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# A CRITICAL SURVEY OF UPPER-ATMOSPHERE DENSITY MEASUREMENTS BY MEANS OF IONIZATION GAUGES

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# ABSTRACT

Recently published data (Horowitz, Newton, and Priester, 1965) indicate that there is a consistent difference, by approximately a factor of 2, between upper-atmosphere densities measured by gauges in an orbiting satellite and densities measured by satellite drag. Prepared for personnel at the Smithsonian Astrophysical Observatory, who have been working with drag measurements, the present report is a discussion of the gauge-measuring technique. The Explorer 17 gauges and interpretation of their data will be specifically considered. The information and data used in this paper are necessarily limited to reports that have been made publically available , to date, by the Aeronomy Group at NASA's Goddard Space Flight Center.

It is shown that the present "state of the art" for vacuum gauges is such that it is difficult to make estimates of the accuracy of measurements made in the upper atmosphere. This is mainly because gauge calibrations for atomic oxygen have never been made. Also, interpretation of gauge readings of pressures caused by gas mixtures is quite difficult. In light of the difficulties associated with gauge measurements it is felt that the factor of 2 between gauge- and drag-determined densities should be described as a difference rather than a disagreement.

# A CRITICAL SURVEY OF UPPER-ATMOSPHERE DENSITY MEASUREMENTS BY MEANS OF IONIZATION GAUGES[1]

M. P. Friedman[2]

## 1. DESCRIPTION OF EXPLORER 17 PRESSURE GAUGES

The Explorer 17 used four vacuum gauges (Horowitz, Newton, and Priester, 1965) to measure upper-atmosphere densities. In this note we will first describe these vacuum gauges and then discuss how the gauge readings are used to obtain atmospheric densities. In addition, difficulties associated with interpretation of gauge readings will be discussed.

Each gauge is basically a collection chamber with an opening to the atmosphere. The ambient atmosphere flows either into or out of the chamber, depending on the relative pressures. An additional effect is the satellite tumbling; this causes the chamber opening to face alternately toward and then away from the flow. The tumbling-induced changes in chamber conditions are used to determine atmospheric densities; the theoretical reasoning behind this is given in the Appendix.

Two of the vacuum gauges were the Bayard-Alpert hot-filament type (BAG) and the remaining two were the Redhead cold-cathode type (RHG). A brief description of their operation will be given here; this information was obtained from Barrington (1963, pp. 82-89).

In the BAG, electrons are emitted from a hot filament; as they move toward a positively charged grid, they collide with gas molecules in the

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gauge, producing positive ions. The positive ions are drawn to a negatively charged collector and the electrons go to the grid. At a constant grid voltage and electron-emission current, the number of positive ions formed is proportional to the gas number density. At constant temperature the number density is proportional to pressure. These relations lead to the equation

$$I = kP \ , \tag{1}$$

where I is positive ion current, P is pressure inside gauge, and k is gauge sensitivity.

In the RHG, the electron discharge is caused by application of a large voltage (~ 5000 v) between the anode and cathode. A magnetic field is also applied; it causes the electrons to spiral around the chamber instead of going directly to the anode. This spiraling increases the probability of an ionizing collision between the atmosphere molecules and the electrons. When sufficient electrons and collisions are present, an equilibrium is attained in which a steady ion current is measured. The pressure (at constant temperature) is related to the ion current; this relation is, however, nonlinear:

$$I = kP^n \ . \tag{2}$$

This expression can be made to appear linear by its being written

$$I = (kP^{n-1})P = \bar{k}P \ . \tag{2a}$$

It seems that the form (2a) was used (Newton _et_ _al_. 1964, p. 5) when Explorer 17 data were interpreted by the Aeronomy Group at NASA's Goddard Space Flight Center.

## 2. GAUGE CALIBRATION

We see, from the previous section, that in order for the gauge readings to be interpreted, their sensitivities must be known. This, in turn, requires a knowledge of the sensitivity for each of the atmospheric constituents, as well as all possible combinations of them. The gauges were laboratory calibrated with the use of $N_2$ and He. Newton <u>et al</u>. (1964) found that the sensitivity for He was 0.17 times that for $N_2$, and that this relation held for both the BAG and the RHG. The scatter in these measurements was ±25% for the BAG, and ±30% for the RHG (Newton <u>et al</u>., 1964). In Table 1 we give some typical $He/N_2$ sensitivity ratios:

Table 1. $He/N_2$ sensitivity ratios

| | Newton et al. (1964) | Barrington (1963) | Dushman and Lafferty (1962) | | | | Cross sections von Engle (1965) |
|---|---|---|---|---|---|---|---|
| $\frac{k_{He}}{k_{N_2}}$ | 0.17 | 0.127 | 0.148 | 0.155 | 0.205 | 0.21 | 0.12 |

The cross-sections entry in Table 1 is the ratio of ionizing cross section of He to $N_2$ at 90 ev, which was the electron energy in the BAG.

The sensitivities for $O_2$ and O were not determined experimentally. These were estimated by first taking the sensitivity ratio $k_{O2}/k_{N2} = 0.77$ as given by Barrington (1963). (The conditions under which this number was determined is nowhere mentioned by Barrington.) In Table 2 some values of $k_{O2}/k_{N2}$ are given:

Table 2. $O_2/N_2$ sensitivity ratios

| | Barrington (1965) | Dushman and Lafferty (1965) | Cross sections von Engle (1965) |
|---|---|---|---|
| $k_{O_2}/k_{N_2}$ | 0.77 | 0.85 1.14 | 1.0 |

For a determination of the sensitivity for O, the sensitivity for $O_2$ was multiplied by the ratio of ionizing cross sections (Fite and Brackman, 1959)

$$k_O = k_{O_2} \times \frac{\sigma_O}{\sigma_{O_2}} = 0.542\ k_{O_2} = 0.542 \times 0.77\ k_{N_2} = 0.42\ k_{N_2} .$$

Since the gauge sensitivity for O has never been measured, the above procedure, utilizing ionizing cross sections, is probably the best currently possible. We see from Tables 1 and 2 that the ionizing cross section is related to sensitivity; however, the exact determination of this relation has never been made. Since O is such a chemically active substance, its behavior in any gauge is very hard to predict, and therefore an error estimate for the above value of $k_O$ would be quite difficult. Unfortunately, due to the abundance of O at satellite altitudes this appears to be an important, unresolved point.

## 3. INTERPRETATION OF GAUGE READINGS

The very nature of the gauge readings makes it difficult to interpret them. This is because the reading consists only of the total ion current that is caused by the atmospheric gas mixture in the gauge. Each constituent of the upper atmosphere has a different gauge sensitivity, and it is the proper combination of the concentrations and sensitivities that gives the final ion current. Furthermore, this "proper combination" is by no means unique; that is, there are many combinations of the individual gas concentrations that could lead to a given ion current.

The situation is further complicated for the Redhead gauges since these gauges have a nonlinear pressure-current relation, as shown in equation (2). For these gauges the sum of the partial pressures is not directly proportional to the sum of the partial currents, which contribute to the total current reading.

## 4. KINETIC THEORY AND GAUGE READINGS

For a collision-free flow each of the atmospheric constituents enters and leaves the gauge chamber independent of the others. In Appendices 1, 2, and 3 it is shown that for the actual gauge dimensions the incoming and outgoing molecules attain an equilibrium in a time much shorter than the satellite tumble time. Therefore, under equilibrium conditions the equation found above (A-9) can be written

$$\rho_0 - \rho_\pi = 2\sqrt{\pi}\,\rho_\infty V_n/C \tag{3}$$

where $V_n$ is the component of satellite velocity normal to the gauge opening, $C = \sqrt{2RT/M}$ is the most probable molecular speed inside the gauge, $\rho_0$ is density inside the gauge when it faces toward the incoming airstream, $\rho_\pi$ is the density inside the gauge when it faces away from the airstream, and $\rho_\infty$ is the ambient atmosphere density.

We can further simplify equation (3) by neglecting $\rho_\pi$ which is several orders of magnitude (at satellite velocities) smaller than $\rho_0$:

$$\rho_0 = 2\sqrt{\pi}\,\rho_\infty V_n \sqrt{M/2RT} \quad . \tag{4}$$

Let us now consider two types of molecules with different molecular weights. We denote by h and l the heavier and lighter molecules respectively, and form the ratio $\rho_h/\rho_l$ using equation (4) and dropping the subscript 0

$$\frac{\rho_h}{\rho_l} = \frac{\rho_{h\infty}}{\rho_{l\infty}}\sqrt{\frac{M_h}{M_l}} \quad . \tag{5}$$

We see then that the density ratio heavy/light inside the gauge differs from that in the atmosphere by the square root of the molecular weights. For example, with nitrogen ($M = 28$) and atomic oxygen ($M = 16$) the concentration of $N_2$ inside the gauge is enhanced by greater than 30% relative to the oxygen concentration.

There is one further kinetic theory effect which bears mentioning; this arises from the tubular construction of the gauges. A schematic diagram (Figure 1) applicable to both gauges is given below:

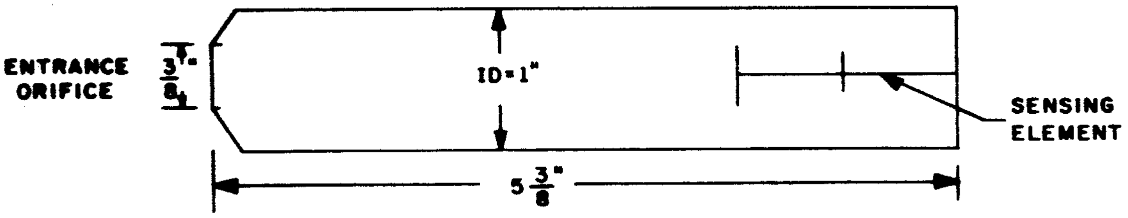

Figure 1. The orifice probe.

Most molecules entering the orifice will strike the walls several times before reaching the sensing element. Each wall collision is followed by a diffuse or a specular reflection. For every diffuse reflection it is just as probable that the molecule will return toward the entrance as it is that it will continue toward the sensing element.

The result is that the number of molecules that actually reaches the sensing element is always less than the number that enters the probe orifice.

The ratio formed by dividing the number of molecules getting to the sensing element by the number entering the probe is called the transmission probability. This probability depends on the size and shape of the probe chamber, the flow speed, the flow direction relative to the orifice, and the molecular weights of the molecules involved.

Ballance (1966) has computed some transmission probabilities for several representative configurations, and has obtained probabilities between .5 and .9 for models approximately the BAG and the RHG. Furthermore, he shows that the transmission probability in flight (with the probe orifice facing the flow) is greater by a factor of 2 or 3 than the probability in still air. This latter condition corresponds to a test chamber on the ground. Also, the value of the transmission probability in flight depends on the direction of the incoming flow relative to the orifice opening. A proper determination of this probability requires an accurate monitoring of the satellite tumbling orientation relative to its orbital velocity vector.

These two effects, the (heavier) mass selection and the transmission probability, are consequences of kinetic theory and are completely independent of the gauge sensitivity problems discussed in the previous sections. Therefore, interpretation of gauge readings must take into account the combined kinetic theory and gauge sensitivity effects.

## 5. CONCLUSION

There is at present a question as to the accuracy of different techniques of determining density in the upper atmosphere. Cook ( 1965a, b) reviewed the satellite drag technique and estimated that the worst possible error in the density estimates is 30%, and that a 10% error is more likely.

The technique of using ionization gauges for measuring the density has been used by the Aeronomy Group at Goddard Space Flight Center. However, some of the details associated with interpretation of measurement data have never been explained. In this report we have attempted to indicate, in general, how the ionization gauges are used, and also what are the possible difficulties in interpretation of gauge data. The following questions remain unanswered:

A. Does a 30% scatter in calibration measurements on the ground imply at most a 30% scatter for in situ satellite measurements?

B. How accurate are the approximations for gauge sensitivity for atomic and molecular oxygen?

C. How strongly do the estimated gauge sensitivities for atomic and molecular oxygen affect the final results?

D. How is the gauge interaction with oxygen accounted for?

E. How is the nonlinearity of the RHG accounted for?

F. How are the kinetic theory effects, discussed in the previous section, accounted for?

G. How are the contributions of the various atmospheric constituents separated with a single gauge reading?

As already mentioned, our comments are necessarily limited to information made available to the public by the GSFC Aeronomy Group.

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# APPENDIX

## 1. THEORY

We will now develop the theory indicating how the gauge pressures are used to determine atmospheric densities. First, a reference coordinate system is fixed on the satellite; the atmosphere then "flows" past the satellite at the satellite speed. Since the mean free path in the atmosphere is large in comparison to satellite dimensions, the flow is assumed to be collision free. We also assume the flow to have a Maxwellian distribution with a mean velocity equal to that of the satellite; then the flow rate, in grams per second, striking a given area is (Patterson, 1964, p. 43)*

$$F_i = \frac{\rho_\infty C_\infty A}{2\sqrt{\pi}} \left\{ \exp(-s^2 \cos^2\psi) + \sqrt{\pi}\, S \cos\psi \,[1 + \mathrm{erf}(s \cos\psi] \right\},$$

$$\equiv \frac{\rho_\infty C_\infty A}{2\sqrt{\pi}} F(s \cos\psi)\ ,\ \mathrm{g/sec} \quad , \qquad \text{(A-1)}$$

where

A = area, $cm^2$,

$\psi$ = angle between normal-to-area and flow velocity vector (see Figure A-1),

$C_\infty$ = $\sqrt{2RT_\infty/M_\infty}$ = thermal speed of atmosphere molecules,

$\rho_\infty$ = density of ambient atmosphere, $g/cm^3$,

$M_\infty$ = molecular weight ambient atmosphere, g,

$T_\infty$ = temperature ambient atmosphere, °K,

R = gas constant, erg/°K/mole,

S = speed ratio = satellite velocity/$C_\infty$.

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*See Section 6. References, p. 10.

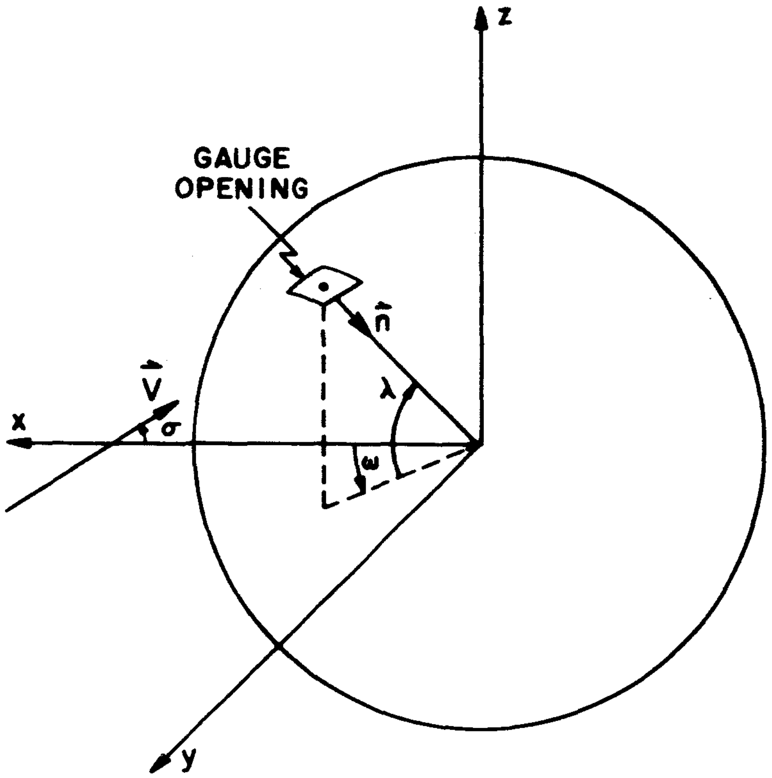

Figure A-1. Satellite gauge configuration.

Let A be the area of the opening to the gauge chamber; the quantity $F_i$ is the flow into the chamber. Once inside the chamber, the molecules, after undergoing several collisions with the wall, accommodate to the wall temperature.

The molecules inside the chamber are assumed to be in equilibrium at chamber conditions; they can pass outward through the area again. The flow rate out is

$$F_o = \frac{\rho C A}{2\sqrt{\pi}} \quad , \qquad \text{g/sec} \quad , \tag{A-2}$$

where C is $\sqrt{2RT/M}$ (thermal speed of molecules inside gauge chamber, in centimeter per second), T is the temperature inside chamber, in °K, M is the molecular weight inside chamber, in grams, and $\rho$ is the density inside chamber, in grams per cubic centimeter.

Therefore, the net change in mass is

$$\dot{\rho} v = (F_i - F_0) \; ,$$

or

$$\dot{\rho} + \rho \frac{CA}{2v\sqrt{\pi}} = \frac{\rho_\infty C_\infty A}{2v\sqrt{\pi}} F(s \cos \psi) \; , \tag{A-3}$$

where v is the chamber volume, in cubic centimeters.

Before proceeding further we must determine the angle $\psi$ between the velocity vector and the inward pointing normal. As the satellite tumbles about the z axis (Figure 1), the unit normal $\vec{n}$ has direction cosines

$$\begin{aligned} n_x &= - \cos \lambda \cos \omega t \; , \\ n_y &= - \cos \lambda \sin \omega t \; , \\ n_z &= - \sin \lambda \; , \end{aligned} \tag{A-4}$$

where $\omega$ is the satellite tumbling rate, in radians per second, and $90° + \lambda$ is the angle between $\vec{n}$ and the z axis. The coordinate system in Figure 1 has been set up such that the atmospheric velocity $\vec{V}$ is in the x-z plane; therefore,

$$\begin{aligned} V_x &= - V \cos \sigma \; , \\ V_y &= 0 \; , \\ V_z &= V \sin \sigma \; , \end{aligned} \tag{A-5}$$

where $\sigma$ is the angle between $\vec{V}$ and the (negative) x axis. Combining equations (A-4) and (A-5), we have

$$\vec{V} \cdot \vec{n} = V \cos \psi = V \cos \sigma \cos \lambda \cos \omega t - V \sin \sigma \sin \lambda ,$$

and

$$\cos \psi = \cos \sigma \cos \lambda \cos \omega t - \sin \sigma \sin \lambda . \qquad \text{(A-6)}$$

## 2. ATMOSPHERIC-DENSITY RELATION TO PROBE MEASUREMENTS

We first take note of the coefficient of $\rho$ in the mass balance equation (A-3). This coefficient is essentially a transpiration frequency; i.e., it gives a time constant for the flow out of the gauge chamber. In order for the gauge to operate properly, the transpiration frequency should be somewhat higher than the tumbling frequency $\omega$.

Referring back to equation (A-3), we now write

$$\dot{\rho} + \alpha\rho = \beta\, F(s \cos \psi) . \tag{A-3}$$

We look for a solution to equation (A-3) for a time that is long with respect to a tumble time but short with respect to an orbit time. For such a time t, $\alpha \gg 1$ and $\alpha$ and $\beta$ can be treated as constants. We integrate equation (A-3), recalling equation (A-6), which gives the time dependence of $\cos \psi$:

$$\rho = \rho_0\, e^{-\alpha t} + e^{-\alpha t} \beta \int_0^t e^{\alpha t}\, F(s \cos \psi)\, d\tau . \tag{A-7}$$

An asymptotic series for large $\alpha$ can be developed from equation (A-7) if we integrate by parts several times and neglect terms of order $\alpha^{-3}$ and $e^{-\alpha t}$ and smaller:

$$\begin{aligned} \rho &= \frac{\beta}{\alpha} \left[ F(s \cos \psi) - \frac{1}{\alpha} \dot{F}(s \cos \psi) + \frac{1}{\alpha^2}\, \ddot{F}(s \cos \psi) \right] , \\ &= \frac{\rho_\infty C_\infty}{C} \left[ F - \frac{\dot{F}}{\alpha} + \frac{\ddot{F}}{\alpha^2} \right] , \end{aligned} \tag{A-8}$$

where

$$\dot{F} = s\sqrt{\pi}\,\omega\ \sin\omega t \cos\sigma\ \cos\lambda\ \{1 + \mathrm{erf}\,(s\cos\psi)\}\ ,$$

$$\ddot{F} = s\sqrt{\pi}\,\omega^2 \cos\omega t \cos\sigma\ \cos\lambda\ \{1 + \mathrm{erf}(s\cos\psi)\}$$

$$+\ 2(s\omega\sin\omega t\cos\sigma\ \cos\lambda)^2 \exp(-s^2\cos^2\psi)\ .$$

Of the four vacuum gauges on Explorer 17, three operated successfully, and two of these were in the satellite equatorial plane, $\lambda = 0$. For this special case, $\lambda = 0$, we form the difference ($\rho$ at $\omega t = 0$) - ($\rho$ at $\omega t = \pi$) using equation (A-8):

$$\rho_0 - \rho_\pi = \frac{\rho_\infty C_\infty}{C}\ (2s\cos\sigma\sqrt{\pi})\left(1 - \frac{\omega^2}{a^2}\right)\ .$$

We solve for $\rho_\infty$, and eliminate $\rho_0$ and $\rho_\pi$ in favor of the corresponding pressures using the ideal gas law, $P = \rho RT/M$:

$$\rho_\infty = \frac{\left(P_0 - P_\pi\right)C}{(2RT/M)\sqrt{\pi}\ \ V\cos\sigma\left(1 - \frac{\omega^2}{a^2}\right)} = \frac{P_0 - P_\pi}{C\ V\sqrt{\pi}\ \cos\sigma\ \left(1 - \frac{\omega^2}{a^2}\right)}\ . \qquad \text{(A-9)}$$

## 3. THE EFFECT OF TUMBLING ON DENSITY DETERMINATION

The only term in equation (A-9) that is affected by satellite tumbling frequency is $\omega^2/\alpha^2$, where $\omega$, the tumbling frequency, is $3\pi$ rad/sec; $\alpha$, the transpiration frequency, is $CA/2v\sqrt{\pi}$; C, the most probable molecular velocity, inside the gauge, is $\sqrt{2RT/M}$; A is the area of the gauge opening; and v is the volume of the gauge.

For the Explorer 17, $A = 0.938\ \text{cm}^2$ and $v = 55\ \text{cm}^3$; in addition, the gauge temperature was about 300° K. If we assume a molecular weight of 20 for the gas inside the gauge, we can calculate

$$\alpha = 1.8 \times 10^2/\text{sec} \quad ,$$

and

$$\frac{\omega}{\alpha} = 5.3 \times 10^{-2} \ .$$

Clearly $\omega^2/\alpha^2$ is negligible with respect to 1. Hence the density formula, equation (9), can be written

$$\rho_\infty = \frac{P_0 - P_n}{C V_n \sqrt{\pi}} \quad , \tag{A-10}$$

where $V_n = V \cos \sigma$, the component of satellite velocity perpendicular to gauge opening. This equation agrees with equation (1) given by Horowitz, Newton, and Priester (1965).