Department of AERONAUTICS and ASTRONAUTICS
STANFORD UNIVERSITY

Tenth Semiannual Status Report

June 1966

on

BASIC STUDIES IN SPACE VEHICLE ATTITUDE CONTROL

in the

Department of Aeronautics and Astronautics
Stanford University

under

Research Grant NsG-133-61

from the

National Aeronautics and Space Administration

This report summarizes progress during the past six months under a continuing research grant for the period beginning December 1965. The initial grant is based on Ref. 1, and its continuation on Ref. 2. The research is supervised by Prof. I. Flügge-Lotz and Prof. R. H. Cannon, Jr., Principal Investigators.

A separate financial accounting will be forwarded by the University.

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SUMMARY

During the present report period, Ph.D. degrees were received by four members of this program, W. G. Eppler, R. Busch, J. L. Almuzara, and K. Hales. The advanced degree of Aeronautical Engineer was received by R. D. Hensley.

Mr. Eppler's thesis [Ref. 4] was reported at length in the preceding status report. It was the only research supervised by Professor Cannon, who is on sabbatical leave this year.

The research on nonlinear and optimal control of planet-pointing space vehicles, begun by Messrs. Busch, Almuzara, and Hales, continues with new Ph.D. candidates under Professor Flügge-Lotz (Sec. A). Basic studies of optimal control, under Professor Franklin, and satellite trajectory studies, under Professor Breakwell, also continue (Secs. A and B).

A. NONLINEAR STUDIES, OPTIMAL CONTROL

(Studies Supervised by Professor Flügge-Lotz)

1. Optimum and Suboptimal Control of the Pitch Motion of a Satellite in Elliptic Orbit; Preliminary Consideration of the Linearized Roll-Yaw Motion (Ph.D. Research of R. Busch)

The report on the work done by Ronald Busch has been finished [Ref. 5]. Due to an overload in the publication office, where the report copies are made, the report has not yet been copied, but it is expected that it can be mailed in about three weeks.

For a satellite in an elliptic orbit in operation for a year or more, the suggested steady-state control of the pitch motion causes a very slowly growing deviation of the roll-yaw attitude from its desired state because of the basic nonlinearity of the system of differential equations describing the attitude of the satellite. In addition, one may consider this also as a consequence of the fact that the desired state is not an equilibrium state. Since the missions of many satellites require very accurate attitude keeping, this problem will be studied further. Mr. W. Boykin, a new Ph.D. candidate, is particularly interested in this problem. Such suggestions as keeping a very accurate attitude only on part of each orbit and relaxing in between, will also be considered.

2. The Complete Attitude Control Problem for an Earth Satellite in Elliptic Orbit (Ph.D. Research of K. Hales)

The report on this problem is finished [Ref. 6]. Copies will soon be ready (see remarks under Sec. 1). The iteration procedure used by K. Hales does not yet seem to be fast enough. We will study the possibility of improving it. Since this procedure requires digital computing equipment, which at this time would be too heavy and voluminous to be carried on board, the question arises whether one can hope to miniaturize the equipment or whether one should consider correcting attitude error by occasional signals from an earth station.

In that case, the fact of delay in the signal transmission has to be considered. A new student will continue this work in the very near future.

3. The Validity of Linearization in Attitude Control (Ph.D. Research of J. L. Almuzara)

A report on this work was finished in the last few weeks [Ref. 7]. Particularly interesting are the comparisons of the systems

$$\left.\begin{array}{l}\ddot{x} + \sin x = u(t) \\ \ddot{x} + x \qquad = u(t)\end{array}\right\} \text{with } |u| \leq A \geq 1$$

and

Diagrams showing the differences in minimum settling time for zeroing a disturbance are very instructive.

Mr. Frank Curtis' studies of the same problem for minimum-fuel criterion continue. His studies benefit from the basic studies of Mr. Almuzara, but may later become more interesting for practical designs.

(Studies Supervised by Professor Franklin)

4. Computation of Optimal Controls by a Method Based on Second Variations (Ph.D. Research of T. E. Bullock)

This work is concerned with developing efficient numerical techniques for generating optimal controls with a special emphasis on second-order direct methods. A continuation of the work reported in the last status report has led to answers to the theoretical problems posed. The theoretical work has been justified from a pragmatic point of view in part by the successful application of the method to several numerical examples, with the aid of a digital computer, the 7090.

For the purpose of this status report, the numerical results of one example will be given. The problem chosen for this study is found in Merriam [Ref. 8] on optimization techniques. In using this problem to illustrate a second-order method described in his book, Merriam states that for a particular control initialization the application of "...the method based on second variations results in complete failure." The difficulty encountered here is a conjugate point in the accessory problem. Application of the theory developed in this research has circumvented these difficulties, as shown in the numerical results presented here.

The problem given by Merriam may be described briefly as follows. A driven nonlinear oscillator of the type studied by van der Pol may be described in state-space notation by

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1 + (1 - x_1^2)\, x_2 + u$$

For the initial conditions inside the stable limit cycle $x_1(0) = 1$, $x_2(0) = 0$, $u(t)$ is to be chosen to minimize the cost

$$J = \frac{1}{2} \int_0^5 (x_1^2 + x_2^2 + u^2)\, d\sigma$$

The final states are unspecified or free in this version of the problem.

The neighboring extremal control law, as described by Breakwell, Speyer, and Bryson [Ref. 10] and Kelley [Ref. 11], is optimal for the given problem and is the best to second order for changes in the state or terminal constraints. This control law is given in the feedback form

$$u(t) = c_1(t)\, x_1(t) + c_2(t)\, x_2(t) + c_3(t)$$

The neighboring extremal control law is an automatic by-product of the computing method based on second variations. The time-varying quantities $c_1(t)$, $c_2(t)$, and $c_3(t)$, which describe the control law for the given example, are shown in Fig. 1. The corresponding optimal trajectories are shown in Fig. 2.

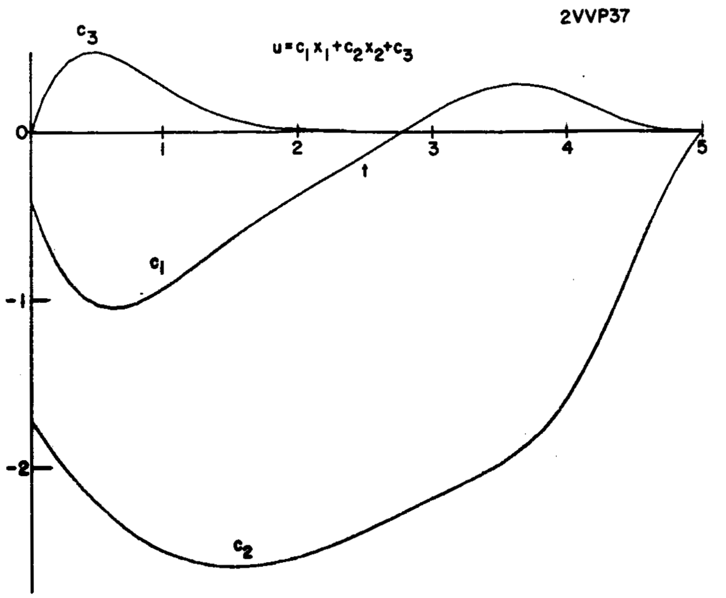

FIG. 1. THE NEIGHBORING EXTREMAL CONTROL LAW.

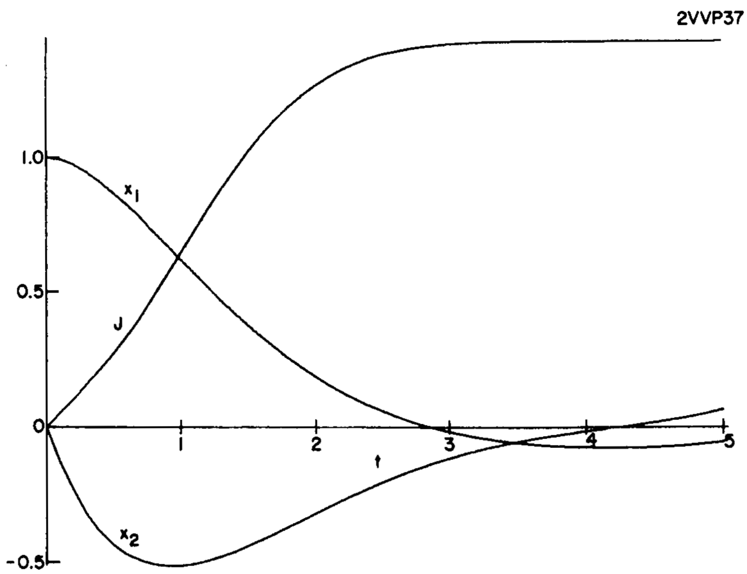

FIG. 2. THE OPTIMAL TRAJECTORIES FOR THE VAN DER POL PROBLEM WITH QUADRATIC LOSS.

The relative convergence of the method based on second variations, computed with the program entitled 2VVP, and the method of steepest descent [Ref. 11], computed with the program SDVP, is shown by comparing Fig. 3 with Fig. 4. Figure 3 shows several successive iterations on the control by SDVP and the corresponding optimal control. The corresponding results for 2VVP are shown in Fig. 4. Since the change in the shape of the control function is quite large from one iteration to the next, some additional information is helpful to distinguish the various iterations. At the bottom of Fig. 4, a sequence of small numbered plots shows the general trend of each iteration. These small figures may be used to help trace out each corresponding curve in the large plot which shows all of the iterations superimposed.

The method based on second variations has a clear advantage in this example. This is illustrated graphically in Figs. 3 and 4 and in the following data. For $u^{(0)} = 0$, SDVP obtained a cost which agreed with the optimal in only two significant figures after 18 iterations. In contrast, 2VVP converged to 5 significant figures in the cost in only 5 iterations. The relative convergence rates may be further compared with reference to Fig. 5. This plot shows the equivalent number of significant figures in J which do not agree with the optimal cost, J^*, for each of the methods.

A numerical example of the type discussed in the last status report [Ref. 3], which has the dimension of the constraint less than the dimension of the state but not equal to zero, is being worked on the computer. The successful running of this program and the subsequent completion of the final report will terminate this project.

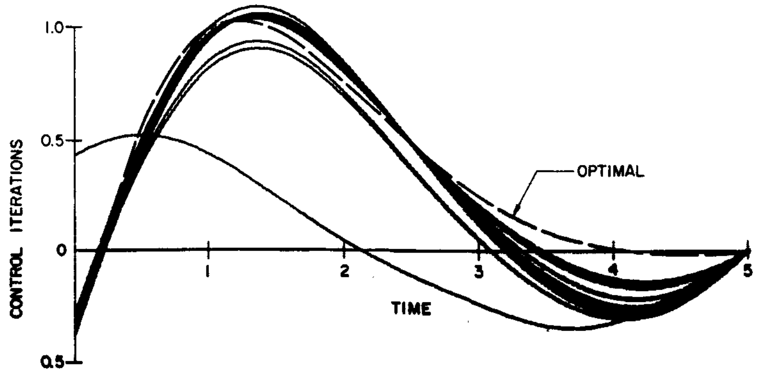

FIG. 3. SEVERAL SUCCESSIVE ITERATIONS ON THE CONTROL BY STEEPEST DESCENT (SDVP) FOR $u^{(0)} = 0$.

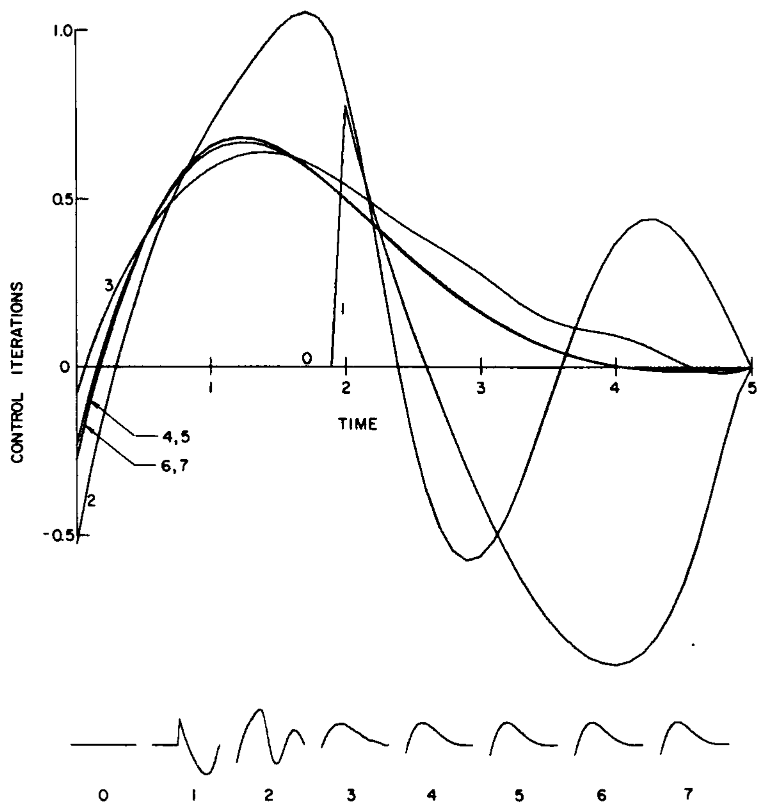

FIG. 4. SEVERAL SUCCESSIVE ITERATIONS ON THE CONTROL BY THE METHOD BASED ON SECOND VARIATIONS (2VVP) FOR $u^{(0)} = 0$.

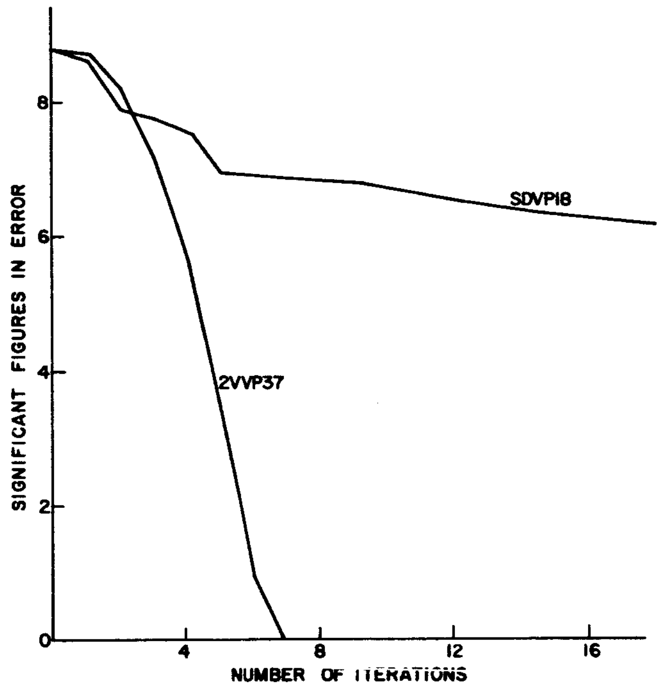

FIG. 5. A COMPARISON OF THE CONVERGENCE RATES OF STEEPEST DESCENT (SDVP) AND SECOND VARIATIONS (2VVP).

B. SATELLITE ORBIT STUDIES
(Studies Supervised by Professor Breakwell)

1. Rigorous Error Bounds on Position and Velocity in Satellite Orbit Theories (Ph.D. Research of J. Vagners)

By utilizing results of Hamiltonian theory and the von Zeipel method for treating artificial satellite orbits, error bounds have been derived for a general class of orbits with eccentricity less than one. In order to extend the error bounds for the general axisymmetric problem to time intervals of the order $1/J_2$, where J_2 is the oblateness parameter, the known integral of energy has been utilized to calibrate the governing differential equation for the rapidly rotating phase. The nonsingular rapid phase in the analysis was taken to be the sum of the mean anomaly, argument of periapsis and the right ascension of the ascending node. A corresponding analysis for the general asymmetric problem (including the fesseral harmonics) has also been given. From the general error analysis, an algorithm was derived for the computation of the correct initial conditions consistent with the expected accuracy of the theory. Numerical results verifying the conclusions of the theory were also presented.

This work is reported in full in Ref. 12, and will be presented at the Space Flight Mechanics Specialist Conference in Denver, Colorado, July 6, 1966.

2. Libration-Point Satellites (Ph.D. Research of R. W. Farquhar)

A comprehensive study of the control and use of libration-point satellites has been initiated. The completed work will eventually appear as a SUDAAR report. However, significant results of this study will be published in brief papers as soon as possible.

The first paper has been finished [Ref. 13] and will be presented at the AAS Space Flight Mechanics Specialist Conference at Denver, Colorado, July 8, 1966. Six copies of this paper have already been sent to NASA Electronics Research Center. This paper is entitled

"Station-Keeping in the Vicinity of Collinear Libration Points with an Application to a Lunar Communications Problem." The content is summarized in the following abstract.

A feedback control system is used to position a satellite in the vicinity of an unstable collinear libration point. It is found that stability can be obtained with a radial-axis control using only range and range-rate measurements. Since the satellite is controlled about an equilibrium path, which is determined by the perturbing accelerations, the station-keeping cost is chiefly a function of the measurement noise. If earth-based measurements are employed, this cost is very small.

A novel method for a direct communications link with the far side of the moon is offered as a possible application for the earth-moon exterior point. Although the solar perturbation on a satellite at this point is nearly resonant, the amplitude of the forced oscillation in the moon's orbital plane and perpendicular to the earth-moon axis is only 4,965 km. By controlling the out-of-plane motion of the communications satellite, it is possible, at moderate cost, to take advantage of the aforementioned forced oscillation and obtain a trajectory where the satellite is never hidden from the earth.

The second phase of this study is concerned with sun-planet libration points and should be finished in time to be included in the next status report.

C. OTHER ACTIVITIES

The four professors participating in this grant will all attend the Third Congress of the International Federation of Automatic Control (IFAC) in London this month. Five Stanford papers will be given. Two of them are based on work sponsored in part by this grant [Refs. 14 and 15]. Professor Flügge-Lotz will be chairman of the technical session on Deterministic Optimal Control (3), and Professor Franklin will be Rapporteur of the technical session on Deterministic Optimal Control (1). A special report on the conference will be forwarded.

Professor Cannon continued his participation with the NASA Research Advisory Committee on Control, Guidance, and Navigation, serving as acting chairman of its meeting at ERC in May. He also continues as chairman of the AIAA Technical Committee on Guidance and Control, which will sponsor both its own Specialist Conference and the Joint Automatic Control Conference in August at the University of Washington.

Professor Breakwell contributed to a six-week summer Institute in Dynamical Astronomy, held at Stanford University, July-August 1966. He presented two papers at the IAF Congress in Athens, Greece, September 1965, one of which was simultaneously presented by the co-author in Monterey, California. He co-authored a third paper also presented at Monterey. He attended a conference on Guidance Theory and Trajectory Analysis, NASA-ERC, November 1965. He attended an AIAA Technical Committee Meeting in Astrodynamics, New York, January 1966, followed by the fourth seminar in celestial mechanics at Yale. He presented some NASA-sponsored Stanford research at a contractors meeting--Guidance Theory and Trajectory Analysis--at NASA-ERC, April 1966. He attended, as co-author of a paper, a seminar at MIT in Guidance Theory and Trajectory Analysis, May 31-June 1, 1966.

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