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SOME PRELIMINARY DESIGN STUDIES FOR A VERY LARGE ORBITING RADIOTELESCOPE

by W. M. Robbins, Jr.

Prepared by
ASTRO RESEARCH CORPORATION
Santa Barbara, Calif.
for

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LIST OF SYMBOLS

A' = constant of integration (see text)

A_c = cross-sectional area of fiber

A_{sc} = area of spherical cap

b = spacing between grid members

c = speed of light

C = defined constant (see text)

d = diameter of grid member (or fiber)

D = aperature diameter of reflector, (or) defined constant (see text)

E = Young's modulus

$\Delta\bar{f}_o$ = acceleration caused by structural forces

F = force

I = second moment of area

K = defined constant (see text)

ℓ = fiber length

m'' = mass per unit area

$\bar{p}$ = vector external force per unit of area

p_ϕ = component of $\bar{p}$ in positive ϕ direction

p_θ = component of $\bar{p}$ in positive θ direction

p_r = component of $\bar{p}$ in positive r direction

$\bar{r}$ = position vector of point relative to c.g. of satellite

R = geometric (power) reflection coefficient

R' = ohmic (power) reflection coefficient

R_a = geometric (amplitude) reflection coefficient

R_s = radius of sphere (from which cap is cut)

$\bar{R}_1$ = component of $\bar{r}$ normal to satellite spin vector

$\bar{R}_2$ = horizontal component of $\bar{r}$

t = thickness of membrane

u = deformation along parallel circle

v = deformation along meridian

w = deformation along outward normal to surface

W = weight

W" = weight per unit area

Z_o = impedance of free space

δ = skin depth

ϵ = strain

θ = longitude of point on spherical cap

θ_R = Rayleigh angle

λ = wavelength

μ_o = permeability of free space

ν = Poisson's ratio

ρ = density

$\bar{\rho}$ = position vector of point relative to center of earth

$\bar{\rho}_o$ = position vector of c.g. of satellite relative to center of earth

$\Delta \rho$ = relative height of point relative to c.g. of satellite

σ = conductivity, (or) tensile stress

σ_S = surface conductivity

τ = shear stress

ϕ = colatitude of point on spherical cap

Φ = value of ϕ at rim of cap

$\bar{\omega}$ = spin vector of satellite

ω_o = angular velocity of satellite orbit

$\bar{1}_u$ = unit vector along parallel circle

$\bar{1}_v$ = unit vector along meridian

$\bar{1}_w$ = unit vector along outward normal to surface

$\bar{1}_{\rho o}$ = unit vector in $\bar{\rho}_o$ direction

ABSTRACT

The size and surface accuracy required of a parabolic reflector to obtain a given angular resolution in a radio telescope is discussed briefly. The use of grating and grid surfaces for the parabolic reflector is examined and the effects of conductor diameter and spacing is given. The effect of ohmic resistance in the reflector surface is analyzed.

The loads, stresses, and deformations caused by gravity gradient in a membrane-like reflector for an orbiting radio telescope with a diameter of 1500 meters is analyzed.

I. INTRODUCTION

There is an ever increasing interest in the study of electromagnetic radiation emanating from various parts of the universe, both for practical reasons and to enhance our basic knowledge of the nature of things. The radiation which can be observed at the surface of the earth by radio telescopes lies in the region between 10 mc and 30,000 mc, since radiation outside this region is strongly absorbed by the atmosphere and requires the use of orbiting satellites which operate above the atmosphere. Orbiting radio antennas operating below 10 mc have, to date, been non-directive and furnish little information about the spatial distribution of low-frequency radiation. Because directivity and wavelength are directly related to antenna size, extremely large structures will be required to fill this need.

The work performed under the present NASA contract concerning the study of advanced structural concepts for space applications has included a number of investigations which are directly applicable to the design of a very large orbiting radio telescope employing a parabolic reflector. The present report deals with two aspects of the parabolic reflector. The first is concerned with the reflection coefficient of grating and grid surfaces; the second with loads, stresses, and deformations of the surface as caused by the earth's gravity gradient.

II. REFLECTOR SIZE AND SURFACE ACCURACY

The limiting angular resolution of a radio telescope (or an optical telescope) is almost entirely determined by the ratio of the telescope aperture to the wavelength, λ, of the radiation being utilized. Although several definitions of resolution are used, a common and conservative one is the Rayleigh angle, θ_R, which is:

$$\theta_R = \frac{\lambda}{D} \cdot 70^\circ = 1.22 \frac{\lambda}{D} \text{ radian} \qquad (1)$$

where D is the diameter of the primary reflecting paraboloid. The diameter required to obtain various angular resolutions as a function of frequency is shown in Figure 1. The required D/λ ratio vs θ_R is shown in Figure 2.

In order that the above resolution be attained, that the antenna gain be maintained satisfactorily high, and that minor lobes be kept low, certain limits must be imposed upon the deviation of the reflector from the desired or nominal parabolic surface. The tolerance on parabolic reflectors is usually specified by defining the RMS deviation to be somewhere between $\lambda/16$ and $\lambda/32$, although antennas give usable results for RMS errors as large as $\lambda/8$ (or slightly larger). According to Reference 1, the principal result of reflector errors of this magnitude is to increase the minor-lobe level and pattern minima. Gain is correspondingly reduced, but beam-angle widening, while inevitable, is not as significant. The required ratio of diameter to RMS deviation as a function of angular resolution is also shown in Figure 2.

III. REFLECTION OF RADIO WAVES BY GRID AND GRATING STRUCTURES

A. GENERAL DISCUSSION:

As is shown in Reference 2, there are considerable structural-weight advantages in using a network rather than a continuous sheet or membrane for the structural component of a very large radio reflector in orbit. The network would, of course, be used to support the reflecting surface and, since the required area density of electrically conducting material turns out to be very low for a continuous sheet of conductor, it becomes apparent that the reflecting surface should be applied only to the structural network rather than as a continuous film. The results of an investigation of the reflecting properties of a conducting gridwork are reported below.

The geometric and ohmic reflection coefficients are examined separately, the geometrical coefficient being determined on the basis of infinite conductivity in the network elements, and the ohmic reflection coefficient on the basis that the conducting material is spread into a uniform conducting sheet. The result of having both types of loss simultaneously has not been investigated but it is reasonable to assume that, if the individual reflection coefficients are each near unity, the overall reflection coefficient is approximately the product of the two.

B. GEOMETRICAL REFLECTION COEFFICIENT:

A unidirectional grating of conductors can be used to reflect radio waves if the grating is oriented so that a grating element lies in the plane determined by the E vector of the incident wave and the normal to the grating "surface" at that point. For receiving a signal of random polarization, this may mean reducing the received power by one-half if the detector itself could respond to signals of arbitrary polarization. According to Reference 3, the amplitude reflection coefficient, R_a, at normal incidence, of an infinite grating of perfectly conducting circular cylinders is

$$R_a = -\frac{1}{1 - \frac{i2b}{\lambda} \ell n \frac{b}{\pi d}} \qquad \text{when } \frac{b}{\lambda} << 1 \qquad (2)$$

where:

R_a = amplitude reflection coefficient

b = spacing between centers of cylinders

d = diameter of cylinder

λ = wavelength

$i = \sqrt{-1}$

It follows that the power reflection coefficient, R, is:

$$R = \frac{1}{1 + \frac{4b^2}{\lambda^2} \ell n^2 \frac{b}{\pi d}} \qquad (3)$$

and is henceforth referred to as the geometrical reflection coefficient.

The geometrical reflection coefficient is shown in Figure 3 as a function of wavelength-to-spacing ratio for various values of spacing-to-diameter ratio. In Figure 4, the wavelength-to-spacing ratio required to give various values of geometrical reflection coefficient is shown vs the spacing-to-diameter ratio. It can be seen that grating spacings as low as $\lambda/8$ give reasonable reflection coefficients when the wires are not too small compared with the spacing, but that spacings as low as $\lambda/32$, or less, might be desirable for wires of very small size compared to the spacing.

Two such gratings at a reasonable angle to each other (greater than 30°, for instance) would reflect waves of any polarization even though the two gratings were not electrically interconnected.

If two coplanar infinite gratings were electrically interconnected at the nodes and subjected to a plane wave at normal incidence, symmetry of the fields about each node would preclude a net current from one grating element to another. The electrical connections would therefore have very little effect on the fields except in the regions very close to the nodes. A similar conclusion seems to be in order for the conditions which exist when a grid surface is used as a parabolic reflector, although no verification of this conclusion has been located in the available literature.

C. OHMIC REFLECTION COEFFICIENT:

A grid or grating which would be a good microwave reflector if the elements were perfect conductors, will be a poor reflector if the elements are made of material with sufficiently high resistivity.

In order to determine the effects of resistance in the reflector, the reflection of a plane wave at normal incidence to a very thin infinite sheet has been analyzed. The plane was considered to have the surface conductivity, σ_s. The ratio, R', of reflected to incident power is given by:

$$R' = \left(\frac{\sigma_s Z_o}{2 + \sigma_s Z_o}\right)^2 \qquad (4)$$

where:

R' = reflection coefficient due to ohmic loss

σ_s = surface conductivity

$Z_o = (\mu_o/\epsilon_o)^{1/2}$ = characteristic impedance (of free space) = 377 ohms

Consider a grating of wires oriented along the E vector of the incident wave and of such a spacing and diameter that the grating would be a good reflector if the wires were of infinite conductivity. If the wires are solid conductors or have a conductive coating, the conductivity of an equivalent conducting sheet is

$$\sigma_s = \sigma \cdot \frac{\pi d \delta}{b} \left\{ \begin{array}{l} \text{for } \delta << d \quad \text{(for solid)} \\ \\ \text{or for } \delta << t \text{ (for coating)} \end{array} \right\} \qquad (5)$$

where:

$$\delta = \left(\frac{\lambda}{\pi \sigma \mu c}\right)^{1/2} = \text{skin depth} \qquad (6)$$

σ_s = equivalent surface conductivity

σ = conductivity of wire (or coating) material

d = diameter of wire

b = wire spacing

t = coating thickness

λ = wavelength

μ = permeability

c = speed of light

If the conductive coating is thin compared with the skin depth

$$\sigma_s = \sigma \frac{\pi dt}{b} \quad \text{for} \quad t << \delta \qquad (7)$$

The skin depth in copper is

$$\delta_{cu} = 3.82 \times 10^{-6} \lambda^{\frac{1}{2}} \text{ meter} \qquad (8)$$

and is shown vs λ in Figure 5. For a "thick" conductive coating or a solid conductor, the spacing-to-diameter ratio vs wavelength is given for various ohmic reflection coefficients, R', in Figure 6. For a "thin" conductive coating, the spacing-to-diameter ratio vs coating thickness for various R' is given in Figure 7.

D. DISCUSSION:

Sufficient information has been collected, or developed herein, to define, for preliminary design purposes, the reflecting properties of a grid surface to be used as the reflector for an orbiting radio telescope.

As an example of their application, suppose that we wished to design an antenna with a network reflecting surface for operation at 4 megacycles (75 meters) with an overall reflection coefficient equal to 0.90 . The requirement could be met by selecting the geometrical and ohmic reflection coefficients each to be equal to 0.95 . The diameter of the structural fibers in the gridwork is likely to be determined by other than electrical requirements (such as resistance to buckling by compressive loads). Suppose that the fiber diameter as determined by these requirements is 0.4 mm (0.016 inches). Then, in order to obtain a geometrical reflection coefficient equal to 0.95 we have, from Figure 4

$$\frac{\lambda}{b} = 60, \quad \frac{b}{d} = 3000, \quad b = 1.25 \text{ meters}$$

Also, from Figure 7, the required surface coating of copper is 0.0035 mm thick.

IV. LOADS, STRESSES AND DEFORMATIONS OF AN ORBITING MEMBRANE-LIKE REFLECTOR

A. PRELIMINARY DISCUSSION:

In order to determine the magnitude of deformation that the earth's gravity gradient will have upon the reflector for a very large orbiting radio telescope, certain preliminary analyses of the problem have been made. By assuming that the reflector is a spherical cap, rather than a paraboloid of revolution, the analysis is considerably simplified. Further, the network structure is approximated by a continuous isotropic membrane and it is assumed that just enough constraint is applied to the rim of the spherical cap to make the system statically determinate.

During the course of the analysis, the problem is restricted to a spherical diameter of 1800 meters, an opening diameter of 1500 meters, and a circular orbit at an altitude of 550 statute miles.

B. EXTERNAL FORCES FROM GRAVITY GRADIENT AND SPIN:

If an earth's satellite is in a circular orbit, the various points within the satellite are subjected to gravity fields of slightly different magnitude and direction. The various points are also subject to different accelerations because of the spin which the satellite may have. As a result of these two causes, certain structural forces must be supplied to each point in order to maintain the relative positions of the various points, since the satellite is assumed to have some specified form.

Let:

ω_o = angular velocity of circular orbit

$\bar{\omega}$ = spin vector of satellite

$\bar{r}$ = position vector of point relative to c.g. of satellite

$\bar{\rho}_o$ = position vector of c.g. of satellite relative to center of earth

$\bar{\rho}$ = position vector of point relative to center of earth

$\bar{R}_1$ = component of $\bar{r}$ normal to $\bar{\omega}$

$\overline{R}_2$ = component of $\overline{r}$ normal to $\overline{\rho}_o$

(i.e. the horizontal component of $\overline{r}$).

$\Delta\rho$ = $\rho - \rho_o$ = vertical position of point

relative to c.g. of satellite

$\overline{1}_{\rho_o}$ = $\frac{\overline{\rho}_o}{\rho_o}$ = unit vector in $\overline{\rho}_o$ direction

$\Delta\overline{f}_o$ = acceleration (of point) which is caused

by structural forces

These various relationships are shown in Figure 8.

Under the condition that the gravity gradient generates no net torques on the satellite (this condition is satisfied for a number of special conditions) the acceleration, $\Delta\overline{f}_o$, which is caused by structural forces has been shown to be

$$\Delta\overline{f}_o = \omega_o^{\,2}\left[\overline{R}_2 - 2\Delta\rho\overline{1}_{\rho o}\right] - \omega^2\overline{R}_1 \qquad (9)$$

Thus the structural forces which are supplied per unit of mass at each point in the structure can be divided into three components which are:

a) an outward-directed horizontal force of magnitude $\omega_o^{\,2}\Delta h$ (where Δh is the horizontal displacement from the c.g.) which results from the gravity gradient,

b) an inward-directed vertical force of magnitude $2\omega_o^2\Delta\rho$ which results from gravity gradient, and

c) a force of magnitude $\omega^2 R_1$, directed toward the inertial spin axis (where R_1 is the distance of the point from the spin axis) and resulting from inertial spin.

If a spherical cap (a spherical shell of radius R_s truncated by a single plane) is orbited with its axis of symmetry normal to the orbital plane, the components of the acceleration caused by the structural forces can be expressed in a spherical coordinate system in which the polar axis is coincident with the axis of symmetry of the cap, as is shown in Figure 9. Letting $\bar{1}_u$, $\bar{1}_v$, and $\bar{1}_w$ be unit vectors in the direction of the parallel circle, the meridian, and the outward normal to the surface, respectively, and θ and ϕ be the longitude and colatitude, respectively, and Φ be the value of ϕ at the rim (at the circle of truncation) then:

$$\Delta \bar{f}_o = \frac{\omega_o^2 R_s}{2} \left[\begin{array}{l} \left\{ \left[3 \sin \phi \right] \sin 2\theta \right\} \bar{1}_u \\ \left(\begin{array}{l} \left[(1 + \cos \Phi) \sin \phi - \frac{3}{2} \sin 2\phi \right] \\ + \left[-\frac{3}{2} \sin 2\phi \right] \cos 2\theta \end{array} \right) \bar{1}_v \\ \left(\begin{array}{l} \left[\frac{1}{2} - (1 + \cos \Phi) \cos \phi + \frac{3}{2} \cos 2\phi \right] \\ + \left[-\frac{3}{2} + \frac{3}{2} \cos 2\phi \right] \cos 2\theta \end{array} \right) \bar{1}_w \end{array} \right] \tag{10}$$

Then $\bar{p} = - m'' \Delta \bar{f}_o$

where: (11)

$\bar{p}$ = vector external force per unit of area

m'' = mass per unit of area

Using the nomenclature of Reference 4:

$$p_\phi = - \frac{m'' \omega_o^2 R_s}{2} \left\{ \begin{array}{l} \left[(1 + \cos \Phi) \sin \phi - \frac{3}{2} \sin 2\phi \right] \\ - \left[\frac{3}{2} \sin 2\phi \right] \cos 2\theta \end{array} \right\} \tag{12a}$$

$$p_\theta = - \frac{m'' \omega_o^2 R_s}{2} \left\{ \left[3 \sin \phi \right] \sin 2\theta \right\} \tag{12b}$$

$$p_r = -\frac{m''\omega_o^2 R_s}{2}\left\{\begin{array}{l}\left[\frac{1}{2} - (1 + \cos\Phi)\cos\phi + \frac{3}{2}\cos 2\theta\right] \\ \\ -\frac{3}{2}\left[1 - \cos 2\phi\right]\cos 2\theta\end{array}\right\} \quad (12c)$$

As a concrete example, let the opening diameter of the spherical cap be 1500 meters, and let the focal ratio (as defined at the center of the spherical cap) be 0.3 . For such a cap

$$f = 0.5R_s$$

$$f/D = 0.3$$

$$R_s = 0.6D = 900\text{m} = 2950\text{ft}$$

$$\cos\Phi = 0.553$$

$$\sin\Phi = 0.833$$

The external force components p_ϕ , p_θ , and p_r , each contain a term which varies sinusoidally with 2θ , and two of them contain a term which is independent of θ .

$$p_\phi = p_{\phi o} + p_{\phi 2}\cos 2\theta \quad (13a)$$

$$p_\theta = p_{\theta o} + p_{\theta 2}\sin 2\theta \quad (13b)$$

$$p_r = p_{ro} + p_{r2}\cos 2\theta \quad (13c)$$

where, for our particular example,

$$p_{\phi o} = -K(1.553\sin\phi - 1.500\sin 2\phi) \quad (14a)$$

$$p_{\theta o} = 0 \quad (14b)$$

$$p_{ro} = - K\ (0.500 - 1.553 \cos \phi + 1.500 \cos 2\phi) \qquad (14c)$$

$$p_{\phi 2} = K\ (3 \sin \phi \cos \phi) \qquad (14d)$$

$$p_{\theta 2} = - K\ (3 \sin \phi) \qquad (14e)$$

$$p_{r2} = K\ (3 \sin^2 \phi) \qquad (14f)$$

$$K = \frac{m'' \omega_o{}^2 R_s{}^2}{2} \qquad (15)$$

C. MEMBRANE STRESSES:

By the methods used in Reference 4, the normal and shearing stresses in the membrane can be found under the assumptions that the meridianal component of stress vanishes at the rim. Both of these conditions at the rim follow from the assumption that the system is made statically determinate by a system at the rim, which can apply forces only in the plane of the rim. Then the components of stress are

$$\sigma_\phi = \sigma_{\phi o} + \sigma_{\phi 2} \cos 2\theta \qquad (16a)$$

$$\tau_{\phi\theta} = \tau_{\phi\theta 2} \sin 2\theta \qquad (16b)$$

$$\sigma_\theta = \sigma_{\theta o} + \sigma_{\theta 2} \cos 2\theta \qquad (16c)$$

where:

$$\sigma_{\phi o} = C\left[\frac{0.345}{(1 + \cos \phi)} - 0.222\right] \qquad (17a)$$

$$\sigma_{\theta o} = C \left[- \frac{0.345}{(1 + \cos\phi)} + 0.444 \right.$$

$$\left. + 0.345 \cos\phi - \frac{2}{3} \cos^2\phi \right] \quad (17b)$$

$$\sigma_{\phi 2} = - C \left[\frac{1.608}{(1 + \cos\phi)^2} - \frac{2}{3} \right] \quad (17c)$$

$$\tau_{\phi \theta 2} = C \left[\frac{1.608}{(1 + \cos\phi)^2} - \frac{2}{3} \cos\phi \right] \quad (17d)$$

$$\sigma_{\theta 2} = C \left[\frac{1.608}{(1 + \cos\phi)^2} - \frac{2}{3} \cos^2\phi \right] \quad (17e)$$

and

$$C = \frac{9R_s K}{2t} = \frac{9}{4} \frac{m''}{t} \omega_o^2 R_s^2 = \frac{9}{4} \rho \omega_o^2 R_s^2 \quad (18)$$

t = thickness of membrane

ρ = density of membrane material

For the circular orbit where $\omega_o = 10^{-3}$ rad/sec (which corresponds to an orbital altitude of approximately 550 statute miles) and for R_s = 2950 feet,

$$\frac{C}{\rho} = 7.31 \frac{\text{lbf/in}^2}{\text{lbm/in}^3} \quad (18a)$$

The values of the above stress components are shown as functions of ϕ in Figure 10. Also shown is an additional stress component, $\sigma_{\theta 3}$, which would be added to $\sigma_{\theta o}$ if the satellite were spinning at an angular frequency equal to the orbital angular frequency.

D. DEFORMATIONS:

According to Reference 4, the differential equation which expresses the meridional deformations of the spherical surface is

$$\frac{d^2 v_n}{d\phi^2} \sin^2 \phi - \frac{dv_n}{d\phi} \cos \phi \sin \phi + v_n (1 - n^2)$$

$$= nR_s \gamma_{\phi\theta n} \sin \phi + R_s \left(\frac{d\epsilon_{\phi n}}{d\phi} - \frac{d\epsilon_{\theta n}}{d\phi} \right) \sin^2 \phi \qquad (19)$$

where the meridional deformation, v, is

$$v = \sum_{n=o}^{\infty} v_n \qquad (20)$$

and the n indicates that the particular component of v corresponds to the particular set of σ's which vary with θ as $\sin n\theta$ and/or $\cos n\theta$. Also the component of deformation in the direction of the outward normal to the surface is

$$w_n = R_s \epsilon_{\phi n} - \frac{dv_n}{d\phi} \qquad (21)$$

and the component of deformation in the direction of the parallel circle is

$$u_n = \frac{R_s}{n}(\epsilon_{\theta n} - \epsilon_{\phi n}) \sin\phi - \frac{1}{n}\left(v_n \cos\phi - \frac{dv_n}{d\phi}\sin\phi\right) \qquad (22)$$

where by Hooke's law:

$$\epsilon_{\phi n} = \frac{1}{E}(\sigma_{\phi n} - \nu\sigma_{\theta n}) \qquad (23)$$

$$\epsilon_{\phi n} - \epsilon_{\theta n} = \frac{(1+\nu)}{E}(\sigma_{\phi n} - \sigma_{\theta n}) \qquad (24)$$

E = Young's modulus

ν = Poisson's ratio

For n = 2 , the total solution to the equation for ν is

$$v_2 = D\left\{\begin{aligned} &\left[A' - \frac{0.804}{(1-\cos^2\phi)} + \frac{2}{3(1-\cos\phi)} - \frac{2}{3}\cos\phi \right.\\ &\left. - \frac{1}{6}\cos^2\phi\right]\frac{\sin^3\phi}{(1+\cos\phi)^2} \\ &+\left[\frac{0.536}{(1+\cos\phi)^3} - \frac{0.804}{(1+\cos\phi)^2} - \frac{2}{3(1+\cos\phi)}\right. \\ &\left. - \frac{2}{3}\cos\phi + \frac{1}{6}\cos^2\phi\right]\frac{\sin^3\phi}{(1-\cos\phi)^2}\end{aligned}\right\} \qquad (25)$$

where

$$D = \frac{R_s C(1 + \nu)}{E} = \frac{9}{2} \cdot \frac{R_s{}^2 K(1 + \nu)}{Et} = \frac{9}{4} \frac{R_s^3 \omega_o{}^2 \rho(1 + \nu)}{E} \qquad (26)$$

In order to evaluate the properties of the membrane of which the spherical cap is assumed to be formed, it is now assumed to be a network of equilateral triangles of identical members of unidirectional bundles of fiberglas filaments. Under these conditions

t = average "thickness" of net

$E = E_m/3$

E_m = longitudinal tensile modulus of fiberglas members

$\nu = \frac{1}{3}$

The following properties are assumed:

$\rho = 0.08$ lbm/in^3

$E_m = 7 \times 10^6$ lbf/in^2

$\omega_o = 10^{-3}$ rad/sec

$R_s = 2950$ ft

Under these conditions

$$D = 0.000985 \text{ ft} = 0.0118 \text{ in}$$

It has already been assumed that the rim constraint for $n = 2$ is such that $\sigma_{\phi 2}(\Phi) = 0$. It is now further assumed that the rim constraint is capable of supplying only an in-plane shear stress and that the constraint is of such a nature that the u_2 displacement of the constraint is proportional to $\tau_{\phi\theta 2}$ at each point on the rim. Any passive constraint will have a positive $\tau_{\phi\theta 2}$ associated with a negative u_2 and vice versa.

Let the rim constraint be formed by a family of filaments in the form of squares, as shown in Figure 11. Such a system is capable of supplying the required in-plane shear stress, since each point is connected to two other points with the same shear stress because of the $\sin 2\theta$ variation of $\sigma_{\phi\theta 2}$.

Let a length on the rim of the spherical cap contain the termination of two orthogonal filaments of the rim-constraint system, as is shown in Figure 12, and let

t_2 be the average thickness of filaments in each of the two directions. Consider one-half of each length to be associated with the region ds. Each of the two half filaments has the length $\frac{\sqrt{2}}{4} \cdot D$ and the cross-section $A_c = t_2 \cdot ds/\sqrt{2}$, where D = opening diameter of reflector = $R_s/0.6$.

Then

$$\frac{dF}{\sqrt{2}} = \epsilon\, E_2 A_c = \frac{\left(\frac{u_2}{\sqrt{2}}\right)}{\left(\frac{D\sqrt{2}}{4}\right)} \cdot E_2 \cdot \frac{t_2 ds}{\sqrt{2}} \tag{27}$$

$$\frac{dF}{ds} = \frac{1.2 u_2 E_2 t_2}{R_s}$$

Since dF is the force of the membrane on the constraint,

$$t_2 \tau_{\phi\theta 2}(\Phi) = -\frac{dF}{ds} = -\frac{1.2 u_2 E_2 t_2}{R_s}$$

Evaluation of $\tau_{\phi\theta 2}$ and u_2 at the rim and substitution in the above yields

$$A' = -1.081 - 0.7765 \frac{Et}{E_2 t_2} \tag{28}$$

If the filaments in the rim constraint were inelastic, then $\frac{E_2 t_2}{Et} = \infty$ and $A' = -1.081$.

However, when $\frac{E_2 t_2}{Et} = 0.5$ (which corresponds to a constraint-system density at the rim of 1/3 the reflector density if the reflector is made of equilateral triangles of equal filaments of the same material), then $A' = -2.634$.

The most important deformation of a reflector is the component, w, normal to the surface, and w_2 has been plotted in Figure 13 for the two values of A' obtained above.

E. ELASTIC STABILITY:

It will be observed from Figure 10 that a considerable portion of the spherical cap is in a state of biaxial compression due to the effects of gravity gradient. Although the magnitudes of the stresses are very low, the diameter of the structural members required to satisfy electrical requirements is also very low, so that failure by elastic instability is likely to be an important design criterion.

Buckling may occur in one of two ways. The first consists of Euler buckling of an individual element which may, for simplicity, be assumed to be pinned at its ends. The buckling stress is obtained from the formula

$$\sigma_1 = \frac{\pi^2 EI}{A\ell^2} \qquad (29)$$

where:

EI = bending stiffness of element

A = cross-sectional area of element

ℓ = length of element

The second type of buckling failure is general instability. The exact determination of the buckling condition for a non-uniform stress distribution, such as that shown in Figure 10, is difficult. In the present instance, however, the diameters of structural members are very small compared to the radius of the spherical cap, so that the elementary formula for the buckling of a sphere due to uniform pressure (see Reference 4, p. 477) can be used, provided that the stress predicted by the formula is interpreted as the maximum compressive stress occuring anywhere on the spherical surface. The formula, as modified to apply to discrete members, is

$$\sigma_2 = \frac{2E}{R_s}\sqrt{\frac{I}{A}} \qquad (30)$$

where I and A have the same meaning as in equation (29).

For the case of round fibers with diameter, d, for equations (29) and (30) become

$$\sigma_1 = \frac{\pi^2}{16} E \cdot \left(\frac{d}{\ell}\right)^2 \qquad (31)$$

$$\sigma_2 = \frac{Ed}{2R_s} \qquad (32)$$

These formulas may be used to size the members of the gridwork. Fiber diameter, d, is determined by equation (32). Mesh spacing is then determined by equation (31). As an example, consider an antenna with the following properties:

Opening diameter of spherical cap = 1500 m = 4920 ft

Radius, R_s = 2950 ft

Material: aluminum ($E = 10^7$, $\rho = 0.1$ lb/in^3)

Orbital altitude: 550 statute miles ($\omega_o = 10^{-3}$ rad/sec)

From Figure 10, the maximum compressive stress at any point is approximately

$$\sigma_{max} = 2.3 \times 0.10 = 0.23 \text{ psi}$$

The fiber diameter, as determined by equation (32), is

$$d = 2R_s \cdot \frac{\sigma_{max}}{E} = 2 \times 2950 \times 12 \times \frac{0.23}{10^7} = 0.00163 \text{ in}$$

The element length, as determined by equation (31) is

$$\ell = \frac{\pi}{4}\sqrt{\frac{E}{\sigma_{max}}}\, d = \frac{\pi}{4} \times \sqrt{\frac{10^7}{0.23}} \times 0.00163 = 26.8 \text{ inches}$$

We may now use the results of sections B and C to determine whether the reflection coefficient of the gridwork is adequate. The distance between parellel fibers, b, is equal to $\frac{1}{2}\sqrt{3} \cdot \ell = 23.2$ inches. The spacing to diameter ratio b/d is equal to 14,200. At a frequency of ten megacycles the wavelength to spacing ratio is 44. From Figure 4 the geometrical reflection coefficient is approximately equal to 0.88 . From Figure 6, using the facts that the wavelength at 10 megacycles is 30 meters and that the conductivity of copper is about 1.6 times the conductivity of aluminum, it is seen that the ohmic reflection coefficient is about 0.95 .

Further study is required to determine whether or not these reflection coefficients are adequate. The overall reflection coefficient could be raised above 0.90 by reducing the mesh spacing, ℓ, to one foot.

It is of interest to compute the structural weight of the spherical cap antenna. The weight per unit area is

$$W'' = \frac{1}{2} \frac{\text{(Weight of three elements)}}{\text{(Area of mesh triangle)}}$$

which, for aluminum fibers of 0.00163" diameters and one foot mesh spacing, is equal to 0.867×10^{-5} lb/ft^2. The area of the spherical cap is

$$A_{sc} = 2\pi R_s^2 (1 - \cos \Phi) = 2\pi \times (2950)^2 (1 - 0.553)$$
$$= 24.4 \times 10^6 \text{ ft}^2$$

The total structural weight of the antenna is

$$W = W'' \quad A_{sc} = 0.867 \times 10^{-5} \times 24.4 \times 10^6 = 212 \text{ lb}$$

Conditions other than those considered here will undoubtedly increase the structural wieght, perhaps by a large factor.

For comparison, the weight of an aluminum membrane that satisfies the general elastic instability requirement will be computed. The required thickness (see Reference 4, p. 477) is

$$t = \sqrt{3(1 - \nu^2)} \cdot R_s \frac{\sigma_{max}}{E} = 0.00135 \text{ in}$$

The weight density is

$$W'' = \rho t = 0.1 \times 0.00135 \times 144 = 0.0194 \text{ lb/ft}^2$$

The total weight is

$$W = W'' A_{sc} = 0.0194 \times 24.4 \times 10^6 = 473{,}000 \text{ lb}$$

It is clear from this calculation that solid membranes are not feasible structures for orbiting antennas of the size considered here.

F. DISCUSSION OF RESULTS:

In the spherical cap considered herein (with an opening diameter of 1500 meters) the equivalent membrane stresses caused by gravity gradient are 0.49 psi in tension, and 0.32 psi in compression, for a material with a density of 0.1 lb/in^3 (aluminum).

If a fiberglas network structure is used to form the spherical cap, the peak second harmonic deflection, w_2 , normal to the surface as caused by gravity gradient, is only 2.5×10^{-3} ft or 0.030 in. Although the symmetrical displacement, w_o , normal to the surface has not been evaluated, it is expected to be lower than w_2 .

On the basis of buckling instabilities, the total structural weight, using aluminum members, is only 212 lb.

Thus, the loads imposed by gravity gradients upon the 1500-meter reflector, when operating in the low megacycle range and orbiting at an altitude of 550 statute miles, will cause only negligible displacement normal to the surface and will not necessitate large structural weights in order to prevent buckling instabilities from compressive loads.

Astro Research Corporation
P. O. Box 4128,
Santa Barbara, California, October 15, 1965

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3. Twersky, Victor, ON SCATTERING OF WAVES BY THE INFINITE GRATING OF CIRCULAR CYLINDERS, Sylvania Engineering Report, EDL-E28, March 1, 1958.

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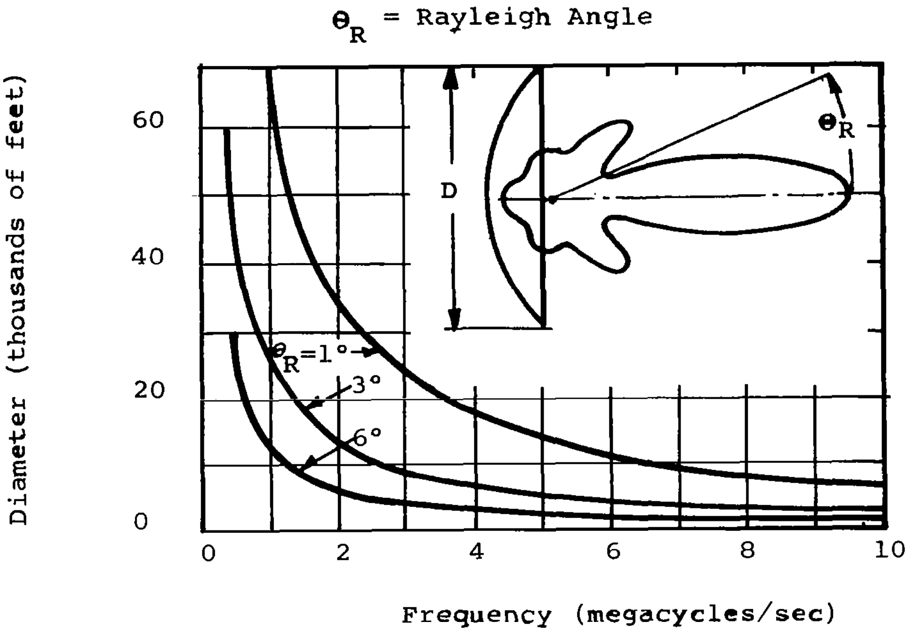


Figure 1. - Antenna Dish Diameter.

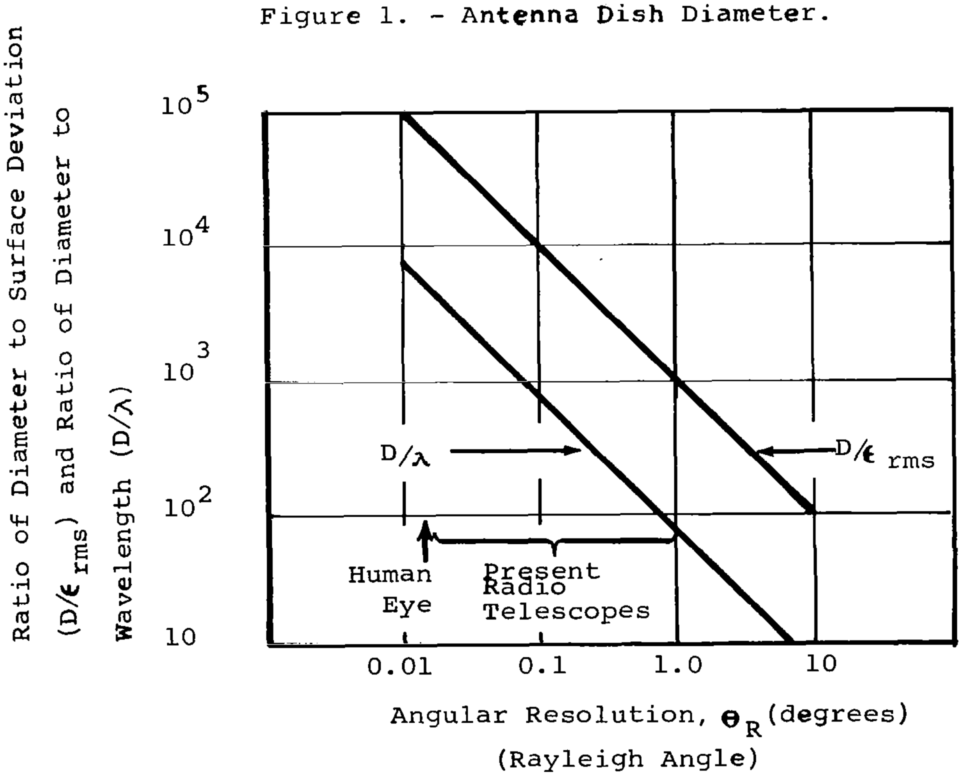


Figure 2. - Size and Accuracy Requirements

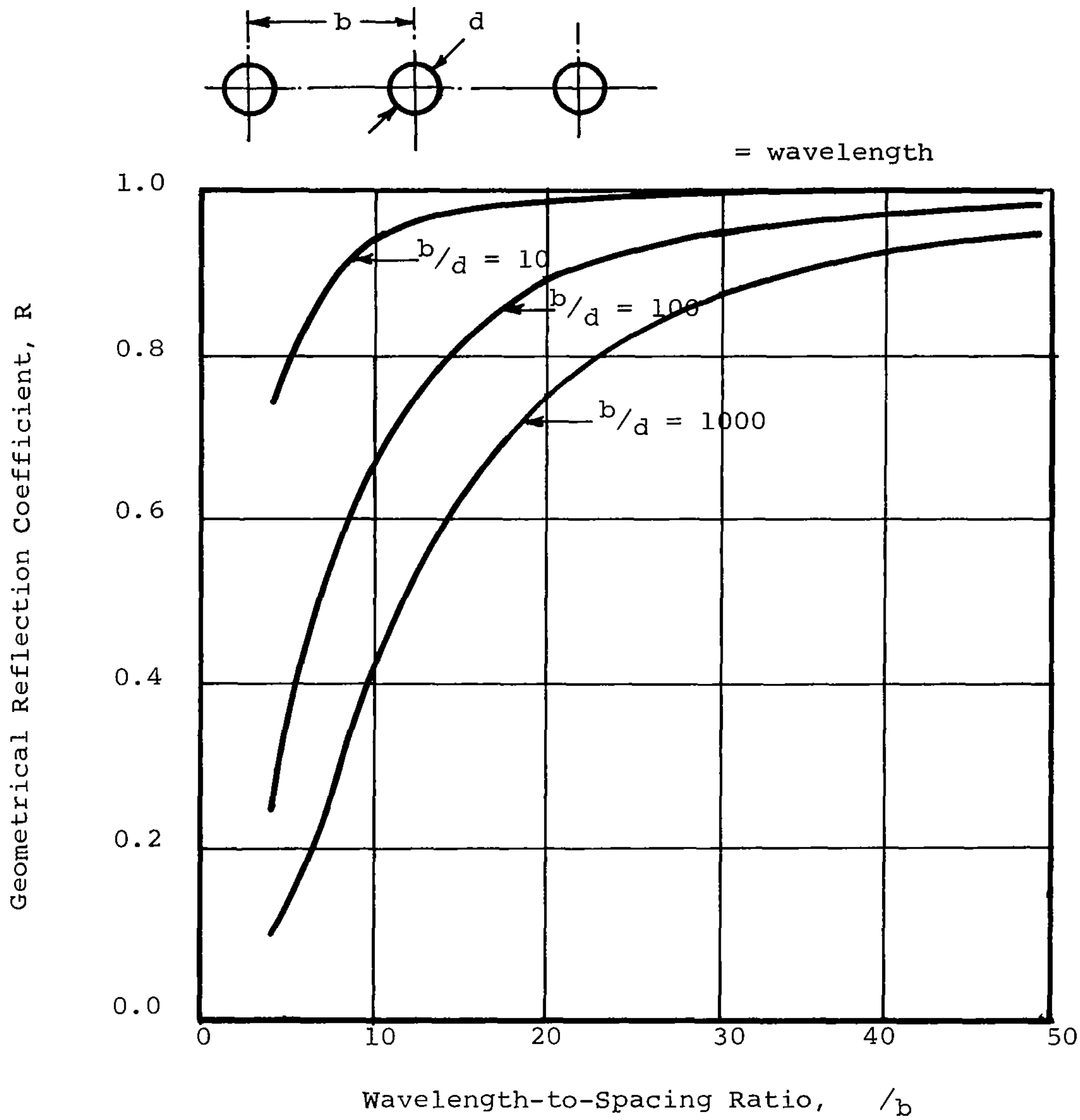


Figure 3. - Geometrical Reflection Coefficient for Grating of Wires.

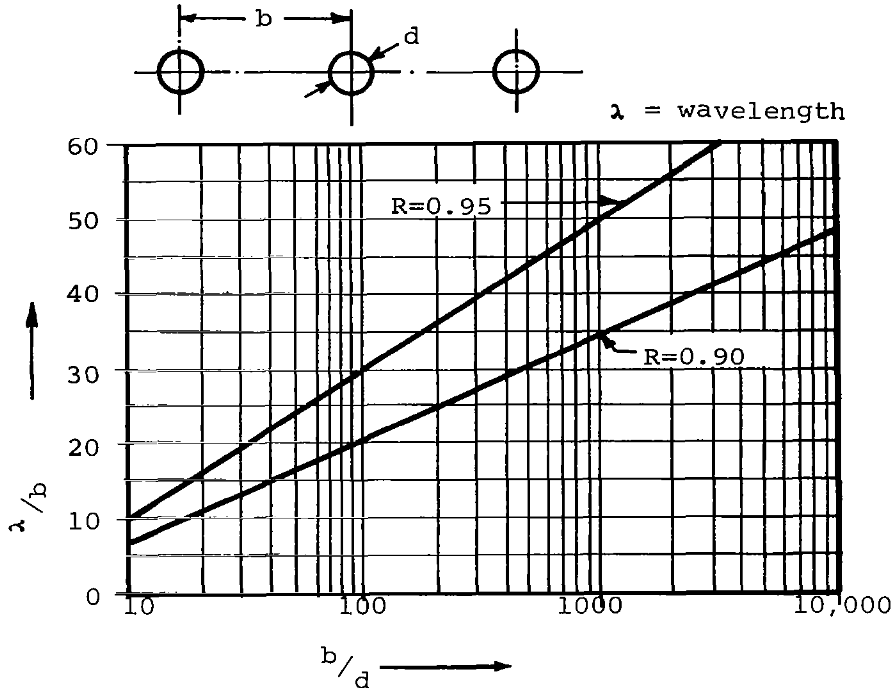


Figure 4. - Grating Geometry Required for Various Geometrical Reflection Coefficients.

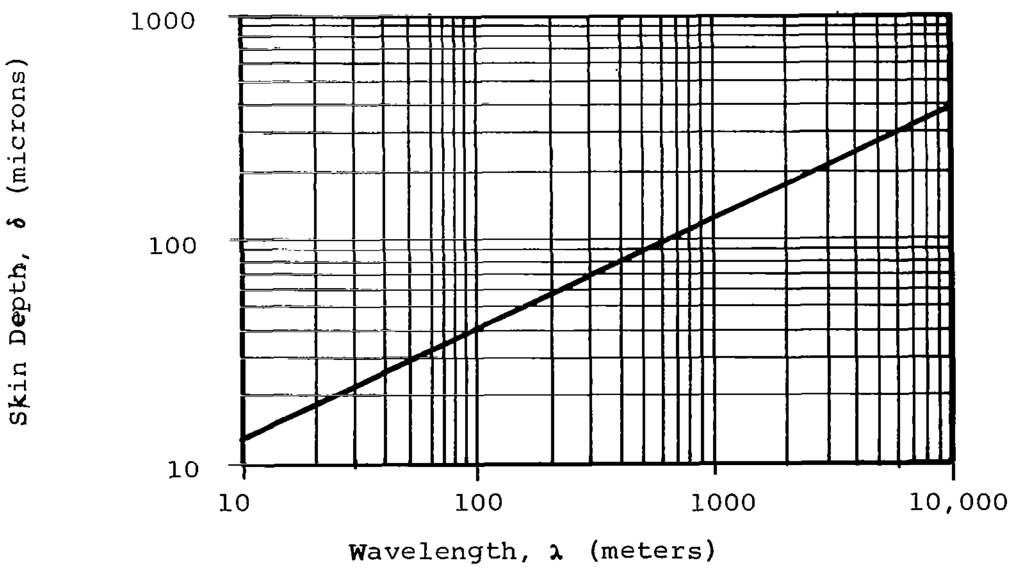


Figure 5. - Skin Depth in Copper.

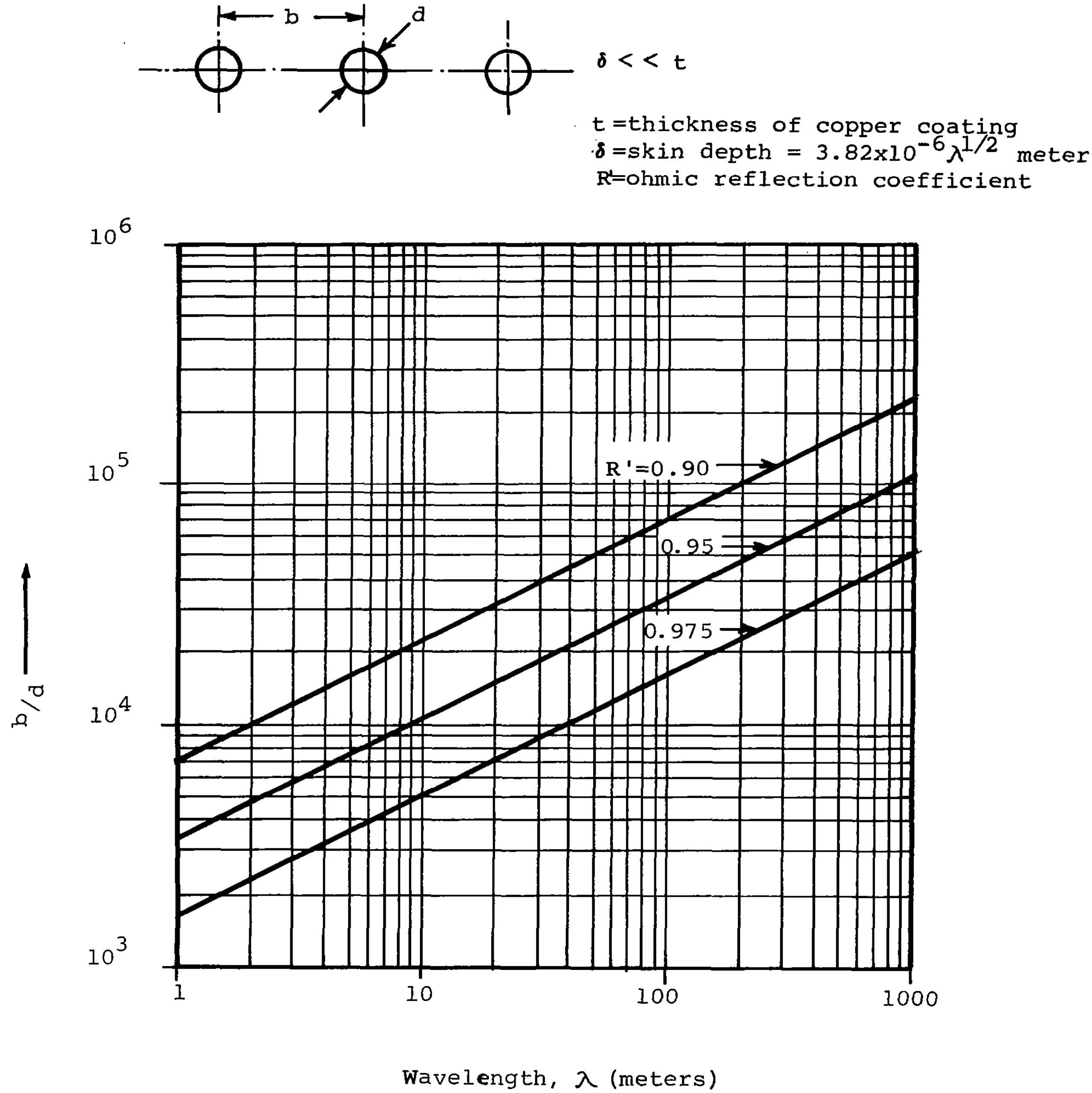
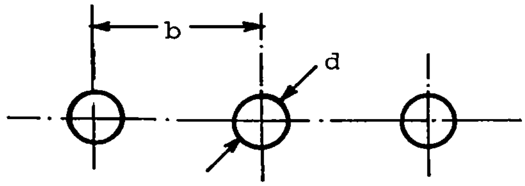


Figure 6. - Ohmic Reflection Coefficient for "Thick" Copper Coating.



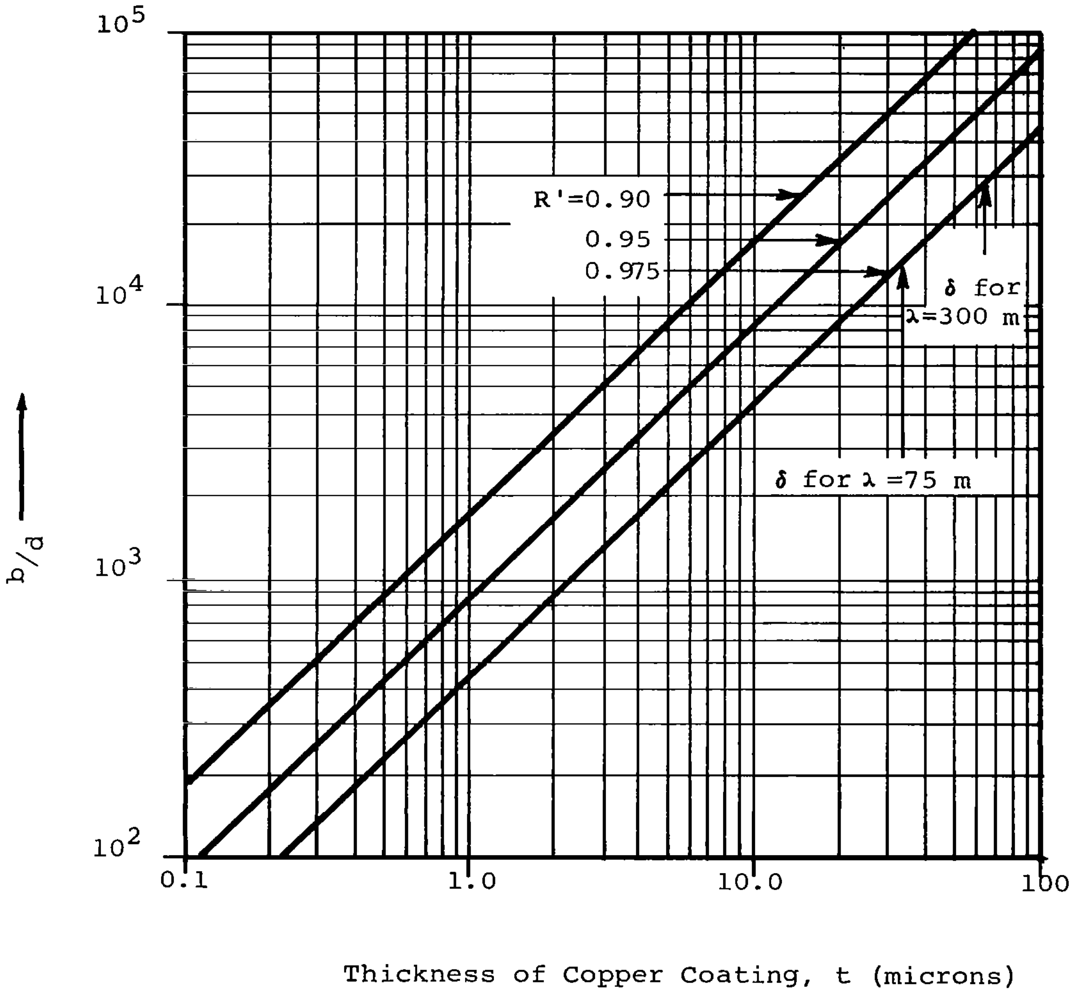


Figure 7. - Ohmic Reflection Coefficient for "Thin" Copper Coating.

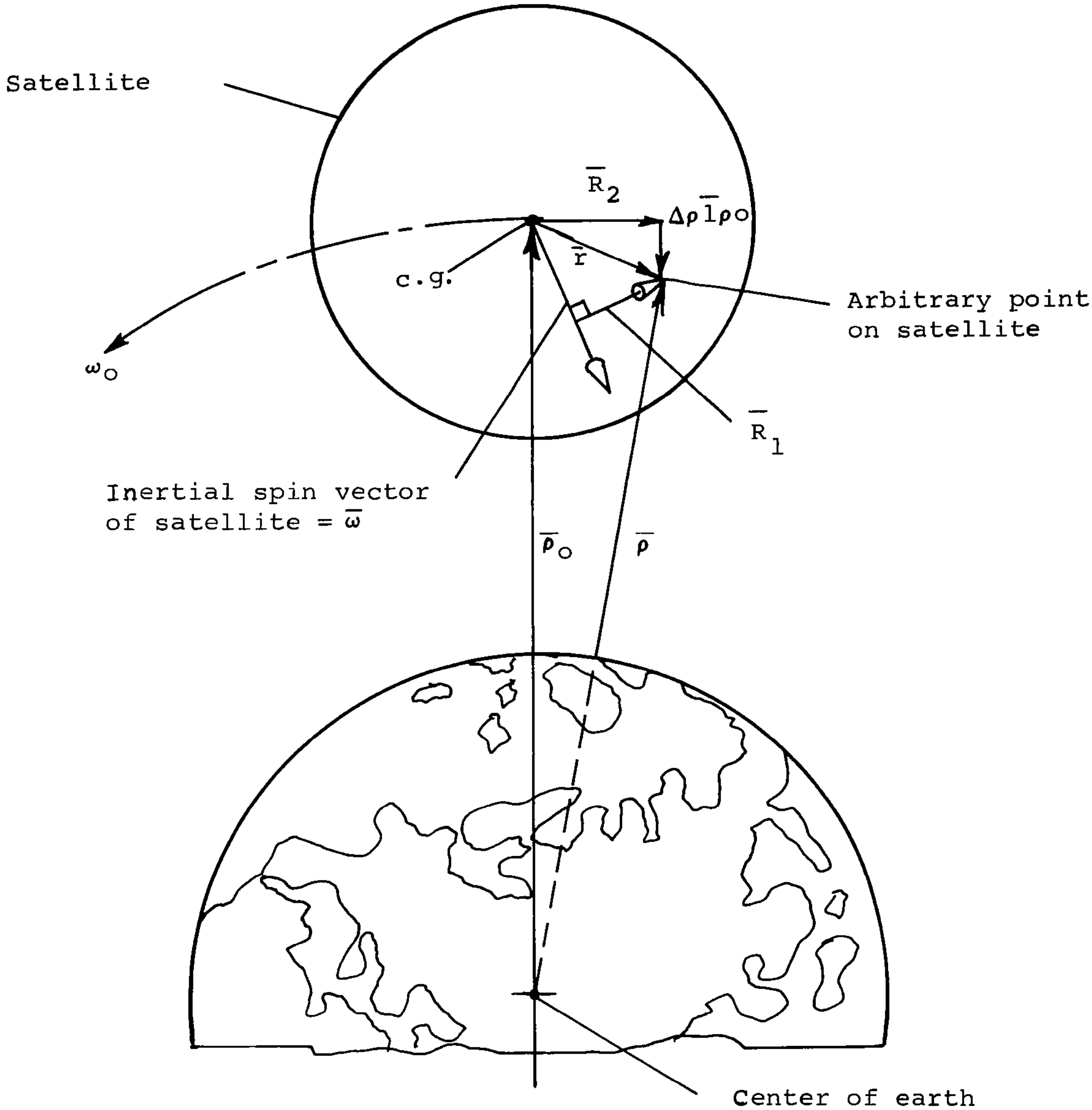


Figure 8.- Location of Arbitrary Point on Satellite.

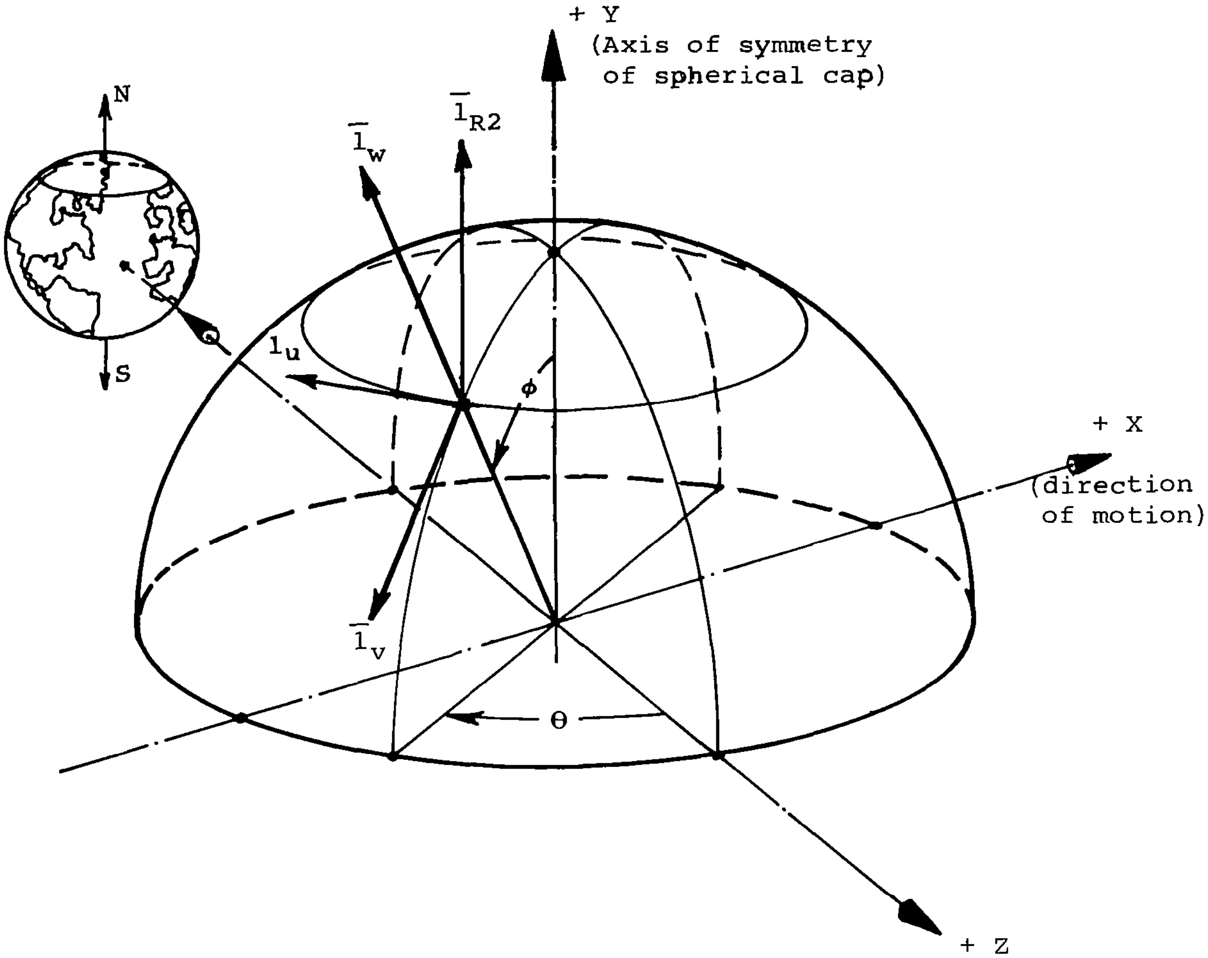


Figure 9. - Coordinate System Used in Analysis of Orbiting Spherical Cap.

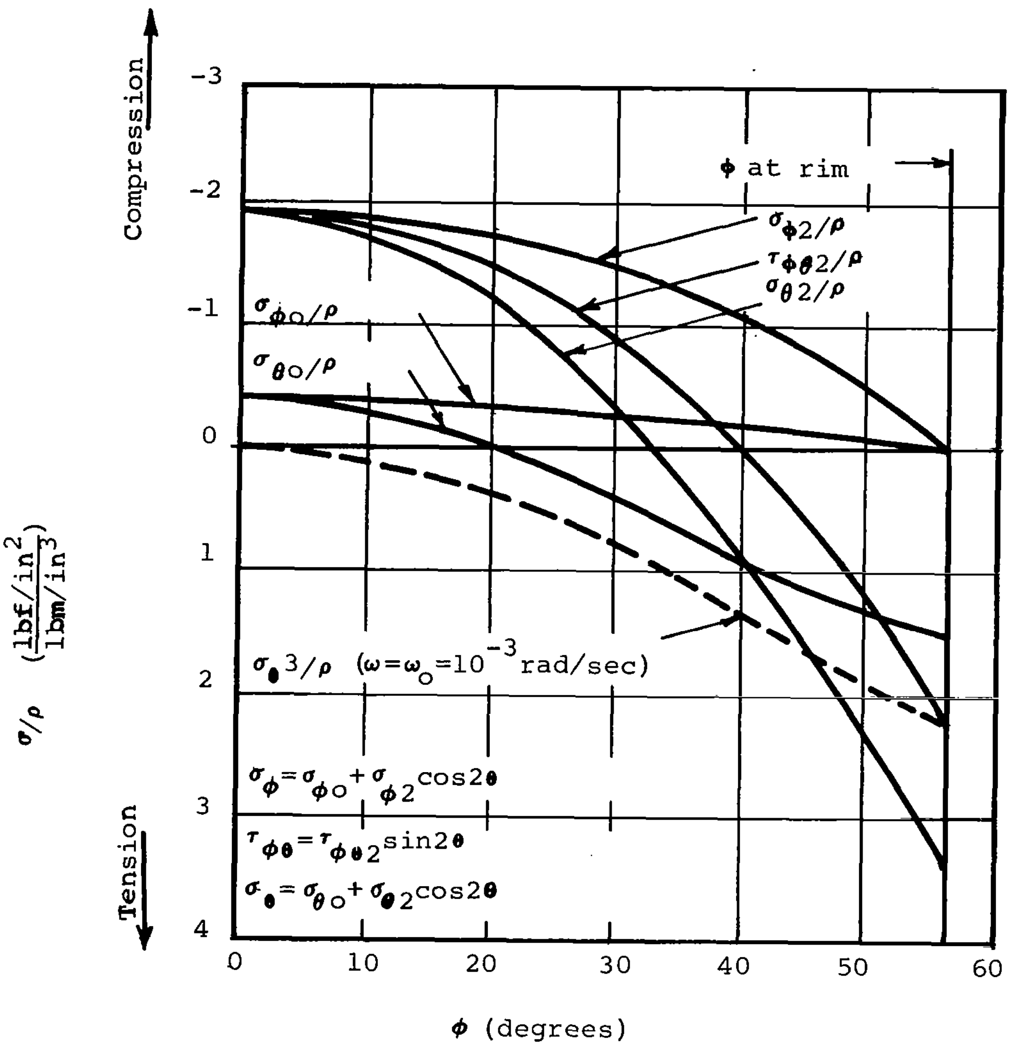


Figure 10. - Stresses in Spherical Cap Due to Gravity Gradient and Spin.

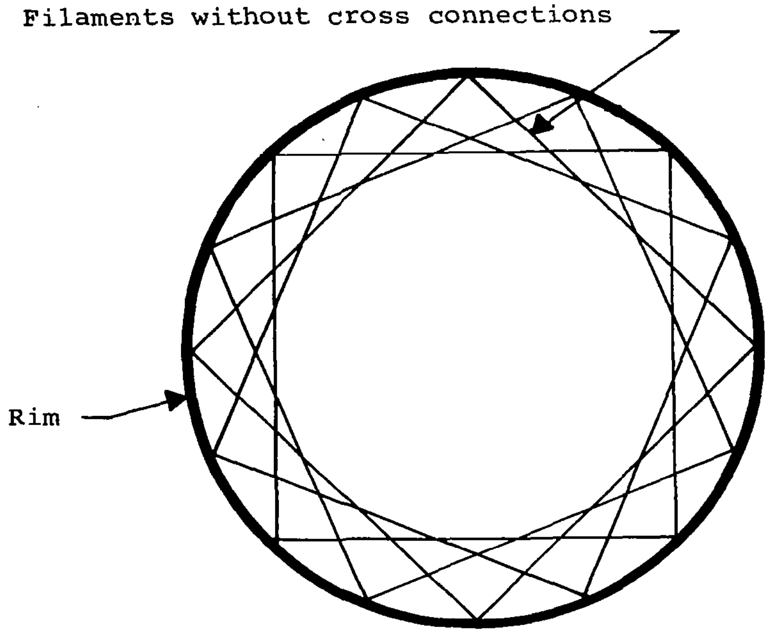


Figure 11. - Rim-Constraint System

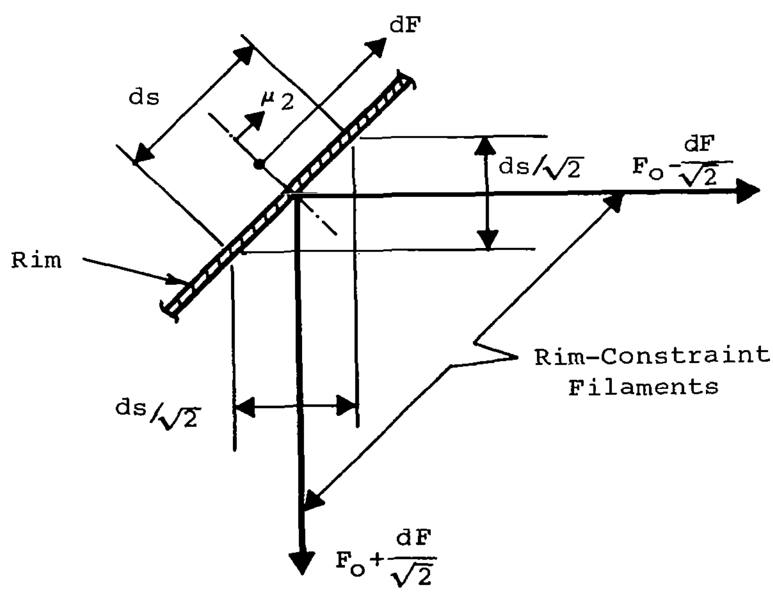


Figure 12. - Geometry at Rim-Constraint Point

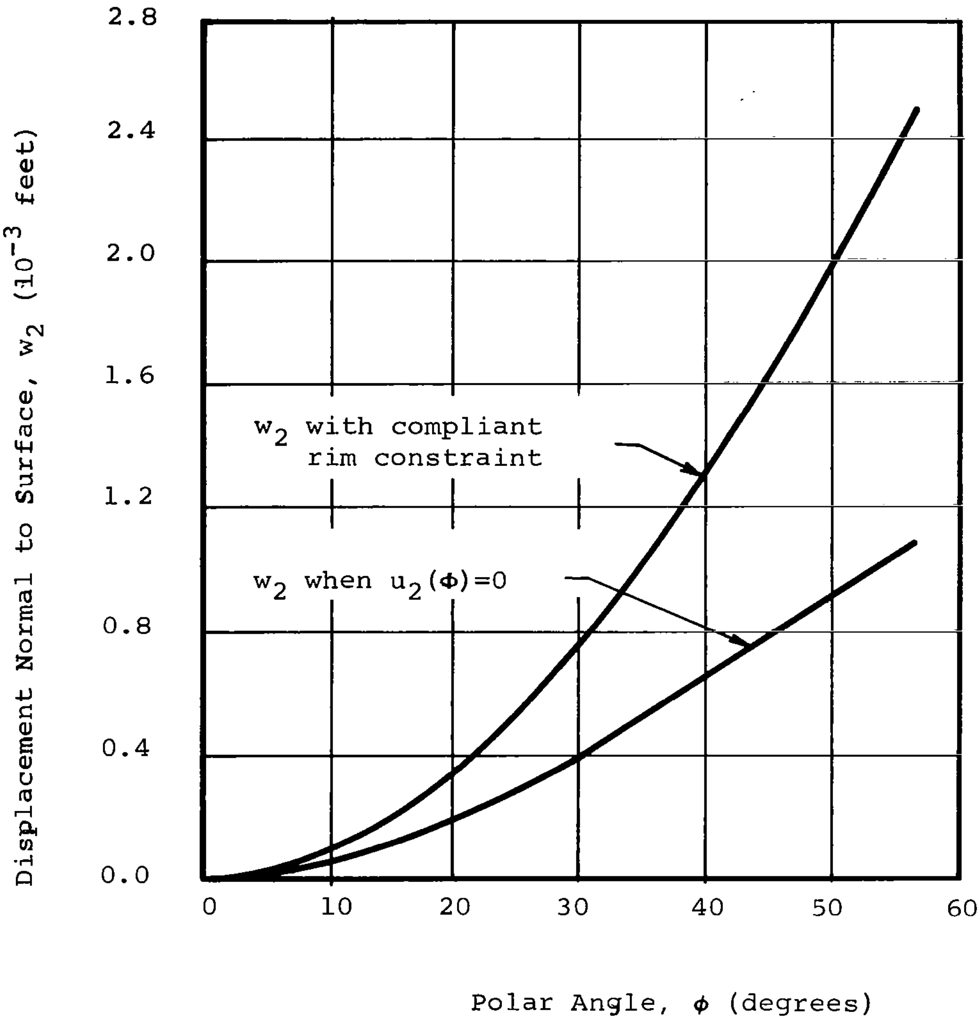


Figure 13. - Displacement Normal to Surface

(for n = 2)

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