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DETERMINATION OF BOUNDARIES OF CLOUDINESS

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DETERMINATION OF BOUNDARIES OF CLOUDINESS

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by I. A. Kibel'

SUMMARY

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The determination of boundaries of cloudiness, as described here, is based upon considering an adiabatic flow, in which are disseminated separate regions with pseudoadiabatic processes, whose boundaries are not known in advance and must be determined alongside with the solution of the problem of motion itself. On that basis, conditions are derived outside and inside the cloud, and a particular example is considered.

author

* * *

In the hyrdodynamic theory of cloudiness occurring from the leeward side of an obstacle, the cloud boundary is usually determined after the adiabatic motion near the obstacle is found as the geometric spot of points, at which the moisture, transferred by the adiabatic flow, begins to saturate the space [1]. Meanwhile, and provided no precipitations are falling, the motion inside the cloud must be viewed as pseudoadiabatic : instead of preservation of potential temperature we must assume that of pseudopotential. The system of hydrodynamics equations will then be modified in its form and all the structure of the solution may be disrupted. A correct statement of the problem would then be the consideration of an adiabatic flow, in which separate regions with a pseudoadiabatic process are dissemniated; the

* OB OPREDELENII GRANITS OBLACHNOSTI

boundaries of these regions are not known in advance and must be determined alongside with the solution of the very problem of motion.

The condition for saturation can be taken as the boundary condition at the edge of these region. Introducing into the consideration the specific moisture q, we shall write at boundary $q = q_{max}(T, p)$, where q_{max} is determined by the well known Magnus formula and is dependent on both, the temperature T and pressure p [2]:

$$q_{\max} = 0{,}23 \cdot 10^{-3} \frac{P}{p} e_{\max}(T); \quad e_{\max}(T) = 6{,}1 \cdot 10^{-3} P \exp 17{,}13 \frac{T-273}{T-38}. \quad (1)$$

We shall consider for the sake of simplicity the case of stationary flow past the crest. The motion takes place in the plane (x, z) (x being the horizontal and z the vertical coordinate). From the correlations

$$u = \frac{\partial\psi}{\partial z}, \quad w = -\frac{\partial\psi}{\partial x}. \quad (2)$$

we may introduce the current function ψ.

We disregard the effect of air compressibility and we limit the motion by the streamlined contour from below, and by the horizontal wall $z = H$ from above. Let then be given for that value u_∞ of the horizontal velocity u at $x = -\infty$ and the value T_∞ of the temperature T: $u_\infty = U(z)$, $T_\infty = T_0 - \gamma z$ (γ, T_0 are constants). The equations of motion, after excluding pressure from them, will give the correlation [3]

$$\frac{\partial\psi}{\partial z}\frac{\partial\Omega}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\Omega}{\partial z} = \frac{g}{T_m}\frac{\partial T'}{\partial x}, \quad (3)$$

where $\Omega = \partial u/\partial z - \partial w/\partial x = \Delta\psi$, T' is the deflection of temperature from T_∞; g is the gravitation acceleration; T_m is the average temperature of the air ($T_m \approx 250^\circ$). The equation of heat inflow will be written in the form:

$$\frac{\partial\psi}{\partial z}\frac{\partial T'}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T'}{\partial z} - (\gamma_a \varepsilon - \gamma)\frac{\partial\psi}{\partial x} = 0, \quad (4)$$

$\gamma_a = [(\varkappa - 1)/\varkappa] g/R$ (R is a gas constant, $\varkappa$ is the heat capacity ratio); at the same time, we have outside the cloud $\varepsilon = 1$ (see [2]) and

$$\varepsilon = \left[1 + \frac{0{,}623}{c_p} L \frac{\varkappa}{\varkappa - 1} \frac{e_{\max}(T)}{pT}\right]\left(1 + \frac{0{,}623}{c_p}\frac{L}{p}\frac{de_{\max}}{dT}\right)^{-1} \quad (5)$$

inside the cloud (L being the latent condensation heat and c_p - the heat capacity of the air at constant pressure).

For the determination of moisture q, we shall take the transfer equation

$$\frac{\partial\psi}{\partial x}\frac{\partial q}{\partial z} - \frac{\partial\psi}{\partial z}\frac{\partial q}{\partial x} = 0, \qquad (6)$$

which is integrable and gives $q = Q(\psi)$, where Q is an arbitrary function of ψ, of which the form is found from the condition $q = q_\infty(z)$ at infinity ahead of the obstacle.

Introducing $Q(\psi)$ into the left-hand part of (1), we shall obtain at cloud boundary an interconnection between ψ, p and T. Then we may substitute with a great degree of precision p in (1) by its value

$$p_\infty = p_0 \exp\left(\frac{g}{R}\int_z^0 \frac{dz}{T_\infty}\right),$$

where p_0 is the pressure at sea level, and write instead of (1):

$$Q(\psi) = 3{,}8\cdot 10^{-3}\frac{P}{p_0}\exp\left(\int_0^{z_{\text{кр}}} \frac{g}{R}\frac{dz}{T_\infty} + 17{,}13\,\frac{T_\infty - 273 + T_{\text{кр}}'}{T_\infty - 38 + T_{\text{кр}}'}\right) \qquad (7)$$

($\psi_{\text{кр}}$, $z_{\text{кр}}$, $T'_{\text{кр}}$ correspondingly to ψ, z, T' at cloud boundary). The equation (7) interconnects $T'_{\text{кр}}$, $z_{\text{кр}}$, $\psi_{\text{кр}}$. However, the temperature T' may be excluded with the help of (4). As is well known, for the region outside the cloud (4) admits the integral

$$T' = -(\gamma_a - \gamma)\,[z - f_1(\psi)], \qquad (8)$$

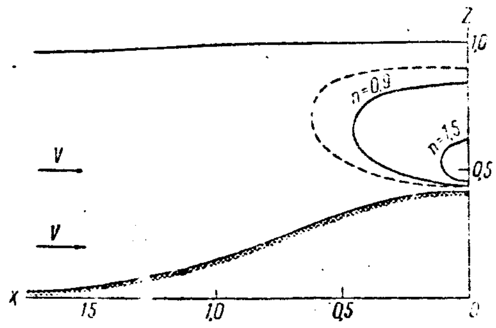

Fig. 1

where f_1 is an arbitrary function of ψ, the form of which shall be found from the condition at infinity. On the other hand, we may substitute, with the same degree of precision with which (5) was written, in ε the function p and T by their values p_∞ and T_∞ *.

* Practically, substituting p_∞ by the approximate formula $p_\infty = p_0 \exp\ (-\frac{g}{RT_m}$ we may obtain for ε the approximate expression

$$\varepsilon \approx \varepsilon_\infty = (1 + 0{,}133\exp\chi)(1 + 0{,}851\exp\chi)^{-1},$$

where

$$\chi = \frac{20{,}2}{T_m}\left[T_0 - 273 - \left(\gamma - \frac{g}{20{,}2\,R}\right)z\right].$$

We may then write for the cloud

$$T' = -(\gamma_a - \gamma)\left[\int_0^z \frac{\varepsilon_\infty \gamma_a - \gamma}{\gamma_a - \gamma}\, dz - F_1(\psi)\right], \qquad (9)$$

where $F_1(\psi)$ is still another arbitrary function of ψ.

Introducing T' from (8) into (3), we shall obtain outside of the cloud a single equation for ψ, admitting an integral of the form

$$\Delta\psi = \left[f_2(\psi) + z\frac{df_1}{d\psi}\right]\frac{g}{T_m}(\gamma_a - \gamma), \qquad (10)$$

where $f_2(\psi)$ is a new arbitrary function of ψ.

Inside the cloud

$$\Delta\psi = \left[F_2(\psi) + z\frac{dF_1}{d\psi}\right]\frac{g}{T_m}(\gamma_a - \gamma), \qquad (11)$$

where $F_2(\psi)$ is an arbitrary function of ψ.

In the following we shall utilize the condition (7) in the approximate form *

$$Q(\psi_{\text{кр}}) = 3.8\cdot 10^{-3}\frac{P}{p_0}\exp(bt_0 + bT_{\text{кр}}' - \alpha z_{\text{кр}}), \qquad (12)$$

where

$$\alpha = b\gamma - \frac{g}{RT_m}, \quad b = \frac{20,2}{T_m}, \quad t_0 = T_0 - 273.$$

If we represent q_∞ in the form $q_\infty = q_0\exp(-\Gamma z)$, where q_0 and Γ are constants, we may write, according to (8), $q_\infty = q_0\exp(-\Gamma f_1(\psi_\infty))$. since T' becomes zero far off the obstacle and we have $[f_1(\psi)]_{x\to-\infty} = z$. Then, $Q(\psi) = q_0\exp(-\Gamma f_1(\psi))$. Substituting this Q into (12), and T' according to (8), we shall have the final condition , linking $z_{\text{кр}}$ and $\psi_{\text{кр}}$ in the form

$$z_{\text{кр}} = mf_1(\psi_{\text{кр}}) + s, \qquad (13)$$

where

$$f_2(\psi) = [\Gamma + (\gamma_a - \gamma)b]\left(b\gamma_a - \frac{g}{RT_m}\right)^{-1}, \quad s = \left(bt_0 + \ln\frac{0,38P}{q_0 p_0}\right)\left(b\gamma_a - \frac{g}{RT_m}\right)^{-1}. \qquad (14)$$

Subsequently, directing x to $-\infty$, we shall have, according to (10)

$$f_2(\psi) = -f_1\frac{df_1}{d\psi} + \frac{T_m}{g(\gamma_a - \gamma)}\left(\frac{dU}{dz}\right)_{z=f_1(\psi)}. \qquad (15)$$

We shall require at cloud boundary the continuity of functions' transfer, and the same goes for the velocity and the vortex Ω. These binding conditions will give us, first of all, the possibility of linking

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* We approximately substitute the integration and we reject T' in the denominator of the right-hand part of (7).

F_1 and F_2 with f_1. equating T' by (8) and (9) at (13), we shall obtain:

$$F_1(\psi) = \int_0^{mf_1(\psi)+s} \frac{\varepsilon_\infty \gamma_a - \gamma}{\gamma_a - \gamma} dz + (1-m) f_1(\psi) - s. \qquad (16)$$

From the equality of the vortices (10) and (11), we shall have at binding

$$F_2(\psi) = [m f_1(\psi) + s]\left(\frac{df_1}{d\psi} - \frac{dF_1}{d\psi}\right) + f_2(\psi). \qquad (17)$$

Therefore, F_1, F_2 and f_2 are expressed through f_1. As to the determination of the latter, and according to (2), the equation

$$\psi = \int_0^{f_1(\psi)} U(z)\, dz. \qquad (18)$$

serves its purpose.

Introducing at the same time dimensionless quantities, we shall finally have outside the cloud

$$\frac{\partial^2\Psi}{\partial X^2} + \frac{\partial^2\Psi}{\partial Z^2} = D^2(Z - f) + \left(\frac{d\bar{U}}{dZ}\right)_{Z=f}; \qquad (19)$$

and inside the cloud

$$\frac{\partial^2\Psi}{\partial X^2} + \frac{\partial^2\Psi}{\partial Z^2} = D^2[(1-mn)Z + Smn - (1-m^2n)f] + \left(\frac{d\bar{U}}{dZ}\right)_{Z=f}, \qquad (20)$$

where

$$n = \frac{\gamma_a}{\gamma_a - \gamma}[1 - \varepsilon_\infty(Z)]_{Z=mf(\Psi)+S},$$

$$D^2 = g\frac{\gamma_a - \gamma}{T_m}\frac{H^2}{V^2}, \quad HZ = z, \quad HX = x, \quad HS = s, \quad Hf = f_1 \qquad (21)$$

$$VH\Psi = \psi, \quad V\bar{U} = U$$

(V being the characteristic velocity).

As an example, we shall consider the case of a very slanting obstacle, when the longwave method can be applied, and in the equations (1) the second derivatives by X can be dropped. Let us assume also that $U(z) = \text{const} = V$. Then $f(\Psi) = \Psi$. We shall take for <u>n</u> a constant mean value. In the absence of cloudiness we shall simply obtain

$$\Psi = Z - Z_0 \csc D(1 - Z_0)\sin D(1 - Z) \qquad (22)$$

($Z = Z_0(X)$ being the equation of the streamlined contour).

But, if a cloud should appear above the obstacle, we would have to estimate : under the cloud

$$\Psi = Z - \csc D(Z_1 - Z_0)\left[Z_0 \sin D(Z_1 - Z) + \left(\frac{1-m}{m}Z_1 - \frac{S}{m}\right)\sin D(Z_0 - Z)\right]; \tag{23}$$

in the cloud

$$\begin{aligned}(1 - m^2 n)\Psi = mnS + (1 - mn)Z + \\ + \csc \widetilde{D}(Z_2 - Z_1)\left[\left(\frac{1-m}{m}Z_1 - \frac{S}{m}\right)\sin \widetilde{D}(Z_2 - Z) + \right. \\ \left. + \left(\frac{1-m}{m}Z_2 - \frac{S}{m}\right)\sin \widetilde{D}(Z - Z_1)\right];\end{aligned} \tag{24}$$

above the cloud

$$\Psi = Z + \left(\frac{1-m}{m}Z_2 - S\right)\csc D(1 - Z_2)\sin D(1 - Z). \tag{25}$$

Here Z_1 and Z_2 are respectively the lower and upper limits of the cloud $\widetilde{D} = \sqrt{1 - m^2 n}\,D$. The quantities Z_1 and Z_2 are determined from a system of two transcendential equations (we shall omit them), which is obtained provided: a) we equate $\partial\Psi/\partial Z$ from (23) and $\partial\Psi/\partial Z$ from (24) at $Z = Z_1$ and b) we equate $\partial\Psi/\partial Z$ from (24) and (25) at $Z = Z_2$.

The solution will have a sharply different character depending upon whether or not we shall have $n > 1$ $(\varepsilon\gamma_a < \gamma)$ and $n < 1$ $(\varepsilon\gamma_a > \gamma)$. In Fig. 1 we gave an example of cloudiness contour at flow past the obstacle $Z_0 = 0.42\,e^{-X}$; it was assumed that $D = 3$, $m = 0.6$, $S = 432$. The cloudiness contour for an all adiabatic flow is given by dashed curve; all the other lines represent the cloudiness at $n = 0.9$ and $n = 1.5$, respectively.

**** THE END ****

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