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TESTS OF SATELLITE DETERMINATIONS OF THE GRAVITY FIELD AGAINST GRAVIMETRY AND THEIR COMBINATION[1]

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Abstract

Six solutions for the variations of the geopotential from satellite orbit analysis were compared to terrestrial gravimetry in the form of mean free air anomalies of 300 n. mi. squares. Statistical parameters calculated were the mean square values of each type as well as the mean square difference for different samples based on the number of observations in the 300 n. mi. square.

The study showed significant variation in quality between different solutions, the best being that recently obtained from camera tracking of satellites by Gaposhkin [1966]. It also showed that the arithmetic mean of four independent satellite solutions was better than any single solution. This combined satellite solution was used in an adjustment with the gravimetry to obtain a single best solution, given herein in the forms of a geoid map; spherical harmonic coefficients to degree & order 12,12; and gravity anomalies at 10° intervals.

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Introduction. The complexity of determining the gravity field from satellite orbit perturbations, especially the tesseral harmonics, has always made it desirable to test the satellite determinations against some independent standard. Various tests which have been proposed and applied in the past [Kaula, 1963, 1966c] include the astro-geodetic geoid; the near-zero harmonic J_{21}, deduced from observations of latitude variation; and the accelerations of the 24-hour synchronous satellite orbits. However, all these tests have been incomplete in some ways, and have left unsure such questions as the maximum degree and order to which satellite determinations are reliable.

The most obvious standard of comparison has always been gravimetry. But it has been an uncertain standard because the high amount of local variability in gravity anomalies has necessitated some degree of combination of the gravimetric data in order that the comparison not be lost in the "noise". Each combination unavoidably entails statistical assumptions, and thus in turn contributes to the unsureness of the gravimetric determination. When the combination is pushed all the way to a spherical harmonic representation [Uotila, 1962; Kaula, 1966a], it becomes evident that the gravimetric determination of the low degree harmonics is quite inferior.

However, there should be some intermediate representation between the noisy point values and the questionable harmonic coefficients which will afford a fairly clear comparison. The information we have of the spectrum [Kaula, 1966c, Table 2] indicates that the satellite determinations of the gravity field should represent roughly 60%, in $mgals^2$, of the variance of 10° square (better 600 n. mi. square, to emphasize 10° arc, not longitude) mean anomalies and roughly 40% of the variance of 300 n. mi. square means. Mainly because of computational convenience, it was decided to use

300 n. mi. square mean anomalies for a comparison of terrestrial gravimetry with satellite solutions. We discuss in turn the formation of the 300 n. mi. mean anomalies; the comparison of the satellite solutions therewith; and their combination to obtain an optimum representation.

Determination of mean gravity anomalies for 300 n. mi. squares. The basic data were mean free air anomalies for rectangles 1° in latitude by 1° in longitude, provided by the USAF Aeronautical Chart & Information Center, St. Louis, Missouri. These anomalies were determined essentially by the techniques described by Uotila [1960].

To obtain a set as nearly uniform as possible in statistical properties, the $1^\circ \times 1^\circ$ means were combined to form mean values for 60 n. mi. x 60 $\pm$ 30 n. mi. areas. The total number of such areas was 16,331, covering 24.6% of the earth's surface. Table I is the result of an autocovariance analysis of this sample, taking only products between values falling within the same 300 n. mi. square.

Of the 300 ($\pm$ 30) n. mi. squares, there were 935, covering 56.5% of the earth's surface, which contained one or more observed 60 n. mi. means. The mean anomalies of the 60 n. mi. areas without observations were estimated by applying linear regression [Kaula, 1966a; Moritz, 1962] to the 60 n. mi. means within the same 300 n. mi. square, using the covariances given in Table I:

$$\hat{g}_i = \sum_{j,k} K_{ik} \left[K_{jk}\right]^{-1} g_j, \tag{1}$$

where the g_j's are observed values, $\left[K_{jk}\right]^{-1}$ is the inverse of their covariance matrix, and K_{ik} is the covariance between the ith unobserved square and the kth observed value. The 300 n. mi. square means were then formed as the arithmetic mean of all the observed and extrapolated 60 n. mi. means they contained.

The set of 935 300 n. mi. square mean anomalies were used in turn in a world-wide covariance analysis to obtain autocovariances $K_o(\psi)$ as given in Table II. These in turn were analyzed to determine the power spectrum, as expressed by the degree variances σ_ℓ^2 [Kaula, 1966a].

$$\sigma_\ell^2 = \frac{2\ell+1}{2} \int_0^\pi P_\ell \, (\cos \psi) \, K_o \, (\psi) \sin \psi \, d\psi, \tag{2}$$

where ψ is arc distance on the sphere and P_ℓ $(\cos \psi)$ is the conventional Legendre zonal harmonic. The degree variances thus determined are given in Table III, together with some calculated from satellite-determined normalized potential coefficients $\bar{C}_{\ell m}$, $\bar{S}_{\ell m}$ by:

$$\sigma_\ell^2 = \gamma^2 \; (\ell-1)^2 \sum_m \left\{\bar{C}^2_{\ell m} + \bar{S}^2_{\ell m}\right\}, \tag{3}$$

where γ is the mean value of the acceleration of gravity.

Further details and theoretical derivations are given in Kaula et al, [1966].

Satellite determinations of the gravity field. All spherical harmonic coefficients given in this paper apply to functions normalized to $2\sqrt{\pi}$: i.e., such that the mean square of the surface harmonic is unity over the unit sphere. Table IV gives the most recently published determinations of the zonal harmonics, together with the mean values used in conjunction with all tesseral harmonic solutions in comparisons with terrestrial data. Table V gives the most recently published determinations of the tesseral harmonic coefficients of the gravitational field by the four principal efforts in this area: the sets A [Anderle, 1966] and G&N [Guier & Newton, 1965], who used Doppler tracking data; and the sets K6 [Kaula, 1966b] and G8 [Gaposhkin, 1966], who used camera tracking data. The solution of Gaposhkin [1966] is a continuation of the work by the late Imre Izsak, and is a distinct improvement over his last result [Izsak, 1966]. In addition to these published solutions, there are given two solutions which probably will not be elsewhere published: K8 and G6, since one of the questions we wish to test is to how high a degree should the determination of the gravity field from satellites be carried.

We also give in Table V a "combined" set C, which is the arithmetic mean of sets A, G&N, K8, and G6. (Set K6 was not used because at the time K8 was believed to be better; set G8 was not used because it did not become available until later.) Set C has been truncated at the highest coefficient common to the four sets: $\bar{C}_{72}$, $\bar{S}_{72}$. Some orbit analysts would express shock at such combination and truncation. The basis of their disapproval is that each solution should be regarded as a complete set, the truncation of which constitutes a different representation of the gravity field than would have been obtained analyzing the same data for a set comprising the same terms as the truncation. However, their objection applies when "optimum representation" is defined as approximating as closely as possible the satellite orbits from which the sets were determined. These orbits in themselves are of rather evanescent interest; it seems geophysically more interesting to define "optimum representation" as approximating as closely as possible the acceleration of gravity at the earth's surface.

The final set in Table V, CA, is a "combined-adjusted" set calculated by a weighted least squares adjustment between the gravimetry and set C, described below.

Comparison of satellite data and gravimetry. For each 300 n. mi. square mean free air gravity anomaly Δg we have two independent estimates, $\hat{g}$, one based on terrestrial gravimetry:

$$\hat{g}_T = g_H + \delta g + \epsilon_T; \tag{4}$$

and one based on satellite orbit perturbations:

$$\hat{g}_S = g_H + \epsilon_S. \tag{5}$$

In (4) and (5), g_H is the true value of the contribution to the gravity anomaly Δg of the geopotential harmonic coefficients $\overline{C}_{\ell m}$, $\overline{S}_{\ell m}$ estimated from satellite orbits:

$$g_H = \gamma \sum_{\ell,m} (\ell-1)\, \overline{P}_{\ell m} (\sin \varphi) \left[\overline{C}_{\ell m} \cos m\lambda + \overline{S}_{\ell m} \sin m\lambda\right], \tag{6}$$

where φ is latitude, λ is longitude, γ is the mean value of the gravity acceleration, and $\overline{P}_{\ell m}$ (sin φ) is the normalized associated function. The quantity δg is the balance of the field:

$$\delta g = \Delta g - g_H \tag{7}$$

ϵ_T and ϵ_S are, respectively, the errors in the estimates $\hat{g}_T$ and $\hat{g}_S$. The satellite error ϵ_S includes only the error of commission: i.e., the consequence of errors in the coefficients $\hat{C}_{\ell m}$, $\hat{S}_{\ell m}$; it does not include the error of omission, δg.

All four of the quantities g_H, δg, ϵ_T, and ϵ_S we should expect to be statistically independent. Hence from sets of estimates $\hat{g}_T$ and $\hat{g}_S$ we can immediately obtain an estimate of the mean square of g_H:

$$\hat{E}\left\{ g_H^2 \right\} = < \hat{g}_T \hat{g}_S >, \tag{8}$$

where the carets denote the mean value of the quantity they enclose. Then we obtain immediately an estimate of the mean square of the satellite error ϵ_S:

$$\hat{E}\left\{ \epsilon_S^2 \right\} = < \hat{g}_S^2 > - < \hat{g}_T \hat{g}_S > \tag{9}$$

To obtain an estimate of the mean square of the gravimetric error ϵ_T, there would have to be used statistics pertaining to the number and distribution of the observations within the square. The complications entailed in obtaining such an estimate appear to be avoidable without too much error by the assumptions:

(1) The mean square error $E\left\{ \epsilon_T^2 \right\}$ is inversely proportionate to the number, n, of observed 60 n. mi. means within the 300 n. mi. square; and

(2) The mean square error $E\left\{ \epsilon_T^2 \right\}$ of a 300 n. mi. square with only one observed 60 n. mi. square is equal to the mean square anomaly,

$E\left\{(g_H + \delta g)^2\right\}$. Whence:

$$\hat{E}\left\{ \epsilon_T^2 \right\} \approx < <g_T^2> / n > \qquad (10)$$

Then:

$$\hat{E}\left\{ \delta g^2 \right\} = \hat{E}\left\{ \hat{g}_T^2 \right\} - \hat{E}\left\{ \epsilon_T^2 \right\} - \hat{E}\left\{ g_H^2 \right\} \qquad (11)$$

If we have several determinations of the field from satellites, the most obvious measure by which to compare them is the mean square difference of their estimates $\hat{g}_S$ from the gravimetric estimate $\hat{g}_T$:

$$\begin{aligned} \hat{E}\left\{(\hat{g}_T - \hat{g}_S)^2\right\} &= < (\hat{g}_T - \hat{g}_S)^2 > \\ &= < \hat{g}_T^2 > - 2 < \hat{g}_T \hat{g}_S > + < \hat{g}_S^2 > \end{aligned} \qquad (12)$$

A non dimensional measure, adjusted to remove the terrestrial contribution ϵ_T to the error, would be the correlation coefficient:

$$r_T = \frac{<\hat{g}_T \hat{g}_S>}{\left[<\hat{g}_S^2> \left(<\hat{g}_T^2> - \hat{E}\left\{\epsilon_T^2\right\}\right)\right]^{\frac{1}{2}}} \qquad (13)$$

The correlation coefficient r_T reflects both omission (δg) and commission (ϵ_S) errors; for a dimensionless measure of commission errors only there is:

$$r_L = \frac{\langle \hat{g}_T \; \hat{g}_S \rangle}{\langle \hat{g}_S^2 \rangle} \qquad (14)$$

Finally, using the gravimetric estimates of degree variances σ_ℓ^2 in Table III, there is another estimate of $E\left\{ g_H^2 \right\}$, which we denote by D:

$$\hat{E}\left\{ g_H^2 \right\} = D = \sum_\ell \frac{n_\ell}{2\ell+1} \hat{\sigma}_\ell^2 , \qquad (15)$$

where n_ℓ is the number of coefficients of degree ℓ included in g_H.

The quantities actually calculated from the 300 n. mi. means were $\hat{\sigma}_\ell^2$; as given in Table III, by equation (2); $\langle \hat{g}_S^2 \rangle$; $\langle \hat{g}_T^2 \rangle$; $\langle (\hat{g}_T - \hat{g}_S)^2 \rangle$; and $E\left\{ \epsilon_T^2 \right\}$, by equation (10). Equations (9), (11), (12), (13), (14), and (15) were then used to derive $\hat{E}\left\{ g_H^2 \right\}$ (or $\langle \hat{g}_T \; \hat{g}_S \rangle$); $\hat{E}\left\{ \sigma_S^2 \right\}$; $\hat{E}\left\{ \delta g^2 \right\}$; r_T; r_L; and D, respectively. The results are given in Table VI for each of three sets of 300 n. mi. squares: 1) the 935 squares with one or more observed 60 n. mi. means; 2) the 369 squares with 10 or more observed 60 n. mi. means; and 3) the 136 squares with 20 or more observed 60 n. mi. means.

The variations between the three sets emphasize the uncertainty in the results because of sample differences from the assumption of complete randomness between the four quantities g_H, δg, ϵ_T, and ϵ_S. The best results are those for solution CA, but it is itself partly dependent on the gravity anomalies against which it is being tested. The best single solution is clearly solution G8; however, it is not significantly better than the more limited solution C, which is the arithmetic mean of fewer terms from four earlier solutions. In any case, the small values of $E\left\{ \epsilon_S^2 \right\}$ and r_L for solutions G8 and C indicate that they are about as good as might be expected for the number of terms they respectively contain.

Combination of satellite data and gravimetry. Given a satellite solution represented by k spherical harmonic coefficients $\hat{x}_j$, and a gravimetric solution represented by n (>k) mean free air anomalies $\hat{g}_{Ti}$, we can write k observation equations of the form:

$$\gamma\ (\ell-1)\left[\hat{x}_j+dx_j\right] = \frac{1}{4\pi}\sum_{i=1}^{n} Q_{ij}\ (\hat{g}_{Ti}+dg_i)A_i, \tag{16}$$

where Q_{ij} is the value of spherical harmonic j (of degree ℓ) for square i, and A_i is the area on the unit sphere represented by $\hat{g}_{Ti}$. The solution to (16) is obtained by minimizing the quadratic sum:

$$\sum_{i=1}^{n} P_i\ dg_i^2 + \sum_{j=1}^{k} W_j\ dx_j^2 = \text{MIN}. \tag{17}$$

The set $\hat{x}_j$ used for a numerical solution was solution C in Table V, plus the mean values of the zonals in Table IV: hence k was 54. The weights W_j were $1/q^2$, where q is the quartile of the range of solutions A, G&N, G6, and K8: e.g., 0.08×10^{-6} for $\bar{S}_{22}$ of the tesserals or 0.02×10^{-6} for $\bar{C}_{40}$ of the zonals.

The set $\hat{g}_{Ti}$ used was a 1654-member set of 300 n. mi. means covering the entire world. 935 members of this set were those containing observations, as described in the preceding sections. The remaining 719 members were obtained by applying a linear regression, as in equation (1), to the residuals of the values g_{Ti} with respect to the low degree field g_H constituted by solution A in Tables IV - V up through the 6th degree. (Solution A was used for this calculation because at the time it was the best by the criterion of comparison with 24-hour orbit accelerations; solutions K6, G6, and G8 had not yet been made.) The autocovariances $K_0(\psi)$ in Table II were correspondingly adjusted by removal of degrees 6 and lower:

$$K_7(\psi) = K_0(\psi) - \sum_{\ell=0}^{6} \sigma_\ell^2 \, P_\ell \, (\cos \psi) \tag{18}$$

In applying the linear regression (1) to estimate a particular value $\hat{g}_i$, in general the g_j used were limited to the nearest and next-to-nearest observed values. As a consequence of this lack of rigour, some variation of degrees 0 - 6 crept back into the estimates $\hat{g}_{Ti}$ used in (16) - (17), before they were added back onto the values based on solution A for degrees 0 - 6.

The weights P_i used in (17) were based on a minor modification of the variances given by (10), in order to give the 719 extrapolated values $\hat{g}_{Ti}$ some weight:

$$P_i = (n_i+1)/< g_T^2 > \tag{19}$$

The results of this adjustment are given in Table V as harmonic coefficients (solution CA); in Table VII as gravity anomalies as 10° intervals; and in Figure 1 as a geoid, calculated by Stokes' formula. In the adjustment, the largest correction to any 300 n. mi. mean was + 37.7 milligals, to an extrapolated value at Lat. 67.5° N, Long. 353.5° E. At the opposite extreme, no estimate based on 25 observed 60 n. mi. means changed more than 1.3 milligals. The changes in general were much smaller than in a previous adjustment where solution A was held rigidly through degree 6, despite the small changes to the harmonic coefficients, as indicated by the differences between solutions C and CA in Table V.

Table VIII gives the spherical harmonic coefficients for the final solution through degree 12. The small differences from solution CA in Table V are apparently a consequence of applying the harmonic analysis to values at 5° intervals interpolated from the 300 n. mi. means.

Conclusions. The principal conclusions indicated by Table VI appear to be:

1. The best published solutions for analysis of satellite orbits are those recently obtained from camera tracking: solution G8 by Gaposhkin [1966], and solution K6 by Kaula [1966b]. The principal indication of this superiority are the smaller estimated mean square errors, $\hat{E}\{\epsilon_S^2\}$. Solution G8 is further superior to K6 because it contains more terms and hence more information, as indicated by the larger estimated mean square contribution to the field, $\langle \hat{g}_T \hat{g}_S \rangle$.

2. An arithmetic mean of different solutions from satellite data is superior to any single solution, as indicated by the smaller estimated mean square error $\hat{E}\{\epsilon_S^2\}$ for solution C, compared to those for solutions A, G&N, G6, and K8.

3. The agreement between terrestrial gravimetry and satellite orbit solutions is quite satisfactory, as indicated by the magnitude of the cross covariance $\langle \hat{g}_T \hat{g}_S \rangle$ compared with the purely gravimetric autocovariance value D and the purely satellite mean square value $\langle \hat{g}_S^2 \rangle$.

4. The terrestrial gravimetry contains appreciable additional information, as indicated by the estimates of higher degree variation $\hat{E}\left\{\delta g^2\right\}$. Hence there should be a range of variation over which gravimetry and satellite data should be of comparable value - probably around those coefficients for which the sum of degree plus order $(\ell+m)$ ranges from 9 to 13.

5. Hence a combination of satellite and terrestrial solutions, such as solution CA, should be superior to either solution alone.

The superiority of the camera solutions G8 and K6 to the Doppler solutions A and G&N is probably due more to their recency than anything else: the camera solutions were completed more than a year later. It is entirely possible that completion of a new cycle of analysis of Doppler tracking data could reverse the situation.

It would have been desirable, of course, to have utilized the better solution G8 in the extrapolation of terrestrial gravimetry and in the combined satellite solution C. However, improved solutions based on Doppler tracking will doubtless come along shortly, at which time a new combination of satellite solutions and extrapolation of gravimetry can be made.

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Table I: Short-Range Autocovariance of Free-Air Gravity Anomalies

Number of Pairs in Sample	Distance Degrees	Covariance $Mgal^2$
16331	0.00°	711
6803	0.92°	421
35225	1.32°	272
43674	2.35°	196
33864	3.35°	162
19414	4.34°	97
2675	5.31°	164
113	6.19°	137

Table II: Long-Range Autocovariance of

Free-Air Gravity Anomalies

Distance Degrees	Covariance Mgals^2	Distance Degrees	Covariance Mgals^2	Distance Degrees	Covariance Mgals^2
0°	274	59°	-13	121°	14
5°	116	64°	-11	126°	11
9°	89	69°	-6	131°	7
13°	51	74°	-5	136°	1
18°	34	80°	-5	141°	-1
23°	20	85°	-3	146°	-8
29°	5	90°	-1	151°	-4
34°	-4	95°	5	156°	-9
39°	-10	100°	9	162°	-2
44°	-9	105°	16	167°	2
49°	-9	111°	17	172°	1
54°	-10	116°	16	175°	1

Table III: Power Spectrum of Free-Air Gravity Anomalies

Degree ℓ	Degree Variance σ_ℓ^2 Mgal^2
0	2.7
1	-0.5
2	6.3
3	31.8
4	18.6
5	8.4
6	22.2
7	11.0
8	9.2
9	10.1

Table IV: Normalized Zonal Harmonic Coefficients, $\bar{C}_{\ell o}$, of the Geopotential

Degree ℓ	Smith [1963, 1965] 10^{-6}	Kozai [1964] 10^{-6}	King-Hele et al [1965] 10^{-6}	Guier & Newton [1965] 10^{-6}	Adopted Mean 10^{-6}
2	-484.172	-484.174	-484.172		-484.17 ± .01
3	0.923	0.963	0.967	1.019	0.97 ± .02
4	0.567	0.550	0.507		0.54 ± .02
5	0.054	0.063	0.045	0.002	0.04 ± .02
6	-0.202	-0.179	-0.158		-0.18 ± .02
7	0.077	0.086	0.114	0.163	+0.11 ± .02
8	0.112	0.065	-0.107		

Table V: Normalized Tesseral Harmonic Coefficients, $\overline{C}_{\ell m}$, $\overline{S}_{\ell m}$, of the Geopotential

Coefficient	Solution							
	A	G&N	G6	G8	K6	K8	C	CA
	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}
C22	2.45	2.38	2.45	2.38	2.43	2.42	2.42	2.42
S22	-1.52	-1.20	-1.34	-1.35	-1.39	-1.39	-1.36	-1.36
C31	2.15	1.84	1.95	1.94	1.94	1.90	1.93	1.79
S31	0.27	0.21	0.29	0.27	0.15	0.11	0.20	0.18
C32	0.98	1.22	0.75	0.73	0.72	0.69	0.91	0.78
S32	-0.91	-0.68	-0.52	-0.54	-0.78	-0.78	-0.72	-0.75
C33	0.58	0.66	0.47	0.56	0.55	0.55	0.56	0.57
S33	1.62	0.98	1.55	1.62	1.24	1.29	1.36	1.42
C41	-0.49	-0.56	-0.56	-0.57	-0.61	-0.59	-0.55	-0.56
S41	-0.57	-0.44	-0.45	-0.47	-0.49	-0.48	-0.48	-0.46
C42	0.27	0.42	0.38	0.33	0.33	0.28	0.34	0.30
S42	0.67	0.44	0.66	0.66	0.71	0.69	0.62	0.60
C43	1.03	0.84	0.86	0.85	0.89	0.89	0.90	0.92
S43	-0.25	0.00	-0.21	-0.19	0.07	0.19	-0.07	-0.19
C44	-0.41	-0.21	0.01	-0.05	-0.31	-0.32	-0.26	-0.06
S44	0.34	0.19	0.37	0.23	0.11	0.00	0.23	0.32
C51	0.03	0.14	-0.09	-0.08	-0.05	-0.01	0.02	0.00
S51	-0.12	-0.17	-0.09	-0.10	0.03	0.02	-0.09	-0.02
C52	0.64	0.27	0.63	0.63	0.75	0.68	0.55	0.44
S52	-0.33	-0.34	-0.23	-0.23	-0.17	-0.25	-0.29	-0.28
C53	-0.39	0.09	-0.77	-0.52	-0.61	-0.67	-0.44	-0.31
S53	-0.12	0.10	-0.03	0.01	0.15	0.12	0.02	0.03
C54	-0.55	-0.49	-0.28	-0.26		0.08	-0.31	0.02
S54	0.15	-0.26	0.05	0.06		0.37	0.08	0.11
C55	0.21	-0.03	0.07	0.16		-0.45	-0.05	0.10
S55	-0.59	-0.67	-0.65	-0.59		-0.21	-0.53	-0.49
C61	-0.08	0.	-0.05	-0.05	-0.18	-0.19	-0.08	-0.10
S61	0.19	0.10	-0.03	-0.03	0.12	0.13	0.10	0.10

Table V

(Continued)

Coefficient	Solution							
	A	G&N	G6	G8	K6	K8	C	CA
	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}
C62	0.13	-0.16	0.09	0.07	0.03	0.08	0.03	-0.01
S62	-0.46	-0.16	-0.35	-0.37	-0.38	-0.41	-0.34	-0.28
C63	-0.02	0.53	-0.01	-0.05	0.12	0.10	0.14	0.09
S63	-0.13	0.05	0.03	0.03	0.35	0.46	0.10	0.04
C64	-0.19	-0.31	0.00	-0.04	0.13	0.08	-0.10	-0.19
S64	-0.32	-0.51	-0.32	-0.52	-0.50	-0.43	-0.40	-0.43
C65	-0.09	-0.18	-0.29	-0.31	-0.11	-0.04	-0.15	-0.17
S65	-0.79	-0.50	-0.45	-0.46	-0.37	-0.38	-0.53	-0.60
C66	-0.32	0.01	-0.02	-0.04		0.15	-0.04	-0.05
S66	-0.36	-0.23	-0.40	-0.16		-0.15	-0.27	-0.26
C71	0.33	0.13	0.22	0.20	0.21	0.06	0.15	0.06
S71	0.08	0.09	0.22	0.16	0.11	0.06	0.11	0.07
C72	0.35	0.46	0.38	0.36		0.31	0.38	0.29
S72	-0.19	0.06	0.16	0.16		0.26	0.07	0.08
C73	0.32	0.39		0.25		-0.03		
S73	0.04	-0.21		0.02		-0.32		
C74	-0.47	-0.14		-0.15		-0.41		
S74	-0.24	0.00		-0.10		0.15		
C75	0.05	-0.06		0.08		0.22		
S75	0.02	-0.19		0.05		-0.31		
C76	-0.48	-0.45		-0.21				
S76	-0.24	-0.75		0.06				
C77		0.09		0.06				
S77		-0.14		0.10				
C81		-0.15	-0.06	-0.08	-0.05	-0.06		
S81		-0.05	0.09	0.07	0.05	0.06		
C82		0.09	0.06	0.03	0.09	0.08		
S82		-0.04	0.04	0.04	-0.07	-0.07		

Table V

(Continued)

Coefficient	Solution							
	A	G&N	G6	G8	K6	K8	C	CA
	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}
C83		-0.05		-0.04		0.08		
S83		0.22		0.00		0.22		
C84		-0.07	-0.15	-0.21		0.08		
S84		-0.04	0.11	-0.01		0.04		
C85		0.08		-0.05		0.03		
S85		0.		0.12		-0.34		
C86		-0.02		-0.02		0.10		
S86		0.67		0.32		0.12		
C87		0.17		-0.01				
S87		-0.07		0.03				
C88		-0.15		-0.25				
S88		0.09		0.10				
C91			0.11	0.12		-0.00		
S91			0.06	0.01		0.03		
C92				0.00		-0.09		
S92				0.04		-0.09		
C101			0.10	0.11		0.02		
S101			-0.12	-0.13		-0.01		
C102				-0.11		0.01		
S102				-0.04		-0.04		
C103				-0.07				
S103				0.03				
C104				-0.07				
S104				-0.11				
C111			-0.04	-0.05		-0.13		
S111			0.08	0.01		-0.07		
C121			-0.21	-0.16		-0.12		
S121			-0.04	-0.07		-0.01		

Table V

(Continued)

Coefficient	Solution							
	A	G&N	G6	G8	K6	K8	C	CA
	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}
C122				-0.10				
S122				-0.01				

Table VI: Comparison of Satellite and Gravimetric Determinations of 300 n. mi. Square Mean Anomalies

$Mgal^2$

Solution	$\langle(\hat{g}_T-\hat{g}_S)^2\rangle$	$\langle\hat{g}_T^2\rangle$	$\langle\hat{g}_S^2\rangle$	D	$\langle\hat{g}_T\hat{g}_S\rangle$	$\hat{E}\{\epsilon_T^2\}$	$\hat{E}\{\delta g^2\}$	$\hat{E}\{\epsilon_S^2\}$	r_T	r_L
		Set [1]: $n \geq 1$; 935 members								
A	278	274	192	97	94	72	104	98	.48	.49
G&N	297	274	199	107	88	72	114	111	.44	.44
G6	254	274	149	97	84	72	118	65	.49	.56
G8	245	274	157	114	93	72	89	64	.52	.59
K6	253	274	104	86	62	72	141	42	.43	.60
K8	274	274	135	106	68	72	134	67	.42	.50
C	236	274	119	91	78	72	124	41	.50	.66
CA	224	274	109	91	80	72	102	29	.54	.73
		Set [2]: $n \geq 10$; 369 members								
A	276	354	167	97	122	23	209	45	.52	.73
G&N	282	354	219	107	145	23	186	74	.54	.66
G6	260	354	143	97	118	23	213	25	.54	.83
G8	239	354	140	114	127	23	204	13	.59	.91
K6	270	354	101	86	92	23	239	9	.50	.91
K8	278	354	134	106	105	23	226	29	.50	.78
C	242	354	120	91	116	23	215	4	.58	.97
CA	226	354	110	91	119	23	212	-9	.62	1.08
		Set [3]: $n \geq 20$; 136 members								
A	249	290	134	97	88	13	189	46	.46	.66
G&N	236	290	207	107	130	13	147	77	.56	.63
G6	224	290	105	97	86	13	191	19	.50	.82
G8	188	290	114	114	108	13	169	6	.61	.95
K6	210	290	103	86	92	13	185	11	.54	.89
K8	261	290	137	106	83	13	194	50	.43	.61
C	204	290	104	91	95	13	182	9	.56	.91
CA	181	290	95	91	102	13	175	-7	.63	1.07

Table VII: Mean Free Air Gravity Anomalies at 10° Intervals,

in Milligals referred to the International Formula

Eastern Hemisphere

Long. East

Lat.	5	15	25	35	45	55	65	75	85	95	105	115	125	135	145	155	165	175	Lat.
85	15	14	13	12	10	9	9	8	5	4	4	7	15	19	19	16	10	8	85
75	33	29	19	20	18	10	11	13	9	7	7	11	16	15	5	11	25	25	75
65	26	7	4	10	11	1	-10	-10	4	-0	2	3	10	18	25	29	28	24	65
55	6	8	2	10	9	8	7	-6	-4	-10	-18	-0	5	8	8	16	16	11	55
45	13	20	26	9	1	-14	-13	-23	-6	-19	-14	0	9	15	3	1	-3	-5	45
35	13	0	-25	15	28	6	-8	-2	2	-22	-16	-16	12	24	3	-5	-15	-6	35
25	11	-16	-7	2	-5	-8	4	-8	-29	-23	-11	-2	3	6	-1	-7	-13	-16	25
15	4	-2	-5	3	-2	-8	-16	-23	-13	-9	-12	10	17	9	2	-2	-8	-6	15
5	3	1	-11	-4	-6	-24	-27	-21	-21	-6	6	13	23	20	10	15	17	0	5
-5	-16	-9	-10	4	-18	-32	-28	-11	-43	-7	17	21	-0	4	-2	-4	16	-4	-5
-15	-4	-5	-1	3	2	-8	-7	-20	-28	-14	-8	-13	7	11	12	8	18	10	-15
-25	5	7	6	3	9	8	1	-6	-10	-16	-18	7	2	-2	9	11	24	13	-25
-35	7	4	7	4	8	6	6	6	5	-3	-17	-19	-22	-12	12	3	3	11	-35
-45	2	3	8	15	18	20	14	14	10	1	-13	-22	-23	-20	-9	3	8	10	-45
-55	8	11	16	22	25	27	26	24	20	11	2	-5	-10	-11	-9	5	11	1	-55
-65	15	15	18	22	18	22	24	24	21	17	10	6	3	-1	0	-4	-8	-11	-65
-75	14	14	13	12	13	15	15	13	12	9	7	5	3	1	-1	1	8	11	-75
-85	-5	-5	-5	-6	-6	-6	-6	-6	-7	-8	-8	-8	-10	-11	-11	-11	-11	-11	-85
	5	15	25	35	45	55	65	75	85	95	105	115	125	135	145	155	165	175	

Table VII

(Continued)

Western Hemisphere

Long. East.

Lat.	185	195	205	215	225	235	245	255	265	275	285	295	305	315	325	335	345	355	Lat.
85	8	9	10	11	11	11	12	13	13	11	6	4	4	7	14	17	17	17	85
75	16	13	7	10	9	3	7	16	10	5	-0	7	16	28	35	30	29	30	75
65	12	13	24	32	15	-6	-12	-12	-5	-0	4	7	1	3	35	35	37	34	65
55	-8	8	4	6	-2	-2	-0	-2	-7	-14	-13	-5	10	26	28	11	12	17	55
45	1	3	2	-5	-13	-4	18	19	4	-8	-7	-5	-1	6	13	25	15	14	45
35	-12	-13	-8	-9	-15	-20	-1	7	-5	-1	-19	-18	-9	3	22	13	8	25	35
25	-9	12	5	-4	-11	-18	-16	6	-1	-2	-30	-22	-18	1	-8	-9	10	-1	25
15	-3	1	15	-12	-12	-19	-14	-3	6	16	-16	-34	-32	-23	-15	-2	11	7	15
5	-8	-3	-12	-13	-10	-18	-18	-9	-3	7	23	4	-14	-25	-4	3	1	10	5
-5	-7	-3	7	9	3	-7	-9	-5	4	-0	5	1	-8	-17	-6	-10	-1	-9	-5
-15	4	-5	3	5	3	-1	-2	3	3	1	14	41	-1	-6	-10	-9	-12	2	-15
-25	8	-7	-5	-1	1	-0	-2	-3	-7	-8	4	17	-5	-9	-12	-11	-15	-11	-25
-35	-4	-2	-2	-0	1	-0	-3	-5	-8	-8	4	10	4	-7	-6	7	0	4	-35
-45	2	-3	-2	-2	1	1	1	0	-1	-4	-6	4	-1	-4	-4	-2	-3	-2	-45
-55	-6	-10	-7	-4	-2	2	6	8	9	10	11	9	10	9	8	5	4	4	-55
-65	-16	-16	-14	-12	-11	-0	5	9	13	13	21	25	25	22	19	15	14	14	-65
-75	-2	-5	-9	-9	-7	-2	-2	-3	-1	11	16	11	10	6	-2	2	11	11	-75
-85	-11	-11	-10	-10	-10	-9	-7	-6	-6	-8	-14	-16	-16	-14	-10	-8	-8	-8	-85
	185	195	205	215	225	235	245	255	265	275	285	295	305	315	325	335	345	355	

Table VIII: Spherical Harmonic Coefficients to Degree 12

Referred to International Gravity Formula

NORMALIZED

L	M	ANOMALY(MGALS) C	ANOMALY(MGALS) S	POTENTIAL C	POTENTIAL S
2	0	0.4382E 01		0.4472E-05	
2	1	0.1737E-01	0.5338E-01	0.1773E-07	0.5447E-07
2	2	0.2356E 01	-0.1314E 01	0.2404E-05	-0.1341E-05
3	0	0.1789E 01		0.9130E-06	
3	1	0.3516E 01	0.4701E-00	0.1794E-05	0.2399E-06
3	2	0.1605E 01	-0.1432E 01	0.8188E-06	-0.7308E-06
3	3	0.1114E 01	0.2826E 01	0.5686E-06	0.1442E-05
4	0	0.7302E 00		0.2484E-06	
4	1	-0.1660E 01	-0.1248E 01	-0.5647E-06	-0.4245E-06
4	2	0.1043E 01	0.1617E 01	0.3546E-06	0.5501E-06
4	3	0.2744E 01	-0.5973E 00	0.9334E-06	-0.2032E-06
4	4	-0.1179E-00	0.9216E 00	-0.4011E-07	0.3135E-06
5	0	0.1844E-00		0.4705E-07	
5	1	-0.1815E-00	-0.1875E-01	-0.4630E-07	-0.4784E-08
5	2	0.1858E 01	-0.1141E 01	0.4740E-06	-0.2910E-06
5	3	-0.1291E 01	-0.4409E-01	-0.3293E-06	-0.1125E-07
5	4	0.3981E-01	-0.4581E-00	0.1016E-07	-0.1169E-06
5	5	0.3200E-00	-0.1873E 01	0.8164E-07	-0.4779E-06
6	0	-0.7054E 00		-0.1440E-06	
6	1	-0.4466E-00	0.8788E 00	-0.9114E-07	0.1793E-06
6	2	0.2409E-01	-0.1488E 01	0.4916E-08	-0.3037E-06
6	3	0.4874E-00	0.3673E-00	0.9948E-07	0.7496E-07
6	4	-0.7731E 00	-0.2201E 01	-0.1578E-06	-0.4491E-06
6	5	-0.6209E 00	-0.2979E 01	-0.1267E-06	-0.6079E-06
6	6	-0.2075E-00	-0.1379E 01	-0.4234E-07	-0.2814E-06
7	0	0.8820E 00		0.1500E-06	
7	1	0.3622E-00	0.5091E 00	0.6160E-07	0.8659E-07
7	2	0.1801E 01	0.2311E-00	0.3062E-06	0.3930E-07
7	3	0.9710E-01	-0.9921E-01	0.1651E-07	-0.1687E-07
7	4	-0.1545E 01	0.3461E-00	-0.2627E-06	0.5887E-07
7	5	0.1924E-00	-0.1009E-00	0.3271E-07	-0.1717E-07
7	6	-0.1009E 01	0.1354E 01	-0.1717E-06	0.2303E-06
7	7	0.1753E-00	0.3353E-00	0.2982E-07	0.5702E-07
8	0	-0.7717E 00		-0.1125E-06	
8	1	0.1489E-00	-0.5681E 00	0.2171E-07	-0.8282E-07
8	2	0.4899E-00	0.3590E-01	0.7142E-07	0.5233E-08
8	3	0.2504E-00	-0.2583E-00	0.3651E-07	-0.3765E-07
8	4	0.3506E-01	-0.1935E-00	0.5111E-08	-0.2821E-07
8	5	-0.6156E 00	-0.1546E-00	-0.8974E-07	-0.2254E-07
8	6	0.2231E-00	0.1353E 01	0.3253E-07	0.1972E-06
8	7	0.4941E-00	0.1951E-01	0.7203E-07	0.2844E-08
8	8	-0.4429E-00	-0.7407E 00	-0.6456E-07	-0.1080E-06

Table VIII

(Continued)

NORMALIZED

L	M	ANOMALY (MGALS) C	ANOMALY (MGALS) S	POTENTIAL C	POTENTIAL S
9	0	0.8506E 00		0.1085E-06	
9	1	0.5216E 00	0.1896E-01	0.6654E-07	0.2419E-08
9	2	0.4185E-00	0.4807E-00	0.5338E-07	0.6131E-07
9	3	-0.6167E 00	0.4061E-01	-0.7866E-07	0.5180E-08
9	4	0.3406E-00	0.8194E-01	0.4344E-07	0.1045E-07
9	5	-0.7764E 00	0.7283E 00	-0.9903E-07	0.9290E-07
9	6	0.2859E-00	0.6148E 00	0.3647E-07	0.7842E-07
9	7	0.3568E-00	-0.4644E-00	0.4551E-07	-0.5923E-07
9	8	0.1074E 01	0.5261E-02	0.1370E-06	0.6711E-09
9	9	0.3833E-00	0.2966E-00	0.4889E-07	0.3783E-07
10	0	-0.4001E-00		-0.4536E-07	
10	1	0.1124E 01	0.4273E-00	0.1274E-06	0.4844E-07
10	2	0.1408E-00	-0.1145E 01	0.1596E-07	-0.1298E-06
10	3	-0.6422E 00	-0.1326E 01	-0.7281E-07	-0.1503E-06
10	4	-0.2296E-00	-0.2295E-00	-0.2604E-07	-0.2603E-07
10	5	0.2088E-00	0.9710E-01	0.2367E-07	0.1101E-07
10	6	-0.6574E 00	-0.4328E-00	-0.7454E-07	-0.4907E-07
10	7	0.3292E-00	0.4658E-01	0.3732E-07	0.5282E-08
10	8	0.5782E 00	-0.3640E-00	0.6556E-07	-0.4127E-07
10	9	0.9998E 00	-0.5413E 00	0.1134E-06	-0.6137E-07
10	10	0.6256E 00	0.9720E-01	0.7094E-07	0.1102E-07
11	0	-0.5234E 00		-0.5340E-07	
11	1	-0.2492E-00	-0.1799E-00	-0.2543E-07	-0.1835E-07
11	2	0.3132E-00	-0.4599E-00	0.3196E-07	-0.4693E-07
11	3	0.2143E-00	-0.5473E 00	0.2186E-07	-0.5585E-07
11	4	-0.4191E-00	-0.2081E-00	-0.4276E-07	-0.2123E-07
11	5	0.5946E-01	-0.5655E-01	0.6068E-08	-0.5771E-08
11	6	0.1568E-01	0.1201E-00	0.1600E-08	0.1226E-07
11	7	-0.5777E-01	-0.3470E-00	-0.5895E-08	-0.3541E-07
11	8	0.2861E-00	-0.4572E-00	0.2920E-07	-0.4665E-07
11	9	0.8759E-01	0.4299E-00	0.8938E-08	0.4387E-07
11	10	-0.4198E-00	-0.1822E-00	-0.4283E-07	-0.1859E-07
11	11	0.1056E 01	0.5341E 00	0.1077E-06	0.5450E-07
12	0	-0.5489E 00		-0.5092E-07	
12	1	0.1668E-00	-0.5367E 00	0.1547E-07	-0.4979E-07
12	2	0.5095E 00	0.6492E 00	0.4727E-07	0.6022E-07
12	3	0.1770E-00	0.9646E 00	0.1642E-07	0.8948E-07
12	4	-0.3779E-00	0.5647E-01	-0.3505E-07	0.5238E-08
12	5	0.2452E-00	-0.2722E-00	0.2274E-07	-0.2525E-07
12	6	-0.3246E-01	0.2846E-00	-0.3011E-08	0.2640E-07
12	7	-0.5684E 00	-0.2217E-00	-0.5273E-07	-0.2057E-07
12	8	-0.4013E-01	-0.1370E-00	-0.3723E-08	-0.1271E-07
12	9	0.7847E-01	0.7032E-01	0.7279E-08	0.6523E-08
12	10	0.1765E-00	0.2197E-00	0.1637E-07	0.2038E-07
12	11	-0.3360E-00	-0.2660E-02	-0.3117E-07	-0.2468E-09
12	12	-0.6740E 00	0.2320E-00	-0.6252E-07	0.2152E-07

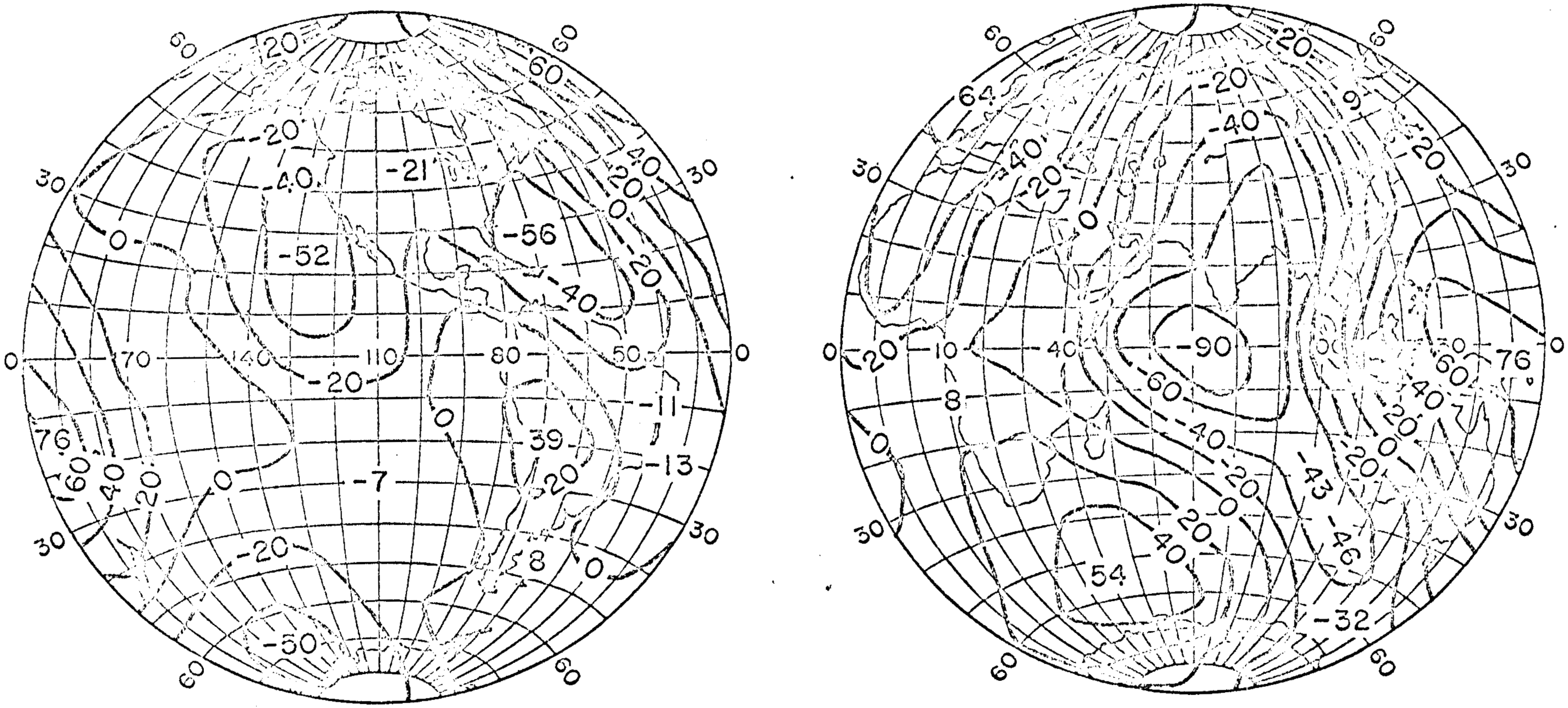

Fig. 1: Geoid heights in meters referred to an ellipsoid of flattening 1/298.25. Calculated by Stokes' formula from 300 n. mi. mean anomalies based on a combination of satellite data and terrestrial gravimetry.