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NA-65-513

PRELIMINARY ANALYSIS

OF THE

METABOLIC RATE MONITOR SYSTEM

NASA CONTRACT NAS4-876

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS

FOREWORD

In compliance with Articles I and II of NASA Contract Letter Document NAS4-876/TRS, dated 16 June 1965, North American Aviation, Inc., Los Angeles Division is forwarding this report to the National Aeronautical and Space Administration for review and acceptance.

PRELIMINARY ANALYSIS

OF

METABOLIC RATE MONITOR SYSTEM

I. INTRODUCTION

The NASA Contract NAS4-367 (Reference (b)) was initiated for the purpose of developing to prototype status certain respiratory sensors and to test these sensors under various environmental conditions with human subjects. Under Phase I of that program a prototype sensor was assembled which utilized a Beckman Instruments, Inc. P_{O_2}, P_{CO_2}, P_T, and temperature sensor and NASA supplied Technology, Inc. mass flowmeters. During testing under sea level and altitude conditions several inherent errors became apparent with regard to the determination of oxygen consumption. However, it required the efforts of a Phase II program with an analytical approach to the thermodynamic and physical characteristics of the system in order to pin-point the basic problems and determine the magnitude of possible errors. The results of the Phase II efforts were summarized in Reference (c). Further efforts utilizing similar equipment were proposed in References (d) and (c) to determine feasibility of measuring oxygen consumption of a human subject by in-flight instrumentation.

A new concept for measuring oxygen consumption designed by Webb Associates, Inc. was presented to NASA and NAA/LAD by W. V. Blockley of Webb Associates on 24 May 1965. As a result of this meeting NAA/LAD proposed (Reference (f)) to perform a theoretical evaluation of the Webb Associates system known as a Metabolic Rate Monitor (MRM) at the request of Dr. James Roman of NASA. This evaluation was to be conducted before any further efforts were spent on the respiratory analyzer system.

The principle tasks of the evaluation as given by the Work Statement are as follows:

1. Basic evaluation of the theoretical concept.
2. Preliminary error analysis to determine the magnitude of errors for extreme ranges of conditions of R.Q., O_2 consumption, CO_2 production, etc.
3. Evaluation of mechanical components of the system for source of error in measuring ability and cursory examination of error magnitudes for both sea level and altitude conditions where data is readily available.
4. Preliminary evaluation of the MRM utilizing flow splitting venturies for minimizing diluent gas flow requirements.
5. Prepare and submit letter report

II. DESCRIPTION OF THE BASIC MRM SYSTEM

The Metabolic Rate Monitor (MRM) system as described at the meeting of May 24 employs a technique which relates a subject's oxygen consumption to the excitation voltage (speed) of a DC blower. The blower speed is servo-controlled by an P_{O_2} sensor (polarographic cell) located downstream of the blower and an O_2 sensor located just downstream of the O_2 or air supply. The difference between the two cell outputs will constitute the error signal for servo control of the blower speed. The flow through the blower is a mixed combination of the subject's exhalation and a diluent of the same composition as the inspired gas. When the subject's oxygen consumption varies, the diluent flow must vary to maintain a constant partial pressure difference of oxygen. The P_{O_2} sensor-servo system maintains the constant partial pressure difference by adjusting the blower speed (excitation voltage). A schematic diagram of the basic MRM system is shown in figure 1. Alternate system concepts brought up

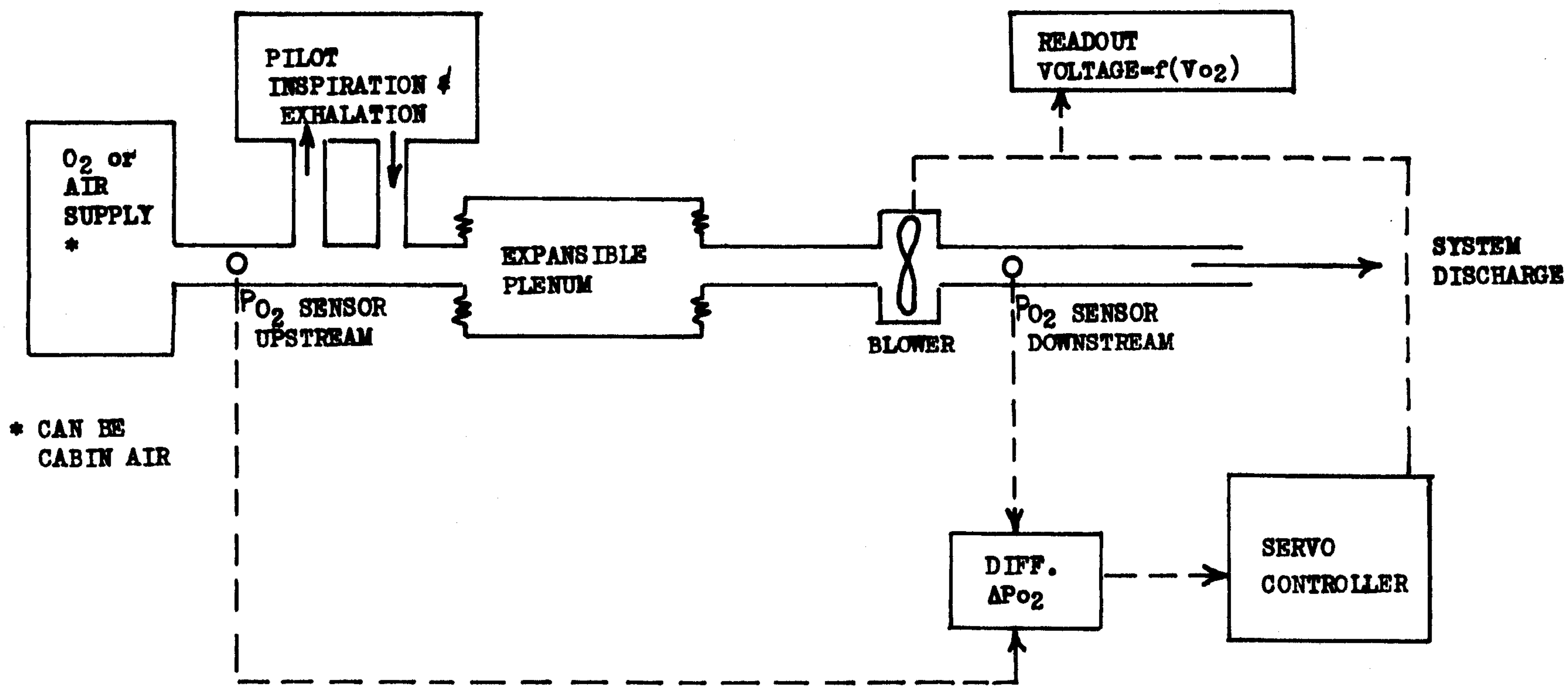
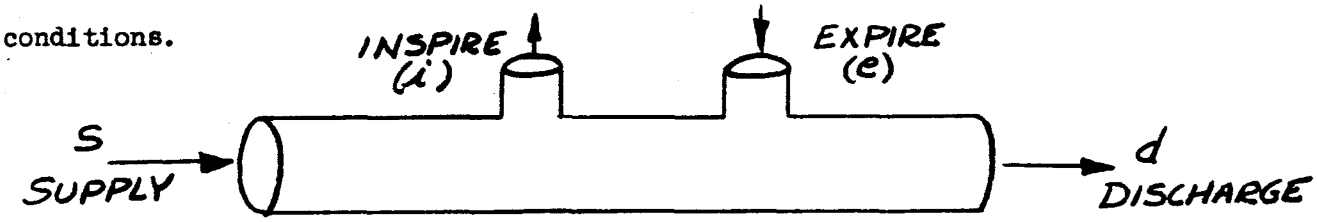


FIGURE 1 SCHEMATIC DIAGRAM OF BASIC METABOLIC RATE MONITOR (MRM) EVALUATED

after the initiation of the evaluation contract are shown in figure 7 and discussed in a latter section of this report.

III. ANALYSIS OF THE BASIC MRM SYSTEM

For analysis purposes the schematic of the system may be reduced to the simple form shown below. The performance of this system (MRM) has been shown to be strongly influenced by the humidity levels of the inspired and expired air. Thus, for purposes of the error analysis it is assumed that the inspired and expired air is dried before making the flow and pressure measurements. It is also assumed that the total pressure and the temperature are uniform throughout the total system. It is most important to note that the $(\dot{V})$ terms used in the nomenclature and subsequent derivations are defined at the local temperature and pressure and should not be confused with $(\dot{V})$ terms at SPTD conditions.



Measured Values:

P_T = Total Pressure, mm Hg

$P_{O_{2s}}$ = Oxygen Partial Pressure - Supply mm Hg

$P_{O_{2d}}$ = Oxygen Partial Pressure - Discharge mm Hg

$\dot{V}_s$ = Volumetric Supply Flow Rate (at local conditions)

$R.Q.$ = Respiratory Quotient

$\dot{V}_{O_2}$ = Volumetric Oxygen Uptake (at local conditions)

$\dot{V}_d$ = Discharge Volume Flow Rate (at local conditions)

Equations describing the operation of the system can be written in several forms, each of which provides insight into the phenomena involved. The following derivation is of interest in examining the nature of the change in oxygen partial pressure. Since this formulation will not be used in the subsequent numerical analysis, the man's expired water vapor is included for clarity.

Beginning with the equation for oxygen uptake:

$$\dot{V}_{O_2} = \dot{V}_{O_{2i}} - \dot{V}_{O_{2e}}$$

also

$$\dot{V}_{O_2} = \dot{V}_{O_{2s}} - \dot{V}_{O_{2d}}$$

then

$$\dot{V}_{O_2} = \left(\dot{V}_s \cdot \frac{P_{O_{2s}}}{P_T}\right) - \left(\dot{V}_d \frac{P_{O_{2d}}}{P_T}\right)$$

and

$$\dot{V}_s - \dot{V}_i = \dot{V}_d - \dot{V}_e$$

so

$$\dot{V}_{O_2} = \frac{P_{O_{2s}}}{P_T}\left(\dot{V}_d + \dot{V}_i - \dot{V}_e\right) - \frac{P_{O_{2d}}}{P_T}\dot{V}_d$$

$$P_T \dot{V}_{O_2} = \left(P_{O_{2s}} - P_{O_{2d}}\right)\dot{V}_d + P_{O_{2s}}\left(\dot{V}_i - \dot{V}_e\right)$$

$$P_T \dot{V}_{O_2} = \left(P_{O_{2s}} - P_{O_{2d}}\right)\dot{V}_d + P_{O_{2s}}\left(\dot{V}_{O_2} + \dot{V}_{CO_2} + \dot{V}_{H_2O} + \dot{V}_{N_2}\right)$$

where $\dot{V}_{O_2}$, $\dot{V}_{CO_2}$, $\dot{V}_{H_2O}$, and $\dot{V}_{N_2}$ are the incremental volume flow rates between the inhaled and exhaled gas.

$$\left(P_T - P_{O_{2s}}\right)\dot{V}_{O_2} = \dot{V}_d\left(P_{O_{2s}} - P_{O_{2d}}\right) + P_{O_{2s}}\left(\dot{V}_{CO_2} + \dot{V}_{H_2O} + \dot{V}_{N_2}\right)$$

For the special case of the supply gas being pure oxygen, $P_{O_{2S}} = P_T$, and the left side of the equation vanishes, and the discharge rate, $\dot{V}_d$, is a measure of the rate of exhalation of nonoxygen gases and is independent of oxygen uptake rate.

Recalling the manner in which the MRM operates, the blower creating the flow ($\dot{V}_S$) or ($\dot{V}_d$) is controlled by a servo which is maintaining a constant differential of oxygen partial pressure between supply and discharge. The oxygen uptake ($\dot{V}_{O_2}$) is then a function of ($\dot{V}_S$) or ($\dot{V}_d$). It is thus desirable to arrive at an equation showing the relationship of ($\dot{V}_S$) to ($\dot{V}_{O_2}$) in a form such as $\dot{V}_S = \dot{V}_{O_2}K$. A derivation accomplishing this is shown below:

$$\dot{V}_{O_2} = \left(\frac{P_{O_2s}}{P_T}\right)\dot{V}_s - \left(\frac{P_{O_2d}}{P_T}\right)\dot{V}_d$$

$$\frac{P_{O_2s}}{P_T}\dot{V}_S = \dot{V}_{O_2} + \left(\frac{P_{O_2d}}{P_T}\right)\dot{V}_d = \dot{V}_{O_2} + \frac{P_{O_2d}}{P_T}\left[\dot{V}_S - (\dot{V}_i - \dot{V}_e)\right]$$

BUT $\dot{V}_i - \dot{V}_e = \dot{V}_{O_2} - (R.Q)\dot{V}_{O_2}$ $\qquad \dot{V}_i - \dot{V}_e = \dot{V}_{O_2}(1 - R.Q.)$

$$\frac{P_{O_2}}{P_T}\dot{V}_S = \dot{V}_{O_2} + \frac{P_{O_2d}}{P_T}\left[\dot{V}_S - (1-R.Q.)\dot{V}_{O_2}\right]$$

$$\frac{P_{O_2s}}{P_T}\dot{V}_s - \frac{P_{O_2d}}{P_T}\dot{V}_s = \dot{V}_{O_2} - \frac{P_{O_2d}}{P_T}(1-R.Q)\dot{V}_{O_2}$$

$$\dot{V}_S\left(\frac{P_{O_2s}}{P_T} - \frac{P_{O_2}d}{P_T}\right) = \dot{V}_{O_2}\left[1 - \frac{P_{O_2d}}{P_T}(1-R.Q.)\right]$$

$$\dot{V}_S = \dot{V}_{O_2}\left[\frac{1 - \frac{P_{O_2}d}{P_T}(1-R.Q.)}{\frac{P_{O_2s} - P_{O_2d}}{P_T}}\right]$$

$$\dot{V}_S = \dot{V}_{O_2}\,\frac{P_T - P_{O_2d}(1-R.Q.)}{P_{O_2s} - P_{O_2d}} = \dot{V}_{O_2}\left[\frac{P_T}{P_{O_2s} - P_{O_2d}} - \frac{P_{O_2d}}{P_{O_2s} - P_{O_2d}}(1-R.Q.)\right]$$

A similar derivation showing the relationship of $(\dot{V}_d)$ to $(\dot{V}_{O_2})$ is shown below:

$$\dot{V}_{O_2} = \left(\frac{P_{O_2s}}{P_T}\right)\dot{V}_S - \left(\frac{P_{O_2d}}{P_T}\right)\dot{V}_d$$

$$\frac{P_{O_2d}}{P_T}\dot{V}_d = \frac{P_{O_2s}}{P_T}\dot{V}_S - \dot{V}_{O_2} = \frac{P_{O_2s}}{P_T}\left[\dot{V}_d + \dot{V}_{O_2}(1-R.Q)\right] - \dot{V}_{O_2}$$

$$\frac{P_{O_2d}}{P_T}\dot{V}_d - \frac{P_{O_2s}}{P_T}\dot{V}_d = \frac{P_{O_2s}}{P_T}\dot{V}_{O_2}(1-R.Q.) - \dot{V}_{O_2} = \dot{V}_{O_2}\left[\frac{P_{O_2s}}{P_T}(1-R.Q)-1\right]$$

$$\dot{V}_d = \dot{V}_{O_2}\left[\frac{\frac{P_{O_2s}}{P_T}(1-R.Q.)-1}{\frac{P_{O_2d}-P_{O_2s}}{P_T}}\right] = \dot{V}_{O_2}\left[\frac{P_{O_2s}(1-R.Q.)-P_T}{P_{O_2d}-P_{O_2s}}\right]$$

The equations derived above represent the subject's oxygen uptake $(\dot{V}_{O_2})$ in terms of the MRM output $(\dot{V}_d)$ or $(\dot{V}_S)$. However $(\dot{V}_{O_2})$ was a volume flow rate at local condition, the expression will have more physiological significance under varying altitude conditions, if $\dot{V}_{O_2}$ is replaced by a mass rate of oxygen uptake $(\dot{U})$. This transformation is made in the following steps where:

X = Oxygen Mol fraction in gas composition

$\dot{U}$ = Mass rate of oxygen uptake, gm/unit time

R = Universal gas constant

then

$$\dot{V}_d = \left[1-(1-R.Q)X_S\right]\frac{\dot{U}}{32}\;\frac{RT}{X_S P_T - P_{O_2d}}$$

$$= \left[1-(1-R.Q)X_S\right]RT\frac{\dot{U}}{32}\;\frac{1}{\frac{P_{O_2s}}{P_T}P_T - P_{O_2d}}$$

so $$\dot{V}_d = \dot{U}\left[1-(1-R.Q)X_S\right]\frac{RT}{32}\frac{1}{\Delta P_{O_2}}$$

The above equation is used as a basis for the error analysis. The parameters considered in the error analysis are:

(a) Deviation from an assumed R.Q. of 1.0

(b) Measurement errors in P_{O2} sensors

(c) Measurement errors in temperature

(d) Measurement errors in barometric pressure level

(e) Measurement errors of $(\dot{V}_d)$

Error Analysis of R.Q. Sensitivity

The above equation is analyzed in Appendix 1 to determine the effect of deviation from a standard assumed R.Q. on oxygen uptake $(\dot{U})$. The results of the analysis are shown in figure 8. It will be noted that the error in oxygen uptake due to a deviation from an assumed R.Q. is a function of the Mol fraction of oxygen in the supply gas. Further, it will be noted that the slope of these curves is identical to the Mol fraction. Therefore, when the supply gas is pure oxygen, the error in oxygen uptake is equal to the error in assumed deviation in R.Q. This correlates with the previously noted fact that with pure oxygen as a supply gas the MRM measures CO_2 production. If it is assumed that the maximum range of R Q's will remain between 0.8 and 1.2, a supply gas having oxygen concentrations up to 50% will allow accuracies within ±10% in oxygen uptake. Typical plots of oxygen uptake as a function $\dot{V}_d$ are shown over this R.Q. range for varying conditions in Appendix 4.

Error Analysis in P_{O2} Sensors

Figure 10 shows the combined error in oxygen uptake as a function of error in the individual measurement of each oxygen partial pressure sensor. This error is seen to be a function of the ratio or the supply oxygen concentration to the discharge oxygen concentration. The curve in this form is general and will apply to all altitudes, supply gas compositions, etc. However, to be more meaningful in terms of MRM the relationship of this ratio to the differential P_{O2} servo setting must be defined. It may be expressed by the equation:

$$\Delta P_{O_2} = X_S P_T \left(1 - \frac{X_D}{X_S}\right)$$

The curve shows corresponding ΔP_{O2} settings for sea level conditions with air as the supply gas. It will be noted that for the curve of $(X_D/X_S = 0.8)$ the ΔP_{O2} setting is 31.8 mm Hg and that this would represent a situation where there is no bypass flow. It is apparent that as the supply flow increases over that of the subject's minute volume that the percent error in oxygen uptake for a given error in P_{O2} increases markedly. For example, with opposing errors of 1% the P_{O2} sensors, the percent error in oxygen uptake can be as much as 64% with the 8 mm Hg. ΔP_{O2} setting, but would only reach 10% in the case of no dilution. Calculation describing figure 10 may be found in Appendix 1.

Error Analysis in Temperature Measurement

The error in oxygen uptake as a function of error in absolute temperature is shown in figure 9. The curve indicates the relative insensitivity of oxygen uptake to state-of-art temperature measurements. Calculations for this curve are shown in Appendix 1.

Error Analysis for Barometric Pressure Measurement

The equation for $(\dot{U})$ is seen to be independent of barometric pressure; therefore, no altitude errors will be experienced by the MRM.

Analysis of Errors in Measurement of $(\dot{V}_d)$

Examination of the basic equation shows that error in $(\dot{U})$ will be directly proportional to errors in measurement of $(\dot{V}_d)$.

IV. ANALYSIS OF THE BLOWER

An evaluation of a blower as the means of establishing and metering volume flow through the MRM has been conducted. It was first considered that a problem might exist in precise fan calibration due to Reynold's Number effects with altitude. The establishment of 16,000 feet as the maximum required operating altitude has eliminated Reynold's Number effects from being important.

Performance curves of the fan currently used with the MRM are shown in figure 2 and 3. The basic concept of the blower metering technique is that volume flow is proportional to rpm. Reference to figure 3, however, shows that the flow through the blower will vary from zero to 28 cfm at constant blower speed due to a static pressure rise across the blower of zero to 0.87 inches of water.

If the blower were used in conjunction with a diluter demand automatic pressure breathing oxygen regulator as described in Specification MIL-R-25916 the suction pressure required (1.0 in. H_2O) to provide a flow of 85 liters/minute from the regulator would be greater than that available from the blower at a rate of zero cfm and at 10,000 rpm. The excellent linear relationship of rpm vs volume flow shown by Webb Associates, Inc. for their MRM configuration was possible because their system was essentially a zero pressure drop device, as can be seen in figure 4.. This problem was brought to the attention of Webb Associates under the terms of their consulting agreement on this contract. They have stated that Globe Associates (manufacturer of the present fan) can provide a higher compression ratio blower which when operated essentially unloaded will not be sensitive to small changes in static pressure. Webb Associates believes that the use of such a blower in conjunction with the Type II miniaturized pressure regulator (MIL-R-19121D) would result in a satisfactory configuration for flight use. No data on this blower has been supplied so a quantitative evaluation cannot be made. It is suggested by NAA that if a further development of the MRM is deemed advisable that the use of positive displacement devices be investigated.

V. ANALYSIS OF A FLOW METER TYPE RESPIRATION ANALYZER

At the request of Dr. James Roman an error analysis was made for the theoretical operation of a flow meter type respiratory analyzer for comparison with the error analysis of the MRM. A simplified schematic of the respiratory analyzer is shown in figure 5. The essential components are shown including the various sensors required by symbolic representation. In this system the inlet gas composition is a mixture of nitrogen and oxygen of either fixed or variable composition. The basic equation for the system for volumetric flow is as follows:

$$\dot{V}_{O_2} = \left[\left(\frac{P_{O_2,1}}{P_B}\right)\left(\frac{P_{T_2} - P_{CO_2} - P_{O_2,2}}{P_{T_1} - P_{O_2,1}}\right) - \left(\frac{P_{O_2,2}}{P_B}\right)\right]\dot{V}_d$$

where

$$P_{N_2,2} = P_{T_2} - P_{CO_{2e}} - P_{O_2,2}$$

and

$$P_{N_2,1} = P_{T_1} - P_{O_2,1}$$

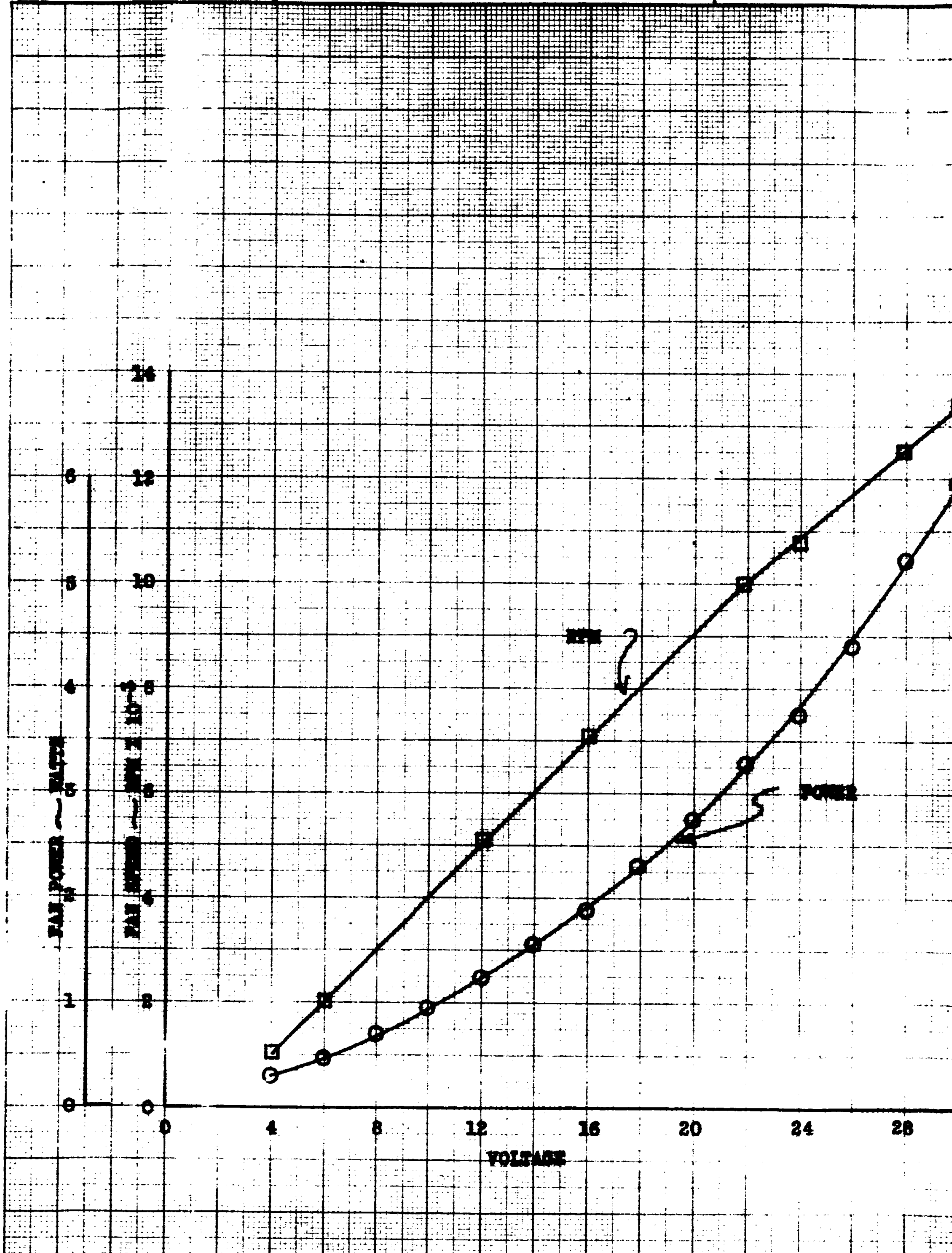
The error analysis of the system was based on the equation given and is found in Appendix 2. In changing to mass uptake the equation takes the form:

$$\dot{U}\frac{M}{M_{ox}} = \left[\frac{P_{ox}\left(P_2 - P_{ox_2} - P_{DIOX_2}\right) - P_{ox_2}\left(P_1 - P_{ox_1}\right)}{P_1 - P_{ox_1}}\right]\dot{m}$$

where $M = \sum P_i MW_i$

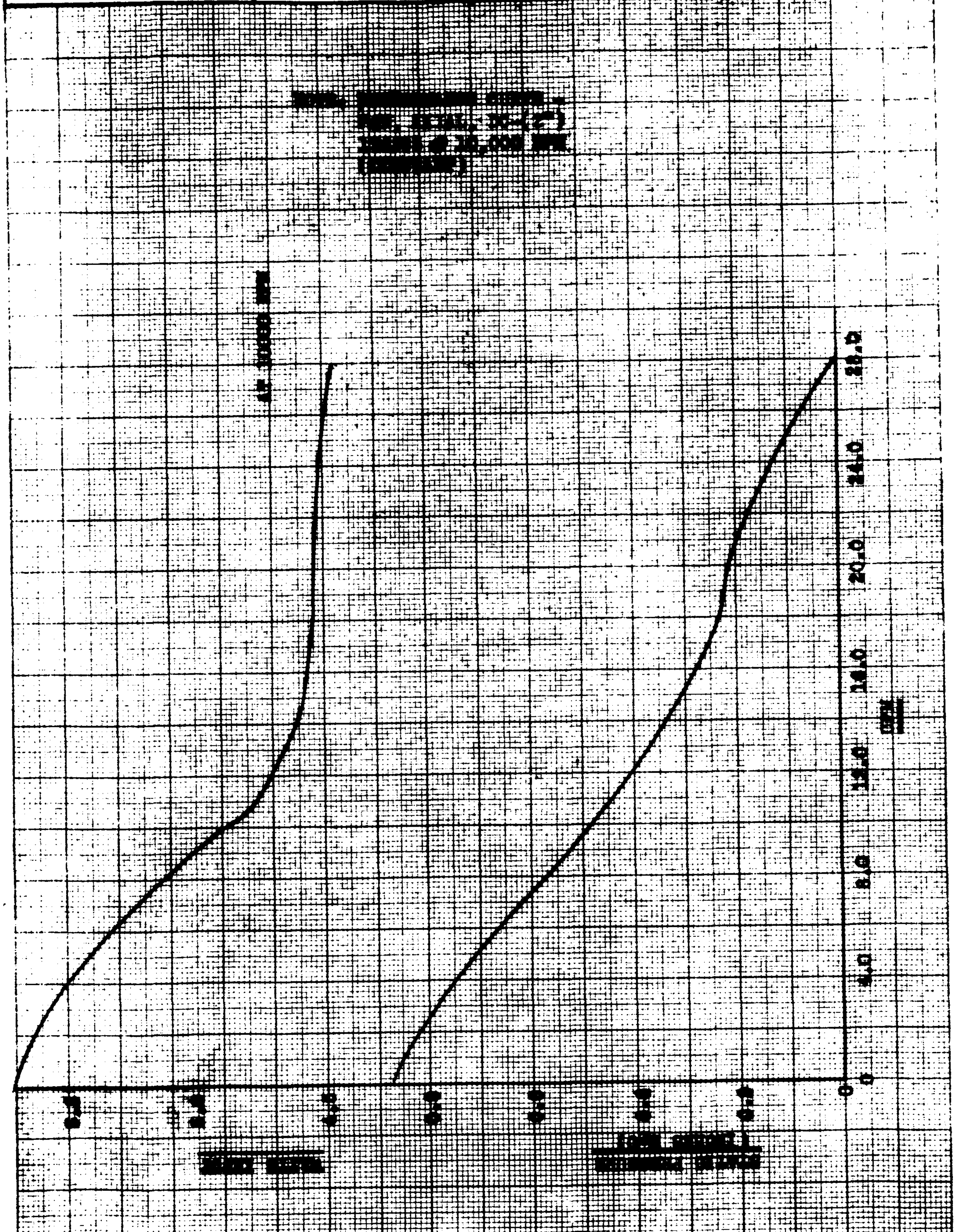
The nomenclature is given in Appendix 2 which contains the development of this equation.

THERMODYNAMICS PREPARED BY:	NORTH AMERICAN AVIATION, INC. FAN POWER AND SPEED VS. VOLTAGE	PAGE NO. 13 OF
CHECKED BY:		REPORT NO. NA-65-513
DATE:	FIGURE 2	MODEL NO.



FORM 1021-B

THERMODYNAMICS PREPARED BY:	**NORTH AMERICAN AVIATION, INC.** FAN PERFORMANCE CURVE	PAGE NO 14 OF
CHECKED BY:		REPORT NO. NA-65-513
DATE:	FIGURE 3	MODEL NO.



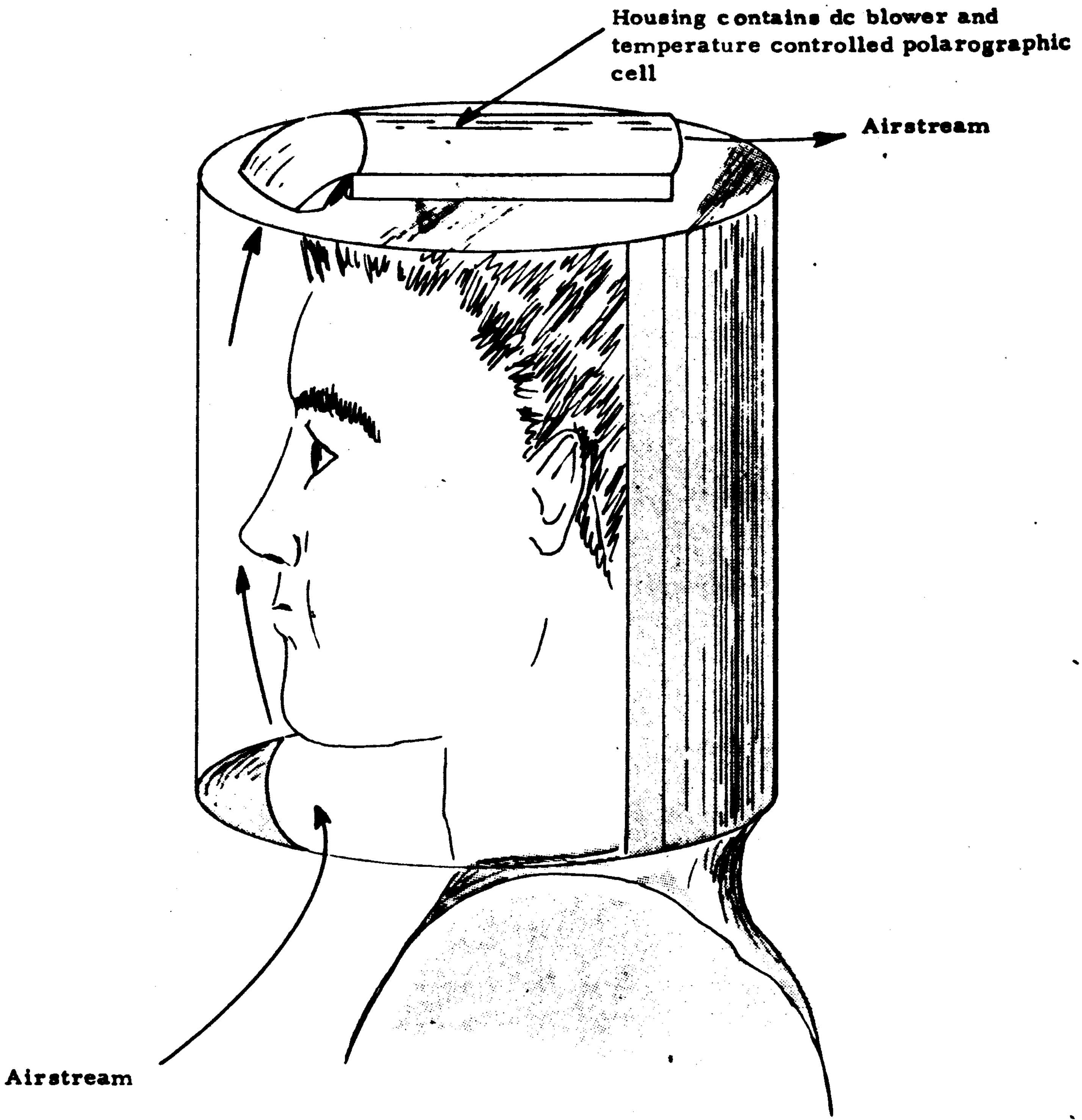


Figure 4 Helmet System Used in Laboratory MRM Device.

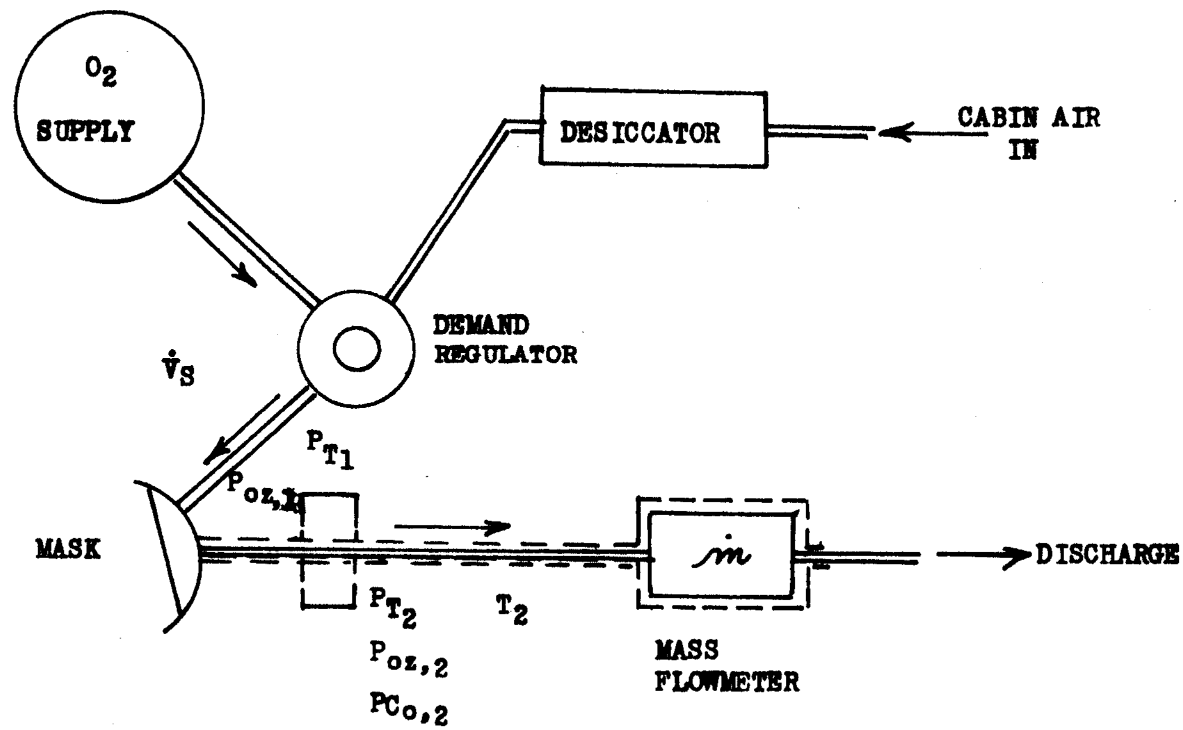


FIGURE 5 SCHEMATIC DIAGRAM OF RESPIRATION ANALYZER

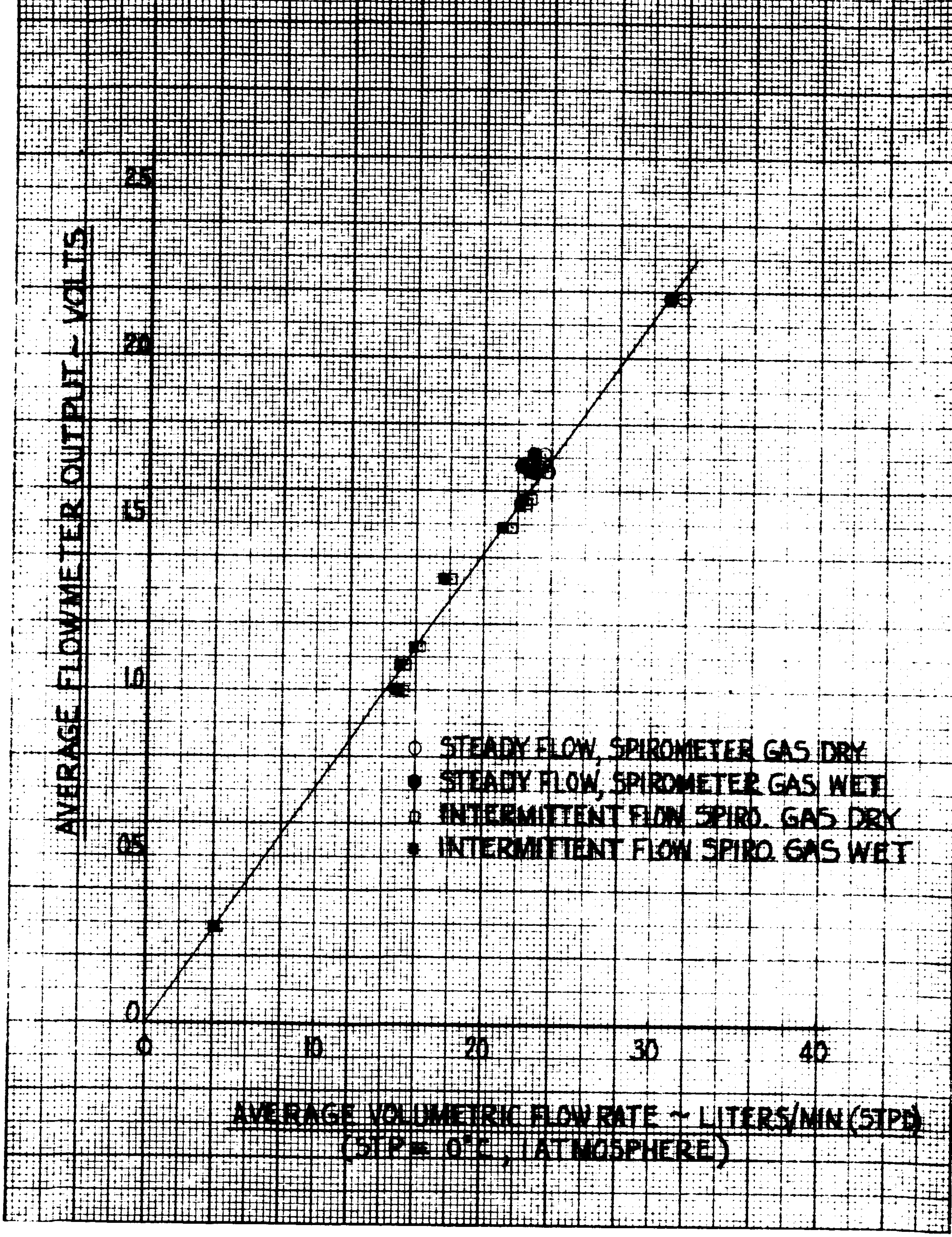
Figures 11 through 13 show the error in oxygen uptake vs. P_{O2}, $P_{O2,2}$ and P_{DIOX_2}. An alternate calculation based on a differentiation of the complete equation was made for comparative purposes and is included in Appendix 3.

Evaluation of the analyzer should be based on a ±5% accuracy of the flowmeter. A simple calibration of a typical meter using pulsitile flow has been conducted. A Collins Spirometer was used as a reference. The results, shown in figure 6, justify the above accuracy statement.

FORM 1006-X (FORMERLY 20147)

PREPARED BY: R.K.B.	**NORTH AMERICAN AVIATION, INC.** CALIBRATION OF TECH., INC. FLOWMETER NO. 1	PAGE NO. 18 OF
CHECKED BY:		REPORT NO. NA-65-513
DATE: FEB. 2, 1965	CALIBRATION GAS - OXYGEN	MODEL NO.

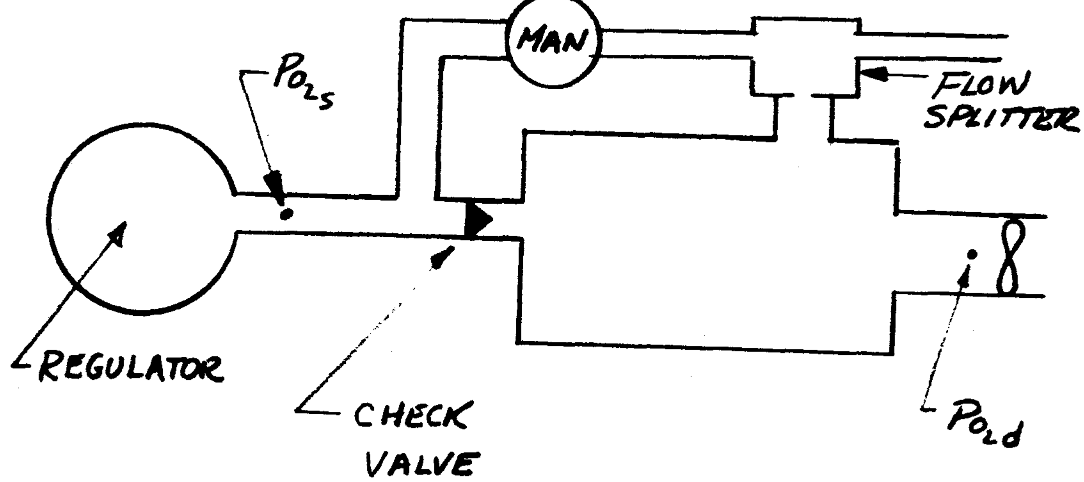
FIGURE



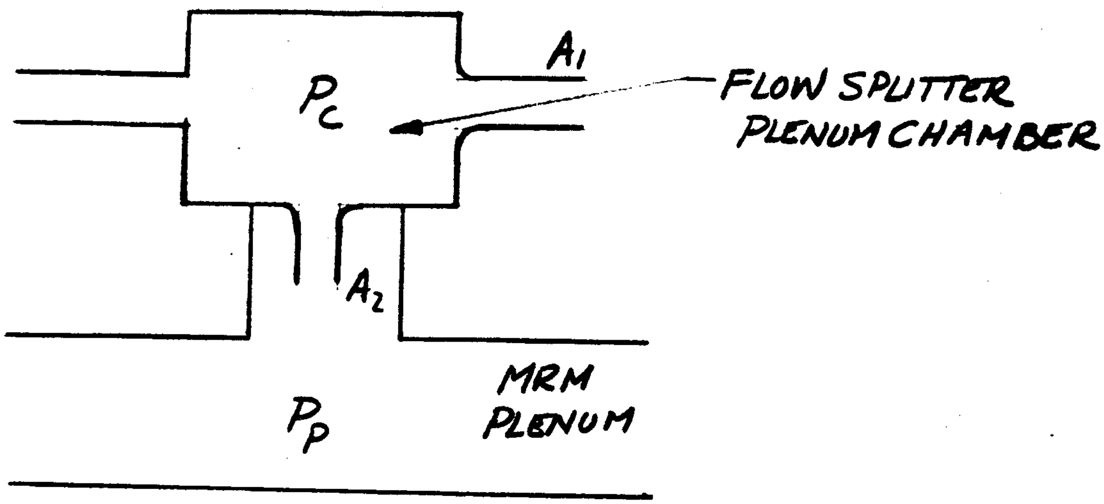
VI. ANALYSIS OF THE VENTURIS

A preliminary look has been made at a concept proposed by Dr. Roman for "sampling" the respiration of a man by means of flow splitting venturis. Thepurpose of this idea is to allow use of "scaled down" MRM to reduce the large quantities of diluent gas required by the system.

Examination of the problem indicates that it is only necessary to sample the man's expired flow providing he breathes gas of the same composition as that supplied to the MRM. A schematic of a suitable arrangement is shown below.



Since with this arrangment it is only necessary to split the expired flow which is always above ambient pressure, a venturi will not be required. A very accurate flow splitter can be made by dumping the expired air into a plenum and discharging it through bell-mouth nozzles sized for the desired flow split. The accuracy of the split will depend only on the accuracy of the nozzle discharge areas and the ability to maintain the plenum pressure of the MRM equal to ambient.



The following derivation shows that the flow split is proportional to the area ratio

By continuity

$$W_1 = \rho_1 A_1 V_1$$
$$W_2 = \rho_2 A_2 V_2$$

then

$$q_1 = P_C - P_B = \frac{\rho_1 V_1^2}{2g}$$
$$q_2 = P_C - P_B = \frac{\rho_2 V_2^2}{2g}$$

For very small changes between P_B & P_P, $\rho_1 = \rho_2$

then $$\frac{P_C - P_B}{P_C - P_P} = \frac{V_1^2}{V_2^2}$$

and

$$\frac{W_1}{W_2} = \frac{\rho_1 A_1 V_1}{\rho_2 A_2 V_2} = \frac{A_1 V_1}{A_2 V_2}$$

$$\frac{V_1^2}{V_2^2} = \left(\frac{A_1 V_1}{A_2 V_2}\right)^2 \times \frac{A_2^2}{A_1^2}$$

$$\frac{P_C - P_B}{P_C - P_P} = \frac{W_1^2}{W_2^2} \times \frac{A_1^2}{A_2^2}$$

finally

$$\frac{W_1}{W_2} = \sqrt{\frac{P_C - P_B}{P_C - P_P}} \times \frac{A_1}{A_2}$$

VII. ALTERNATE CONCEPTS OF THE MRM

After the contractural efforts began several alternate concepts were introduced which modified the basic design of the MRM. Unfortunately, there was insufficient time to explore all of these concepts. The concept shown in figure 7 was brought out in time for a cursory evaluation of the concept to be made. As indicated in the schematic diagram the system utilizes a separate source for the diluent which may be air or 100 percent nitrogen. This enables the pilot (subject) to breath 100 percent oxygen which is a desirable feature. This system also differs from the basic in that a flowmeter is used to measure the oxygen inspired flow rate, and the blower readout is a function of the expired oxygen. Instead of a (ΔP_{O_2}) set-point, the servo-controller can operate with an altitude compensated P_{O_2} sensor only.

This concept retains some of the basic problems of taking differences from large numbers to obtain oxygen uptake, as seen in the following analysis:

$$\dot{V}_{O_2} = \dot{V}_{O_{2i}} - (\dot{V}_e - \dot{V}_{CO_2})$$

$\dot{V}_{O_2} =$ Oxygen Uptake

$\dot{V}_{O_{2i}} =$ Inspired Oxygen Volumetric Flowrate, L/M

$\dot{V}_e =$ Expired Gas Volumetric Flowrate, L/M

$\dot{V}_{CO_2} =$ Expired CO_2 Volumetric Flowrate, L/M

then $$\dot{V}_{O_2} = \dot{V}_{O_{2i}} - \dot{V}_F \frac{P_{O_{2,2}}}{P_B}$$

$P_{O_{2,2}} =$ Discharge Partial Pressure of O_2

$\dot{V}_F =$ Fan Volumetric Flowrate, L/M

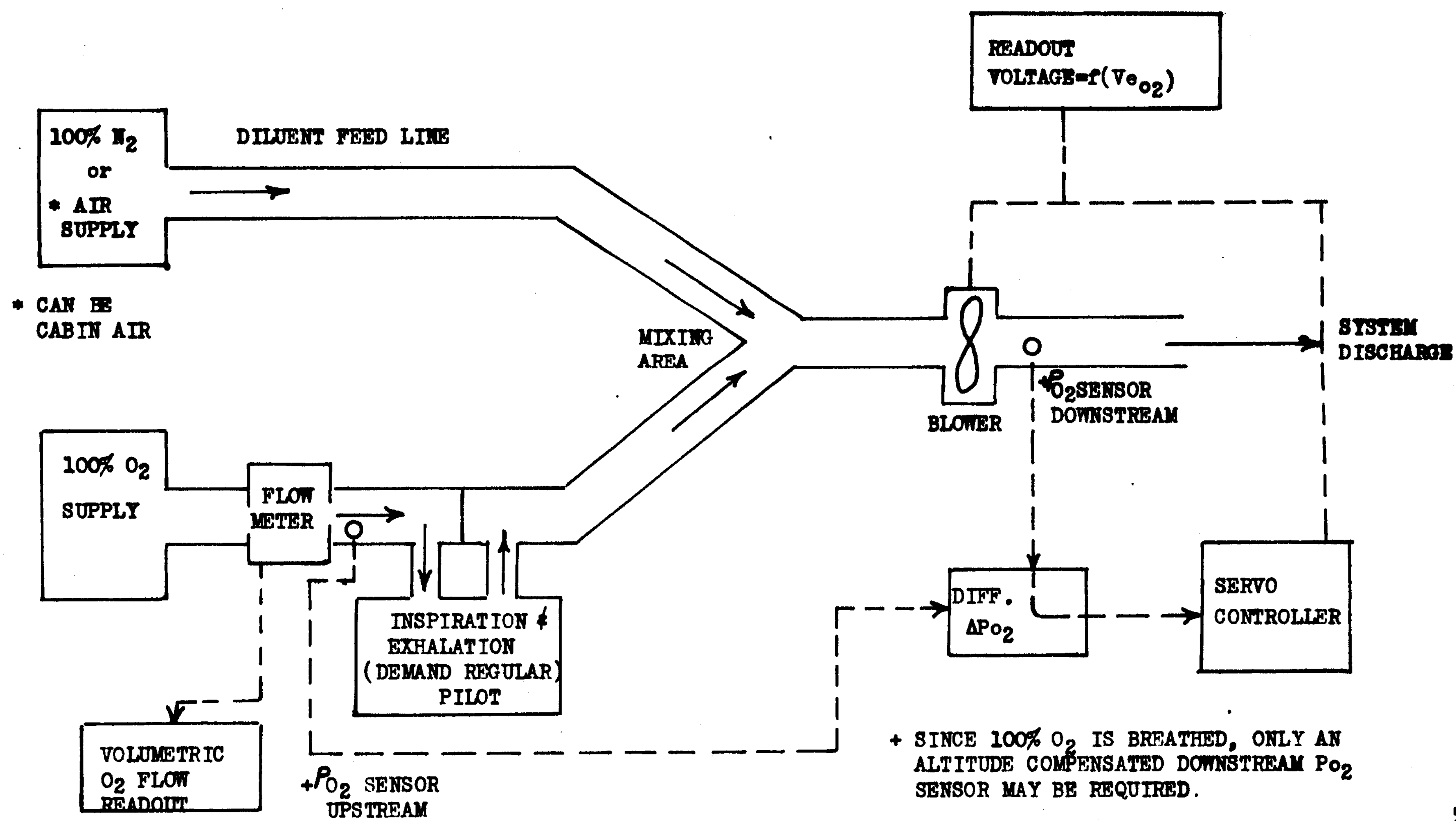


FIGURE 7 SCHEMATIC DIAGRAM OF ALTERNATE MRM CONCEPTS

$$\dot{V}_{O_{2i}} \equiv X \dot{V}_F$$

AND $$\dot{V}_{O_2} = X \dot{V}_F - \dot{V}_F \left(\frac{P_{O_2}}{P_B}\right)$$

CONSIDER $X = 0.2$

$$\dot{V}_{O_2} = 0.2 \dot{V}_F - \dot{V}_F \left(\frac{P_{O_2}}{P_B}\right)$$

$$\dot{V}_{O_2} = .2(100) - (100)\left(\frac{P_{O_2}}{P_B}\right)$$

If the fan volumetric flowrate is 100 liters/min

ASSUMING $\left(\frac{P_{O_2}}{P_B}\right) = 0.19$

$$\dot{V}_{O_2} = .2(100) - (100)(.19) = 20 - 19 = 1.0$$

FOR A ONE PERCENT ERROR IN FAN FLOW

$$\dot{V}_{O_2} = 20 - 99 \times .19 = 20 - 18.81 = 1.19$$

GIVING A 19 % ERROR IN $\dot{V}_{O_2}$

FOR A 5% ERROR IN FLOWMETER MEASUREMENT

$$\dot{V}_{O_2} = (20 + .05 \times 20) - 19 = 21 - 19$$

$$= 2.0$$ OR A 100% ERROR IN $\dot{V}_{O_2}$

The added complexity of the additional flowmetering device did not improve the accuracy of this system.

VIII. CONCLUSIONS

It is concluded by the North American Aviation, Inc. evaluators that on a basis of error susceptibility and complexity of configuration that the respiratory analyzer using the MRM concepts represents a higher development risk than the respiratory analyzer concept using a single flowmeter.

IX. REFERENCES

A. NASA Contract Letter Document NAS4-876/TRS, dated 16 June 1965

B NASA Contract NAS4-367

C NAA Report NA-65-22, dated 25 January 1965

D NAA Proposal NA-65-51, dated 15 February 1965

E NAA Proposal NA-65-51-1, dated 15 February 1965

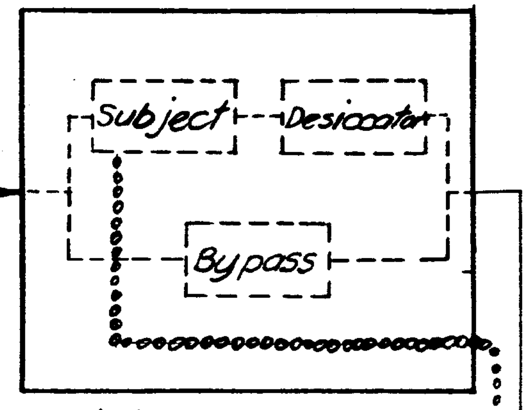
F NAA Price Proposal 65-1799-3-JP-00, dated 26 May 1965

MODEL

Supply

Dry Air at:

- Pressure P_1
- Temperature T_1
- Oxygen fraction (Vol. or mol.) x_1
- Volumetric flow rate, $\dot{V}_1$



Discharge

Gaseous Mixture of Air and Expirate at:

- Pressure P_2
- Temperature T_2
- Partial pressure oxygen P_{ox_2}
- Volumetric flow rate $\dot{V}_2$

Uptake:

Pure Oxygen at:

- Mass uptake rate $\dot{U}$

RQ: respiration quotient; ox: oxygen; non-ox: non-oxygen

$\dot{n}$: molar flow rate; exp.: expiration; insp: inspiration; resp: respiration.

Analysis

$$\dot{U} = 32\left[\frac{\chi_1 P_1 \dot{V}_1}{RT_1} - \frac{P_{ox_2}\dot{V}_2}{RT_2}\right]$$

$$= 32\left[\chi_1\left(\frac{P_2\dot{V}_2}{RT_2} + \frac{\dot{U}}{32} - \Delta\dot{n}_{non\text{-}ox}\right) - \frac{P_{ox_2}\dot{V}_2}{RT_2}\right]$$

$$\dot{U}(1-\chi_1) = 32\left[(\chi_1 P_2 - P_{ox_2})\frac{\dot{V}_2}{RT_2} - \chi_1\Delta\dot{n}_{non\text{-}ox}\right]$$

Assume $\Delta\dot{n}_{non\text{-}ox} = \Delta\dot{n}_{CO_2} = \frac{\dot{U}}{32} \times RQ$

$$\dot{U}[1-(1-RQ)\chi_1] = \frac{32}{RT_2}(\chi_1 P_2 - P_{ox_2})\dot{V}_2$$

$$\dot{n}_2 - \dot{n}_1 = \dot{n}_{exp} - \dot{n}_{insp}$$

$$= \Delta\dot{n}_{resp}$$

$$= \Delta\dot{n}_{ox} + \Delta\dot{n}_{non\text{-}ox}$$

$$= -\frac{\dot{U}}{32} + \Delta\dot{n}_{non\text{-}ox}$$

$$\frac{P_2\dot{V}_2}{RT_2} - \frac{P_1\dot{V}_1}{RT_1} = -\frac{\dot{U}}{32} + \Delta\dot{n}_{non\text{-}ox}$$

$$\frac{P_1\dot{V}_1}{RT_1} = \frac{P_2\dot{V}_2}{RT_2} + \frac{\dot{U}}{32} - \Delta\dot{n}_{non\text{-}ox}$$

For $P_1 = P_2 = P$ and $P_{ox_1} - P_{ox_2} = \Delta P_{ox}$

$$\dot{U}[1-(1-RQ)\chi_1] = \frac{32}{RT_2}\Delta P_{ox}\dot{V}_2$$

$$\dot{V}_2 = \frac{\dot{U}}{32} RT_2 \left[\frac{1-(1-RQ)\chi_1}{\Delta P_{ox}}\right]$$

Errors

Effect of error in temperature measurement

For $P_S = P_d = P_T$

$$\dot{V}_d = \frac{RT_d}{32}\,\frac{[1-(1-RQ)\chi_{O_2 s}]}{\Delta P_{O_2}}\,\dot{U} \qquad \text{where } \chi_{O_2 s} = \frac{P_{O_2 s}}{P_T}$$

$$\epsilon_T \equiv \frac{|\Delta T_d|}{T_d}$$

$$\dot{V}_{d\,observed} = \frac{R(1 \pm \epsilon_T)T_d}{32}\,\frac{[1-(1-RQ)\chi_{O_2} s]}{\Delta P_{O_2}}\,\dot{U}$$

$$\dot{U}_{calculated} = (1 \pm \epsilon_T)\dot{U}$$

$$\therefore \quad \epsilon_U = \frac{\dot{U}_{calc} - \dot{U}}{\dot{U}}$$

$$= \pm \epsilon_T$$

Effect of deviation of subject's RQ from reference (Assumed) RQ

For $P_S = P_d = P_T$

$$\dot{V}_{d\,obs} = \frac{RT_d}{32}\left[1 + (RQ_{subject} - 1)\frac{P_{O_2 s}}{P_T}\right]\frac{1}{\Delta P_{O_2}}\,\dot{U}$$

$$\dot{U}_{calc} = \left[1 + (RQ_{subject} - 1)\frac{P_{O_2 s}}{P_T}\right]\left[1 + (RQ_{reference} - 1)\frac{P_{O_2 s}}{P_T}\right]^{-1}\dot{U}$$

$$\epsilon_U \equiv \frac{\dot{U}_{calc} - \dot{U}}{\dot{U}} = \frac{P_T + (R.Q._{SUB} - 1)P_{O_2 s} - P_T - (R.Q_{REF} - 1)P_{O_2 s}}{P_T + (R.Q_{REF.} - 1)P_{O_2 s}}$$

$$= \frac{P_{O_2 s}\,(RQ_{subj} - RQ_{ref})}{P_T + P_{O_2 s}\,(RQ_{ref} - 1)}$$

$$= \frac{\chi_{O_2 s}\,(RQ_{subj} - RQ_{ref})}{1 + \chi_{O_2 s}(RQ_{ref} - 1)} \qquad \text{where } \chi_{O_2 s} \equiv \frac{P_{O_2 s}}{P_T}$$

For the case $RQ_{ref} = 1$, the above reduces to

$$\epsilon_U = (RQ_{subj} - 1)\chi_{O_2 s}$$

Errors (Cont.)

This relationship is plotted on figure 8.

<u>Effect of error in oxygen partial pressure measurements.</u>

$$\dot{U}\left[1+(RQ-1)\chi_{O_2s}\right] = \frac{32}{RT_d}\,\Delta P_{O_2}\,\dot{V}_d$$

The following subscripts are used:

exp: experiment ; set: set point

$$P_{O_2s\,exp} = (1 \pm \epsilon_s)\,P_{O_2s\,set}$$

$$P_{O_2d\,exp} = (1 \pm \epsilon_d)\,P_{O_2d\,set}$$

When $|\epsilon_s| = |\epsilon_d| = |\epsilon_{O_2}|$

(WORST CASE) $$\Delta P_{O_2\,exp} = P_{O_2d\,exp} - P_{O_2d\,exp} = \Delta P_{O_2\,set} \pm \epsilon_{O_2}\,(P_{O_2s}+P_{O_2d})$$

$$\dot{V}_{d\,exp} = RT_2\,\frac{\dot{U}}{32}\,\frac{1+(RQ-1)\chi_{O_2s}}{\Delta P_{O_2}\,exp} = RT_2\,\frac{\dot{U}}{32}\,\frac{1+(RQ-1)\chi_{O_2s}}{\Delta P_{O_2}\,set \pm \epsilon_{O_2}(P_{O_2s}-P_{O_2d})}$$

$$\dot{U}_{exp} = \frac{\Delta P_{O_2}\,set}{\Delta P_{O_2}\,set \pm \epsilon_{O_2}\,(P_{O_2s}+P_{O_2d})}\,\dot{U}$$

$$\epsilon_U \equiv \frac{\Delta\dot{U}}{\dot{U}} = \frac{\Delta P_{O_2}set - \left[\Delta P_{O_2}set \pm \epsilon_{O_2}\,(P_{O_2s}set + P_{O_2}set)\right]}{\Delta P_{O_2}set \pm \epsilon_{O_2}\,(P_{O_2s}set + P_{O_2d}set)}$$

$$= -\frac{\pm\epsilon_{O_2}\,(P_{O_2s}set + P_{O_2d}set)}{\Delta P_{O_2}set \pm \epsilon_{O_2}\,(P_{O_2s}set + P_{O_2d}set)}$$

$$= -\frac{\pm\epsilon_{O_2}\,(\chi_{O_2s} + \chi_{O_2d})}{(\chi_{O_2s} - \chi_{O_2d}) \pm \epsilon_{O_2}\,(\chi_{O_2d} + \chi_{O_2s})}$$

$$= -\frac{\pm\left(1+\frac{\chi_{O_2d}}{\chi_{O_2s}}\right)\epsilon_{O_2}}{\left(1-\frac{\chi_{O_2d}}{\chi_{O_2s}}\right) \pm \epsilon_{O_2}\left(1+\frac{\chi_{O_2d}}{\chi_{O_2s}}\right)}$$

Numerical Calculations

Effect of error in oxygen partial pressure measurements

① ϵ_{O_2}	② $\frac{\chi_{O_2 d}}{\chi_{O_2 s}}$	③ $(1+\frac{\chi_{O_2 d}}{\chi_{O_2 s}})\epsilon_{O_2}$	④ $1-②-③$	⑤ $\epsilon_U = \frac{③}{④}$
0.01	0.95	.0195	.0305	0.635
0.02		.039	.0110	3.550
0.03		.0585		
0.04		.0780		
0.05		.0975		
0.01	0.90	.019	.081	.235
0.02		.038	.062	.613
0.03		.057	.043	1.325
0.04		.076	.024	3.17
0.05		.095	.005	19.0
0.01	0.85	.0185	.1315	.141
0.02		.0370	.113	.327
0.03		.0555	.0945	.587
0.04		.0740	.076	.974
0.05		.0925	.0575	1.61
0.01	0.80	.018	.182	.099
0.02		.036	.164	.22
0.03		.054	.146	.37
0.04		.072	.128	.56
0.05		.090	.100	.90

FORM 1040-B

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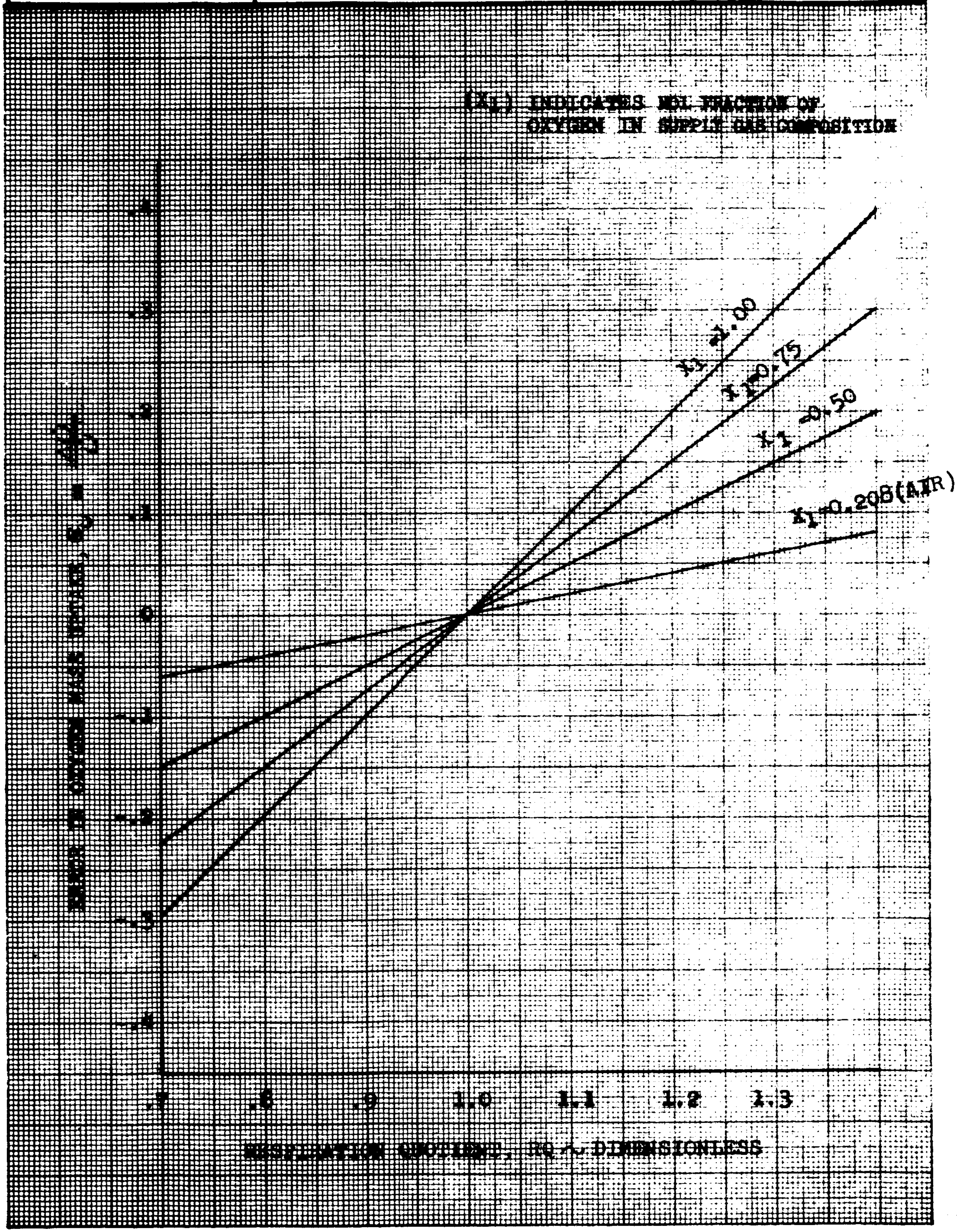
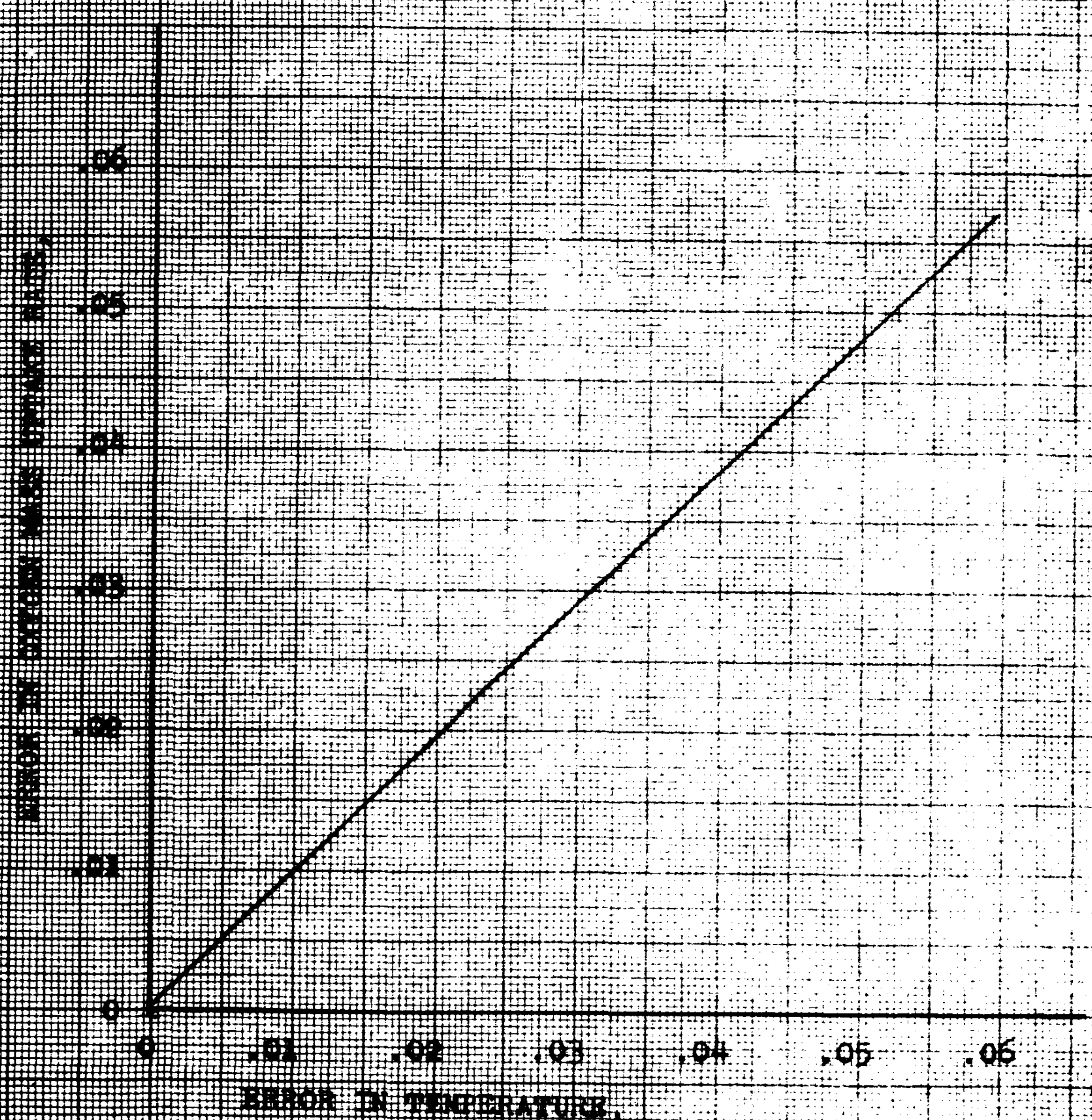
CHECKED BY:		NA-65-513 REPORT NO.
DATE:	FIGURE 8	MODEL NO.



PREPARED BY:	**NORTH AMERICAN AVIATION, INC.** ERROR IN OXYGEN UPTAKE FOR ERROR IN TEMPERATURE	PAGE NO. 33 OF
CHECKED BY:		NA-65-513 REPORT NO.
DATE:	FIGURE 9	MODEL NO.



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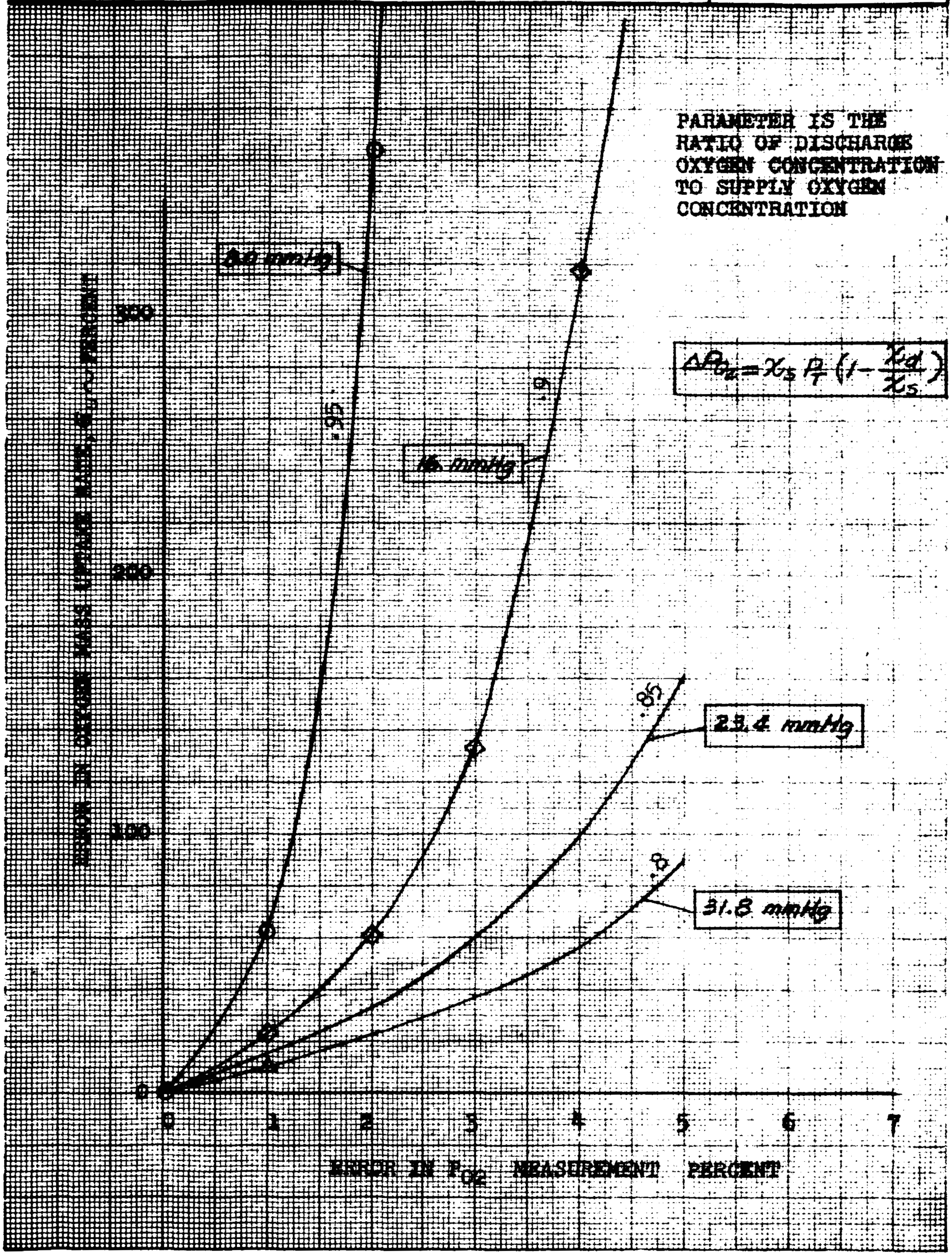
CHECKED BY:		NA-65-513 REPORT NO.
DATE:	FIGURE 10	MODEL NO.



<u>Analysis</u>

FOR MODEL AND NOTATION SEE PAGE 39

$$\frac{\dot{U}}{MW_{OX}} = \dot{n}_{OX1} - \dot{n}_{OX2}$$
$$= \chi_{OX1}\,\dot{n}_{OX1} - \chi_{OX2}\,\dot{n}_2$$

Assume

$$\dot{n}_{NIT1} = \dot{n}_{NIT2}$$
$$\chi_{NIT1}\,\dot{n}_1 = \chi_{NIT2}\,\dot{n}_2$$

$$\frac{\dot{U}}{MW_{OX}} = \left(\chi_{OX1}\frac{\chi_{NIT2}}{\chi_{NIT1}} - \chi_{OX2}\right)\dot{n}_2$$

$$= \left(\chi_{OX1}\frac{\chi_{NIT2}}{\chi_{NIT1}} - \chi_{OX2}\right)\frac{\dot{m}_2}{MW_2}$$

$$= \left(\chi_{OX1}\frac{\chi_{NIT2}}{\chi_{NIT1}} - \chi_{OX2}\right)\frac{\dot{m}_2}{\Sigma_i\,\chi_{i2}\,MW_i}$$

$$= \left(\frac{P_{OX1}\,P_{NIT2}}{P_{NIT1}} - P_{OX2}\right) \times \frac{\dot{m}_2}{\Sigma_i\,P_{i_2}\,MW_i}$$

If supply contains only oxygen and nitrogen and expirate contains only oxygen, nitrogen and carbon dioxide; then :

$$\frac{\dot{U}}{MW_{OX}} = \frac{P_{OX_1}(P_2 - P_{OX_2} - P_{DIOX_2}) - P_{OX_2}(P_1 - P_{OX_1})}{(P_1 - P_{OX_1})[P_{OX_2}MW_{OX} + P_{DIOX_2}MW_{DIOX} + (P_2 - P_{OX_2} - P_{DIOX_2})MW_{NIT}]}\dot{m}_2$$

$$= \frac{P_{OX_1}(P_2 - P_{DIOX_2}) - P_{OX_2}P_1}{(P_1 - P_{OX_1})[(MW_{OX} - MW_{NIT})P_{OX_2} + (MW_{DIOX} - MW_{NIT})P_{DIOX_2} + MW_{NIT}P_2]}\dot{m}_2$$

$$\dot{U} = 8\,\frac{P_{OX_1}(P_2 - P_{DIOX_2}) - P_{OX_2}P_1}{(P_1 - P_{OX_1})(P_{OX_2} + 4P_{DIOX_2} + 7P_2)}\,\dot{m}_2$$

Errors

In following, assume there is no error other than that being examined. ϵ indicates absolute value of fractional error.

Effect of error in measurement of discharge partial pressure of carbon dioxide.

For $P_1 = P_2 = P$,

$$\dot{U} = 8\,\frac{\chi_{ox_1}(1-\chi_{DIOX_2}) - \chi_{ox_2}}{(1-\chi_{ox_1})(\chi_{ox_2} + 4\chi_{DIOX_2} + 7)}\,\dot{m}_2$$

$$\epsilon_{DIOX} \equiv \frac{|\Delta P_{DIOX_2}|}{P_{DIOX_2}} = \frac{|\Delta \chi_{DIOX_2}|}{\chi_{DIOX_2}}$$

$$\epsilon_U \equiv \frac{\Delta \dot{U}}{\dot{U}}$$

$$= \frac{\chi_{ox_1}[1-(1\pm\epsilon_{DIOX_2})\chi_{DIOX_2}] - \chi_{ox_2}] - \chi_{ox_2}}{[\chi_{ox_2} + 4(1\pm\epsilon_{DIOX_2})\chi_{DIOX_2} + 7]} \times$$

$$\frac{(\chi_{ox_2} + 4\chi_{DIOX_2} + 7)}{\chi_{ox_1}(1-\chi_{DIOX_2}) - \chi_{ox_2}} - 1$$

$$= \mp\,\frac{11\chi_{ox_1} + \chi_{ox_1}\chi_{ox_2} - 4\chi_{ox_2}}{\chi_{ox_1}(1-\chi_{DIOX_2}) - \chi_{ox_2}} \cdot \frac{\epsilon_{DIOX_2}\,\chi_{DIOX_2}}{\chi_{ox_2} + 4(1\pm\epsilon_{DIOX_2})\chi_{DIOX_2} + 7}$$

Numerical Calculations

Assume compositions:

Constituent	x_i	
	supply	discharge
oxygen	0.20955	0.1627
carbon dioxide		0.0407

$$\frac{11\,x_{ox_1} + x_{ox_1} x_{ox_2} - 4\,x_{ox_2}}{x_{ox_1}(1 - x_{diox_2}) - x_{ox_2}}\, x_{diox_2} = 1.79314285$$

$$x_{ox_2} + 4\,x_{diox_2} + 7 = 7.3255$$

① ϵ_{diox_2}	② 7.3255 − 0.1628 × ①	③ ϵ_U 1.79314285 × ①/②
0.01	7.323872	0.0024483536
0.025	7.32143	0.0061229256
0.05	7.31736	0.0122326615
0.075	7.31329	0.018389222
0.09	7.310848	0.022074437
0.10	7.30922	0.024532616

FORM 1002-X (FORMERLY ED147)

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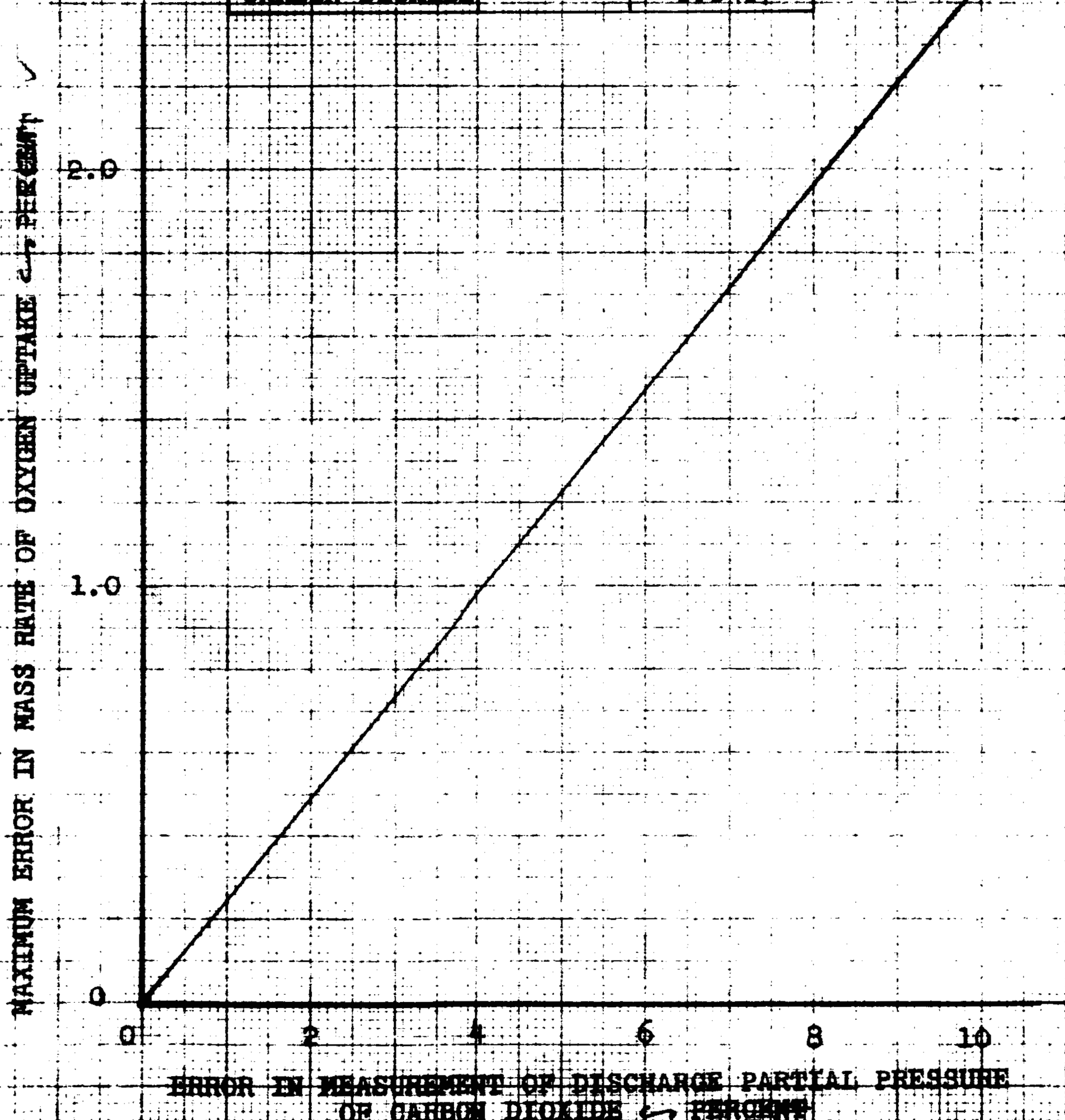
CHECKED BY:	PARTIAL PRESSURE MEASUREMENT	NA-65-513 REPORT NO.
DATE:	ON OXYGEN UPTAKE RATE	MODEL NO.

FIGURE 11

COMPOSITIONS

CONSTITUENT	CONCENTRATION (MOL. FRACTION)	
	SUPPLY	DISCHARGE
OXYGEN	0.20955	0.1627
CARBON DIOXIDE		0.0407



MODEL

APPENDIX 2

Supply

Dry gaseous mixture of oxygen and inerts at

Oxygen fraction (vol. or mol)		X_{OX_1}
Nitrogen fraction (vol. or mol)		X_{NIT_1}
Molar flow rate (total)		$\dot{n}_1$
Oxygen molor flow rate		$\dot{M}_{OX_1}$
Oxygen molar flow rate		$\dot{m}_{OX_1}$
Nitrogen molar flow rate		$\dot{m}_{NIT_1}$
Pressure	MEASURED	P_1
Partial pressure oxygen	MEASURED	P_{OX_1}

Uptake

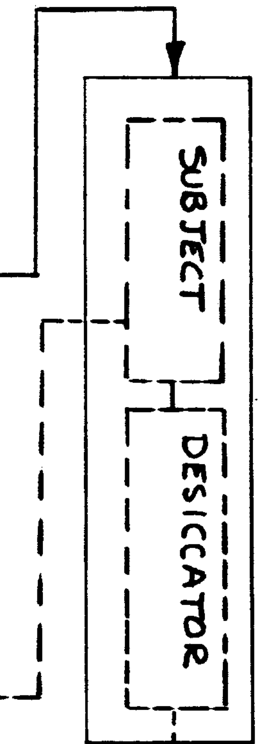
Pure oxygen at

Mass uptake rate (to be determined)		$\dot{U}$

Discharge

Dry expirate at

Mass flow rate (total)	(MEASURED)	$\dot{m}_2$
Oxygen fraction (vol. or mol)		X_{OX_2}
Nitrogen fraction (vol. or mol)		X_{NIT_2}
Carbon dioxide		X_{DIOX_2}
Molar flow rate (total)		$\dot{n}_2$
Oxygen molar flow rate		$\dot{M}_{OX_2}$
Nitrogen molar flow rate		$\dot{M}_{NIT_2}$
Pressure	MEASURED	P_2
Partial pressure oxygen	MEASURED	P_{OX_2}
Partial pressure carbon dioxide	MEASURED	P_{DIOX_2}



Effect of error in supply oxygen partial pressure measurement

From

$$\dot{U} = 8\,\frac{x_{ox_1}(1-x_{diox_2}) - x_{ox_2}}{(1-x_{ox_1})(x_{ox_2} + 4x_{diox_2} + 7)}\,\dot{m}_2$$

$$\epsilon_{ox_1} \equiv \frac{\Delta P_{ox_1}}{P_{ox_1}} = \frac{\Delta x_{ox_1}}{x_{ox_1}}$$

$$\epsilon_U \equiv \frac{\Delta \dot{U}}{\dot{U}}$$

$$= \frac{8\cdot\frac{(1\pm\epsilon_{ox_1})x_{ox_1}(1-x_{diox_2}) - x_{ox_2}}{[1-(1\pm\epsilon_{ox_1})x_{ox_1}](x_{ox_2}+4x_{diox_2}+7)}\dot{m}_2 - 8\cdot\frac{x_{ox_1}(1-x_{diox_2})-x_{ox_2}}{(1-x_{ox_1})(x_{ox_2}+4x_{diox_2}+7)}\dot{m}_2}{8\cdot\frac{x_{ox_1}(1-x_{diox_2}) - x_{ox_2}}{(1-x_{ox_1})(x_{ox_2}+4x_{diox_2}+7)}\dot{m}_2}$$

$$= \frac{[(1\pm\epsilon_{ox_1})x_{ox_1}(1-x_{diox_2}) - x_{ox_2}]\,(1-x_{ox_1})}{[1-(1\pm\epsilon_{ox_1})x_{ox_1}][x_{ox_1}(1-x_{diox_2})-x_{ox_2}]} - 1$$

Numerical Calculation

Constituent	x_i	
	Supply	Discharge
Oxygen	0.20955	0.1627
Carbon dioxide		0.0407

$1 - x_{diox_2} = .9593$

$1 - x_{ox_1} = .79045$

Effect of error in supply oxygen partial pressure measurement

1 ϵ_{ox_1}	2 $1+$ 1	3 $1-$ 1	4 .20102×2 .20102×3	5 4 $-.1627$	6 .20955×2 .20955×3	7 $1-$ 6	8 5/7	9 20.628×8	10 9 -1 ϵ_{U}
.001	1.001		.20122	.03852	.20976	.79024	.04874	1.0054	+.0054
.001		.999	.20082	.03812	.20934	.79066	.04821	.9945	−.0055
.005	1.005		.20203	.03933	.21060	.78940	.04982	1.0277	+.0277
.005		.995	.20001	.03731	.20850	.79150	.04713	.9722	−.0278
.01	1.01		.20303	.04033	.21165	.78835	.05115	1.0551	+.0551
.01		.99	.19901	.03631	.20745	.79255	.04581	.9450	−.0550
.03	1.03		.20705	.04435	.21584	.78416	.05655	1.1665	+.1665
.03		.97	.19499	.03229	.20326	.79674	.04052	.8358	−.1642

FORM 1021-S

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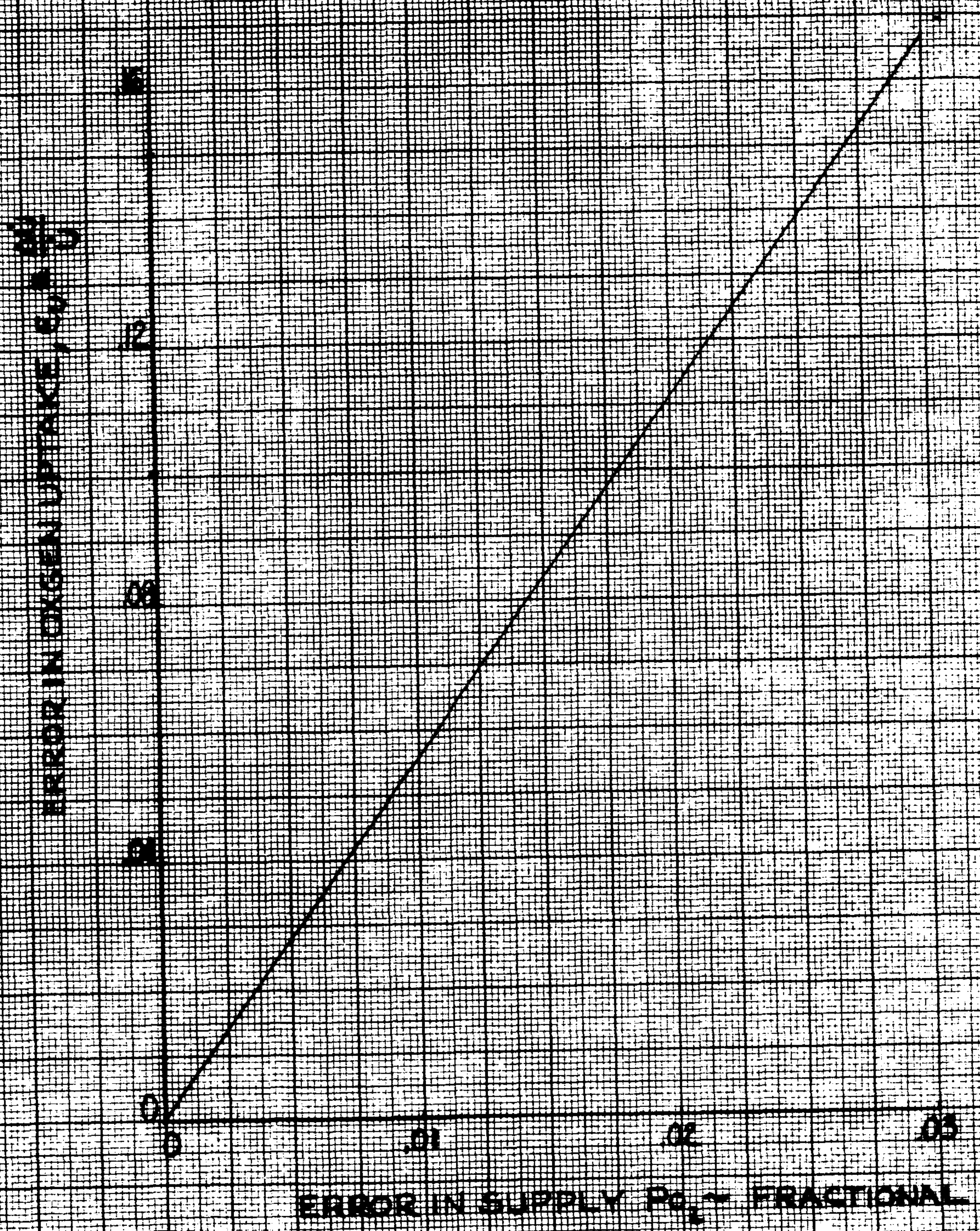
CHECKED BY:		NA-65-513 REPORT NO.
DATE:	FIGURE 12	MODEL NO.



Effect of error in discharge oxygen partial pressure measurement

From

$$\dot{U} = 8\,\frac{\chi_{OX_1}(1-\chi_{DIOX_2}) - \chi_{OX_2}}{(1-\chi_{OX_1})(\chi_{OX_2}+4\chi_{DIOX_2}+7)}\,\dot{m}_2$$

$$\epsilon_{OX_2} \equiv \frac{\Delta P_{OX_1}}{P_{OX_1}} = \frac{\Delta \chi_{OX_1}}{\chi_{OX_1}}$$

$$\epsilon_U \equiv \frac{\Delta \dot{U}}{\dot{U}}$$

$$= \frac{8\,\dfrac{\chi_{OX_1}(1-\chi_{DIOX_2}) - (1\pm\epsilon_{OX_2})\chi_{OX_2}}{(1-\chi_{OX_1})\left[(1\pm\epsilon_{OX_2})\chi_{OX_2}+4\chi_{DIOX_2}+7\right]}\,\dot{m}_2 - 8\,\dfrac{\chi_{OX_1}(1-\chi_{DIOX_2})-\chi_{OX_2}}{(1-\chi_{OX_1})(\chi_{OX_2}+4\chi_{DIOX_2}+7)}\,\dot{m}_2}{8\,\dfrac{\chi_{OX_1}(1-\chi_{DIOX_2})-\chi_{OX_2}}{(1-\chi_{OX_1})(\chi_{OX_2}+4\chi_{DIOX_2}+7)}\,\dot{m}_2}$$

$$= \frac{\chi_{OX_1}(1-\chi_{DIOX_2}) - (1\pm\epsilon_{OX_2})\chi_{OX_2}}{(1\pm\epsilon_{OX_2})\chi_{OX_2} + 4\chi_{DIOX_2} + 7} \cdot \frac{\chi_{OX_2}+4\chi_{DIOX_2}+7}{\chi_{OX_1}(1-\chi_{DIOX_2})-\chi_{OX_2}} - 1$$

Effect of error in discharge oxygen partial pressure measurement

Numerical Calculation

Constituent	χ_i	
	Supply	Discharge
Oxygen	0.20955	0.1627
Carbon dioxide		0.0407

$1-\chi_{DIOX2} = .9593$

$4\,\chi_{DIOX2} = .1628$

$4\chi_{DIOX2}+7 = 7.1628$

$\chi_{OX_2}+4\chi_{DIOX2}+7 = 7.3255$

$\chi_{OX_1}(1-\chi_{DIOX2}) = .2010213$

$\chi_{OX_1}(1-\chi_{DIOX2})-\chi_{OX_2} = .0383213$

$$\epsilon_U = \frac{.201021 - (1\pm\epsilon_{OX_2})\times .1627}{7.1628+(1\pm\epsilon_{OX_2})\times .1627}\;\frac{7.3255}{.0383213} - 1$$

①	② $1+\epsilon_{i2}$	③	④ $.1627\times②$ / $.1627\times③$	⑤ $.2010213-④$	⑥ $7.1628+④$	⑦ $\frac{7.3255}{⑥}$	⑧ $\frac{④}{.0383213}$	⑨ ⑦ × ⑧	⑩ ⑨ − 1 ϵ_U
.001	1.001		.16286	038161	7.3257	.999726	.9958169	.99554	-.00446
.001		.999	.16254	038481	7.3253	1.000273	1.0041674	1.00444	+.00444
.005	1.005		.16351	037511	7.3263	.9998908	.9788511	.97874	-.02126
.005		.995	.16189	039131	7.3247	1.0001092	1.0211292	1.02124	+.02124
.010	1.01		.16433	036691	7.3271	.9997816	.9574570	.95724	-.04276
.010		.99	.16107	039951	7.3239	1.0002184	1.0425273	1.04275	+.04275
.030	1.03		.16758	033441	7.3304	.9993315	.8726478	.87206	-.12794
.030		.97	.15782	043201	7.3206	1.0006693	1.1273365	1.12809	+.12809

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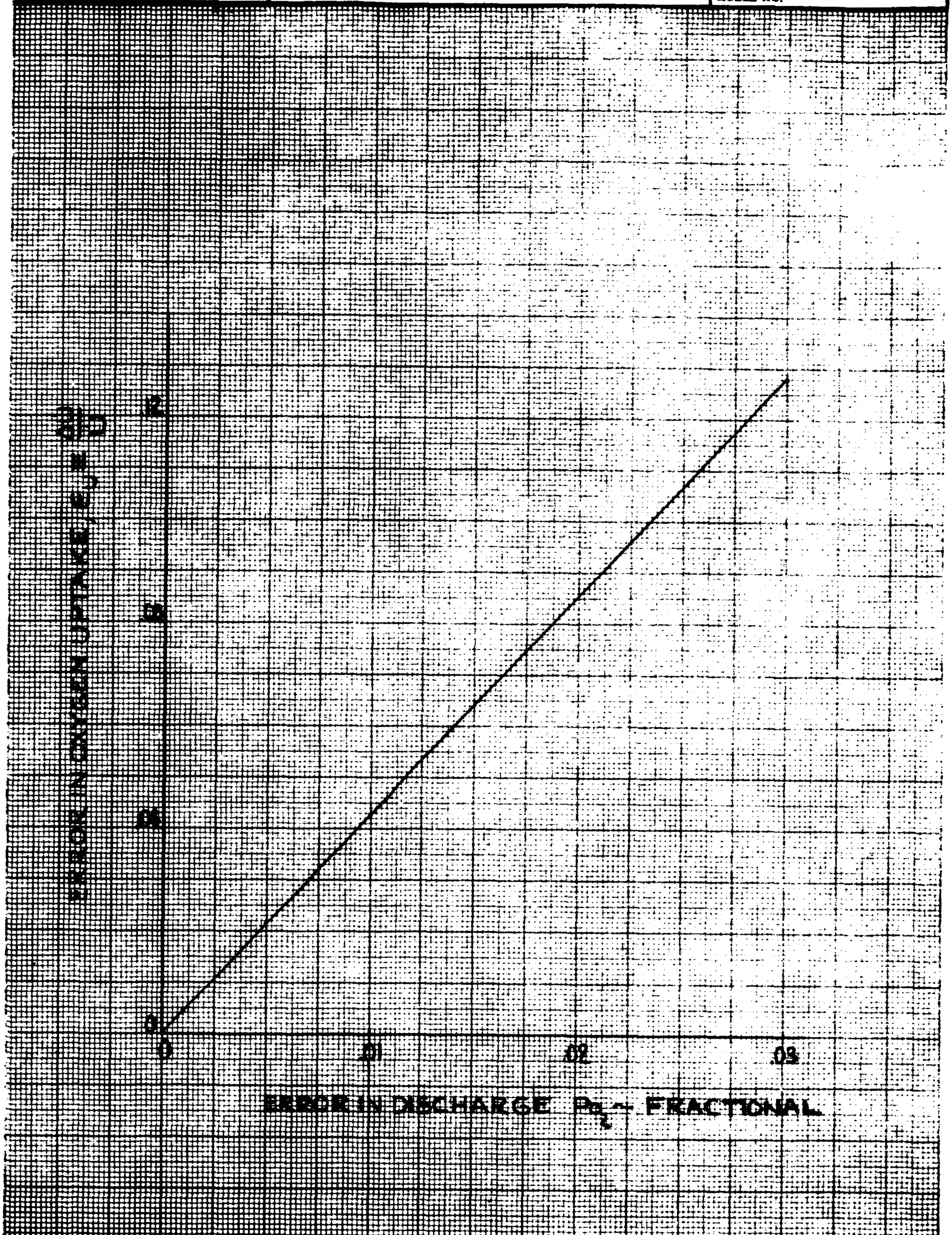
CHECKED BY:	EFFECT OF ERROR IN DISCHARGE O_2 PARTIAL PRESSURE MEASUREMENT	NA-65-513 REPORT NO.
DATE:	FIGURE 13	MODEL NO.



Error

An alternate method of error analysis is presented in which the product of two or more errors is neglected. (This is implicit in taking the differentials.)

Let $\quad M = \Sigma P_i MW_i$

$P_{ox_1} = x \qquad P_1 = \alpha$

$P_{ox_2} = y \qquad P_2 = \beta$

$P_{OrOx_2} = z$

Substituting the above symbols into the equation

$$\dot{U}\frac{M}{M_{ox}} = \frac{x(\beta - y - z) - y(\alpha - x)}{\alpha - x} \cdot \dot{m} = \frac{\beta x - xz - \alpha y}{\alpha - x}\,\dot{m}$$

Take the differential of both sides of the equation

$$d\left[\dot{U}\frac{M}{M_{ox}}\right] = d\left[\frac{\beta x - xz - \alpha y}{\alpha - x}\,\dot{m}\right]$$

$$\frac{M}{M_{ox_1}}\,d\dot{U} + \frac{\dot{U}}{M_{ox}}\,dM = \frac{\beta x - xz - \alpha y}{\alpha - x}\,d\dot{m} + \dot{m}\,d\left[\frac{\beta x - xz - \alpha y}{\alpha - x}\right]$$

$$d\left[\frac{\beta x - xz - \alpha y}{\alpha - x}\right] = \frac{(x\,d\beta + \beta\,dx) - (z\,dx + x\,dz) - (y\,d\alpha + \alpha\,dy)}{\alpha - x} - \frac{(\beta x - xz - \alpha y)(d\alpha - dx)}{(\alpha - x)^2}$$

$$= \frac{(x\,d\beta + \beta\,dx)(\alpha - x) - (z\,dx + x\,dz)(\alpha - x) - (y\,d\alpha + \alpha\,dy)(\alpha - x) - (\beta x - xz - \alpha y)(d\alpha - dx)}{(\alpha - x)^2}$$

Error (cont)

Collecting terms of like differential

$$= \left\{ \begin{array}{l} \dfrac{\alpha x\, d\beta + [(\beta - z)(\alpha - x) + (\beta x - xz - \alpha y)]dx - \alpha(\alpha - x)dy - x(\alpha - x)dz}{(\alpha - x)^2} \\ - \dfrac{[y(\alpha - x) + (\beta x - xz - \alpha y)]\, d\alpha}{(\alpha - x)^2} \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \dfrac{\alpha x\, d\beta + [\alpha\beta - \alpha z \cancel{-\beta x} \cancel{+ xz} \cancel{+\beta x} \cancel{- xz} - \alpha y]dx - \alpha(\alpha - x)dy - x(\alpha - x)dz}{(\alpha - x)^2} \\ - \dfrac{[\cancel{\alpha y} - xy + \beta x - xz \cancel{- \alpha y}]\, d\alpha}{(\alpha - x)^2} \end{array} \right\}$$

$$= \left\{ \frac{\alpha x\, d\beta + \alpha[\beta - z - y]dx - \alpha(\alpha - x)dy - x(\alpha - x)dz - x(\beta - z - y)d\alpha}{(\alpha - x)^2} \right\}$$

$$= \frac{\alpha x}{(\alpha - x)^2}\left\{ d\beta + (\beta - z - y)\frac{dx}{x} - \frac{(\alpha - x)}{x}dy - \frac{\alpha - x}{\alpha}dz - (\beta - z - y)\frac{d\alpha}{\alpha} \right\}$$

$$= \frac{\alpha x}{(\alpha - x)^2}\left\{ \beta\frac{d\beta}{\beta} + (\beta - z - y)\frac{dx}{x} - \frac{(\alpha - x)y}{x}\frac{dy}{y} - \frac{(\alpha - x)z}{\alpha}\frac{dz}{z} - (\beta - z - y)\frac{d\alpha}{\alpha} \right\}$$

$$\therefore \frac{M}{M_{ox_1}}d\dot{U} + \frac{\dot{U}}{M_{ox_1}}dM = \frac{\beta x - xz - \alpha y}{\alpha - x}d\dot{m} + \dot{m}\frac{\alpha x}{(\alpha - x)^2}\left\{ \quad \right\}$$

$$\frac{\dfrac{M}{M_{ox_1}}d\dot{U} + \dfrac{\dot{U}}{M_{ox_1}}dM}{\dot{U}\dfrac{M}{M_{ox_1}}} = \frac{\dfrac{\beta x - xz - \alpha y}{\alpha - x}d\dot{m} + \dot{m}\dfrac{\alpha x}{(\alpha - x)^2}\left\{ \quad \right\}}{\dot{m}\,\dfrac{\beta x - xz - \alpha y}{\alpha - x}}$$

$$\frac{d\dot{U}}{\dot{U}} + \frac{dM}{M} = \frac{d\dot{m}}{\dot{m}} + \frac{\alpha x}{(\alpha - x)(\beta x - xz - \alpha y)}\left\{ \quad \right\}$$

Error (Cont)

The quantity

$$\frac{dM}{M} = \frac{d\,\Sigma P_i MW_i}{\Sigma P_i MW_i}$$

$$= \frac{d\left[32y + 44z + (\beta - y - z)\,28\right]}{32y + 44z + (\beta - y - z)\,28}$$

$$= \frac{d\left[4y + 16z + 28\beta\right]}{4y + 16z + 28\beta}$$

$$= \frac{dy + 4dz + 7d\beta}{y + 4z + 7\beta}$$

$$= \frac{\frac{1}{\beta z}\frac{dy}{y} + \frac{4}{\beta y}\frac{dz}{z} + \frac{7}{yz}\frac{d\beta}{\beta}}{\frac{1}{\beta z} + \frac{4}{\beta y} + \frac{7}{yz}}$$

$\therefore$

$$\frac{d\dot{U}}{\dot{U}} = -\frac{\frac{1}{\beta z}\frac{dy}{y} + \frac{4}{\beta y}\frac{dz}{z} + \frac{7}{yz}\frac{d\beta}{\beta}}{\frac{1}{\beta z} + \frac{4}{\beta y} + \frac{7}{yz}} + \frac{d\dot{m}}{\dot{m}} + \frac{\alpha x}{(\alpha - x)(\beta x - xz - \alpha y)}\left\{\quad\right\}$$

Where the quantity $\{\ \}$:

$$\{\ \} = \beta\frac{d\beta}{\beta} + (\beta - z - y)\frac{dx}{x} - \frac{(\alpha - x)y}{x}\frac{dy}{y} - \frac{(\alpha - x)z}{\alpha}\frac{dz}{z} - (\beta - z - y)\frac{d\alpha}{\alpha}$$

Error (Cont)

Confining the numerical error calculations to the cases in which one group of instrument has zero error and the other group has the same error magnitude but may have different sign and for the following conditions:

$$x = P_{ox_1} = 158 \text{ mm Hg}$$
$$y = P_{ox_2} = 127.6 \text{ mmHg}$$
$$z = P_{DIOX_2} = 30.4 \text{ mm Hg}$$
$$\alpha = \beta = P_1 = P_2 = 760 \text{ mmHg}.$$

Effect of all instrument error having the same sign and magnitude

If $\frac{d\alpha}{\alpha} = \frac{d\beta}{\beta} = \frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z} = \frac{d\dot{m}}{\dot{m}} = \pm \epsilon$

$$\frac{d\dot{u}}{\dot{u}} = \mp \frac{dM}{M} \pm \frac{d\dot{m}}{\dot{m}} + \frac{\alpha x}{(\alpha - x)(\beta x - xz - \alpha y)} \left\{ \qquad \right\}$$

$$\frac{dM}{M} = \frac{\frac{1}{\beta z}\frac{dy}{y} + \frac{4}{\beta y}\frac{dz}{z} + \frac{7}{yz}\frac{d\beta}{\beta}}{\left(\frac{1}{\beta z} + \frac{4}{\beta y} + \frac{7}{yz}\right)}$$

$$\frac{dy}{y} = \frac{dz}{z} = \frac{d\beta}{\beta} = \pm \epsilon$$

$$\therefore \frac{dM}{M} = \frac{\left(\frac{1}{\beta z} + \frac{4}{\beta y} + \frac{7}{yz}\right)(\pm\epsilon)}{\left(\frac{1}{\beta z} + \frac{4}{\beta y} + \frac{7}{yz}\right)} = (\pm \epsilon)$$

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REPORT NO.

MODEL NO.

Error (Contd)

<u>Effect of all instrument error having the same sign and magnitude</u>

$$\frac{d\dot{m}}{\dot{m}} = \pm\epsilon$$

$$\{\ \} = (\beta - z - y)\frac{dx}{x} - \frac{(\alpha - x)y}{x}\frac{dy}{y} - \frac{(\alpha - x)z}{\alpha}\frac{dz}{z} - (\beta - z - y)\frac{d\alpha}{\alpha} + \frac{(\alpha - x)\beta}{\alpha}\frac{d\beta}{\beta}$$

$$= \left[\cancel{(\beta - z - y)} - \frac{(\alpha - x)y}{x} - \frac{(\alpha - x)z}{\alpha} - \cancel{(\beta - z - y)} + \frac{(\alpha - x)\beta}{\alpha}\right](\pm\epsilon)$$

$$= [-(\alpha - x)]\left[\frac{y}{x} + \frac{z}{\alpha} - \frac{\beta}{\alpha}\right](\pm\epsilon)$$

$$\therefore \quad \frac{d\dot{U}}{\dot{U}} = -\cancel{(\pm\epsilon)} + \cancel{(\pm\epsilon)} + \frac{\alpha x\ (\pm\epsilon)}{\cancel{(\alpha - x)}(\beta x - xz - \alpha y)}[-\cancel{(\alpha - x)}]\left[\frac{y}{x} + \frac{z}{\alpha} - \frac{\beta}{\alpha}\right]$$

$$= -\frac{\alpha x}{(\beta x - xz - \alpha y)}\left[\frac{y}{x} + \frac{z}{\alpha} - \frac{\beta}{\alpha}\right](\pm\epsilon)$$

<u>Numerical Calculation</u>

$$\epsilon_U = \frac{d\dot{U}}{\dot{U}} = -\frac{760 \times 158\ (\pm\epsilon)}{760 \times 158 - 158 \times 30.4 - 760 \times 127.6}\left[\frac{127.6}{158} + \frac{30.4}{760} - \frac{760}{760}\right]$$

$$= -\frac{760 \times 158\ (\pm\epsilon)}{760(158 - 127.6) - 158 \times 30.4}[.808 + .04 - 1]$$

$$= -\frac{760 \times 158\ (\pm\epsilon)}{760(30.4) - 158 \times 30.4}[-.152]$$

$$= -\frac{760 \times 158 \times (-.152)}{602 \times 30.4}(\pm\epsilon) = .9973(\pm\epsilon)$$

Effect of worst combination of all sensor error

When all error terms are additive there results the highest error in oxygen uptake

$$\frac{d\dot{U}}{\dot{U}} \leq \epsilon + \epsilon + \frac{\alpha x \, \epsilon}{(\alpha - x)(\beta x - xz - \alpha y)} \Big\{ \quad \Big\}$$

where

$$\Big\{ \quad \Big\} = \Big\{ 2(\beta - z - y) + (\alpha - x)\left(\frac{y}{x} + \frac{z}{\alpha} + \frac{\beta}{\alpha}\right) \Big\}$$

Numerical Calculation

$$2(\beta - z - y) = 2(760 - 30.4 - 127.6) = 2 \times 602 = 1204$$

$$(\alpha - x) = (760 - 158) = 602$$

$$\frac{y}{x} + \frac{z}{\alpha} + \frac{\beta}{\alpha} = \left(\frac{127.6}{158} + \frac{30.4}{760} + 1\right) = 1.846$$

$$\Big\{ 2(\beta - z - y) + (\alpha - x)\left(\frac{y}{x} + \frac{z}{\alpha} + \frac{\beta}{\alpha}\right) \Big\} = 1204 + 602 \times 1.846$$

$$= 1204 + 1110$$

$$= 2314$$

$$\frac{\alpha x}{(\alpha - x)(\beta x - xz - \alpha y)} = \frac{760 \times 158}{(760 - 158) \times 602 \times 30.4}$$

$$= \frac{760 \times 158}{602^2 \times 30.4} = .01088$$

$$\therefore \quad \frac{d\dot{U}}{\dot{U}} \leq \epsilon + \epsilon + .01088 \times 2314 \, \epsilon = 27.2 \, \epsilon$$

Effect of error in oxygen partial pressure measurement

When P_{ox1} and P_{ox2} alone has measuring error and $\frac{dy}{y} = -\frac{dx}{x}$, then

$$\frac{d\dot{U}}{\dot{U}} = -\frac{dM}{M} \pm \frac{d\dot{m}}{\dot{m}} + \frac{\alpha x}{(\alpha - x)(\beta x - xz - \alpha y)} \left\{ \quad \right\}$$

$$\frac{dM}{M} = \frac{\frac{1}{\beta z}\frac{dy}{y} + \frac{4}{\beta y}\cancel{\frac{dz}{z}}^{\to 0} + \frac{7}{yz}\cancel{\frac{d\beta}{\beta}}^{\to 0}}{\frac{1}{\beta z} + \frac{4}{\beta y} + \frac{7}{yz}}$$

$$= \frac{\frac{1}{\beta z}\frac{dy}{y}}{\frac{1}{\beta z} + \frac{4}{\beta y} + \frac{7}{yz}}$$

$$\frac{d\dot{m}}{\dot{m}} = 0$$

$$\left\{ \quad \right\} = (\beta - z - y)\frac{dx}{x} - \frac{(\alpha - x)y}{x}\frac{dy}{y} - \frac{(\alpha - x)z}{\alpha}\cancel{\frac{dz}{z}}^{\to 0}$$
$$-(\beta - z - y)\cancel{\frac{d\alpha}{\alpha}}^{\to 0} + \frac{(\alpha - x)}{\alpha}\beta\cancel{\frac{d\beta}{\beta}}^{\to 0}$$

$$\frac{d\dot{U}}{\dot{U}} = -\frac{\frac{1}{\beta z}\frac{dy}{y}}{\frac{1}{\beta z} + \frac{4}{\beta y} + \frac{7}{yz}} + \frac{\alpha x}{(\alpha - x)(\beta x - xz - \alpha y)}\left[(\beta - z - y)\frac{dx}{x} - \left(\frac{\alpha - x}{x}\right)\frac{dy}{y}\right]$$

Substituting $\frac{dx}{x} = -\frac{dy}{y}$ and $\frac{dy}{y} = (\pm\epsilon)$

$$\frac{d\dot{U}}{\dot{U}} = -\frac{\frac{1}{\beta z}(\pm\epsilon)}{\frac{1}{\beta z} + \frac{4}{\beta y} + \frac{7}{yz}} + \frac{\alpha x\,(\pm\epsilon)}{(\alpha - x)(\beta x - xz - \alpha y)} - \left[(\beta - z - y) + \frac{(\alpha - x)y}{x}\right]$$

Numerical Calculations

$$\frac{\frac{1}{\beta z}}{\frac{1}{\beta z}+\frac{4}{\beta y}+\frac{7}{yz}} = \frac{\frac{1}{30.4\times 760}}{\frac{1}{30.4\times 760}+\frac{4}{760\times 127.6}+\frac{7}{127.6\times 30.4}}$$

$$= \frac{\frac{1}{23100}}{\frac{1}{23100}+\frac{4}{97000}+\frac{7}{3880}}$$

$$= .0228$$

$$\frac{\alpha x}{(\alpha - x)(\beta x - xz - \alpha y)} = \frac{760\times 158}{(760-158)(760\times 158 - 158\times 30.4 - 760\times 127.6)}$$

$$= \frac{760\times 158}{602\,(602\times 30.4)} = .0109$$

$$(\beta - z - y) = (760 - 30.4 - 127.6) = 602$$

$$\frac{(\alpha - x)\,y}{x} = \frac{760-158}{158}\times 127.6 = 486$$

$$\therefore \quad \frac{d\dot{U}}{\dot{U}} = .0228(\pm\epsilon) + .0109\times(602+486)(\pm\epsilon)$$

$$= .0228(\pm\epsilon) + 11.87(\pm\epsilon) = 11.9(\pm\epsilon)$$

Effect of error in oxygen differential pressure measurement

$$\Delta P_{ox} = P_{ox_1} - P_{ox_2}$$

$$\frac{d(\Delta P_{ox})}{\Delta P_{ox}} = \frac{d(P_{ox_1} - P_{ox_2})}{P_{ox_1} - P_{ox_2}}$$

$$= \frac{dP_{ox_1} - dP_{ox_2}}{P_{ox_1} - P_{ox_2}}$$

$$= \frac{P_{ox_1}\frac{dP_{ox_1}}{P_{ox_1}} - P_{ox_2}\frac{dP_{ox_2}}{P_{ox_2}}}{P_{ox_1} - P_{ox_2}}$$

When $\frac{dP_{ox_1}}{P_{ox_1}} = -\frac{dP_{ox_2}}{P_{ox_2}} = (\pm\epsilon)$

then

$$\frac{d\Delta P_{ox}}{\Delta P_{ox}} = \frac{\left(1 + \frac{P_{ox_2}}{P_{ox_1}}\right)(\pm\epsilon)}{1 - \frac{P_{ox_2}}{P_{ox_1}}}$$

or $\epsilon_{\Delta p} \equiv \frac{d(x-y)}{x-y} = \frac{\left(1+\frac{y}{x}\right)(\pm\epsilon)}{\left(1-\frac{y}{x}\right)}$

$$= \frac{\left(1+\frac{127.6}{158}\right)(\pm\epsilon)}{\left(1-\frac{127.6}{158}\right)} = \frac{1.809\,(\pm\epsilon)}{.1925}$$

$$= 9.4\,(\pm\epsilon)$$

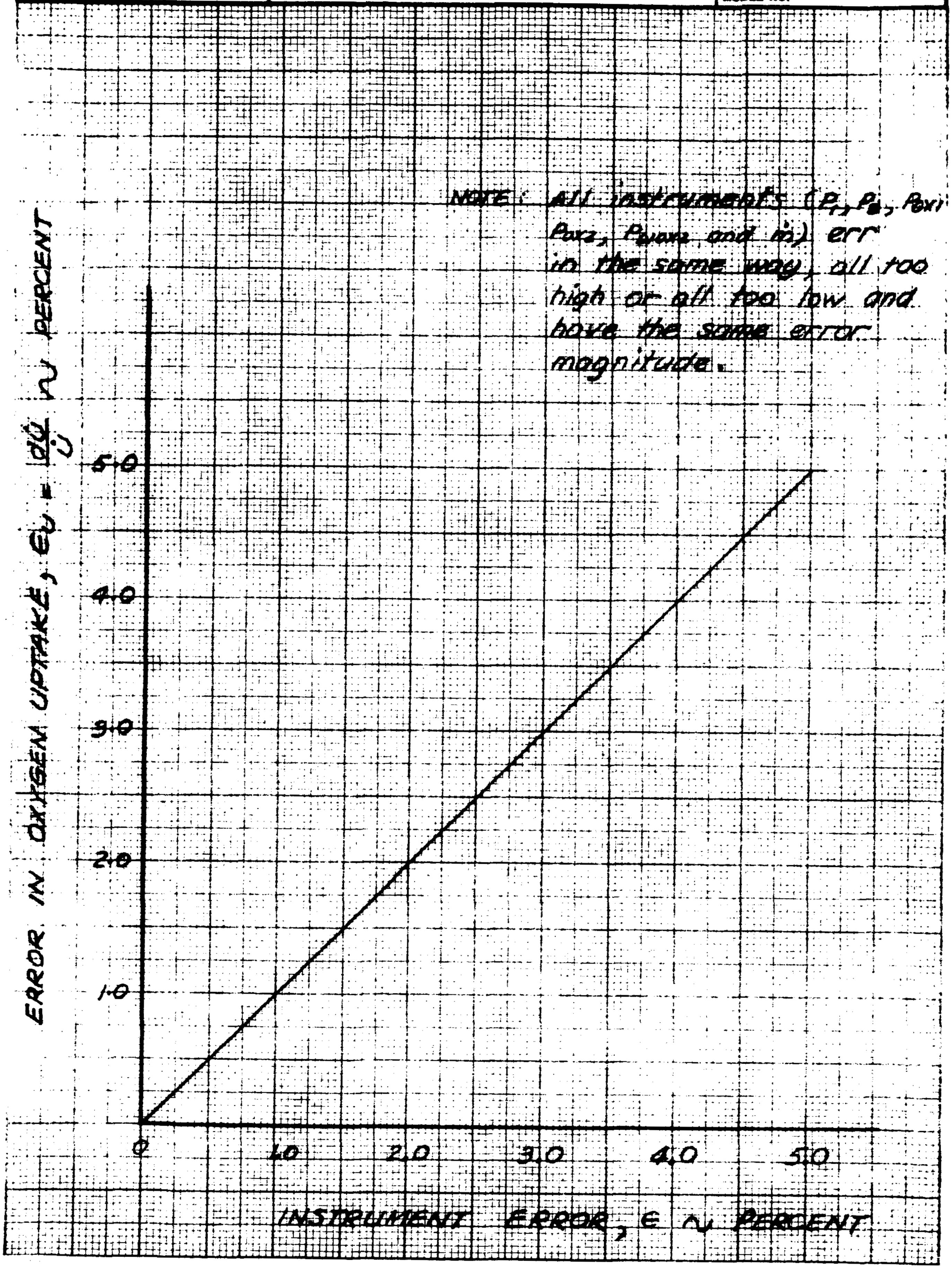
or

$$\epsilon = \frac{\epsilon_{\Delta p}}{9.4}$$

$$\therefore \frac{d\dot{w}}{\dot{w}} = \frac{11.9}{9.4}(\pm\epsilon_{\Delta p}) = 1.269(\pm\epsilon_{\Delta p})$$

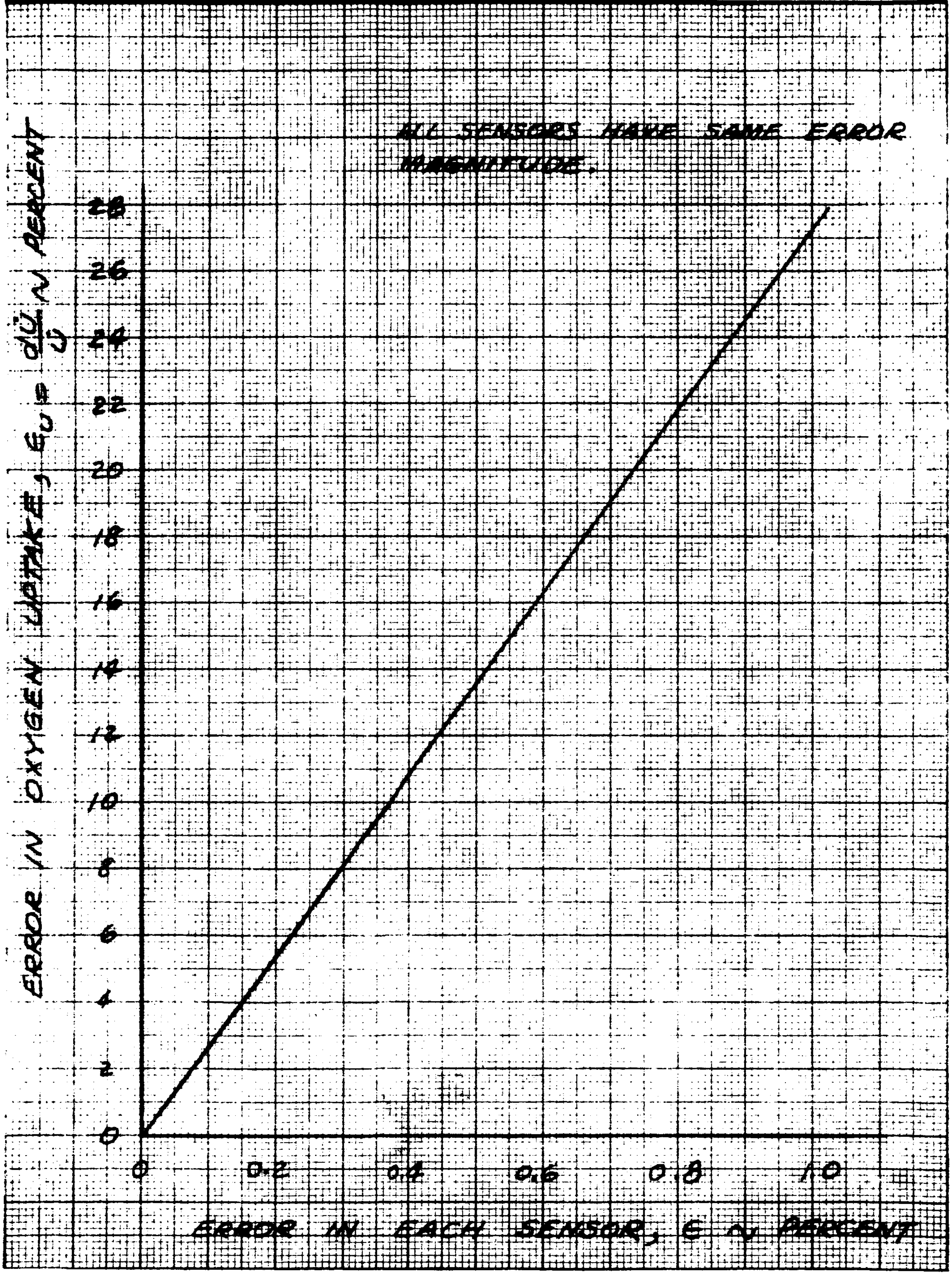
FORM 1002-X (FORMERLY ED147)

PREPARED BY:	NORTH AMERICAN AVIATION, INC. EFFECT OF ERROR OF ALL SENSORS ON O_2 UPTAKE	PAGE NO. 55 OF
CHECKED BY:		REPORT NO.
DATE:	FIGURE 14	MODEL NO.

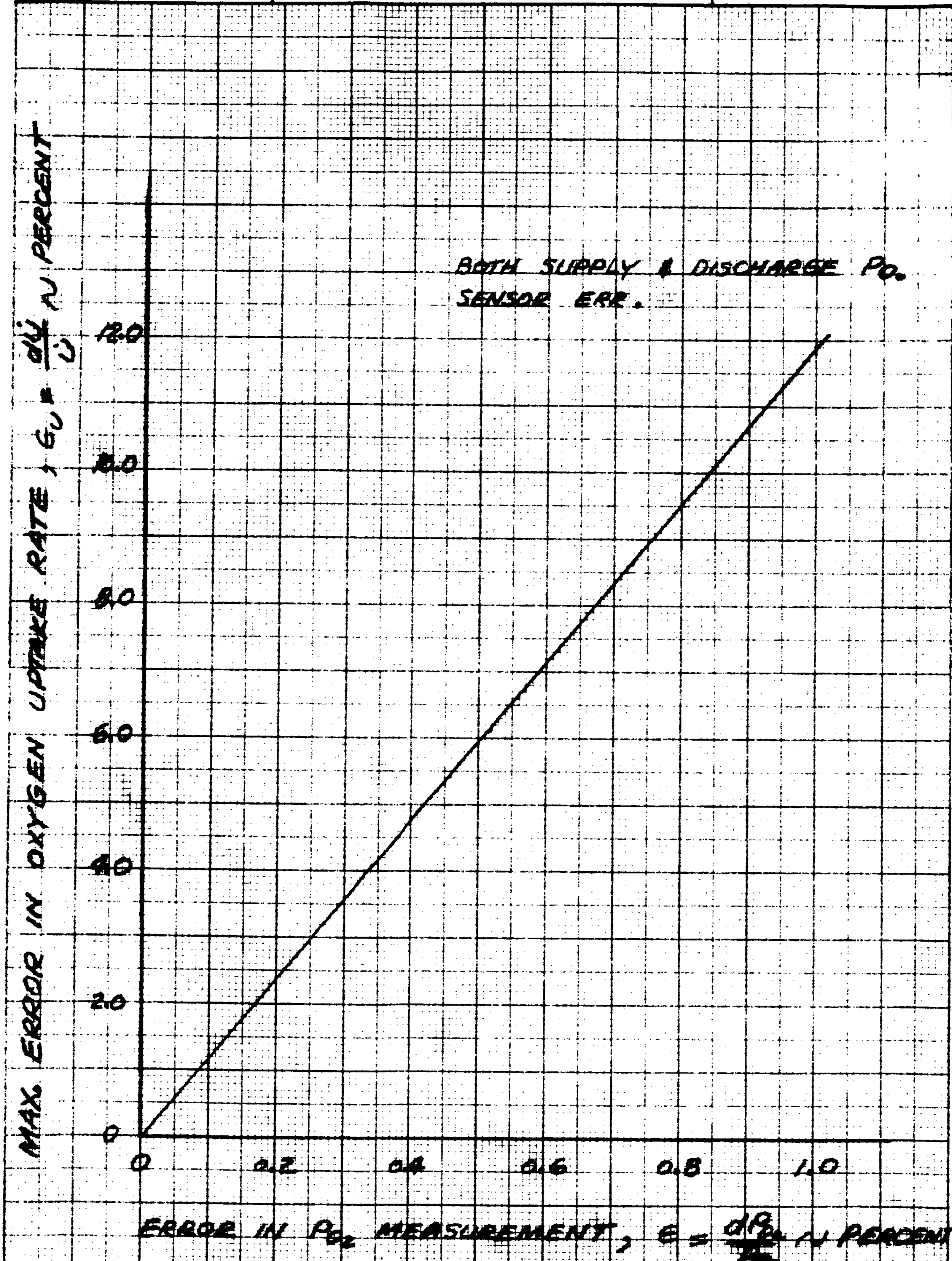


FORM 1002-X (FORMERLY SD147)

PREPARED BY:	**NORTH AMERICAN AVIATION, INC.** EFFECT OF WORST ERROR COMBINATION ON O_2 UPTAKE	PAGE NO. 56 OF
CHECKED BY:		REPORT NO.
DATE:	FIGURE 15	MODEL NO.

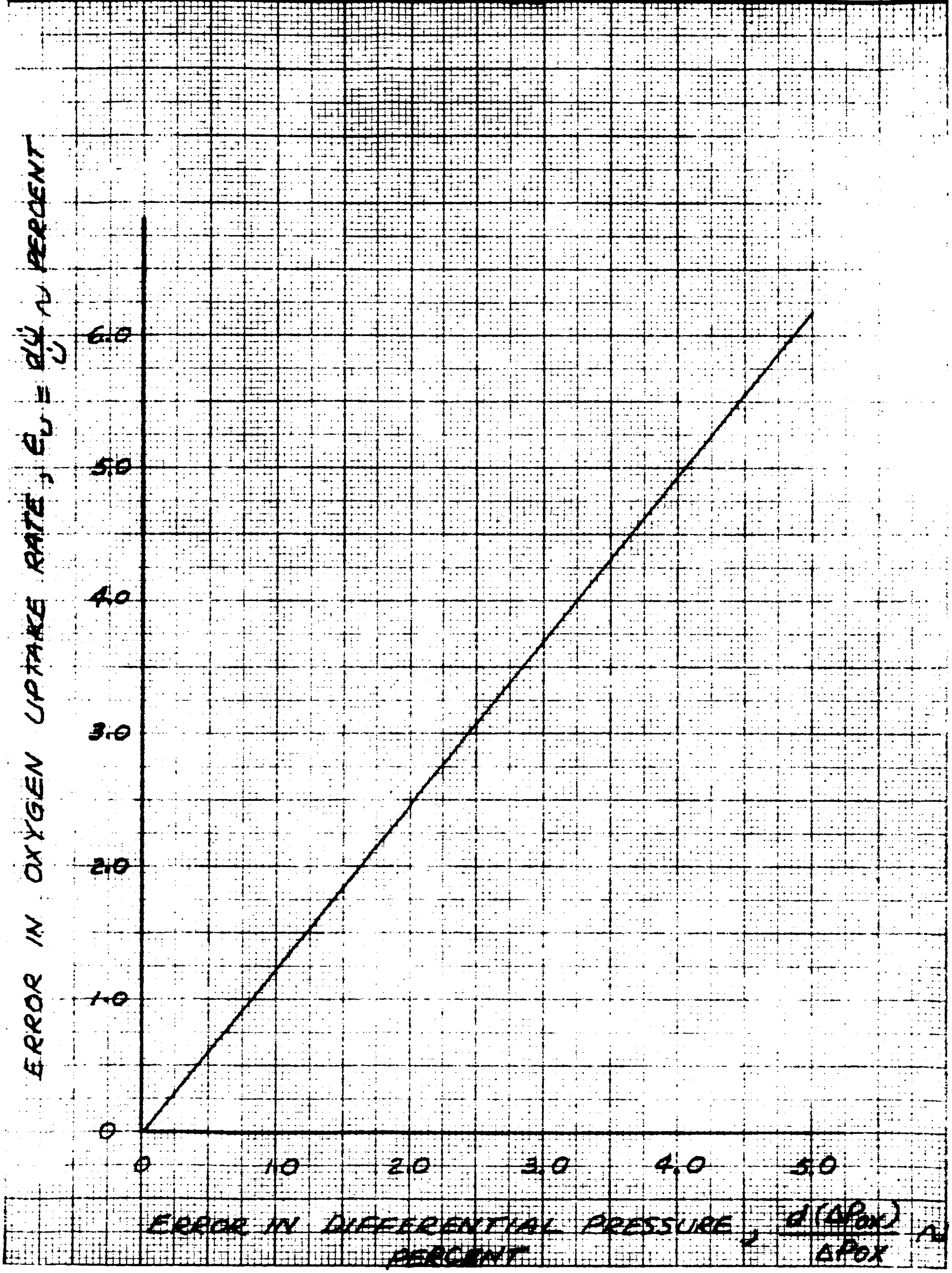


PREPARED BY:	NORTH AMERICAN AVIATION, INC. EFFECT OF ERROR IN P_{O_2} MEASUREMENT ON O_2 UPTAKE	PAGE NO. 57 OF
CHECKED BY:		REPORT NO.
DATE:	FIGURE 16	MODEL NO.



FORM 1002-X (FORMERLY ED147)

PREPARED BY:	NORTH AMERICAN AVIATION, INC. EFFECT OF (ΔP_{O_2}) ERROR	PAGE NO. 58 OF
CHECKED BY:	MEASUREMENT ON O_2 UPTAKE	REPORT NO.
DATE:	FIGURE 17	MODEL NO.



Numerical Calculation

From ·

$$\dot{V}_d = \frac{\dot{U}}{32}\,\frac{RT\,[1-(1-RQ)\chi_s]}{\chi_s P_{Td} - P_{O_2 d}}$$

where

$$\chi_s = \frac{P_{O_2 s}}{P_{Ts}}$$

If

$$P_{Ts} = P_{Td} = P_T$$

then

$$\dot{V}_d = \frac{\dot{U}}{32}\cdot\frac{RT\,[1-(1-RQ)\chi_s]}{P_{O_2 s} - P_{O_2 d}}$$

$$P_T \dot{V}_{O_2} = \frac{\dot{U}}{32} RT$$

$$\frac{\dot{U}}{32} = \frac{P_T \dot{V}_{O_2}}{RT} = \frac{\dot{V}_{O_2} P_T(ALT)}{RT}$$

$$\therefore\quad \dot{V}_d = \frac{\dot{V}_{O_2} P_T(ALT)}{RT}\,\frac{RT\,[1-(1-RQ)\chi_s]}{P_{O_2 s} - P_{O_2 d}}$$

$$\therefore\quad \dot{V}_d = \dot{V}_{O_2}\,\frac{[1-(1-RQ\chi_s)]}{P_{O_2 s} - P_{O_2 d}}\,P_T(ALT)$$

DRY AIR

Case 1 $\quad \dot{V}_{O_2} = 40.587$ std c.c. ; $\dot{U} = \frac{40.587}{22414} \times 32 = .05835638$ gm

$$\chi_s = \frac{158}{760} = .2078947; \quad \frac{\dot{V}_{O_2}}{\Delta P_{O_2}} = \frac{40.587}{7.71156} = 5.263137 \frac{cc}{mmHg}$$

$$\Delta P_{O_2} = P_{O_2 s} - P_{O_2 d} = 7.71156 \text{ mm Hg}$$

① RQ	② 1-RQ	③ (1 - RQ) × χ_s	④ 1 - ③	⑤ 5.263137 × ④
.8	.2	.04157894	.9584211	5.044302
.9	.1	.02078947	.9792105	5.153719
1.0	0	.0	1.000000	5.263137
1.1	-.1	-.02078947	1.020789	5.372555
1.2	-.2	.04157894	1.041579	5.481972

Numerical Calculation (cont)

DRY OXYGEN

Case 1

$$\dot{V}_{O_2} = 40.587 \text{ Std c.c.}$$

$$\dot{U} = \frac{40.587}{22414} \times 32 = .05835638 \text{ gm}$$

$$\Delta P_{O_2} = P_{O_2}s - P_{O_2}d = 7.71156 \text{ mmHg}$$

$$x_s = \frac{P_{O_2s}}{P_{Ts}} = 1 \quad (\text{Pure } O_2)$$

$$\dot{V}_d = 40.587 \frac{[1-(1-RQ)\times 1]}{7.71156} \times P_T(\text{Alt})$$

$$= 40.587 \frac{[RQ]}{7.71156} \times P_T(\text{Alt})$$

$$= 5.263137 [RQ] \times P_T(\text{Alt})$$

RQ	SL $\dot{V}_d = 5.263137 \times 760 \times (RQ)$		16000' $\dot{V}_d = 5.263137 \times 412 \times (RQ)$	
.8	3200	c.c	1734.7	c.c.
.9	3600	"	1951.6	"
1.0	4000	"	2168.4	"
1.1	4400	"	2385.3	"
1.2	4800	"	2602.1	"

DRY OXYGEN

Case 2

$\dot{V}_{O_2} = 100$ std. c.c.

$\dot{U} = \frac{100}{22414} \times 32 = .1427679$ gm

$\Delta P_{O_2} = P_{O_2}s - P_{O_2}d = 19$ mmHg

$\dot{V}_d = 100 \frac{[RQ]}{19} \times P_T(Alt)$

$\dot{V}_d = 5.263157 \times P_T(Alt) \times RQ$

RQ	SL $\dot{V}_d = 5.263157 \times 760 \times RQ$		16000' $\dot{V}_d = 5.263157 \times 412 \times RQ$	
.8	3200	c.c.	1734.7	c.c.
.9	3600	"	1951.6	"
1.0	4000	"	2168.4	"
1.1	4400	"	2385.3	"
1.2	4800	"	2602.1	"

DRY AIR (Cont)

Case 1

P_T (Sea Level) = 760 mmHg ; P_T (16000') = 412 mmHg

① RQ	Sea Level $\dot{V}_d = 760 \times$ ⑤		16000 $\dot{V}_d = 412 \times$ ⑤	
.8	3833.7	c.c.	2078.3	c.c.
.9	3916.8	"	2123.3	"
1.0	4000.0	"	2168.4	"
1.1	4083.1	"	2213.5	"
1.2	4166.3	"	2258.6	"

Case 2

$$\dot{V}_{O_2} = 100 \text{ std. c.c.}$$

$$\dot{U} = \frac{100}{22414} \times 32 = .1427679 \text{ gm}$$

$$\Delta P_{O_2} = P_{O_2 s} - P_{O_2 d} = 158 - 139 = 19 \text{ mmHg}$$

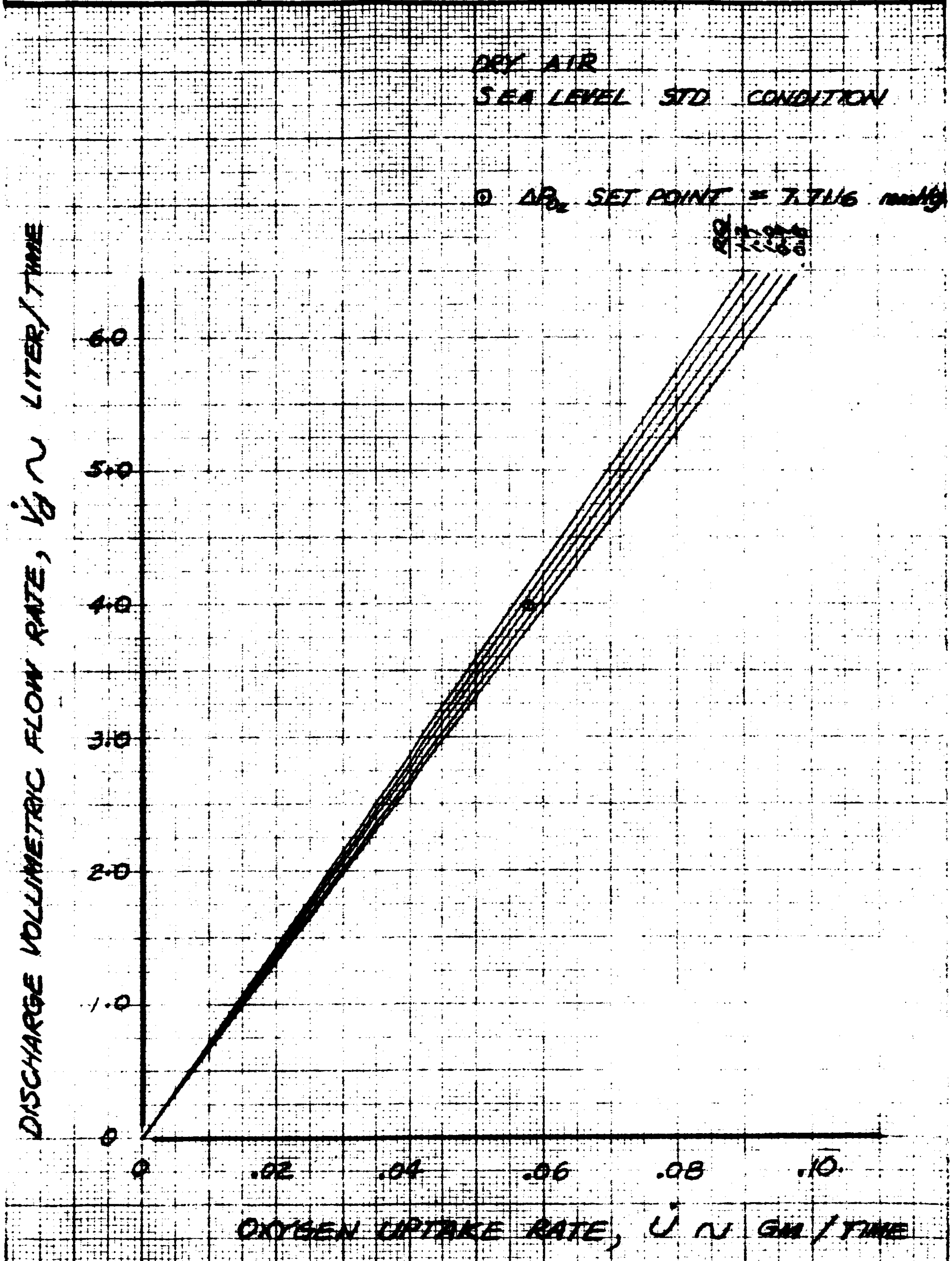
$$\dot{V}_d = 100 \frac{[1-(1-RQ)\times .2078947]}{19} P_T(Alt)$$

$$= 5.263157 [1-(1-RQ)\times .2078947] P_T(Alt)$$

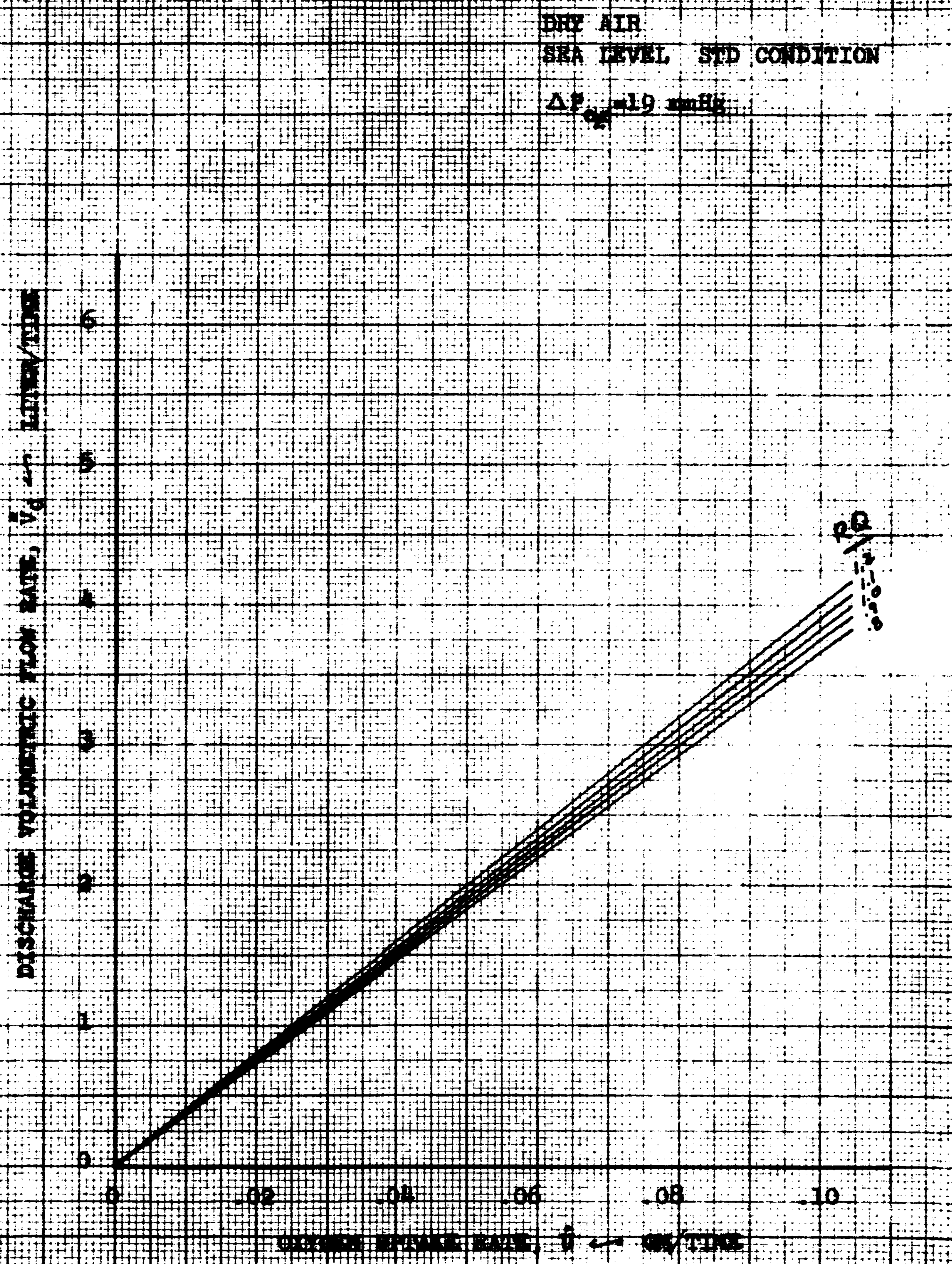
① RQ	⑤ 5.263157×[④ of page 1]	SL $\dot{V}_d = 760 \times$ ⑤		16000' $\dot{V}_d = 412 \times$ ⑤	
.8	5.0443216	3833.7	c.c.	2078.2	c.c.
.9	5.1537395	3916.8	"	2123.3	"
1.0	5.2631570	4000.0	"	2168.4	"
1.1	5.3725764	4083.2	"	2213.5	"
1.2	5.4819943	4166.3	"	2258.6	"

FORM 1000-X (FORMERLY ED147)

THERMODYNAMICS	NORTH AMERICAN AVIATION, INC.	PAGE NO. 63 OF
PREPARED BY: RT	DISCHARGE VOLUMETRIC FLOW RATE VS. O_2 UPTAKE	NA-65-513 REPORT NO.
CHECKED BY:		
DATE: 6/23/65	FIG 1 FIGURE 18	MODEL NO.

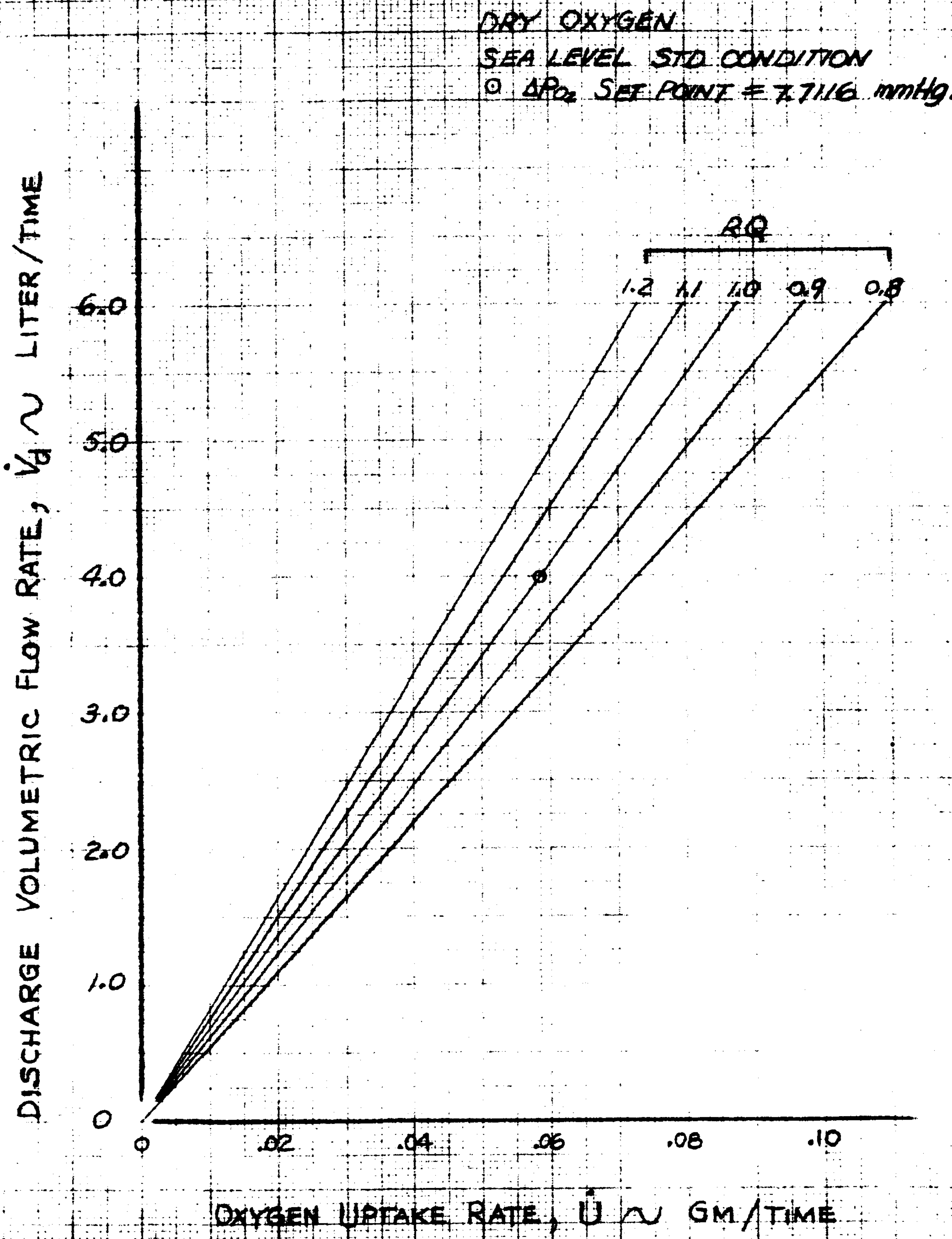


THERMODYNAMICS PREPARED BY: RT	**NORTH AMERICAN AVIATION, INC.** DISCHARGE VOLUMETRIC FLOW RATE VS. O_2 UPTAKE	PAGE NO. 64 OF
CHECKED BY:		NA-65-513 REPORT NO.
DATE: 6/23/65	FIGURE 19	MODEL NO.

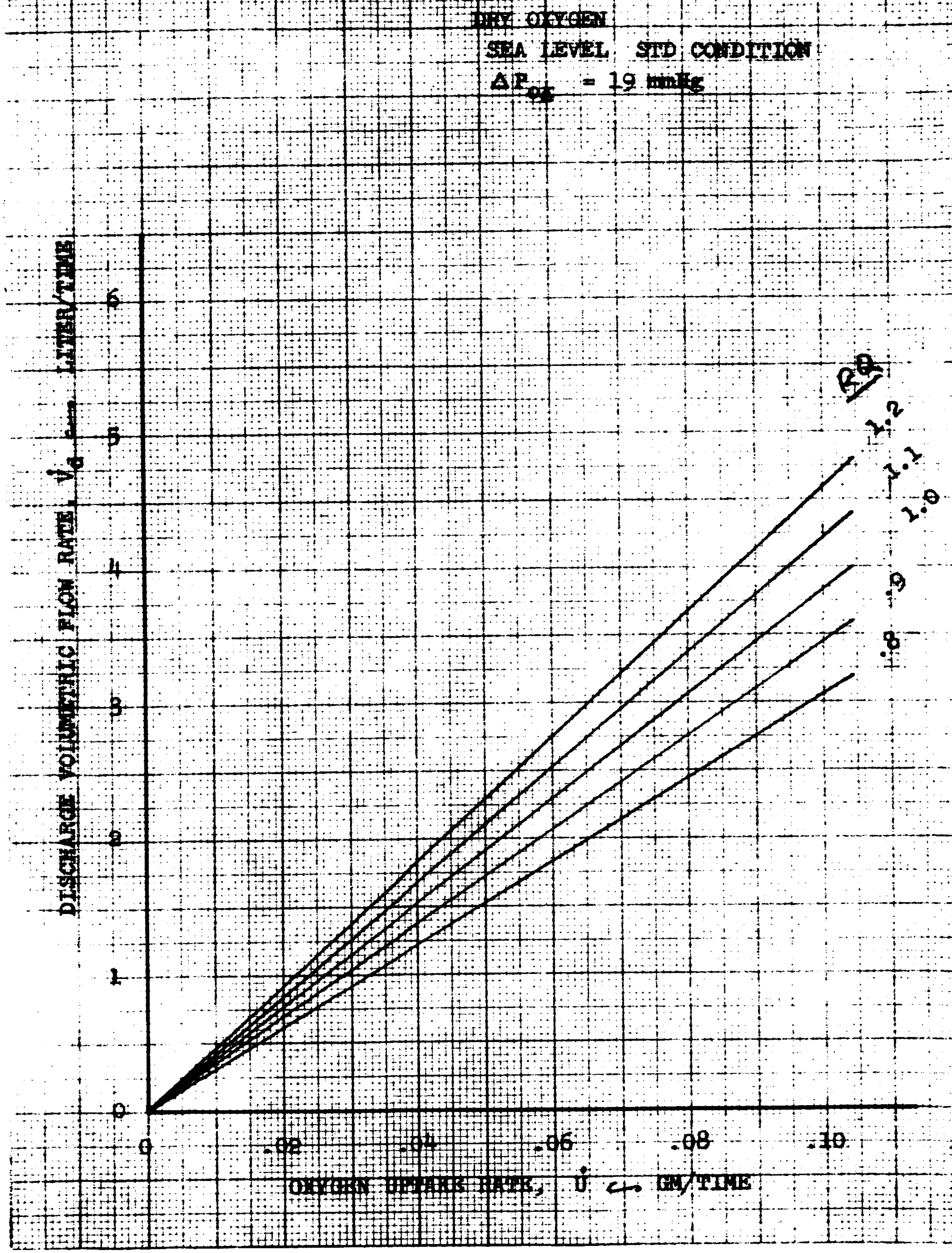


FORM 1000-X (FORMERLY ED147)

THERMODYNAMIC PREPARED BY: RT	**NORTH AMERICAN AVIATION, INC.** DISCHARGE VOLUMETRIC FLOW RATE VS. O_2 UPTAKE	PAGE NO. 65 OF
CHECKED BY:		NA-65-513 REPORT NO.
DATE: 6/23/65	FIGURE 20	MODEL NO.

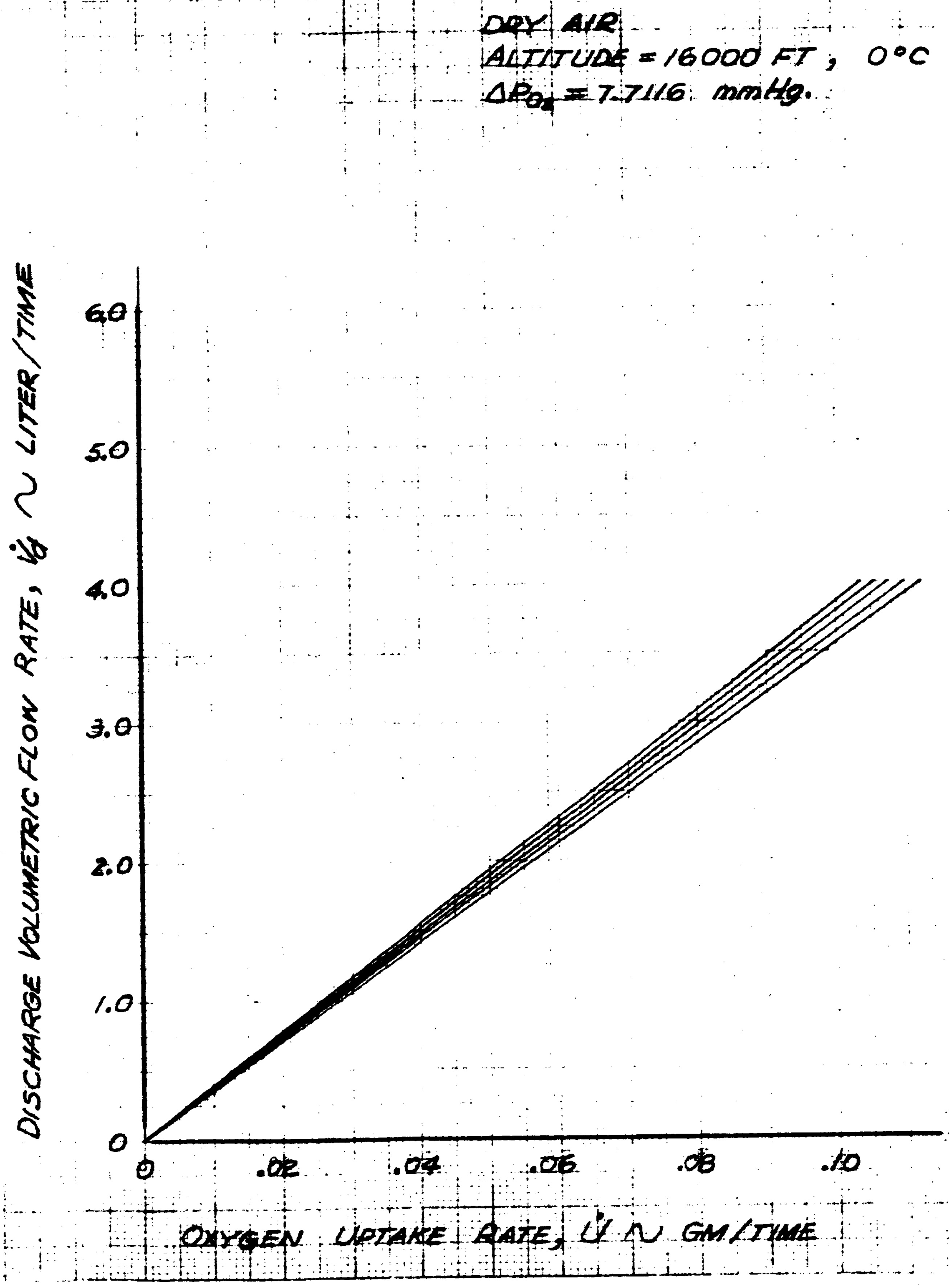


THERMODYNAMICS	NORTH AMERICAN AVIATION, INC.	PAGE NO. 66 OF
PREPARED BY: R.T.	DISCHARGE VOLUMETRIC FLOW RATE VS. O_2 UPTAKE	NA-65-513 REPORT NO.
CHECKED BY:		
DATE: 6/23/65	FIGURE 21	MODEL NO.

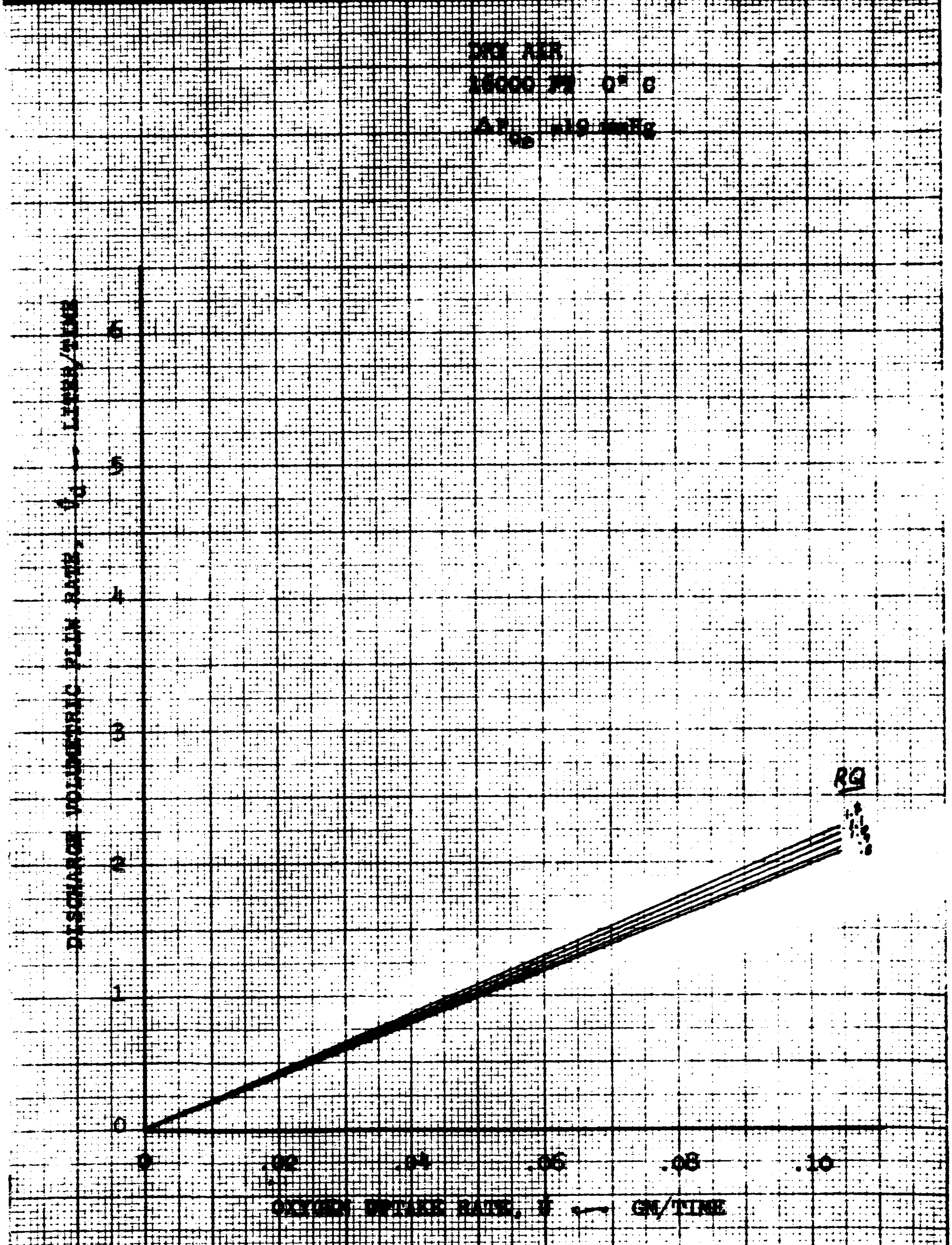


FORM 1006-X (FORMERLY ED147)

THERMODYNAMICS PREPARED BY: RT	NORTH AMERICAN AVIATION, INC. DISCHARGE VOLUMETRIC FLOW RATE VS. O_2 UPTAKE	PAGE NO. 67 OF
CHECKED BY:		MA-65-513 REPORT NO.
DATE:	FIGURE 22	MODEL NO.

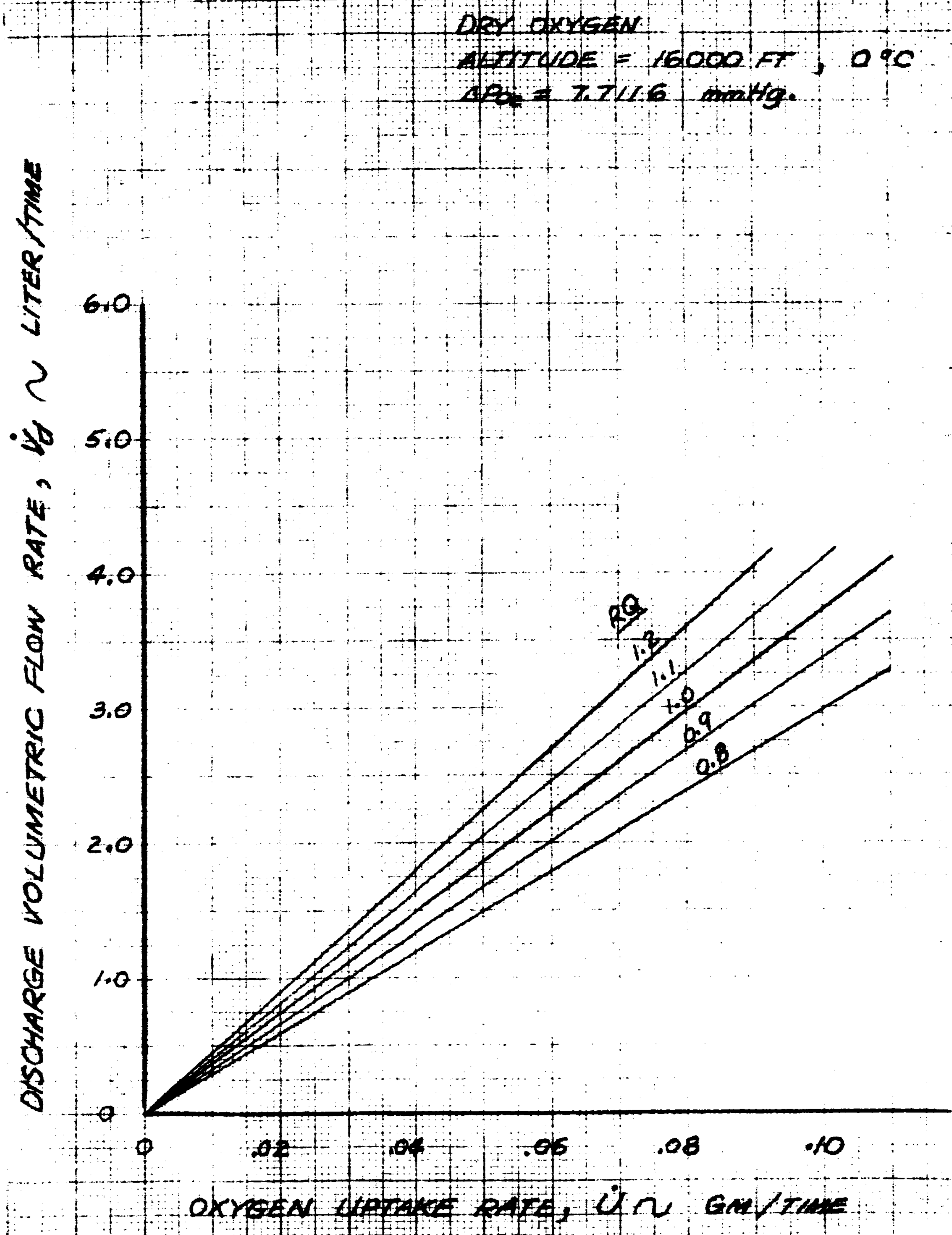


THERMODYNAMICS R.T.	NORTH AMERICAN AVIATION, INC.	PAGE NO. 68 OF
PREPARED BY:	DISCHARGE VOLUMETRIC FLOW RATE VS. O_2 UPTAKE	NA-65-513 REPORT NO.
CHECKED BY:		
DATE: 6/23/65	FIGURE 23	MODEL NO.



FORM 1008-X (FORMERLY ED147)

THERMODYNAMICS PREPARED BY: RT	**NORTH AMERICAN AVIATION, INC.** DISCHARGE VOLUMETRIC FLOW RATE VS. O_2 UPTAKE	PAGE NO. 69 OF
CHECKED BY:		MA-65-513 REPORT NO.
DATE: 6/23/65	FIGURE 24	MODEL NO.



FORM 1008-X (FORMERLY ED167)

THERMODYNAMICS PREPARED BY: R.T.	NORTH AMERICAN AVIATION, INC. DISCHARGE VOLUMETRIC FLOW RATE VS. O_2 UPTAKE	PAGE NO. 70 OF
CHECKED BY:		NA-65-513 REPORT NO.
DATE: 6/23/65	FIGURE 25	MODEL NO.