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CHAIN SAMPLING INSPECTION PLANS--ChSP-0,4 AND ChSP-1,4

by

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INTRODUCTION

This report extends the work on two-stage chain sampling inspection plans to those included in the subsets designated ChSP-0,4 and ChSP-1,4; i.e., plans with acceptance numbers of C_1, C_2 = 0,4 and 1,4. The format is the same as the preceding Technical Report No. N-22[1], which presented similar information on the subsets, ChSP-0,3 and ChSP-1,3. Earlier reports, Nos. N-20[2] and N-21[3], contain the general description of the plans and the theoretical development as well as results on the subsets, ChSP-0,1, and ChSP-0,2, ChSP-1,2, respectively.

EVALUATING OPERATING CHARACTERISTICS

In this section Markov chains are presented for the following five sampling plans selected from those that have been evaluated:

Plan	k_1,	k_2;	C_1,	C_2
(1)	1,	2;	0,	4
(2)	1,	2;	1,	4
(3)	1,	3;	0,	4
(4)	2,	3;	0,	4
(5)	2,	3;	1,	4

* This report is based in part on work being done in preparation of a doctoral thesis at Rutgers--The State University.

An algebraic solution for P_a, the proportion of lots expected to be accepted, is given for plan (1) by solving the Markov chain for the limiting probability of the rejection (R) state.

Plan (1): 1,2; 0,4

State at ith trial

State at (i-1)st trial	0	1	2	3	4	R
0	P_0	P_1	P_2	P_3	P_4	$1 - \sum_{i=0}^{4} P_i$
1	P_0	P_1	P_2	P_3	·	$1 - \sum_{i=0}^{3} P_i$
2	P_0	P_1	P_2	·	·	$1 - \sum_{i=0}^{2} P_i$
3	P_0	P_1	·	·	·	$1 - \sum_{i=0}^{1} P_i$
4	P_0	·	·	·	·	$1 - P_0$
R	P_0	·	·	·	·	$1 - P_0$

Fig. 1. Markov Chain for ChSP-0,4 Plan: $(k_1,k_2;\ C_1,C_2 = 1,2;\ 0,4)$

Proceeding as in former reports[1,2], the solution of the limiting probability of state "R" from which P_a is obtained is as follows:

$$P_0 + P_1 + P_2 + P_3 + P_4 + P_R = 1 \tag{1}$$

$$P_0 = P_0 \tag{2}$$

$$P_1 = P_1P_0 + P_1P_1 + P_1P_2 + P_1P_3 \tag{3}$$

$$P_2 = P_2P_0 + P_2P_1 + P_2P_2 \tag{4}$$

$$P_3 = P_3P_0 + P_3P_1 \tag{5}$$

$$\rho_4 = P_4\rho_0 \tag{6}$$

Combining (2), (3), (4), and (5) results in the following:

$$\rho_1 = \frac{P_0P_1+P_0P_1P_3-P_0P_1P_2P_3}{1-P_1-P_2-P_1P_3+P_1P_2P_3} \tag{7}$$

Combining (1), (3), (6), and (7) leads to,

$$\rho_R = 1 - P_0P_4 - \frac{P_0+P_0P_3-P_0P_2P_3}{1-P_1-P_2-P_1P_3+P_1P_2P_3} \tag{8}$$

from which we get,

$$\rho_a = (P_0+P_0P_3+P_0P_4+P_0P_1P_4+ P_0P_2P_3+ P_0P_2P_4+ P_0P_1P_3P_4-P_0P_1P_2P_3P_4)/ (1-P_1-P_2-P_1P_3+P_1P_2P_3) \tag{9}$$

Plan (2): 1,2; 1,4

State at ith trial

State at (i-1)st trial	0	1	2	3	4	R
0	P_0	P_1	P_2	P_3	P_4	$1 - \sum_{i=0}^{4} P_i$
1	P_0	P_1	P_2	P_3	·	$1 - \sum_{i=0}^{3} P_i$
2	P_0	P_1	P_2	·	·	$1 - \sum_{i=0}^{2} P_i$
3	P_0	P_1	·	·	·	$1 - \sum_{i=0}^{1} P_i$
4	P_0	·	·	·	·	$1 - P_0$
R	P_0	P_1	·	·	·	$1 - \sum_{i=0}^{1} P_i$

Fig. 2. Markov Chain for ChSP-1,4 Plan: $(k_1,k_2;\ C_1,C_2 = 1,2;\ 1,4)$

Plan (3): 1,3; 0,4

State at ith trial

State at (i-1)st trial	00	01	10	11	02	20	12	21	22	03	30	13	31	04	40	R0	R
00	P_0	P_1	·	·	P_2	·	·	·	·	P_3	·	·	·	P_4	·	·	$1-\sum_{i=0}^{4}P_i$
01	·	·	P_0	P_1	·	·	P_2	·	·	·	·	P_3	·	·	·	·	$1-\sum_{i=0}^{3}P_i$
10	P_0	P_1	·	·	P_2	·	·	·	·	P_3	·	·	·	·	·	·	$1-\sum_{i=0}^{3}P_i$
11	·	·	P_0	P_1	·	·	P_2	·	·	·	·	·	·	·	·	·	$1-\sum_{i=0}^{2}P_i$
02	·	·	·	·	·	P_0	·	P_1	P_2	·	·	·	·	·	·	·	$1-\sum_{i=0}^{2}P_i$
20	P_0	P_1	·	·	P_2	·	·	·	·	·	·	·	·	·	·	·	$1-\sum_{i=0}^{2}P_i$
12	·	·	·	·	·	P_0	·	P_1	·	·	·	·	·	·	·	·	$1-\sum_{i=0}^{1}P_i$
21	·	·	P_0	P_1	·	·	·	·	·	·	·	·	·	·	·	·	$1-\sum_{i=0}^{1}P_i$
22	·	·	·	·	·	P_0	·	·	·	·	·	·	·	·	·	·	$1-P_0$
03	·	·	·	·	·	·	·	·	·	·	P_0	·	P_1	·	·	·	$1-\sum_{i=0}^{1}P_i$
30	P_0	P_1	·	·	·	·	·	·	·	·	·	·	·	·	·	·	$1-\sum_{i=0}^{1}P_i$
13	·	·	·	·	·	·	·	·	·	·	P_0	·	·	·	·	·	$1-P_0$
31	·	·	P_0	·	·	·	·	·	·	·	·	·	·	·	·	·	$1-P_0$
04	·	·	·	·	·	·	·	·	·	·	·	·	·	·	P_0	·	$1-P_0$
40	P_0	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	$1-P_0$
R0	P_0	P_1	·	·	P_2	·	·	·	·	P_3	·	·	·	P_4	·	·	$1-\sum_{i=0}^{4}P_i$
R	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	P_0	$1-P_0$

Fig. 3. Markov Chain for ChSP-0,4 Plan: $(k_1,k_2;\ C_1,C_2 = 1,3;\ 0,4)$

Plan (4): 2,3,; 0,4

State at ith trial

State at (i-1)st trial	00	01	10	11	02	20	12	21	22	03	30	13	31	04	40	R0	R
00	P_0	P_1	·	·	P_2	·	·	·	·	P_3	·	·	·	P_4	·	·	$1 - \sum_{i=0}^{4} P_i$
01	·	·	P_0	P_1	·	·	P_2	·	·	·	·	P_3	·	·	·	·	$1 - \sum_{i=0}^{3} P_i$
10	P_0	P_1	·	·	P_2	·	·	·	·	P_3	·	·	·	·	·	·	$1 - \sum_{i=0}^{3} P_i$
11	·	·	P_0	P_1	·	·	P_2	·	·	·	·	·	·	·	·	·	$1 - \sum_{i=0}^{2} P_i$
02	·	·	·	·	·	P_0	·	P_1	P_2	·	·	·	·	·	·	·	$1 - \sum_{i=0}^{2} P_i$
20	P_0	P_1	·	·	P_2	·	·	·	·	·	·	·	·	·	·	·	$1 - \sum_{i=0}^{2} P_i$
12	·	·	·	·	·	P_0	·	P_1	·	·	·	·	·	·	·	·	$1 - \sum_{i=0}^{1} P_i$
21	·	·	P_0	P_1	·	·	·	·	·	·	·	·	·	·	·	·	$1 - \sum_{i=0}^{1} P_i$
22	·	·	·	·	·	P_0	·	·	·	·	·	·	·	·	·	·	$1 - P_0$
03	·	·	·	·	·	·	·	·	·	·	P_0	·	P_1	·	·	·	$1 - \sum_{i=0}^{1} P_i$
30	P_0	P_1	·	·	·	·	·	·	·	·	·	·	·	·	·	·	$1 - \sum_{i=0}^{1} P_i$
13	·	·	·	·	·	·	·	·	·	·	P_0	·	·	·	·	·	$1 - P_0$
31	·	·	P_0	·	·	·	·	·	·	·	·	·	·	·	·	·	$1 - P_0$
04	·	·	·	·	·	·	·	·	·	·	·	·	·	·	P_0	·	$1 - P_0$
40	P_0	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	$1 - P_0$
R0	P_0	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	$1 - P_0$
R	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	P_0	$1 - P_0$

Fig. 4. Markov Chain for ChSP-0,4 Plan: $(k_1,k_2;\ C_1,C_2 = 2,3;\ 0,4)$

Plan (5): 2,3; 1,4

State at ith trial

State at (i-1)st trial	00	01	10	11	02	20	12	21	22	03	30	13	31	04	40	R0	R1	R
00	P_0	P_1	·	·	P_2	·	·	·	·	P_3	·	·	·	P_4	·	·	·	$1-\sum_{i=0}^{4}P_i$
01	·	·	P_0	P_1	·	·	P_2	·	·	·	·	P_3	·	·	·	·	·	$1-\sum_{i=0}^{3}P_i$
10	P_0	P_1	·	·	P_2	·	·	·	·	P_3	·	·	·	·	·	·	·	$1-\sum_{i=0}^{3}P_i$
11	·	·	P_0	P_1	·	·	P_2	·	·	·	·	·	·	·	·	·	·	$1-\sum_{i=0}^{2}P_i$
02	·	·	·	·	·	P_0	·	P_1	P_2	·	·	·	·	·	·	·	·	$1-\sum_{i=0}^{2}P_i$
20	P_0	P_1	·	·	P_2	·	·	·	·	·	·	·	·	·	·	·	·	$1-\sum_{i=0}^{2}P_i$
12	·	·	·	·	·	P_0	·	P_1	·	·	·	·	·	·	·	·	·	$1-\sum_{i=0}^{1}P_i$
21	·	·	P_0	P_1	·	·	·	·	·	·	·	·	·	·	·	·	·	$1-\sum_{i=0}^{1}P_i$
22	·	·	·	·	·	P_0	·	·	·	·	·	·	·	·	·	·	·	$1-P_0$
03	·	·	·	·	·	·	·	·	·	·	P_0	·	P_1	·	·	·	·	$1-\sum_{i=0}^{1}P_i$
30	P_0	P_1	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	$1-\sum_{i=0}^{1}P_i$
13	·	·	·	·	·	·	·	·	·	·	P_0	·	·	·	·	·	·	$1-P_0$
31	·	·	P_0	·	·	·	·	·	·	·	·	·	·	·	·	·	·	$1-P_0$
04	·	·	·	·	·	·	·	·	·	·	·	·	·	·	P_0	·	·	$1-P_0$
40	P_0	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	$1-P_0$
R0	P_0	P_1	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	$1-\sum_{i=0}^{1}P_i$
R1	·	·	P_0	·	·	·	·	·	·	·	·	·	·	·	·	·	·	$1-P_0$
R	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	P_0	P_1	$1-\sum_{i=0}^{1}P_i$

Fig. 5. Markov Chain for ChSP-1,4 Plan: $(k_1,k_2;\ C_1,C_2 = 2,3;\ 1,4)$

DISCUSSION OF OPERATING CHARACTERISTICS

Again, as in N-22[1], operating characteristics for four different sample sizes (n = 10,20,50 and 100) for a wide range of ChSP-0,4 and ChSP-1,4 sampling plans are presented in the Appendix . The OC curves for each sample size are presented on a single page. Each of Figs. 6,7,8 and 9 contains three sets of OC curves, one for each of three values of k_2(k_2 = 2,3 and 5). The OC curves are of Type B*, based on probabilities of sampling from a process.

The OC curves for ChSP-0,4 and ChSP-1,4 are completely analogous to those of ChSP-0,3 and ChSP-1,3 as well as ChSP-0,2 and ChSP-1,2 presented in Technical Report Nos. N-22[1] and N-21[2] respectively. Again there are close similarities in the effects of changes in the parameters k_1, k_2, and n; hence they will only be summarized here.

Effect of k_1

This is shown for each k_2 within each of the individual charts e.g., Figs. 6.1, 6.2 and 6.3 for n = 10. The range in k_1 is normally from 0 to k_2-1. However the curve for k_1 = 0 has been shown for k_2 = 2 only. These curves consistently result in poor discrimination between good and bad quality. The k_1 = 1, k_2 = 5 curve for C_1 = 1, C_2 = 4 has also been omitted. Increasing k_1 has the effect of tightening (i.e. lowering) the OC curves, again more so for ChSP-0,4 than for ChSP-1,4.

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* See Reference (4), pp. 56-60.

Effect of k_2

This effect is shown between the three charts on a single page for each n, e.g. between Figs. 6.1, 6.2 and 6.3 for n = 10. Three values of k_2 i.e., k_2 = 2,3 and 5 are shown. In general, increasing k_2 also has the effect of tightening the OC curves.

Effect of n

Again as in ChSP-0,2 and 1,2 as well as ChSP-0,3 and 1,3, there is a close similarity in the patterns of the ChSP-0,4 and 1,4 plans for different sample sizes as can be seen among Figs. 6, 7, 8 and 9. In presenting the OC curves for n = 100 in Fig. 9, a double horizontal scale, fraction defective, p, and number defective, pn, has again been used. For binomial sampling these OC curves provide suitable approximations for sample sizes of n = 50 and larger by using the lower or pn scale. For Poisson sampling Fig. 9 is applicable for all sample sizes.

COMPARISON OF ChSP-0,4, ChSP-1,4 AND OTHER ChSP PLANS

On each of the charts of the four figures are shown the ChSP-0,3, ChSP-0,2 and ChSP-0,1 plans having the stated chart-values of k_2 and having $k_1 = k_2 - 1$. In each case these curves are shown by solid lines and are the last three numbered curves of each set, ChSP-0,3, ChSP-0,2 and ChSP-0,1 respectively. Also shown on each chart, with a thin dashed line, is the OC curve for the single sampling plan: given sample size, n, and c = 0. This is again shown for reference purposes.

These new charts indicate that the ChSP-0,4 plans are even more effective in adding a swelling on the underlying OC curve of the c = 0 plan than the corresponding 0,3, 0,2 and 0,1 plans. How this swelling increases by going successively from a 0,1 to a 0,2 and to a 0,3 and now to a 0,4 plan,

can be seen, for example on each chart by comparing the plans having $k_1 = k_2 - 1$. The improvement in the "operating ratio," O.R., is again roughly comparable to that obtained in single sampling by going to a larger acceptance number, c; in this case from c = 1, c = 2 and c = 3 to c = 4.

The relationship between ChSP-1,4 and ChSP-1,3 (See Reference 1) is shown graphically for one case only--n = 100, $k_1 = 1$, $k_2 = 2$ in Fig. 9.1.

REFERENCES

1. Stephens, K. S. and H. F. Dodge, "Chain Sampling Inspection Plans--ChSP-0,3 and ChSP-1,3, "Rutgers University Statistics Center, Technical Report No. N-22, December, 1965.

2. Dodge, H. F. and K. S. Stephens, "A General Family of Chain Sampling Inspection Plans," Rutgers University Statistics Center, Technical Report No. N-20, December 30, 1964.

3. Stephens, K. S. and H. F. Dodge, "Chain Sampling Inspection Plans--ChSP-0,2 and ChSP-1,2," Rutgers University Statistics Center, Technical Report No. N-21, May, 1965.

4. Dodge, H. F. and H. G. Romig, Sampling Inspection Tables, John Wiley and Sons, Inc., New York, 2nd Edition, 1959.

Appendix: Operating Characteristic Curves for ChSP-0,4 and ChSP-1,4 Plans

OC curves for the following chain sampling plans are presented here.

Figs. 6-9 give OC curves for sample sizes, 10, 20, 50 and 100 respectively.

Figs. 6.1-9.1				Figs. 6.2-9.2				Figs. 6.3-9.3			
k_1	k_2	C_1	C_2	k_1	k_2	C_1	C_2	k_1	k_2	C_1	C_2
0	2	0	4	1	3	1	4	2	5	1	4
1	2	1	4	2	3	1	4	4	5	1	4
1	2	1	3*	1	3	0	4	1	5	0	4
1	2	0	4	2	3	0	4	2	5	0	4
1	2	0	3**	2	3	0	3**	4	5	0	4
1	2	0	2**	2	3	0	2**	4	5	0	3**
1	2	0	1**	2	3	0	1**	4	5	0	2**
								4	5	0	1**

* This ChSP-1,3 plan is presented in Fig. 9.1 only for n = 100 in order to show the typical effect of changing from a 1,3 to a 1,4 plan.

** These ChSP-0,3, 0,2 and 0,1 plans along with the single sampling plans for c = 0 are presented for comparison with the ChSP-0,4 and ChSP-1,4 plans.

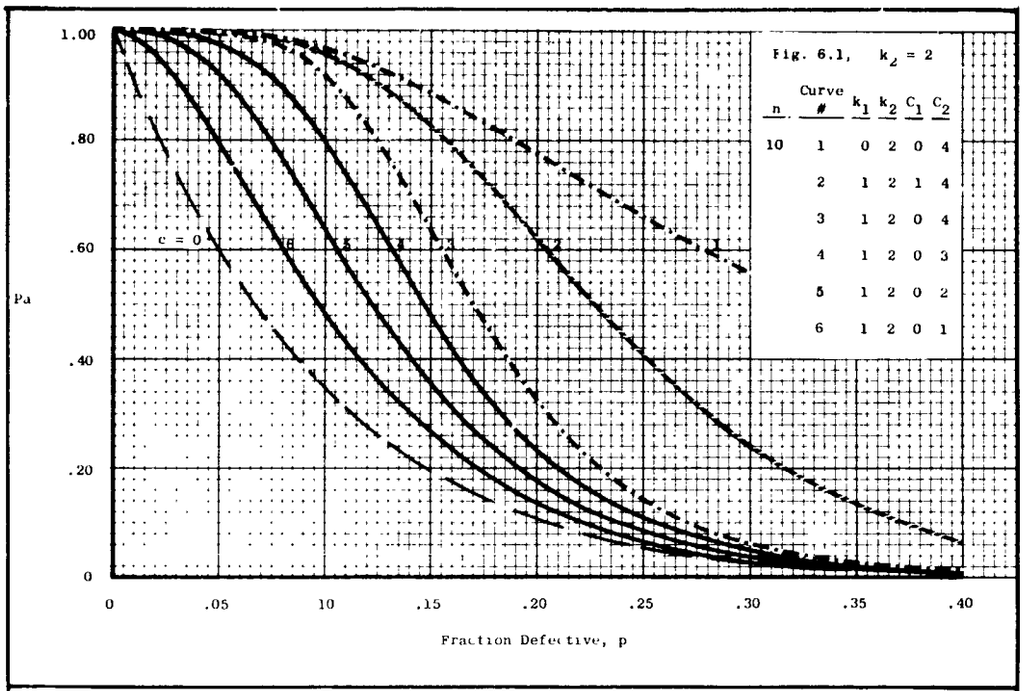


Fig. 6.1, $k_2 = 2$

n	Curve #	k_1	k_2	C_1	C_2
10	1	0	2	0	4
	2	1	2	1	4
	3	1	2	0	4
	4	1	2	0	3
	5	1	2	0	2
	6	1	2	0	1

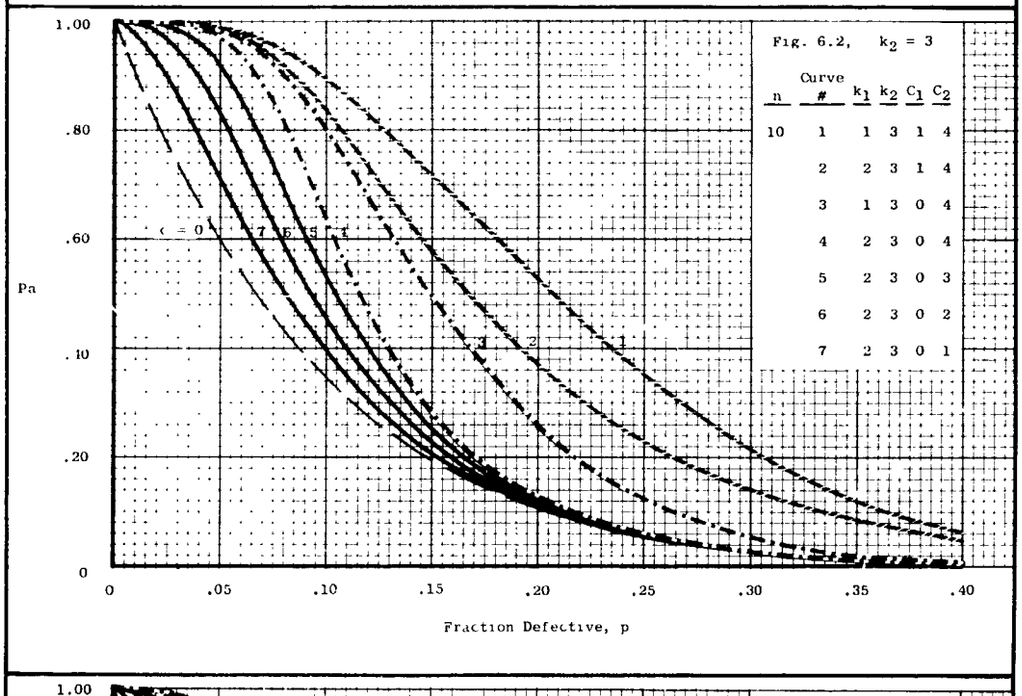


Fig. 6.2, $k_2 = 3$

n	Curve #	k_1	k_2	C_1	C_2
10	1	1	3	1	4
	2	2	3	1	4
	3	1	3	0	4
	4	2	3	0	4
	5	2	3	0	3
	6	2	3	0	2
	7	2	3	0	1

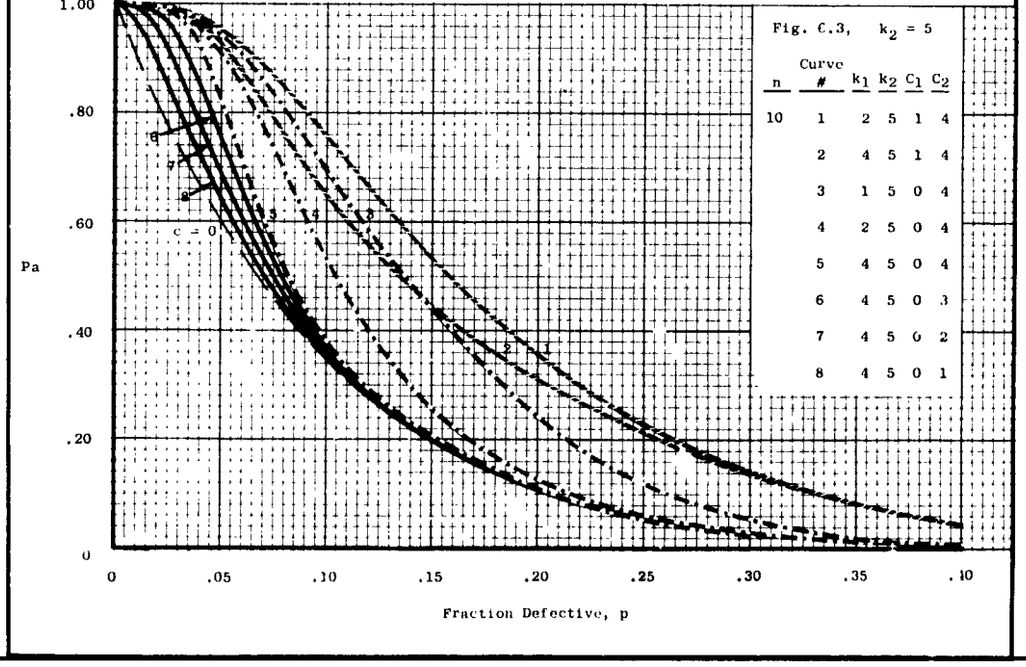
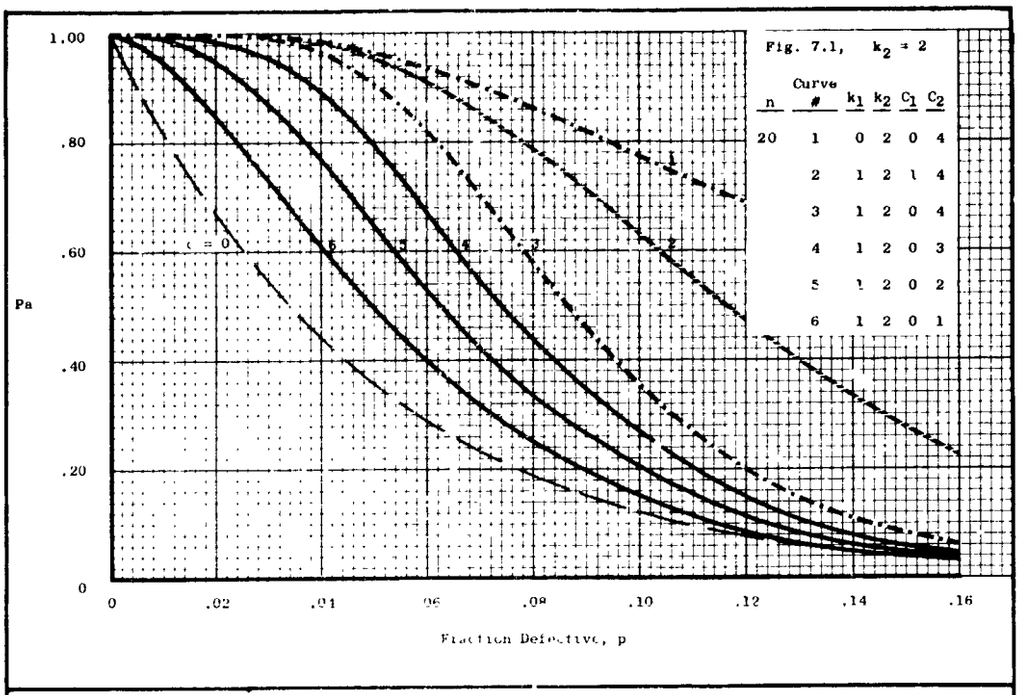
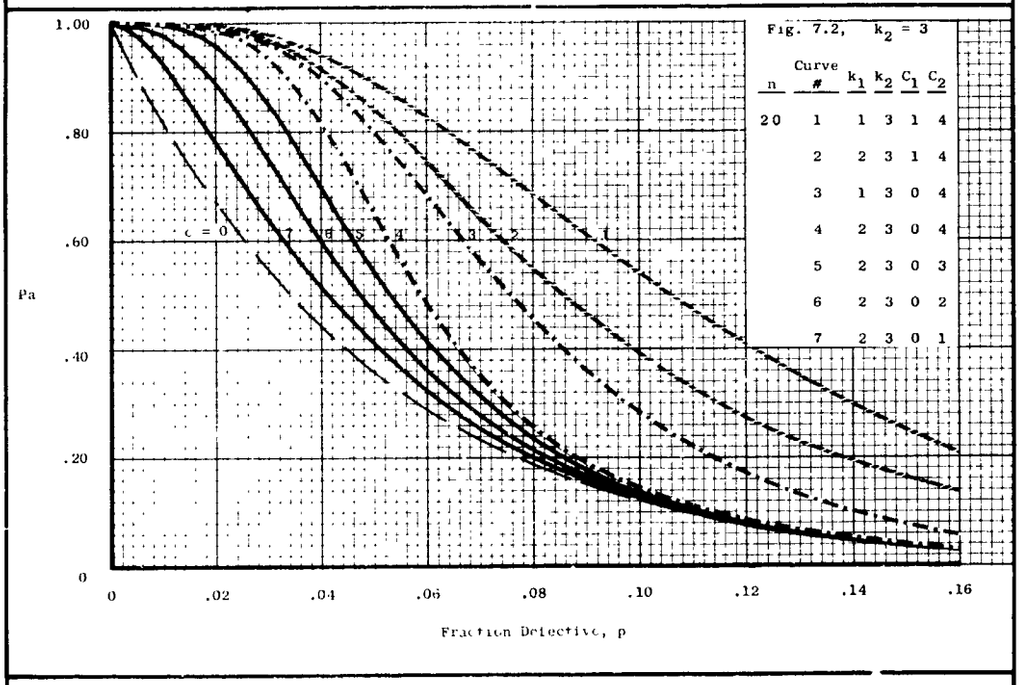


Fig. 6.3, $k_2 = 5$

n	Curve #	k_1	k_2	C_1	C_2
10	1	2	5	1	4
	2	4	5	1	4
	3	1	5	0	4
	4	2	5	0	4
	5	4	5	0	4
	6	4	5	0	3
	7	4	5	0	2
	8	4	5	0	1





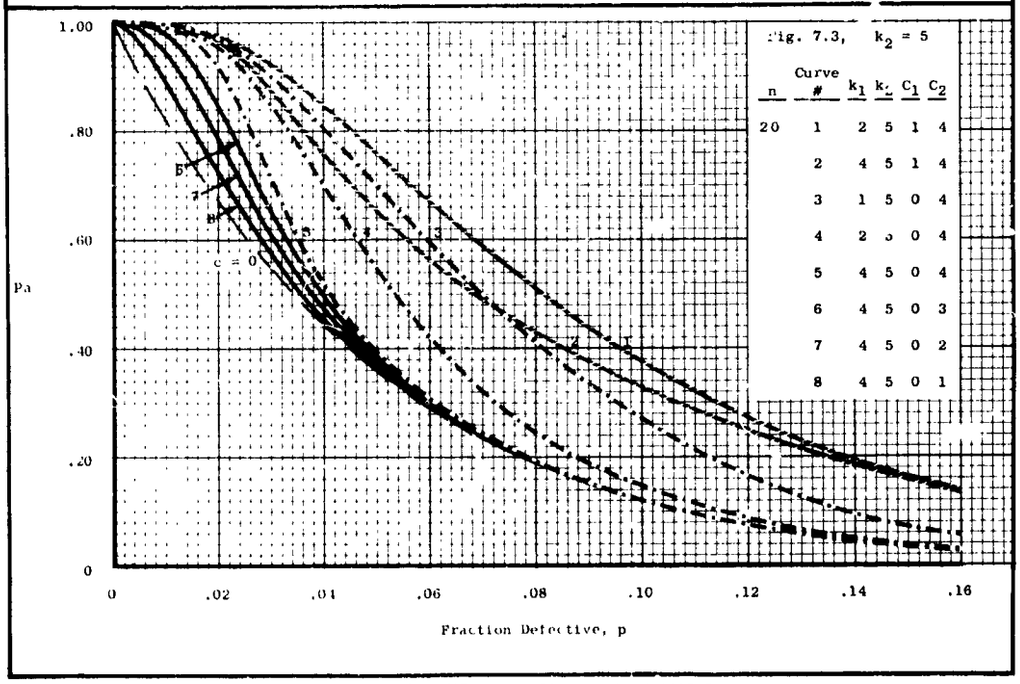


FIG. 7 OC CURVES--- n - 20, k_2 = 2, 3 AND 5

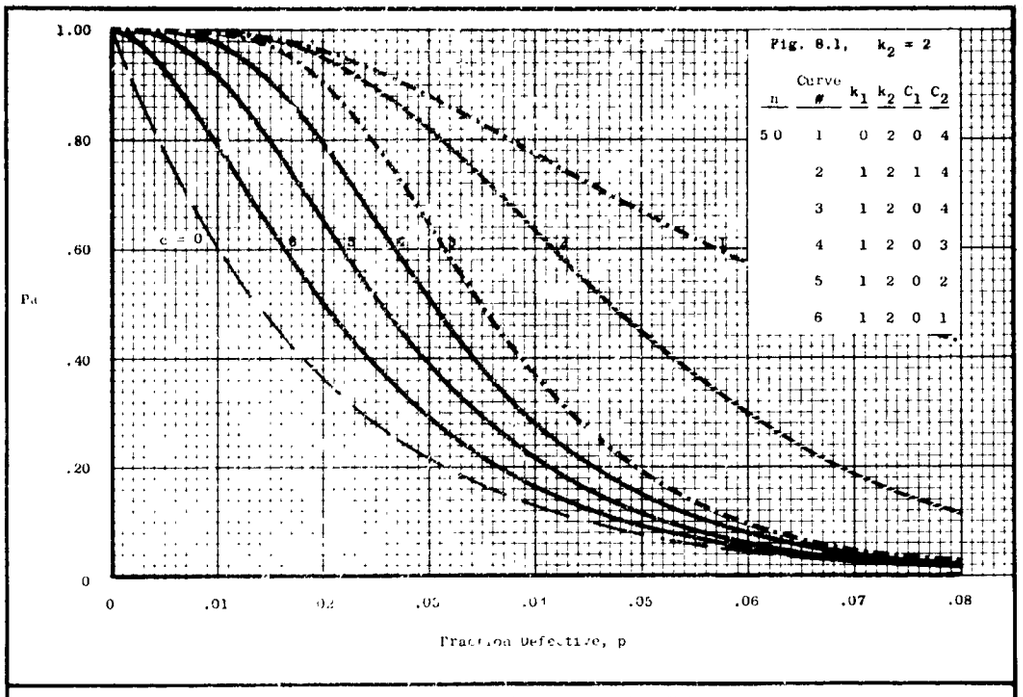


Fig. 8.1, $k_2 = 2$

n	Curve #	k_1	k_2	C_1	C_2
50	1	0	2	0	4
	2	1	2	1	4
	3	1	2	0	4
	4	1	2	0	3
	5	1	2	0	2
	6	1	2	0	1

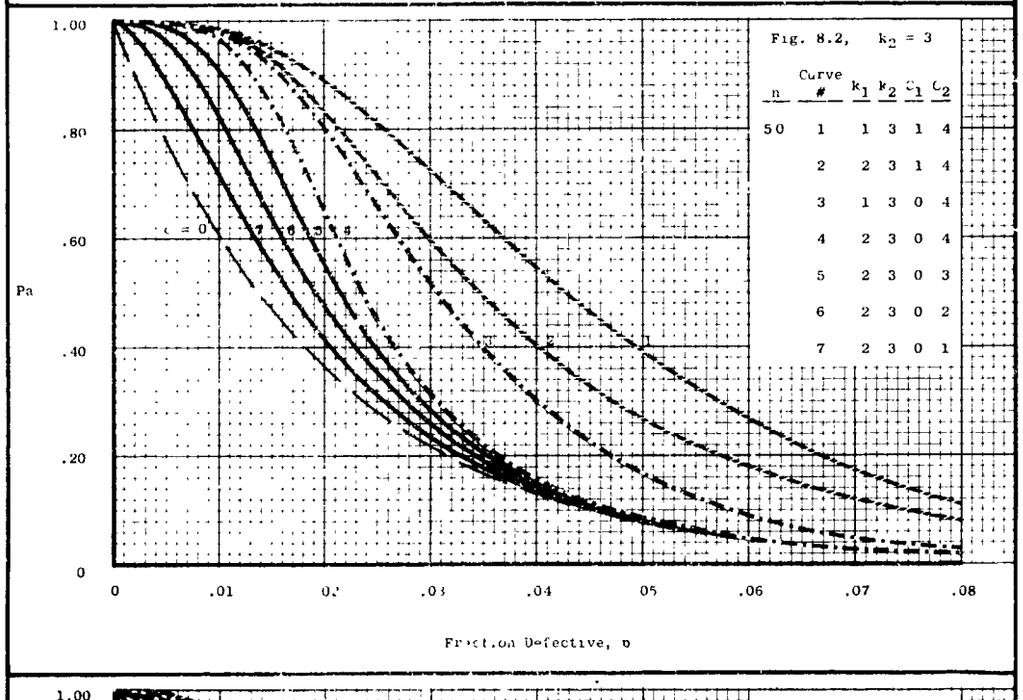


Fig. 8.2, $k_2 = 3$

n	Curve #	k_1	k_2	C_1	C_2
50	1	1	3	1	4
	2	2	3	1	4
	3	1	3	0	4
	4	2	3	0	4
	5	2	3	0	3
	6	2	3	0	2
	7	2	3	0	1

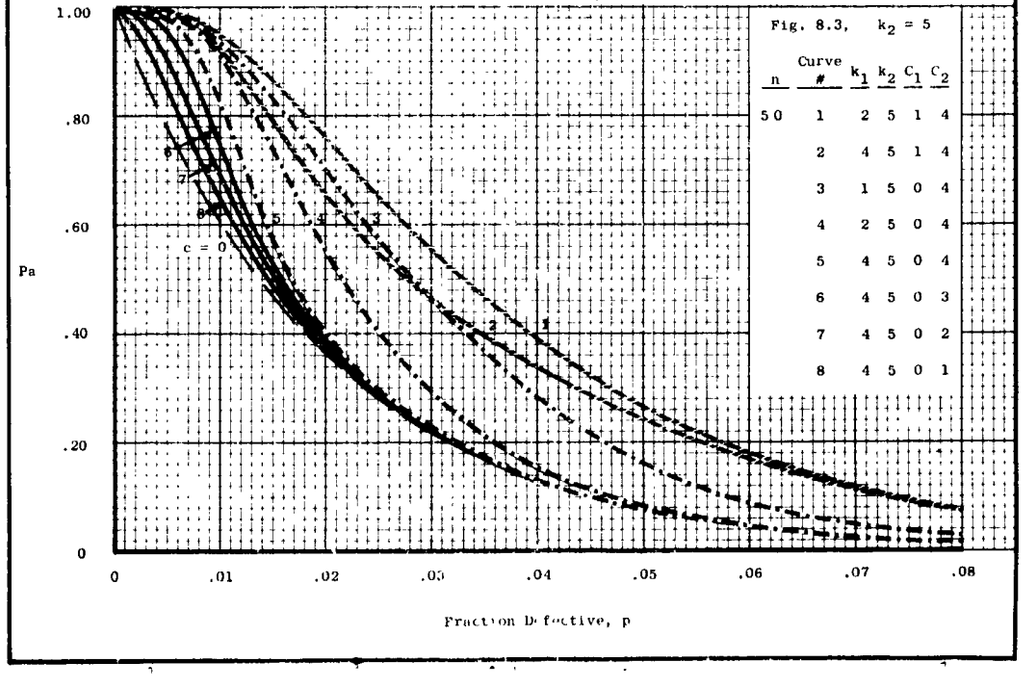
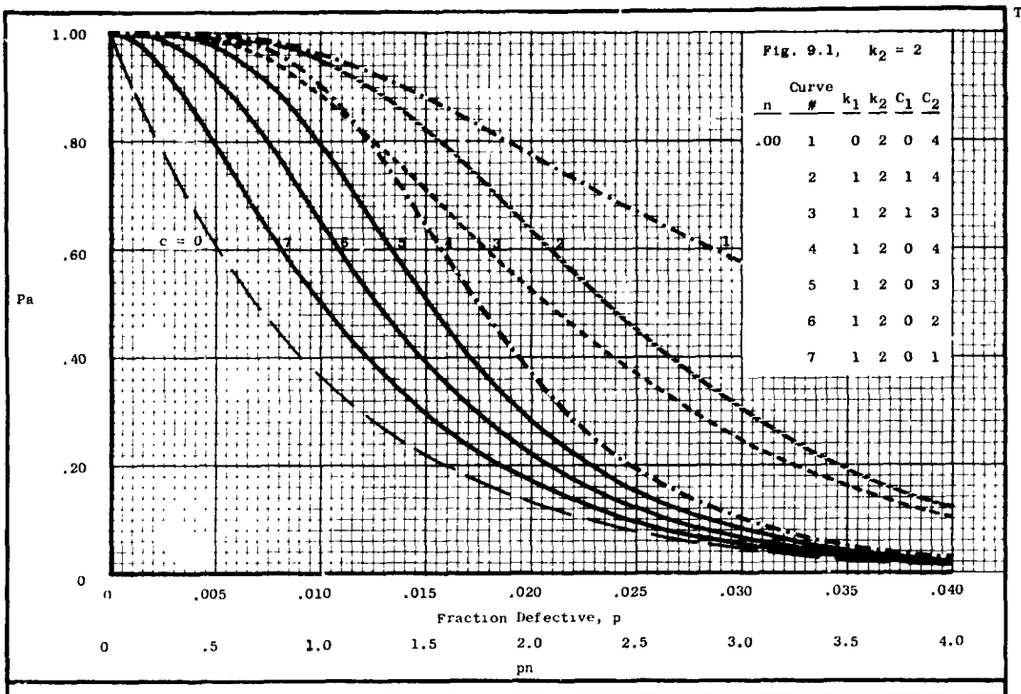
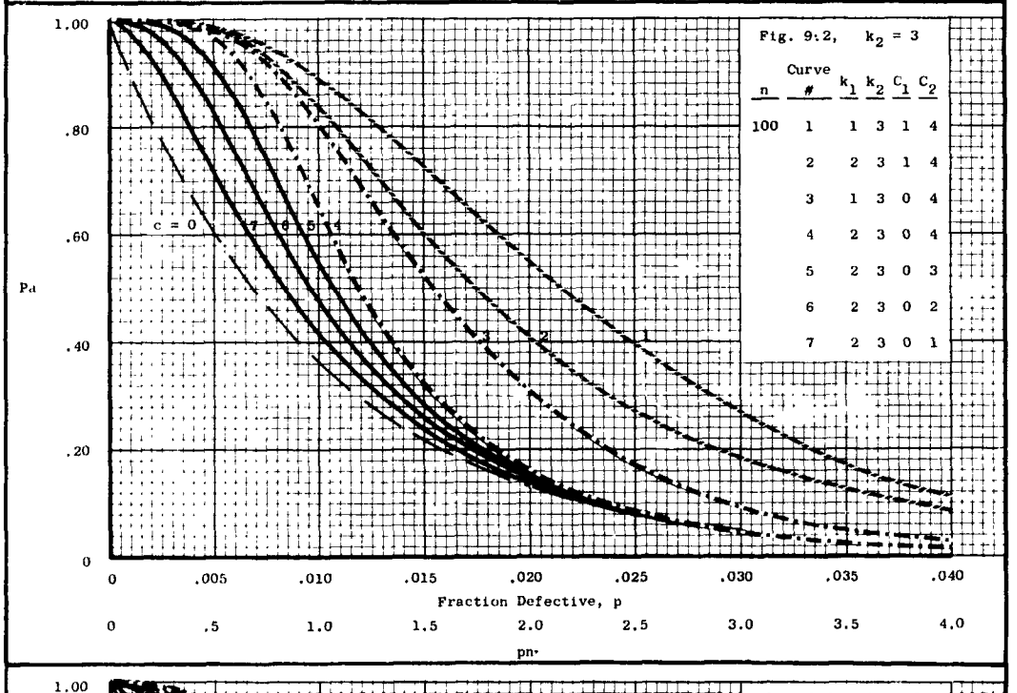


Fig. 8.3, $k_2 = 5$

n	Curve #	k_1	k_2	C_1	C_2
50	1	2	5	1	4
	2	4	5	1	4
	3	1	5	0	4
	4	2	5	0	4
	5	4	5	0	4
	6	4	5	0	3
	7	4	5	0	2
	8	4	5	0	1





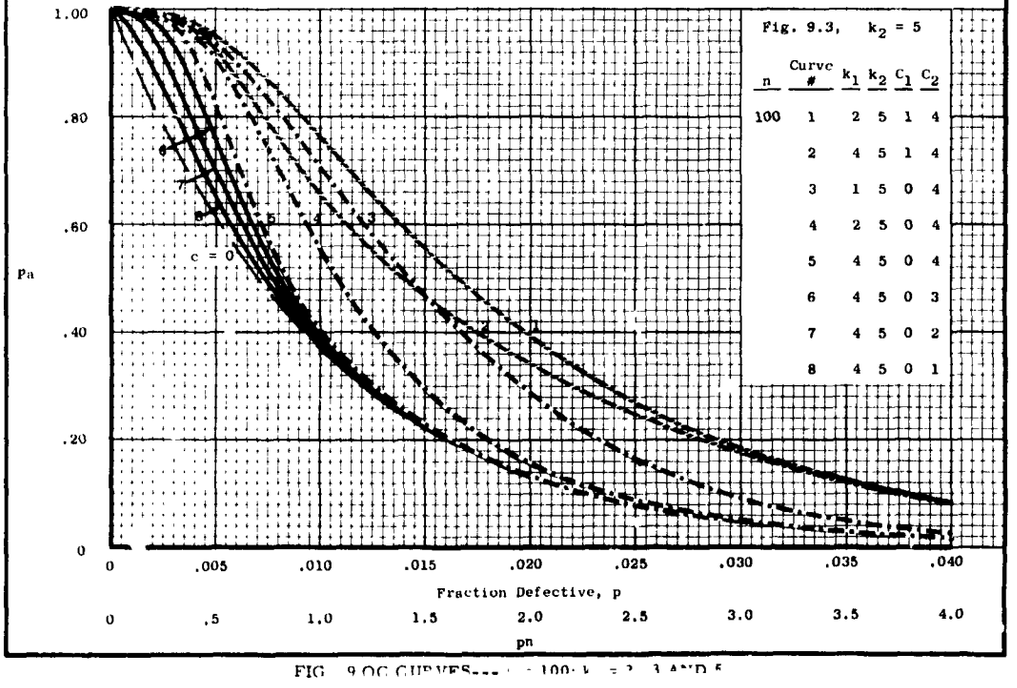


FIG 9 OC CURVES--- n = 100; k_2 = 2, 3 AND 5