3). The reason for this will be appreciated from Figure 1: Two branches of the dispersion curves meet at the axis with a non-zero slope as $k_\perp \rightarrow 0$. This implies that in a ringing experiment the ringing might be somewhat weaker at this point, since it is possible for some of the energy to propagate away. Reference to the experimental results of Calvert and Goe [1963, see their Figure 4] suggests that the ringing was indeed weak at this point.

One way in which the curves of Figure 2 may be understood is that they represent the time response at the receiving antenna to a delta-

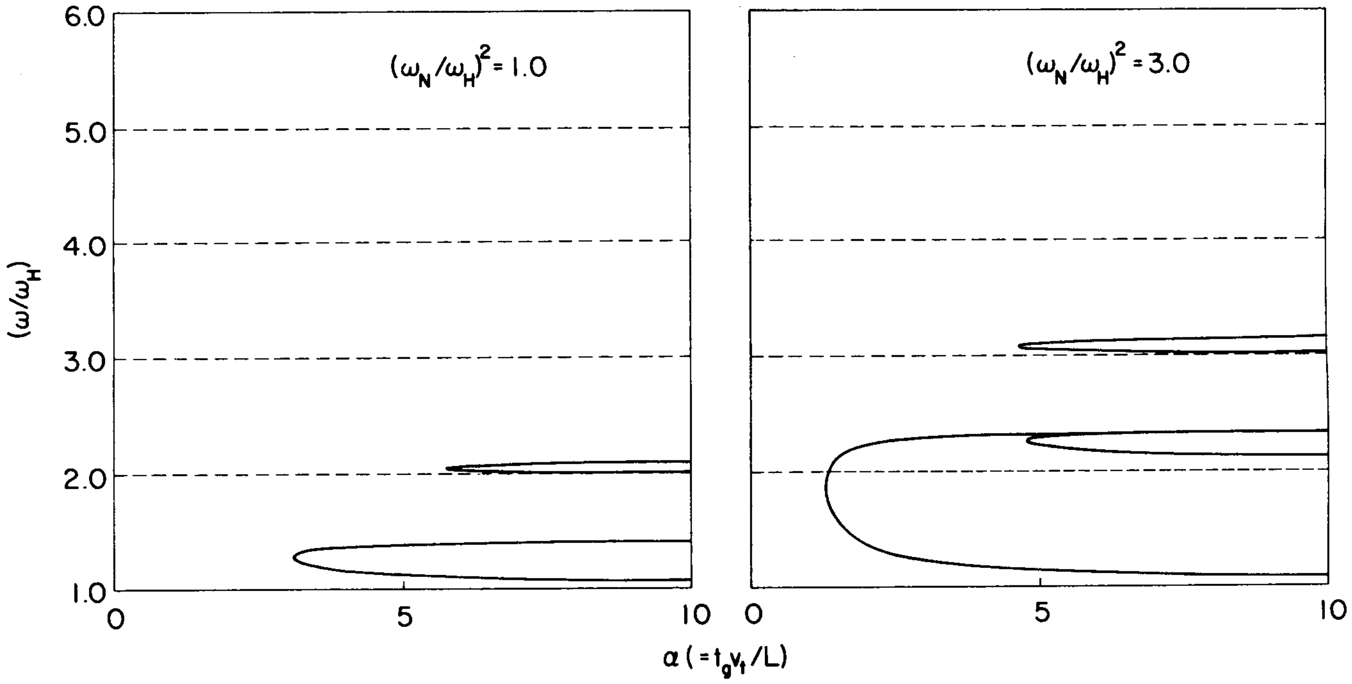


Fig. 2(a). Group delay, t_g, for a signal to propagate between antennas L apart and parallel to the magnetic field, normalized to the transit time (L/v_t) at the thermal velocity.

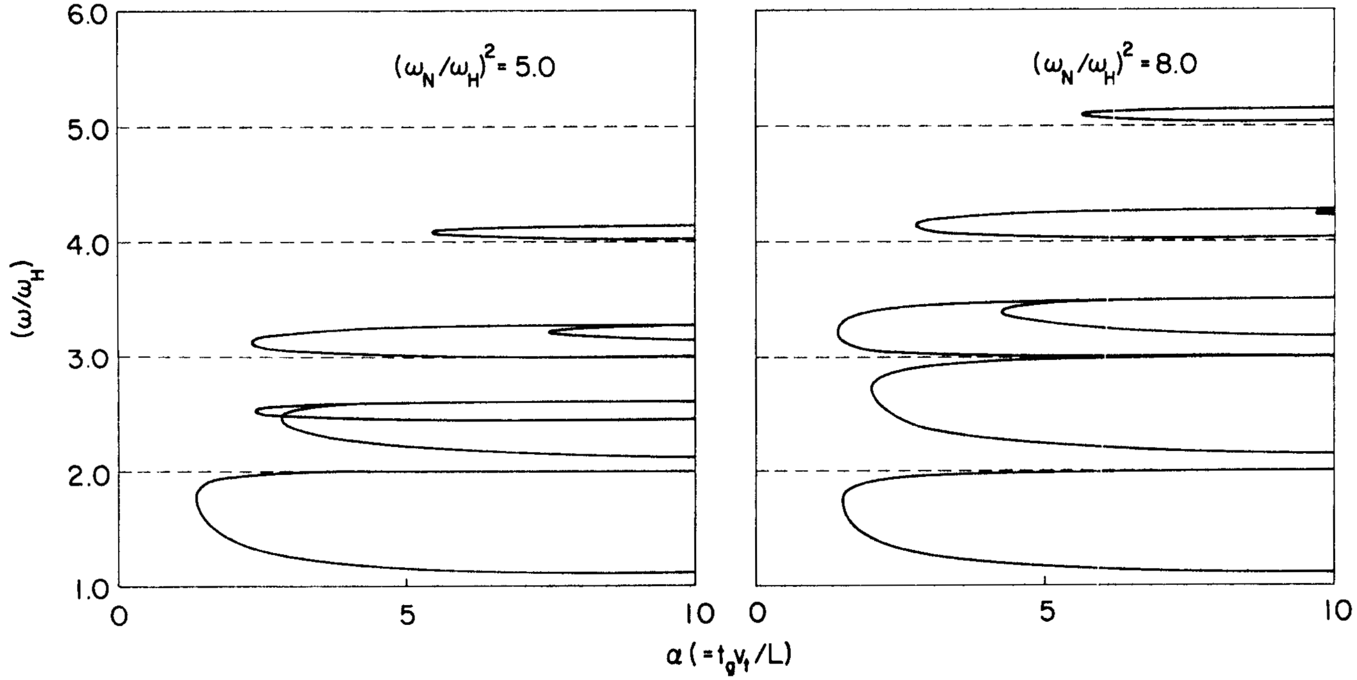


Fig. 2(b). Group delay, t_g, for a signal to propagate between antennas L apart and parallel to the magnetic field, normalized to the transit time (L/v_t) at the thermal velocity.

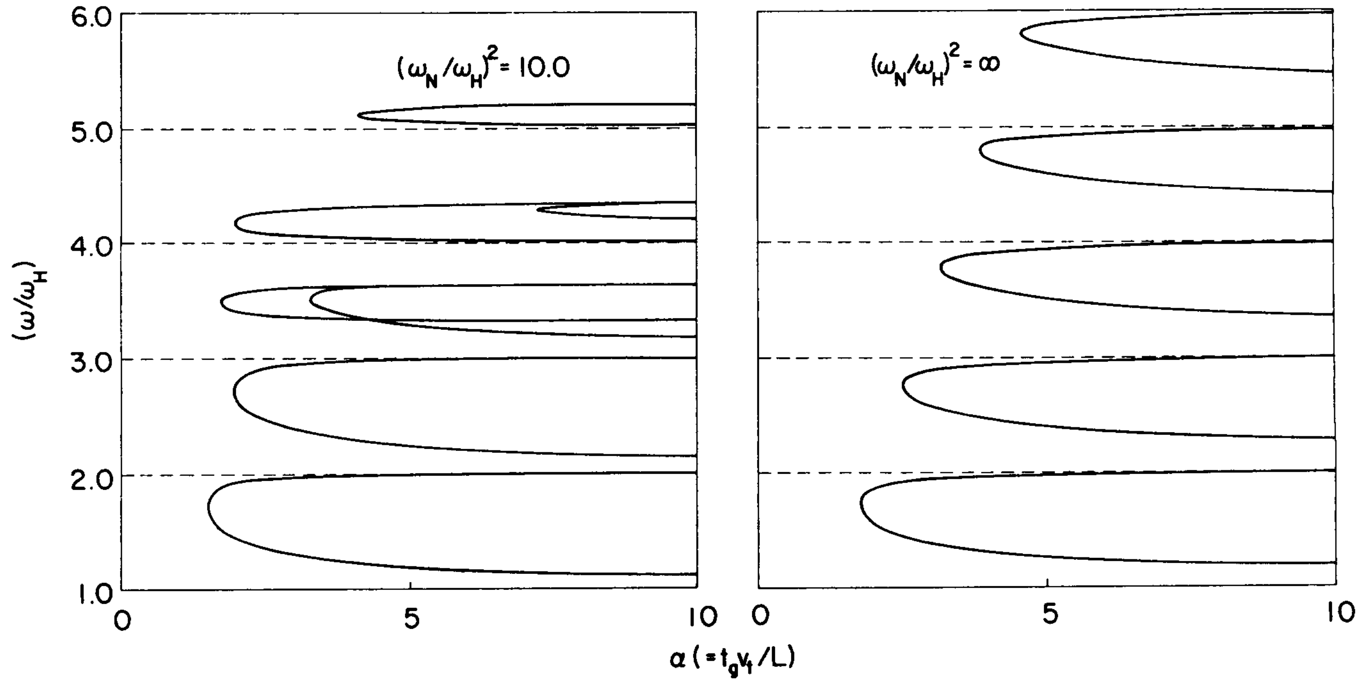


Fig. 2(c). Group delay, t_g, for a signal to propagate between antennas L apart and parallel to the magnetic field, normalized to the transit time (L/v_t) at the thermal velocity.

function pulse applied at the transmitting antenna. In practice, however, an rf pulse would probably be applied, similar to that employed by Alouette I. The extent to which this would allow the various noses of Figure 2 to be traced out would consequently depend on the bandwidth of the transmitted signal and the receiver. Assume, for example, that the rf transmitter pulse has an approximately Gaussian envelope, $f(t) = \exp(-t^2/t_p^2) \exp i\omega_o t$. The amplitudes of the Fourier components of this signal vary as $\exp[-(\omega-\omega_o)^2(t_p/2)^2]$, and the transmission will be substantially free from distortion provided that t_g varies little over the frequency range $\omega_o[1\pm(2/\omega_o t_p)]$. Outside this bandwidth, the Fourier component amplitudes are less than e^{-1} of the central amplitude corresponding to ω_o .

The regions of propagation in which t_g varies least are clearly those near its minima. This implies that the lower the passband the greater is the permissible bandwidth. To obtain some numerical values, we may consider the worst conditions of Figure 2, which occur for $\omega_N^2/\omega_H^2 = 1.0$. Measurements of $(\alpha)_{min}$ with pulse dispersion as little as 10 percent would be possible in the lowest passband for $\omega_o t_p \approx 45$. For $\omega_N^2/\omega_H^2 = 10.0$, measurements with a pulse of the same bandwidth would be possible up to the third passband, and a value of $\omega_o t_p \approx 25$ would be adequate in the lowest passband.

The foregoing discussion suggests the feasibility of utilizing the group delay method to measure ionospheric or laboratory plasma parameters, and in the following section experiments will be described designed to validate the proposed method. In practice, observations of minimum group delay should give the electron thermal velocity accurately, and hence the electron temperature, while observations of ringing, i.e. the approach to infinite group delay, should define the location of the upper hybrid frequency and the gyrofrequency harmonics. From the latter, an estimate of the gyrofrequency, ω_H , can be obtained. Substitution of this in the expression $(\omega_N^2 + \omega_H^2)^{1/2}$ for the upper hybrid frequency, will then yield the plasma frequency ω_N .

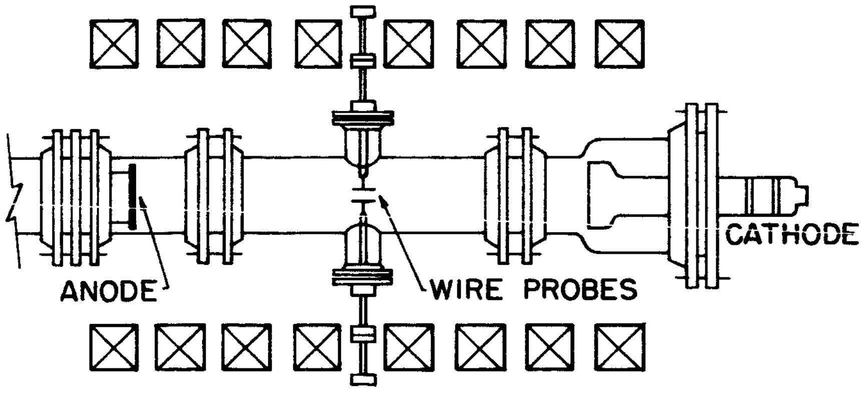
3. EXPERIMENTAL OBSERVATIONS OF GROUP DELAY

The experimental set-up

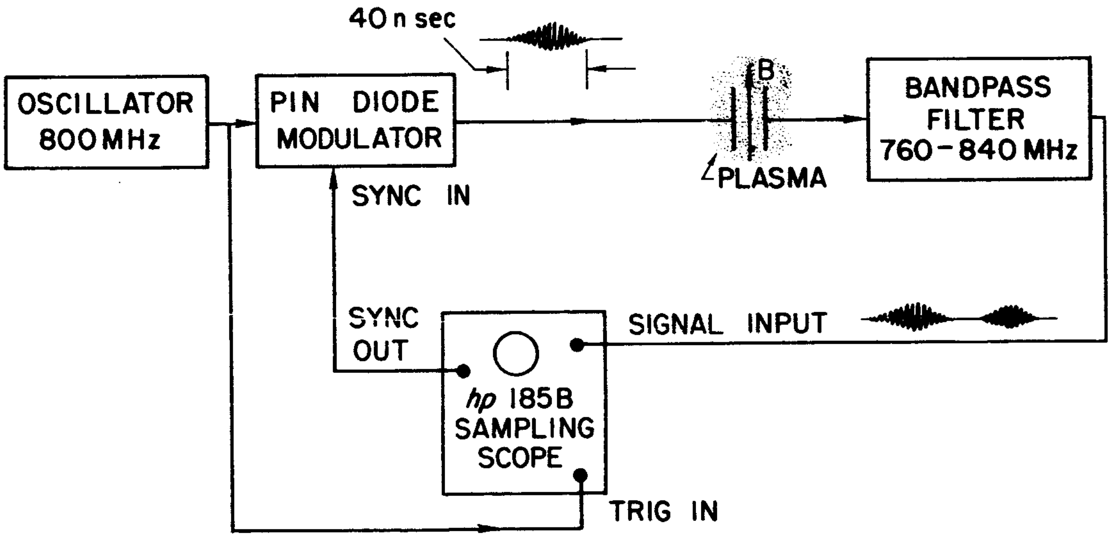
A laboratory plasma experiment has been carried out in the apparatus shown schematically in Figure 3. RF pulses were applied to a cylindrical wire antenna immersed in a low-pressure argon discharge and a transmitted signal was detected on a similar antenna aligned parallel to the first and to a variable dc magnetic field. A large oxide-coated cathode was used to provide a cylindrical column of substantially uniform plasma, and both antennas were movable within this to a precision of 0.01 mm over radial distances of several centimeters. The plasma frequency could be controlled by variation of the discharge current.

The rf pulses were formed by means of an 800 Mc CW oscillator and a commercial PIN-diode modulator modified to produce a 40 nsec pulse (see Figure 3b). The received signal was taken through a bandpass filter which improved the pulse shape by restricting the bandwidth. The final envelope approached closely the ideal Gaussian profile with $\omega_o t_p \approx 56$. It was very important to keep the transmission line length between modulator and input antenna, and between output antenna and oscilloscope, as short as possible ($\approx$ 90 cm total). Minimum reflection at the oscilloscope input was ensured by special choice of line lengths and connectors. A graphical recorder was used to make permanent records.

In our experiments, the working frequency, ω, is dictated by the plasma parameters and its physical dimensions. Considerations of the microwave circuit and its matching, and the nature of the detection system, demand that ω should be fixed while ω_N and ω_H are varied. In this respect the experiment is carried out analogously to making observations with a fixed-frequency satellite sounder. It is not possible to make accurate observations of Alouette-type ringing using only one antenna since, on the nanosecond time-scale of these experiments, unwanted line reflection and ringing effects due to the applied pulse persist for comparatively long times. On the millisecond time-scale of the plasma ringing occurring in the Alouette experiments, these were completely negligible, of course.



(a) DISCHARGE CHAMBER



(b) CIRCUIT SCHEMATIC

Fig. 3. Experimental set-up for laboratory measurements of group delay. (Antennas were tungsten wires 5.0 cm long x 0.45 mm diameter. The plasma column was 50 cm long and uniform over a diameter of about 6 cm, equal to that of the cathode).

Calibrations

To check the group delay measurements, it is necessary to have independent calibrations for the magnetic field, electron plasma frequency, and electron temperature. The first of these was established very precisely by means of a proton resonance gaussmeter, while the electron temperature was obtained to an accuracy of about ±10 percent by Langmuir probe methods, and was shown to vary very little at a given pressure over wide ranges of discharge current and magnetic field. The most difficult parameter to obtain accurately is the electron plasma frequency, since probe measurements of electron density tend to be unreliable in the presence of a magnetic field. The determination was accomplished by the following method: The transfer admittance between the two antennas was measured qualitatively by applying a fixed signal to the transmitting antenna and observing the signal at the receiving antenna as magnetic field was varied at fixed discharge current. A typical record obtained like this is shown in Figure 4. The point 'X' may be identified as the upper hybrid frequency [Harp, 1966] while the various peaks located near the cyclotron harmonics may be interpreted by the admittance theory discussed in Section 2. From a series of such records, the curve shown in Figure 5 has been constructed, showing how the upper hybrid frequency varies with discharge current.

Reference to Figure 5 shows how it may be used to determine the plasma frequency at a given current and magnetic field: The quantity $(\omega_H/\omega)^2$ is always known accurately. Now, on the curve, $1 = (\omega_H/\omega)^2 + (\omega_N/\omega)^2$. This yields immediately the value of $(\omega_N/\omega)^2$ corresponding to the current I_T . Since $I \propto \omega_N^2$ in the positive column, we have

$$\left(\frac{\omega_N}{\omega_H}\right)^2_I = \left(\frac{I}{I_T}\right)\left(\frac{\omega^2}{\omega_H^2} - 1\right) , \tag{2}$$

and,

$$\left(\frac{\omega_T}{\omega_H}\right)_I = \left[1 + \left(\frac{\omega_N}{\omega_H}\right)^2_I\right]^{1/2} . \tag{3}$$

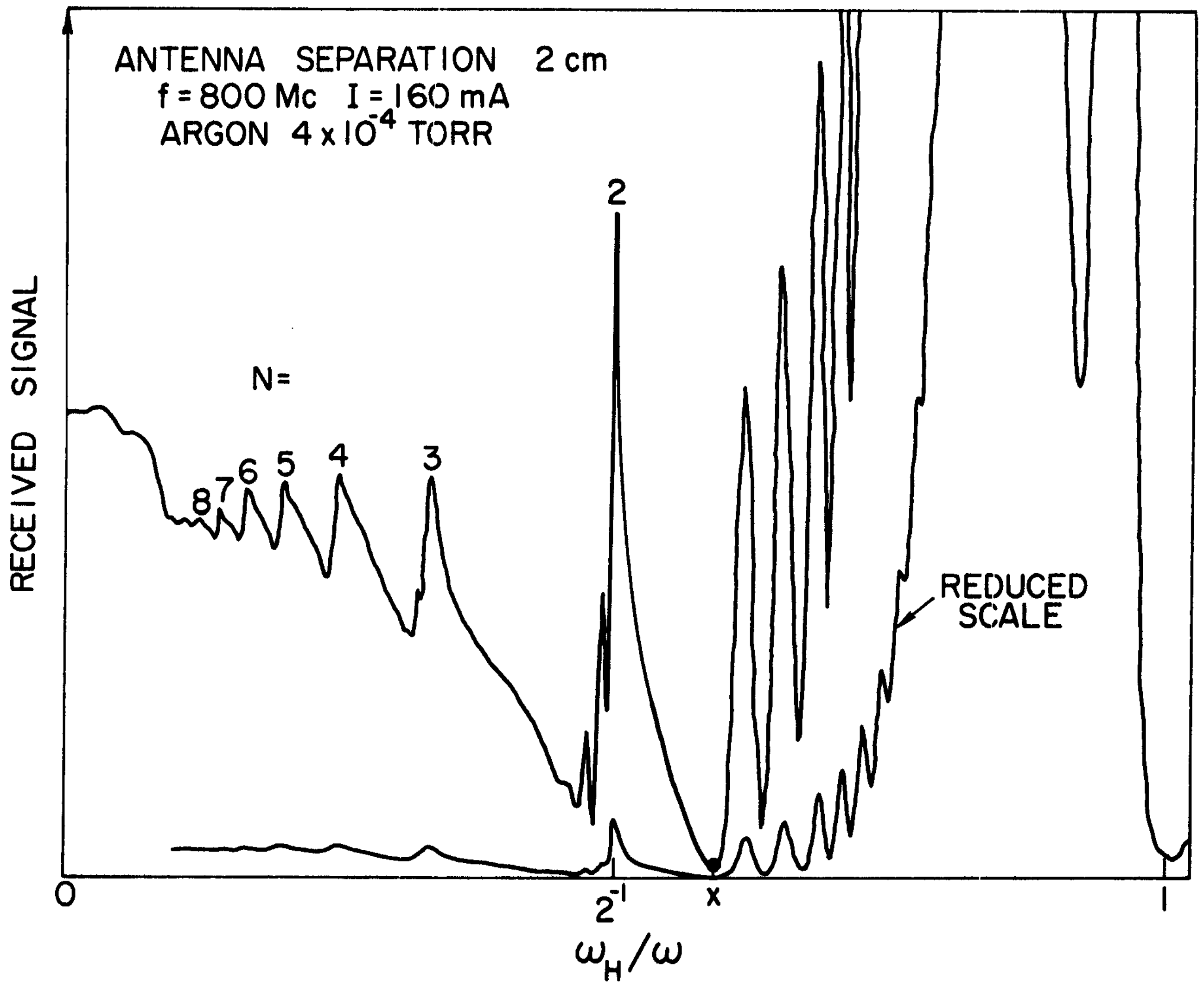


Fig. 4. Rf transmission record with varying magnetic field.

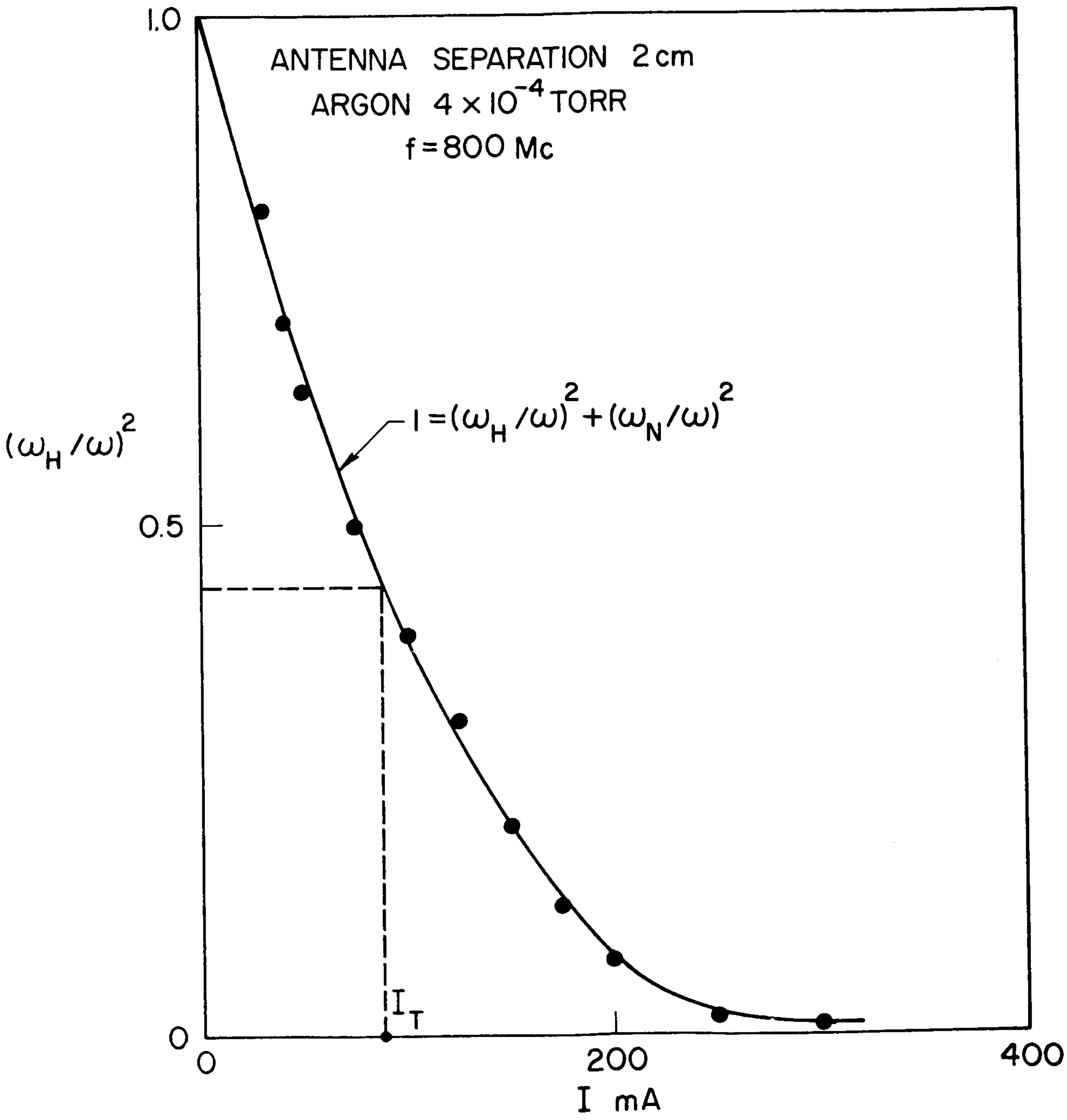


Fig. 5. Variation of upper hybrid frequency with discharge current.

This calibration procedure has been used throughout. The results suggest that it generally defines $(\omega_N/\omega_H)^2_I$ and $(\omega_T/\omega_H)_I$ to within about 5 percent error.

Measurements with variable antenna spacing

Figure 6 shows results obtained for pulse transmission with fixed ω, ω_N and ω_H, and variable antenna separation. The interpretation of the records is as follows. In each case, a direct pulse travelling at the velocity of light is picked up on the receiving antenna. This serves to indicate the undistorted pulse shape and to define the time scale. The direct pulse is following after an appropriate group delay by the wave-packet travelling in the electrostatic wave mode. At large probe separations, the two pulses are well separated, and group delay measurements may be made from the records. When the probes are close together, the direct and delayed pulses interfere with each other. Measurements at short spacings would be possible if the direct signal were discriminated against. Although this procedure was not adopted in our laboratory experiments, at the comparatively low frequencies applicable to ionospheric work, it should be feasible to null out the direct signal, either partially or completely, by applying an antiphase signal to the receiving antenna.

The group delay data of Figure 6 indicate that, to within the limits of experimental error, a straight line can be drawn through the points marking the centers of the wave-packets, and that it extrapolates back to zero delay at zero antenna separation. This would be expected for infinite planar geometry, but is not strictly correct for a wave with cylindrical symmetry, such as is excited in these experiments. The agreement is good simply because the antenna spacing is typically of the order of tens of wavelengths. At a few wavelengths from the exciting antenna, the asymptotic solutions for the wave propagation apply. These vary as $[\exp i(\omega t-k_\perp r)/r^{1/2}]$ so that the group velocity is similar to that of a plane wave. When experimental conditions are such that the probe separation is only a few wavelengths, the linear relation breaks down.

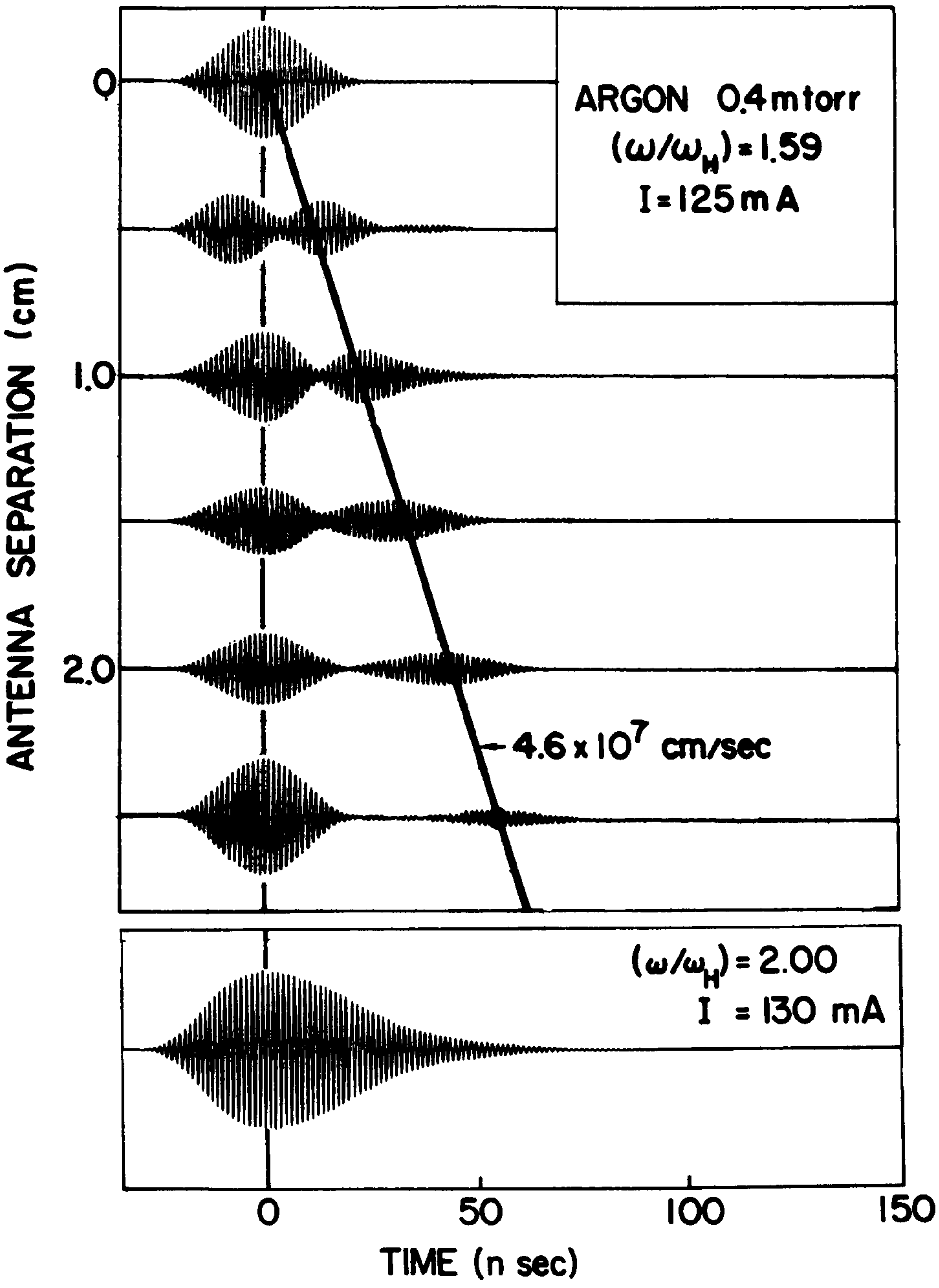


Fig. 6. Typical group delay and ringing records.

Measurements near group delay minima

Since the bandwidth of our experimental pulse was about ± 4 percent, it was not feasible to follow accurately the detailed shapes of noses for which the group delay varied rapidly for frequency changes of this order. The first passband offers entirely suitable conditions, however, and Figure 7 presents typical experimental data for a series of values of $(\omega_N/\omega_H)^2$. To obtain these, ω and the antenna separation were maintained constant while group delay records were taken as (ω/ω_H) was varied. This involved making use at each point of the calibration procedure outlined earlier.

In higher passbands, the noses are generally sharper, and it was only possible to make satisfactory measurements in the second and third for $(\omega_N/\omega_H)^2 \gtrsim 6$. These agreed with theory to about the same accuracy as the data presented in Figure 7. The implication of the results is that, if $(\omega_N/\omega_H)^2$ is known from ringing measurements, then T_e can be obtained with less than 10 percent error by use of the nose data. In an ionospheric experiment, where narrower bandwidth could be employed, this precision could probably be substantially improved.

Observations of ringing

It was found experimentally that as the magnetic field and discharge parameters were varied so as to approach closely the conditions $\omega = \omega_T$, or $\omega = n\omega_H$, the direct pulse component began to lengthen. The record shown in Figure 6 illustrates the prolonged ringing typically observed at the upper bybrid frequency, and the gyrofrequency harmonics. The interpretation of this phenomenon is that, as the group velocity for electrostatic waves tends to zero near these frequencies, they cannot carry away rapidly the energy pulsed into the plasma. Consequently, it stays in the vicinity of the antenna and manifests its presence in the ringing. The received signal is observed with effectively zero delay, through direct capacitive coupling, independent of the antenna separation.

It is a very interesting point that these ringing observations, simulating the "Alouette I" results, are obtained under purely

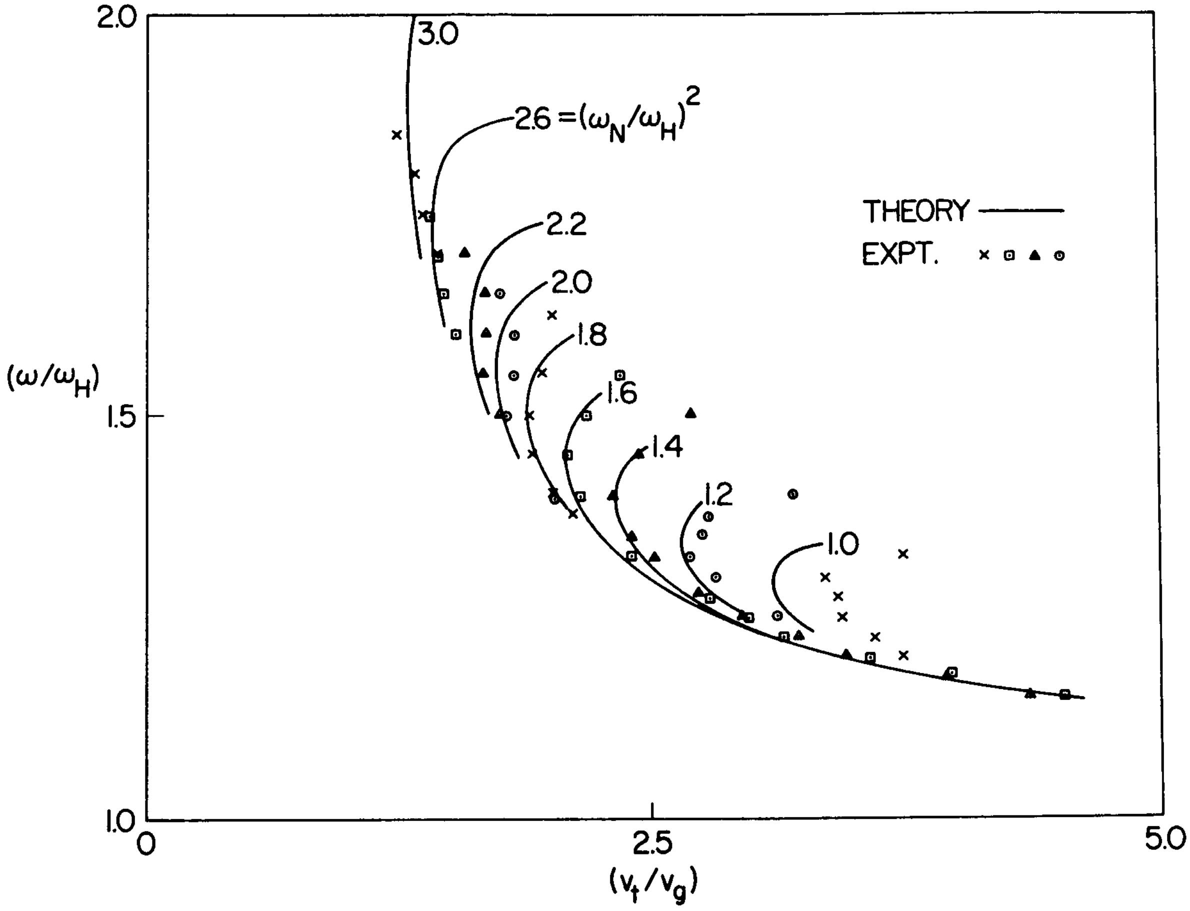


Fig. 7. Comparison of experimental and theoretical group delays in the vicinity of the group delay minima (Antenna separation 2.5 cm; argon 4×10^{-4} Torr; f = 800 Mc).

quasistatic conditons. This follows since the free space wavelength at our working frequency is about six column diameters. On the other hand, experiments in which a pulsed, effectively plane, electromagnetic wave has been applied to a plasma small compared with a free space wavelength have shown no ringing [S. J. Tetenbaum, private communication]. The implication is that, in practice, the electromagnetic wave corrections to the ringing theory proposed by Shkarofsky and Johnston [1965] may not be necessary.

4. APPLICABILITY OF THE GROUP DELAY METHOD TO THE IONOSPHERE

The experiments described in Section 3 indicate that very good agreement can be obtained between theory and experiment under controlled laboratory conditions. In this section, we shall consider the various factors relevant to making swept-frequency measurements by means of ionospheric sounding satellites.

Pulse shape

To minimize the bandwidth of the wave packet, it will be desirable to approach a Gaussian envelope for the rf pulses. A relatively high value of $\omega_o t_p$ will be required to trace out nose shapes accurately in the various passbands as the satellite moves through regions of varying electron density and magnetic field. For example, if $T_p \approx 3 t_p$ is regarded as an adequate transmitter pulse length to achieve this, then for the frequency range 0.5 - 3.5 Mc of most interest in the Alouette I experiments [Calvert and Goe, 1963], and a value of $\omega_o t_p \approx 120$, we have $T_p = 125$ µsec at the lowest frequency and $T_p \approx 17$ µsec at the highest, compared with the fixed pulsed length of 100 µsec of the Alouette I transmitter.

Antenna separation

Unless the direct signal received at the second antenna is to be nulled out, the separation, L, between the two antennas must be such as to give a minimum group delay of T_p. This will prevent the direct and delayed pulses from interfering with each other. It follows that,

$$L \approx \frac{T_p v_t}{\alpha_{min}} , \tag{4}$$

where $(\omega/\omega_H)_{min}$ and α_{min} correspond to the point of minimum group velocity under study. We see from Figure 2 that for $T_e = 2000°$ K ($v_t = 1.74 \times 10^5$ m/s) and $f_0 = 0.5$ Mc we have $L \approx 20$ m at $(\omega_N^2/\omega_H^2) = 1.0$, and $L \approx 40$ m at $(\omega_N^2/\omega_H^2) = 10.0$. These are quite practical values. If the fixed value $L \approx 40$ m were taken, with $\omega_o t_p = 120$, accurate measurement could easily be made over the entire range of parameters encountered by Alouette I. The only place where difficulties might be encountered is in the ranges where t_g is double-valued (t_{g1}, t_{g2}) for a given frequency. Observation of the arrival of two distinct pulses could be expected only if $|t_{g1} - t_{g2}| < T_p$.

If provision were made for reducing the amplitude of the direct signal component at the receiver to well below that of the delayed signal, the antenna separation could probably be reduced by a factor of ten. If a second transmitter located at the receiving antenna were used to relay the signal back to the sending antenna, the group delay could be effectively doubled for a given separation. The reflection could also be obtained, with or without amplification, from another vehicle, for example in mother-daughter satellite experiments.

Effects of satellite velocity

We should consider two possible effects occurring due to the motion of the satellite. The first is due to the component of antenna velocity, $v_{\parallel}$, parallel to the magnetic field lines. For a signal to be received on the second antenna, the antenna length, ℓ , must satisfy the inequality $(\ell/L) > (v_{\parallel}/v_g) = \alpha(v_{\parallel}/v_t)$. If we take as a convenient value for $v_{\parallel}$ a satellite velocity of 7.5 km/sec, and assume $T_e = 2000^\circ K$, then $(\ell/L) > (\alpha/23)$ is required. This is easily satisfied and, in practice, the ratio would probably be $(\ell/L) \approx 1$.

Motion perpendicular to the magnetic field constitutes a second possible source of error. The measurement of group delay will be subject to an error factor of $[1 \pm (v_{\perp}/v_g)]$. If the plasma temperature and satellite velocity used above are again taken as suitable limits, this factor becomes $[1 \pm (\alpha/23)]$. Figure 2 shows that values of α are high enough for this to be appreciable. The error could be compensated for from knowledge of the satellite velocity and its orientation with respect to the magnetic field, or additional measurements could be made with the roles of the transmitting and receiving antennas reversed. This would allow determination of $v_{\perp}$ and elimination of the error. In experiments where the wave-packet is reflected back to the transmitting antenna, the effect will be automatically cancelled out.

Effects of collisions

So far we have ignored completely the possible effects of collisions. It may be shown that the effect of elastic electron/neutral encounters can be introduced into the dispersion relation (1) by replacing ω_N^2 by $\omega_N^2 \left(1 - (i\nu/\omega)\right)$, and ω by $(\omega - i\nu)$, where ν is the electron/neutral collision frequency [Crawford and Tataronis, 1966]. To a good approximation the attenuation per unit distance is given by $k_{i\perp} = \nu/v_g = \alpha(\nu/v_t)$ [Crawford, 1965]. In the laboratory experiments described in Section 3 we had $\nu \approx 10^6$/sec, $v_T \approx 10^8$ cm/sec, and α of the order of unity, so that the attenuation due to collisions should have been only a few db up to distances of many centimeters. Under conditions similar to those of the Alouette ionospheric experiments, the ratio (ν/v_t) will be several orders of magnitude lower than in our laboratory experiments. This will effectively compensate for the two or three orders of magnitude greater antenna separation required for ionospheric group delay measurements.

It may be remarked that collisional effects should have least effect on measurements near the noses, where the group velocity is maximum. They would be most likely to introduce effects into the ringing. Since, however, prolonged ringing has already been observed to occur very close to the cyclotron harmonics in the Alouette I experiments, it is implied that the practical effects of this correction may be negligible.

5. DISCUSSION

The main purpose of this work was to validate the interpretation of ionospheric ringing measurements as a plasma excitation effect in regions of near-zero group velocity. As will be appreciated from Section 3, our experimental work goes far beyond this, serving to confirm the detailed theory of perpendicularly-propagating, warm magnetoplasma waves, and providing a firm foundation for a powerful new diagnostic technique based on group delay measurement. Although simultaneous measurements of signals at both the sending and receiving antennas might be useful, an extremely important intrinsic advantage of the group delay method is that the majority of the plasma volume under study is remote from the vehicle; its surrounding sheaths and plasma inhomogeneities, and possible stray magnetic fields. For this reason, measurements of a quantity such as ω_N are more likely to be reliable than those obtained by alternative "local" techniques such as ringing, Langmuir probes, or resonance probes. In fact, measurements at several different antenna spacings could probably eliminate the influence of local effects almost completely.

In the ionosphere, a number of independent checks on the quantities of interest, ω_N , ω_H and T_e are possible without much additional complication. For example, the observed magnetic field can be checked against spherical harmonic expansions, and against deductions from electromagnetic resonances when they appear in the ionograms [Calvert and Goe, 1963]. It may also be compared with the value obtained from ringing observations made simultaneously at the transmitting antenna, as in the Alouette I experiments. The plasma frequency can also be checked by ringing observations made when the antennas are aligned perpendicular to the magnetic field. Electron temperature measurements could be made reliably by using the antennas as Langmuir probes.

Since the ringing at ω_N just referred to has been observed strongly in satellite experiments, it is interesting to speculate on whether it might be possible to measure group delay between two antennas in this mode using the set-up for the perpendicular propagation measurements described in Section 4. As derived by Bohm and Gross [1949], the dispersion relation may be written,

$$\omega^2 = \omega_N^2 + 3v_t^2 k_{\parallel}^2 \quad , \qquad (5)$$

where Landau damping has been neglected. Effectively this confines the validity of (5) to $v_t k_{\parallel} < 0.3\,\omega_N$ [Fejer and Calvert, 1964]. Using (3) this indicates that group delay should be observable over the range, $0 < (v_g/v_t) < 1$. This corresponds to values of α similar to those for perpendicular propagation, so that measurements should be feasible with the same set-up. More detailed theory [Derfler and Simonen, 1966] indicates that the range to be expected experimentally is rather wider than this.

It may be remarked that a group delay experiment confirming (5) would be an important contribution to basic plasma physics. A great many efforts to verify this dispersion relation in the laboratory have failed due to such factors as plasma inhomogeneity, and the influence of electron/neutral collisions, and it is only very recently that strong support has been obtained for it in conditions approaching a homogeneous collisionless plasma [Derfler and Simonen, 1966; van Hoven, 1966].

Throughout this paper, we have discussed electron resonance effects. It might be possible, under some circumstances, to make similar measurements of group delay and ringing related to the lower hybrid frequency and ion gyrofrequency harmonics. The physical lengths involved would be much greater, owing to the relatively large ion gyroradius, and the phenomena might be studied best initially in mother-daughter satellite experiments.

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