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CORRECTIONS FOR LINE-OF-SIGHT VELOCITIES ON ACCOUNT OF THE DAILY ROTATION AND ANNUAL REVOLUTION OF THE EARTH DURING SOLAR OBSERVATIONS

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SUMMARY

The question of line-of-sight velocity corrections for the relative motion of the observer is considered here with more detail than in preceding works. The author has recourse to a more rational scheme for their calculation, which is thus proposed for practical purposes. An example of complete calculations is given for a single case.

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* *

It is well known that the measured shifts of spectral lines on negatives represent only the reflection of heavenly body's line-of-sight velocity relative to observer. In order to bring the measured spectral line shifts to Sun, in particular, it is necessary to free them from the influence of observer's motion relative to the Sun by way of introduction of corresponding corrections to the measured line-of-sight velocities.

A series of works have been spotted in the AO of the Leningrad university, concerned with the study of the shift of Fraunhofer lines in the solar spectrum. But it was found that in order to speed up the computations there arose the necessity of a more rational scheme. This is precisely the object of the present work, which, among other things, proposes a practical scheme for the computation of corrections of line-of-sight velocities. (See [1]).

* O POPRAVKAKH LUCHEVYKH SKOROSTEY ZA SCHET SUTOCHNOGO VRASHCHENIYA I GODICHNOGO OBRASHCHENIA ZEMLI PRI NABLYUDENIYAKH SOLNTSA.

If V is the true line-of-sight velocity at any point Q on the Sun's surface (Fig. 1), we have

$$V = V^* + C_1 - C_0 - \Delta_1.$$

Here V^* is the measured line-of-sight velocity, C_1 is the projection of observer's motion velocity on a line, linking him with the observed point Q on the solar disk at time of observation, C_0 is the projection of observer's motion velocity on a line, linking him with the point — the visible center of the image — at the moment of time of solar disk's center observation, Δ_1 is the limb effect *.

The motion of the observer relative to Sun is determined by two parts: the first is conditioned by the daily rotation of the Earth and the second — by the annual revolution of the Sun around the Earth. Let us pause at these two corrections.

1. - The correction for the daily rotation of the Earth, expressed in km/sec and added to the observed velocity, is given by the expressions

$$C_1' = \frac{2\pi r \cos\varphi'}{86164{,}1} \cos\delta \sin H = 0.2334 \cos\delta \sin H \quad \text{(for the A O of LGU)} \qquad (2)$$

Here φ' is the heliocentrical latitude of the observer, r is the distance from the observer to the center of the Earth, taking into account his location above sea level (expressed in km). If the latter's height is h, we have

$$r = r_0 + h, \qquad (3)$$

where r_0 is the distance from the point at sea level to the center of the Earth. The value of r_0 is found from the expression :

$$r_0 = a\,(0.998320 + 0.001684 \cos 2\varphi - 0.000004 \cos 4\varphi). \qquad (4)$$

In formula (4) a is the equatorial radius of the Earth expressed in km, φ is the geographical latitude of the place of observation; as to φ', entering in (2), it is determined from the expression:

$$\varphi' = \varphi - 695.65 \sin 2\varphi + 1'',17 \sin 4\varphi. \qquad (5)$$

In formula 2 : 1) the quantity 86164.1 is the number of mean solar seconds in sidereal days; 2) δ is the visible declination of the point Q

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* By limb effect we understand the difference in the position of spectral lines on the limb and at Sun's center.

at time of observation; 3) $H = s - \alpha$ is the hour angle; 4) α - is the visible right ascension of the point Q at the moment of observation; 5) s is the mean sidereal time.

Note that δ and α, determined relative to the true equator and equinox, are found by adding corrections $\Delta\alpha$ and $\Delta\delta$ to the visible equatorial coordinates of the center of the solar disk, α_0 and δ_0; these corrections are given by the correlations :

$$\Delta\alpha \simeq -\theta \cos\psi \sec\delta_0, \quad \Delta\delta \simeq \theta \sin\psi. \qquad (6)$$

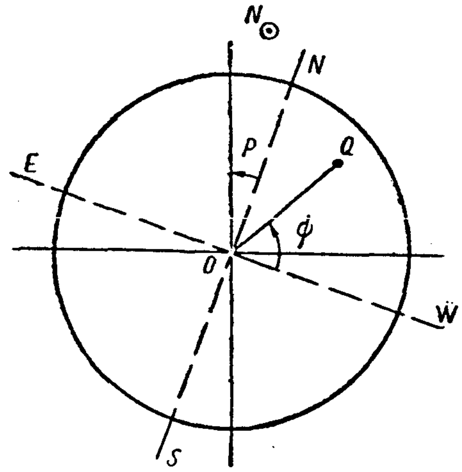

Fig. 1

Here ψ is the position angle of the point Q, measured from the west end of the daily parallel (Fig. 1), θ is the angle formed by rays linking the observer with the points Q and the visible center of Sun's image O. If ρ is the distance from Q to the center of Sun's image, ρ_0 is the radius of Sun's image and S is the angular radius of the Sun, we have

$$\theta \simeq \left(\frac{\rho}{\rho_0}\right) S.$$

It should be noted, that when the Sun's height is less than 30°, account should be taken of the differential refraction when determining ρ and θ.

For a fixed place φ' is constant so that the correction C_1', which has a maximum value of $\pm 0{,}46$ km/sec, can be tabulated for every day by the arguments δ and H.

2. - The correction, expressed in km/sec, for the orbital rotation of the Earth, which must also be added to the observed velocity, is, according to F. Sclesinger [2], given by the expression :

$$C_1'' = -1732{,}34\,(\Delta X \cos\delta' \cos\alpha' + \Delta Y \cos\delta' \sin\alpha' + \Delta Z \sin\delta'). \qquad (7)$$

Here X, Y, Z are the geocentrical rectangulgar equatorial coordinates of the center of the Sun, determined relative to mean equator and equinox and related to a certain epoch, for example in the case of interest to us, to the year 1963. ΔX, ΔY, ΔZ are the velocity components of this motion, interpolated to the moment of observation respectively in the directions: 1) parallel to the equinoctial line, 2) parallel to the equatorial plane, 3) perpendicular to the equatorial plane.

All these six quantities are tabulated in the Astronomical Yearbook for every day at 00 00 hours ephemeris time. α' and δ' in formula (7) are the equatorial coordinates of the point of observation, determined relative the same equator and equinox as X, Y, Z.

According to [1], we have for the center of the Sun :

$$\operatorname{tg} \alpha_0' = \frac{Y}{X}, \ \sin \delta_0' = Z. \qquad (8)$$

The corresponding equatorial coordinates α' and δ' of the point Q are found by adding the quantities

$$\Delta\alpha' \cong -\theta \cos \psi \sec \delta_0', \quad \Delta\delta' \cong \theta \sin \psi \qquad (6')$$

to the visible equatorial coordinates of the center of the Sun α_0' and δ_0' at time of observation of the point Q.

Taking into account the above-said, the correction in km/sec for the motion velocity component of the observer along the visual ray to the point Q will have the form

$$C_1 = C_1' + C_1'' = -\frac{2\pi r \cos \varphi'}{86164{,}1} \cos \delta \sin H - 1732{,}34 (\Delta X \cos \delta' \cos \alpha' + \Delta Y \cos \delta' \sin \alpha' + \Delta Z \sin \delta'). \qquad (9)$$

The correction C_0 in the expression (1) for observer's motion along the line linking him with the point O (at the time of observation of the disk's center) is found by substituting into formula (9) the respective equatorial coordinates of the Sun's center and of the quantities ΔX, ΔY, ΔZ at the time when the Sun's spectrum is observed.

Summarizing the above-expounded method of determination of the correction for observer's motion velocity component along the visual ray toward the point of observation, we may reduce it to the performing of the following operations:

1. - α_o and δ_o of the center of Sun's disk are interpolated to the moments of time of observations *.

2.- The position angle ψ of the point of observation on the Sun's disk is measured (for the points of the eastern and western ends of the equator the angle is respectively equal to $180^o + P$ and P, where P is the position angle of the Sun's rotation axis, computed from the northern meridian point).

3.- The corrections $\Delta\alpha$ and $\Delta\delta$ are computed by formulas (6).

4.- The equatorial coordinates α and δ of the observation point are obtained by adding $\Delta\alpha$ and $\Delta\delta$ to the above-obtained values of α_o and δ_o.

5.- The sidereal time s is determined for every moment of time of observation, and then the hour angles $H = s - \alpha$ of the observed points on the Sun's disk at that moment.

6.- r and φ' are determined from the expressions (3), (4), (5).

7.- Finally the value of

$$\frac{2\pi r \cos\varphi'}{86164{,}1},$$

being constant for the place of observation, is computed.

8.- After finding the respective trigonometric functions of δ and H entering into formula (2), we compute the correction C'_1.

9.- From the Astronomical Yearbook we write the coordinates X, Y, Z and their first differences ΔX, ΔY, ΔZ, on the day of observation at 00 00 hours ephemeris time; then these quantities are interpolated to the times of observations of the Sun's disk points.

10. - By formulas (8) we determine α'_o and δ'_o.

11.- By formulas (6) we determine $\Delta\alpha'$ and $\Delta\delta'$ and then by adding them to the above-obtained values of α'_o and δ'_o, we find α' and δ'.

12.- Upon finding the respective trigonometric functions of α' and δ' entering into formula (7), we compute the correction C''_1.

13.- The final correction C_1, being the sum of C'_1 and C''_1, is then determined.

14.- Finally the correction C_o is determined by substitution of the corresponding equatorial coordinates of Sun's center $(\alpha_0, \delta_0, \alpha'_0, \delta'_0)$ and of the quantities ΔX, ΔY, ΔZ into formula (9) at the moment of time when the spectrum of the Sun's center is observed (for the study of shifts).

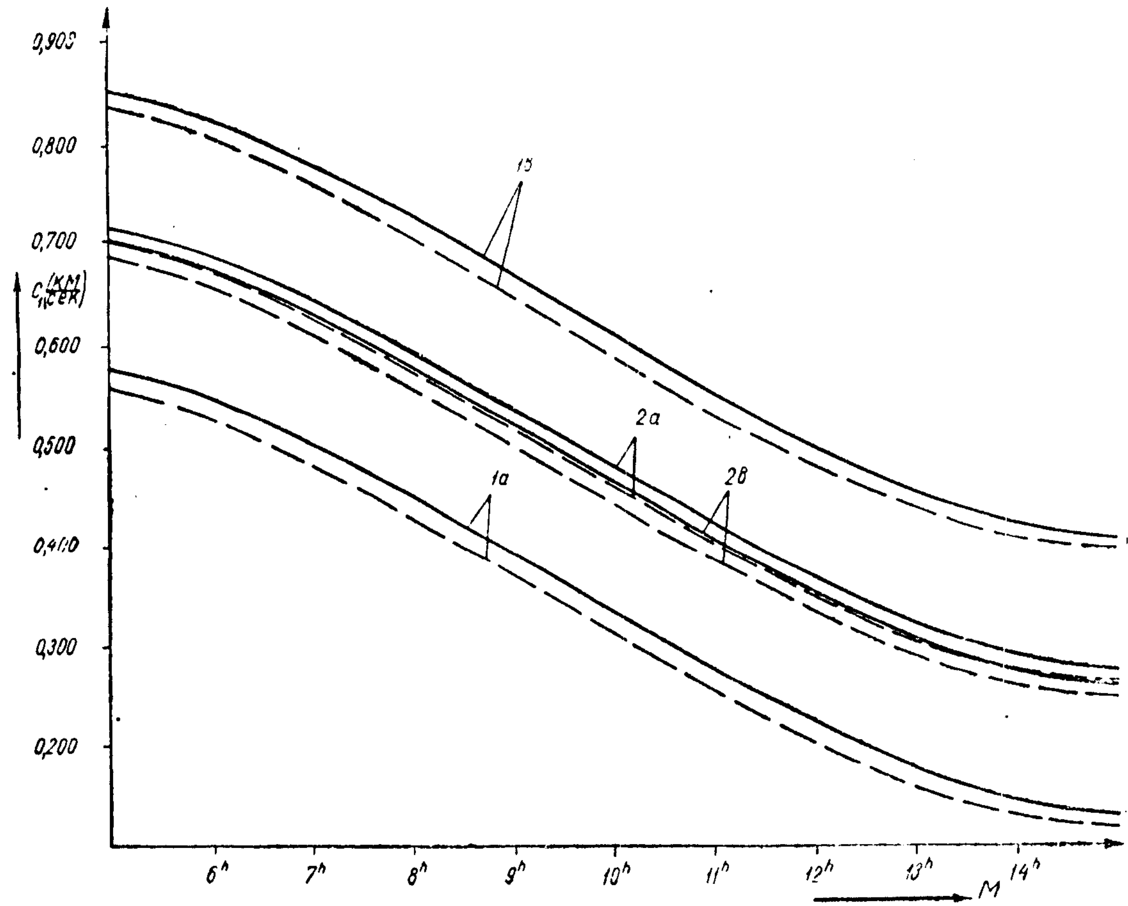

Fig. 2

VARIATION OF THE CORRECTION C FOR THE RELATIVE MOTION OF THE OBSERVER AS A FUNCTION OF TIMES OF OBSERVATION.

1a — Eastern end of the equator of the Sun; 1 — Western end of the Sun's equator; 2a - southern end of Sun's pole; 2 —northern end of Sun's pole.

——————— 4 October 1963

- - - - - - - - 10 October 1963

The curves of C_1 variation for the eastern and western ends of the equator and northern and southern ends of the poles of the Sun are plotted in Fig. 2 as a function of observation time, which in the given case correspond to 4 and 10 October 1963. As may be seen from the drawing, the curves 1 and 2 shifted little relative to one another within the limits of six days, and that is why the values of C_1 (and even of C_o) for the intermediate days are found by interpolation.

In reality correction for the motion of the center of gravity of the system Earth-Moon should be effected. However, this correction is small and is disregarded in this case.

In conclusion, we present, as an example, a complete calculation of two indicated corrections for the eastern end of the equator on 4 October 1963, using the scheme proposed by us, which is practical for calculations (see Tables 1 and 2).

*** THE END ***

TABLE 1

M	6^h	9^h	12^h	15^h
s	$9^h04^m50^s.3$	$12^h05^m19^s.9$	$15^h05^m49^s.4$	$18^h06^m19^s.0$
δ_0	$-4°04'33''.5$	$4°07'27''.3$	$-1°10'21''.1$	$-4°13'14''.8$
+ $\Delta\delta$	$-07'04''.2$	$-07'04''.2$	$-07'04''.2$	$-07'04''.2$
δ	$-4°11'37''.7$	$-4°11'31''.5$	$4°17'25''.3$	$-4°20'19''.0$
α_0	$12^h37^m49^s.8$	$12^h38^m17^s.1$	$12^h38^m44^s.3$	$12^h39^m11^s.6$
+ $\Delta\alpha$	$+57^s.5$	$+57^s.5$	$+57^s.5$	$+57^s.5$
α	$12^h38^m47^s.3$	$12^h39^m14^s.6$	$12^h39^m41^s.8$	$12^h40^m09^s.1$
H	$20^h26^m03^s.0$	$23^h26^m05^s.3$	$02^h26^m07^s.6$	$05^h26^m09^s.9$
$\sin H$	-0.8037	0.1473	$+0.5953$	$+0.9891$
$\cos\delta$	$+0.9973$	$+0.9973$	$+0.9972$	$+0.9971$
$\cos\delta\sin H$	-0.8015	-0.1469	$+0.5936$	$+0.9862$
C' (km/sec)	$+0.187$	$+0.034$	-0.138	0.230

../.. cont'd

TABLE 2

M	6^h	9^h	12^h	15^h
X	−0.984 2070	−0.983 7858	−0.983 3601	−0.982 9298
Y	−0.163 9537	0.165 8867	−0.167 8190	−0.169 7506
Z	−0.071 1015	−0.071 9398	−0.072 7777	−0.073 6153
δ_0'	−4°04′38.″3	−4°07′31.″5	−4°10′24.″8	−4°13′18.″0
+ $\Delta\delta'$	−07′04.″2	−07′04.″2	−07′04.″2	−07′04.″2
δ'	−4°11′42.″5	−4°14′35.″7	−4°17′29.″0	−4°20′22.″2
− lg Y	$\bar{1}$.2146	$\bar{1}$.2198	$\bar{1}$.2248	$\bar{1}$.2297
lg X	$\bar{1}$.9931	$\bar{1}$.9929	$\bar{1}$.9927	$\bar{1}$.9925
lg tg α_0'	$\bar{1}$.2215	$\bar{1}$.2269	$\bar{1}$.2321	$\bar{1}$.2372
α_0'	$12^h37^m49.^s1$	$12^h38^m17.^s0$	$12^h38^m44.^s0$	$12^h39^m11.^s0$
+ $\Delta\alpha'$	$+57.^s5$	$+57.^s5$	$+57.^s5$	$+57.^s5$
α'	$12^h38^m46.^s6$	$12^h39^m14.^s5$	$12^h39^m41.^s5$	$12^h40^m08.^s5$
sin α'	−0.1683	−0.1704	−0.1723	−0.1742
cos α'	−0.9857	−0.9854	−0.9851	−0.9847
a = sin δ'	−0.0731	−0.0740	−0.0748	−0.0757
cos δ'	+0.9973	+0.9973	+0.9972	+0.9971
b = cos δ' cos α'	−0.9830	−0.9827	−0.9823	−0.9818
c = cos δ' sin α'	−0.1678	−0.1699	−0.1718	−0.1737
ΔX	+0.003 3518	+0.003 3879	+0.003 4240	+0.003 4601
ΔY	−0.015 4675	−0.015 4614	−0.015 4553	−0.015 4492
ΔZ	−0.006 7073	−0.006 7047	−0.006 7020	−0.006 6993
$\Delta X \cdot b$	−0.003 2948	−0.003 3293	−0.003 3634	−0.003 3971
+ $\Delta Y \cdot c$	+0.002 5954	+0.002 6269	+0.002 6552	+0.002 6835
+ $\Delta Z \cdot a$	+0.000 4903	+0.000 4961	+0.000 5013	+0.000 5071
Σ	−0.000 2091	−0.000 2063	−0.000 2069	−0.000 2065
C_1^* (км/sec)	+0.362	+0.357	+0.358	+0.358
C_1 (км/sec)	0.549	+0.391	+0.220	+0.128

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