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ABSTRACT

The fields of a dipole antenna immersed in a lossy uniaxial medium are determined. A quantity named "effective resistance" is defined in terms of the surface integral of the Poynting vector on a sphere centered at the antenna, and computations of this are presented as plots of the effective resistance against the radius of the sphere for various antenna lengths and medium losses. At frequencies below the plasma frequency, the effective resistance is found to be inversely proportional to antenna length near the antenna but proportional to the square of the length at large distances. These results are used to provide an explanation for the paradox of increasing radiation resistance with decreasing antenna length which occurs in lossless hyperbolic anisotropic plasmas.

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1. INTRODUCTION

The theory of dipole radiation in anisotropic media has been extensively investigated, and several methods have been used to calculate the dipole's radiation resistance. Balmain [1964] used a quasi-static approximation to the problem and used the induced E. M. F. method to calculate the input impedance of a dipole oriented in the z direction. His relationship, valid for short dipoles, is

$$Z_{in} = \frac{1}{j\omega\pi\epsilon_0 K' L}\left[\ln\frac{L}{b} - 1 + \ln a\right], \qquad (1.1)$$

where Z_{in} is the input impedance, ω is the operating angular frequency, ϵ_0 is the permittivity of vacuum, L is the half-length of the antenna, b is the radius of the antenna, $a = \sqrt{K'/K_o}$, and where K' and K_o are terms in the relative permittivity matrix K_m characterizing the medium. The expression for K_m is

$$K_m = \begin{bmatrix} K' & jK'' & 0 \\ -jK'' & K' & 0 \\ 0 & 0 & K_o \end{bmatrix}. \qquad (1.2)$$

When the anisotropic medium is a lossless plasma operating at a frequency which is below both its gyroresonant frequency and plasma frequency, K'/K_o will be real and negative. Under this condition, the real part of (1.1) will be

$$R_{in} = \frac{\eta}{2K' k_o L}, \tag{1.3}$$

where R_{in} is the real part of the input impedance, η is the characteristic impedance of vacuum, and k_o is the free space wave number. Rationalized M.K.S. units are used throughout. Hence the input resistance R_{in} is proportional to L^{-1} in this case, whereas in vacuum R_{in} is known to be proportional to L^2. Because the medium is lossless the input resistance is equal to the radiation resistance, leading to the surprising result that a short antenna has a larger radiation resistance than a longer one!

If K'/K_o were positive, however, Z_{in} of (1.1) would be purely imaginary, so the input resistance in this case would be zero. This corresponds to the vacuum space behavior of L^2, however, because the higher order terms in (1.1) have been effectively neglected by the quasi-static approach.

The same problem was investigated by Staras [1964] who used a volume source distribution which was of finite diameter and had a rather elaborate form as an approximation to the dipole current distribution. His results indicated that the radiation resistance approaches zero as L^2 when K'/K_o is negative, in contradiction to (1.3) which essentially assumed a linear filamentary current distribution.

Further studies have been made for the uniaxial medium in which

K'' of (1.2) is zero. Because a scalar factor of the matrix K_m can be removed by a simple scaling procedure, no loss of generality results from also setting K' = 1. The relative permittivity matrix of (1.2) would then acquire the form

$$K_m = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & K \end{bmatrix} . \tag{1.4}$$

Seshadri [1965] investigated the radiation resistance of a dipole in this uniaxial medium by using a Fourier transform for the z coordinate, finding the field components in this transform domain, and then expressing the power as an integral in the transform domain with the aid of Parseval's theorem. His result for the case where K is negative is

$$R_{in} = \frac{\eta}{2 k_0 L} , \tag{1.5}$$

where again (and throughout this study) the dipole was in the z direction. Equation (1.5) corresponds to Balmain's relation of (1.3) for this medium. For K positive, Seshadri found R_{in} to be proportional to L^2, corresponding to its behavior in isotropic media.

Mittra [1965] studied this phenomenon in a manner similar to that of Seshadri and arrived at the same results. He pointed out that the radiation resistance is independent of the medium parameter K except for the fact that it is equal to its free space value when K is positive, but is a completely different value when K is negative. This curious behavior was attributed to the fact that this simple model is unrealistic because the medium was considered to be infinite in extent and because plasma sheath effects near the antenna were ignored. He also remarked that when the medium is lossy the input resistance can no longer be called the radiation resistance.

The behavior of the radiation resistance of short dipoles with length is related to the power delivered by an infintesimal dipole of finite moment. Let R(L) be the radiation resistance of a dipole with prescribed current distribution as a function of its length. The power radiated by the dipole can be written as

$$P = \frac{I^2}{2} R(L) \tag{1.6}$$

where I is the peak value of the current at the center of the antenna. The peak value convention for alternating currents and fields is used throughout this study. But M = AIL where M is the dipole moment and A is a proportionality constant which depends on the current distribution. So, in terms of the dipole moment, (1.6) becomes

$$P = \frac{M^2}{2A^2L^2} R(L), \tag{1.7}$$

It is seen from (1.7) that for P to approach a non-zero, finite limit as L approaches zero, R(L) must approach zero as L^2.

The work by Balmain and Seshadri thus implies that the power radiated from an infintesimal dipole in an anisotropic medium characterized by (1.2) is infinite. This phenomenon has been observed by other authors, and has acquired the name "infinity catastrophe." (See, for example, Kogelnick [1960] and Arbel and Felsen [1963]). To resolve this, Lee and Papas [1965] have recently presented a new theory of antenna radiation. They proposed that the power obtained by the "conventional procedure" is composed of two parts, P_{rev} and P_{irr}, and that only P_{irr} is actually absorbed by the shpere at infinity. They contend that P_{irr} is finite for an infintesimal dipole immersed in the anisotropic medium even when K'/K_o is negative, so from (1.7), and using this method of calculation, the radiation resistance of short dipoles must be proportional to L^2. Furthermore, a more recent paper by the same authors [Lee and Papas, 1966] contends that even using the "conventional method" of calculation, the power radiated by the unit infintesimal dipole in a uniaxial medium (characterized by (1.4) with K negative) is equal to the power radiated in vacuum. Hence, the radiation resistance of short dipoles in this medium would be proportional to L^2, in contradiction to the results of Seshadri, etc.

It should be apparent at this point that there is some doubt as to the radiation resistance of short dipoles in this uniaxial medium. In order to add some new insight to this quandary, this study investigated the problem by a different method. All of the above investigators used either near field methods or Fourier transform methods to compute the power delivered to the medium by the source. In contrast, Poynting's theorem was used here to compute the power flowing out of a sphere centered at the antenna. Some comments are in order, however, regarding the validity of this procedure before continuing this discussion.

The fields for the infintesimal dipole in a uniaxial medium are given in Clemmow [1963] and the fields are seen to be infinite on a surface known as the characteristic cone when K is negative. Therefore, the fields cannot satisfy the homogenous Maxwell's equations there because they are not differentiable and hence do not have a curl. Some doubt thus exists as to the validity of this solution or of the solution corresponding to any source which has singularities of the field in source-free regions. These singularities occur with many dipole current distributions in uniaxial media with K negative, so it is not really appropriate to talk of the radiation resistance in these circumstances. It is, however, instructive to investigate the "radiation resistance" of these sources as if the fields obtained were valid. In this study, quotes are used around "radiation resistance" whenever it is only the formal quantity obtained from the surface integral of the Poynting vector which is under discussion, and with no implications that it is to be interpreted as the radiation effectiveness of the antenna.

In this study, a filamentary dipole with finite length immersed in a uniaxial medium was considered. The current on the antenna was assumed to be sinusoidal; that is, expressible as

$$I(z) = I_m \sin k_o(L - |z|). \qquad (1.8)$$

The exact field expressions for the antenna were found in closed form. Although these fields are singular on three characteristic cones when K is negative, the surface integral of the Poynting vector was determined with the aid of a computer to arrive at a plot of the "radiation resistance" against the length of the antenna. The problem was also approached by introducing a slight amount of loss, resulting in fields which remain finite, and examining the power radiated through spheres of various radii as the loss approached zero.

The introduction of loss was accomplished by allowing K of (1.4) to become complex, or of the form $\alpha - j\beta$ where β is positive. A medium which has (1.4) as its relative permittivity matrix could be a plasma with an infinite static magnetic field applied in the z direction. K being real would correspond to a plasma with no collisions between electrons, whereas K would be complex if the plasma had a finite collision frequency. No loss would occur in the x or y directions because the electrons cannot move in a direction perpendicular to the infinite magnetic field, so no collisions would occur as the result of an applied electric field in these directions.

Briefly, then, the problem studied here was to find the "radiation resistance" of a z directed dipole immersed in a lossless uniaxial medium by Poynting vector methods without questioning the validity of the fields obtained, and to study the power flow from the dipole when the medium is lossy, representing an environment closer to reality, in order to present results which are more physically meaningful than those obtained for the lossless case.

2. THEORETICAL DISCUSSION

The most convenient method to obtain the fields of a localized current distribution in a uniaxial medium would be to use the principle of scaling discussed by Clemmow [1963] and by Bates and Mittra [1966]. Both papers are primarily concerned with the case where K is real and positive. Clemmow mentioned that the scaling procedure gives correct results when K is negative, but neither paper discusses complex K. Furthermore, application of the scaling procedure for complex K is difficult for many sources. For example, when scaling a line source such as $\overline{J} = \hat{z}I(z)\delta(x)\delta(y)$ terms such as $\delta(Kx)$ appear. If K is real, it is well known that $\delta(Kx) = (1/|K|)\delta(x)$; however, for complex K one must use $\delta(Kx) = (1/K)\delta(x)$ to achieve results which satisfy Maxwell's equations. A valid criticism for this procedure is that the definition of the Dirac delta function is not valid for complex arguments. The purpose of this section, then, is to develop a procedure to find the uniaxial fields of a line source by scaling only the vacuum fields and not the source itself.

2.1. Justification of the Scaling Procedure

Maxwell's equations for a uniaxial medium with $e^{j\omega t}$ time dependence assumed are

$$\nabla \times \overline{H} = j\omega\epsilon_0 K_m \overline{E} + \overline{J} \qquad (2.1)$$

$$-\nabla \times \overline{E} = j\omega\mu_o \overline{H}, \tag{2.2}$$

where

$$K_m = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & K \end{bmatrix}, \tag{2.3}$$

and μ_o is the permeability of vacuum, $\overline{E}$ is the electric field intensity vector, $\overline{H}$ is the magnetic field intensity vector, and $\overline{J}$ is the current density vector. Eliminating $\overline{E}$ from (2.1) and (2.2) results in

$$\nabla \times (K_m^{-1} \nabla \times \overline{H}) - k_o^2 \overline{H} = \nabla \times (K_m^{-1} \overline{J}). \tag{2.4}$$

Define the Fourier transform of the vector field components as

$$\tilde{A}_\xi(\ell, m, n) = \frac{k_o^3}{(2\pi)^3} \iiint_{-\infty}^{+\infty} A_\xi(x, y, z)\, e^{-jk_o(\ell x + my + nz)}\, dx\, dy\, dz, \tag{2.5}$$

where ξ = x, y, or z, and $\overline{A}$ is a source or field vector. Transforming both sides of (2.4) results in the matrix equation

$$\begin{bmatrix} 0 & +jk_o n & -jk_o m \\ -jk_o n & 0 & +jk_o \ell \\ +jk_o m & -jk_o \ell & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{K} \end{bmatrix} \begin{bmatrix} 0 & +jk_o n & -jk_o m \\ -jk_o n & 0 & +jk_o \ell \\ +jk_o m & -jk_o \ell & 0 \end{bmatrix} \begin{bmatrix} \tilde{H}x \\ \tilde{H}y \\ \tilde{H}z \end{bmatrix}$$

$$-k_o^2 \begin{bmatrix} \tilde{H}x \\ \tilde{H}y \\ \tilde{H}z \end{bmatrix} = \begin{bmatrix} 0 & +jk_o n & -jk_o m \\ -jk_o n & 0 & +jk_o \ell \\ +jk_o m & -jk_o \ell & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{K} \end{bmatrix} \begin{bmatrix} \tilde{J}x \\ \tilde{J}y \\ \tilde{J}z \end{bmatrix}. \tag{2.6}$$

Considering only z directed currents and simplifying,

$$\begin{bmatrix} (n^2+\frac{1}{K}m^2-1) & (-\frac{1}{K}\ell m) & (-\ell n) \\ (-\frac{1}{K}\ell m) & (n^2+\frac{1}{K}\ell^2-1) & (-mn) \\ (-\ell n) & (-mn) & (m^2+\ell^2-1) \end{bmatrix} \begin{bmatrix} \tilde{H}x \\ \tilde{H}y \\ \tilde{H}z \end{bmatrix} = \frac{j}{K\,k_o} \begin{bmatrix} -m\tilde{J} \\ +\ell\tilde{J} \\ 0 \end{bmatrix}, \tag{2.7}$$

where $\bar{J}(x, y, z) = \hat{z}J(x, y, z)$. Multiplying both sides of (2.7) by the inverse of the matrix on the left results in the equations:

$$\tilde{H}_x = \frac{jm(\ell^2+m^2+n^2-1)}{K k_o \Delta} \tag{2.8}$$

$$\tilde{H}_y = \frac{-j\ell(\ell^2+m^2+n^2-1)}{K k_o \Delta} \tag{2.9}$$

$$\tilde{H}_z = 0 \tag{2.10}$$

where

$$\Delta = (\ell^2+m^2+n^2-1)(1-\frac{1}{K}\ell^2-\frac{1}{K}m^2-n^2)\,. \tag{2.11}$$

The above equations imply the existence of a potential $\bar{A}(x, y, z) = \hat{z}A(x, y, z)$ such that $\bar{H}=\nabla\times\bar{A}$. From (2.8), (2.9), (2.11), and the matrix which corresponds to the curl operator, the potential in the transform domain is recognized as

$$\tilde{A} = \frac{1}{K k_o^2(\frac{1}{K}\ell^2+\frac{1}{K}m^2+n^2-1)} \tag{2.12}$$

Let $J=\delta(x)\delta(y)\delta(z)$, corresponding to a unit infintesimal dipole in the z direction. In the transform domain, this source is

$$\tilde{J} = \frac{k_0^3}{(2\pi)^3} \,. \qquad (2.13)$$

From (2.12), (2.13), and the inversion integral for (2.5), the z component of the vector potential corresponding to this source can be written in the (x, y, z) domain as

$$A_d = \iiint_{-\infty}^{+\infty} \frac{k_0}{(2\pi)^3 K(\frac{1}{K}\ell^2 + \frac{1}{K}m^2 + n^2 - 1)} e^{jk_0(\ell x + my + nz)} \, d\ell \, dm \, dn \,. \qquad (2.14)$$

Rearranging (2.14) results in the equation

$$A_d = \frac{k_0}{(2\pi)^3 K} \iint_{-\infty}^{+\infty} e^{jk_0(\ell x + my)} \left[\int_{-\infty}^{+\infty} \frac{e^{jk_0 nz}}{(n^2 + \frac{1}{K}\ell^2 + \frac{1}{K}m^2 - 1)} dn \right] d\ell \, dm \,. \qquad (2.15)$$

The integral in brackets can be integrated directly using Cauchy's integral formula. The integrand has two poles, one at $n = -\sqrt{1 - \frac{1}{K}\ell^2 - \frac{1}{K}m^2}$ and the other at $n = +\sqrt{1 - \frac{1}{K}\ell^2 - \frac{1}{K}m^2}$. Defining the square root of the complex number $re^{j\varphi}$ as $\sqrt{r}e^{j\frac{\varphi}{2}}$ for $-\pi < \varphi \leq \pi$, and noting that K will have a negative imaginary part, the poles will be in the second and fourth quadrants of the complex plane for all values of ℓ and m. Closing the contour on the top will then allow the integral to be evaluated for $z > 0$. Equation (2.15) then becomes

$$A_d = \frac{-jk_0}{8\pi^2 K} \iint_{-\infty}^{+\infty} \frac{e^{jk_0\left(\ell x + my - z\sqrt{1 - \frac{1}{K}(\ell^2 + m^2)}\right)}}{\sqrt{1 - \frac{1}{K}(\ell^2 + m^2)}} \, d\ell \, dm \qquad (2.16)$$

Perform a change of variable on (2.16), letting $\ell = \frac{1}{K}\rho\cos\varphi$, $m = \frac{1}{K}\rho\sin\varphi$, $x = r\cos\theta$, and $y = r\sin\theta$. Then,

$$A_d = \frac{-j}{8\pi^2 K}\int_0^\infty\int_{-\pi}^{\pi}\frac{e^{j\left(r\rho\cos(\varphi-\theta) - z\sqrt{k_o^2 - \frac{1}{K}\rho^2}\right)}}{\sqrt{k_o^2-\frac{1}{K}\rho^2}}\,\rho\, d\rho\, d\varphi \tag{2.17}$$

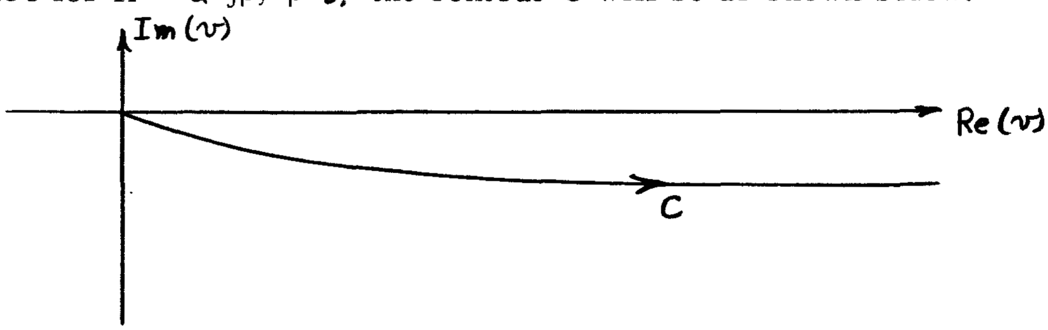
Equation (2.17) can be easily integrated with respect to φ, resulting in the one dimensional integral

$$A_d = \frac{1}{4\pi\sqrt{K}}\int_0^\infty \frac{e^{-\frac{z}{\sqrt{K}}\sqrt{\rho^2 - K k_o^2}}}{\sqrt{\rho^2 - K k_o^2}}\,\rho J_o(r\rho)\, d\rho \tag{2.18}$$

Performing another change of variable, with $v = \sqrt{\rho^2 - K k_o^2} - \sqrt{-K k_o^2}$, results in the contour integral

$$A_d = 4\pi\sqrt{K}\int_C e^{-j\frac{z}{\sqrt{K}}\left(v+\sqrt{-K k_o^2}\right)}\, J_o\left(r\sqrt{v^2 + 2v\sqrt{-K k_o^2}}\right) dv \tag{2.19}$$

where for $K = \alpha - j\beta$, $\beta > 0$, the contour C will be as shown below.



This contour can be deformed onto the positive real axis because the integrand is analytic. The integral in (2.19) then becomes an integral from zero to infinity, a definite integral which is given in Bateman [1954]. The result is

$$A_d = \frac{e^{jk_0\sqrt{z^2 + K(x^2+y^2)}}}{4\pi\sqrt{z^2 + K(x^2+y^2)}} . \tag{2.20}$$

Equation (2.20) is then recognized as the vector potential for the same source in vacuum after a suitable change of variable. If $A_d^o(x, y, z)$ designates the vector potential for the unit z directed dipole in vacuum, then

$$A_d(x, y, z) = A_d^o(\sqrt{K}x, \sqrt{K}y, z) . \tag{2.21}$$

All of the above steps are valid for the imaginary part of K strictly negative and for z positive. Equations (2.20) and (2.21) also hold for z negative by symmetry.

The vector potential for a line source $\bar{J}=\hat{z}I(z)\delta(x)\delta(y)$ is easily found by superposition using (2.21). Expressing the superposition in integral form results in the equation

$$A(x, y, z) = \int_{-\infty}^{+\infty} I(\alpha)A_d(x, y, z-\alpha)d\alpha . \tag{2.22}$$

Substituting (2.21) into (2.22) results in

$$A(x, y, z) = \int_{-\infty}^{+\infty} I(\alpha)A_d^o(\sqrt{K}x, \sqrt{K}y, z-\alpha)d\alpha . \tag{2.23}$$

Finally, recognizing (2. 23) as a scaled version of the vector potential for the same line source in vacuum results in

$$A(x, y, z) = A^{o}(\sqrt{K}x, \sqrt{K}y, z) \ . \tag{2.24}$$

where $A^{o}(x, y, z)$ is the free space vector potential for the line source.

The scaling procedure for the field components can be derived similarly using the equations $\overline{H} = \nabla \times \overline{A}$ and $\overline{E} = \frac{1}{j\omega\epsilon_0} K_m^{-1} \nabla \times \overline{H}$ with (2. 24), resulting in the equations:

$$E_x(x, y, z) = \sqrt{K}\ E_x^{o}(\sqrt{K}x, \sqrt{K}y, z) \tag{2.25}$$

$$E_y(x, y, z) = \sqrt{K}\ E_y^{o}(\sqrt{K}x, \sqrt{K}y, z) \tag{2.26}$$

$$E_z(x, y, z) = E_z^{o}(\sqrt{K}x, \sqrt{K}y, z) \tag{2.27}$$

$$H_x(x, y, z) = \sqrt{K}\ H_x^{o}(\sqrt{K}x, \sqrt{K}y, z) \tag{2.28}$$

$$H_y(x, y, z) = \sqrt{K}\ H_y^{o}(\sqrt{K}x, \sqrt{K}y, z) \ . \tag{2.29}$$

Equations (2. 25) through (2. 29) represent a means of obtaining the fields of a line source in the z direction immersed in a uniaxial medium from the vacuum fields of the same source. The procedure is valid for all K with a negative imaginary part, as demonstrated above. The validity of the procedure for real K can be established by allowing K to approach the real axis from below. If the fields approach a limit uniformly as K becomes real, the procedure of (2. 25) through (2. 29) should be valid for K real. Unfortunately, this is not the case for some sources when K is negative as the fields approach infinity in certain directions. This point will be discussed further in section 4.

2.2. Dependence of the Radiation Resistance on the Medium Parameter K

The above development of a scaling procedure for K complex essentially completes the theoretical background necessary for this study. However, analysis of the preliminary numerical results has led to an interesting theoretical discussion regarding the behavior of the radiation resistance of a filamentary current distribution with changes in the medium parameter K. The result of this discussion (which appears below) is that multiplying K by a real and positive constant does not affect the radiation resistance of z directed line sources in the medium. The discussion to follow, however, departs from consideration of line sources exclusively in order to develop the phenomenon in full generality, and then proceeds to line sources as a special case.

If the sources considered are only those for which the scaling procedure outlined by Clemmow [1963] can by applied without difficulty, the method would relate a source and field distribution in a uniaxial medium to a corresponding vacuum source and field. If the fields are transverse magnetic with respect to z, the equivalence is particularly simple and is repeated below for the relative permittivity matrix of (2.3). Here the quantities without superscript refer to the uniaxial medium whereas the superscript "o" designates free space quantities.

$$E_x(x, y, z) = \sqrt{K}\, E_x^o(\sqrt{K}x, \sqrt{K}y, z) \qquad (2.30)$$

$$E_y(x, y, z) = \sqrt{K}\ E_y^o(\sqrt{K}x, \sqrt{K}y, z) \tag{2.31}$$

$$E_z(x, y, z) = E_z^o(\sqrt{K}x, \sqrt{K}y, z) \tag{2.32}$$

$$H_x(x, y, z) = \sqrt{K}\ H_x^o(\sqrt{K}x, \sqrt{K}y, z) \tag{2.33}$$

$$H_y(x, y, z) = \sqrt{K}\ H_y^o(\sqrt{K}x, \sqrt{K}y, z) \tag{2.34}$$

$$H_z(x, y, z) = 0 \tag{2.35}$$

$$J_x(x, y, z) = \sqrt{K}\ J_x^o(\sqrt{K}x, \sqrt{K}y, z) \tag{2.36}$$

$$J_y(x, y, z) = \sqrt{K}\ J_y^o(\sqrt{K}x, \sqrt{K}y, z) \tag{2.37}$$

$$J_z(x, y, z) = K\ J_z^o(\sqrt{K}x, \sqrt{K}y, z)\ . \tag{2.38}$$

If one considers two unaixial media with relative permittivity matrices

$$K_{m1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & K_1 \end{bmatrix} \tag{2.39}$$

and

$$K_{m2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & K_2 \end{bmatrix}, \tag{2.40}$$

then for each admissible source-field pair in medium 1 there is a corresponding pair in medium 2. The relationship can be made explicit by relating fields and sources in each medium to free space using equations (2.30) through (2.38) and effecting a change of variable to obtain the relationship between quantities in mediums 1 and 2. The results are:

$$E_x''(x, y, z) = b\ E_x'(bx, by, z) \tag{2.41}$$

$$E''_y(x, y, z) = b\, E'_y(bx, by, z) \tag{2.42}$$

$$E''_z(x, y, z) = E'_z(bx, by, z) \tag{2.43}$$

$$H''_x(x, y, z) = b\, H'_x(bx, by, z) \tag{2.44}$$

$$H''_y(x, y, z) = b\, H'_y(bx, by, z) \tag{2.45}$$

$$H''_z(x, y, z) = H'_z(x, y, z) = 0 \tag{2.46}$$

$$J''_x(x, y, z) = b\, J'_x(bx, by, z) \tag{2.47}$$

$$J''_y(x, y, z) = b\, J'_y(bx, by, z) \tag{2.48}$$

$$J''_z(x, y, z) = b^2\, J'_z(bx, by, z)\ , \tag{2.49}$$

where

$$b = \sqrt{\frac{K_2}{K_1}} \tag{2.50}$$

and the '' and ' refer to mediums 2 and 1 respectively. Considering only real and positive b, the power delivered by a source in medium 2 can be related to the power delivered by the equivalent source in medium 1. The power is defined as

$$P'' = \iiint_{-\infty}^{\infty} \overline{E}''(x, y, z) \cdot \overline{J}''^{*}(x, y, z)\, dxdydz\ . \tag{2.51}$$

Using equations (2.41) through (2.49), and noting that since b is real the scale factors do not affect the conjugation,

$$P'' = \iiint_{-\infty}^{+\infty} b^2 [E'_x(bx, by, z) J'_x{}^{*}(bx, by, bz) + \ldots]\, dxdydz\ . \tag{2.52}$$

Let $x' = bx$ and $y' = by$. Then,

$$P'' = \iiint_{-\infty}^{+\infty} [E'_x(x', y', z) J'_x{}^{*}(x', y', z) + \ldots]\, dx'dy'dz\ . \tag{2.53}$$

The right side of (2.53) is then recognized as P'. Thus in a uniaxial medium the power delivered by a source distribution which produces a transverse magnetic field is the same as the power delivered by its corresponding source in another uniaxial medium as long as the ratio $\frac{K_2}{K_1}$ is real and positive.

A simplification results if the source is a z directed line source; that is, expressible as

$$\overline{J}'(x, y, z) = \hat{z}I(z)\delta(x)\delta(y) , \qquad (2.54)$$

The results of section 2.1 are then applicable and a development parallel to that above except using (2.25) through (2.29) would show that the power delivered by a line source in medium 1 would be equal to that delivered by the same source in medium 2, again as along as $\frac{K_2}{K_1}$ is real and positive. If in addition K_1 and K_2 are real it is meaningful to speak of radiation resistance. Because the radiation resistance depends only on the power and current, both of which are the same in the two media, the radiation resistance of the source must then be the same.

In particular the radiation resistance of a z directed dipole in a uniaxial medium with K real and positive is the same as it is in vacuum if the current distribution of the dipole is the same in the two media. Furthermore, if K is negative and the procedure of (2.25) through (2.29) is valid, the "radiation resistance" of z directed dipoles in this medium would be the same as it is for $K = -1$. In numerical computations, then, only the case where $K = -1$ needs investigation. This invariance of the radiation resistance under multiplication of K by a real and positive constant is in agreement with the results by Seshadri, etc.

3. NUMERICAL PROCEDURE

As stated before, the objective of this research is to study the "radiation resistance" of dipole antennas in uniaxial media using Poynting vector methods. The first step, then, would be to find the fields of the dipole. The scaling procedure outlined in equations (2.25) through (2.29) relates the desired fields to those of the same source in vacuum, hence reducing the problem to that of obtaining the vacuum fields for the source.

Consider, then, a dipole antenna in the z direction with half-length L. If the dipole is assumed to have a sinusoidal current distribution, that is

$$\bar{J} = \hat{z}\, I_m \sin k_0 (L - |z|)\, \delta(x)\, \delta(y), \tag{3.1}$$

then the exact fields of the source in vacuum are known [Jordan, 1950]. A simple, but algebraically involved, application of (2.25) through (2.29) to the vacuum fields results in the uniaxial fields, which are:

$$H = \hat{\varphi} \frac{j I_m}{4\pi r \sin\theta} \left(e^{-jk_0 r_1} + e^{-jk_0 r_2} - 2\cos k_0 L\, e^{-jk_0 r_0} \right) \tag{3.2}$$

$$\bar{E} = \frac{jk_0 I_m}{4\pi\omega\epsilon_0}\left\{\hat{r}\left(-\frac{L}{r}\frac{e^{-jk_0r_1}}{r_1} + \frac{L}{r}\frac{e^{-jk_0r_2}}{r_2}\right) + \frac{\hat{\theta}}{\sin\theta}\left[\left(1-\frac{L}{r}\cos\theta\right)\frac{e^{-jk_0r_1}}{r_1}\right.\right.$$

$$\left.\left. + \left(1+\frac{L}{r}\cos\theta\right)\frac{e^{-jk_0r_2}}{r_2} - 2\cos k_0 L\,\frac{e^{-jk_0r_0}}{r_0}\right]\right\} \qquad (3.3)$$

where:

$$r_1 = r\sqrt{(\cos\theta - \tfrac{L}{r})^2 + K\sin^2\theta} \qquad (3.4)$$

$$r_0 = r\sqrt{\cos^2\theta + K\sin^2\theta} \qquad (3.5)$$

$$r_2 = r\sqrt{(\cos\theta + \tfrac{L}{r})^2 + K\sin^2\theta}, \qquad (3.6)$$

and where r, θ, and φ are the polar coordinates variables.

Having the uniaxial fields for the source, the next step would be to compute the Poynting vector $\bar{P}$, which is defined as

$$\bar{P} = \frac{1}{2}\,Re(\bar{E}\times\bar{H}^*). \qquad (3.7)$$

The power delivered by the antenna would then be

$$P_o = \int_0^{\pi}\int_{-\pi}^{+\pi} \bar{P}(r,\theta,\varphi)\cdot\hat{r}\; r^2\sin\theta\, d\varphi\, d\theta \qquad (3.8)$$

where the Poynting vector has been integrated on a sphere of radius r. The Poynting vector is independent of φ, and is symmetric in θ about $\theta = \frac{\pi}{2}$. Equation (3.8) can then be rewritten as

$$P_o = 4\pi\int_0^{\frac{\pi}{2}} P_r(r,\theta)\, r^2\sin\theta\, d\theta, \qquad (3.9)$$

where $P_r(r, \theta)$ is the radial component of the Poynting vector. However, to form $P_r(r, \theta)$ it is necessary to conjugate $\overline{H}$, and thus one must know the value of K because r_0, r_1, and r_2 may be real, imaginary, or complex. For this reason, the problem will be approached differently depending on whether K is real or complex.

3.1. Lossless Media

The medium parameter K is real if and only if the medium is lossless. There is no need to investigate the case where K is positive, because this was shown to be equivalent to vacuum. K will then be assumed negative, and set equal to -1 for convenience. Equations (3.4) through (3.6) then become

$$r_1 = r\sqrt{\left(\cos\theta - \frac{L}{r}\right)^2 - \sin^2\theta} \qquad (3.10)$$

$$r_0 = r\sqrt{\cos^2\theta - \sin^2\theta} \qquad (3.11)$$

$$r_2 = r\sqrt{\left(\cos\theta + \frac{L}{r}\right)^2 - \sin^2\theta} \qquad (3.12)$$

Equations (3.10) through (3.12) show that r_0, r_1, and r_2 may be positive or imaginary depending on the value of r and θ. Figure 1 shows the antenna, the sphere of integration, and the four regions where different ones of the variables r_0, r_1, and r_2 are imaginary, hence making the Poynting vector acquire a different form. These regions are labelled 1 through 4 in order of increasing θ. Using (3.7) and noting which of

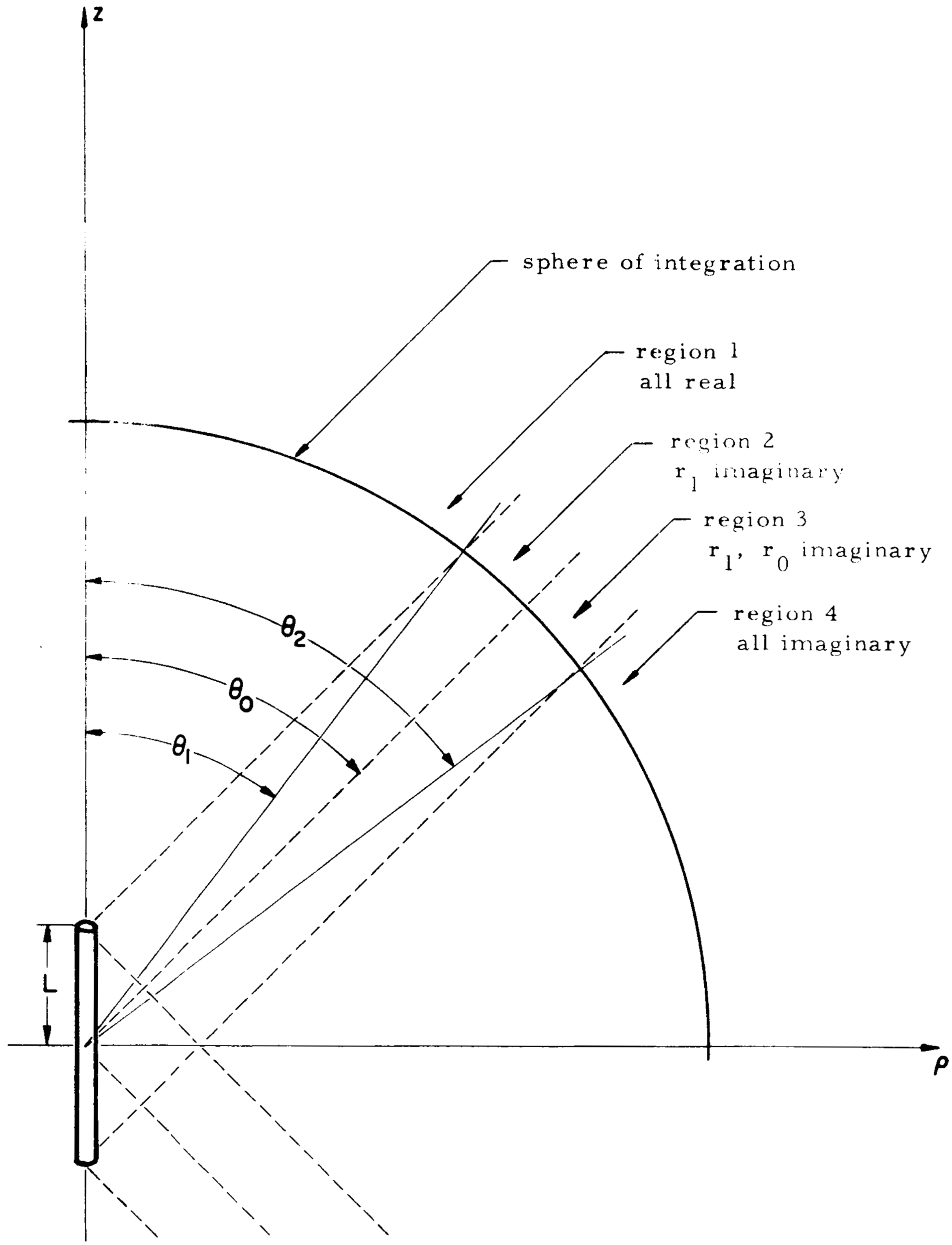


Figure 1 Geometry of the Antenna and its Environment

r_0, r_1, and r_2 are imaginary allows $P_r(r, \theta)$ to be computed in each region, resulting in the formulas:

Region 1:

$$P_r = \frac{k_0^2 I_m^2}{32\pi^2 r \omega \epsilon_0 \sin^2\theta}\left[\left(1-\frac{L}{r}\cos\theta\right)\frac{1+\cos(a_2-a_1)-2\cos\ell\cos(a_0-a_1)}{a_1}\right.$$

$$+\left(1+\frac{L}{r}\cos\theta\right)\frac{1+\cos(a_1-a_2)-2\cos\ell\cos(a_0-a_2)}{a_2}$$

$$\left.+(-2\cos\ell)\frac{\cos(a_1-a_0)+\cos(a_2-a_0)-2\cos\ell}{a_0}\right] \tag{3.13}$$

Region 2:

$$P_r = \frac{k_0^2 I_m^2}{32\pi^2 r \omega \epsilon_0 \sin^2\theta}\left[\left(1-\frac{L}{r}\cos\theta\right)\frac{(2\cos\ell\sin a_0-\sin a_2)e^{-b_1}}{b_1}\right.$$

$$+\left(1+\frac{L}{r}\cos\theta\right)\frac{e^{-b_1}\cos a_2+1-2\cos\ell\cos(a_0-a_2)}{a_2}$$

$$\left.+(-2\cos\ell)\frac{e^{-b_1}\cos a_0+\cos(a_2-a_0)-2\cos\ell}{a_0}\right] \tag{3.14}$$

Region 3:

$$P_r = \frac{k_0^2 I_m^2}{32\pi^2 r \omega \epsilon_0 \sin^2\theta}\left[\left(1-\frac{L}{r}\cos\theta\right)\frac{-e^{-b_1}\sin a_2}{b_1}\right.$$

$$+\left(1+\frac{L}{r}\cos\theta\right)\frac{e^{-b_1}\cos a_2+1-2\cos\ell\, e^{-b_0}\cos a_2}{a_2}$$

$$\left.+(-2\cos\ell)\frac{-e^{-b_0}\sin a_2}{b_0}\right] \tag{3.15}$$

Region 4:

$$P_r = 0 , \tag{3.16}$$

where $\ell = k_o L$, $a = k_o r$, $a_1 = k_o r_1$, $a_0 = k_o r_0$, $a_2 = k_o r_2$,

$b_1 = k_o r \sqrt{\sin^2 \theta - (\cos \theta - \frac{L}{r})^2}$, and $b_0 = k_o r \sqrt{\sin^2 \theta - \cos^2 \theta}$.

The "radiation resistance," then, is given by $R = \frac{2P_o}{I^2}$, where I is the current at the center of the antenna, or $I = I_m \sin (k_o L)$. Using (3.9) and (3.13) through (3.16), an integral expression for the "radiation resistance" can be derived. This formula, after simplification, is

$$R = \frac{\eta}{4\pi \sin^2 \ell} \left\{ \int_0^{\theta_1} \left[\frac{(1-\frac{\ell}{a}\cos\theta)[1+\cos(a_2-a_1) - 2\cos\ell \cos(a_0-a_1)]}{a_1/a} \right. \right.$$

$$+ \frac{(1+\frac{\ell}{a}\cos\theta)[1+\cos(a_1-a_2) - 2\cos\ell\cos(a_0-a_2)]}{a_2/a}$$

$$\left. + \frac{2\cos\ell[2\cos\ell - \cos(a_1-a_0) - \cos(a_2-a_0)]}{a_0/a} \right] \frac{d\theta}{\sin\theta}$$

$$+ \int_{\theta_1}^{\theta_0} \left[\frac{(1-\frac{\ell}{a}\cos\theta)(2\cos\ell \sin a_0 - \sin a_2)\, e^{-b_1}}{b_1/a} \right.$$

$$+\frac{(1+\frac{\ell}{a}\cos\theta)\left[1+e^{-b_1}\cos a_2-2\cos\ell\cos(a_0-a_2)\right]}{a_2/a}$$

$$\left.+\frac{2\cos\ell\left[2\cos\ell-e^{-b_1}\cos a_0-\cos(a_2-a_0)\right]}{a_0/a}\right]\frac{d\theta}{\sin\theta}$$

$$+\int_{\theta_0}^{\theta_2}\left[\frac{-(1-\frac{\ell}{a}\cos\theta)e^{-b_1}\sin a_2}{b_1/a}\right.$$

$$+\frac{(1+\frac{\ell}{a}\cos\theta)\left[1+e^{-b_1}\cos a_2-2\cos\ell e^{-b_0}\cos a_2\right]}{a_2/a}$$

$$\left.\left.+\frac{2\cos\ell\, e^{-b_0}\sin a_2}{b_0/a}\right]\frac{d\theta}{\sin\theta}\right\} . \qquad (3.17)$$

In (3.17) and in Figure 1, the angles θ_1, θ_0, and θ_2 are those where r_1, r_0, and r_2 become zero, respectively.

It can be seen from close inspection of (3.17) that the integrand approaches infinity at θ_1, θ_0, and θ_2. (Throughout the following discussion, r is assumed to be fixed). In each region, the integrand is composed of three terms. At each end point, one of these terms becomes infinite because its denominator approaches zero as $\sqrt{\theta-\theta_i}$, where θ_i is the angle of the singularity. Thus, the integral exists, although its complexity defies attempts to integrate it directly. The integral as written cannot be performed numerically on a computer either because the integrand is unbounded. However, a method can be devised whereby a known integral is subtracted off and the remaining integral evaluated numerically. To demonstrate this method, computation of the first integral in (3.17) will be explained.

The integral can be written as

$$R_1 = \int_0^{\theta_1} \left[\frac{f_1(\theta)}{g_1(\theta)} + f_2(\theta) \right] d\theta \tag{3.18}$$

where $f_1(\theta)$ and $f_2(\theta)$ are analytic in $[0, \theta_1]$ and $g_1(\theta)$ approaches zero at θ_1. (Note that the integrand approaches zero as θ approaches zero in spite of the term "sin (θ)" in the denominator). Explicitly,

$$f_1(\theta) = \frac{\eta}{4\pi \sin^2 \ell} \; \frac{(1 - \frac{\ell}{a}\cos\theta)\left[1 + \cos(a_2 - a_1) - 2\cos\ell \cos(a_0 - a_1)\right]}{\sin\theta} \tag{3.19}$$

$$f_2(\theta) = \frac{\eta}{4\pi \sin^2 \ell \sin\theta} \left[\frac{(1 + \frac{\ell}{a}\cos\theta)(1 + \cos(a_1 - a_2) - 2\cos\ell \cos(a_0 - a_2))}{a_2/a} + \frac{2\cos\ell\left[2\cos\ell - \cos(a_1 - a_0) - \cos(a_2 - a_0)\right]}{a_0/a} \right] \tag{3.20}$$

$$g_1(\theta) = a_1/a \tag{3.21}$$

Equation (3.18) can be rearranged as

$$R_1 = \int_0^{\theta_1} \left\{ \left[\frac{f_1(\theta)}{g_1(\theta)} - \frac{f_1(\theta_1)}{\sqrt{\theta_1 - \theta}} \right] + f_2(\theta) \right\} d\theta + \int_0^{\theta_1} \frac{f_1(\theta_1)}{\sqrt{\theta_1 - \theta}} \, d\theta \, . \qquad (3.22)$$

Or, letting $f_3(\theta) = \frac{f_1(\theta)}{g_1(\theta)} - \frac{f_1(\theta_1)}{\sqrt{\theta_1 - \theta}}$ and evaluating the second integral,

$$R_1 = \int_0^{\theta_1} [f_3(\theta) + f_2(\theta)] d\theta + 2\sqrt{\theta_1} \, f_1(\theta_1) \, . \qquad (3.23)$$

Analysis will reveal that $f_3(\theta)$ approaches zero as θ approaches θ_1, so the integrand $(f_3(\theta) + f_2(\theta))$ is now bounded and continuous, and hence can be evaluated numerically on a digital computer.

All of the difficulties of (3. 7) were handled in a manner like that of the above example. The radius of the sphere on which the integral is performed does not affect the result, as long as the sphere encloses the antenna, so r was set equal to 4L in the computations. The results are presented in Figure 2 as a plot of the "radiation resistance" against the length of the antenna, with (1. 5), the equation obtained by Seshadri, also plotted for comparison. As can be seen from the slightly erratic behavior of the computer results, some difficulty was encountered in the numerical evaluation. This is thought to be because the integrand was still very badly behaved even after subtracting off its singular behavior, and because forming the integrand involved finding a small difference between large numbers and hence stressed the accuracy of the computer used. However, in spite of the erratic behavior, Figure 2 still demonstrates that this method yields results which are quite close to those obtained by Seshadri.

3.2. Lossy Media

When K becomes complex, radiation resistance no longer has meaning because power is lost to the medium. However, it is helpful to define "effective resistance" as a useful extension of the term as

$$R = \frac{2}{I^2} \int_0^{\pi} \int_{-\pi}^{+\pi} P_r(r, \theta)\, r^2 \sin\theta\, d\theta \,, \qquad (3.24)$$

where again the integral is performed on the surface of a sphere of radius r centered at the antenna, I is the current at the center of the antenna, and $P_r(r, \theta) = 1/2\, \mathrm{Re}(\overline{E} \times \overline{H}^*) \cdot \hat{r}$. R is then a function of the antenna length and the distance from the antenna (radius of the sphere of integration), and can be thought of as the power radiated by the antenna through a sphere of radius r when it has $\sqrt{2}$ amperes of current at its center.

Because $P_r(r, \theta)$ is continuous and bounded when K is not real and negative, evaluation of (3.24) is straightforward in this case. The computer was programmed to evaluate E_θ and H_φ from (3.2) and (3.3), compute P_r, and then integrate it to form R from (3.24).

Figure 3 is a plot of R as a function of the distance from the antenna for an antenna with a total length of .02 free space wavelengths ($k_oL = .01$) and for $K = -1 - j\beta$ for several values of β. It demonstrates that R approaches the "radiation resistance" of the antenna (for K = -1) for every fixed r as β approaches zero.

Figure 4 is a plot of R as a function of r for antennas of several lengths, where K = -1-j.03. It demonstrates that although the effective resistance near the antenna varies inversely with L as in the lossless case, the curves cross at larger distances until eventually the longest antenna has the largest effective resistance.

Figure 5 plots R as a function of antenna length for r = .02 free space wavelengths, and demonstrates that although R increases with decreasing length at first, it eventually decreases again as the length approaches zero. The straight line again represents the equation $R = \frac{\eta}{2k_o L}$, the result obtained by Seshadri for a lossless medium with K negative.

The fact that the curves cross in Figure 4 is quite interesting, and its implications are discussed in section 4. It means that the attenuation of the fields of a shorter antenna is more rapid, at least in the range plotted in the figure. The reasons for this are not evident from (3.2) and (3.3), so further study was made of this phenomenon. In particular, a computer program was written to compute the power per unit solid angle radiating from the antenna, which is defined as

$$P_{\Omega}(r, \theta) = r^2 Pr(r, \theta) . \tag{3.25}$$

The results of this investigation are Figures 6, 7, 8, and 9, each of which represents a different antenna length and is a plot of P_{Ω} as a function of θ at several radii. The medium parameter K was -1-j.03 in each case. These curves show that the power attenuates more rapidly in all directions for smaller antennas.

Discussion of these figures is deferred to section 4. However, it should be pointed out here that some of the curves would behave very erratically (particularly in Figure 5) if they were drawn through all of the points where computations were made, so they were extended as dotted lines in a reasonable manner. These points are assumed to be erroneous output resulting from computer round-off error and the inability of the integration routine used to handle the functions involved, which Figure 6 shows are very badly behaved.

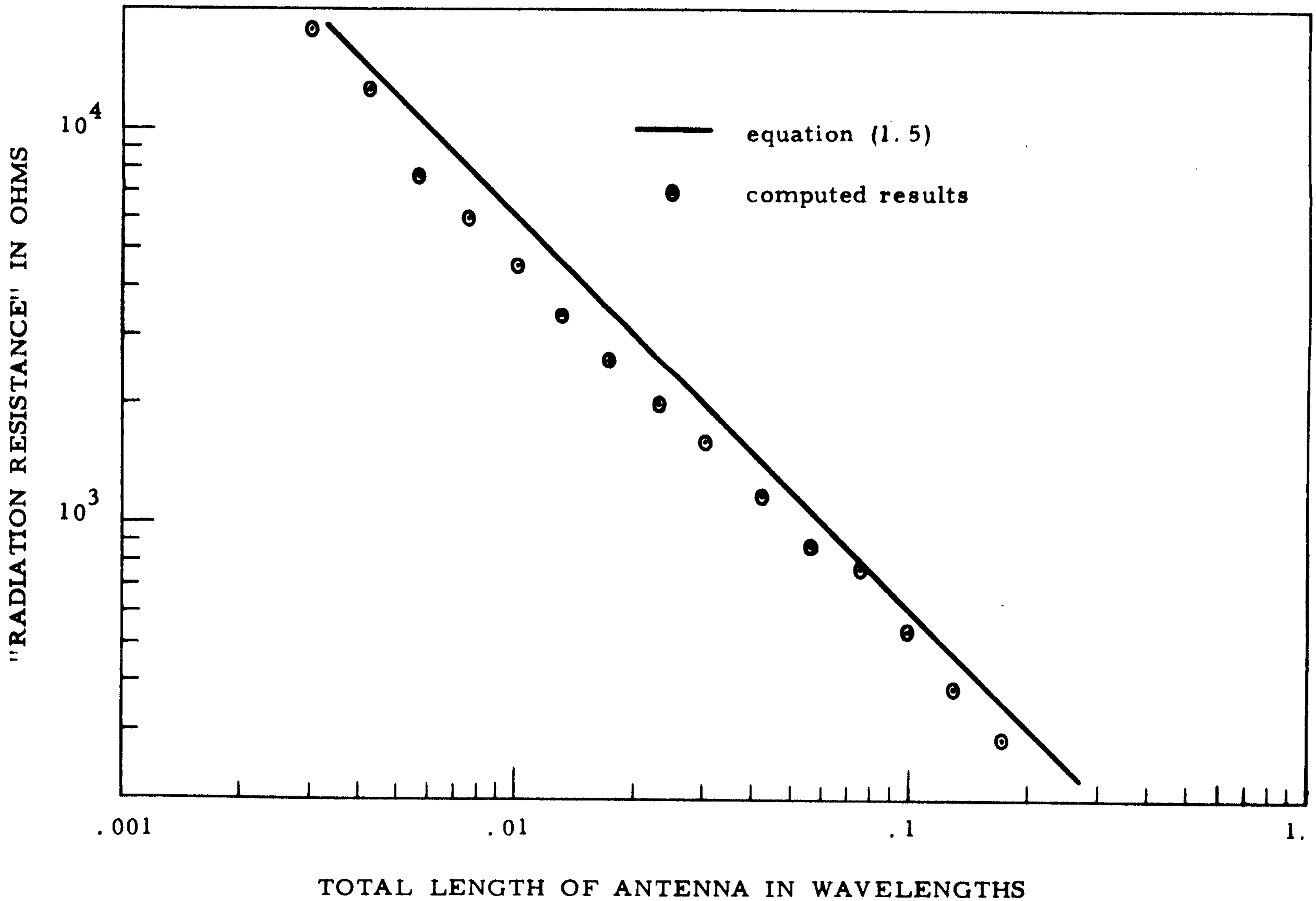


Figure 2 "Radiation Resistance" for a Lossless Medium

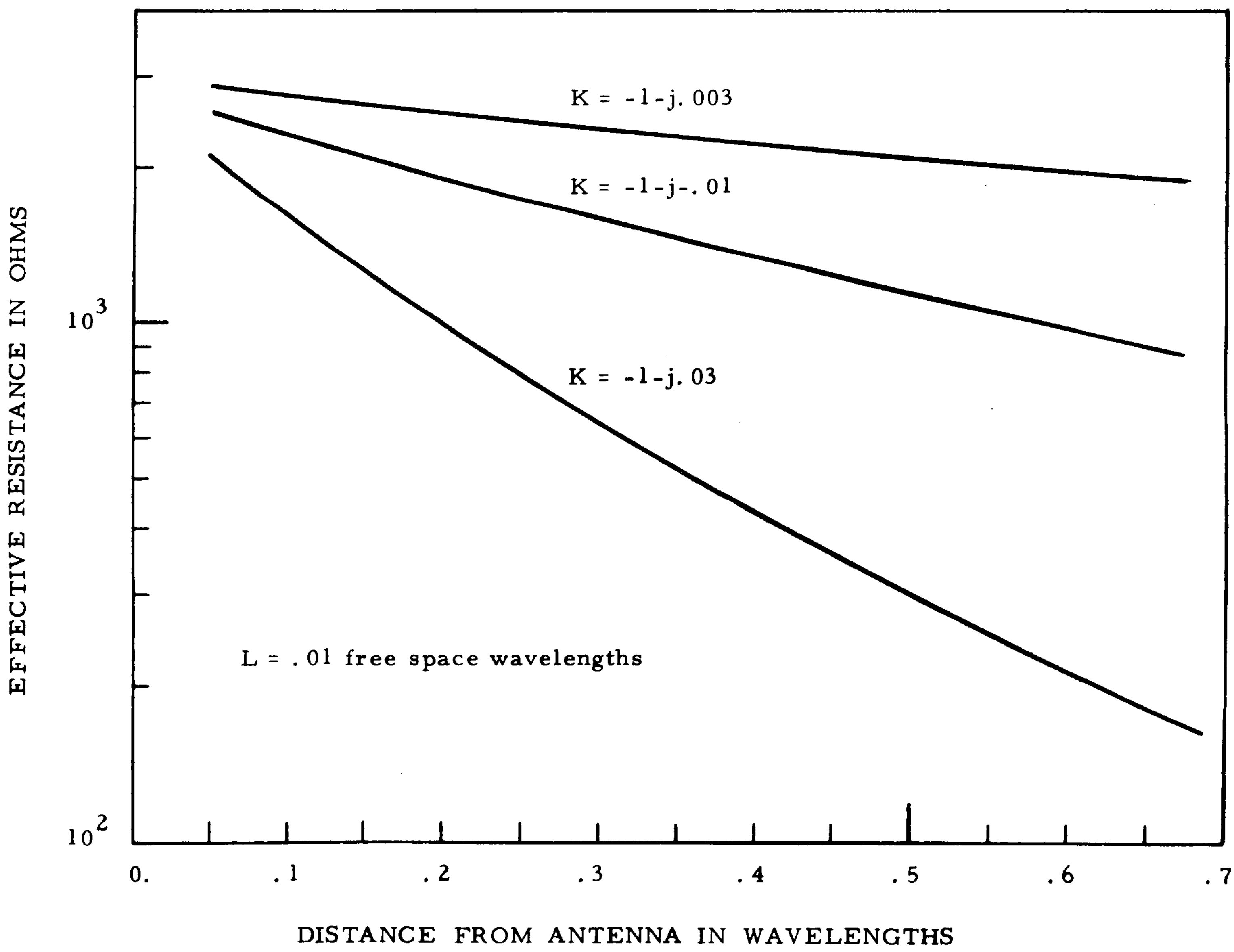


Figure 3 Effective Resistance for Different Amounts of Loss

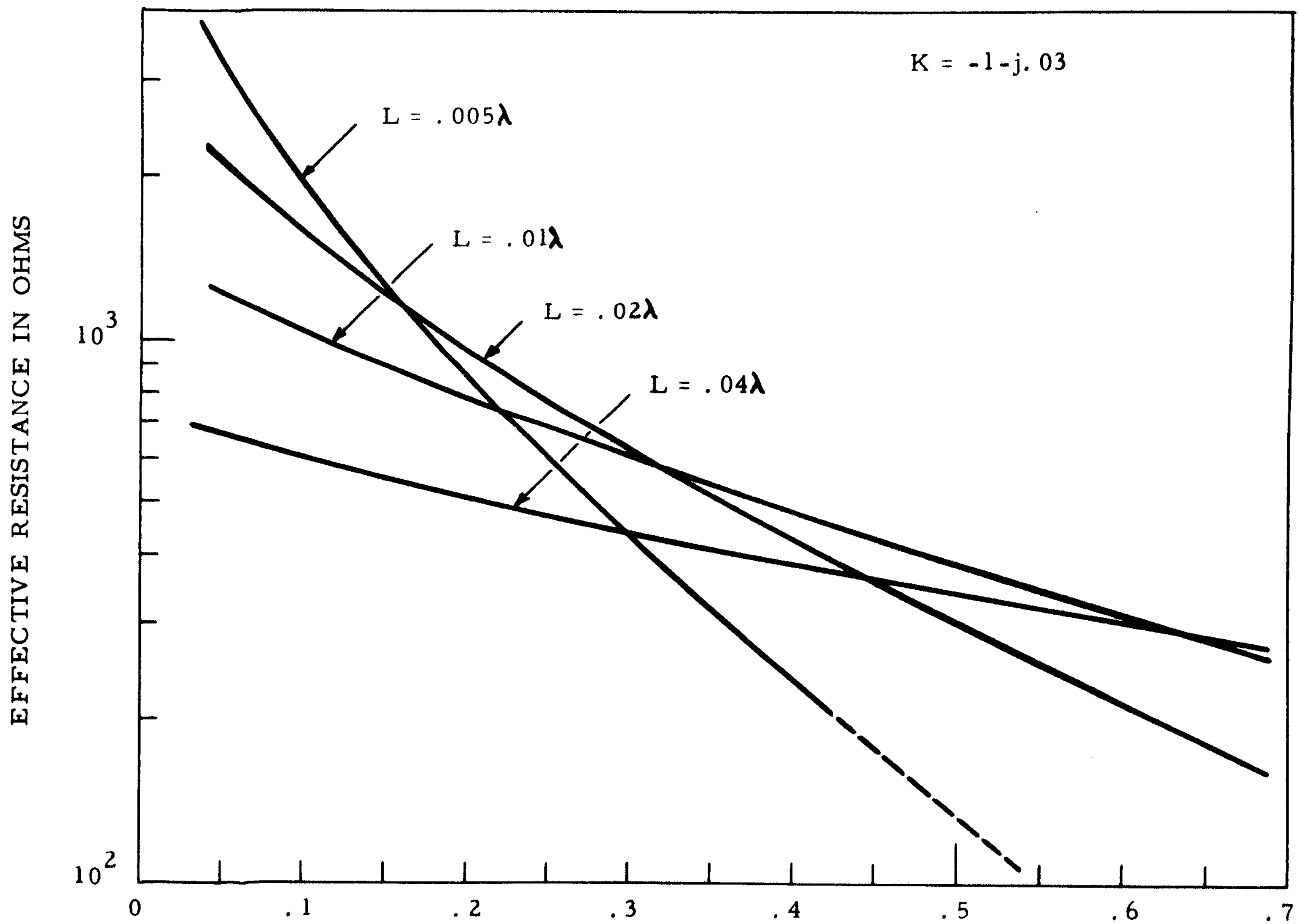


Figure 4 Effective Resistance for Antennas of Different Lengths

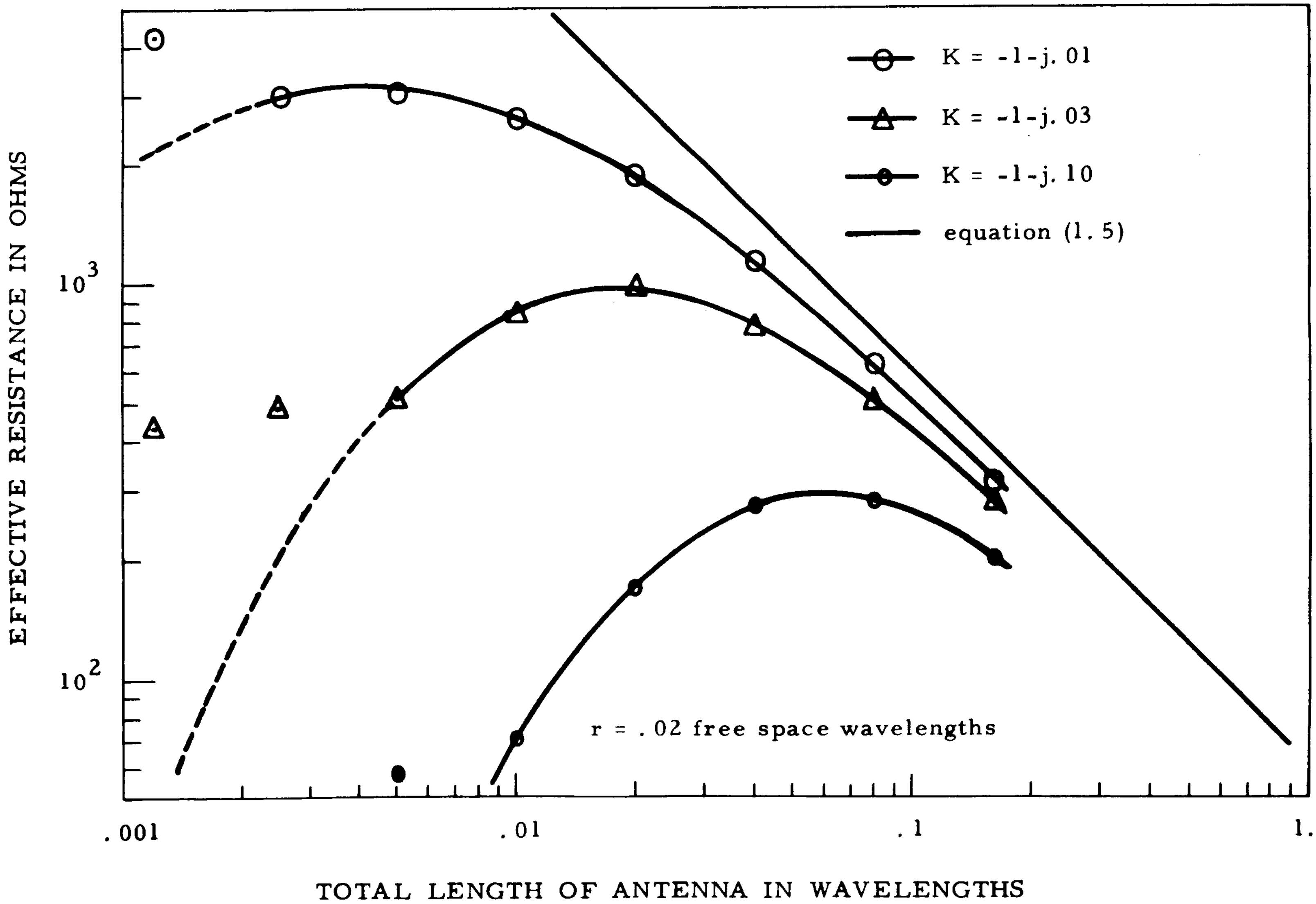


Figure 5 Effective Resistance versus Antenna Length for Several Lossy Media

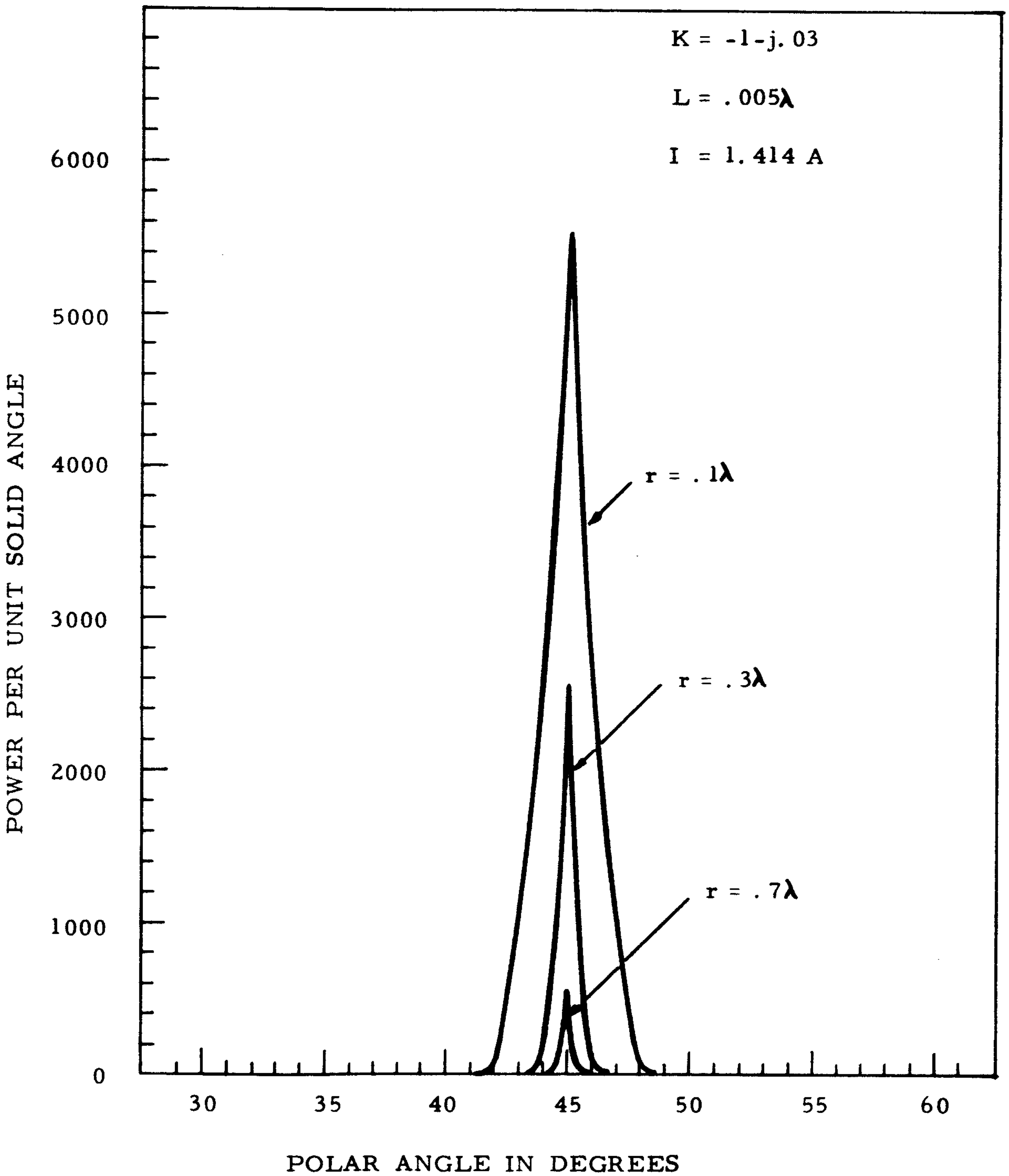


Figure 6 Power per Unit Solid Angle for L = . 005λ

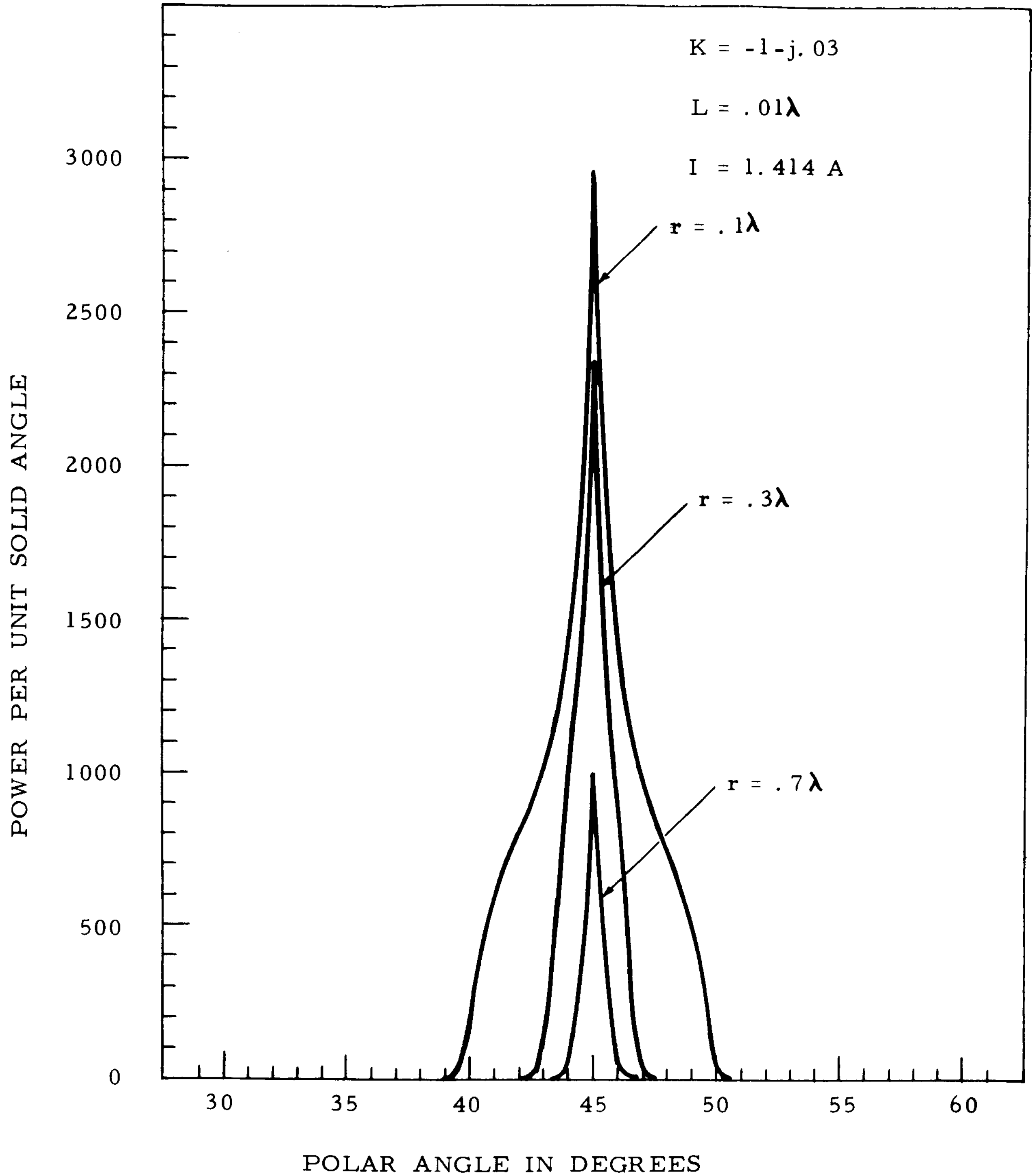


Figure 7 Power per Unit Solid Angle for L = . 01λ

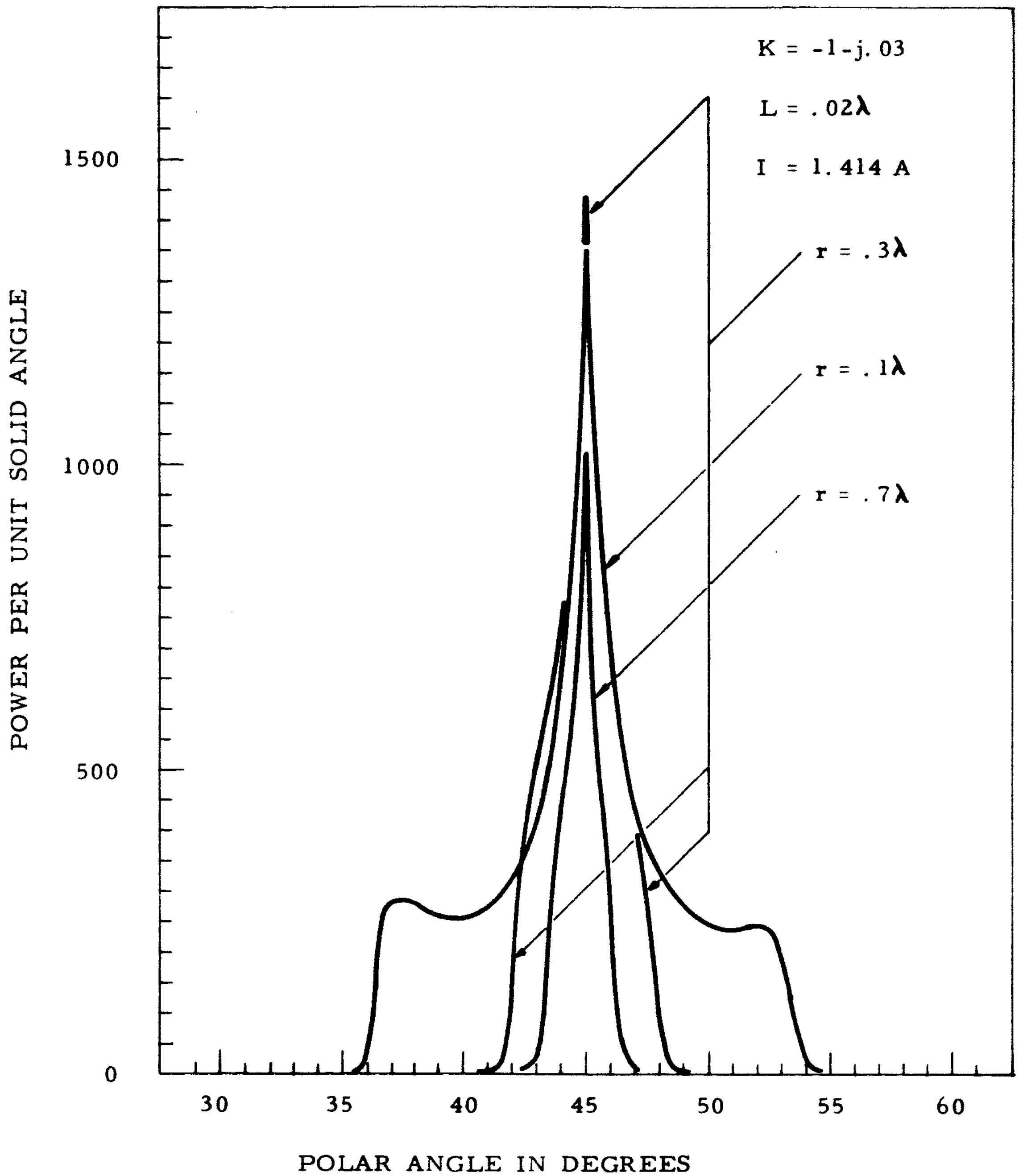


Figure 8 Power per Unit Solid Angle for L = .02λ

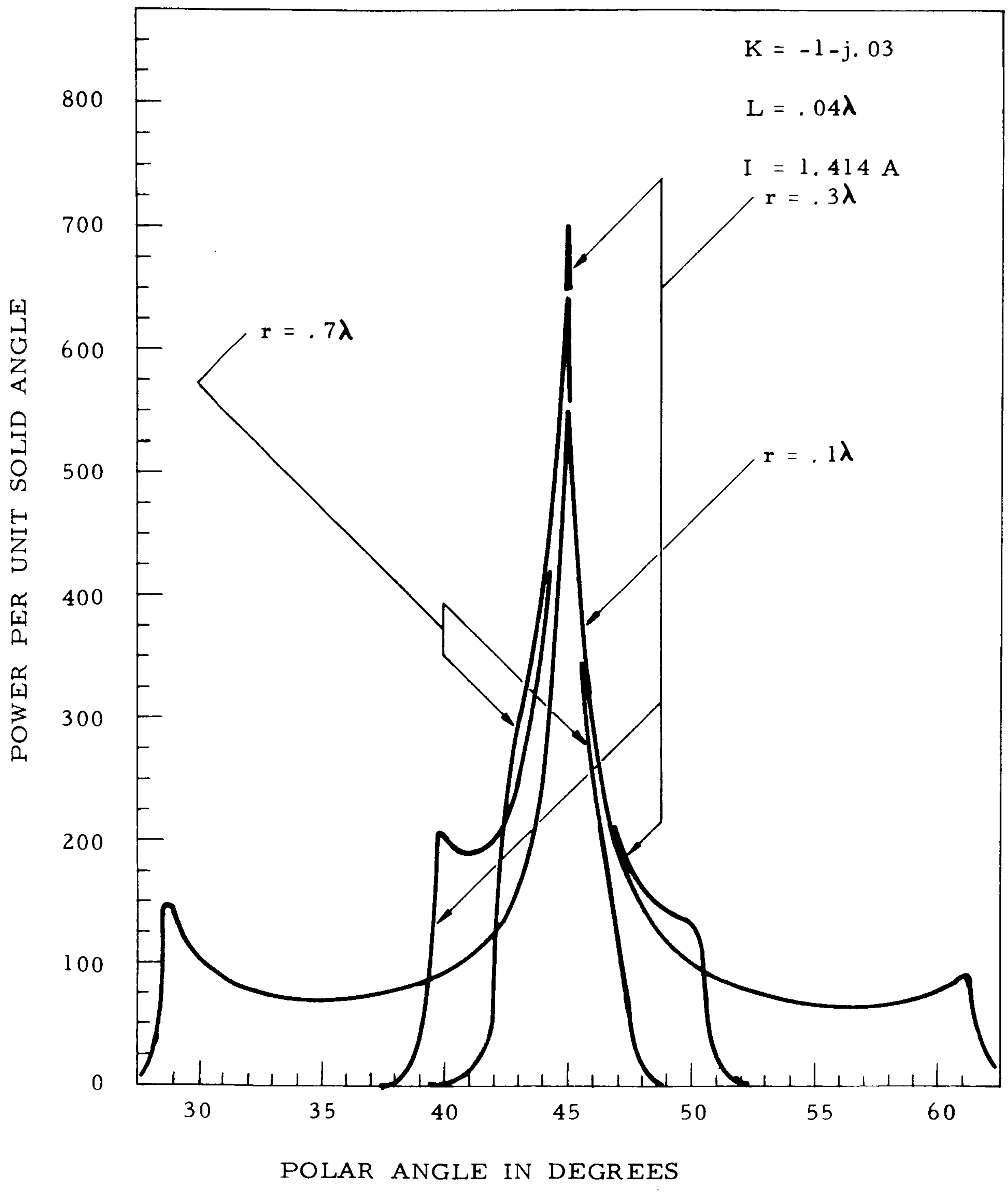


Figure 9 Power per Unit Solid Angle for L = . 04λ

4. DISCUSSION OF RESULTS AND CONCLUSIONS

The paradoxes of increasing radiation resistance with decreasing antenna length and of the infinite power radiated from the unit dipole, which occur in anisotropic media at frequencies below the plasma and gyroresonant frequencies, have received a great deal of attention in the literature. The two phenomena have been shown in section 1 to be related, and both also occur in uniaxial media.

Although on one hand there is not a great deal of application for uniaxial medium theory, the study of dipole radiation in this medium has been undertaken because this is the simplest of anisotropic media and hence the theory can be developed more fully, adding insight to the more complicated problems more closely related to the physical world. Mittra [1965] also pointed out that when solving problems related to short dipoles in the anisotropic media characterized by (1.2), the simplifying quasi-static approximations employed because the antenna is short give rise to fields which are independent of K''. One can thus comment that these assumptions are in many ways equivalent to the assumption that the medium is uniaxial. Therefore, many of the results obtained for the uniaxial medium carry over also to the more general anisotropic medium.

Several unphysical results occur, however, when the radiation resistance of a dipole with sinusoidal current distribution immersed in a uniaxial medium with K negative is investigated. First of all, the fields obtained are singular on three cones which extend to infinity. As pointed out, this casts serious doubt as to whether this solution is even valid. Furthermore, Figure 2 shows that if the "radiation resistance" is calculated anyway, it exhibits a curious L^{-1} behavior as L approaches zero.

To explain these results, one would then look for errors in the development. One possible error is that the validity of the scaling procedure has not been demonstrated for K real and negative. However, when K is slightly complex the fields are continuous and bounded so the validity of the fields is certain, and the scaling procedure is fully justified. But Figure 3 shows that the power radiating through a sphere of any fixed radius approaches the "power" obtained when K = -1 as the lossless limit is approached, showing that this cannot be a cause. The choice of a sinusoidal current distribution is not extremely critical either because Balmain [1964] still found the L^{-1} behavior for a "smoothed" current distribution for which the fields remained finite.

One must then conclude that the problem is not realistic in some other aspect. Actually, the problem is unrealistic in many ways, as brought out by several authors. Staras [1966] commented that the Appleton-Hartree tensor of (1.2) and (1.4) may not be appropriate to the problem because the antenna has negligible radius, violating a condition for the development of the tensor. Mittra [1965] pointed out that the infinite extent of the medium and the fact that plasma sheath effects have been ignored also make the problem unrealistic. Another defect, the one considered here, is that for any physical plasma there will be electron collisions and hence some loss. To what extent does the paradox exist when the medium is slightly lossy? The following paragraphs answer this question by discussing the power delivered by antennas with different lengths but the same driving point current for lossy media.

Figure 4 demonstrates that when there is some loss, as the length of the antenna decreases the power near the antenna exhibits the L^{-1} behavior of the lossless case. However, the power radiated through large spheres decreases as the antenna becomes shorter! In fact, scrutiny of Figure 4 would reveal that the shortest antenna is the <u>first</u> of the group to be radiating less power than the longest! It is also important to note that even when the loss tangent is as small as .03, as in Figure 4, this reversal in the amount of power obtained occurs in what must be considered the "near field." It was observed in computations not presented here that halving the loss tangent approximately doubled the distance where the power curves of two antennas crossed, so with any reasonable amount of loss this reversal in antenna effectiveness would occur at distances much less than those normally used between the transmitter and receiver of a communications system.

Even the L^{-1} behavior of the power travelling near the antenna is not completely unfamiliar, for consider the case of a short dipole of finite radius immersed in a lossy isotropic medium. Deschamps [1962] presented a method whereby the driving point impedance of an antenna immersed in a lossy medium could be obtained from its impedance (both resistance and reactance) in a lossless medium. The input resistance of a short dipole in an isotropic lossless medium has an $\omega^2 L^2$ behavior. As for the reactance, a quasi-static approach reveals that if the radius remains proportional to the length, the short antenna behaves as a capacitor whose size is proportional to L. The impedance, then, as a function of length and frequency for a lossless medium is approximately

$$Z(\omega, L) = K_1 \omega^2 L^2 - j \frac{K_2}{\omega L} , \qquad (4.1)$$

where K_1 and K_2 are positive constants.

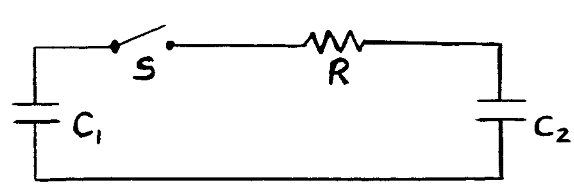
Application of Deschamps' procedure to (4.1) shows that in a medium with $\epsilon = \epsilon_0(1 - \delta)$ and $\mu = \mu_0$, the input resistance is approximately

$$R_{in}(\omega, L) = K_1 \omega^2 L^2 + \frac{K_2}{(1+\delta^2)\omega L} . \qquad (4.2)$$

This equation shows that the input resistance, and hence the power flowing near the antenna, has an L^{-1} behavior even in an isotropic lossy dielectric. Unfortunately, the similarity ends when the lossless limit is approached, because the isotropic case shows L^2 behavior for the radiation resistance whereas Figures 3 and 4 show that this is not the case for uniaxial media.

Returning again to the lossy uniaxial medium, Figure 4 shows that the power attenuates more rapidly for a short dipole than for a long one. The reason for this is not apparent from the field expressions (3.2) and (3.3), so computations of the power per unit solid angle were carried out and presented in Figures 6, 7, 8, and 9. These figures show that the power attenuates more rapidly for a shorter antenna in all directions, and that the power for shorter antennas tends to be concentrated near the characteristic cone emanating from the center of the antenna. The side-lobes apparent in Figure 9 result from power traveling along the other two characteristic cones from the top and bottom of the antenna, but this power decays (or redistributes) rapidly causing the sidelobes to disappear in the curves representing larger distances.

Therefore, it is apparent that when the medium is considered to be lossy, the unphysical results which occur when the medium is lossless largely disappear. This is not to say that the results obtained for K = -1 are valid, but rather that the simple model used is not appropriate for the lossless case. A similar situation exists in circuit theory when the circuit below is considered.



C_1 and C_2 are capacitors of equal value. C_1 is initially charged to a potential V, and C_2 is uncharged. When the switch is closed, current flows until ultimately the potential across each capacitor is V/2. However, if R=0 (lossless case) the above solution contradicts the law of conservation of energy, and analagously a paradox results. The usual explanation for this is that the model neglects the effects of radiation.

The conclusions for this study, then, are:

(1) The radiation resistance of a filamentary dipole immersed in a uniaxial medium with K positive is equal to its radiation resistance in vacuum.

(2) If the dipole is assumed to have a sinusoidal current distribution, the fields obtained for K negative are infinite along three characteristic cones, causing serious doubt as to the acceptability of these solutions. Similar remarks hold for other current distributions for which the fields are singular outside the source region.

(3) If the "radiation resistance" is computed for this case anyway, it is seen to be proportional to L^{-1} for short dipoles, creating the unphysical result that the "radiation resistance" increases with decreasing antenna length.

(4) One of the ways this dilemma may be resolved is to introduce a slight loss to the medium, in which case the behavior of the

power transmitted bears striking similarity to the case where the medium is isotropic and lossy; that is, increasing with decreasing length near the antenna but decreasing at large distances.

(5) The paradox of increasing "radiation resistance" with decreasing length for the lossless case can thus be viewed as a lossy medium input resistance phenomenon that has been carried over to the lossless limit because the model is inaccurate.

(6) Hence, the input resistance of a dipole antenna in a lossless anisotropic medium must not be used as a measure of its effectiveness in a communications system when the frequency of operation is to be below the plasma and gyroresonant frequencies. This is because the actual environment will have some loss, and although a shorter antenna will have a high input resistance, a longer antenna will actually transmit more power at large distances.

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