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SUMMARY REPORT

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AN ANALYTICAL APPROACH TO SOLUTION OF TWO-POINT BOUNDARY CONDITION PROBLEMS IN OPTIMAL GUIDANCE

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SUMMARY

This report summarizes accomplishments under Contract NASW-1165, "Analytical Research in Guidance Theory", for the period of May 1965 to April 1966. It is intended to be a complete, self-standing document. Therefore, material is repeated here which can be found in a previous Northrop report, "Analytical and Mathematical Studies for Direct Solutions for Path-Adaptive Guidance Functions" (ref. 1). For those familiar with that report and nomenclature, or the guidance problem itself, the first three sections of this report may be ignored.

Subsequent sections of this report contain detailed discussions of findings and related conclusions and suggestions. Major conclusions derived from the study may be summarized as follows:

- This analytical approach to optimal guidance functions has been verified as mathematically feasible. Answers obtained from this approach agree with those from numerical methods.
- An analytical solution (quasi-closed form) of the nonlinear algebraic equations in the Lagrange multipliers is possible.
- The guidance functions which result from the analytical approach can be rapidly evaluated; with due consideration of their (present) complexity. Time required on the IBM 7094 computer is only a few seconds.
- The elimination of insignificant or non-essential terms in the analytical formulas is difficult and time consuming. What can be dropped from the formulas for one case cannot always be dropped for another. This study area will require much more effort to set down standards for the elimination of insignificant terms.

- The critical expression for time remaining to cutoff is best derived from the final velocity condition specified for the trajectory. A series of a relatively large number of terms is needed for flights of over 300 seconds or for large altitude changes. Increasing the number of terms in this series causes a large amplification of the complexity of the guidance function.

- The availability of fairly reliable and easy to use computer languages for algebraic manipulations and analytical differentiation contributes to the feasible undertaking of a large-scale analytical approach to problems such as this. On a long-term basis, human blunders are minimized, thus saving thousands of man-hours for worthwhile actitivies. However, there are still many shortcomings in the use of these languages.

During the period of performance reported, several presentations of progress and results were made to the NASA Electronics Research Center at Cambridge, Massachusetts, and Headquarters, NASA, at Washington, D. C. A paper was also presented at the Third Aerospace Sciences Meeting of the American Institute of Aeronautics and Astronautics in New York in January 1966 (ref. 2).

Work currently in progress is primarily a continuation and refinement of what is reported herein. No major extensions of this approach to other cases are contemplated for the next few months.

SECTION I

PATH-ADAPTIVE GUIDANCE CONCEPT

For a rocket vehicle to be steered optimally from some initial point to some prescribed terminal condition requires that there be a means to determine the thrust direction (and possibly magnitude) such that the vehicle will reach the desired terminal condition by a path which makes some quantity an extremum. In cases of powered flight for transferring from some specified point to an orbit, it is desired to use a minimum of propellant.

"Path-Adaptive" guidance has been proposed as a general and versatile approach to optimal space flight guidance (refs. 4 and 5). The Path-Adaptive concept is quite simply stated: The local state of the vehicle is sensed at some instant during flight, and based on this information an optimal steering direction is computed. The vehicle is continually adapting according to the local "environment" by seeking the optimal path from its present conditions to the terminal conditions. The "environment" or local state can be described by the state variables of position, velocity, etc.

No attempt is made to return to any standard or reference trajectory. A new path is determined solely on the local state and required terminal conditions, which implies that the optimal guidance problems must be repeatedly solved during powered flight. Because these computations take place on-board the vehicle, it is desirable to have <u>general solutions</u> for the optimal guidance which are amenable to implementation with on-board computers.

If one is willing to trade precision and flexibility for "immediate" results, there are solutions available. One such solution comes from a purely empirical approach. For a given class of missions (trajectories and vehicles), a family of numerical solutions to the guidance problem is obtained. These solutions are then synthesized into a functional model which is supposed to characterize the usual behavior of that family of solutions. The model is based on "curve-fitting" procedures and results are valid only for cases near those from which the solution is constructed. This approach is discussed in references 6 and 7.

Another approach is to make assumptions about the physical aspects of the guidance problem such as "flat earth" and "small range angle". Under these restrictions, solutions to the guidance problem come about more easily. The approximations are then improved or upgraded by various schemes to yield approximate solutions.

However, it is implicit in the Path-Adaptive concept that a general approach be used, and solutions should be obtained which are free of approximations and restrictions.

This report summarizes the results obtained to date of an investigation of an analytical approach to Path-Adaptive guidance functions. The particular problem considered has been relatively simple, yet realistic. The mathematical approach is entirely analytical and without any gross approximations. The results shown are quite general and are a first step toward the solution of more complicated optimal guidance problems.

SECTION II

AN ANALYTICAL APPROACH TO OPTIMAL GUIDANCE FUNCTIONS

2.1 CALCULUS OF VARIATIONS AS THE OPTIMIZATION METHOD

The optimal guidance problem to be considered can be stated as follows: "Given a single-stage vehicle at some initial position and velocity, find the path which transfers the vehicle to a specified terminal condition and which requires a minimum amount of propellant".

Use of the calculus of variations does not lead directly to a solution; instead, a set of necessary conditions to be satisfied by the solution is developed. Among these necessary conditions is a set of differential equations, known as the Euler-Lagrange equations, which are related to the equations of motion for the vehicle. The control variables are those which may be deliberately changed during flight to control the vehicle's performance and thrust direction. In addition to these conditions there are others which are related to the specified initial and terminal conditions on the flight and are known as transversality conditions. The sets of differential equations involve the state variables and control variables.

2.2 THE TWO-POINT BOUNDARY CONDITION PROBLEM

The guidance problem is now expressed as a set of necessary conditions to be satisfied by the unknown guidance function. The control variable (the angle associated with thrust direction, for example) will appear (implicitly) in the differential equations. The equations can be integrated, subject to the boundary conditions, to obtain a value for the control variable. In such cases the differential equations are subject to boundary conditions at both the initial and final values of the independent variable. Usually some of the initial or final values of the control variables are not known and must be guessed. A trial integration of the differential equations is then carried out until the boundary conditions are approximately satisfied. The integration is then repeated, using adjusted guesses, until the boundary conditions are satisfied within some required tolerance. This procedure is, of course, numerical and is sometimes called "shooting" which probably best describes the process.

2.3 AN ANALYTICAL APPROACH TO SOLUTION

The approach described above is essentially a trial-and-error method designed to compute numerical solutions. Path-Adaptive guidance demands answers repeatedly during the flight. The time required to carry out the numerical solution to the two-point boundary value problem is too slow for purposes of guidance, even if there is assurance of convergence within a reasonable number of iterations. It is therefore desirable to have an analytical expression for the guidance function which can be evaluated at any time from measured values for the state variables.

The analytical approach used was to expand the terminal conditions of the flight in Taylor series about an interval of time, which is the flight time. The coefficients of these series may then be evaluated in terms of the known values of the state variables at the initial time. The differential equations are substituted into the coefficients of the Taylor series to form an algebraic system of equations from which the control variable may be obtained. The solution of these algebraic equations for the control variable is in terms of the coefficients of the series expansions for the terminal conditions, and these coefficients are in terms of quantities which can be measured at the initial time.

The details of this analytical approach are more clearly described in Section III in which the approach is used to solve for the guidance function for optimal ascent to circular orbit.

SECTION III

AN ANALYTICAL APPROACH TO OPTIMAL ASCENT TO CIRCULAR ORBIT

3.1 PROBLEM FORMULATION

The problem may be stated as follows: "For a single-stage rocket vehicle at some initial position and velocity, what is the thrust direction required to transfer it from that point to a circular orbit, such that the propellant expended is a minimum?" The assumptions regarding the vehicle and its flight are itemized as follows:

- Out of atmosphere flight
- Non-rotating, spherical earth
- Thrust magnitude and mass flow rate are constant
- Trajectory and final circular orbit are coplanar.

The equations of motion for the vehicle are, in first-order form:

$$\dot{u} = \frac{F}{m} \sin\chi - V_x \qquad (1)$$

$$\dot{v} = \frac{F}{m} \cos\chi - V_y \qquad (2)$$

$$\dot{x} = u \qquad (3)$$

$$\dot{y} = v \qquad (4)$$

where the angle χ is the steering angle (control variable) and is measured from the launch vertical to the local thrust direction as shown schematically in Figure 3-1. The terms V_x and V_y are the x and y components of the gravitational acceleration.

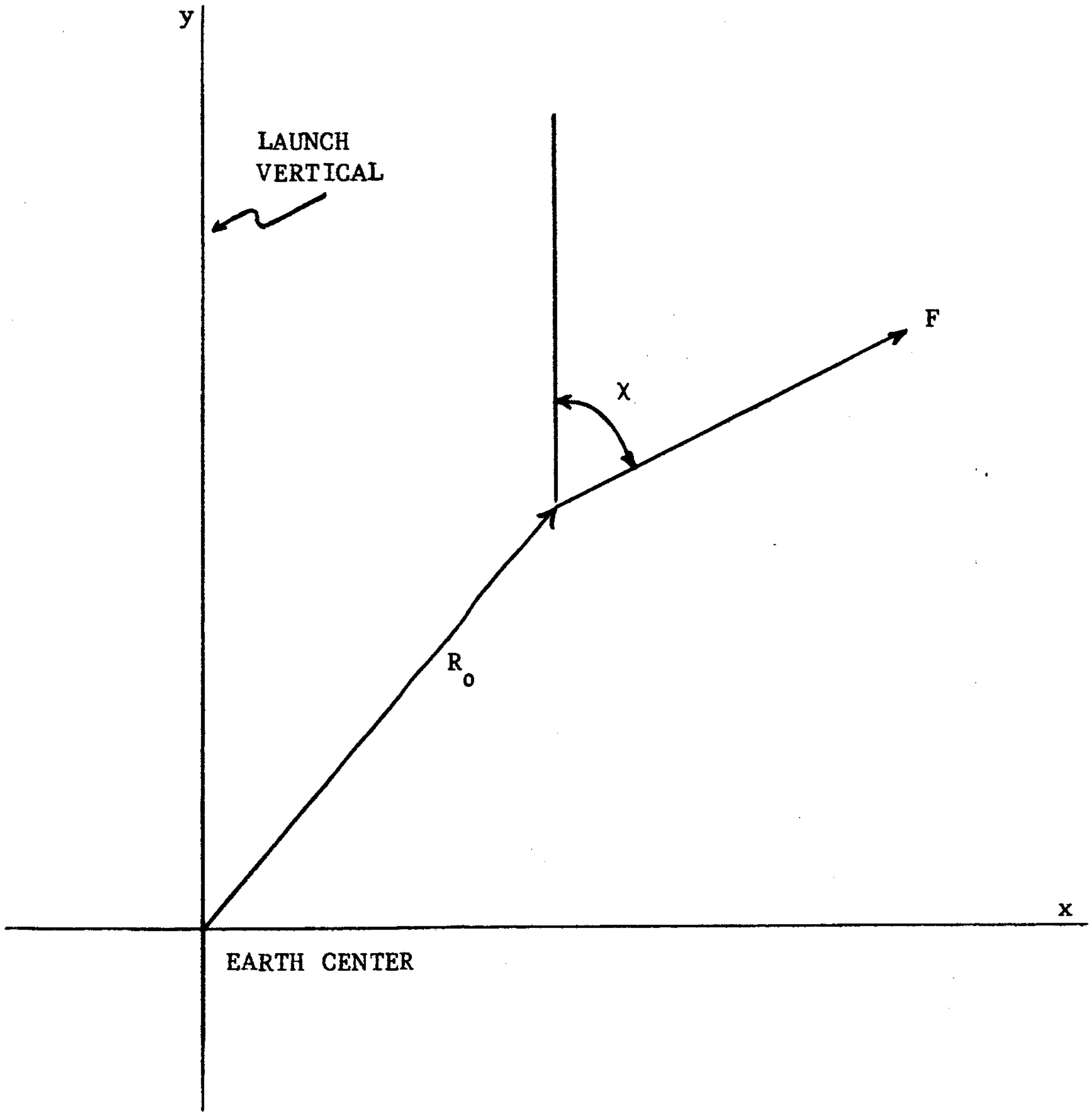

Figure 3-1. DEFINITION OF STEERING ANGLE

The Euler-Lagrange equations for this problem are:

$$\dot{\lambda}_1 = -\lambda_3 \qquad (5)$$

$$\dot{\lambda}_2 = -\lambda_4 \qquad (6)$$

$$\dot{\lambda}_3 = \lambda_1 V_{xx} + \lambda_2 V_{xy} \qquad (7)$$

$$\dot{\lambda}_4 = \lambda_1 V_{xy} + \lambda_2 V_{yy} \qquad (8)$$

$$0 = \lambda_1 \cos\chi - \lambda_2 \sin\chi \qquad (9)$$

Equation (9) yields

$$\operatorname{Tan}\chi = \frac{\lambda_1}{\lambda_2} \qquad (10)$$

Further requirements are that

$$\lambda_1^2 + \lambda_2^2 = 1 \qquad (11)$$

$$\lambda_1 v - \lambda_2 u + \lambda_3 y - \lambda_4 x = 0 \qquad (12)$$

must hold at the initial point of the flight. These two equations will be called the "Scaling" and "Transversality" equations, respectively.

The terminal conditions on the flight are given as three functions which describe a circular orbit.

$$F_1 = (x^2 + y^2 - R_{co}^2)_{co} = 0 \quad \text{"Radius"} \qquad (13)$$

$$F_2 = (u^2 + v^2 - V_{co}^2)_{co} = 0 \quad \text{"Velocity"} \qquad (14)$$

$$F_3 = (xu + yv)_{co} = 0 \quad \text{"Orthogonality"} \qquad (15)$$

where the subscript "co" indicates a value at cutoff time. These functions are given names because they are often referred to in the following parts of this report.

Equations (10) and (11) imply that

$$\sin\chi = \lambda_1 \Big/ \sqrt{\lambda_1^2 + \lambda_2^2} \qquad (16)$$

$$\cos\chi = \lambda_2 \Big/ \sqrt{\lambda_1^2 + \lambda_2^2} \qquad (17)$$

and

$$(\lambda_1)_o = \sin \chi_o \quad \text{or} \quad \chi_o = \sin^{-1}(\lambda_1)_o \qquad (18)$$

$$(\lambda_2)_o = \cos \chi_o \quad \text{or} \quad \chi_o = \cos^{-1}(\lambda_2)_o \qquad (19)$$

where the subscript "o" indicates a value at the initial time, t_o.

Therefore a solution for one of the Lagrange multipliers $(\lambda_i)_o$ is a solution for χ_o. The solution must satisfy equations (1) through (9) for any time. At the initial time, equations (11) and (12) must be satisfied, and equations (13) through (15) must be satisfied at the terminal time.

The terminal time with respect to the initial time is unknown. Denoting this unknown time as t_{co} and the initial time as t_o, define

$$\Delta t = (t_{co} - t_o).$$

3.2 APPROACH TO SOLUTION

It can be seen that a solution for one of the multipliers as an expression explicitly in terms of the state variables (evaluated at initial time) and the vehicle and mission parameters such as $\dot{m}$, F, R_{co}, and V_{co} is the desired Path-Adaptive guidance function.

$$\lambda_i = F(x,y,u,v,m,F,\dot{m},V_{co},R_{co})_o$$

or

$$\chi = G(x,y,u,v,m,F,\dot{m},V_{co},R_{co})_o \tag{20}$$

To obtain this expression, four simultaneous algebraic equations in the four unknown Lagrange multipliers are developed. The multipliers occur explicitly and their coefficients are in terms of the variables shown in equation (20).

Two of the four equations needed are already available; they are the scaling and transversality equations, equations (11) and (12). The other two are obtained from equations (13) through (15) in the following manner:

Equations (13) through (15) are expanded in Taylor series about the interval $\Delta t = (t_{co} - t_o)$. These three terminal conditions are then expressed in terms of initial conditions as

$$F_1 = \sum_n^{o,p} \frac{1}{n!} \frac{d^n}{dt^n} (x^2+y^2-R_{co}^2)_o (\Delta t)^n = 0 \tag{21}$$

$$F_2 = \sum_n^{o,p} \frac{1}{n!} \frac{d^n}{dt^n} (u^2+v^2-V_{co}^2)_o (\Delta t)^n = 0 \tag{22}$$

$$F_3 = \sum_n^{o,p} \frac{1}{n!} \frac{d^n}{dt^n} (xu+yv)_o (\Delta t)^n = 0 \tag{23}$$

The coefficients of Δt in equations (21) through (23) are evaluated at t_o, but the value for Δt is not known.

If equation (22), for example, is written as the finite series or polynomial,

$$\sum_{n}^{o,p} A_n(\Delta t)^n = 0 \qquad (22')$$

the unknown Δt may be solved for in terms of the known A_n. The technique for obtaining an analytical expression for Δt is known as "series reversion" or "series inversion". The mechanics of this procedure are given in reference 8 and further discussed in Section V. Reversion of equation (22') for Δt yields

$$\Delta t = \sum_{n}^{o,q} B_n Z^n \qquad (24)$$

where

$$Z = \frac{V_{co}^2 - (u^2+v^2)_o}{2(u\dot{u} + v\dot{v})_o}$$

$$A_1 = 1$$

$$A_2 = (u\ddot{u} + v\ddot{v} + \dot{u}^2 + \dot{v}^2)/2(u\dot{u} + v\dot{v})$$

$$A_3 = (\tfrac{1}{3} u\dddot{u} + \tfrac{1}{3} v\dddot{v} + \dot{u}\ddot{u} + \dot{v}\ddot{v})/2(u\dot{u} + v\dot{v})$$

$$B_1 = 1$$

$$B_2 = -A_2$$

$$B_3 = 2A_2^2 - A_3$$

$$\vdots$$

Tables 3-1 through 3-3 list the expansions of equations (21) through (23). An expression for Δt in terms of variables which can be evaluated at the initial time has now been obtained and can be substituted into the two series given by equations (21) and (23). Now, Δt no longer appears explicitly in the equations.

The result of the substitution reduces equations (21) through (23) to two equations.

$$F_1 = \sum_n^{o,p} \left[\frac{1}{n!} \frac{d^n}{dt^n} (x^2+y^2-R_{co}^2) \left(\sum_n^{o,q} B_n Z^n \right)^n \right]_o = 0 \qquad (25)$$

$$F_2 = \sum_n^{o,p} \left[\frac{1}{n!} \frac{d^n}{dt^n} (xu+yv) \left(\sum_n^{o,q} B_n Z^n \right)^n \right]_o = 0 \qquad (26)$$

These two equations can be put in explicit terms of the Lagrange multipliers by repeated substitutions of equations (1) through (8); and together with the scaling and transversality equation they form a system of four algebraic equations in the four multipliers.

$$\begin{aligned} \sum R_{ijk\ell}\, \lambda_1^i \lambda_2^j \lambda_3^k \lambda_4^\ell &= 0 \quad \text{from equation (25)} \\ \sum Q_{ijk\ell}\, \lambda_1^i \lambda_2^j \lambda_3^k \lambda_4^\ell &= 0 \quad \text{from equation (26)} \\ (\lambda_1^2 + \lambda_2^2 - 1) &= 0 \quad \text{from equation (11)} \\ \lambda_1 v - \lambda_2 u + \lambda_3 y - \lambda_4 x &= 0 \quad \text{from equation (12)} \end{aligned} \qquad (28)$$

where $i+j+k+\ell = 0,1,2,\ldots$ and $R_{ijk\ell} = f(x,y,u,v,m,F,\dot{m},R_{co},V_{co})$ and similarly for $Q_{ijk\ell}$.

Equations (25) and (26) are put in the form shown in equations (28)by putting the Taylor Series coefficients in equations (21) through (23) in terms of the multipliers. This involves a large amount of algebra, and the end result is a polynomial-type expression (in the multipliers) for each time derivative of u and v up through the nth order. As an example of this, consider the first three terms of the expansion of equation (21)

$$F_1 = (x^2+y^2-R_{co}^2)_o + 2(xu+yv)_o(\Delta t) + (x\dot{u}+y\dot{v}+u^2+v^2)_o(\Delta t)^2 + \ldots$$

From equations (1) and (2), the $\dot{u}$ and $\dot{v}$ terms are:

$$\dot{u} = \frac{F}{m} \sin\chi - V_x$$

$$\dot{v} = \frac{F}{m} \cos\chi - V_y$$

With equations (16) and (17) these become

$$\dot{u} = \frac{F}{m} \frac{\lambda_1}{\sqrt{\lambda_1^2 + \lambda_2^2}} - V_x \qquad (29)$$

$$\dot{v} = \frac{F}{m} \frac{\lambda_2}{\sqrt{\lambda_2^2 + \lambda_2^2}} - V_y \qquad (30)$$

at any time. For time t_o, equation (11) applies and the $\dot{u}$ and $\dot{v}$ become

$$\dot{u}_o = \frac{F}{m} \lambda_1 - V_x$$

$$\dot{v}_o = \frac{F}{m} \lambda_2 - V_y.$$

Higher order derivatives of u and v are obtained by differentiating equations (29) and (30) and applying equation (11) after the differentiation to evaluate the formula at t_o. Equations (5) through (8) are applied to eliminate time derivatives of the multipliers. The λ coefficients are in terms of F, m, $\dot{m}$, x, y, u, and v. When all n orders of these derivatives have been obtained they may be substituted into equations (24) through (26) and then brought to the form shown in equations (28). A considerable amount of labor is required to obtain these latter equations.

3.3 SOLUTION OF THE RESULTING ALGEBRAIC EQUATIONS

Equations (28) are nonlinear. It is desired to have an analytical expression for the unknown multipliers in terms of their coefficients, so that an expression of the form shown in equation (20) is obtained.

A method for the solution of these equations was developed which yields an analytical solution. It has been tested numerically and compares well with conventional numerical methods, and a proof of its validity has been given (ref. 9). For simplicity, the method is called "Successive Substitutions". It is best illustrated by an example.

Suppose it is required to solve the equations

$$J_1 = a_{10} x_1 + a_{01} x_2 + a_{20} x_1^2 + a_{11} x_1 x_2 + a_{02} x_2^2 + \dots$$

$$J_2 = b_{10} x_1 + b_{01} x_2 + b_{20} x_1^2 + b_{11} x_1 x_2 + b_{02} x_2^2 + \dots$$

Let an approximate solution for x_1 and x_2 be given by

$$x_1^{(1)} = \frac{\begin{vmatrix} J_1 & a_{01} \\ J_2 & b_{01} \end{vmatrix}}{\begin{vmatrix} a_{10} & a_{01} \\ b_{10} & b_{01} \end{vmatrix}} \quad \text{and} \quad x_2^{(1)} = \frac{\begin{vmatrix} a_{10} & J_1 \\ b_{10} & J_2 \end{vmatrix}}{\begin{vmatrix} a_{10} & a_{01} \\ b_{10} & b_{01} \end{vmatrix}}$$

This "first approximation" obtained from linear terms only is then substituted into the second-degree terms, and the higher degree terms are ignored. This gives

$$J_1^{(1)} = J_1 - \left[a_{20}x_1^{(1)^2} + a_{11}x_1^{(1)}\ x_2^{(1)} + a_{02}x_2^{(1)^2}\right] = a_{10}x_1 + a_{01}x_2$$

$$J_2^{(1)} = J_2 - \left[b_{20}x_1^{(1)^2} + b_{11}x_1^{(1)}\ x_2^{(1)} + b_{02}\ x_2^{(1)^2}\right] = b_{10}x_1 + b_{01}\ x_2$$

and again the linear terms are solved to obtain a "second approximation".

$$x_1^{(2)} = \frac{\begin{vmatrix} J_1^{(1)} & a_{01} \\ J_2^{(1)} & b_{01} \end{vmatrix}}{\begin{vmatrix} a_{10} & a_{01} \\ b_{10} & b_{01} \end{vmatrix}} \quad \text{and} \quad x_2^{(2)} = \frac{\begin{vmatrix} a_{10} & J_1^{(1)} \\ b_{10} & J_2^{(1)} \end{vmatrix}}{\begin{vmatrix} a_{10} & a_{01} \\ b_{10} & b_{01} \end{vmatrix}}$$

In general, the (k+1)st approximation is obtained by treating the linear terms as unknowns and substituting the k^{th} approximation into all other terms up through the (k+1)st degree. Any order "approximation" can be displayed in an analytical form.

This technique was applied to equations (28), and good results were obtained. Before it could be applied, however, the determinant of linear terms in the multipliers had to be made non-zero. This was done by judicious substitutions of equation (11) to ensure that linear terms in all four multipliers would occur explicitly in the radius and orthogonality equations. The scaling equation was expanded in a binomial series to the sixth degree in λ_2,

$$\lambda_1 = (1 - \lambda_2^2)^{\frac{1}{2}} \simeq 1 - \frac{1}{2}\lambda_2^2 - \frac{1}{8}\lambda_2^4 - \frac{1}{16}\lambda_2^6 \; .$$

Because the angle χ usually runs from around 70° to 140°, the series is quite accurate (for the number of terms taken) around 90° because λ_2 is small.

This method for solving the algebraic equations is fairly simple. It is based on the fact that the multipliers are all much smaller than their coefficients, and these coefficients remain about the same magnitude while products of the multipliers become progressively smaller in magnitude as their degree increases.

Table 3-1. SERIES EXPANSION FOR RADIUS EQUATION

$$0 = F_1 = \sum_n^{0,6} \frac{1}{n!} \frac{d^n}{dt^n} (x^2 + y^2 - R_{co}^2)_o (\Delta t)^n$$

OR

$$F_1 = \sum_n^{0,6} \bar{A}_n (\Delta t)^n$$

$$\bar{A}_o = (x^2 + y^2 - R_{co}^2)_o$$

$$\bar{A}_1 = 2(xu + yv)_o$$

$$\bar{A}_2 = (x\dot{u} + y\dot{v} + u^2 + v^2)_o$$

$$\bar{A}_3 = (\frac{1}{3} x\ddot{u} + \frac{1}{3} y\ddot{v} + u\dot{u} + v\dot{v})_o$$

$$\bar{A}_4 = (\frac{1}{12} x\dddot{u} + \frac{1}{12} y\dddot{v} + \frac{1}{3} \dot{u}\ddot{u} + \frac{1}{3} \dot{v}\ddot{v} + \frac{1}{4} \dot{u}^2 + \frac{1}{4} \dot{v}^2)_o$$

$$\bar{A}_5 = (\frac{1}{60} x\, u^{IV} + \frac{1}{60} y\, v^{IV} + \frac{1}{12} \dot{u}\dddot{u} + \frac{1}{12} \dot{v}\dddot{v} + \frac{1}{6} \dot{u}\ddot{u} + \frac{1}{6} \dot{v}\ddot{v})_o$$

$$\bar{A}_6 = (\frac{1}{360} x\, u^{V} + \frac{1}{360} y\, v^{V} + \frac{1}{60} u\, u^{IV} + \frac{1}{60} v\, v^{IV} + \frac{1}{24} \dot{u}\, \dddot{u}$$

$$+ \frac{1}{24} \dot{v}\, \dddot{v} + \frac{1}{36} \ddot{u}^2 + \frac{1}{36} \ddot{v}^2)_o$$

Table 3-2. SERIES EXPANSION FOR VELOCITY EQUATION

$$0 = F_1 = \sum_n^{0,6} \frac{1}{n!} \frac{d^n}{dt^n} (x^2 + y^2 - R_{co}^2)_o (\Delta t)^n$$

OR

$$F_1 = \sum_n^{0,6} \bar{B}_n (\Delta t)^n$$

$$\bar{B}_o = (u^2 + v^2 - R_{co}^2)_o$$

$$\bar{B}_1 = 2(u\dot{u} + v\dot{v})_o$$

$$\bar{B}_2 = (u\ddot{u} + v\ddot{v} + \dot{u}^2 + \dot{v}^2)_o$$

$$\bar{B}_3 = \left(\frac{1}{3} u \dddot{u} + \frac{1}{3} v \dddot{v} + \dot{u}\ddot{u} + \dot{v}\ddot{v}\right)_o$$

$$\bar{B}_4 = \left(\frac{1}{12} u u^{IV} + \frac{1}{12} v v^{IV} + \frac{1}{3} \dot{u}\dddot{u} + \frac{1}{3} \dot{v}\dddot{v} + \frac{1}{4} \ddot{u}^2 + \frac{1}{4} \ddot{v}^2\right)_o$$

$$\bar{B}_5 = \left(\frac{1}{60} u u^{V} + \frac{1}{60} v v^{V} + \frac{1}{12} \dot{u} u^{IV} + \frac{1}{12} \dot{v} v^{IV} + \frac{1}{6} \ddot{u}\dddot{u} + \frac{1}{6} \ddot{v}\dddot{v}\right)_o$$

$$\bar{B}_6 = \left(\frac{1}{360} u u^{VI} + \frac{1}{360} v v^{VI} + \frac{1}{60} \dot{u} u^{V} + \frac{1}{60} \dot{v} v^{V} + \frac{1}{24} \ddot{u} u^{IV} + \frac{1}{24} \ddot{v} v^{IV} + \frac{1}{36} \dddot{u}^2 + \frac{1}{36} \dddot{v}^2\right)_o$$

Table 3-3. SERIES EXPANSION FOR ORTHOGONALITY EQUATION

$$0 = F_1 = \sum_n^{0,6} \frac{1}{n!} \frac{d^n}{dt^n} (x^2 + y^2 - R_{co}^2)_o (\Delta t)^n$$

OR

$$F_1 = \sum_n^{0,6} \bar{C}_n (\Delta t)^n$$

$$\bar{C}_o = (xu + yv)_o$$

$$\bar{C}_1 = (x\dot{u} + y\dot{v} + u^2 + v^2)_o$$

$$\bar{C}_2 = (\tfrac{1}{2}\, x\, \ddot{u} + \tfrac{1}{2}\, y\, \ddot{v} + \tfrac{3}{2}\, u\, \dot{u} + \tfrac{3}{2}\, v\, \dot{v})_o$$

$$\bar{C}_3 = (\tfrac{1}{6}\, x\, \dddot{u} + \tfrac{1}{6}\, y\, \dddot{v} + \tfrac{2}{3}\, u\, \ddot{u} + \tfrac{2}{3}\, v\, \ddot{v} + \tfrac{1}{2}\, \dot{u}^2 + \tfrac{1}{2}\, \dot{v}^2)_o$$

$$\bar{C}_4 = (\tfrac{1}{24}\, x\, \overset{IV}{u} + \tfrac{1}{24}\, y\, \overset{IV}{v} + \tfrac{5}{24}\, u\, \dddot{u} + \tfrac{5}{24}\, v\, \dddot{v} + \tfrac{5}{12}\, \dot{u}\, \ddot{u} + \tfrac{5}{12}\, \dot{v}\, \ddot{v})_o$$

$$\bar{C}_5 = (\tfrac{1}{120}\, x\, \overset{V}{u} + \tfrac{1}{120}\, y\, \overset{V}{v} + \tfrac{1}{20}\, u\, \overset{IV}{u} + \tfrac{1}{20}\, v\, \overset{IV}{v} + \tfrac{1}{8}\, \dot{u}\, \dddot{u} + \tfrac{1}{8}\, \dot{v}\, \dddot{v}$$

$$+ \tfrac{1}{12}\, \ddot{u}^2 + \tfrac{1}{12}\, \ddot{v}^2)_o$$

$$\bar{C}_6 = (\tfrac{1}{720}\, x\, \overset{VI}{u} + \tfrac{1}{720}\, y\, \overset{VI}{v} + \tfrac{7}{720}\, u\, \overset{V}{u} + \tfrac{7}{720}\, v\, \overset{V}{v} + \tfrac{7}{240}\, \dot{u}\, \overset{IV}{u}$$

$$+ \tfrac{7}{240}\, \dot{v}\, \overset{IV}{v} + \tfrac{7}{144}\, \ddot{u}\, \dddot{u} + \tfrac{7}{144}\, \ddot{v}\, \dddot{v})_o$$

SECTION IV

SOLUTION FOR STEERING FUNCTION

4.1 INTRODUCTION

So far, the basic analytical approach to the guidance problem has been given. In Section III, when the Δt expression was obtained by reversion and then substituted to get equations (25) and (26), a natural division of the study was possible. Instead of carrying through all the algebraic substitutions immediately, the study was divided into two parts. One part was to investigate the Δt expression with the use of true values for the multipliers. The other part was to study the solution for the multipliers with the use of true values for Δt.

When satisfactory solutions for each sub-problem had been accomplished, the algebraic expressions for Δt and those used for the multipliers would be combined. If the combination is done first, it is inconvenient to carry out any appreciable analysis. For this reason, the substitution of the Δt expression was postponed until both problems could be analyzed independently. One series for Δt, valid for a certain class of trajectories, was chosen for substitution. However, this selection is not as accurate as would be desired for long flight times. These results are discussed in Section VI.

In this section the results of the study of the equations used to obtain the multipliers are summarized. The analysis shown here is based on equations where true Δt values are assumed. The Δt studies are reported in Section V.

4.2 USE OF NOMINAL DATA

Before proceeding further, it should be made clear how nominal data is used to test the guidance formulas. First, a nominal flight is computed numerically by the methods described in Section 2.2. An initial point is chosen, and the circular orbit radius is specified along with the vehicle's thrust, specific impulse, weight, etc. The differential equations are then integrated, and a print-out of the numerical results is obtained. Among the quantities of interest printed are the multipliers, steering angle, mass,position, velocity, acceleration, etc. These are printed at two-second intervals so that a time history of these quantities is available. This set of data is called a "nominal" trajectory.

Now, using the data for position, velocity, mass, and the cutoff and vehicle parameters at any point on this nominal, the analytically derived guidance formulas may be numerically evaluated and the analytically predicted values compared with those predicted by the nominal at the same point. From one nominal time history a number of cases can be obtained by testing the guidance formulas for a sequence of time points beginning at the start of the nominal trajectory and going up until near the cutoff time. The nominal data is used strictly for comparison, not for generation of any guidance formulas.

Generally the nominal data is regarded as exact, but in fact it is not. It is not known what errors might exist in the nominal values, but they should be very small. Thus, nominal values and those predicted by analytical formulas which disagree in the neighborhood of one percent or so may be essentially the same values.

A computer program named "TOPSY" was used to process this nominal data and make necessary comparisons between nominal and analytically predicted values.

Further details on this program are given in Section VIII.

4.3 DEVELOPMENT OF THE ROTS EQUATIONS

Equations (28) are known as the "ROTS" equations because they come from the Radius, Orthogonality, Transversality, and Scaling equations. The velocity equation was used to solve for Δt. Depending upon which series was used to obtain the Δt, there were two other systems of equations possible, "VOTS" and "RVTS"; both analogous to the ROTS equations. In the VOTS equations the radius equation was used to get the Δt expression, and in the RVTS the orthogonality equation was used to get Δt.

Further investigation of the VOTS and RVTS equations confirmed what had been reported earlier in reference 1. These two systems of equations do not generally produce accurate solutions for the multipliers in comparison to the ROTS equations. In general, use of the velocity condition to get Δt and the ROTS equations to obtain the multipliers was determined to be the best choice.

The orders of series used in equations (21) and (23) ranged from third through fifth. Corresponding to each order of series used, a different set of ROTS equations was derived. Table 4-1 shows the "R" and "Q" coefficients as derived from a fifth-order series for the radius condition and a fourth-order series for the orthogonality condition. The algebra through which these coefficients were derived was explained in Section 3.2. The notation "ROTS (I,J)" denotes the system of algebraic equations in the multipliers that is obtained from Taylor series of order "I" for the radius conditions and order "J" for the orthogonality condition. Note that Δt appears explicitly in these formulas, and also that some of these coefficients are linear combinations of each other.

Because the formulas involved in this study are so complicated, it is generally difficult to predict or even speculate accurately about the interrelationships of the above parameters. From the series expansions it would seem that the interval Δt should be selected as a criterion for the orders of series required. However, Δt is itself an unknown parameter in the problem and ultimately depends upon the initial and terminal flight conditions and vehicle parameters. An attempt was made to relate errors in the X values with Δt, ΔR, and the orders of series used. A corresponding analysis is made in Section V for the Δt expression. The groups of nominal cases used for the error analysis are displayed in Table 4-2.

A topic that needs clarification is that of the relative errors of solutions. The relative errors are determined in the conventional manner as:

$$\text{Relative Error} = \frac{\text{True - Approximate}}{\text{True}}$$

where "True" is the nominal value and "Approximate" is the value obtained from the analytical solution. Percent error is 100 times the relative error.

Percent errors are generally used in this report because they are immediately meaningful in the sense of "goodness" of results. Thus a one percent error indicates a good correlation between nominal values and analytical results, while a 30 percent error does not. It should be understood that the errors reported cannot be exact and have no genuine usefulness when considered individually. At most they should be regarded as a relative index to the correlation of nominal and analytical results. Suppose, for example, that one set of ROTS equations gives an error of five percent and another gives an error of one percent in its solution. The percentages are taken to mean that both solutions are fairly good and the latter is the better.

4.4 SOLUTION OF THE ROTS EQUATIONS

Two methods were used to obtain numerical solutions with the nominal values of the multipliers used as initial approximations. These solutions are considered to be the actual roots of the equations which were closest to the desired roots. No attempt was made to find other sets of roots. The method of Successive Substitutions, described in Section 3.3, was also used to solve for the multipliers. This method was programmed to produce numerical results, and the actual development of an explicit expression for the multipliers was not done. To do so is straightforward but laborious, and the same information could be obtained numerically without going through a mass of algebra. In most cases, both the Newton-Raphson and Successive Substitution methods yield values for the multipliers which are near the nominal values. For cases where the solutions are not close to nominal values both methods produce solutions which are near each other.

In some cases the Newton-Raphson method did not converge to any value, while the Successive Substitution method did. In some of these cases the results of the Successive Substitutions were quite close to nominal values. Time has not permitted a detailed investigation of this non-convergence. It was probably caused by too small a tolerance specified for the iterates in the convergence tests. Other cases of non-convergence are due to inaccurate ROTS equations. This is discussed in the following section.

4.5 ERROR ANALYSIS

In this section the behavior of the solutions of the ROTS equations are described and analyzed. In particular, the effects of Δt, series truncation error, and change in altitude, ΔR, are investigated. There are several reasons for this interpretation. First, the nominal data is not exact and the maximum possible error in it is not known. Secondly, digital computers truncate numbers instead of rounding after an arithmetical operation, and an unfortunate sequence of truncations can degrade accuracy. The relative error, which is directly related to the number of significant digits compared, is also determined to some extent by the iteration tolerance specified by the Newton-Raphson algorithm.

Generally, good correlation was obtained between nominal values for the multipliers and those predicted by the analytical solution. Figures 4-1 through 4-4 are representative time histories of the percent errors in λ_1 and λ_2 as obtained by the Newton-Raphson and Successive Substitution methods. The independent variable is Δt, and it goes from a maximum at the origin to zero at cutoff. Actually, each of these figures represents a number of cases; each of which is different with respect to initial conditions and time-to-cutoff, but having the same terminal conditions and vehicle specifications.

Tables 4-3 through 4-162 are tabulations representative of the actual numerical values obtained for the four multipliers and the steering angle. The solutions are shown to several digits and should give an indication of accuracy in terms of significant digits instead of relative errors. These tables are for sample cases taken from those shown in Table 4-2. (In Table 4-11, an unintentional "perturbation" was introduced through a key punch error. At $t_o = 130$ sec. the nominal value for the x-coordinate is 0.26303181×10^7, and the value used was 0.29303181×10^7. This caused a large error in the λ solutions, but had a comparatively small effect on the χ value.)

A characteristic of the solutions is the oscillation of their errors as shown in Figures 4-1 through 4-4. The magnitude of the errors appears to be a sinusoidal function of time. The apparent discontinuity in the λ_2 error is a large relative error of several hundred percent. This behavior was noted for all cases. The cause of the errors in the solutions is the accuracy of the scaling equation when expanded in the binomial series. The expansion of λ_1 into terms of λ_2 is accurate for λ_1 in the neighborhood of 90 degrees. Recall that $\lambda_1 = \sin \chi$ and $\lambda_2 = \cos \chi$; yet, while λ_1 is well represented, for the number of terms taken, λ_2 is less accurately represented near 90 degrees. The result is a sudden and large error in the solution for λ_2 as the steering angle, χ , goes through 90 degrees.

In Figure 4-5 a plot of λ_1, as calculated from the expanded scaling equation, is shown as a function of χ. In Figure 4-6 a plot of λ_2 as a function of χ is shown.

There are additional sources of error in computing the nominal values of the multipliers from the nominal values for χ's. The FORTRAN routines for sine and cosine lose accuracy when these functions are close to zero. The nominal χ values are also in error by some small amount. Errors are also introduced because of the inability to compute precisely the coefficients Q_{oooo} and R_{oooo} for values of Δt near zero. In these coefficients the term, $xu+yv$, occurs and should be zero at $\Delta t = 0$. Because of computer truncation error, it never quite becomes zero and the error in these coefficients affects the solution of the ROTS equations. Its effect is evident when the error increases near cutoff instead of becoming zero.

Three of the equations used in the ROTS equations are truncated series. The remainder terms for the radius and orthogonality series, as evaluated from nominal data, are shown in Figures 4-7 and 4-8. Nominal data for these figures corresponds with that for Figures 4-1 through 4-4. Remainder terms for the radius series are well behaved and drop to comparatively small values as Δt decreases. The wild variations in the remainder terms for the orthogonality series have been traced to the miscalculation of the term $xu + yv$ described above. There seems to be only a slight effect of this error in the solution for the multipliers.

We assert that the "approximate" error in λ_1 for the condition of maximum Δt is, in fact, the largest error. As Δt and ΔR decrease, the errors in the solutions of the ROTS equations also decrease.

In order to determine the relationships between the accuracy of the solutions of the ROTS equations and the parameters of ΔR, Δt, and orders of series used, a number of solutions were obtained and compared with each other. The nominal cases used for numerical tests were those shown in Table 4-2. The assertion that the maximum error in the solution of the ROTS equations occurs for maximum Δt on any nominal case is essential in the subsequent analysis. However, this assumption could be replaced with the use of an average error for a nominal flight or use of the maximum error, should the initial error not be the apparent maximum. In either case the behavior of the relationships leads to the same conclusions. The Newton-Raphson solution for λ_1, at the time of maximum Δt on a nominal flight, was chosen for comparison when changes were made in ΔR, Δt, and orders of series used.

Figure 4-9 shows the improvement in accuracy as higher order series are used in the ROTS equations. In these cases the changes in altitude (radius at ignition to radius at cutoff) range from slightly over ten kilometers to almost 20 kilometers, and the corresponding flight times are from 170.34 seconds to 178.71 seconds. There is a significant improvement in accuracy from the ROTS (3, 3) to ROTS (4, 3) equations. For this range of ΔR, not much improvement is obtained from the ROTS (4, 4) to ROTS (5, 4) equations. The contribution of the extra term in the radius series is still not apparent when larger ΔR values are considered, as shown in Figure 4-10. But when ΔR increases over 50 kilometers, the effect of the extra term is noticeable. Usually the Newton-Raphson method fails to converge to any root when applied to the ROTS (4, 3) and ROTS (4, 4) equations for ΔR over 50 kilometers. For the ROTS (5, 4) equations, solutions can still be obtained, although their errors begin to increase rapidly.

To show the effect of Δt on the errors of the solutions, nominal cases were obtained which have a constant ΔR but varying thrust levels to cause different flight times. Table 4-163 shows the increase in error in λ_1 as flight time is increased.

Figure 4-11 is a plot of the initial percent error of the Newton-Raphson solution versus ΔR. The equations solved were the ROTS (4, 3) equations for cases where varying thrust levels were used to obtain varying flight times. Similar results were obtained for other orders of ROTS equations. The points at R = 19.78 Km correspond with those shown in Table 4-163. As the figure indicates, very little increase in error is incurred until the thrust drops to 100,000 pounds. For that case the error increases a relatively large amount.

A final test of the accuracy of the steering angle was the simulation of a flight, using the solutions of the ROTS equations computed on the basis of nominal Δt values. This was accomplished by integrating the equations of motion from nominal ignition conditions to the specified terminal conditions. The steering angle values were obtained from the ROTS equations at each time step. Coefficients of the multipliers in these equations were evaluated from current values for the state variables as predicted by the numerical integration from the previous time point. The integration method used was the conventional fourth-order Runge-Kutta method. Integration time steps were 0.1 seconds. Difficulties were encountered in computing solutions near the terminal conditions because of the truncation errors mentioned previously. Instead of flying all the way to orbit, a "target point" was selected which was a position and velocity a few seconds prior to injection. At this point comparisons were made between nominal values and those achieved by flying according to the analytical solution. As a check on the accuracy of the simulation, nominal data (including the steering angle) was used to integrate the equations of motion for a test case with the aim of reproducing the nominal case. The nominal was not exactly reproduced, it was in error at the target point by about ten to fifteen meters in x and y and less than one meter per second in u and v. It was determined that this error was caused by the integration step size and an attempt was made to reduce the error by altering the step size. However, step sizes smaller or larger than one-tenth second caused an increase in error. More accuracy could have been obtained by use of extended precision arithmetic in the integration subroutine but storage limitations precluded this being done conveniently.

Because of this situation, the errors in the simulation results should not be taken as absolutely accurate. In most cases they are probably more accurate than indicated.

Some sample results of the simulation of a flight according to the analytical solution are shown in Table 4-164. The ROTS equations of different orders were used to obtain the chi values for substitution into the equations of motion. In addition, solutions of the ROTS equations obtained from various substitution steps were used. Table 4-164 shows results for the solutions of the ROTS (4,3) equations for four nominal cases: AA-1, AA-24, AA-1C, and AA-24C.

The solution of a differential equation is dependent upon the initial conditions specified. It can be seen that the steering angle computed from the third substitution of the Successive Substitution method leads to somewhat better accuracy at the target point than do the higher order substitution steps. A comparison of the relative errors of the substitution steps in Figures 4-1 through 4-4 shows that the third substitution has a small error initially and thus does not propagate as large an integration error to the target point as do the higher order steps. As expected, accuracy was degraded as ΔR and Δt increased and improved as the order of the ROTS equations increased. The effect of Δt was probably to allow for more propagated integration errors, rather than being an intrinsic cause of inaccurate steering angles computed from the ROTS equations.

4.6 DISCUSSION OF RESULTS

The ultimate success of this formal analytical approach depends upon the convergence of the series used to develop a solution for the initial Lagrange multipliers. At the beginning there is no definite standard for "convergence". A qualitative requirement is that the series have sufficient terms to permit the multipliers to be accurately calculated from them. What orders of series are necessary is determined by using numerically integrated, or nominal, trajectories as guidelines.

For a high-thrust, chemical rocket, series of order four or five provide good accuracy for altitude changes of up to 50 kilometers. The series also become progressively more accurate as Δt (and ΔR) decrease during a flight. As a convenient index to the orders of series required, the parameter ΔR presently seems to be a good choice. This quantity should be predictable in advance of a flight and determine orders of series needed. Further investigation of these topics would be desirable, particularly for cases where low thrust is involved. For certain ranges of thrust values, Δt probably becomes a primary influence on the choice of series, rather than ΔR (c.f., Figure 4-11).

Accurate values for X can be obtained either numerically or through an explicit formula in terms of the initial conditions and specified terminal conditions. The calculations for λ_2 are inaccurate for X values around 90 degrees. However, this inaccuracy shows little influence on λ_1. When X values begin to approach 180 degrees an alternate set of formulas could be used where λ_2 is expanded in terms of λ_1.

In Tables 4-3 through 4-162 the Successive Substitution method applied to the ROTS (5, 3) and ROTS (5, 4) equations does not always yield values as accurate as those for the ROTS (4, 3) or ROTS (3, 3) equations. This method depends upon the degree of the terms in the multipliers; and intuitively, one expects better convergence if the lower degree terms are the largest in magnitude. Ideally, one would like to have the linear terms to account for most of the sum of the terms in the equations:

$$\sum R_{ijk\ell} \lambda_1^i \lambda_2^j \lambda_3^k \lambda_4^\ell = 0$$

$$\sum Q_{ijk\ell} \lambda_1^i \lambda_2^j \lambda_3^k \lambda_4^\ell = 0.$$

To some extent the terms of these equations can be rearranged by substituting the scaling equation "in a judicious manner". For example, the term R_{2001} can be replaced by two terms when the scaling equation is substituted. Let $\lambda_1^2 = 1 - \lambda_2^2$ and $R_{2001} \lambda_1^2 \lambda_4 = R_{2001} (1 - \lambda_2^2) \lambda_4 = R_{2001} \lambda_4 - R_{2001} \lambda_2^2 \lambda_4$. The result is two terms; one of them a linear term, and the other one smaller than the original term for values of chi around 90 degrees. This substitution can also be used to simplify the ROTS equations by combination of terms after the substitution. The ROTS (4, 3) equations were developed with this term-rearrangement used to assure dominant linear and quadratic terms. The method is based on trial-and-error and thus takes time. Because of the time required, the extensions of the series to the ROTS (5, 3) and ROTS (5, 4) have not been adjusted to assure the desired dominant terms. Thus the Successive Substitution method is not being used on the best arrangement of terms.

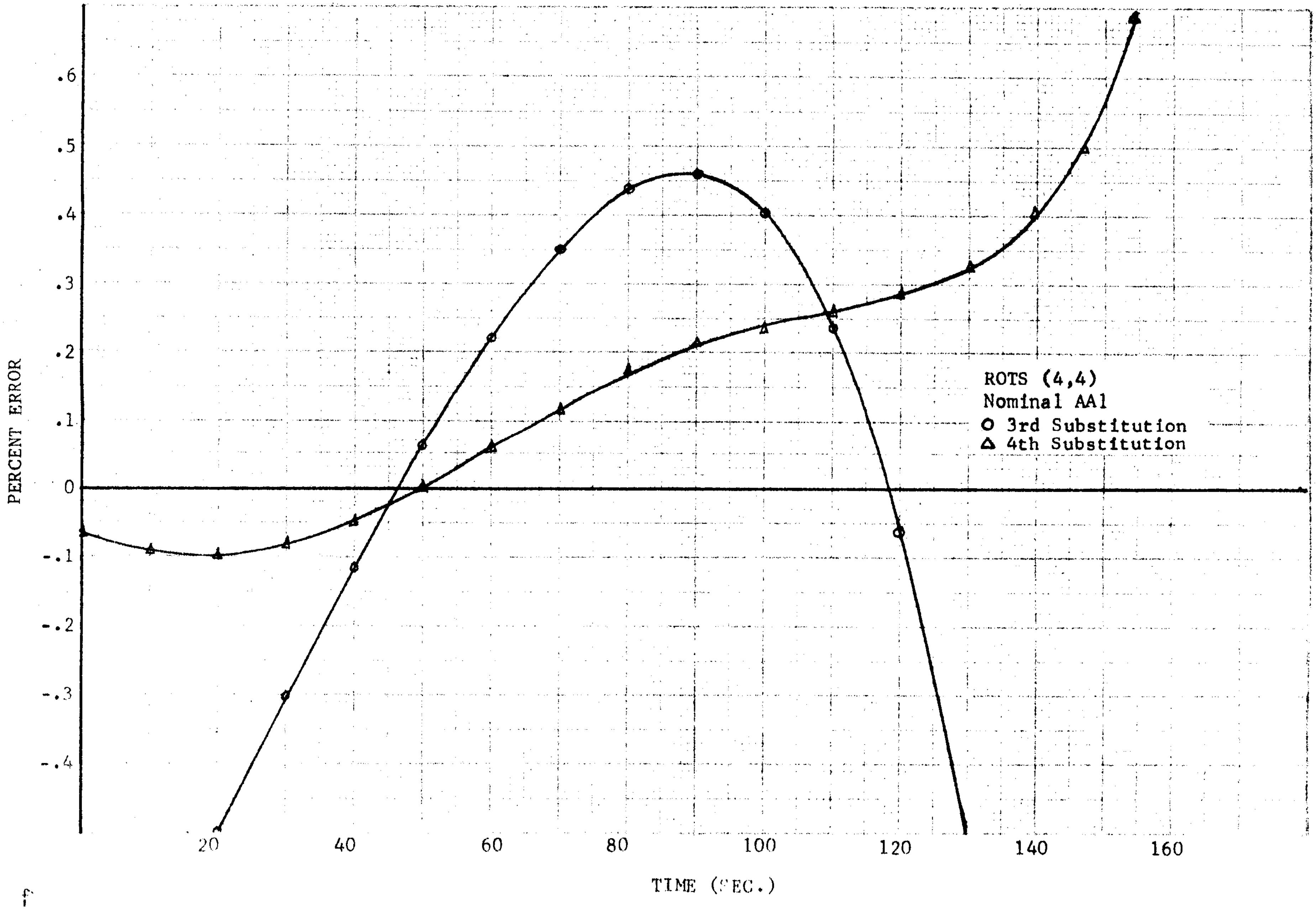

Figure 4-1. TIME HISTORY OF PERCENT ERROR λ_1

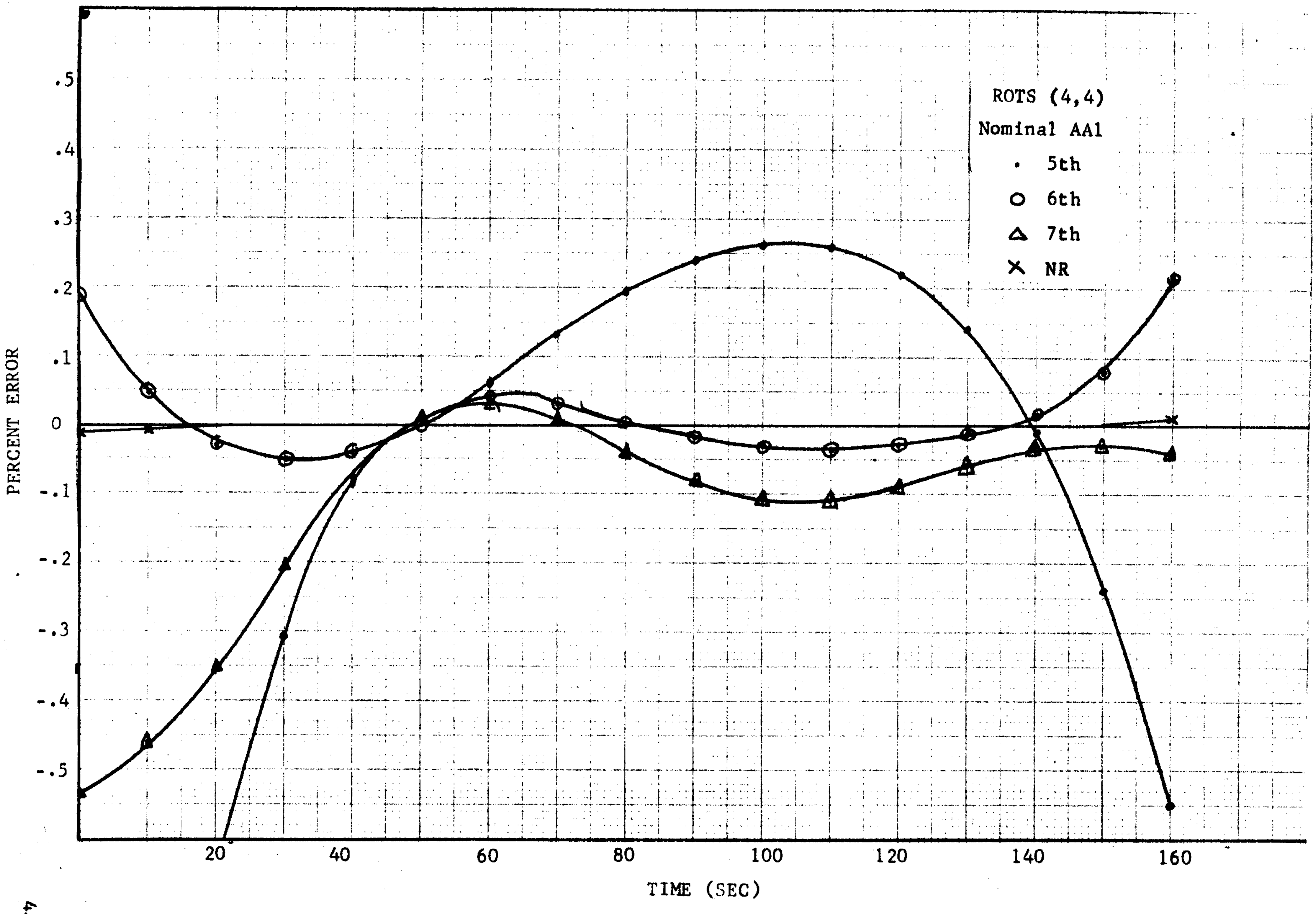

Figure 4-2. TIME HISTORY OF PERCENT ERROR λ_1

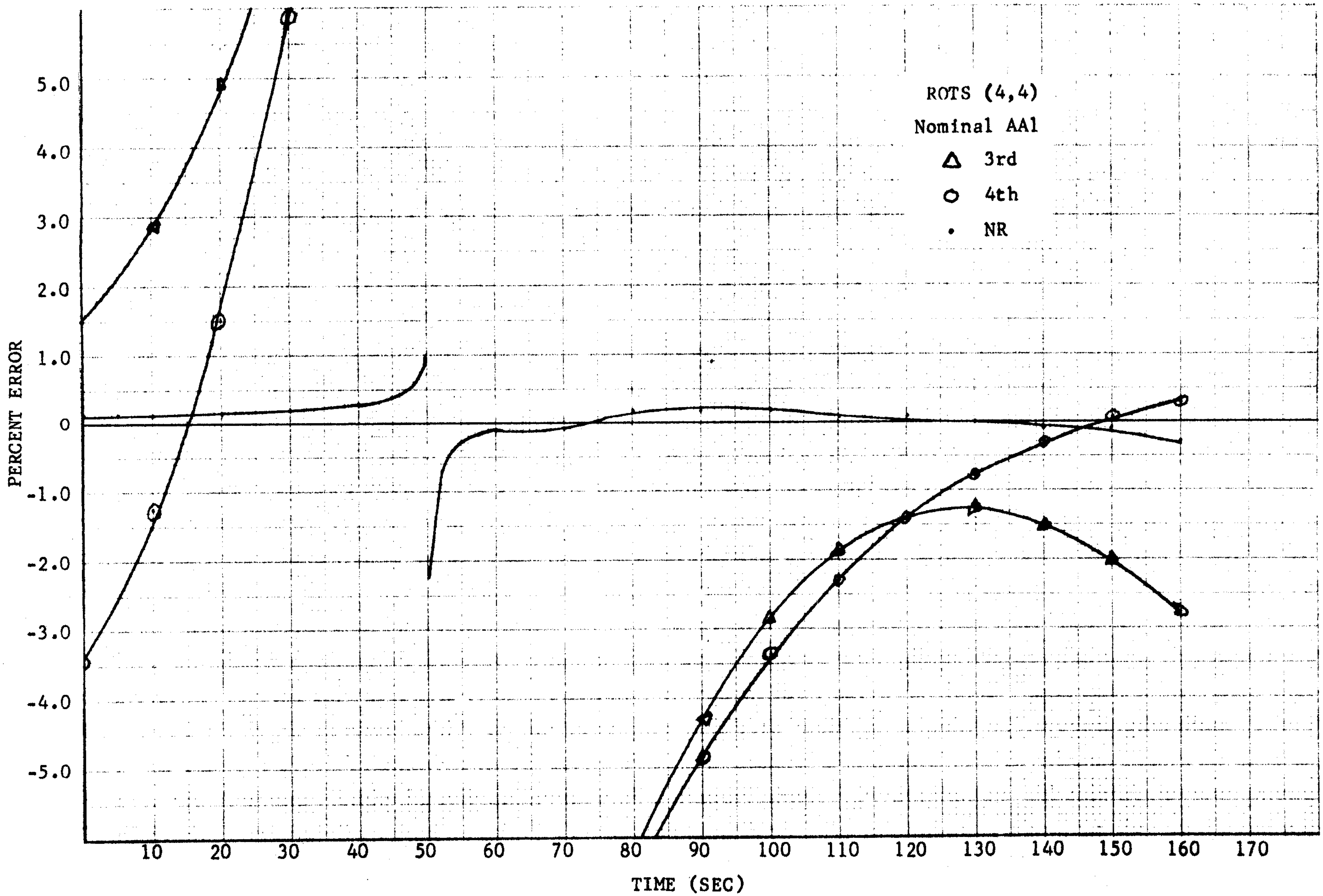

Figure 4-3. TIME HISTORY OF PERCENT ERROR λ_2

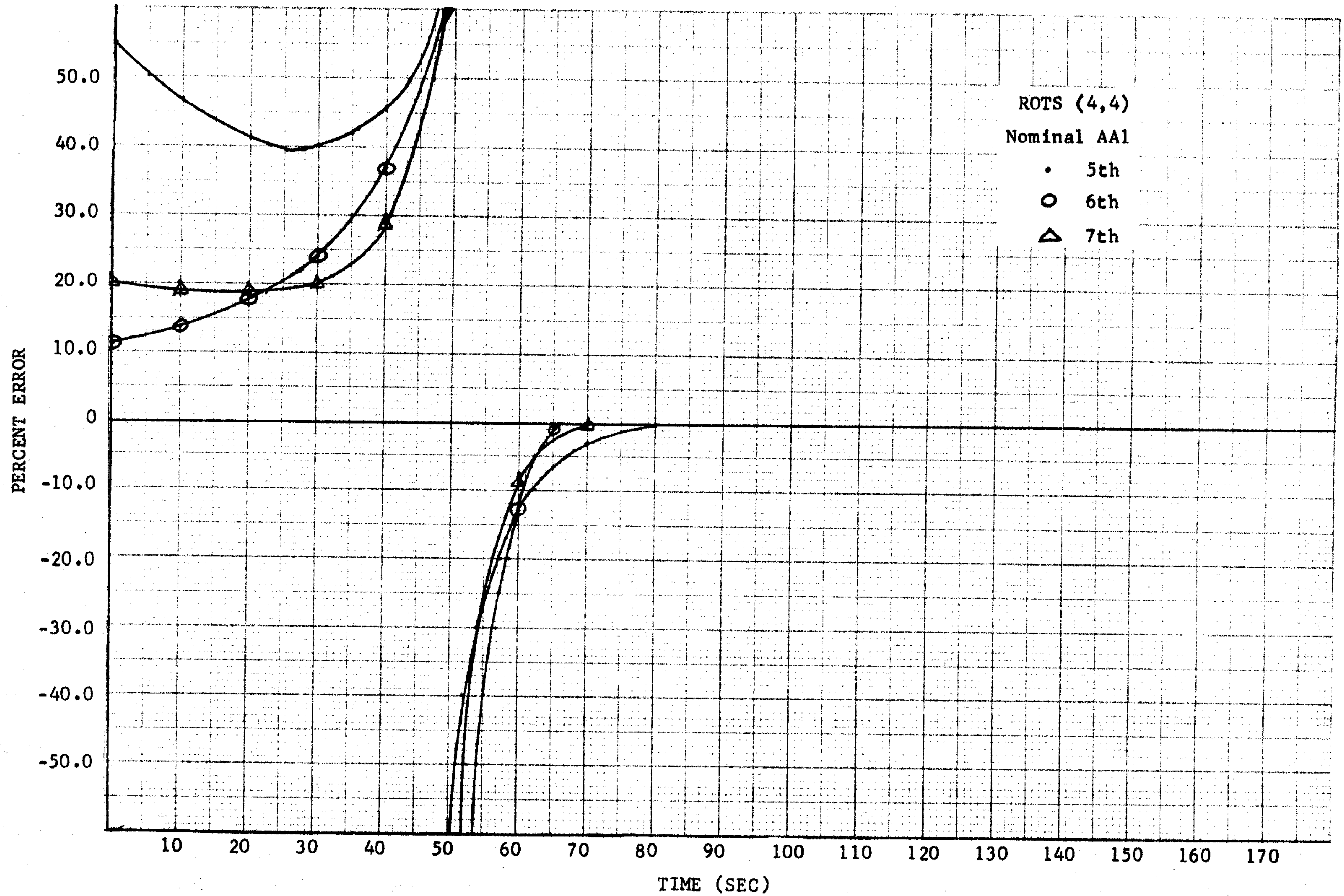

Figure 4-4. TIME HISTORY OF PERCENT ERROR λ_2

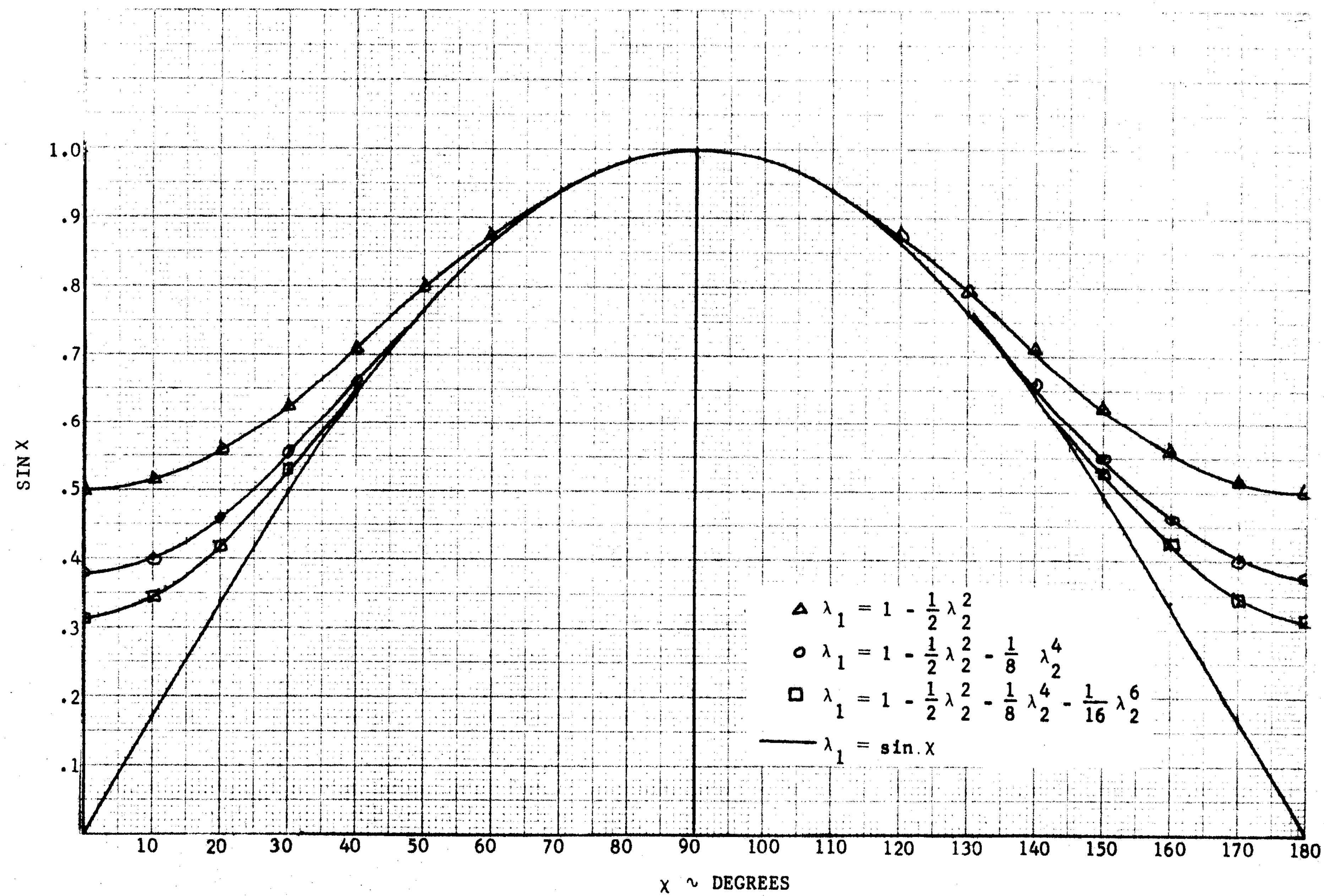

Figure 4-5. BINOMIAL EXPANSION OF SCALING EQUATION COMPARED TO SIN X

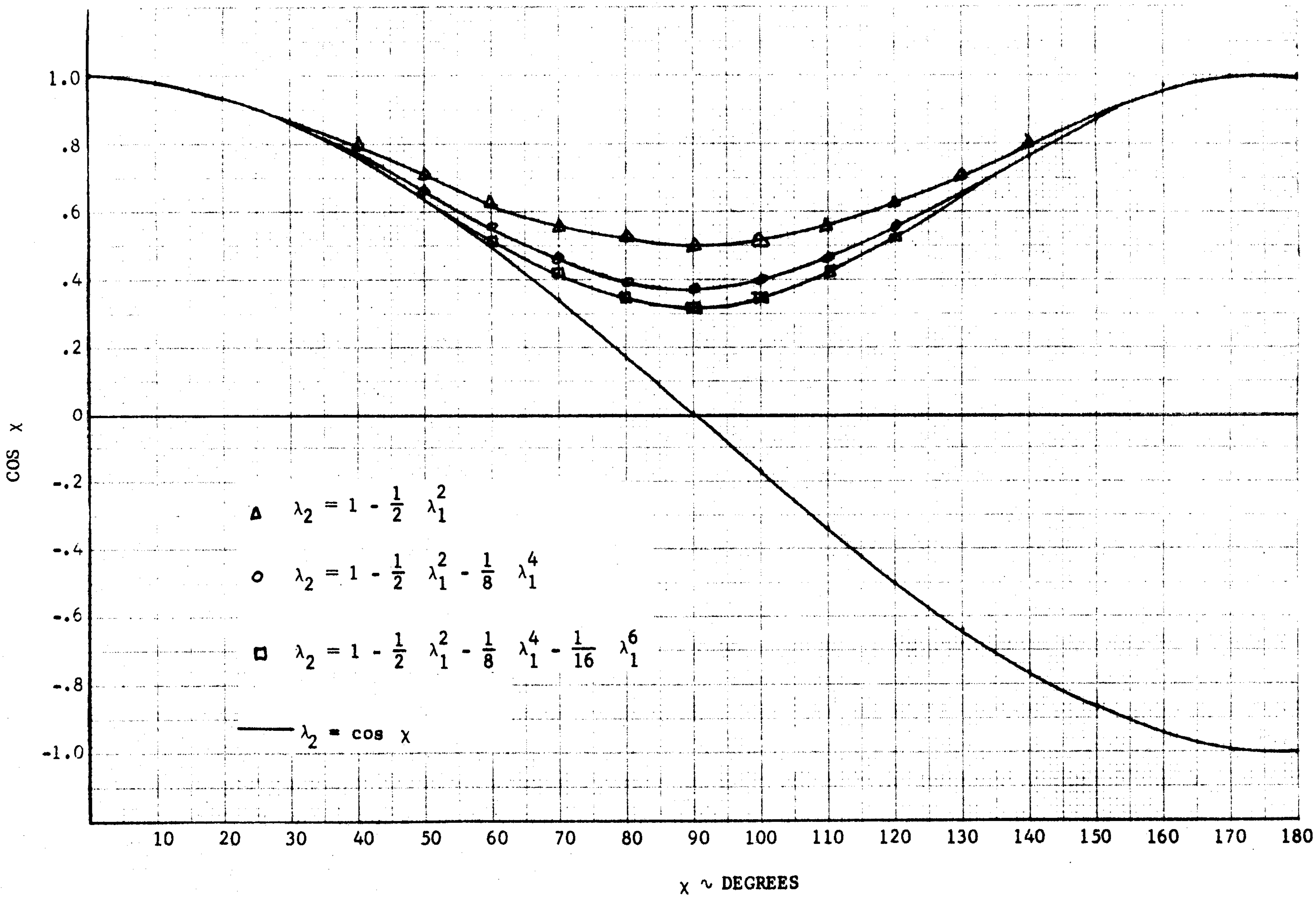

Figure 4-6. BINOMIAL EXPANSION OF SCALING EQUATION COMPARED TO Cos χ

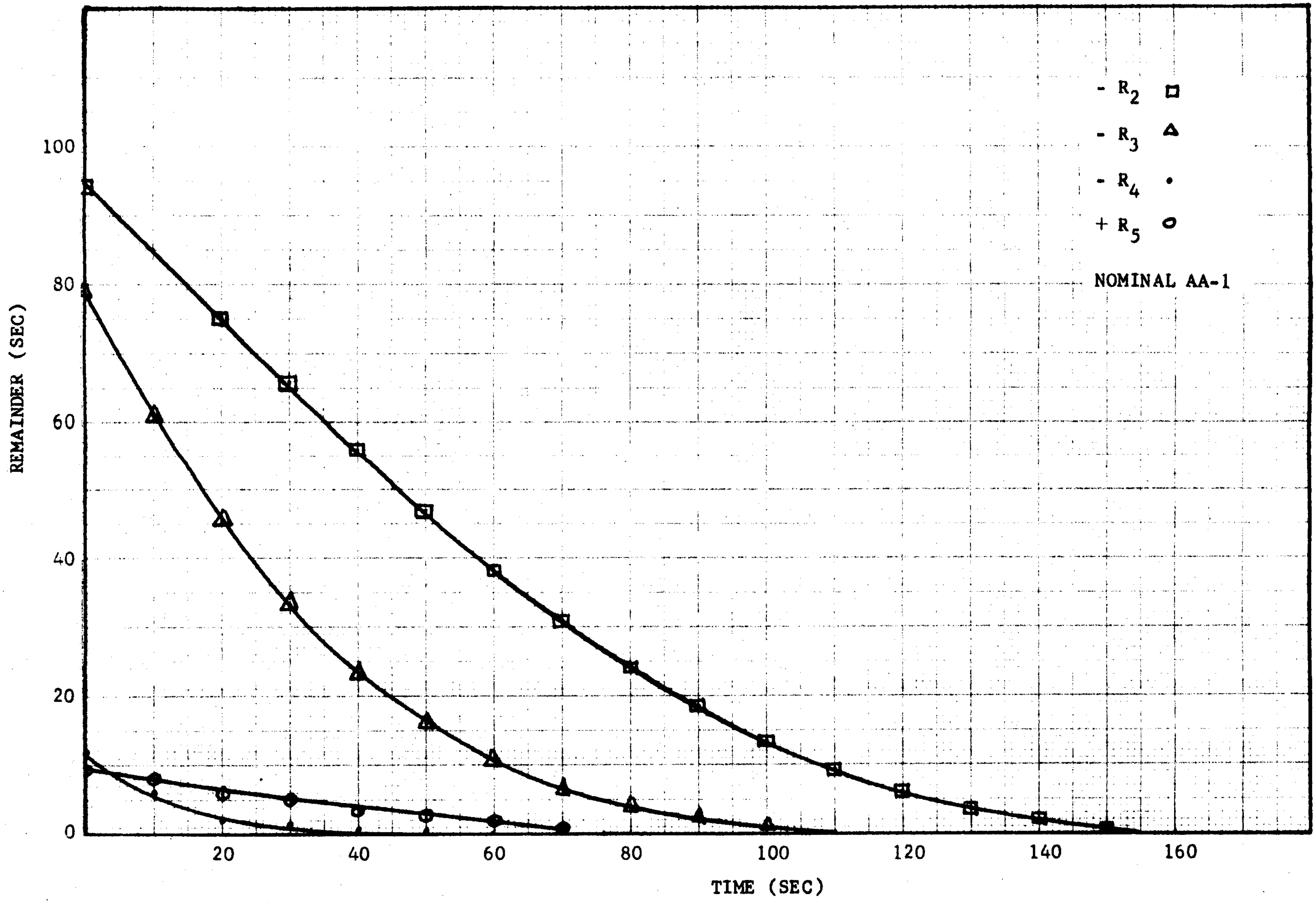

Figure 4-7. TIME HISTORY OF REMAINDER TERMS OF RADIUS EQUATION

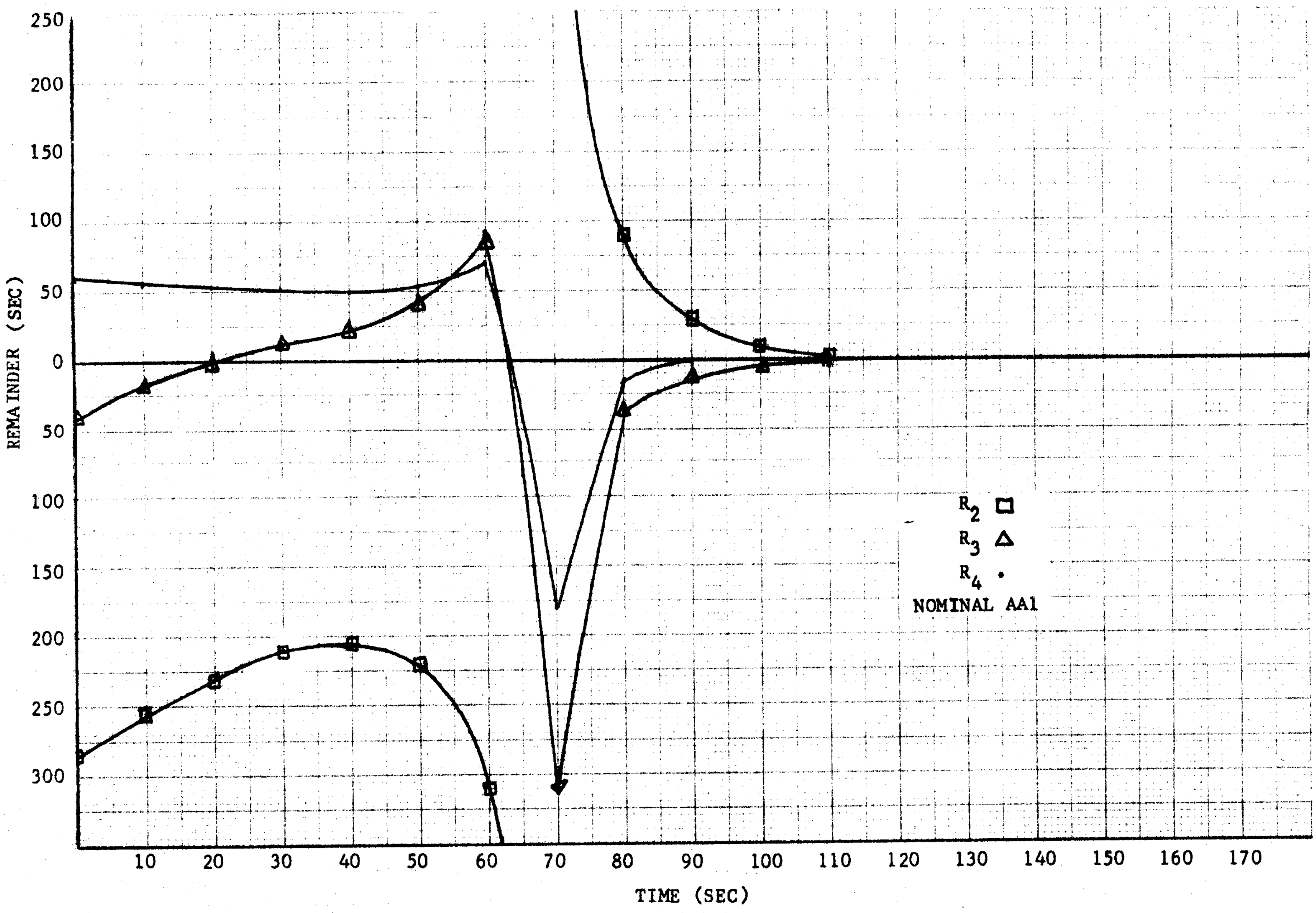

Figure 4-8. TIME HISTORY OF REMAINDER TERMS OF ORTHOGONALITY EQUATION

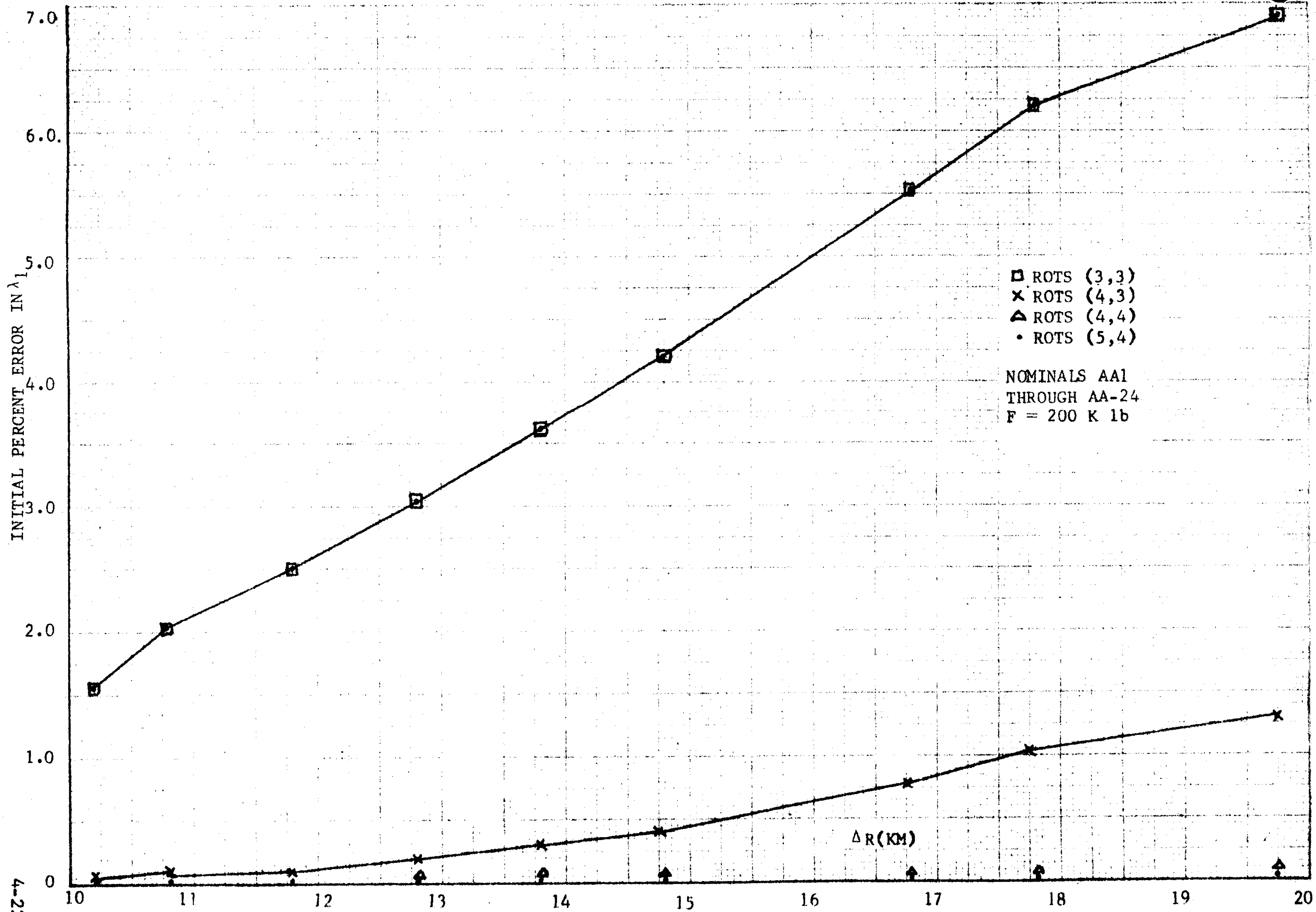

Figure 4-9. ABSOLUTE VALUE OF INITIAL PERCENT ERROR IN λ_1 FROM ROTS EQUATION VS. ΔR

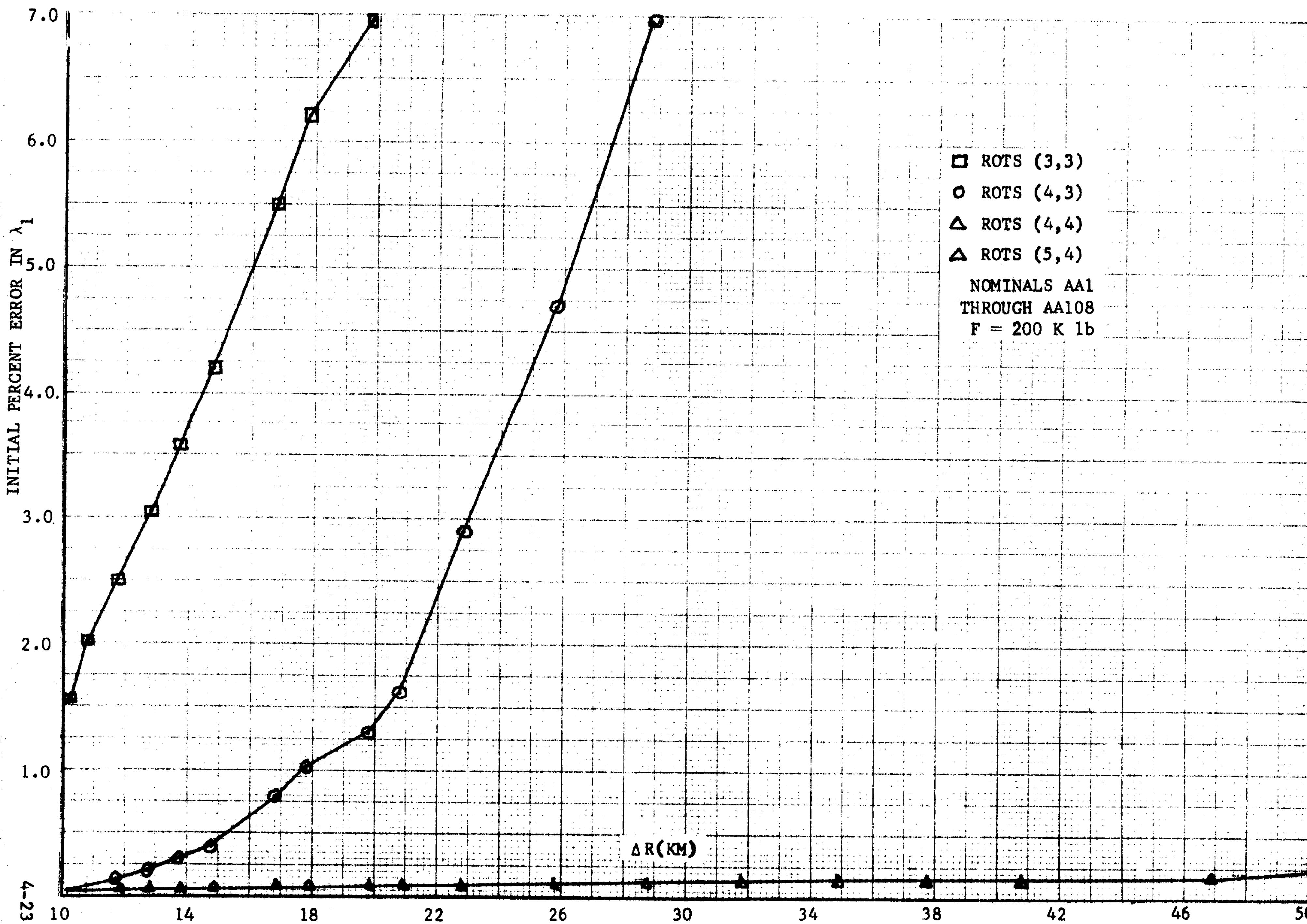

Figure 4-10. ABSOLUTE VALUE OF INITIAL PERCENT ERROR IN λ_1 FROM ROTS EQUATIONS VS. ΔR

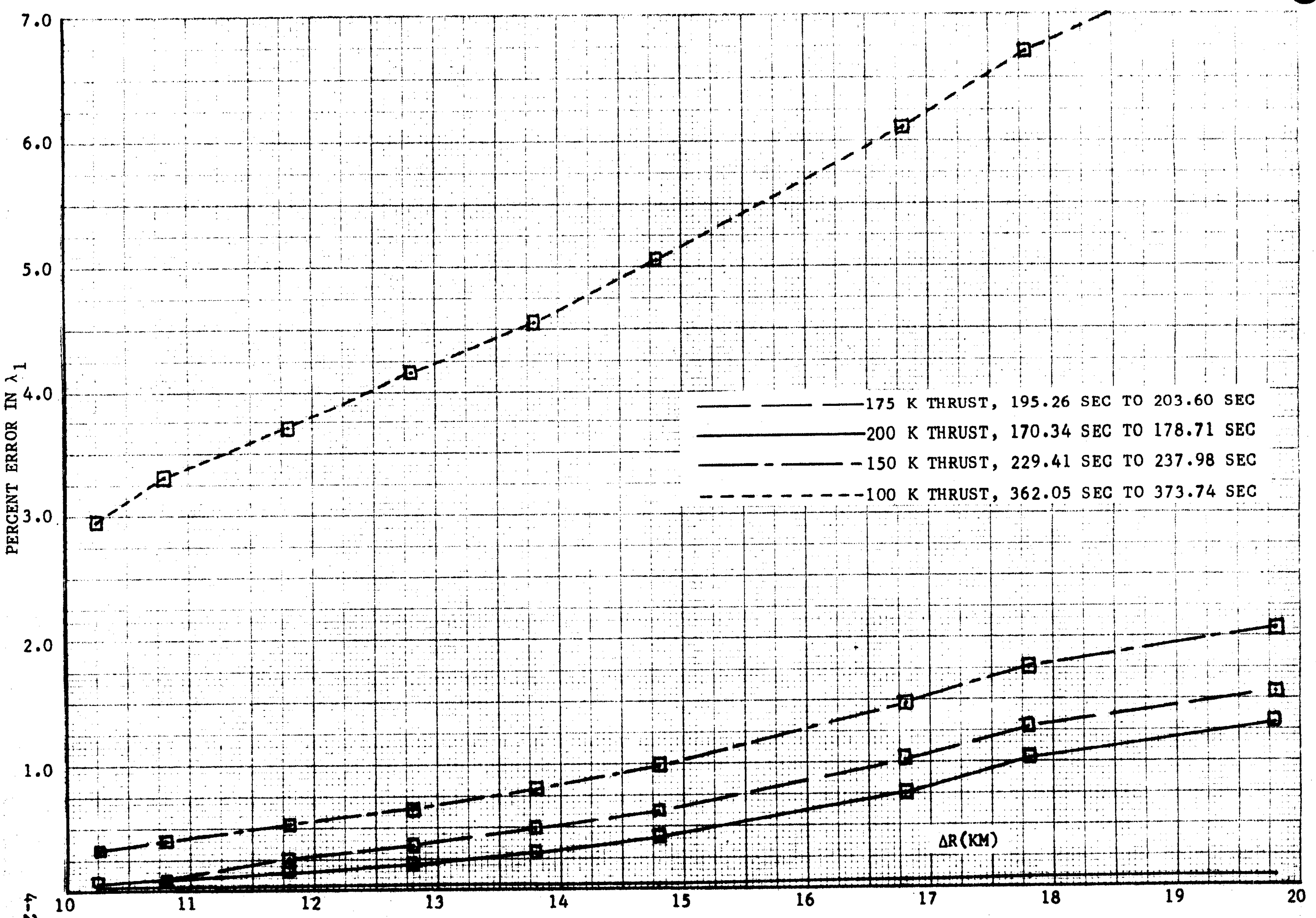

Figure 4-11. ABSOLUTE VALUE OF INITIAL PERCENT ERROR IN λ_1 FROM (4,3) ROTS EQUATIONS VS. ΔR

Table 4-1. R AND Q COEFFICIENTS IN TERMS OF STATE VARIABLES AND ΔT, FOR FIFTH-ORDER RADIUS SERIES AND FOURTH-ORDER ORTHOGONALITY SERIES

$$R_{oooo} = (R_{\alpha}^2 - R_{co}^2) + 2(xu + yv)(\Delta t) + (V_o^2 - xV_x$$

$$- yV_y)(\Delta t)^2 + (- \frac{1}{3} xuV_{xx} - \frac{1}{3} xvV_{xy} - \frac{1}{3} yuV_{xy}$$

$$- \frac{1}{3} yvV_{yy} - uV_x - vV_y)(\Delta t)^3 + (\frac{1}{12} xV_xV_{xx}$$

$$+ \frac{1}{12} xV_yV_{xy} - \frac{1}{12} xu^2V_{xxx} - \frac{1}{6} xuvV_{xxy} - \frac{1}{12} xv^2V_{xyy}$$

$$+ \frac{1}{12} yV_xV_{xy} + \frac{1}{12} yV_yV_{yy} - \frac{1}{12} yu^2V_{xxy} - \frac{1}{6} yuvV_{xyy}$$

$$- \frac{1}{12} yv^2V_{yyy} - \frac{1}{3} u^2V_{xx} - \frac{1}{3} uvV_{xy} - \frac{1}{3} uvV_{xy}$$

$$- \frac{1}{3} v^2V_{yy} + \frac{1}{4} V_x^2 + \frac{1}{4} V_y^2)(\Delta t)^4 + (\frac{1}{60} xuV_{xx}^2$$

$$+ \frac{1}{60} xvV_{xx}V_{xy} + \frac{1}{60} xvV_{xy}^2 + \frac{1}{60} xvV_{xy}V_{yy} + \frac{1}{20} xuV_xV_{xxx}$$

$$+ \frac{1}{20} xvV_xV_{xxy} + \frac{1}{20} xuV_yV_{xxy} + \frac{1}{20} xvV_yV_{xyy}$$

$$- \frac{1}{60} xu^3V_{xxxx} - \frac{1}{20} xu^2vV_{xxxy} - \frac{1}{20} xuv^2V_{xxyy}$$

$$- \frac{1}{60} xv^3V_{xyyy} + \frac{1}{60} yuV_{xx}V_{xy} + \frac{1}{60} yvV_{xy}^2 + \frac{1}{60} yuV_{xy}V_{yy}$$

$$+ \frac{1}{60} yvV_{yy}^2 + \frac{1}{20} yuV_xV_{xxy} + \frac{1}{20} yvV_xV_{xyy} + \frac{1}{20} yuV_yV_{xyy}$$

$$+ \frac{1}{20} yvV_yV_{yyy} - \frac{1}{60} yu^3V_{xxxy} - \frac{1}{20} yu^2vV_{xxyy} - \frac{1}{20} yuv^2V_{xyyy}$$

$$- \frac{1}{60} yv^3V_{yyyy} + \frac{1}{4} uV_xV_{xx} + \frac{1}{4} vV_xV_{xy} + \frac{1}{4} vV_yV_{yy}$$

$$+ \frac{1}{4} uV_yV_{xy} - \frac{1}{4} vu^2V_{xxy} - \frac{1}{4} uv^2V_{xyy} - \frac{1}{12} u^3V_{xxx}$$

$$- \frac{1}{12} v^3V_{yyy})(\Delta t)^5$$

Table 4-1. (Continued)

$$R_{1000} = (x \frac{F}{M})(\Delta t)^2 + (-\frac{1}{3} x \frac{F}{M}\frac{\dot{M}}{M} + u \frac{F}{M})(\Delta t)^3 + (\frac{1}{6} x \frac{F}{M}\frac{\dot{M}^2}{M^2}$$

$$- \frac{1}{12} x \frac{F}{M} V_{xx} - \frac{1}{12} y \frac{F}{M} V_{xy} - \frac{1}{3} u \frac{F}{M}\frac{\dot{M}}{M} - \frac{1}{2}\frac{F}{M} V_x)(\Delta t)^4$$

$$+ (-\frac{1}{20} x \frac{F}{M} u V_{xxx} - \frac{1}{20} x \frac{F}{M} v V_{xxy} + \frac{1}{60} x \frac{F}{M} V_{xx}\frac{\dot{M}}{M}$$

$$- \frac{1}{10} x \frac{F}{M}\frac{\dot{M}^3}{M^3} - \frac{1}{20} y \frac{F}{M} u V_{xxy} - \frac{1}{20} y \frac{F}{M} v V_{xyy} + \frac{1}{60} y V_{xy}\frac{\dot{M}}{M}\frac{F}{M}$$

$$+ \frac{1}{6} u \frac{F}{M}\frac{\dot{M}^2}{M^2} - \frac{1}{12} u \frac{F}{M} V_{xx} - \frac{1}{12} v \frac{F}{M} V_{xy} - \frac{1}{6}\frac{F}{M} u V_{xx}$$

$$- \frac{1}{6} v V_{xy}\frac{F}{M} + \frac{1}{6} V_x \frac{F}{M}\frac{\dot{M}}{M})(\Delta t)^5$$

$$R_{0100} = (y \frac{F}{M})(\Delta t)^2 + (-\frac{1}{3} y \frac{F}{M}\frac{\dot{M}}{M} + v \frac{F}{M})(\Delta t)^3 + (-\frac{1}{12} x \frac{F}{M} V_{xy}$$

$$+ \frac{1}{6} y \frac{F}{M}\frac{\dot{M}^2}{M^2} - \frac{1}{12} y \frac{F}{M} V_{yy} - \frac{1}{3} v \frac{F}{M}\frac{\dot{M}}{M} - \frac{1}{2}\frac{F}{M} V_y)(\Delta t)^4$$

$$+ (-\frac{1}{20} x \frac{F}{M} u V_{xxy} - \frac{1}{20} x \frac{F}{M} v V_{xyy} + \frac{1}{60} x \frac{F}{M} V_{xy}\frac{\dot{M}}{M}$$

$$- \frac{1}{20} y \frac{F}{M} u V_{xyy} - \frac{1}{20} y \frac{F}{M} v V_{yyy} + \frac{1}{60} y \frac{F}{M} V_{yy}\frac{\dot{M}}{M} - \frac{1}{10} y \frac{F}{M}\frac{\dot{M}^3}{M^3}$$

$$- \frac{1}{12} u \frac{F}{M} V_{xy} + \frac{1}{6} v \frac{F}{M}\frac{\dot{M}^2}{M^2} - \frac{1}{12} v \frac{F}{M} V_{yy} + \frac{1}{6} V_y \frac{F}{M}\frac{\dot{M}}{M}$$

$$- \frac{1}{6}\frac{F}{M} u V_{xy} - \frac{1}{6}\frac{F}{M} v V_{yy})(\Delta t)^5$$

$$R_{2010} = (\frac{1}{3} x \frac{F}{M})(\Delta t)^3 + (-\frac{1}{30} y \frac{F}{M} V_{xy})(\Delta t)^5$$

Table 4-1. (Continued)

$$R_{1101} = (\frac{1}{3} x \frac{F}{M})(\Delta t)^3 + (-\frac{1}{6} x \frac{F}{M}\frac{\dot{M}}{M} + \frac{1}{3} u \frac{F}{M})(\Delta t)^4 + (-\frac{1}{15} x \frac{F}{M} V_{xx}$$

$$- \frac{1}{15} x \frac{F}{M} V_{yy} + \frac{1}{10} x \frac{F}{M}\frac{\dot{M}^2}{M^2} - \frac{1}{12} y \frac{F}{M} V_{xy} - \frac{1}{6} u \frac{F}{M}\frac{\dot{M}}{M} - \frac{1}{6}\frac{F}{M} V_x)(\Delta t)^5$$

$$R_{0010} = (-\frac{1}{3} x \frac{F}{M})(\Delta t)^3$$

$$R_{1110} = (\frac{1}{3} y \frac{F}{M})(\Delta t)^3 + (-\frac{1}{6} y \frac{F}{M}\frac{\dot{M}}{M} + \frac{1}{3} v \frac{F}{M})(\Delta t)^4$$

$$+ (-\frac{1}{12} x \frac{F}{M} V_{xy} - \frac{1}{15} y \frac{F}{M} V_{xx} - \frac{1}{15} y \frac{F}{M} V_{yy} + \frac{1}{10} y \frac{F}{M}\frac{\dot{M}^2}{M^2}$$

$$- \frac{1}{6} v \frac{F}{M}\frac{\dot{M}}{M} - \frac{1}{6}\frac{F}{M} V_y)(\Delta t)^5$$

$$R_{0201} = (\frac{1}{3} y \frac{F}{M})(\Delta t)^3 + (-\frac{1}{30} x \frac{F}{M} V_{xy})(\Delta t)^5$$

$$R_{0001} = (-\frac{1}{3} y \frac{F}{M})(\Delta t)^3$$

$$R_{2100} = (\frac{1}{12} x \frac{F}{M} V_{xy} + \frac{1}{12} y \frac{F}{M} V_{xx} - \frac{1}{12} y \frac{F}{M} V_{yy})(\Delta t)^4 + (\frac{1}{60} x \frac{F}{M} u V_{xxy}$$

$$+ \frac{1}{60} x \frac{F}{M} v V_{xyy} - \frac{1}{20} x \frac{F}{M} V_{xy}\frac{\dot{M}}{M} + \frac{1}{60} y \frac{F}{M} u V_{xxx} + \frac{1}{60} y \frac{F}{M} v V_{xxy}$$

$$- \frac{1}{60} y \frac{F}{M} u V_{xyy} - \frac{1}{60} y \frac{F}{M} v V_{yyy} - \frac{1}{20} y \frac{F}{M} V_{xx}\frac{\dot{M}}{M} + \frac{1}{20} y \frac{F}{M} V_{yy}\frac{\dot{M}}{M}$$

$$+ \frac{1}{12} u \frac{F}{M} V_{xy} + \frac{1}{12} v \frac{F}{M} V_{xx} - \frac{1}{12} v \frac{F}{M} V_{yy})(\Delta t)^5$$

Table 4-1. (Continued)

$$R_{2111} = (\frac{1}{2} x \frac{F}{M})(\Delta t)^4 + (-\frac{3}{10} x \frac{F}{M}\frac{\dot{M}}{M} + \frac{1}{2} u \frac{F}{M})(\Delta t)^5$$

$$R_{1200} = (-\frac{1}{12} x \frac{F}{M} V_{xx} + \frac{1}{12} x \frac{F}{M} V_{yy} + \frac{1}{12} y \frac{F}{M} V_{xy})(\Delta t)^4 + (-\frac{1}{60} x \frac{F}{M} u V_{xxx}$$
$$- \frac{1}{60} x \frac{F}{M} v V_{xxy} + \frac{1}{60} x \frac{F}{M} u V_{xyy} + \frac{1}{60} x \frac{F}{M} v V_{yyy} + \frac{1}{20} x \frac{F}{M} V_{xx} \frac{\dot{M}}{M}$$
$$- \frac{1}{20} x \frac{F}{M} V_{yy} \frac{\dot{M}}{M} + \frac{1}{60} y \frac{F}{M} u V_{xxy} - \frac{1}{60} y \frac{F}{M} v V_{xyy} - \frac{1}{20} y \frac{F}{M} V_{xy} \frac{\dot{M}}{M}$$
$$- \frac{1}{12} u \frac{F}{M} V_{xx} + \frac{1}{12} u \frac{F}{M} V_{yy} + \frac{1}{12} v \frac{F}{M} V_{xy})(\Delta t)^5$$

$$R_{1220} = (-\frac{1}{4} x \frac{F}{M})(\Delta t)^4 + (\frac{3}{20} x \frac{F}{M}\frac{\dot{M}}{M} - \frac{1}{4} u \frac{F}{M})(\Delta t)^5$$

$$R_{1202} = -R_{1220}$$

$$R_{1002} = \frac{1}{3} R_{1220}$$

$$R_{0300} = (-\frac{1}{12} x \frac{F}{M} V_{xy})(\Delta t)^4 + (-\frac{1}{60} x \frac{F}{M} u V_{xxy} - \frac{1}{60} x \frac{F}{M} v V_{xyy}$$
$$+ \frac{1}{20} x V_{xy} \frac{\dot{M}}{M}\frac{F}{M} - \frac{1}{12} u \frac{F}{M} V_{xy})(\Delta t)^5$$

$$R_{0111} = (-\frac{1}{6} x \frac{F}{M})(\Delta t)^4 + (\frac{1}{10} x \frac{F}{M}\frac{\dot{M}}{M} - \frac{1}{6} u \frac{F}{M})(\Delta t)^5$$

Table 4-1. (Continued)

$$R_{0210} = \left(\frac{1}{6} x \frac{F}{M} \frac{\dot{M}}{M} - \frac{1}{3} u \frac{F}{M}\right)(\Delta t)^4 + \left(\frac{1}{30} x \frac{F}{M} V_{xx} - \frac{1}{20} x \frac{F}{M} V_{yy} - \frac{1}{10} x \frac{F}{M} \frac{\dot{M}^2}{M^2} - \frac{1}{30} y \frac{F}{M} V_{xy} + \frac{1}{6} u \frac{F}{M} \frac{\dot{M}}{M} + \frac{1}{6} \frac{F}{M} V_x\right)(\Delta t)^5$$

$$R_{3000} = \left(-\frac{1}{12} y \frac{F}{M} V_{xy}\right)(\Delta t)^4 + \left(-\frac{1}{60} y \frac{F}{M} u V_{xxy} - \frac{1}{60} y \frac{F}{M} v V_{xyy} + \frac{1}{20} y \frac{F}{M} V_{xy} \frac{\dot{M}}{M} - \frac{1}{12} v \frac{F}{M} V_{xy}\right)(\Delta t)^5$$

$$R_{2120} = \left(\frac{1}{4} y \frac{F}{M}\right)(\Delta t)^4 + \left(-\frac{3}{20} y \frac{F}{M} \frac{\dot{M}}{M} + \frac{1}{4} v \frac{F}{M}\right)(\Delta t)^5$$

$$R_{2102} = -R_{2120}$$

$$R_{2001} = \left(\frac{1}{6} y \frac{F}{M} \frac{\dot{M}}{M} - \frac{1}{3} v \frac{F}{M}\right)(\Delta t)^4 + \left(-\frac{1}{30} x \frac{F}{M} V_{xy} + \frac{1}{30} y \frac{F}{M} V_{yy} - \frac{1}{20} y \frac{F}{M} V_{xx} - \frac{1}{10} y \frac{F}{M} \frac{\dot{M}^2}{M^2} + \frac{1}{6} v \frac{F}{M} \frac{\dot{M}}{M} + \frac{1}{6} \frac{F}{M} V_y\right)(\Delta t)^5$$

$$R_{1211} = \left(\frac{1}{2} y \frac{F}{M}\right)(\Delta t)^4 + \left(-\frac{3}{10} y \frac{F}{M} \frac{\dot{M}}{M} + \frac{1}{2} v \frac{F}{M}\right)(\Delta t)^5$$

$$R_{1011} \doteq \left(-\frac{1}{6} y \frac{F}{M}\right)(\Delta t)^4 + \left(\frac{1}{10} y \frac{F}{M} \frac{\dot{M}}{M} - \frac{1}{6} v \frac{F}{M}\right)(\Delta t)^5$$

Table 4-1. (Continued)

$$R_{0120} = \frac{1}{2} R_{1011}$$

$$R_{2000} = (\frac{1}{4} \frac{F^2}{M^2})(\Delta t)^4 + (- \frac{1}{6} \frac{F}{M} \frac{F}{M} \frac{\dot{M}}{M})(\Delta t)^5$$

$$R_{0200} = R_{2000}$$

$$R_{4030} = - \frac{3}{4} R_{0010}$$

$$R_{3110} = (\frac{3}{20} x \frac{F}{M} V_{xy} + \frac{3}{20} y \frac{F}{M} V_{xx})(\Delta t)^5$$

$$R_{3121} = 3 R_{4030} = - \frac{9}{4} R_{0010}$$

$$R_{3101} = (\frac{3}{20} x \frac{F}{M} V_{xx} - \frac{3}{20} y \frac{F}{M} V_{xy})(\Delta t)^5$$

$$R_{2210} = (- \frac{3}{20} x \frac{F}{M} V_{xx} + \frac{3}{20} x \frac{F}{M} V_{yy} + \frac{3}{10} y \frac{F}{M} V_{xy})(\Delta t)^5$$

$$R_{2212} = 3 R_{4030} = - \frac{9}{4} R_{0010}$$

$$R_{2201} = (\frac{3}{10} x \frac{F}{M} V_{xy} + \frac{3}{20} y \frac{F}{M} V_{xx} - \frac{3}{20} y \frac{F}{M} V_{yy})(\Delta t)^5$$

$$R_{2030} = - \frac{12}{10} R_{4030} = \frac{36}{40} R_{0010}$$

Table 4-1. (Continued)

$$R_{1121} = -\frac{9}{5} R_{4030} = \frac{27}{20} R_{0010}$$

$$R_{1103} = -\frac{3}{5} R_{4030} = \frac{9}{20} R_{0010}$$

$$R_{1310} = \left(-\frac{3}{20} x \frac{F}{M} V_{xy} + \frac{3}{20} y \frac{F}{M} V_{yy}\right)(\Delta t)^5$$

$$R_{1301} = \left(\frac{3}{20} x \frac{F}{M} V_{yy} + \frac{3}{20} y \frac{F}{M} V_{xy}\right)(\Delta t)^5$$

$$R_{1303} = R_{4030} = -\frac{3}{4} R_{0010}$$

$$R_{0030} = \frac{1}{5} R_{4030} = -\frac{3}{20} R_{0010}$$

$$R_{0012} = -\frac{4}{10} R_{4030} = \frac{12}{40} R_{0010}$$

$$R_{3130} = -\frac{3}{4} R_{0001}$$

$$R_{2221} = 3 R_{3130} = -\frac{9}{4} R_{0001}$$

$$R_{1112} = -\frac{9}{5} R_{3130} = \frac{27}{20} R_{0001}$$

$$R_{1130} = -\frac{3}{5} R_{3130} = \frac{9}{20} R_{0001}$$

Table 4-1. (Continued)

$$R_{1312} = 3\ R_{3130} = -\ \frac{9}{4}\ R_{0001}$$

$$R_{0403} = R_{3130} = -\ \frac{3}{4}\ R_{0001}$$

$$R_{0203} = -\ \frac{12}{10}\ R_{3130} = \frac{36}{40}\ R_{0001}$$

$$R_{0021} = -\ \frac{4}{10}\ R_{3130} = \frac{12}{40}\ R_{0001}$$

$$R_{0003} = \frac{1}{5}\ R_{0403} = \frac{1}{5}\ R_{3130} = -\ \frac{3}{20}\ R_{0001}$$

Table 4-1. (Continued)

$$Q_{oooo} = (xu + yv) + (- xV_x - yV_y + V_o^2)(\Delta t) + (- \tfrac{1}{2} xu\, V_{xx}$$

$$- \tfrac{1}{2} xv\, V_{xy} - \tfrac{1}{2} yu\, V_{xy} - \tfrac{1}{2} yv\, V_{yy} - \tfrac{3}{2} v\, V_y - \tfrac{3}{2} u\, V_x)\ (\Delta t)^2$$

$$+ (\tfrac{1}{6} x\, V_x\, V_{xx} + \tfrac{1}{6} x\, V_y\, V_{xy} - \tfrac{1}{6} xu^2\, V_{xxx} - \tfrac{1}{3} xuv\, V_{xxy}$$

$$- \tfrac{1}{6} xv^2\, V_{xyy} + \tfrac{1}{6} y\, V_x\, V_{xy} + \tfrac{1}{6} y\, V_y V_{yy} - \tfrac{1}{6} yu^2\, V_{xxy} - \tfrac{1}{3} yuv\, V_{xyy}$$

$$- \tfrac{1}{6} yv^2\, V_{yyy} - \tfrac{2}{3} u^2\, V_{xx} - \tfrac{2}{3} uv\, V_{xy} - \tfrac{2}{3} vu\, V_{xy} - \tfrac{2}{3} v^2\, V_{yy} + \tfrac{1}{2} \tfrac{F^2}{M^2}$$

$$+ \tfrac{1}{2} V_x^2 + \tfrac{1}{2} V_y^2)(\Delta t)^3 + (\tfrac{1}{24} xu\, V_{xx}^2 + \tfrac{1}{24} xv\, V_{xx}\, V_{xy} + \tfrac{1}{24} xu\, V_{xy}^2$$

$$+ \tfrac{1}{24} xv\, V_{xy}\, V_{yy} + \tfrac{1}{8} xu\, V_x V_{xxx} + \tfrac{1}{8} xv\, V_x V_{xxy} + \tfrac{1}{8} xu\, V_y V_{xxy} + \tfrac{1}{8} xv\, V_y\, V_{xyy}$$

$$- \tfrac{1}{24} xu^3 V_{xxxx} - \tfrac{1}{8} xu^2 v\, V_{xxxy} - \tfrac{1}{8} xuv^2\, V_{xxyy} - \tfrac{1}{24} xv^3\, V_{xyyy} + \tfrac{1}{24} yuV_{xx} V_{xy}$$

$$+ \tfrac{1}{24} yvV_{xy}^2 + \tfrac{1}{24} yuV_{xy} V_{yy} + \tfrac{1}{24} yv\, V_{yy}^2 + \tfrac{1}{8} yuV_x V_{xxy} + \tfrac{1}{8} yvV_x V_{xyy}$$

$$+ \tfrac{1}{8} yuV_y\, V_{xyy} + \tfrac{1}{8} yv\, V_y\, V_{yyy} - \tfrac{1}{24} yu^3\, V_{xxxy} - \tfrac{1}{8} yu^2 vV_{xxyy} - \tfrac{1}{8} yuv^2 V_{xyyy}$$

$$- \tfrac{1}{24} yv^3 V_{yyyy} + \tfrac{5}{12} u\, V_x V_{xx} + \tfrac{5}{12} u\, V_y V_{xy} - \tfrac{5}{12} u^3\, V_{xxx} - \tfrac{5}{6} u^2 v\, V_{xxy}$$

$$- \tfrac{5}{12} uv^2 V_{xyy} + \tfrac{5}{24} v\, V_x V_{xy} + \tfrac{5}{24} v\, V_y\, V_{yy} - \tfrac{5}{24} vu^2 V_{xxy} - \tfrac{5}{12} uv^2\, V_{xyy}$$

$$- \tfrac{5}{24} v^3\, V_{yyy} + \tfrac{5}{12} V_x\, u\, V_{xx} + \tfrac{5}{12} V_x\, v\, V_{xy} + \tfrac{5}{12} V_y\, u\, V_{xy}$$

$$+ \tfrac{5}{12} V_y\, u\, V_{yy})(\Delta t)^4$$

Table 4-1. (Continued)

$$
\begin{aligned}
Q_{1000} = {} & \left(x \frac{F}{M}\right)(\Delta t) + \left(- \frac{1}{2} x \frac{F}{M} \frac{\dot{M}}{M} + \frac{3}{2} u \frac{F}{M}\right)(\Delta t)^2 + \left(\frac{1}{3} x \frac{F}{M} \frac{\dot{M}^2}{M^2}\right. \\
& \left. - \frac{1}{6} x \frac{F}{M} V_{xx} - \frac{1}{3} y \frac{F}{M} V_{xy} - \frac{2}{3} u \frac{F}{M} \frac{\dot{M}}{M} - \frac{F}{M} V_x\right)(\Delta t)^3 \\
& + \left(- \frac{1}{8} x \frac{F}{M} u V_{xxx} - \frac{1}{8} x \frac{F}{M} v V_{xxy} + \frac{1}{24} x \frac{F}{M} V_{xx} \frac{\dot{M}}{M} - \frac{1}{4} x \frac{F}{M} \frac{\dot{M}^3}{M^3}\right. \\
& + \frac{5}{12} u \frac{F}{M} \frac{\dot{M}^2}{M^2} - \frac{5}{24} u \frac{F}{M} V_{xx} - \frac{5}{24} v \frac{F}{M} V_{xy} - \frac{5}{12} \frac{F}{M} u V_{xx} \\
& - \frac{5}{12} \frac{F}{M} v V_{xy} + \frac{5}{12} V_x \frac{F}{M} \frac{\dot{M}}{M} - \frac{1}{8} y \frac{F}{M} u V_{xxy} - \frac{1}{8} y \frac{F}{M} v V_{xyy} \\
& \left. + \frac{1}{24} y V_{xy} \frac{\dot{M}}{M} \frac{F}{M}\right)(\Delta t)^4
\end{aligned}
$$

$$
\begin{aligned}
Q_{0100} = {} & \left(y \frac{F}{M}\right)(\Delta t) + \left(- \frac{1}{2} y \frac{F}{M} \frac{\dot{M}}{M} + \frac{3}{2} v \frac{F}{M}\right)(\Delta t)^2 + \left(- \frac{1}{3} x \frac{F}{M} V_{xy}\right. \\
& \left. + \frac{1}{3} y \frac{F}{M} \frac{\dot{M}^2}{M^2} - \frac{1}{6} y \frac{F}{M} V_{yy} - \frac{2}{3} v \frac{F}{M} \frac{\dot{M}}{M} - \frac{F}{M} V_y\right)(\Delta t)^3 \\
& + \left(- \frac{1}{8} x \frac{F}{M} u V_{xxy} - \frac{1}{8} x \frac{F}{M} v V_{xyy} + \frac{1}{24} x \frac{F}{M} V_{xy} \frac{\dot{M}}{M} - \frac{1}{8} y \frac{F}{M} u V_{xyy}\right. \\
& - \frac{1}{8} y \frac{F}{M} v V_{yyy} + \frac{1}{24} y \frac{F}{M} V_{yy} \frac{\dot{M}}{M} - \frac{1}{4} y \frac{F}{M} \frac{\dot{M}^3}{M^3} - \frac{5}{24} u \frac{F}{M} V_{xy} \\
& + \frac{5}{12} v \frac{F}{M} \frac{\dot{M}^2}{M^2} - \frac{5}{24} v \frac{F}{M} V_{yy} + \frac{5}{12} V_y \frac{F}{M} \frac{\dot{M}}{M} - \frac{5}{12} \frac{F}{M} u V_{xy} \\
& \left. - \frac{5}{12} \frac{F}{M} v V_{yy}\right)(\Delta t)^4
\end{aligned}
$$

$$
Q_{0010} = \left(- \frac{1}{2} x \frac{F}{M}\right)(\Delta t)^2 + \left(\frac{1}{3} x \frac{F}{M} \frac{\dot{M}}{M} - \frac{2}{3} u \frac{F}{M}\right)(\Delta t)^3
$$

$$
Q_{0001} = \left(- \frac{1}{2} y \frac{F}{M}\right)(\Delta t)^2 + \left(\frac{1}{3} y \frac{F}{M} \frac{\dot{M}}{M} - \frac{2}{3} v \frac{F}{M}\right)(\Delta t)^3
$$

Table 4-1. (Continued)

$$Q_{1110} = (\frac{1}{2} y \frac{F}{M})(\Delta t)^2 + (-\frac{1}{3} y \frac{F}{M}\frac{\dot{M}}{M} + \frac{2}{3} v \frac{F}{M})(\Delta t)^3 + (-\frac{5}{24} x \frac{F}{M} V_{xy}$$

$$- \frac{1}{6} y \frac{F}{M} V_{xx} - \frac{1}{6} y \frac{F}{M} V_{yy} + \frac{1}{4} y \frac{F}{M}\frac{\dot{M}^2}{M^2} - \frac{5}{12} v \frac{F}{M}\frac{\dot{M}}{M} - \frac{5}{12}\frac{F}{M} V_y)(\Delta t)^4$$

$$Q_{0201} = (\frac{1}{2} y \frac{F}{M})(\Delta t)^2 + (-\frac{1}{3} y \frac{F}{M}\frac{\dot{M}}{M} + \frac{2}{3} v \frac{F}{M})(\Delta t)^3 + (-\frac{1}{12} x \frac{F}{M} V_{xy})(\Delta t)^4$$

$$Q_{1101} = (\frac{1}{2} x \frac{F}{M})(\Delta t)^2 + (-\frac{1}{3} x \frac{F}{M}\frac{\dot{M}}{M} + \frac{2}{3} u \frac{F}{M})(\Delta t)^3 + (-\frac{1}{6} x \frac{F}{M} V_{xx}$$

$$- \frac{1}{6} x \frac{F}{M} V_{yy} + \frac{1}{4} x \frac{F}{M}\frac{\dot{M}^2}{M^2} - \frac{5}{24} y \frac{F}{M} V_{xy} - \frac{5}{12} u \frac{F}{M}\frac{\dot{M}}{M}$$

$$- \frac{5}{12}\frac{F}{M} V_x)(\Delta t)^4$$

$$Q_{2010} = (\frac{1}{2} x \frac{F}{M})(\Delta t)^2 + (-\frac{1}{3} x \frac{F}{M}\frac{\dot{M}}{M} + \frac{2}{3} u \frac{F}{M})(\Delta t)^3 + (-\frac{1}{12} y \frac{F}{M} V_{xy})(\Delta t)^4$$

$$Q_{2212} = 15\ Q_{0030}$$

$$Q_{2201} = (\frac{3}{4} x \frac{F}{M} V_{xy})(\Delta t)^4 + (\frac{3}{8} y \frac{F}{M} V_{xx} - \frac{3}{8} y \frac{F}{M} V_{yy})(\Delta t)^4$$

$$Q_{2030} = -6\ Q_{0030}$$

Table 4-1. (Continued)

$$Q_{2001} = \left(-\frac{1}{12} x \frac{F}{M} V_{xy} + \frac{1}{12} y \frac{F}{M} V_{yy} - \frac{1}{8} y \frac{F}{M} V_{xx} - \frac{1}{4} y \frac{F}{M} \frac{\dot{M}^2}{M^2}\right.$$

$$\left. + \frac{5}{12} v \frac{F}{M} \frac{\dot{M}}{M} + \frac{5}{12} \frac{F}{M} V_y\right)(\Delta t)^4$$

$$Q_{1121} = -9\, Q_{0030}$$

$$Q_{1103} = -3\, Q_{0030}$$

$$Q_{1310} = \left(-\frac{3}{8} x \frac{F}{M} V_{xy} + \frac{3}{8} y \frac{F}{M} V_{xy}\right)(\Delta t)^4$$

$$Q_{1301} = \left(\frac{3}{8} x \frac{F}{M} V_{yy} + \frac{3}{8} y \frac{F}{M} V_{xy}\right)(\Delta t)^4$$

$$Q_{1303} = 5\, Q_{0030}$$

$$Q_{0300} = \left(-\frac{1}{24} x \frac{F}{M} u V_{xxy} - \frac{1}{24} x \frac{F}{M} v V_{xyy} + \frac{1}{8} x \frac{F}{M} V_{xy} \frac{\dot{M}}{M}\right.$$

$$\left. - \frac{5}{24} u \frac{F}{M} V_{xy}\right)(\Delta t)^4$$

$$Q_{0030} = \left(\frac{1}{8} x \frac{F}{M}\right)(\Delta t)^4$$

Table 4-1. (Continued)

$$Q_{0210} = \left(\frac{1}{12} x \frac{F}{M} V_{xx} - \frac{1}{8} x \frac{F}{M} V_{yy} - \frac{1}{4} x \frac{F}{M} \frac{\dot{M}^2}{M^2} - \frac{1}{12} y \frac{F}{M} V_{xy} + \frac{5}{12} u \frac{F}{M} \frac{\dot{M}}{M} + \frac{5}{12} V_x\right)(\Delta t)^4$$

$$Q_{0012} = -\frac{1}{2} Q_{0030}$$

$$Q_{3000} = \left(- \frac{1}{24} y \frac{F}{M} u V_{xxy} - \frac{1}{24} y \frac{F}{M} v V_{xyy} + \frac{1}{8} y \frac{F}{M} V_{xy} \frac{\dot{M}}{M} - \frac{5}{24} v \frac{F}{M} V_{xy}\right)(\Delta t)^4$$

$$Q_{3130} = 5 Q_{0003}$$

$$Q_{2221} = 15 Q_{0003}$$

$$Q_{1112} = -9 Q_{0003}$$

$$Q_{2111} = \left(x \frac{F}{M}\right)(\Delta t)^3 + \left(- \frac{3}{4} x \frac{F}{M} \frac{\dot{M}}{M} + \frac{5}{4} u \frac{F}{M}\right)(\Delta t)^4$$

$$Q_{1202} = - Q_{1220}$$

Table 4-1. (Continued)

$$Q_{2100} = \left(\frac{1}{3} x \frac{F}{M} V_{xy} + \frac{1}{6} y \frac{F}{M} V_{xx} - \frac{1}{6} y \frac{F}{M} V_{yy}\right)(\Delta t)^3 + \left(\frac{1}{24} x \frac{F}{M} u V_{xxy} + \frac{1}{24} x \frac{F}{M} v V_{xyy} - \frac{1}{8} x \frac{F}{M} V_{xy} \frac{\dot{M}}{M} + \frac{1}{24} y \frac{F}{M} u V_{xxx} + \frac{1}{24} y \frac{F}{M} v V_{xxy} - \frac{1}{24} y \frac{F}{M} u V_{xyy} - \frac{1}{24} y \frac{F}{M} v V_{yyy} - \frac{1}{8} y \frac{F}{M} V_{xx} \frac{\dot{M}}{M} + \frac{1}{8} y \frac{F}{M} V_{yy} \frac{\dot{M}}{M} + \frac{5}{24} u \frac{F}{M} V_{xy} + \frac{5}{24} v \frac{F}{M} V_{xx} - \frac{5}{24} v \frac{F}{M} V_{yy}\right)(\Delta t)^4$$

$$Q_{1200} = \left(-\frac{1}{6} x \frac{F}{M} V_{xx} + \frac{1}{6} x \frac{F}{M} V_{yy} + \frac{1}{3} y \frac{F}{M} V_{xy}\right)(\Delta t)^3 + \left(-\frac{1}{24} x \frac{F}{M} u V_{xxx} - \frac{1}{24} x \frac{F}{M} v V_{xxy} + \frac{1}{24} x \frac{F}{M} u V_{xyy} + \frac{1}{24} x \frac{F}{M} v V_{yyy} + \frac{1}{8} x \frac{F}{M} V_{xx} \frac{\dot{M}}{M} - \frac{1}{8} x V_{yy} \frac{\dot{M}}{M} \frac{F}{M} + \frac{1}{24} y \frac{F}{M} u V_{xxy} + \frac{1}{24} y \frac{F}{M} v V_{xyy} - \frac{1}{8} y \frac{F}{M} V_{xy} \frac{\dot{M}}{M} - \frac{5}{24} u \frac{F}{M} V_{xx} + \frac{5}{24} u \frac{F}{M} V_{yy} + \frac{5}{24} v \frac{F}{M} V_{xy}\right)(\Delta t)^4$$

$$Q_{1220} = \left(-\frac{1}{2} x \frac{F}{M}\right)(\Delta t)^3 + \left(\frac{3}{8} x \frac{F}{M} \frac{\dot{M}}{M} - \frac{5}{8} u \frac{F}{M}\right)(\Delta t)^4$$

$$Q_{1002} = 3\, Q_{1220}$$

$$Q_{0111} = \frac{2}{3} Q_{1220}$$

$$Q_{2120} = \left(\frac{1}{2} y \frac{F}{M}\right)(\Delta t)^3 + \left(-\frac{3}{8} y \frac{F}{M} \frac{\dot{M}}{M} + \frac{5}{8} v \frac{F}{M}\right)(\Delta t)^4$$

Table 4-1. (Continued)

$$Q_{1202} = -Q_{2120}$$

$$Q_{1211} = 2\ Q_{2120}$$

$$Q_{1011} = \frac{2}{3}\ Q_{2120}$$

$$Q_{0120} = -\ \frac{1}{3}\ Q_{2120}$$

$$Q_{4030} = 5\ Q_{0030}$$

$$Q_{3110} = (\frac{3}{8}\ x\ \frac{F}{M}\ V_{xy} + \frac{3}{8}\ y\ \frac{F}{M}\ V_{xx})(\Delta t)^4$$

$$Q_{3121} = 15\ Q_{0030}$$

$$Q_{3101} = (\frac{3}{8}\ x\ \frac{F}{M}\ V_{xx} - \frac{3}{8}\ y\ \frac{F}{M}\ V_{xy})(\Delta t)^4$$

$$Q_{2210} = (-\ \frac{3}{8}\ x\ \frac{F}{M}\ V_{xx} + \frac{3}{8}\ x\ \frac{F}{M}\ V_{yy} + \frac{3}{4}\ y\ \frac{F}{M}\ V_{xy})(\Delta t)^4$$

$$Q_{1130} = -3\ Q_{0003}$$

$$Q_{1312} = 15\ Q_{0003}$$

Table 4-1. (Concluded)

$$Q_{0403} = 5\ Q_{0003}$$

$$Q_{0203} = -6\ Q_{0003}$$

$$Q_{0021} = -2\ Q_{0003}$$

$$Q_{0003} = (\frac{1}{8}\ y\ \frac{F}{M})(\Delta t)^4$$

$$Q_{2000} = Q_{0200}$$

$$Q_{0200} = (-\frac{5}{12}\ \frac{F}{M}\ \frac{F}{M}\ \frac{\dot{M}}{M})(\Delta t)^4$$

Table 4-2. DESCRIPTION OF NOMINAL CASES USED IN ERROR ANALYSES

NOMINAL CASE NUMBER	THRUST (K lb)	ΔR (KM)	FLIGHT TIME (SEC)
STD. AA-1 to STD. AA-24 (9 cases)	200	9.78 (MIN) to 18.78 (MAX)	170.34 (MIN) to 178.71 (MAX)
STD. AA-1A to STD. AA-24A (9 cases)	175	9.78 (MIN) to 18.78 (MAX)	195.26 (MIN) to 203.60 (MAX)
STD. AA-1B to STD AA-24B (9 cases)	150	9.78 (MIN) to 18.78 (MAX)	229.41 (MIN) to 237.98 (MAX)
STD. AA-1C to STD. AA-24C (9 cases)	100	9.78 (MIN) to 18.78 (MAX)	362.05 (MIN) to 373.74 (MAX)
STD. AA-27 to STD. AA-151 (29 cases)	200	19.77 (MIN) to 121.77 (MAX)	179.81 (MIN) 301.53 (MAX)

All cases begin flight at same position and velocity coordinates,

$R_{ignition} = 6555931.8$ m

$V_{ignition} = 6780.6832$ m/sec

$I_{sp} = 420$ sec

TABLES 4-3 THROUGH 4-162

The following tables display solutions of the ROTS equations for λ_1, λ_2, λ_3, and λ_4 as obtained by the method of Successive Substitutions and by the Newton-Raphson method, denoted by the subscript "n-r". Values for Chi are shown as computed from $\sin^{-1} \lambda_1$ where λ_1 is calculated by the Newton-Raphson method. True, or nominal, values are shown for λ's and Chi for comparison.

Results are shown for tests against eight nominal trajectories: AA-1, AA-24, AA-1A, AA-24A, AA-1B, AA-24B, AA-1C, and AA-24C. These are the same nominal cases described in Table 4-2.

TABLE 4-3

LAMBDA 1 ROTS (3,3) STD. AA1

		SUCCESSIVE SUBSTITUTIONS								
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.97372	1.0000	.97666	.97517	.98689	.98862	.99000	.98892	76.84	81.46
10	.98231	1.0000	.98461	.98378	.99332	.99351	.99432	.99380	79.21	83.62
20	.98952	1.0000	.99122	.99085	.99765	.99715	.99757	.99738	81.69	85.85
30	.99508	1.0000	.99620	.99610	.99978	.99938	.99952	.99948	84.31	88.16
40	.99867	1.0000	.99923	.99922	.99964	.99998	.99995	.99995	87.04	90.55
50	.99999	1.0000	.99994	.99994	.99718	.99870	.99866	.99861	89.90	93.02
60	.99874	1.0000	.99798	.99803	.99239	.99528	.99546	.99526	92.86	95.58
70	.99463	1.0000	.99301	.99331	.98526	.98947	.99014	.98974	95.93	98.92
80	.98740	1.0000	.98466	.98566	.97580	.98103	.98245	.98181	99.10	100.95
90	.97686	1.0000	.97264	.97508	.96396	.96977	.96214	.97128	102.34	103.76
100	.96288	1.0000	.95666	.96164	.94971	.95559	.95893	.95756	105.65	106.67
110	.94542	1.0000	.93652	.94555	.93293	.93850	.94357	.94151	109.01	109.68
120	.92456	1.0000	.91208	.92713	.91348	.91858	.92287	.92205	112.39	112.77
130	.90043	1.0000	.88329	.90678	.89114	.89605	.89971	.89913	115.78	115.96
140	.87331	1.0000	.85019	.88506	.96567	.87120	.87311	.87284	119.15	119.21
150	.84352	1.0000	.81282	.86247	.83668	.84432	.84314	.84332	122.48	122.51
160	.81147	1.0000	.77105	.83948	.80356	.81553	.80985	.81082	125.76	125.82
170	.77758	1.0000	-.12575	.55549	-.31084	.99826	-.26269	.42329	128.95	154.96

TABLE 4-4

LAMBDA 2 ROTS (3,3) STD. AA1

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS						NEW-RAPH
		FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	
0	.227749	.216050	.222866	.161423	.150426	.141058	.152949	.148475
20	.144370	.132505	.135248	.068510	.075389	.069622	.073862	.072393
40	.051507	.039346	.039605	-.026933	-.006851	-.009569	-.009769	-.009560
60	-.050004	-.063405	-.062690	-.123105	-.096948	-.095097	-.098310	-.097156
80	-.158201	-.175100	-.169213	-.218689	-.193853	-.186495	-.191266	-.189867
100	-.269914	-.294404	-.276961	-.313309	-.294808	-.223625	-.287880	-.286900
120	-.381036	-.419321	-.381758	-.407584	-.395842	-.385152	-.387564	-.387958
140	-.487157	-.547367	-.479456	-.502685	-.492789	-.487784	-.488224	-.487998
160	-.584391	-.676680	-.566591	-.600333	-.583112	-.587641	-.584838	-.585289

TABLE 4-5

LAMBDA 3 ROTS (3,3) STD. AA1

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.17809	.16956	.16773	.16564	.5576	.14814	.15313	.15151
20	.18963	.18016	.17940	.16382	.16139	.15774	.15969	.15907
40	.20091	.19090	.19083	.16103	.16767	.16792	.16754	.16769
60	.21139	.120158	.120136	.15893	.17386	.17955	.17704	.17778
80	.22042	.21200	.21023	.15971	.17927	.19282	.18879	.18977
100	.22736	.22199	.21650	.16556	.18392	.20660	.20306	.20368
120	.23167	.23143	.21921	.17774	.18917	.21874	.21804	.21827
140	.23309	.24023	.21737	.19587	.19808	.22711	.22896	.22948
160	.23168	.24203	.20361	.21170	.21859	.22276	.22347	.22428

TABLE 4-6

LAMBDA 4 ROTS (3,3) STD. AA1

				SUCCESSIVE SUBSTITUTIONS				
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.45610	.42724	.42048	.43597	.40348	.37948	.39279	.38878
20	.47794	.44961	.44702	.41867	.40751	.39749	.40241	.40090
40	.49872	.47175	.47152	.40166	.41546	.41709	.41595	.41636
60	.51715	.49304	.49243	.38950	.42478	.44022	.43400	.43580
80	.53176	.51283	.50802	.38716	.43329	.46685	.45730	.45960
100	.54110	.53047	.51654	.39842	.44060	.49368	.48563	.48703
120	.54408	.54535	.51636	.42403	.44929	.51542	.51394	.51444
140	.54026	.55699	.50618	.46062	.46519	.52744	.53142	.53254
160	.52998	.55202	.47195	.48903	.48256	.51173	.51320	.51488

TABLE 4-7

LAMBDA 1 ROTS (3,3) STD. AA24

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.89650	1.0000	.90558	.89319	.95121	.96711	.95892	.95830	63.70	73.40
10	.91598	1.0000	.92498	.91592	.96923	.97474	.97016	.97034	66.34	76.01
20	.93512	1.0000	.94385	.93780	.98357	.98174	.98111	.98088	69.24	78.78
30	.95337	1.0000	.96152	.95798	.99267	.98819	.99042	.98954	72.43	81.71
40	.96999	1.0000	.97712	.97545	.99910	.99387	.99693	.99589	75.92	84.80
50	.98404	1.0000	.98958	.98905	.99959	.99815	.99986	.99943	79.75	88.08
60	.99436	1.0000	.99764	.99758	.99505	.99999	.99899	.99964	83.91	91.53
70	.99961	1.0000	.99991	.99991	.98559	.99801	.99448	.99594	88.41	95.16
80	.99839	1.0000	.99494	.99513	.97137	.99071	.98642	.98776	93.25	98.97
90	.98933	1.0000	.98140	.98278	.95258	.97672	.97440	.97458	98.37	102.95
100	.97142	1.0000	.95825	.96297	.92922	.95513	.95731	.95590	103.72	107.08
110	.94414	1.0000	.92488	.93647	.90107	.92573	.93363	.93129	109.24	111.36
120	.90767	1.0000	.88129	.90472	.86769	.88930	.90201	.90025	114.81	115.81
130	.86298	1.0000	.82811	.86959	.82850	.84753	.86183	.86211	120.34	120.45
140	.81163	1.0000	.76655	.83320	.78300	.80273	.81334	.81596	125.74	125.32
150	.75559	1.0000	.69794	.79748	.73068	.75708	.75704	.76098	130.92	130.45
160	.69691	1.0000	.62374	.74408	.67113	.71250	.69364	.69911	135.82	135.64
170	.63745	1.0000	.54480	.73411	.60345	.67037	.62379	.63507	140.39	140.57

TABLE 4-8

LAMBDA 2 ROTS (3,3) STD. AA24

				SUCCESSIVE SUBSTITUTIONS				
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.443029	.434546	.466217	.308691	.254403	.283674	.297070	.285751
20	.354322	.335088	.352693	.180530	.190207	.193450	.166720	.194608
40	.243120	.213895	.221556	.423223	.110520	.078291	.098600	.090569
60	.106005	.068615	.069456	-.099280	.002891	-.044818	-.014061	-.002671
80	-.056708	-.100531	-.098613	-.237586	-.135959	-.164232	-.150991	-.155934
100	-.237340	-.288962	-.272138	-.369959	-.296321	-.289044	-.296449	-.293670
120	-.419666	-.487241	-.436514	-.499100	-.458608	-.431814	-.437275	-.435359
140	-.584159	-.683297	-.577566	-.628477	-.601504	-.582745	-.580772	-.578106
160	-.717156	-.867476	-.686896	-.758330	-.714120	-.724935	-.713830	-.715009

TABLE 4-9

LAMBDA 3 ROTS (3,3) STD. AA24

				SUCCESSIVE SUBSTITUTIONS				
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.24128	.24254	.23515	.23690	.19626	.19630	.20531	.20044
20	.26716	.26253	.25771	.22924	.21052	.20936	.21144	.21043
40	.29526	.28230	.28015	.21369	.23058	.21655	.22598	.22216
60	.32388	.30035	.30011	.19479	.25073	.22658	.24503	.23717
80	.34975	.31476	.31419	.18225	.26333	.25286	.26174	.25804
100	.36818	.32374	.31858	.18704	.26419	.29424	.28588	.28852
120	.27482	.32647	.31037	.21242	.25811	.32896	.32965	.33187
140	.36822	.32365	.28883	.24892	.26063	.34303	.36491	.37827
160	.35087	.31276	.25095	.27731	.28141	.33329	.35177	.36757

TABLE 4-10

LAMBDA 4 ROTS (3,3) STD. AA24

				SUCCESSIVE SUBSTITUTIONS				
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.61036	.60766	.58037	.64493	.51358	.50125	.52939	.51623
20	.66867	.65352	.63704	.60320	.53316	.52837	.53420	.53160
40	.73178	.69845	.69162	.54591	.57318	.54108	.56313	.55409
60	.79540	.73891	.73820	.48834	.61742	.56242	.60590	.58721
80	.85181	.77045	.76888	.45603	.64593	.62418	.64453	.63593
100	.88895	.78887	.77555	.47215	.64880	.71947	.70032	.70633
120	.89763	.79179	.75310	.53647	.63733	.79479	.79649	.80141
140	.87469	.78032	.70213	.61933	.64395	.81951	.86631	.89493
160	.82660	.74900	.61925	.67553	.68398	.79004	.82803	.86065

TABLE 4-11

LAMBDA 1 ROTS (3,3) STD. AA1A

		SUCCESSIVE SUBSTITUTIONS								
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.97061	1.0000	.97340	.97146	.98150	.98565	.98675	.98557	76.07	80.26
10	.97848	1.0000	.98056	.97938	.98852	.99064	.99131	.99060	78.09	82.14
20	.98541	1.0000	.98697	.98630	.99396	.99465	.99497	.99466	80.20	84.08
30	.99122	1.0000	.99234	.99203	.99770	.99762	.99775	.99766	82.40	86.08
40	.99572	1.0000	.99646	.99636	.99968	.99944	.99948	.99947	84.70	88.15
50	.99871	1.0000	.99910	.99908	.99984	.99999	.99999	.99998	87.09	90.27
60	.99997	1.0000	.99999	.99999	.99814	.99909	.99913	.99907	89.57	92.47
70	.99929	1.0000	.99890	.99891	.99456	.99658	.99677	.99660	92.15	94.73
80	.99646	1.0000	.99553	.99569	.98911	.99227	.99276	.99243	94.81	97.05
90	.99130	1.0000	.98963	.99020	.98175	.98598	.98693	.98642	97.56	99.45
100	.98363	1.0000	.98093	.98240	.97254	.97754	.97913	.97842	100.37	101.92
110	.97333	1.0000	.96918	.97228	.96140	.96684	.96915	.96829	103.26	104.47
120	.96031	1.0000	.95418	.95992	.94828	.95380	.95683	.95588	106.19	107.08
130	.94454	1.0000	-.59643	-.33468	-.21267	-.10643	-.11697	.88639	109.17	117.58
140	.92604	1.0000	.91368	.92917	.91572	.92074	.92441	.92357	112.17	112.55
150	.90491	1.0000	.88792	.91130	.89593	.90089	.90405	.90341	115.18	115.39
160	.88128	1.0000	.85842	.89226	.87349	.87907	.88084	.88050	118.20	118.30
170	.85537	1.0000	.82506	.87242	.84803	.85544	.85472	.85482	121.19	121.26
180	.82743	1.0000	.78792	.85227	.81918	.83028	.82581	.82664	124.16	124.24
190	.79773	1.0000	.74467	.83113	.78454	.80231	.79246	.79454	127.08	127.39

TABLE 4-12

LAMBDA 2 ROTS (3,3) STD. AA1A

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.240655	.231071	.238874	.191449	.168787	.160982	.144125	.169258
20	.170187	.161413	.165493	.109723	.103218	.100094	.104914	.103116
40	.092349	.084068	.085251	.025123	.033162	.032199	.032327	.032329
60	.009357	-.001235	-.001235	-.060936	-.042480	-.041489	-.043986	-.043045
80	-.083985	-.094489	-.092793	-.147169	-.124060	-.120085	-.124172	-.122817
100	-.180159	-.195273	-.187584	-.232754	-.210739	-.203220	-.207913	-.206585
120	-.278911	-.302711	-.283112	-.317618	-.300567	-.290645	-.294696	-.293748
140	-.377409	-.415495	-.376374	-.402472	-.390741	-.381427	-.383952	-.383409
160	-.472582	-.532126	-.464187	-.488624	-.478299	-.473623	-.474278	-.474041
180	-.561567	-.651275	-.543548	-.577729	-.560961	-.564701	-.562326	-.562728

TABLE 4-13

LAMBDA 3 ROTS (3,3) STD. AA1A

				SUCCESSIVE SUBSTITUTIONS				
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.16226	.15320	.15109	.15744	.14549	.13823	.14324	.14170
20	.17216	.16273	.16160	.15766	.15007	.14672	.14883	.14888
40	.18206	.17260	.17226	.15665	.15554	.15527	.15527	.15528
60	.19168	.18272	.18272	.15522	.16127	.16438	.16276	.16226
80	.20065	.19297	.19246	.15461	.16702	.17452	.17159	.17238
100	.20853	.20325	.20087	.15641	.17229	.18575	.18206	.18289
120	.21490	.21345	.20721	.16216	.17713	.19734	.19419	.19470
140	.21941	.22344	.21064	.17278	.18243	.20787	.20674	.20691
160	.22181	.23269	.20980	.18773	.18794	.21536	.21623	.21651
180	.22208	.23570	.19828	.20044	.19657	.21247	.21302	.21360

TABLE 4-14

LAMBDA 4 ROTS (3,3) STD. AA1A

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.39489	.36302	.35524	.39579	.36039	.33685	.34964	.34622
20	.41153	.38183	.37798	.38439	.36070	.35064	.35592	.35440
40	.42786	.40098	.39992	.37054	.36484	.36465	,35459	.36463
60	.44322	.42015	.42015	.35969	.37154	.38005	.37619	.37732
80	.45680	.43893	.45754	.35193	.37915	.39776	.39104	.39281
100	.46770	.45688	.45082	.35144	.38637	.41756	.40943	.41119
120	.47504	.47354	.45869	.36097	.39302	.42744	.43077	.43182
140	.47814	.48843	.45983	.38073	.40066	.45418	.45190	.45223
160	.47664	.50016	.45216	.40809	.41194	.46367	.46545	.46602
180	.47059	.49800	.42416	.42880	.42118	.45206	.45311	.45425

TABLE 4-15

LAMBDA 1 ROTS (3,3) STD. AA24A

		SUCCESSIVE SUBSTITUTIONS								
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.90226	1.0000	.91065	.89924	.94368	.96524	.95866	.95607	64.45	72.96
10	.91826	1.0000	.92602	.91713	.96042	.97294	.96736	.96677	66.67	75.19
20	.93402	1.0000	.94017	.93466	.97471	.97977	.97627	.97641	69.07	77.53
30	.94922	1.0000	.95574	.95136	.98613	.98583	.98467	.98477	71.66	79.99
40	.96347	1.0000	.96927	.96666	.99432	.99114	.99182	.99159	74.46	82.57
50	.97628	1.0000	.98120	.97991	.99896	.99555	.99700	.99659	77.49	85.27
60	.98702	1.0000	.99083	.99038	.99986	.99896	.99969	.99945	80.76	88.11
70	.99500	1.0000	.99739	.99732	.99693	.99999	.99958	.99982	84.27	91.08
80	.99940	1.0000	.99999	.99999	.99018	.99868	.99658	.99732	88.02	94.19
90	.99937	1.0000	.99768	.99775	.97973	.99385	.99068	.99159	92.02	97.44
100	.99407	1.0000	.98955	.99015	.96569	.98462	.98176	.98223	96.24	100.81
110	.98274	1.0000	.97473	.97704	.94817	.97025	.96938	.96890	100.66	104.32
120	.96484	1.0000	.95254	.95859	.92713	.95022	.95281	.95125	105.23	107.96
130	.94013	1.0000	.92254	.93539	.90237	.92450	.93113	.92894	109.92	111.73
140	.90875	1.0000	.88459	.90835	.87352	.89355	.90354	.90156	114.66	115.64
150	.87122	1.0000	.93892	.87872	.84008	.85840	.86955	.86869	119.39	119.69
160	.82842	1.0000	.78604	.84787	.80149	.82047	.82906	.82978	124.06	123.92
170	.78149	1.0000	.72678	.81728	.75723	.78137	.78230	.78460	128.60	128.32
180	.73169	1.0000	.66204	.78823	.70674	.74261	.72951	.73392	132.97	132.78
190	.68027	1.0000	.59230	.76167	.64910	.70539	.67063	.67983	137.13	137.17
200	.62838	1.0000	.50966	.73550	.57529	.66700	.59841	.61856	141.06	141.79

TABLE 4-16

LAMBDA 2 ROTS (3,3) STD. AA24A

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.431192	.422711	.448904	.331092	.261406	.284533	.308796	.293108
20	.357209	.343000	.361482	.223499	.200131	.216553	.219394	.215897
40	.267784	.247883	.258198	.106412	.132762	.127606	.130510	.129352
60	.160551	.135376	.138654	-.016320	.051029	.024680	.037421	.032970
80	.034446	.004361	.004365	-.139757	-.051354	-.082597	-.067772	-.073086
100	-.108731	-.144517	-.140299	-.259743	-.174612	-.190123	-.186898	-.187632
120	-.262823	-.308065	-.287752	-.375305	-.311729	-.303575	-.311140	-.308400
140	-.417325	-.480425	-.428126	-.488580	-.450142	-.428595	-.434483	-.432637
160	-.560093	-.654155	-.551579	-.602242	-.575812	-.559871	-.559588	-.558076
180	-.681635	-.822137	-.650787	-.720511	-.679349	-.687098	-.678169	-.679225
200	-.777901	-.990284	-.727319	-.848448	-.762534	-.811719	-.774932	-.785735

TABLE 4-17

LAMBDA 3 ROTS (3,3) STD. AA24A

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.21347	.21097	.20393	.21584	.18013	.17636	.18815	.18273
20	.23313	.22754	.22246	.21436	.18874	.18895	.19286	.19116
40	.25440	.24463	.24173	.20722	.20181	.19993	.20077	.20051
60	.27660	.26159	.26064	.19557	.21763	.20819	.21348	.21151
80	.29837	.27751	.27751	.18327	.23239	.22066	.22836	.22530
100	.31756	.29131	.29003	.17686	.24186	.24339	.24387	.24357
120	.33148	.30198	.29567	.18291	.24412	.27399	.26608	.26841
140	.33777	.30896	.29221	.20354	.34226	.30074	.29924	.30081
160	.33544	.31222	.27832	.23356	.24528	.31471	.32779	.33382
180	.32543	.31121	.25271	.26228	.26224	.31380	.32849	.33838
200	.31003	ERROR IN DATA.						

TABLE 4-18

LAMBDA 4 ROTS (3,3) STD. AA24A

		SUCCESSIVE SUBSTITUTIONS						
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.51448	.49888	.47287	.56048	.45391	.42967	.46355	.45022
20	.55489	.53467	.51732	.53910	.45831	.45600	.46502	.46060
40	.59855	.57125	.56206	.50566	.47755	.47377	.47546	.47496
60	.64379	.60712	.60433	.46537	.50697	.48809	.49927	.49499
80	.68742	.64016	.64016	.42939	.53636	.51315	.52966	.52295
100	.72456	.66787	.66458	.41309	.55547	.56165	.56225	.56165
120	.74930	.68785	.67263	.42886	.55997	.62587	.60893	.61387
140	.75655	.69854	.66072	.47640	.55651	.67880	.67596	.67910
160	.74453	.69944	.62766	.53953	.56285	.70199	.72834	.74050
180	.71569	.68922	.57399	.59348	.59328	.69274	.72129	.74048
200	.67542	.12051	-.05270	.03189	-.01103	.01177	-.00811	-.00421

TABLE 4-19

LAMBDA 1 ROTS (3,3) STD. AA1B

TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
				SUCCESSIVE SUBSTITUTIONS						
0	.96285	1.0000	.96604	.96340	.97073	.97944	.98089	.97686	74.33	78.15
10	.97038	1.0000	.97282	.97086	.97871	.98488	.98544	.98414	76.02	79.78
20	.97732	1.0000	.97914	.97778	.98555	.98953	.98959	.98892	77.77	81.46
30	.98358	1.0000	.98489	.98403	.99116	.99338	.99322	.99294	79.60	83.19
40	.98903	1.0000	.98995	.98946	.99546	.99641	.99620	.99614	81.50	84.96
50	.99354	1.0000	.99415	.99392	.99837	.99856	.99840	.99843	83.48	86.79
60	.99697	1.0000	.99735	.99728	.99983	.99976	.99969	.99972	85.53	88.66
70	.99917	1.0000	.99936	.99935	.99978	.99993	.99996	.99994	87.67	90.59
80	.99999	1.0000	.99999	.99999	.99822	.99895	.99909	.99899	89.88	92.57
90	.99928	1.0000	.99904	.99906	.99510	.99670	.99698	.99677	92.16	94.60
100	.99688	1.0000	.99629	.99642	.99044	.99303	.99351	.99317	94.52	96.70
110	.99264	1.0000	.99153	.99198	.98421	.98780	.98858	.98810	96.95	98.85
120	.98643	1.0000	.98453	.98567	.97643	.98088	.98206	.98143	99.44	101.06
130	.97812	1.0000	.97507	.97747	.96706	.97215	.97383	.97305	102.00	103.33
140	.96763	1.0000	.96295	.96738	.95608	.96152	.96376	.96286	104.61	105.66
150	.95489	1.0000	.94797	.95551	.94342	.94893	.95169	.95072	107.27	108.06
160	.93987	1.0000	.92994	.94196	.92897	.93437	.93747	.93651	109.96	110.53
170	.92258	1.0000	.90875	.92696	.91260	.91790	.92101	.92014	112.69	113.05
180	.90308	1.0000	.88424	.91075	.89410	.89961	.80216	.90150	115.43	115.64
190	.88146	1.0000	.85638	.89367	.87325	.87969	.88091	.88058	118.18	118.29
200	.85784	1.0000	.82502	.87603	.84968	.85827	.85713	.85731	120.92	120.98
210	.83240	1.0000	.79012	.85824	.82304	.83563	.83083	.83184	123.65	123.71
220	.80533	ERROR IN DATA								

TABLE 4-20

LAMBDA 2 ROTS (3,3) STD. AA1B

		SUCCESSIVE SUBSTITUTIONS						
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.270017	.260601	.270543	.240207	.201712	.194529	.211476	.205381
20	.211729	.204237	.210783	.169360	.144298	.143891	.151492	.148429
40	.147697	.141773	.145145	.095103	.084640	.087079	.088573	.087783
60	.077777	.072758	.073705	.018409	.021635	.024670	.022803	.023365
80	.002083	-.003129	-.003127	-.059615	-.045691	-.042508	-.046018	-.044806
100	-.078883	-.086035	-.084539	-.137933	-.117841	-.113718	-.118059	-.116612
120	-.164177	-.175867	-.169239	-.215850	-.194587	-.188541	-.193228	-.191812
140	-.252353	-.272192	-.255385	-.293215	-.274814	-.266772	-.271156	-.270011
160	-.341515	-.374303	-.340698	-.370582	-.356662	-.348080	-.351410	-.350629
180	-.429457	-.481147	-.422481	-.449025	-.437712	-.431497	-.433196	-.432763
200	-.513909	-.591573	-.497921	-.429996	-.515541	-.515487	-.514725	-.514785
220	-.592814	ERROR IN DATA						

TABLE 4-21

LAMBDA 3 ROTS (3,3) STD. AA1B

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.15020	.13963	.13693	.15257	.13879	.13116	.13677	.13527
20	.15898	.14829	.14647	.15489	.14260	.13887	.14193	.14100
40	.16796	.15741	.5644	.15600	.14716	.14651	.4740	.14708
60	.17698	.16695	.16677	.15615	.15244	.15420	.15345	.15364
80	.18586	.17688	.17688	.15584	.15820	.16225	.16034	.16087
100	.19434	.18716	.18670	.15595	.16408	.17102	.16822	.16896
120	.20212	.19770	.19563	.15762	.16972	.18069	.17731	.17807
140	.20888	.20844	.20308	.16206	.17501	.19102	.18770	.18828
160	.21433	.21927	.20831	.17018	.18025	.20120	.19895	.19921
180	.21822	.23008	.21048	.18225	.18641	.21006	.20955	.20964
200	.22040	.24009	.20801	.19705	.19450	.21568	.21616	.21645
220	.22085	ERROR IN DATA						

TABLE 4-22

LAMBDA 4 ROTS (3,3) STD. AA1B

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.34150	.30341	.29343	.36104	.32522	.29966	.31333	.31042
20	.35437	.31922	.31301	.35546	.32262	.31000	.31751	.31555
40	.36741	.33570	.33267	.34769	.23222	.32034	.32260	.32189
60	.38029	.35270	.35189	.33866	.32676	.33090	.32933	.32970
80	.39262	.37004	.37003	.32995	.33243	.34229	.33816	.33926
100	.40388	.38745	.38629	.32361	.33917	.35521	.34932	.35080
120	.41349	.40463	.39969	.32193	.34598	.36983	.36293	.36444
140	.42088	.42136	.40923	.32693	.35240	.38541	.37886	.37997
160	.42549	.43692	.41377	.33959	.35885	.40012	.39586	.39632
180	.42694	.45116	.41217	.35932	.36688	.41164	.41073	.41090
200	.42504	.46233	.40217	.38274	.37787	.41650	.41735	.41789
220	.41985	ERROR IN DATA						

TABLE 4-23

LAMBDA 1 ROTS (3,3) STD. AA24B

TIME	TRUE	FIRST	SUCCESSIVE SUBSTITUTIONS SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.90245	1.0000	.91131	.90001	.93017	.96014	.95489	.94979	64.48	71.77
10	.91548	1.0000	.92317	.91374	.94574	.96811	.96177	.95950	66.27	73.64
20	.92840	1.0000	.93506	.92749	.95992	.97511	.96896	.96851	68.18	75.58
30	.94105	1.0000	.94681	.94101	.97237	.98125	.97623	.97671	70.22	77.61
40	.95324	1.0000	.95820	.95404	.98282	.98659	.98320	.98394	72.41	79.72
50	.96474	1.0000	.96898	.96624	.99100	.99118	.98948	.99006	74.74	81.92
60	.97525	1.0000	.97882	.97723	.99665	.99498	.99462	.99489	77.22	84.21
70	.98444	1.0000	.98733	.98657	.99959	.99788	.99820	.99823	79.88	86.60
80	.99191	1.0000	.99405	.99380	.99968	.99963	.99989	.99987	82.70	89.08
90	.99720	1.0000	.99846	.99843	.99685	.99989	.99946	.99957	85.71	91.67
100	.99981	1.0000	.99999	.99999	.99111	.99820	.99677	.99709	88.89	94.36
110	.99922	1.0000	.99801	.99807	.98253	.99402	.99175	.99219	92.25	97.16
120	.99490	1.0000	.99188	.99232	.97118	.98679	.98431	.98460	95.78	100.07
130	.98636	1.0000	.98079	.98258	.95714	.97597	.97426	.97406	99.47	103.08
140	.97319	1.0000	.96470	.96883	.94043	.96115	.96126	.96033	103.29	106.19
150	.95511	1.0000	.94259	.95131	.92095	.94208	.94482	.94315	107.23	109.41
160	.93203	1.0000	.91429	.93046	.89850	.91881	.92438	.92227	111.24	112.74
170	.90403	1.0000	.87962	.90697	.87269	.89168	.89941	.89740	115.30	116.18
180	.87143	1.0000	.83859	.88169	.84397	.86137	.86954	.86824	119.37	119.75
190	.83472	1.0000	.79143	.85562	.80908	.82888	.83457	.83450	123.41	123.44
200	.79458	1.0000	.73852	.82980	.77010	.79536	.79440	.79610	127.38	127.24
210	.75177	1.0000	.68022	.80518	.72538	.76193	.74882	.75328	131.25	131.12
220	.70710	1.0000	.61731	.78279	.67437	.73006	.69786	.70744	135.00	134.97
230	.66137	1.0000	.54936	.76306	.61537	.70069	.64036	.65954	138.59	138.74

TABLE 4-24

LAMBDA 2 ROTS (3,3) STD. AA24B

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.430788	.421145	.447180	.367547	.279607	.296956	.332407	.312864
20	.371568	.360761	.380809	.280383	.221725	.247181	.257731	.248949
40	.302193	.289103	.303164	.184533	.163189	.182481	.179139	.178465
60	.221068	.205788	.213384	.081707	.099996	.10354	.098928	.100912
80	.126904	.109053	.111321	-.025285	.027020	.014186	.015679	.016020
100	.019236	-.002001	-.002000	-.132988	-.059948	-.080284	-.074897	-.076103
120	-.100846	-.127358	-.123879	-.238373	-.162007	-.176437	-.174942	-.174807
140	-.229989	-.265677	-.249656	-.340257	-.276109	-.275625	-.281189	-.278850
160	-.362368	-.414108	-.372908	-.440024	-.395297	-.381504	-.3888908	-.386538
180	-.490519	-.568168	-.486429	-.540797	-.510199	-.494148	-.497496	-.496143
200	-.607154	-.723147	-.583423	-.645315	-.611769	-.608693	-.605281	-.605161
220	-.707108	-.874856	-.659103	-.754999	-.690145	-.720652	-.701617	-.706770

TABLE 4-25

LAMBDA 3 ROTS (3,3) STD. AA24B

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.18868	.18236	.17531	.19753	.16637	.15973	.17341	.16808
20	.20371	.19602	.19032	.20084	.17146	.17221	.17863	.17534
40	.21996	.21051	.20655	.20059	.17926	.18338	.18373	.18303
60	.23722	.22563	.22344	.19652	.19000	.19274	.19068	.19144
80	.25504	.24105	.24038	.18940	.20271	.20122	.20061	.20105
100	.27261	.25629	.25629	.18149	.21528	.21174	.21286	.21256
120	.28872	.27079	.26972	.17667	.22518	.22776	.22652	.22693
140	.30187	.28390	.27889	.17939	.23074	.24975	.24346	.24532
160	.31054	.29515	.28198	.19249	.23252	.27297	.26689	.26849
180	.31365	.30419	.27735	.21501	.23451	.29064	.29319	.29467
200	.31099	.31102	.26392	.24203	.24336	.29930	.30968	.31435
220	.30331	.31240	.23768	.26386	.26050	.29346	.30086	.30601

TABLE 4-26

LAMBDA 4 ROTS (3,3) STD. AA24B

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.42576	.39690	.37096	.48123	.39920	.36594	.40234	.39101
20	.45294	.42411	.40483	.47515	.39482	.38692	.40516	.39737
40	.48239	.45280	.44025	.46095	.39926	.40529	.40780	.40577
60	.51362	.48247	.47603	.43927	.41291	.41986	.41540	.41695
80	.54559	.51234	.51051	.41322	.43319	.43293	.43083	.43190
100	.57659	.54132	.54132	.38897	.45489	.45085	.45233	.45186
120	.60410	.56805	.56548	.37519	.47335	.48047	.47740	.47832
140	.62507	.59104	.57969	.38017	.48162	.52162	.50899	.51264
160	.63652	.60904	.58098	.40720	.48414	.56361	.55199	.55500
180	.63650	.62106	.56723	.45115	.48716	.59281	.59773	.60050
200	.62484	.62675	.53779	.49982	.50181	.60517	.62199	.63054
220	.60327	.62031	.48725	.53446	.52842	.58595	.59887	.60797

TABLE 4-27

LAMBDA 1 ROTS (3,3) STD. AA1C

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.90859	1.0000	.92516	.91648	.89820	.93555	.93277	.92802	65.31	68.13
20	.92267	1.0000	.93529	.92781	.91721	.95127	.94571	.94244	67.32	70.47
40	.93652	1.0000	.94565	.93949	.93552	.96482	.95751	.95579	69.47	72.90
60	.94995	1.0000	.95610	.95133	.95268	.97614	.96826	.96791	71.79	75.45
80	.96270	1.0000	.96645	.96307	.96816	.98522	.97797	.97855	74.30	78.11
100	.97443	1.0000	.97640	.97428	.98132	.99212	.98645	.98745	77.01	80.91
120	.98468	1.0000	.98548	.98440	.99146	.99689	.99335	.99427	79.95	83.87
140	.99286	1.0000	.99306	.99269	.99790	.99951	.99811	.99861	83.15	86.98
160	.99826	1.0000	.99827	.99823	.99999	.99982	.99999	.99999	86.62	90.27
180	.99997	1.0000	.99997	.99997	.99723	.99736	.99822	.99784	90.38	93.76
200	.99697	1.0000	.99667	.99681	.98930	.99138	.99204	.99154	94.45	97.46
220	.98812	1.0000	.98658	.98779	.97600	.98076	.98085	.98036	98.83	101.37
240	.97224	1.0000	.96760	.97229	.95709	.96423	.96403	.96350	103.53	105.53
260	.94829	1.0000	.93741	.95040	.93199	.94067	.94084	.94011	108.50	109.93
280	.91549	1.0000	.89362	.92321	.89935	.90961	.91008	.90925	113.72	114.60
300	.87356	1.0000	.83398	.89297	.85656	.87187	.86985	.86994	119.12	119.55
320	.82280	1.0000	.75674	.86327	.79944	.83061	.81732	.82145	124.63	124.77
340	.76422	1.0000	.66035	.83841	.72133	.79202	.74680	.76354	130.16	130.22
360	.69937	1.0000	.50032	.81707	.56670	.76076	.59994	.67251	135.62	137.74

TABLE 4-28

LAMBDA 2 ROTS (3,3) STD. AA1C

				SUCCESSIVE SUBSTITUTIONS				
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.417679	.386876	.408687	.440641	.353533	.360478	.380264	.372523
40	.350603	.329692	.347872	.353616	.262983	.288289	.304921	.294022
80	.270545	.259002	.271759	.250366	.171260	.208744	.214618	.205988
120	.174357	.170380	.176604	.130346	.078766	.115068	.109815	.106850
160	.058916	.058656	.059482	-.003779	-.018880	.003047	-.006357	-.004796
200	-.077688	-.081553	-.079777	-.145860	-.131002	-.125859	-.132068	-.129769
240	-.233963	-.254539	-.235388	-.289915	-.265115	-.265771	-.269331	-.267690
280	-.402325	-.461242	-.391891	-.438246	-.416259	-.414516	-.417440	-.416244
320	-.568322	-.697507	-.522933	-.606113	-.560455	-.577071	-.567218	-.570272
360	-.714753	-.999679	-.604854	-.855837	-.657145	-.810448	-.682431	-.740087

TABLE 4-29

LAMBDA 3 ROTS (3,3) STD. AA1C

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.13846	.11867	.11254	.14974	.13749	.13537	.14043	.13946
40	.15426	.13331	.12800	.16409	.14517	.14637	.15398	.15050
80	.17193	.15039	.14657	.17760	.15383	.15947	.16498	.16192
120	.19152	.17042	.16851	.18884	.16511	.17408	.17615	.17400
160	.21280	.19385	.19359	.19679	.18029	.18931	.18773	.18737
200	.23485	.23097	.22041	.20264	.19868	.20550	.20237	.20313
240	.25582	.25172	.24550	.21185	.21724	.22564	.22205	.22296
280	.27282	.28564	.26255	.23246	.23343	.25152	.24780	.24832
320	.28275	.32125	.26148	.26662	.24883	.27619	.27375	.27446
360	.28383	-1.84229	-1.98249	-1.94566	-2.09835	-1.89651	-1.96101	-1.92468

TABLE 4-30

LAMBDA 4 ROTS (3,3) STD. AA1C

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.25000	.18135	.15879	.28110	.27141	.25747	.26855	.26843
40	.26603	.20165	.18501	.29502	.26641	.25890	.27467	.27074
80	.28454	.22598	.21556	.30554	.26364	.26631	.28022	.27439
120	.30552	.25477	.25020	.31043	.26647	.27793	.28456	.28013
160	.32839	.28820	.28765	.30901	.27742	.29146	.29026	.28915
200	.35171	.32589	.32481	.30501	.29522	.30604	.30213	.30314
240	.37269	.36655	.35588	.30842	.31425	.32822	.32280	.32409
280	.38732	.40784	.37213	.33059	.33091	.35731	.35204	.35273
320	.39157	.44591	.36217	.37052	.34568	.38294	.37947	.38948
360	.38354	-2.30012	-2.47757	-2.43084	-2.62422	-2.36877	-2.45055	-2.40451

TABLE 4-31

LAMBDA 1 ROTS (3,3) STD. AA24C

				SUCCESSIVE SUBSTITUTIONS						
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.86585	1.0000	.88984	.87423	.85646	.91695	.90407	.89917	59.98	64.05
20	.88190	1.0000	.90076	.88642	.87960	.93631	.92015	.91674	61.87	66.46
40	.89828	1.0000	.91246	.89975	.90285	.95317	.93463	.93346	63.93	68.98
60	.91487	1.0000	.92493	.91416	.92567	.96734	.94791	.94912	66.18	71.65
80	.93149	1.0000	.93809	.92951	.94729	.97874	.96036	.96347	68.66	74.47
100	.94782	1.0000	.95171	.94544	.96674	.98746	.97212	.97615	71.40	77.46
120	.96340	1.0000	.96538	.96135	.98282	.99373	.98295	.98674	74.45	80.66
140	.97755	1.0000	.97840	.97631	.99432	.99782	.99212	.99466	77.83	84.08
160	.98931	1.0000	.98968	.98895	.99969	.99982	.99830	.99922	81.61	87.74
180	.99736	1.0000	.99759	.99750	.99815	.99943	.99985	.99957	85.83	91.68
200	.99995	1.0000	.99986	.99986	.98905	.99557	.99517	.99469	90.53	95.90
220	.99497	1.0000	.99349	.99390	.97237	.98633	.98329	.98343	95.74	100.44
240	.98002	1.0000	.97475	.97799	.94834	.96900	.96388	.96450	101.47	105.31
260	.95278	1.0000	.93947	.95183	.91663	.94090	.93649	.93657	107.67	110.52
280	.91153	1.0000	.88354	.91727	.87526	.90078	.89925	.89829	114.28	116.07
300	.85576	1.0000	.80362	.87868	.81953	.85094	.84809	.84820	121.15	121.98
320	.78663	1.0000	.69791	.84266	.74159	.79913	.77604	.78488	128.12	128.29
340	.70694	1.0000	.56636	.81656	.62993	.75938	.66925	.70756	135.01	134.96
360	.62061	1.0000	.40899	.80664	.46654	.75213	.49192	.61855	141.63	141.79

TABLE 4-32

LAMBDA 2 ROTS (3,3) STD. AA24C

		SUCCESSIVE SUBSTITUTIONS						
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.500288	.469364	.501525	.518635	.399638	.427482	.455882	.437581
40	.439417	.418425	.447768	.430890	.302574	.355648	.383382	.358671
80	.363760	.351873	.375464	.320581	.205120	.278753	.288558	.267807
120	.268046	.263124	.277994	.184453	.111786	.183827	.168950	.162308
160	.145765	.143664	.148624	.024887	.018484	.058121	.032805	.039410
200	-.009380	-.016327	-.016157	-.147534	-.093920	-.098108	-.109057	-.102862
240	-.198888	-.224713	-.209789	-.317446	-.247086	-.266322	-.265543	-.264055
280	-.411226	-.482602	-.406758	-.485381	-.435273	-.437547	-.441656	-.439401
320	-.617421	-.777288	-.560958	-.680567	-.606538	-.632392	-.612785	-.619645
360	-.784113	-.010871	-.621852	-.935602	-.667836	-.891739	-.685608	-.785744

TABLE 4-33

LAMBDA 3 ROTS (3,3) STD. AA24C

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS						NEW-RAPH
		FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	
0	.15157	.13393	.12492	.16514	.14868	.15086	.15967	.15644
40	.17077	.15216	.14365	.18301	.15674	.16300	.17544	.16936
80	.19319	.17387	.16686	.20006	.16562	.17915	.19027	.18257
120	.21936	.19975	.19524	.21346	.17845	.19837	.20181	.19639
160	.24945	.23037	.22874	.23001	.19880	.21753	.21136	.21160
200	.28239	.26529	.26527	.22002	.22582	.23483	.22694	.22995
240	.31464	.30352	.29875	.22387	.25124	.25820	.25281	.25483
280	.33933	.34223	.31735	.24958	.26708	.29703	.28952	.29125
320	.34873	.37809	.30492	.39808	.28177	.33242	.33577	.33644
360	.34024	.38617	.22311	.32584	.29088	.32225	.30518	.31516

TABLE 4-34

LAMBDA 4 ROTS (3,3) STD. AA24C

		SUCCESSIVE SUBSTITUTIONS						
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.27059	.20540	.17220	.31155	.29858	.28996	.31230	.30800
40	.29268	.23168	.20492	.33300	.29315	.29016	.32145	.31080
80	.31945	.26373	.24456	.35048	.28887	.30101	.33005	.31492
120	.35151	.30229	.29144	.35910	.29211	.31992	.23301	.32144
160	.38883	.34757	.34439	.35440	.30984	.34034	.33327	.33217
200	.42952	.39851	.39847	.34015	.34110	.35835	.34572	.35018
240	.46809	.45186	.44361	.33593	.37248	.38578	.37699	.38010
280	.49457	.50172	.46296	.36835	.39125	.43506	.42436	.42674
320	.49834	.54097	.43801	.43127	.40800	.47652	.48067	.48170
360	.47669	.53586	.32845	.45923	.41466	.45451	.43252	.44531

TABLE 4-35

LAMBDA 1 ROTS (4,3) STD. AA1

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.97372	1.0000	.98194	.98351	.97387	.97197	.97312	.97335	76.84	76.74
10	.98231	1.0000	.98860	.98995	.98282	.98164	.98242	.98258	79.21	79.29
20	.98952	1.0000	.99399	.99501	.99017	.98953	.98998	.99007	81.69	81.92
30	.99507	1.0000	.99782	.99842	.99566	.99538	.99558	.99561	84.31	84.63
40	.99867	1.0000	.99980	.99994	.99902	.99894	.99898	.99899	87.04	87.43
50	.99999	1.0000	.99959	.99932	.99997	.99998	.99998	.99998	89.90	90.32
60	.99874	1.0000	.99685	.99636	.99827	.99826	.99834	.99835	92.86	93.29
70	.99463	1.0000	.99126	.99088	.99368	.99358	.99386	.99390	95.93	96.33
80	.98740	1.0000	.98247	.98279	.98601	.98579	.98635	.98642	99.10	99.45
90	.97686	1.0000	.97019	.97205	.97510	.97480	.97567	.97578	102.34	102.64
100	.96288	1.0000	.95415	.95872	.96084	.96056	.96170	.96183	105.65	105.88
110	.94542	1.0000	.93414	.94296	.94318	.94312	.94439	.94452	109.01	109.17
120	.92456	1.0000	.91000	.92503	.92212	.92362	.92374	.92386	112.39	112.50
130	.90043	1.0000	.88165	.90526	.89766	.89928	.89982	.89993	115.78	115.85
140	.87331	1.0000	.84907	.88410	.86989	.87347	.87278	.87296	119.15	119.19
150	.84352	1.0000	.81222	.86200	.83880	.84556	.84276	.84318	122.48	122.52
160	.81147	1.0000	.77086	.83936	.84013	.81590	.80965	.81073	125.76	125.83
170	.77758	1.0000	-.12575	.55549	-.19382	.69574	.67474	.23622	128.95	166.34

TABLE 4-36

LAMBDA 2 ROTS (4,3) STD. AA1

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.227749	.190043	.181593	.227143	.235146	.230272	.229180	.229293
20	.144370	.109631	.099879	.139828	.144324	.141156	.140484	.140573
40	.051507	.019863	.010523	.044150	.045847	.044945	.044723	.044758
60	-.050004	-.079278	-.085283	-.059697	-.058965	-.057502	-.057298	-.057331
80	-.158201	-.187227	-.185505	-.166663	-.167932	-.164608	-.164117	-.164189
100	-.269914	-.302802	-.287306	-.277175	-.278152	-.274082	-.273567	-.273628
120	-.381036	-.424244	-.387217	-.387460	-.386205	-.383045	-.382687	-.382717
140	-.487157	-.549408	-.481444	-.495149	-.488669	-.488385	-.487693	-.487781
160	-.584391	-.676954	-.566815	-.599522	-.582568	-.587924	-.584889	-.585416

TABLE 4-37

LAMBDA 3 ROTS (4,3) STD. AA1

		SUCCESSIVE SUBSTITUTIONS						
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.17809	.16917	.15380	.18670	.19084	.18103	.18250	.18218
20	.18963	.17054	.16512	.19180	.19448	.18752	.18855	.18833
40	.20091	.18117	.17646	.19522	.19720	.19465	.19499	.19493
60	.21139	.19194	.18729	.19738	.19926	.20237	.20212	.20215
80	.22042	.20274	.19697	.19917	.20105	.21037	.21005	.21008
100	.22736	.21351	.20472	.20183	.20320	.21804	.21835	.21834
120	.23167	.22426	.20970	.20654	.20658	.22450	.22567	.22574
140	.23309	.23512	.21098	.21395	.21229	.22862	.22994	.23018
160	.23168	.23996	.20121	.21737	.21485	.22223	.22233	.22293

TABLE 4-38

LAMBDA 4 ROTS (4,3) STD. AA1

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.45610	.40324	.38536	.48620	.49905	.46587	.47146	.47022
20	.47714	.42567	.41172	.48615	.49395	.47205	.47565	.47490
40	.49872	.44820	.43677	.48349	.48913	.48157	.48267	.48248
60	.51715	.47035	.45920	.47992	.48520	.49366	.49287	.49298
80	.53176	.49162	.47752	.47781	.48289	.50689	.50585	.50593
100	.54110	.51156	.49017	.47956	.48304	.51915	.51967	.51964
120	.54408	.52982	.49566	.48664	.48673	.52796	.53051	.53062
140	.54026	.54621	.49268	.49875	.49505	.53068	.53350	.53402
160	.52998	.54778	.46705	.50064	.49538	.51067	.51086	.51212

TABLE 4-39

LAMBDA 1 ROTS (4,3) STD. AA24

TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH	CHI_{ndm}	CHI_{nr}
		SUCCESSIVE SUBSTITUTIONS								
0	.89650	1.0000	.91552	.91049	.88392	.88197	.88528	.88484	63.70	62.23
10	.91598	1.0000	.93349	.93052	.90932	.90824	.91188	.91133	66.34	65.69
20	.93512	1.0000	.95087	.94964	.93324	.93270	.93611	.93550	69.24	69.31
30	.95337	1.0000	.96702	.96705	.95490	.95467	.95742	.95700	72.43	73.14
40	.96999	1.0000	.98108	.98179	.97341	.97334	.97519	.97493	75.92	77.14
50	.98404	1.0000	.99203	.99282	.98778	.98780	.98873	.98862	79.75	81.35
60	.99436	1.0000	.99865	.99902	.99698	.99703	.99727	.99724	83.91	85.75
70	.99961	1.0000	.99959	.99937	.99998	.99998	.99998	.99998	88.41	90.34
80	.99839	1.0000	.99347	.99307	.99584	.99568	.99605	.99603	93.25	95.10
90	.98934	1.0000	.97901	.97971	.98379	.98339	.98472	.98471	98.34	100.03
100	.97142	1.0000	.95522	.95942	.96330	.96275	.96542	.96553	103.72	105.09
110	.94414	1.0000	.92154	.93296	.93421	.93392	.93786	.93822	109.24	110.24
120	.90767	1.0000	.87799	.90163	.89670	.89768	.90214	.90284	114.81	115.47
130	.86298	1.0000	.82516	.86717	.85128	.85547	.85875	.85976	120.34	120.71
140	.81163	1.0000	.76420	.83155	.79876	.80936	.80852	.80989	125.74	125.91
150	.75559	1.0000	.69635	.79654	.73982	.76162	.75217	.75452	130.92	131.02
160	.69691	1.0000	.62293	.76369	.67502	.71472	.69031	.69550	135.82	135.93
170	.63745	1.0000	.54460	.73403	.60417	.67084	.62286	.63439	140.39	140.63

TABLE 4-40

LAMBDA 2 ROTS (4,3) STD. AA24

		SUCCESSIVE SUBSTITUTIONS						
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.443039	.411027	.423094	.469081	.472813	.465236	.466815	.465890
20	.354322	.313447	.317333	.359613	.361035	.351730	.353691	.353318
40	.243120	.194490	.190790	.229095	.229371	.221341	.222831	.222486
60	.106005	.051856	.044183	.077616	.076969	.073809	.074219	.074150
80	-.056708	-.114226	-.117687	-.091018	-.092803	-.088788	-.088955	-.088978
100	-.237340	-.299248	.284856	-.268487	-.270450	-.260685	-.260042	-.260286
120	-.419666	-.493977	-.443552	-.443734	-.441724	-.431541	-.429937	-.429964
140	-.584158	-.686725	-.580421	-.607060	-.592073	-.589501	-.586560	-.586572
160	-.717156	-.868408	-.687471	-.754223	-.711641	-.728281	-.716098	-.718522

TABLE 4-41

LAMBDA 3 ROTS (4,3) STD. AA24

		SUCCESSIVE SUBSTITUTIONS						
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.24128	.23439	.22339	.28794	.29118	.27784	.27969	.27891
20	.26715	.25385	.24501	.29717	.29631	.27787	.28267	.28191
40	.29526	.27314	.26663	.29916	.29876	.28079	.28661	.28511
60	.32388	.29085	.28609	.29409	.29836	.28886	.29193	.29116
80	.34975	.30516	.30026	.28555	.29478	.30307	.30124	.30160
100	.36818	.31443	.30560	.27995	.28892	.32023	.31818	.31824
120	.37482	.31805	.29936	.28204	.28490	.33360	.33900	.33959
140	.36822	.31695	.28082	.29017	.28921	.33835	.34982	.35449
160	.35087	.30886	.24679	.29317	.30032	.32661	.33499	.34110

TABLE 4-42

LAMBDA 4 ROTS (4,3) STD. AA24

		SUCCESSIVE SUBSTITUTIONS						
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.61036	.58718	.55175	.76654	.77943	.73461	.74033	.73792
20	.66867	.63227	.60654	.76490	.76320	.70533	.72025	.71790
40	.73178	.67661	.65967	.74723	.74673	.69404	.71128	.70681
60	.79540	.71684	.70570	.71825	.73091	.70462	.71328	.71108
80	.85151	.74873	.73728	.69003	.71500	.73608	.73119	.73214
100	.88895	.76832	.74681	.67737	.70003	.77599	.77039	.77058
120	.89763	.77375	.72939	.68607	.69281	.80475	.81656	.81788
140	.87469	.76636	.68538	.70542	.70311	.81012	.83489	.84508
160	.82660	.74110	.61082	.70767	.72236	.77668	.79398	.80672

TABLE 4-43

LAMBDA 1 ROTS (4,3) STD. AA1A

		SUCCESSIVE SUBSTITUTIONS								
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.97061	1.0000	.97906	.98095	.97049	.96807	.96912	.96938	76.07	75.79
10	.97848	1.0000	.98522	.98690	.97870	.97705	.97785	.97802	78.09	77.97
20	.98541	1.0000	.99056	.99196	.98584	.98478	.98534	.98545	80.20	80.21
30	.99122	1.0000	.99490	.99593	.99172	.99112	.99146	.99151	82.40	82.53
40	.99572	1.0000	.99803	.99865	.99617	.99589	.99604	.99607	84.70	84.92
50	.99871	1.0000	.99974	.99992	.99899	.99890	.99894	.99895	87.09	87.38
60	.99997	1.0000	.99980	.99958	.99999	.99999	.99999	.99999	89.57	89.90
70	.99929	1.0000	.99796	.99747	.99899	.99900	.99904	.99905	92.15	92.49
80	.99646	1.0000	.99397	.99345	.99581	.99576	.99594	.99595	94.81	95.15
90	.99130	1.0000	.98758	.98741	.99030	.99016	.99053	.99056	97.56	97.88
100	.98363	1.0000	.97853	.97931	.98232	.98209	.98269	.98275	100.37	100.66
110	.97333	1.0000	.96660	.96913	.97174	.97147	.97231	.97239	103.26	103.49
120	.96031	1.0000	.95157	.95692	.95828	.95933	.95933	.95941	106.19	106.38
130	.94454	ERROR IN DATA								
140	.92604	1.0000	.91148	.92697	.92378	.92437	.92533	.92540	112.17	112.27
150	.90491	1.0000	.88612	.90964	.90222	.90386	.90430	.90438	115.18	115.26
160	.88128	1.0000	.85710	.89115	.87782	.88125	.88044	.88082	118.20	118.26
170	.85537	1.0000	.82426	.87181	.85048	.85677	.85436	.85479	121.19	121.26
180	.82743	1.0000	.78757	.85204	.82012	.83084	.82555	.82655	124.16	124.25
190	.79773	1.0000	.74462	.83110	.78458	.80234	.79243	.79452	127.08	127.39

TABLE 4-44

LAMBDA 2 ROTS (4,3) STD. AA1A

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.240655	.204597	.995187	.241188	.250738	.246554	.245538	.245559
20	.170187	.137370	.126791	.167696	.173772	.170569	.169887	.169952
40	.092349	.062703	.051952	.087341	.090536	.088838	.088498	.088541
60	.007357	-.019665	-.028793	.000663	.001631	.001735	.001715	.001719
80	-.083985	-.109746	-.114437	-.091343	-.091885	-.090012	-.089778	-.089814
100	-.180159	-.207184	-.203394	-.187219	-.188424	-.185232	-.184870	-.184925
120	-.278911	-.311223	-.293505	-.285183	-.285926	-.282290	-.281967	-.282012
140	-.377409	-.420757	-.382165	-.383439	-.382003	-.379206	-.378968	-.378994
160	-.472582	-.534585	-.466562	-.480630	-.471899	-.473987	-.473349	-.473443
180	-.561567	-.651805	-.543981	-.576340	-.560099	-.565083	-.562345	-.562862

TABLE 4-45

LAMBDA 3 ROTS (4,3) STD. AA1A

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.16226	.14470	.13805	.17976	.17521	.16744	.6845	.16817
20	.17216	.15390	.14811	.17666	.17961	.17338	.17421	.17403
40	.18206	.16350	.15846	.18132	.18334	.17965	.18012	.18003
60	.19168	.17347	.16883	.18484	.18650	.18633	.18636	.18635
80	.20065	.18375	.17883	.18756	.18923	.19234	.19307	.19311
100	.20853	.19429	.18795	.19014	.19180	.20049	.20026	.20029
120	.21490	.20510	.19558	.19344	.19460	.20738	.20759	.20758
140	.21941	.21621	.20099	.19826	.19818	.21338	.21416	.21417
160	.22181	.22723	.20292	.20468	.20279	.21721	.21810	.21822
180	.22208	.23289	.19500	.20721	.20361	.21218	.21220	.21261

TABLE 4-46

LAMBDA 4 ROTS (4,3) STD. AA1A

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.39489	.34237	.32416	.42412	.43755	.41151	.41540	.41437
20	.41153	.36091	.34634	.42664	.43496	.41550	.41843	.41780
40	.42786	.37996	.36814	.42692	.43235	.42157	.42310	.42280
60	.44322	.39930	.38883	.42570	.43011	.42958	.42969	.42967
80	.45680	.41863	.40748	.42419	.42857	.43905	.43824	.43825
100	.46770	.43767	.42301	.42393	.42810	.44908	.44835	.44841
120	.47504	.45613	.43429	.42645	.42922	.45841	.45871	.45868
140	.47814	.47372	.44144	.43268	.43250	.46541	.46697	.46701
160	.47664	.48936	.43849	.44167	.43770	.46738	.46915	.46941
180	.47059	.49260	.41784	.44184	.43474	.45152	.45154	.45235

TABLE 4-47

LAMBDA 1 ROTS (4,3) STD. AA24A

			SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.90226	1.0000	.91977	.91608	.88935	.88620	.88834	.88827	64.45	62.66
10	.91826	1.0000	.93412	.93186	.90984	.90783	.91037	.91015	66.67	65.53
20	.93402	1.0000	.94819	.94720	.92948	.92827	.93085	.93057	69.07	68.52
30	.94922	1.0000	.96163	.96165	.94781	.94715	.94946	.94914	71.66	71.65
40	.96347	1.0000	.97398	.97467	.96431	.96399	.96583	.96561	74.46	74.93
50	.97628	1.0000	.98470	.98567	.97841	.97829	.97955	.97944	77.49	78.35
60	.98702	1.0000	.99312	.99398	.98948	.98947	.99014	.99007	80.76	81.92
70	.99500	1.0000	.99848	.99891	.99687	.99691	.99712	.99710	84.27	85.64
80	.99940	1.0000	.99993	.99980	.99995	.99995	.99996	.99996	88.02	89.50
90	.99937	1.0000	.99657	.99610	.99807	.99799	.99813	.99812	92.02	93.51
100	.99407	1.0000	.98753	.98741	.99070	.99046	.99113	.99111	96.24	97.64
110	.98274	1.0000	.97199	.97361	.97739	.97698	.97852	.97851	100.66	101.90
120	.96484	1.0000	.94930	.95489	.95784	.95740	.95996	.96001	105.23	106.26
130	.94013	1.0000	.91906	.93176	.93192	.93185	.93528	.93546	109.92	110.70
140	.90875	1.0000	.88115	.90518	.89965	.90083	.90450	.90487	114.66	115.19
150	.87122	1.0000	.83579	.87617	.86121	.86532	.86705	.86845	119.39	119.72
160	.82842	1.0000	.78346	.84605	.81687	.82631	.82568	.82667	124.06	124.24
170	.78149	1.0000	.72491	.81615	.76697	.78568	.77842	.78041	128.60	128.70
180	.73169	1.0000	.66093	.78768	.71170	.74511	.72635	.73976	132.97	133.05
190	.68027	1.0000	.59187	.76149	.65068	.70629	.66911	.67863	137.13	137.26
200	.62838	1.0000	.50963	.73549	.57523	.66700	.59836	.61853	141.06	141.79

TABLE 4-48

LAMBDA 2 ROTS (4,3) STD. AA24A

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.431192	.400567	.409679	.458522	.464676	.459345	.460151	.459323
20	.357209	.321892	.324940	.369302	.372339	.365417	.366535	.366096
40	.267784	.228097	.225035	.264843	.265993	.259146	.260228	.259959
60	.160551	.117280	.109682	.144661	.144688	.140051	.140685	.140560
80	.034446	-.011606	-.019607	.009900	.008996	.008681	.008716	.008711
100	-.108731	-.157892	-.158661	-.136003	-.137767	-.132821	-.132965	-.133035
120	-.262823	-.318430	-.300351	-.287392	-.288872	-.280123	-.279780	-.279940
140	-.417325	-.487531	-.435476	-.437631	-.435164	-.426567	-.425628	-.425688
160	-.560093	-.658086	-.554873	-.581102	-.567035	-.564904	-.562508	-.562674
180	-.681635	-.823484	-.651643	-.715011	-.676354	-.690565	-.680448	-.682632
200	-.777901	-.990321	-.727325	-.848494	-.762538	-.811759	-.774942	-.785758

TABLE 4-49

LAMBDA 3 ROTS (4,3) STD. AA24A

		SUCCESSIVE SUBSTITUTIONS						
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.21347	.20410	.19331	.25174	.25616	.24798	.24880	.24814
20	.23313	.22010	.21096	.26238	.26277	.25082	.25329	.25262
40	.25440	.23659	.22934	.26873	.26761	.25457	.25811	.25725
60	.27660	.25299	.24751	.27014	.27069	.26042	.26347	.26272
80	.29837	.26846	.26391	.26733	.27178	.26950	.27017	.27002
100	.31756	.28202	.27647	.26300	.27076	.28196	.27998	.28031
120	.33148	.29282	.28287	.26111	.26837	.29573	.29427	.29432
140	.33777	.30047	.28107	.26453	.26716	.30701	.31025	.31052
160	.33544	.30516	.36978	.27258	.27094	.31244	.21995	.32228
180	.32543	.30660	.24767	.28019	.28120	.30845	.31488	.31875
200	.31003	.00998	.08278	.03791	.06063	.04848	.05906	.05703

TABLE 4-50

LAMBDA 4 ROTS (4,3) STD. AA24A

		SUCCESSIVE SUBSTITUTIONS						
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.51448	.48250	.44848	.64015	.65636	.62933	.63177	.62971
20	.55489	.51732	.49116	.64637	.64838	.61128	.61883	.61681
40	.59855	.55296	.53427	.64252	.63974	.60185	.61221	.60968
60	.64379	.58802	.57532	.62942	.63144	.60339	.61189	.60979
80	.68742	.62055	.61067	.61125	.62343	.61740	.61919	.61878
100	.72456	.64823	.63579	.59564	.61528	.64212	.63709	.63793
120	.74930	.66896	.64609	.59068	.60791	.67025	.66643	.66657
140	.75655	.68147	.63819	.59938	.60520	.69142	.69799	.69856
160	.74453	.68562	.61087	.61608	.61250	.69779	.71300	.71781
180	.71569	.68044	.56337	.62765	.62945	.68269	.69521	.70284
200	.67542	.11893	-.05421	.02954	-.01288	.00980	-.00995	-.00615

TABLE 4-51

LAMBDA 1 ROTS (4,3) AA1B

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.96285	1.0000	.97246	.97470	.96214	.95870	.95959	.95985	74.33	73.71
10	.97038	1.0000	.97836	.98047	.97002	.96744	.96823	.96842	76.02	75.56
20	.97732	1.0000	.98380	.98571	.97726	.97540	.97606	.97619	77.77	77.47
30	.98358	1.0000	.98868	.99033	.98373	.98246	.98296	.98305	79.60	79.44
40	.98903	1.0000	.99288	.99420	.98931	.98851	.98885	.98890	81.50	81.46
50	.99354	1.0000	.99624	.99719	.99386	.99341	.99361	.99364	83.48	83.54
60	.99697	1.0000	.99863	.99916	.99726	.99704	.99713	.99714	85.53	85.67
70	.99917	1.0000	.99986	.99998	.99935	.99928	.99930	.99930	87.67	87.87
80	.99999	1.0000	.99977	.99951	.99999	.99999	.99999	.99999	89.88	90.12
90	.99928	1.0000	.99816	.99764	.99905	.99906	.99909	.99909	92.16	92.43
100	.99688	1.0000	.99483	.99425	.99639	.99635	.99648	.99649	94.52	94.80
110	.99264	1.0000	.98958	.98925	.99188	.99177	.99204	.99205	96.95	97.23
120	.98643	1.0000	.98220	.98258	.98540	.98521	.98566	.98569	99.44	99.70
130	.97812	1.0000	.97249	.97421	.97685	.97661	.97726	.97729	102.00	102.23
140	.96763	1.0000	.96023	.96416	.96614	.96590	.96675	.96678	104.61	104.81
150	.95489	1.0000	.94526	.95249	.95319	.95309	.95406	.95408	107.27	107.43
160	.93987	1.0000	.92738	.93929	.93792	.93816	.93912	.93915	109.96	110.09
170	.92258	1.0000	.90646	.92476	.92028	.92122	.92195	.92199	112.69	112.78
180	.90308	1.0000	.88233	.90906	.90018	.90234	.90230	.90258	115.43	115.50
190	.88146	1.0000	.85493	.89250	.87760	.88175	.88082	.88105	118.18	118.23
200	.85784	1.0000	.82406	.87534	.85234	.85963	.85683	.85740	120.92	120.97
210	.83240	1.0000	.78963	.85792	.82427	.83631	.83055	.83178	123.65	123.72
220	.80533	ERROR IN DATA								

TABLE 4-52

LAMBDA 2 ROTS (4,3) STD. AA1B

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.270017	.234676	.224911	.272623	.284527	.281381	.280590	.280508
20	.211729	.179955	.169041	.212069	.220436	.217501	.216911	.216909
40	.147697	.119310	.107660	.145808	.151149	.148890	.148513	.148547
60	.077774	.052336	.040855	.073934	.076796	.075603	.075427	.075446
80	.002083	-.021256	-.031004	-.003182	-.002240	-.002115	-.002118	-.002117
100	-.078883	-.101604	-.107205	-.084885	-.085284	-.083791	-.083669	-.083690
120	-.164177	-.188639	-.186633	-.170236	-.171322	-.168686	-.168509	-.168543
140	-.252353	-.281995	-.267694	-.258057	-.258955	-.255718	-.255571	-.2555603
160	-.341515	-.381099	-.388425	-.347170	-.346502	-.343588	-.343488	-.343505
180	-.429457	-.485110	-.426468	-.436521	-.431974	-.430790	-.430448	-.430503
200	-.513909	-.593187	-.499317	-.525579	-.513212	-.515991	-.514312	-.514639
220	.592814	ERROR IN DATA						

TABLE 4-53

LAMBDA 3 ROTS (4,3) STD. AA1B

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.15020	.13237	.13519	.15910	.16419	.15860	.15921	.15898
20	.15898	.14067	.13422	.16584	.16938	.16425	.16485	.16469
40	.16796	.14942	.14374	.17177	.17411	.17003	.17052	.17042
60	.17698	.15863	.15361	.17679	.17839	.17602	.17629	.17624
80	.18586	.16830	.16362	.18093	.18226	.18228	.18229	.18229
100	.190434	.17844	.17349	.18440	.18580	.18882	.18860	.18863
120	.20212	.18901	.18282	.18762	.18912	.19554	.19527	.19530
140	.20888	.20007	.19112	.19119	.19243	.20224	.20216	.20216
160	.21433	.21160	.19779	.19573	.19600	.20850	.20880	.20878
180	.21822	.22362	.20207	.20171	.20022	.21374	.21432	.21432
200	.22040	.23553	.20245	.20866	.20483	.21657	.21684	.21700
220	.22085	ERROR IN DATA						

TABLE 4-54

LAMBDA 4 ROTS (4,3) STD. AA1B

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.34150	.28702	.26766	.37126	.38636	.36783	.37020	.36939
20	.35437	.30236	.28643	.37615	.38585	.37005	.37217	.37161
40	.36741	.31841	.30546	.37923	.38520	.37346	.37505	.37473
60	.38029	.33509	.32435	.38068	.38459	.37818	.37902	.37886
80	.39262	.35227	.34256	.38095	.38421	.38422	.38425	.38424
100	.40388	.36979	.35945	.38080	.38422	.39139	.39081	.39089
120	.41349	.38745	.37421	.38122	.38475	.39926	.39852	.39860
140	.42088	.40509	.38601	.38332	.38608	.40712	.40683	.40682
160	.42549	.42244	.39384	.38789	.38846	.41394	.41445	.41441
180	.42694	.43926	.39665	.39519	.39221	.41846	.41950	.41951
200	.42504	.45416	.39219	.40358	.39639	.41811	.41858	.41889
220	.41985	ERROR IN DATA						

TABLE 4-55

LAMBDA 1 ROTS (4,3) STD. AA24B

TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
			SUCCESSIVE SUBSTITUTIONS							
0	.90245	1.0000	.91937	.91651	.88807	.88335	.88351	.88377	64.48	62.10
10	.91548	1.0000	.93061	.93872	.90446	.90114	.90173	.90204	66.27	64.43
20	.92840	1.0000	.94184	.94086	.92051	.91828	.91947	.91946	68.18	66.85
30	.94105	1.0000	.95287	.95270	.93596	.93456	.93592	.93584	70.22	69.37
40	.95324	1.0000	.96349	.96398	.95054	.94972	.95107	.95097	72.41	71.98
50	.96474	1.0000	.97345	.97439	.96394	.96351	.96470	.96456	74.74	74.70
60	.97525	1.0000	.98341	.98356	.97579	.97561	.97654	.97645	77.22	77.54
70	.98444	1.0000	.99000	.99110	.98572	.98568	.98630	.98624	79.88	80.49
80	.99191	1.0000	.99578	.99658	.99333	.99337	.99369	.99659	82.70	83.54
90	.99720	1.0000	.99925	.99957	.99824	.99827	.99836	.99835	85.71	86.72
100	.99981	1.0000	.99984	.99964	.99999	.99999	.99999	1.0000	88.89	90.00
110	.99922	1.0000	.99696	.99644	.99821	.99813	.99824	.99823	92.25	93.40
120	.99490	1.0000	.99002	.98968	.99251	.99233	.99277	.99274	95.78	96.91
130	.98636	1.0000	.97840	.97923	.98257	.98226	.98227	.98322	99.47	100.51
140	.97319	1.0000	.96157	.96509	.96813	.96775	.96947	.96941	103.29	104.21
150	.95511	1.0000	.93908	.94749	.94897	.94874	.95119	.95115	107.23	107.98
160	.93203	1.0000	.91064	.92686	.92499	.92536	.92831	.92833	111.24	111.82
170	.90403	1.0000	.87604	.90382	.89608	.89797	.90080	.80094	115.30	115.72
180	.87143	1.0000	.83532	.87914	.86224	.86717	.86874	.86912	119.37	119.64
190	.83472	1.0000	.78867	.85375	.82347	.83385	.83225	.83318	123.41	123.57
200	.79458	1.0000	.73643	.82859	.77974	.79915	.79143	.79360	127.38	127.48
210	.75177	1.0000	.67886	.80453	.73081	.76434	.74611	.75093	131.25	131.33
220	.70710	1.0000	.61664	.78253	.67661	.73116	.69616	.70614	135.00	135.08
230	.66137	1.0000	.54921	.76302	.61574	.70090	.63994	.65929	138.59	138.75

TABLE 4-56

LAMBDA 2 ROTS (4,3) STD. AA24B

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.430788	.401560	.408613	.461042	.470180	.468613	.468818	.467905
20	.371568	.341046	.343904	.391295	.395546	.393211	.393582	.393163
40	.302192	.270186	.268365	.310757	.313265	.308960	.309492	.309282
60	.221068	.187521	.181287	.218722	.219540	.215319	.215866	.215738
80	.126904	.091777	.082638	.115115	.114953	.112143	.112497	.112431
100	.019236	-.017857	-.026522	-.008063	-.000064	-.000087	-.000068	-.000072
120	-.100846	-.141306	-.143597	-.122129	-.123613	-.120013	-.120242	-.120239
140	-.229989	-.277221	-.264220	-.250497	-.251943	-.245173	-.245346	-.245400
160	-.362368	-.422752	-.382442	-.380489	-.379571	-.371832	-.371681	-.371739
180	-.490519	-.573896	-.491630	-.508688	-.500024	-.495567	-.494490	-.494586
200	-.607154	-.726041	-.585497	-.632794	-.606512	-.612629	-.607347	-.608437
220	-.707108	-.875614	-.659498	-.752683	-.692859	-.722377	-.702415	-.708071

TABLE 4-57

LAMBDA 3 ROTS (4,3) STD. AA24B

TRUE	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.18868	.17704	.16612	.22024	.22604	.22412	.22406	.22339
20	.20371	.19016	.18042	.23169	.23372	.22886	.22946	.22907
40	.21996	.20405	.19576	.24101	.24035	.23350	.23485	.23449
60	.23722	.21852	.21182	.24744	.24594	.23857	.24036	.23994
80	.25504	.23238	.22799	.25059	.25044	.24473	.24623	.24589
100	.27261	.24794	.24332	.25085	.25367	.25268	.25297	.25291
120	.28872	.26198	.25651	.24967	.25543	.26276	.26154	.26168
140	.30187	.27490	.26588	.24937	.25593	.27426	.27350	.27267
160	.31054	.28636	.27007	.25225	.25632	.28527	.28549	.28547
180	.31365	.29623	.26728	.25907	.25838	.29339	.29693	.29748
200	.31099	.30471	.25658	.26844	.26480	.29666	.30169	.30353
220	.30331	.30880	.23390	.27416	.27304	.28936	.29216	.29515

TABLE 4-58

LAMBDA 4 ROTS (4,3) STD. AA24B

				SUCCESSIVE SUBSTITUTIONS				
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.42576	.38511	.35144	.53623	.54649	.54065	.54036	.53825
20	.45294	.41137	.38392	.53762	.54461	.52985	.53159	.53044
40	.48239	.43903	.41781	.54337	.54190	.52226	.52611	.52508
60	.51362	.46764	.45212	.54284	.53908	.51921	.52409	.52296
80	.54559	.49651	.48536	.53659	.53656	.52202	.52593	.52503
100	.57659	.52464	.51541	.52689	.53417	.53170	.53243	.53227
120	.60410	.55085	.53958	.51780	.53153	.54796	.54486	.54540
140	.62507	.57386	.55489	.51409	.52872	.56792	.56386	.56425
160	.63652	.59267	.55863	.51896	.52729	.58642	.58654	.58649
180	.63650	.60658	.54880	.53151	.53005	.59794	.60455	.60561
200	.62484	.61555	.52471	.54676	.53976	.59864	.60781	.61124
220	.60327	.61408	.48071	.55228	.55014	.57892	.58379	.58912

TABLE 4-59

LAMBDA 1 ROTS (4,3) STD. AA1C

		SUCCESSIVE SUBSTITUTIONS								
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.90859	1.0000	.92936	.93087	.90015	.88859	.88251	.88170	65.31	61.85
20	.92267	1.0000	.93905	.94056	.91470	.90633	.90175	.90130	67.32	64.33
40	.93652	1.0000	.94905	.95066	.92943	.92374	.92042	.92021	69.47	66.96
60	.94995	1.0000	.95918	.96084	.94405	.94050	.93819	.83813	71.79	69.74
80	.96270	1.0000	.96920	.97108	.95816	.95621	.95470	.95471	74.30	72.69
100	.97443	1.0000	.97877	.98064	.97128	.97040	.96950	.96954	77.01	75.82
120	.98468	1.0000	.98739	.98906	.98279	.98254	.98209	.98212	79.95	79.15
140	.99286	1.0000	.99440	.99560	.99198	.99201	.99184	.99185	83.15	82.68
160	.99826	1.0000	.99892	.99941	.99802	.99809	.99806	.99806	86.62	86.44
180	.99997	1.0000	.99980	.99953	.99998	.99997	.99997	.99997	90.38	90.42
200	.99697	1.0000	.99563	.99499	.99687	.99671	.99672	.99672	94.45	94.64
220	.98812	1.0000	.98469	.98497	.98765	.98732	.98741	.98739	98.83	99.11
240	.97224	1.0000	.96498	.96899	.97122	.97085	.97113	.97109	103.53	103.81
260	.94829	1.0000	.93433	.94719	.94640	.94659	.94703	.94695	108.50	108.75
280	.91549	1.0000	.89051	.92059	.91174	.91441	.91422	.91428	113.72	113.90
300	.87356	1.0000	.83132	.89127	.86524	.87533	.87163	.87266	119.12	119.23
320	.82280	1.0000	.75498	.86248	.80392	.83259	.81741	.82221	124.63	124.69
340	.76422	1.0000	.65967	.83823	.72254	.79267	.74647	.76348	130.16	130.23
360	.69937	1.0000	.50031	.81707	.56721	.76027	.60012	.67210	135.62	137.77

TABLE 4-60

LAMBDA 2 ROTS (4,3) STD. AA1C

		SUCCESSIVE SUBSTITUTIONS						
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.417679	.375846	.371827	.436594	.560012	.470491	.472779	.471805
40	.350603	.319213	.314113	.369419	.383533	.390984	.391835	.391485
80	.270545	.248165	.240489	.286320	.292814	.297569	.297641	.297527
120	.174357	.158764	.147901	.184689	.186029	.188399	.188233	.188234
160	.058916	.046459	.034233	.062832	.061624	.062199	.062129	.062137
200	-.077688	-.093444	-.100011	-.078992	-.081043	-.080926	-.080922	-.080926
240	-.233963	-.264643	-.244901	-.238209	-.239710	-.238513	-.238730	-.238702
280	-.403285	-.467937	-.398504	-.410505	-.405471	-.405278	-.404973	-.405064
320	-.568322	-.700017	-.524441	-.599828	-.557383	-.576937	-.565586	-.569177
360	-.714753	-.999687.	-.604855	-.855401	-.657759	-.810302	-.683511	-.740457

TABLE 4-61

LAMBDA 3 ROTS (4,3) STD. AA1C

				SUCCESSIVE SUBSTITUTIONS				
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.13846	.11658	.10613	.14803	.15750	.16592	.16697	.16638
40	.15426	.13101	.12119	.16628	.17239	.18005	.18035	.18004
80	.17193	.14760	.13909	.18520	.18746	.19355	.19332	.19323
120	.19152	.16681	.16011	.20362	.20267	.20638	.20601	.20603
160	.21280	.18915	.18408	.22002	.21789	.21893	.21879	.21881
200	.23485	.21504	.20986	.23346	.23254	.23225	.23233	.23232
240	.25582	.24475	.23453	.24515	.24544	.24770	.24753	.24750
280	.27282	.27846	.25261	.25915	.25570	.26498	.26447	.26435
320	.28275	.31585	.25498	.27923	.26298	.27994	.27747	.27818
360	.28383	-1.84270	-1.98287	-1.92501	-2.10612	-1.90809	-2.00565	-1.95808

TABLE 4-62

LAMBDA 4 ROTS (4,3) STD. AA1C

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.25000	.17795	.14908	.27505	.30338	.33082	.33434	.33267
40	.26603	.19797	.17457	.29552	.31199	.33379	.33477	.33395
80	.28454	.22156	.20409	.21489	.32047	.33570	.33521	.33499
120	.30552	.24918	.23738	.33114	.32921	.33745	.33665	.33670
160	.32839	.28108	.27330	.34263	.33857	.34062	.34035	.34038
200	.35171	.31715	.30920	.34926	.34805	.34749	.34764	.34761
240	.37269	.35660	.34008	.35566	.35620	.35987	.35958	.35953
280	.38732	.39798	.35834	.36729	.36191	.37588	.37506	.37489
320	.39157	.43863	.35355	.38719	.36440	.38792	.38439	.38540
360	.38354	COMPUTER	TRUNCATION ERROR					

TABLE 4-63

LAMBDA 1 ROTS (4,3) STD. AA24C

			SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.86585	1.0000	.89021	.88607	.83694	.82172	.80370	.80228	59.98	53.35
20	.88190	1.0000	.90115	.89721	.85523	.84493	.83014	.82978	61.87	56.08
40	.89828	1.0000	.91303	.90975	.87470	.86859	.85975	.85703	63.93	58.99
60	.91487	1.0000	.92579	.92336	.89506	.89229	.88308	.88369	66.18	62.09
80	.93149	1.0000	.93925	.93787	.91590	.91550	.90862	.90932	68.66	65.41
100	.94782	1.0000	.95315	.95285	.93660	.93759	.93276	.93339	71.40	68.97
120	.96340	1.0000	.96697	.96762	.95631	.95781	.95474	.95521	74.45	72.79
140	.97755	1.0000	.97995	.98118	.97394	.97530	.97364	.97393	77.83	76.89
160	.98931	1.0000	.99093	.99216	.98815	.98902	.98835	.98847	81.61	81.29
180	.99736	1.0000	.99825	.99885	.99738	.99770	.99757	.99759	85.83	86.03
200	.99995	1.0000	.99962	.99930	.99986	.99980	.99981	.99981	90.53	91.10
220	.99497	1.0000	.99210	.99162	.99375	.99347	.99350	.99350	95.74	96.53
240	.98002	1.0000	.97213	.97452	.97712	.97675	.97705	.97698	101.47	102.31
260	.95278	1.0000	.93577	.94793	.94787	.94799	.94891	.94870	107.67	108.43
280	.91153	1.0000	.87918	.91377	.90354	.90689	.90757	.90746	114.28	114.84
300	.85576	1.0000	.79930	.87623	.84066	.85590	.85106	.85269	121.15	121.49
320	.78663	1.0000	.69446	.84143	.75391	.80243	.77512	.78487	128.12	128.29
340	.70694	1.0000	.56440	.81623	.63464	.76075	.66744	.70581	135.01	135.10
360	.62061	1.0000	.40855	.80664	.46701	.75337	.49134	.61804	141.63	141.83

TABLE 4-64

LAMBDA 2 ROTS (4,3) STD. AA24C

TIME	TRUE	FIRST	SECOND	SUCCESSIVE APPROXIMATIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.500288	.468589	.477337	.550585	.573952	.596155	.601529	.596946
40	.439417	.417941	.424848	.486414	.497491	.516130	.517919	.515259
80	.363760	.348543	.352497	.402057	.402990	.417693	.417171	.416082
120	.268046	.257003	.254459	.292477	.287501	.297438	.296050	.295902
160	.145765	.134623	.125201	.153434	.147767	.152171	.151236	.151354
200	-.009380	-.027362	-.037356	-.016243	-.019633	-.019242	-.019266	-.019270
240	-.198888	-.236056	-.225725	-.212715	-.214405	-.213009	-.213342	-.213285
280	-.411226	-.491549	-.415266	-.429427	-.422222	-.419973	-.420118	-.420119
320	-.617421	-.781715	-.563136	-.665646	-.601930	-.633446	-.612265	-.619656
360	-.784113	-.010876	-.621859	-.935253	-.667535	-.892140	-.686173	-.786145

TABLE 4-65

LAMBDA 3 ROTS (4,3) STD. AA24C

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.15157	.13379	.12051	.17059	.18165	.20340	.20945	.20711
40	.17077	.15187	.13889	.19412	.20014	.22065	.22513	.22316
80	.19319	.17305	.16141	.21927	.21881	.23795	.23878	.23762
120	.21936	.19794	.18869	.24395	.23764	.25191	.25088	.25059
160	.24945	.22698	.22063	.26474	.25684	.26378	.26261	.26273
200	.28239	.26013	.25534	.27862	.27553	.27561	.27567	.27565
240	.31464	.29642	.28731	.28738	.29039	.29245	.29212	.29214
280	.33933	.33389	.30584	.30002	.29857	.31592	.31460	.31456
320	.34873	.37056	.29618	.32238	.30479	.33509	.33555	.33641
360	.34024	.38329	.22046	.32829	.29592	. 32090	.29779	.30901

TABLE 4-66

LAMBDA 4 ROTS (4,3) STD. AA24C

TIME	TRUE	FIRST	SECOND	SECCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	NEW-RAPH
0	.27059	.20517	.16533	.31805	.35394	.42518	.44665	.44012
40	.29268	.23123	.19759	.34832	.36686	.42778	.43915	.43391
80	.31945	.26246	.23633	.37808	.37884	.42622	.42930	.42641
120	.35151	.29955	.28170	.40326	.39074	.42196	.42019	.41948
160	.38883	.34270	.33239	.41912	.40413	.41771	.41553	.41574
200	.42953	.39104	.38409	.42386	.41866	.41873	.41885	.41881
240	.46809	.44189	.42740	.42476	.42994	.43324	.43271	.43274
280	.49457	.49042	.44718	.43680	.43444	.46048	.45838	.45833
320	.49834	.53114	.42652	.46301	.43807	.48006	.48037	.48166
360	.47669	.53226	.32515	.46229	.42097	.45282	.42328	.43762

TABLE 4-67

LAMBDA 1 ROTS (5,3) STD. AA1

TIME	TRUE	SECOND	THIRD	SUCCESSIVE SUBSTITUTIONS FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.97371	.98224	.98419	.97328	.97205	.94794	.91536	.97364	76.84	76.82
20	.98952	.99413	.99530	.98985	.98930	.98331	.97995	.98950	81.69	81.69
40	.99867	.99982	.99996	.99893	.99885	.99832	.99820	.99867	87.04	87.05
60	.99874	.99679	.99623	.99836	.99836	.99833	.99833	.99874	92.86	92.87
80	.98740	.98237	.98262	.98618	.98598	.98566	.98567	.98738	99.10	99.11
100	.96288	.95406	.95859	.96102	.96072	.96071	.96071	.96284	105.65	105.67
120	.92456	.90995	.92495	.92222	.92270	.92304	.92309	.92450	112.39	112.41
140	.87331	.84905	.88408	.86993	.87349	.87254	.87292	.87326	119.15	119.16
160	.81147	.77086	.83935	.80413	.81591	.80964	.81181	.81135	125.76	125.77

TABLE 4-68

LAMBDA 2 ROTS (5,3) STD. AA1

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.227749	.188450	.177778	.229645	.234813	.318438	.402705	.391618	.228071
20	.144370	.108287	.096882	.142051	.145842	.181914	.199206	.182983	.144523
40	.051507	.018799	.008308	.046042	.047877	.057838	.059847	.054119	.051518
60	-.050004	-.080055	-.086795	-.057198	-.057242	-.057735	-.057695	-.058953	-.050095
80	-.158201	-.187738	-.186427	-.165628	-.166819	-.168699	-.168672	-.168820	-.158348
100	-.269914	-.303090	-.287784	-.276579	-.277598	-.277555	-.277528	-.277536	-.270068
120	-.381036	-.424370	-.387406	-.387203	-.386048	-.384736	-.384615	-.384633	-.381163
140	-.487157	-.599441	-.481488	-.495086	-.488630	-.488084	-.488130	-.488291	-.487246
160	-.584391	-.676956	-.566816	-.599519	-.582567	-.587939	-.584900	-.585945	-.584555

TABLE 4-69

LAMBDA 3 ROTS (5,3) STD. AA1

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.17809	.15959	.15247	.18737	.19092	.21279	.24968	.25236	.18171
20	.18963	.16997	.16389	.19257	.19572	.20464	.21400	.20698	.19004
40	.20091	.18064	.17536	.19607	.19832	.20106	.20253	.19956	.19830
60	.21139	.19147	.18635	.19824	.20036	.20222	.20188	.20118	.20654
80	.22042	.22235	.19622	.19994	.20193	.20727	.20649	.20647	.21645
100	.22736	.21322	.20419	.20241	.20379	.21459	.21402	.21405	.22224
120	.23167	.22408	.20938	.20690	.20691	.22209	.22242	.22243	.22839
140	.23309	.23504	.21084	.21409	.21242	.22760	.22856	.22861	.23190
160	.23168	.23995	.20120	.21739	.21497	.22212	.22220	.22220	.23212

TABLE 4-70

LAMBDA 4 ROTS (5,3) STD. AA1

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.45610	.40177	.38201	.48762	.49953	.54695	.65028	.66729	.46896
20	.47794	.42426	.40868	.48791	.49610	.51465	.54024	.52297	.47922
40	.49872	.44692	.43411	.48548	.49191	.49708	.50100	.49375	.49068
60	.51715	.46924	.45699	.48193	.48782	.49329	.49321	.49069	.50326
80	.53176	.49073	.47581	.47959	.48491	.49979	.49771	.49769	.51635
100	.54110	.51091	.48899	.48088	.48435	.51146	.51004	.51011	.52831
120	.54408	.52943	.49497	.48741	.48743	.52274	.52345	.52346	.53641
140	.54026	.54604	.49240	.49905	.49532	.52852	.53057	.53070	.53766
160	.52998	.54775	.46701	.50068	.49542	.51044	.51059	.51060	.53094

TABLE 4-71

LAMBDA 1 ROTS (5,3) STD. AA24

TIME	TRUE	SECOND	THIRD	SUCCESSIVE SUBSTITUTIONS FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.89650	.91620	.91225	.88294	.90467	.05693	******	******	63.70	*****
20	.93512	.95133	.95072	.93324	.94095	.68819	******	.93455	69.24	69.16
40	.96999	.98132	.98229	.97368	.97508	.92077	.62566	.96981	75.92	75.89
60	.99436	.99870	.99910	.99710	.99711	.99342	.99044	.99436	83.91	83.91
80	.99839	.99340	.99293	.99571	.99572	.99492	.99483	.99836	93.25	93.28
100	.97142	.95510	.95924	.96303	.96279	.96045	.95998	.97123	103.72	103.78
120	.90767	.87789	.90150	.89646	.89759	.89784	.89711	.90734	114.81	114.86
140	.81163	.76414	.83150	.79866	.80924	.80700	.80857	.81130	125.74	125.78
160	.69691	.62292	.76368	.67501	.71469	.69022	.70225	.69656	135.82	135.85

TABLE 4-72

LAMBDA 2 ROTS (5,3) STD. AA24

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.443039	.409369	.4018917	.470968	.427006	.011183	.001131	*******	********
20	.354322	.311978	.313917	.359614	.338807	.730396	.034199	*******	.355812
40	.243120	.193272	.188153	.227957	.221868	.390157	.788614	.651559	.243838
60	.106005	.050922	.042301	.076045	.075923	.114468	.137913	.119892	.106004
80	-.056708	-.114876	-.118891	-.092467	-.082403	-.100624	-.101490	-.103057	-.057244
100	-.237340	-.299643	-.285514	-.269482	-;270314	-.278442	-.280041	-.279299	-.238122
120	-.419666	-.494173	-.443836	-.444214	-.441901	-.440447	-.441936	-.441560	-.420379
140	-.584159	-.686792	-.580503	-.607193	-.592241	-.591615	-.589427	-.590097	-.584626
160	-.717156	-.868417	-.687480	-.754334	-.711675	-.728364	-.716143	-.721927	-.717493

TABLE 4-73

LAMBDA 3 ROTS (5,3) STD. AA24

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.24128	.23382	.22207	.28813	.27562	.49450	*******	*******	*******
20	.26716	.25326	.24374	.29681	.28747	.42687	*******	*******	.35581
40	.29526	.27257	.26544	.29841	.29518	.36020	.57443	.78516	.29586
60	.32388	.29032	.28504	.29313	.29772	.31198	.32669	.31533	.30855
80	.34975	.30470	.29939	.28456	.29504	.29474	.29340	.29254	.32211
100	.36818	.31407	.30494	.27910	.28904	.30415	.30060	.30149	.33750
120	.37482	.31780	.29894	.28145	.28465	.32243	.32232	.32258	.35310
140	.36822	.31682	.28061	.28989	.28883	.33408	.34260	.34365	.35999
160	.35087	.30883	.24674	.29311	.30016	.32618	.33456	.33388	.34886

TABLE 4-74

LAMBDA 4 ROTS (5,3) STD. AA24

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.61036	.58574	.54848	.76635	.74078	1.26629	*******	*******	*******
20	.66867	.63083	.60346	.76362	.74163	1.06671	4.27578	*******	.72121
40	.73178	.67524	.65686	.74525	.73815	.88304	1.41754	2.14136	.73342
60	.79540	.71561	.70325	.71598	.72938	.75834	.79365	.76694	.75140
80	.85151	.74769	.73531	.68782	.71562	.71723	.71397	.71213	.77756
100	.88895	.76753	.74537	.67552	.70033	.74060	.73237	.73445	.81214
120	.89763	.77323	.72849	.68482	.69229	.78081	.78093	.78153	.84667
140	.87469	.76608	.68495	.70484	.70232	.80123	.81979	.82212	.85660
160	.82660	.74103	.61071	.70755	.72202	.77581	.79311	.79172	.82253

TABLE 4-75

LAMBDA 1 ROTS (5,3) STD. AA1A

TIME	TRUE	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
				SUCCESSIVE SUBSTITUTIONS						
0	.97061	.97924	.98180	.96941	.96710	.93662	.88443	.97053	76.07	76.06
20	.98541	.99067	.99231	.98520	.98401	.97370	.96535	.98537	80.20	80.19
40	.99572	.99808	.99876	.99589	.99551	.99336	.99254	.99571	84.70	84.70
60	.99997	.99979	.99953	.99999	.99998	.99995	.99994	.99997	89.57	89.58
80	.99646	.99391	.99331	.99599	.99597	.99587	.99588	.96460	94.81	94.82
100	.98363	.97845	.97915	.98356	.98233	.98206	.98207	.98361	100.37	100.39
120	.96031	.95149	.95680	.95872	.95847	.95847	.95848	.96028	106.19	106.20
140	.92604	.91143	.92690	.92391	.92447	.92469	.92474	.92600	112.17	112.18
160	.88128	.85709	.89113	.87787	.88128	.88939	.88073	.88124	118.20	118.21
180	.82743	.78757	.85203	.82012	.83084	.82553	.82738	.82732	124.16	124.17

TABLE 4-76

LAMBDA 2 ROTS (5,3) STD. AA1A

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.240655	.203733	.191814	.245491	.254437	.350361	.466857	.457911	.240975
20	.170187	.136497	.123958	.171412	.178070	.227794	.260960	.238955	.170383
40	.092349	.061896	.049681	.090470	.094573	.114972	.121867	.110522	.092433
60	.007357	-.020352	-.030512	.003180	.004857	.009695	.010486	.006615	.007348
80	-.083985	-.110282	-.115653	-.089462	-.089665	-.090705	-.090657	-.091610	-.084062
100	-.180159	-.207560	-.204158	-.185956	-.187119	-.188545	-.188480	-.188630	-.180274
120	-.278911	-.311451	-.293925	-.284463	-.285291	-.285176	-.285136	-.285152	-.279032
140	-.377409	-.420867	-.382346	-.383120	-.381765	-.380749	-.380645	-.380664	-.377514
160	-.472582	-.534620	-.466613	-.480541	-.474132	-.474459	-.473820	-.473981	-.472668
180	-.561567	-.651808	-.543985	-.576333	-.560095	-.565121	-.562374	-.563319	-.561717

TABLE 4-77

LAMBDA 3 ROTS (5,3) STD. AA1A

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.16226	.14442	.13700	.17195	.17670	.20085	.24603	.25657	.16659
20	.17216	.15358	.14709	.17786	.18144	.19445	.20974	.20243	.17420
40	.18206	.16316	.15750	.18254	.18525	.19080	.19482	.18985	.18174
60	.19168	.17312	.16796	.18602	.18825	.19033	.19085	.18889	.18924
80	.20065	.18342	.17808	.18865	.19067	.19296	.19252	.19202	.19669
100	.20853	.19400	.18734	.19016	.19285	.19806	.19738	.19735	.20399
120	.21490	.20488	.19513	.19412	.19528	.20463	.20414	.20416	.21082
140	.21941	.21606	.20071	.19868	.19857	.20132	.21143	.21143	.21659
160	.22181	.22715	.20278	.20485	.20296	.21260	.21676	.21679	.22044
180	.22208	.23288	.19473	.20724	.20365	.21199	.21197	.21205	.22106

TABLE 4-78

LAMBDA 4 ROTS (5,3) STD. AA1A

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.39489	.34170	.32163	.42672	.44164	.49280	.61453	.66086	.41029
20	.41153	.36015	.34394	.42931	.43959	.46555	.50573	.49002	.41821
40	.42786	.37917	.36594	.42962	.46394	.44739	.45772	.44632	.42684
60	.44322	.39852	.38688	.42832	.43415	.43862	.43987	.43545	.43621
80	.45680	.41792	.40582	.42657	.43175	.43820	.43704	.43596	.44621
100	.46770	.43706	.42170	.42590	.43035	.44387	.44218	.44214	.45632
120	.47504	.45566	.43335	.42788	.43064	.45266	.45151	.45156	.46545
140	.47814	.47342	.43956	.43335	.43328	.46124	.46143	.46143	.47193
160	.47664	.48921	.42823	.44203	.43803	.46537	.46649	.46657	.47380
180	.47059	.49257	.41779	.44190	.43480	.45115	.45109	.45125	.46863

TABLE 4-79

LAMBDA 1 ROTS (5,3) STD. AA24A

TIME	TRUE	SECOND	THIRD	SUCCESSIVE SUBSTITUTIONS FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.90226	.92003	.91747	.88357	.89664	******	******	.90143	64.45	64.35
20	.93402	.94844	.94817	.92599	.93134	.61286	******	.93353	69.07	68.99
40	.96347	.97416	.97524	.96251	.96387	.87089	-.38346	.96325	74.46	74.42
60	.98702	.99320	.99420	.98881	.98891	.97211	.94039	.98697	80.76	80.74
80	.99940	.99992	.99977	.99991	.99991	.99965	.99952	.99941	88.02	88.03
100	.99407	.98746	.98725	.99092	.99077	.98951	.98941	.99402	96.24	96.27
120	.96484	.94920	.95472	.95805	.95769	.95574	.95540	.96468	105.23	105.27
140	.90875	.88107	.90506	.89975	.90093	.90082	.90034	.90851	114.66	114.70
160	.82842	.78341	.84600	.81690	.82629	.82406	.82530	.82813	124.06	124.09
180	.73169	.66092	.78767	.71171	.74509	.72611	.73530	.73145	132.97	132.99
200	.62838	.50963	.73549	.57523	.66700	.59836	.64111	.62039	141.06	141.66

TABLE 4-80

LAMBDA 2 ROTS (5,3) STD. AA24A

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.431192	.399903	.406267	.469761	.443861	1.150056	*******	*******	.432922
20	.357209	.321106	.321945	.378023	.364538	.799595	*******	*******	.358492
40	.267784	.227292	.222522	.271317	.266426	.491753	1.267043	2.584764	.268579
60	.160551	.116538	.107690	.149176	.148475	.234493	.340102	.348581	.160855
80	.034446	-.012227	-.021071	.012763	.012701	.026371	.030953	.019880	.034316
100	-.108731	-.158359	-.159634	-.134436	-.135487	-.144453	-.145130	-.145869	-.109171
120	-.262823	-.318738	-.300913	-.286708	-.287922	-.294212	-.295310	-.294779	-.263400
140	-.417325	-.487699	-.435739	-.437418	-.434952	-.434309	-.435303	-.435023	-.417858
160	-.560093	-.658153	-.554961	-.581103	-.567059	-.567279	-.565466	-.566056	-.560473
180	-.681635	-.823496	-.651657	-.715008	-.676375	-.690824	-.680655	-.685336	-.681888
200	-.777901	-.990321	-.727335	-.848494	-.762538	-.811758	-.774942	-.797873	-.784292

TABLE 4-81

LAMBDA 3 ROTS (5,3) STD. AA24A

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.21347	.20390	.19230	.25493	.25099	.45772	4.00438	*******	.23761
20	.23313	.21982	.20995	.26519	.26089	.40252	1.82541	*******	.24965
40	.25440	.23626	.22836	.27116	.26840	.34884	.71215	1.13128	.26127
60	.27660	.25264	.24658	.27216	.27281	.30520	.36211	.37685	.27263
80	.29837	.26811	.26307	.26891	.27400	.27936	.28228	.27576	.28408
100	.31756	.28170	.27577	.26410	.27236	.27381	.27205	.27182	.29614
120	.33148	.29255	.28232	.26173	.26922	.28332	.28030	.28093	.30906
140	.33777	.30027	.28070	.26479	.26744	.29781	.29710	.29724	.32131
160	.33544	.30504	.26958	.27263	.27091	.30817	.31307	.31343	.32781
180	.32543	.30656	.24760	.28019	.28113	.30753	.31355	.31331	.32305
200	.31003	.00998	-.08279	-.03791	-.06063	-.04849	-.05907	-.05305	.01092

TABLE 4-82

LAMBDA 4 ROTS (5,3) STD. AA24A

TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
		SUCCESSIVE SUBSTITUTIONS							
0	.51448	.48201	.44610	.64723	.64617	1.12346	9.92221	*******	.60048
20	.55489	.51667	.48884	.65256	.64571	.96309	4.48829	*******	.60929
40	.59855	.55221	.53206	.64783	.64222	.81621	1.69137	2.67878	.61948
60	.64379	.58724	.57327	.63382	.63644	.70289	.83539	.87834	.63230
80	.68742	.61979	.60887	.61465	.62832	.63871	.64533	.63098	.64910
100	.72456	.64755	.63430	.59798	.61836	.62480	.62070	.62034	.67082
120	.74930	.66840	.64495	.59198	.60963	.64492	.63800	.63937	.69651
140	.75655	.68107	.63744	.59990	.60575	.67292	.67162	.67190	.72015
160	.74453	.68538	.61048	.61618	.61245	.68942	.69950	.70023	.72868
180	.71569	.68036	.56324	.62764	.62932	.68094	.69287	.69220	.71108
200	.67542	COMPUTER TRUNCATION ERROR							

TABLE 4-83

LAMBDA 1 ROTS (5,3) STD. AA1B

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.96285	.97232	.97521	.96006	.95558	.90340	.74006	.96275	74.33	74.31
20	.97732	.98376	.98606	.97588	.97319	.94983	.91329	.97727	77.77	77.76
40	.98903	.99288	.99440	.98851	.98719	.97881	.97188	.98900	81.50	81.50
60	.99697	.99863	.99923	.99692	.99650	.99457	.99373	.99696	85.53	85.53
80	.99999	.99976	.99947	.99999	.99999	.99995	.99994	.99999	89.88	89.88
100	.99688	.99481	.99414	.99661	.99663	.99662	.99663	.99688	94.52	94.53
120	.98643	.98216	.98245	.98571	.98555	.98529	.98530	.98641	99.44	99.45
140	.96763	.96019	.96405	.96644	.96618	.96600	.96601	.96760	104.61	104.62
160	.93987	.92734	.93922	.93813	.93833	.93840	.93841	.93984	109.96	109.98
180	.90308	.88231	.90902	.90028	.90241	.90206	.90223	.90304	115.43	115.44
200	.85784	.82405	.87533	.85237	.85965	.85670	.85772	.85779	120.92	120.93
220	.80533	COMPUTER TRUNCATION ERROR								

TABLE 4-84

LAMBDA 2 ROTS (5,3) STD. AA1B

TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
				SUCCESSIVE SUBSTITUTIONS					
0	.270017	.235282	.222644	.279884	.294842	.428898	.675293	.612180	.270373
20	.211729	.180191	.166919	.218332	.230041	.312750	.407367	.383600	.211992
40	.147697	.119290	.105758	.151131	.159495	.204742	.235474	.211022	.147870
60	.077774	.052158	.039229	.078347	.083510	.103991	.111739	.098614	.077863
80	.002083	-.021510	-.032320	.000336	.002713	.008992	.010246	.004997	.002098
100	-.078883	-.101869	-.108201	-.082236	-.081970	-.082057	-.081955	-.083607	-.078925
120	-.164177	-.188870	-.187324	-.168403	-.169344	-.170844	-.170776	-.171165	-.164259
140	-.252353	-.282166	-.268122	-.256934	-.257923	-.258521	-.258471	-.258531	-.252453
160	-.341515	-.381205	-.348649	-.346597	-.346054	-.345571	-.345530	-.345543	-.341613
180	-.429457	-.485159	-.426557	-.436305	-.431826	-.431706	-.431354	-.431441	-.429540
200	-.513909	-.593200	-.499337	-.525534	-.513185	-.516214	-.514506	-.515059	-.513984
220	.592814	COMPUTER TRUNCATION ERROR							

TABLE 4-85

LAMBDA 3 ROTS (5,3) STD. AA1B

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.15020	.13254	.12453	.16101	.16754	.20038	.28252	.32371	.15579
20	.15898	.14074	.13355	.16769	.17280	.19453	.23064	.23225	.16299
40	.16796	.14941	.14307	.17355	.17741	.19013	.20392	.19621	.17016
60	.17698	.15856	.15295	.17849	.18140	.18773	.19200	.18666	.17729
80	.18586	.16818	.16300	.18251	.18482	.18764	.18845	.18596	.18439
100	.19434	.17829	.17292	.18582	.18783	.18989	.18961	.18874	.19146
120	.20212	.18886	.18233	.18882	.19060	.19418	.19359	.19340	.19844
140	.20888	.19992	.19073	.19211	.19342	.19996	.19937	.19937	.20517
160	.21433	.21148	.19751	.19635	.19661	.20635	.20606	.20607	.21131
180	.21822	.22354	.20190	.20204	.20053	.21231	.21244	.21245	.21631
200	.22040	.23549	.20238	.20877	.20495	.21597	.21608	.21616	.21951
220	.22085	COMPUTER TRUNCATION ERROR							

TABLE 4-86

LAMBDA 4 ROTS (5,3) STD. AA1B

					Successive Substitutions				
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.34150	.28741	.26616	.37539	.39474	.46309	.67119	.85867	.36144
20	.35437	.30253	.28494	.38006	.39399	.43759	.52686	.54419	.36760
40	.36741	.31840	.30400	.38297	.39275	.41731	.45062	.43556	.37413
60	.38029	.33494	.33295	.38421	.39121	.40315	.41323	.40200	.38114
80	.39262	.35202	.34127	.38420	.38965	.39539	.39719	.39204	.38867
100	.40388	.36949	.35830	.38367	.38838	.39359	.39286	.39113	.39665
120	.41349	.38714	.37324	.38359	.38770	.39658	.39519	.39485	.40483
140	.42088	.40481	.38525	.38510	.38800	.40287	.40141	.40144	.41265
160	.42549	.42222	.39331	.38906	.38960	.40990	.40927	.40929	.41917
180	.42694	.43911	.39633	.39560	.39279	.41583	.41604	.41607	.42317
200	.42504	.45409	.39206	.40379	.39660	.41704	.41720	.41737	.42340
220	.41985	COMPUTER TRUNCATION ERROR							

TABLE 4-87

LAMBDA 1 ROTS (5,3) STD. AA24B

		SUCCESSIVE SUBSTITUTIONS								
TIME	TRUE	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.90245	.91892	.91730	.87733	.87986	-.40013	******	.90177	64.48	64.39
20	.92840	.94163	.94153	.91283	.91322	.39955	******	.92793	68.18	68.12
40	.95324	.96344	.96448	.94551	.94493	.75205	******	.95297	72.41	72.36
60	.97525	.98242	.98387	.97294	.97233	.91190	.49083	.97513	77.22	77.20
80	.99191	.99580	.99670	.99218	.99194	.97995	.95990	.99188	82.70	82.70
100	.99981	.99983	.99961	.99996	.99996	.99972	.99956	.99981	88.89	88.90
120	.99490	.98997	.98956	.99308	.99292	.99214	.99207	.99487	95.78	95.80
140	.97319	.96151	.96494	.96884	.96837	.96636	.96627	.97309	103.29	103.32
160	.93203	.91057	.92674	.92552	.92574	.92464	.92434	.93186	111.24	111.27
180	.87143	.83527	.87908	.86251	.86732	.86632	.86657	.87122	119.37	119.40
200	.79458	.73641	.82857	.77982	.79918	.79056	.79455	.79439	127.38	127.40
220	.70710	.61664	.78252	.67662	.73116	.69607	.71533	.70690	135.00	135.02

TABLE 4-88

LAMBDA 2 ROTS (5,3) STD. AA24B

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.430788	.402688	.406693	.481554	.476807	1.271769	*******	*******	.432214
20	.371568	.341650	.341960	.409062	.408170	.950624	7.021923	*******	.372733
40	.302192	.270402	.266500	.325813	.327489	.661484	2.632462	*******	.303063
60	.221068	.187473	.179591	.231085	.233647	.410477	.892489	.338654	.221624
80	.126904	.091572	.081186	.124812	.126699	.199205	.280336	.265769	.127145
100	.019236	-.018129	-.027683	.007919	.008193	.023599	.029389	.016069	.019190
120	-.100846	-.141577	-.14448	-.117391	-.118732	-.125071	-.125635	-.127550	-.101118
140	-.229989	-.277444	-.264777	-.247745	-.249552	-.257182	-.257521	-.257329	-.230393
160	-.362368	-.422905	-.382754	-.379173	-.378638	-.380875	-.381608	-.381295	-.262795
180	-.490519	-.573978	-.491769	-.508216	-.499764	-.499798	-.499368	-.499515	-.490875
200	-.607154	-.726071	-.585538	-.632688	-.606476	-.613770	-.608495	-.610670	-.607402
220	-.707108	-.875618	-.659502	-.752675	-.692859	-.722470	-.702454	-.713367	-.707311

TABLE 4-89

LAMBDA 3 ROTS (5,3) STD. AA24B

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.18868	.17734	.16553	.22567	.22965	.44288	4.04137	*******	.21049
20	.20371	.19035	.17979	.23693	.23882	.39919	2.45374	*******	.22143
40	.21996	.20412	.19511	.24602	.24648	.35519	1.13187	*******	.23212
60	.23722	.21850	.21116	.25213	.25238	.31522	.53225	.62392	.24257
80	.25504	.23319	.22734	.25486	.25635	.28397	.32593	.32593	.25285
100	.27261	.24779	.24271	.25455	.25839	.26506	.26856	.26141	.26315
120	.28872	.26181	.25596	.25264	.25872	.25951	.25811	.25713	.27373
140	.30187	.27472	.26554	.25151	.25793	.26498	.26265	.26306	.28472
160	.31054	.28621	.26971	.25356	.25730	.27632	.27420	.27447	.29558
180	.31365	.29612	.26704	.25970	.25886	.28763	.28842	.28841	.30414
200	.31099	.30464	.25646	.26865	.26496	.29421	.29802	.29802	.30695
220	.30331	.30879	.23387	.27419	.27306	.28892	.29169	.29069	.30150

TABLE 4-90

LAMBDA 4 ROTS (5,3) STD. AA24B

TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
		SUCCESSIVE SUBSTITUTIONS							
0	.42576	.38579	.35007	.53800	.55806	1.02670	9.02424	*******	.50348
20	.45294	.41177	.38254	.54879	.55824	.90183	5.66051	*******	.51136
40	.48239	.43919	.41642	.55385	.55667	.78290	2.55380	*******	.51962
60	.51362	.46761	.45075	.55250	.55354	.67987	1.16163	1.63816	.52882
80	.54559	.49632	.48405	.54523	.54915	.60234	.69275	.69906	.53964
100	.57659	.53436	.51419	.53424	.54380	.55635	.56362	.54929	.55278
120	.60410	.55052	.53850	.52360	.53797	.54152	.53842	.53662	.56869
140	.62507	.57353	.55400	.51818	.53252	.55017	.54529	.54616	.58698
160	.63652	.59239	.55796	.52141	.52926	.56974	.56559	.56610	.60521
180	.63650	.60637	.54837	.53267	.53093	.58746	.58905	.58904	.61774
200	.62484	.61543	.52450	.54713	.54005	.59430	.60126	.60128	.61734
220	.60327	.61405	.48066	.55232	.55017	.57815	.58297	.58124	.60013

TABLE 4-91

LAMBDA 1 ROTS (5,3) STD. AA1C

				SUCCESSIVE SUBSTITUTIONS						
TIME	TRUE	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.90859	.92465	.92765	.88443	.84590	-.37846	******	.90826	65.31	65.27
40	.93652	.94620	.94881	.91876	.89409	.37456	******	.93628	69.47	69.44
80	.96270	.96771	.97020	.95158	.93819	.74342	******	.96256	74.30	74.27
120	.98468	.98680	.98877	.97946	.97411	.91788	.46971	.98461	79.95	79.94
160	.99826	.99882	.99938	.99713	.99616	.98815	.97295	.99824	86.62	86.61
200	.99697	.99572	.00501	.99752	.99771	.99842	.99860	.99698	94.45	94.45
240	.97224	.96508	.96898	.97235	.97211	.97099	.97111	.97222	103.53	103.53
280	.91549	.89056	.92056	.91245	.91498	.91269	.91331	.91543	113.72	113.73
320	.82280	.75499	.86247	.80405	.83269	.81688	.82487	.82273	124.63	124.64
360	.69937	.50031	.81707	.56719	.76027	.60020	.73272	.67292	135.62	137.71

TABLE 4-92

LAMBDA 2 ROTS (5,3) STD. AA1C

				SUCCESSIVE SUBSTITUTIONS					
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.417679	.388197	.380383	.468102	.536210	1.265617	*******	*******	.418393
40	.350603	.328020	.319940	.395414	.449058	.965550	6.336871	*******	.351231
80	.270545	.254098	.244113	.307564	.346430	.671473	2.672134	*******	.271067
120	.174357	.162469	.149865	.201655	.226110	.396910	.906718	.401033	.174744
160	.058916	.048545	.035075	.075691	.087507	.153431	.230982	.186687	.059144
200	-.077688	-.092434	-.099806	-.070304	-.067583	-.056056	-.052733	-.066642	-.077626
240	-.233963	-.264257	-.249066	-.233546	-.234536	-.239092	-.238615	-.240112	-.234039
280	-.402325	-.467841	-.398572	-.409917	-.404165	-.408711	-.407319	-.407753	-.402460
320	-.568322	-.700008	-.524460	-.599641	-.557226	-.577701	-.566087	-.571957	-.568431
360	-.714753	-.999687	-.604855	-.855424	-.657760	-.810235	-.683543	-.788427	-.739712

TABLE 4-93

LAMBDA 3 ROTS (5,3) STD. AA1C

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.13846	.11892	.10711	.15414	.17302	.32580	1.20278	*******	.15217
40	.15426	.13294	.12198	.17215	.18783	.31421	1.59576	*******	.16800
80	.17193	.14913	13971	.19086	.20239	.29668	.89236	*******	.18425
120	.19152	.16796	.16056	.20903	.21622	.27577	.46497	.52287	.20076
160	.21280	.18996	.18434	.22505	.22897	.25672	.29465	.28901	.21742
200	.23485	.21554	.21001	.23781	.24030	,24597	.24895	.24212	.23428
240	.25582	.24502	.23456	.24834	.24990	.24804	.24741	.24659	.25141
280	.27282	.27856	.25258	.26083	.25770	.26175	.26063	.26087	.26806
320	.28275	.31587	.25495	.27963	.26255	.27844	.27544	.27684	.28071
360	.28383	COMPUTER TRUNCATION ERROR							

TABLE 4-94

LAMBDA 4 ROTS (5,3) STD. AA1C

TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
		SUCCESSIVE SUBSTITUTIONS							
0	.25000	.18176	.14987	.28556	.33225	.61274	3.23279	*******	.29890
40	.26603	.20106	.17541	.30541	.33951	.56466	2.83229	*******	.30820
80	.28454	.22398	.20487	.32413	.34600	.50827	1.56002	*******	.31758
120	.30552	.25096	.23801	.33970	.35152	.44988	.77497	1.08947	.32736
160	.32839	.28230	.27373	.35032	.35611	.39968	146286	.46278	.33811
200	.35171	.31789	.30941	.35598	.35985	.36815	.37303	.36298	.35061
240	.37269	.35698	.34012	.36022	.36269	.36042	.35938	.35825	.36517
280	.38732	.39812	.35830	.36960	.36469	.37145	.36973	.37011	.38001
320	.39157	.43866	.35351	.38771	.36515	.38594	.38170	.38368	.38876
360	.38354	COMPUTER TRUNCATION ERROR							

TABLE 4-95

LAMBDA 1 ROTS (5,3) STD. AA24C

				SUCCESSIVE SUBSTITUTIONS						
TIME	TRUE	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.86585	.88347	.88112	.80190	.72446	******	******	.86508	59.98	59.89
40	.89828	.90871	.90667	.84791	.79316	******	******	.89767	63.93	63.85
80	.93149	.93679	.93622	.89674	.86278	******	******	.93106	68.66	68.60
120	.96340	.96582	.96695	.94436	.923784	.35334	******	.96316	74.45	74.40
160	.98931	.99059	.99201	.98289	.97814	.88311	******	.98923	81.61	81.59
200	.99995	.99965	.99931	.99997	.99991	.99734	.99006	.99995	90.53	90.53
240	.98002	.97225	.97451	.97994	.97946	.97724	.97727	.97998	101.47	101.48
280	.91153	.87925	.91373	.90588	.90838	.90245	.90410	.91136	114.28	114.31
320	.78663	.69447	.84141	.75452	.80277	.77272	.79024	.78639	128.12	128.15
360	.62061	.40855	.80664	.49701	.75238	.49127	.73039	.62005	141.63	141.68

TABLE 4-96

LAMBDA 2 ROTS (5,3) STD. AA24C

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.500288	.482753	.487590	.602662	.700602	2.643503	*******	*******	.501623
40	.439417	.427273	.432039	.532913	.614786	2.118592	*******	*******	.440658
80	.363760	.355538	.357135	.443652	.507748	1.548701	6.562116	*******	.364857
120	.268046	.261448	.257094	.329147	.373418	.977640	8.950356	*******	.268927
160	.145765	.137176	.126407	.184159	.207960	.469358	1.780303	*******	.146339
200	-.009380	-.026094	-.037018	.006627	.013278	.072889	.140578	.100468	-.009186
240	-.198888	-.235553	-.225774	-.199281	-.201624	-.212110	-.211954	-.216018	-.199062
280	-.412226	-.491414	-.415374	-.424395	-.418965	-.430891	-.427408	-.428386	-.411600
320	-.617421	-.781700	-.563178	-.664896	-.601445	-.636540	-.614183	-.627341	-.617725
360	-.784113	-1.087610	-.621860	-.935251	-.667527	-.892187	-.686123	-.874836	-.784557

TABLE 4-97

LAMBDA 3 ROTS (5,3) STD. AA24C

					SUCCESSIVE SUBSTITUTIONS				
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.15157	.13640	.12149	.18018	.20615	.60397	-3.64909	*******	.17715
40	.17077	.15404	.13969	.20408	.22638	.58513	-.71316	*******	.19762
80	.19319	.17478	.16205	.22975	.24642	.54034	-1.00054	*******	.21861
120	.21936	.19925	.18917	.25498	.26501	.47006	3.03356	*******	.23967
160	.24945	.22791	.22096	.27602	.28091	.28824	.94642	-.35483	.26051
200	.28239	.26072	.25551	.28926	.29309	.32144	.36076	.35171	.28120
240	.31464	.29673	.28734	.29577	.30050	.29401	.29290	.29062	.30427
280	.33933	.33401	.30580	.30467	.30319	.30656	.30455	.30517	.32470
320	.34873	.37058	.29612	.32362	.30636	.33047	.32904	.32943	.34176
360	.34024	.38329	.22045	.32830	.29599	.32061	.29771	.31634	.33331

TABLE 4-98

LAMBDA 4 ROTS (5,3) STD. AA24C

TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
				SUCCESSIVE SUBSTITUTIONS					
0	.27059	.20928	.16570	.33357	.39822	1.10999	*******	*******	.36186
40	.29268	.23461	.19816	.36430	.41262	1.03839	-1.56880	*******	.37513
80	.31945	.26513	.23698	.39458	.42525	.92287	-4.38084	*******	.38760
120	.35151	.30154	.28230	.42019	.43511	.77001	4.88863	*******	.39958
160	.38883	.34407	.33285	.43594	.44078	.60924	1.49875	*******	.41211
200	.42952	.39189	.38434	.43926	.44507	.48636	.54705	.53907	.42721
240	.46809	.44233	.42745	.43653	.44447	.43544	.43376	.43064	.44733
280	.49457	.49059	.44712	.44311	.44074	.44776	.44471	.44566	.47205
320	.49834	.53117	.42645	.46463	.44011	.47404	.47185	.47253	.48867
360	.47669	.53226	.32514	.46230	.42106	.45246	.42318	.44702	.46802

TABLE 4-99

LAMBDA 1 ROTS (4,4) STD. AA1

TIME	TRUE	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{ndm}	CHI_{nr}
		SUCCESSIVE SUBSTITUTIONS								
0	.97371	.98143	.98242	.97452	.97186	.99470	.97894	.97379	76.84	76.86
10	.98231	.98822	.98918	.98333	.98186	.99500	.98685	.98336	79.21	79.22
20	.98952	.99373	.99452	.99054	.98984	.99636	.99302	.98956	81.69	81.71
30	.99507	.99768	.99818	.99590	.99564	.99816	.99720	.99509	84.31	84.33
40	.99867	.99976	.99990	.99912	.99907	.99961	.99948	.99869	87.04	87.06
50	.99999	.99963	.99943	.99996	.99996	.99993	.99993	.99999	89.90	89.91
60	.99874	.99696	.99657	.99815	.99811	.99838	.99840	.99874	92.86	92.87
70	.99463	.99141	.99115	.99348	.99334	.99436	.99451	.99463	95.93	95.94
80	.98740	.98264	.98307	.98577	.98552	.98736	.98776	.98741	99.10	99.10
90	.97686	.97036	.97232	.97484	.97453	.97704	.97770	.97687	102.34	102.35
100	.96288	.95431	.95896	.96060	.96033	.96321	.96397	.96289	105.65	105.66
110	.94542	.93427	.94314	.94298	.94295	.94577	.94549	.94544	109.01	109.01
120	.92456	.91010	.92515	.92196	.92250	.92484	.92538	.92456	112.39	112.40
130	.90043	.88171	.90533	.89756	.89922	.90055	.90093	.90043	115.78	115.78
140	.87331	.84910	.88414	.86984	.87344	.87316	.87359	.87329	119.15	119.16
150	.84352	.81223	.86202	.83878	.84555	.84286	.84379	.84349	122.48	122.49
160	.81147	.77086	.83936	.80413	.81590	.80967	.81183	.81135	125.76	125.77
170	.77758	-.12575	.55548	-.27812	.66778	-.17637	.99126	.29104	128.95	163.08

TABLE 4-100

LAMBDA 2 ROTS (4,4) STD. AA1

		Successive Substitutions							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.227749	.192715	.187499	.224303	.235588	.102816	.204102	.183614	.227410
20	.144370	.111929	.104607	.137180	.142155	.851660	.117914	.117437	.144103
40	.051507	.021714	.014085	.041775	.042955	.027842	.032012	.036715	.051343
60	-.050004	-.077901	-.082808	-.060645	-.061423	-.056726	-.056512	-.054719	-.050057
80	-.158201	-.186307	-.183965	-.168078	-.169527	-.158466	-.155924	-.155594	-.158171
100	-.269914	-.302275	-.286493	-.278019	-.278950	-.268786	-.265979	-.265617	-.269861
120	-.381036	-.424010	-.386889	-.387832	-.386529	-.380393	-.379073	-.378887	-.381016
140	-.487157	-.549346	-.481366	-.495243	-.488724	-.487700	-.486913	-.487030	-.487188
160	-.584391	-.676951	-.566811	-.599526	-.582569	-.587898	-.584869	-.585909	-.584550

TABLE 4-101

LAMBDA 3 ROTS (4,4) STD. AA1

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.17809	.16167	.15698	.18520	.19101	.10960	.17215	.15655	.17363
20	.18963	.17205	.16815	.19004	.19306	.15090	.17648	.17371	.18432
40	.20091	.18262	.17922	.19327	.19503	.18149	.18617	.18926	.19564
60	.21139	.19325	.18969	.19539	.19705	.20324	.20278	.20460	.20721
80	.22042	.20384	.19890	.19738	.19924	.21770	.21960	.22008	.18041
100	.22736	.21433	.20612	.20047	.20195	.22628	.23065	.23141	.22660
120	.23167	.22479	.21054	.20574	.20588	.23036	.23465	.23542	.23174
140	.23309	.23536	.21134	.21363	.21201	.23118	.23358	.23390	.23335
160	.23168	.23999	.20126	.21734	.21482	.22252	.22267	.22269	.23231

TABLE 4-102

LAMBDA 4 ROTS (4,4) STD. AA1

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS							
		FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.46610	.40762	.39463	.48201	.49942	.25720	.44152	.39477	.44023
20	.47794	.42987	.42014	.48129	.48997	.37012	.44284	.43417	.46040
40	.49872	.45204	.44413	.47824	.48338	.44665	.45964	.46762	.48254
60	.51715	.47367	.46530	.47479	.47963	.49593	.49452	.49919	.50520
80	.53176	.49428	.48222	.47341	.47854	.52475	.52889	.53005	.52536
100	.54110	.51349	.49343	.47637	.48018	.53835	.54821	.54993	.53919
120	.54408	.53100	.49755	.48482	.48517	.54105	.55053	.55225	.54423
140	.54026	.54672	.49345	.49807	.49445	.53616	.54130	.54199	.54085
160	.52998	.54785	.46715	.50057	.49531	.51126	.51156	.51160	.53134

TABLE 4-103

LAMBDA 1 ROTS (4,4) STD. AA24

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.89650	.91437	.90773	.88437	.84335	-.30098	******	.89705	63.70	63.77
10	.91598	.93250	.92829	.90910	.88392	.27482	******	.91649	66.34	66.42
20	.93512	.95007	.94791	.93765	.91876	.83287	.99996	.93557	69.24	69.32
30	.95337	.96641	.96579	.95418	.94761	.96600	.98180	.95374	72.43	72.50
40	.96999	.98066	.98097	.97274	.97033	.99605	.98003	.97027	75.92	75.99
50	.98404	.99179	.99237	.98729	.98683	.99996	.98991	.98422	79.75	79.81
60	.99436	.99856	.99888	.99873	.99685	.99995	.99803	.99444	83.91	83.96
70	.99961	.99963	.99946	.99999	.99998	.99988	.99993	.99963	88.41	88.45
80	.99839	.99361	.99331	.99609	.99564	.99749	.99739	.99837	93.25	93.26
90	.98933	.97931	.98002	.98419	.98334	.98937	.99087	.98935	98.34	98.37
100	.97142	.95544	.95974	.96378	.96274	.97339	.97731	.97152	103.72	103.71
110	.94414	.92175	.93324	.93469	.93400	.94551	.95278	.94430	109.24	109.21
120	.90767	.87817	.90184	.89710	.89785	.90878	.91582	.90785	114.81	114.79
130	.86298	.82529	.86732	.85157	.85569	.86337	.86845	.86307	120.34	120.34
140	.81163	.76429	.83163	.79892	.80956	.81100	.81465	.81159	125.74	125.75
150	.75559	.69640	.79658	.73989	.76175	.75306	.75824	.75542	130.92	130.94
160	.69691	.62294	.76370	.67504	.71477	.69045	.70238	.69660	135.82	135.84
170	.63745	.54460	.73403	.60417	.67085	.62285	.64952	.63668	140.39	140.46

TABLE 4-104

LAMBDA 2 ROTS (4,4) STD. AA24

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.443039	.413827	.429563	.468230	.540349	*******	*******	*******	.441927
20	.354322	.315991	.322747	.361166	.395411	-.554128	-.008230	.330027	.353131
40	.243120	.196649	.195055	.231928	.241806	-.088734	.198823	.121208	.242023
60	.106005	.053536	.047283	.080720	.079240	.009780	.062645	.049798	.105237
80	-.056708	-.113041	-.115671	-.088395	-.093072	-.070715	-.072193	-.068666	-.056898
100	-.237340	-.298518	-.283736	-.262752	-.270493	-.233346	-.211772	-.206955	-.236951
120	-.419666	-.493611	-.443061	-.442907	-.441374	-.417363	-.401635	-.393036	-.401929
140	-.584159	-.686599	-.580278	-.606828	-.591784	-.586032	-.580876	-.579941	-.584217
160	-.717156	-.868391	-.687455	-.754304	-.711585	-.728140	-.716013	-.721455	-.717453

TABLE 4-105

LAMBDA 3 ROTS (4,4) STD. AA24

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.24128	.23590	.28812	.32741	.87420	*******	*******	*******	.23641
20	.26716	.25544	.24818	.29851	.31791	-.28759	.09467	.26544	.25396
40	.29526	.27474	.26964	.30133	.30799	.05064	.27888	.20784	.27398
60	.32388	.29234	.28881	.29668	.30029	.23159	.28889	.26834	.29790
80	.34975	.30646	.30255	.28816	.29430	.32303	.31664	.32219	.32795
100	.36818	.31546	.30733	.28220	.28873	.35889	.38390	.38874	.36168
120	.37482	.31876	.30049	.28360	.28563	.36120	.39744	.41702	.37944
140	.36822	.31733	.28138	.29092	.29023	.34910	.37077	.37976	.37092
160	.35087	.30897	.24694	.29333	.30075	.32771	.33611	.33706	.35007

TABLE 4-106

LAMBDA 4 ROTS (4,4) STD. AA24

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.61036	.59155	.56096	.76777	.88417	*******	*******	*******	.59325
20	.66867	.63667	.61523	.76900	.82291	.85292	.23322	.66494	.62529
40	.73178	.68081	.66758	.75306	.77115	.08725	.69328	.50153	.66659
60	.79540	.72061	.71255	.72472	.73577	.56016	.70788	.65308	.72106
80	.85151	.75187	.74281	.69622	.71379	.78426	.76737	.78122	.79311
100	.88895	.77070	.75083	.68250	.69953	.86540	.92115	.93144	.87249
120	.89763	.77532	.73190	.68951	.69439	.86599	.94601	.98907	.90841
140	.87469	.76716	.68658	.70702	.70528	.83303	.87956	.89884	.88068
160	.82660	.74133	.61112	.70799	.72323	.77894	.79627	.79822	.82504

TABLE 4-107

LAMBDA 1 ROTS (4,4) STD. AA1A

		SUCCESSIVE SUBSTITUTIONS								
TIME	TRUE	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.97061	.97868	.97985	.97184	.96938	.99346	.97591	.97068	76.07	76.09
10	.97848	.98490	.98606	.97977	.97826	.99389	.98330	.97853	78.09	78.11
20	.98541	.99031	.99135	.98665	.98580	.99508	.98953	.98545	80.20	80.22
30	.99122	.99471	.99553	.99229	.99188	.99673	.99431	.99135	82.40	82.42
40	.99572	.99792	.99844	.99653	.99637	.99841	.99762	.99574	84.70	84.71
50	.99871	.99971	.99987	.99916	.99912	.99964	.99951	.99872	87.09	87.10
60	.99997	.99983	.99966	.99999	.99999	.99996	.99997	.99997	89.57	89.58
70	.99929	.99804	.99764	.99885	.99883	.99891	.99891	.99929	92.15	92.16
80	.99646	.99409	.99369	.99557	.99548	.99607	.99610	.99646	94.81	94.82
90	.99130	.99773	.98768	.98999	.98982	.99107	.99123	.99130	97.56	97.56
100	.98363	.97869	.97958	.98198	.98174	.98362	.98396	.98364	100.37	100.38
110	.97333	.96676	.96938	.97141	.97115	.97352	.97400	.97334	103.26	103.26
120	.96031	.95171	.95714	.95820	.95802	.96062	.96116	.96032	106.19	106.19
130	.94454	ERROR IN DATA								
140	.92604	.91157	.92710	.92359	.92423	.92632	.92670	.92695	112.17	112.17
150	.90491	.88619	.90973	.90209	.90377	.90500	.90530	.90490	115.18	115.19
160	.88128	.85714	.89120	.87775	.88121	.88105	.88144	.88126	118.20	118.20
170	.85537	.82428	.87182	.85045	.85675	.85454	.85536	.85533	121.19	121.20
180	.82743	.78757	.85204	.82011	.83083	.82567	.82744	.82733	124.16	124.17
190	.79773	.74462	.83110	.78458	.80234	.79243	.79633	.79693	127.16	127.16

TABLE 4-108

LAMBDA 2 ROTS (4,4) STD. AA1A

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.240655	.206478	.200724	.235650	.245598	.114130	.218158	.198788	.240371
20	.170187	.139211	.131517	.162857	.167893	.098974	.144316	.141249	.169943
40	.092349	.064380	.055803	.083199	.085042	.056341	.068850	.073734	.092167
60	.007357	-.018245	-.025834	-.002731	-.002756	-.007960	-.006964	-.002868	-.007251
80	-.083985	-.108640	-.112330	-.093932	-.094910	-.088539	-.088190	-.086806	-.084016
100	-.180159	-.206408	-.202040	-.188986	-.190209	-.180204	-.178365	-.178055	-.180138
120	-.278911	-.310751	-.292757	-.286207	-.286801	-.277856	-.275966	-.275743	-.278876
140	-.377409	-.420528	-.381835	-.383897	-.382333	-.376771	-.375828	-.375708	-.377397
160	-.472582	-.534513	-.466469	-.480758	-.474266	-.473222	-.472500	-.472616	-.472613
180	-.561567	-.651797	-.543973	-.576350	-.560104	-.565021	-.562294	-.563230	-.561707

TABLE 4-109

LAMBDA 3 ROTS (4,4) STD. AA1A

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.16226	.14563	.14072	.16813	.17250	.10181	.15694	.14505	.15926
20	.17216	.15495	.15074	.17393	.17620	.13270	.16159	.15801	.16830
40	.18206	.16462	.16098	.17855	.17972	.15831	.16834	.17075	.17782
60	.19168	.17458	.17116	.18211	.18313	.17891	.17996	.18308	.18769
80	.20065	.18479	.18088	.18504	.18645	.19482	.19450	.19592	.19757
100	.20853	.19520	.18963	.18802	.18973	.20636	.20793	.20840	.20672
120	.21490	.20582	.19684	.19188	.19323	.21402	.21722	.21773	.21422
140	.21941	.21670	.20180	.19733	.19739	.21843	.22163	.22213	.21932
160	.22181	.22747	.20330	.20428	.20244	.21975	.22165	.22189	.22182
180	.22208	.23295	.19508	.20714	.20355	.21267	.21279	.21288	.22135

TABLE 4-110

LAMBDA 4 ROTS (4,4) STD. AA1A

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.39489	.34582	.33168	.41684	.42962	.22534	.38189	.34823	.38422
20	.41153	.36376	.35345	.41936	.42559	.30540	.38449	.37427	.39881
40	.42786	.38285	.37467	.41974	.42295	.36641	.39300	.39891	.41484
60	.44332	.40206	.39461	.41891	.42181	.41130	.41402	.42169	.43181
80	.45680	.42112	.41235	.41816	.42203	.44260	.44160	.44500	.44855
100	.46770	.43974	.42685	.41906	.42345	.46246	.46570	.46674	.46314
120	.47504	.45769	.43705	.42302	.42624	.47294	.47968	.48077	.47341
140	.47814	.47474	.44183	.43071	.43084	.47603	.48265	.48369	.47795
160	.47664	.48986	.43927	.44088	.43699	.47249	.47632	.47679	.47668
180	.47059	.49272	.41801	.44172	.43461	.45246	.45268	.45287	.46921

TABLE 4-111

LAMBDA 1 ROTS (4,4) STD. AA24A

		SUCCESSIVE SUBSTITUTIONS								
TIME	TRUE	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.90226	.91910	.91373	.89684	.87388	.53400	******	.90265	64.45	64.51
10	.91826	.93347	.92985	.91575	.90066	.82661	.99998	.91864	66.67	66.73
20	.93402	.94760	.94553	.93400	.92488	.94123	.99068	.93436	69.07	69.13
30	.94922	.96112	.96033	.95114	.94620	.98428	.97086	.94952	71.66	71.72
40	.96347	.97357	.97369	.96665	.96438	.99753	.97315	.96372	74.46	74.52
50	.97628	.98440	.98500	.97992	.97915	.99996	.98223	.97646	77.49	77.54
60	.98702	.99293	.99359	.99034	.99026	.99985	.99195	.98714	80.75	80.80
70	.99500	.99839	.99876	.99725	.99734	.99990	.99825	.99506	84.27	84.31
80	.99940	.99994	.99985	.99998	.99999	.99992	.99999	.99942	88.02	88.05
90	.99937	.99667	.99629	.99789	.99771	.99839	.99823	.99936	92.02	92.04
100	.99407	.98769	.98768	.99041	.99001	.99336	.99352	.99406	96.24	96.25
110	.98274	.97218	.97392	.97708	.97650	.98298	.98474	.98276	100.66	100.65
120	.96484	.94950	.95519	.95758	.95700	.96594	.96765	.96491	105.23	105.22
130	.94013	.91925	.93206	.93172	.93159	.94162	.94667	.94024	109.92	109.91
140	.90875	.88132	.90539	.89953	.90070	.91011	.91497	.90886	114.66	114.65
150	.87122	.83592	.87632	.86115	.86521	.87204	.87579	.87128	119.39	119.39
160	.82842	.78355	.84614	.81655	.82635	.82826	.83125	.82841	124.06	124.06
170	.78149	.72496	.81620	.76697	.78573	.77965	.78384	.78139	128.60	128.61
180	.73169	.66095	.78769	.71170	.74514	.72674	.73582	.73151	132.97	132.99
190	.68027	.59187	.76149	.65468	.70629	.66916	.68910	.67994	137.13	137.16
200	.62838	.50963	.73549	.57523	.66700	.59836	.64111	.62039	141.06	141.66

TABLE 4-112

LAMBDA 2 ROTS (4,4) STD. AA24A

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.431192	.402227	.415366	.443441	.487914	-.861866	*******	*******	.430363
20	.357209	.323711	.330034	.357627	.380763	-.337761	.106193	.337280	.356308
40	.267784	.229887	.229381	.256169	.264590	-.070176	.230160	.152082	.266905
60	.160551	.118892	.113172	.138593	.139221	.016863	.126605	.082060	.159817
80	.034446	-.010277	-.017011	.006040	.003761	-.012336	-.003647	.003145	.034001
100	-.108731	-.156902	-.156918	-.138107	-.140967	-.115018	-.113644	-.112116	-.108780
120	-.262823	-.317781	-.299334	-.288289	-.290189	-.258756	-.244042	-.240910	-.262522
140	-.417325	-.487118	-.434994	-.437888	-.435429	-.414441	-.403573	-.398891	-.417095
160	-.560093	-.657948	-.554711	-.581178	-.566974	-.561070	-.556577	-.556000	-.560118
180	-.681635	-.823457	-.651617	-.715011	-.676316	-.690137	-.680072	-.684419	-.681823
200	-.777901	-.990321	-.727335	-.848493	-.762538	-.811758	-.774942	-.797873	-.784292

TABLE 4-113

LAMBDA 3 ROTS (4,4) STD. AA24A

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.21347	.20490	.19589	.24494	.26598	.38328	*******	*******	.21117
20	.23313	.22110	.21360	.25626	.26661	-.13490	.12348	.23567	.22535
40	.25440	.23773	.23198	.26236	.26642	.04526	.24225	.18818	.24095
60	.27660	.25419	.25004	.26559	.26666	.16897	.25960	.21782	.25820
80	.29837	.26964	.26622	.26374	.26736	.25130	.26114	.26573	.27922
100	.31756	.28310	.27846	.26052	.26748	.30155	.29892	.30152	.30327
120	.33148	.29371	.28442	.25975	.26663	.32516	.34313	.34704	.32691
140	.33777	.30112	.28213	.26404	.26665	.32941	.35443	.36539	.33971
160	.33544	.30555	.27036	.27254	.27108	.32307	.33998	.34601	.33712
180	.32543	.30674	.24787	.28023	.28140	.31078	.31809	.31915	.32494
200	.31003	.09987	.08278	.03790	.06063	.04848	.05906	.05305	.01092

TABLE 4-114

LAMBDA 4 ROTS (4,4) STD. AA24A

					SUCCESSIVE SUBSTITUTIONS				
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.51448	.48474	.45569	.62158	.68218	-.01138	*******	*******	.50652
20	.55489	.52000	.49822	.63032	.65774	-.42307	.26738	.56456	.52936
40	.59855	.55588	.54108	.62890	.63629	.06466	.56997	.43140	.55738
60	.64379	.59097	.58152	.61826	.62149	.37876	.60414	.49873	.59232
80	.68742	.62333	.61612	.60274	.61310	.57470	.59859	.60891	.63615
100	.72456	.65067	.64028	.58996	.60797	.68649	.67912	.68522	.68863
120	.74930	.67090	.64946	.58767	.60418	.73412	.77172	.77970	.73842
140	.75655	.68283	.64041	.59834	.60414	.73813	.78995	.81257	.76080
160	.74453	.68638	.61204	.61599	.61279	.71914	.75320	.76535	.74805
180	.71569	.68071	.56275	.62773	.62984	.68718	.70142	.70346	.71480
200	.67542	.11894	-.05420	.02955	-.01287	.00980	-.00994	.00128	.12003

TABLE 4-115

LAMBDA 1 ROTS (4,4) STD. AA1B

TIME	TRUE	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
			SUCCESSIVE SUBSTITUTIONS							
0	.96285	.97239	.97367	.96478	.96255	.99238	.96850	.96292	74.33	74.35
10	.97038	.97824	.97958	.97220	.97073	.99263	.97567	.97044	76.02	76.03
20	.97732	.98367	.98499	.97901	.97811	.99343	.98227	.97737	77.77	77.79
30	.98358	.98855	.98976	.98510	.98458	.99468	.98796	.98362	79.60	79.62
40	.98903	.99276	.99379	.99033	.99008	.99618	.99260	.98906	81.50	81.52
50	.99354	.99615	.99692	.99457	.99448	.99773	.99611	.99356	83.48	83.49
60	.99697	.99857	.99902	.99768	.99767	.99905	.99850	.99698	85.53	85.55
70	.99917	.99984	.99996	.99953	.99953	.99987	.99977	.99917	87.67	87.68
80	.99999	.99978	.99959	.99996	.99996	.99989	.99991	.99999	89.88	89.89
90	.99928	.99822	.99780	.99886	.99881	.99881	.99881	.99928	92.16	92.17
100	.99688	.99492	.99446	.99607	.99598	.99635	.99634	.99688	94.52	94.53
110	.99264	.98970	.98948	.99147	.99133	.99255	.99227	.99264	96.95	96.95
120	.98643	.98234	.98384	.98496	.98477	.98626	.98638	.98643	99.44	99.45
130	.97812	.97263	.97447	.97641	.97619	.97819	.97842	.97812	102.00	102.01
140	.96763	.96037	.96440	.96572	.96555	.96789	.96821	.96764	104.61	104.62
150	.95489	.94394	.95269	.95282	.95280	.95527	.95561	.95490	107.27	107.27
160	.93987	.92749	.93945	.93762	.93795	.94026	.94957	.93988	109.96	109.97
170	.92258	.90655	.92487	.92006	.92107	.92291	.92316	.92259	112.69	112.69
180	.90308	.88239	.90913	.90003	.90225	.90321	.90346	.90308	115.43	115.43
190	.88146	.85497	.89255	.89751	.88171	.88126	.88171	.88144	118.18	118.18
200	.85784	.82498	.87536	.85230	.85961	.85705	.85804	.85781	120.92	120.93
210	.83240	.78964	.85793	.82426	.83630	.83062	.83275	.83234	123.65	123.66
220	.80533	ERROR IN DATA								

TABLE 4-116

LAMBDA 2 ROTS (4,4) STD. AA1B

TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
		SUCCESSIVE SUBSTITUTIONS							
0	.270017	.234988	.229438	.263126	.271161	.123169	.248989	.228097	.269771
20	.211729	.189679	.173254	.203807	.208080	.114356	.187468	.178275	.211497
40	.147697	.120280	.111432	.138719	.140444	.087227	.121400	.123192	.147495
60	.077774	.053402	.044084	.067995	.068193	.043482	.054677	.060783	.077617
80	.002083	-.020223	-.028383	-.007974	-.008590	-.014804	-.013256	-.008289	.001982
100	-.078883	-.100701	-.105213	-.088532	-.089538	-.085275	-.085468	-.083192	-.078927
120	-.164177	-.187927	-.182425	-.172585	-.173870	-.165194	-.164481	-.163834	-.164176
140	-.252353	-.281499	-.266829	-.259631	-.260284	-.251368	-.250136	-.249926	-.252331
160	-.341515	-.380809	-.347970	-.347976	-.347081	-.340455	-.339683	-.339471	-.341498
180	-.429457	-.484980	-.426287	-.436826	-.432164	-.429307	-.428763	-.428775	-.429466
200	-.513909	-.593154	-.499276	-.525641	-.513248	-.515623	-.513965	-.514486	-.513955
220	-.592814	COMPUTER TRUNCATION ERROR							

TABLE 4-117

LAMBDA 3 ROTS (4,4) STD. AA1B

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.15020	.13251	.12713	.15512	.15814	.09037	.14577	.13648	.14862
20	.15898	.14102	.13625	.16190	.16317	.11414	.15123	.14604	.15654
40	.16796	.14996	.14580	.16789	.16807	.13605	.15633	.15666	.16483
60	.17698	.15931	.15565	.17302	.17287	.15576	.16369	.16723	.17349
80	.18586	.16907	.16558	.17737	.17756	.17301	.17427	.17791	.18245
100	.19434	.17923	.17529	.18119	.18207	.18754	.18703	.18910	.19148
120	.20212	.18977	.18438	.18491	.18637	.19917	.19964	.20040	.20016
140	.20888	.20073	.19238	.18912	.19056	.20789	.20985	.21022	.20787
160	.21433	.21210	.19869	.19437	.19484	.21384	.21642	.21679	.21401
180	.21822	.22394	.20261	.20100	.19960	.21736	.21941	.21967	.21818
200	.22040	.23567	.20268	.20843	.20460	.21811	.21886	.21900	.22032
220	.22085	COMPUTER TRUNCATION ERROR							

TABLE 4-118

LAMBDA 4 ROTS (4,4) STD. AA1B

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.34150	.28739	.27293	.36072	.36936	.18167	.33047	.30735	.33592
20	.35437	.30328	.29169	.36602	.36937	.23938	.33546	.32257	.34636
40	.36741	.31976	.31059	.36963	.36998	.28872	.33944	.34021	.35784
60	.38029	.33671	.32922	.37170	.37135	.32982	.34901	.35735	.37032
80	.39262	.35403	.34704	.37278	.37343	.36302	.36598	.37429	.38349
100	.40388	.37154	.36340	.37370	.37603	.38864	.38736	.39194	.39670
120	.41349	.38906	.37752	.37546	.37897	.40698	.40774	.40935	.40886
140	.42088	.40643	.38858	.37909	.38230	.41864	.42244	.42318	.41862
160	.42549	.42344	.39562	.38531	.38620	.42442	.42937	.43007	.42480
180	.42694	.42988	.39768	.39385	.39105	.42529	.42913	.42961	.42686
200	.42504	.45442	.39260	.40316	.39597	.42091	.42225	.42251	.42489
220	.41985	COMPUTER TRUNCATION ERROR							

TABLE 4-119

LAMBDA 1 ROTS (4,4) STD. AA24B

		SUCCESSIVE SUBSTITUTIONS								
TIME	TRUE	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.90245	.91949	.91478	.90202	.89046	.90139	.99999	.90273	64.48	64.52
10	.91538	.93059	.92713	.91642	.90853	.95219	.98489	.91576	66.27	66.31
20	.92840	.94171	.93941	.93058	.92559	.97993	.96507	.92867	68.18	68.23
30	.94105	.95268	.95144	.94426	.94142	.99337	.95950	.94130	70.22	70.27
40	.95324	.96237	.96291	.95719	.95582	.99866	.96303	.95346	72.41	72.45
50	.96474	.97323	.97353	.97908	.96859	.99996	.97073	.96492	74.74	74.78
60	.97525	.98221	.98291	.97956	.97952	.99984	.98019	.97540	77.22	77.27
70	.98444	.98984	.99065	.98829	.98841	.99962	.98921	.98455	79.88	79.92
80	.99191	.99568	.99633	.99486	.99498	.99974	.99588	.99198	82.70	82.74
90	.99720	.99920	.99948	.99889	.99894	.99998	.99937	.99723	85.71	85.74
100	.99981	.99985	.99970	.99995	.99994	.99971	.99986	.99982	88.89	88.92
110	.99922	.99704	.99651	.99766	.99759	.99801	.99788	.99921	92.25	92.27
120	.99490	.99014	.98992	.99173	.99152	.99382	.99357	.99489	95.78	95.79
130	.98636	.97856	.97952	.98153	.98139	.98606	.98644	.98636	99.47	99.47
140	.97319	.96175	.96539	.96707	.96694	.97382	.97540	.97321	103.29	103.29
150	.95511	.93927	.94777	.94801	.94807	.95645	.95921	.95517	107.23	107.22
160	.93203	.91081	.92711	.92420	.92486	.93371	.93705	.93210	111.24	111.23
170	.90403	.87619	.90401	.89549	.89764	.90561	.90879	.90410	115.30	115.30
180	.87143	.83544	.87928	.86185	.86698	.87248	.87514	.87146	119.37	119.37
190	.83472	.78876	.85385	.82324	.83376	.83476	.83738	.83471	123.41	123.41
200	.79458	.73648	.82864	.77963	.79912	.79283	.79704	.79452	127.38	127.39
210	.75177	.67889	.80455	.73077	.76434	.74670	.75575	.75165	131.25	131.27
220	.70710	.61665	.78253	.67660	.73116	.69631	.71543	.70692	135.00	135.01
230	.66137	.54921	.76302	.61574	.70091	.63995	.67777	.66083	138.59	138.64

TABLE 4-120

LAMBDA 2 ROTS (4,4) STD. AA24B

TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
		SUCCESSIVE SUBSTITUTIONS							
0	.430788	.401249	.412827	.432648	.456317	-.433113	.003965	.393439	.430199
20	.371568	.341411	.348091	.366501	.379023	-.199295	.261971	.335683	.370907
40	.302192	.271004	.272329	.289556	.294087	-.051587	.269385	.199885	.301503
60	.221068	.188587	.184852	.201138	.201326	.017702	.198022	.121893	.220412
80	.126904	.092914	.085665	.101178	.100007	.022650	.090597	.069868	.126357
100	.019236	-.016792	-.024123	-.009522	-.010578	-.023764	-.016424	-.008710	.018878
120	-.100846	-.140414	-.141851	-.129069	-.129883	-.110978	-.113142	-.109112	-.100955
140	-.229989	-.276558	-.263083	-.254549	-.255053	-.227318	-.220425	-.219723	-.229877
160	-.362368	-.422329	-.381807	-.382426	-.380793	-.358054	-.349218	-.346519	-.362190
180	-.490519	-.573679	-.491349	-.509376	-.500356	-.488908	-.484119	-.482837	-.390454
200	-.607154	-.725966	-.585414	-.632941	-.606551	-.610779	-.605135	-.606834	-.607326
220	-.707108	-.875604	-.659489	-.752692	-.692856	-.722222	-.702343	-.713092	-.707287

TABLE 4-121

LAMBDA 3 ROTS (4,4) STD. AA24B

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.18868	.17691	.16791	.20859	.21838	-.15492	.02838	.18020	.18920
20	.20371	.19033	.18236	.22021	.22410	-.05165	.15528	.19926	.20074
40	.21996	.20448	.19782	.22983	.22926	.04029	.20956	.17612	.21310
60	.23722	.21917	.21393	.23677	.23447	.11802	.23121	.18237	.22653
80	.25504	.22409	.23008	.24074	.23993	.18213	.23370	.21585	.24139
100	.27261	.24882	.24532	.24222	.24521	.23365	.24067	.24640	.25812
120	.28872	.26286	.25832	.24268	.24945	.27197	.26785	.27250	.27677
140	.30187	.27571	.26750	.24435	.25218	.29595	.30281	.30382	.29557
160	.31054	.28703	.27123	.24921	.25414	.30660	.32305	.32825	.30974
180	.31365	.29670	.26804	.25764	.25740	.30751	.32302	.32831	.31484
200	.31099	.30496	.25696	.26801	.26449	.30278	.31184	.31397	.31148
220	.30331	.30887	.23400	.27412	.27302	.29049	.29341	.29294	.30227

TABLE 4-122

LAMBDA 4 ROTS (4,4) STD. AA24B

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.42576	.38747	.35628	.49521	.52279	.48366	.00989	.39945	.42781
20	.45294	.41181	.38893	.50831	.51777	-.19558	.32515	.44694	.44330
40	.48239	.44011	.42288	.51595	.51330	.04374	.45642	.37975	.46147
60	.51362	.46919	.45712	.51766	.51133	.23314	.50089	.38598	.48309
80	.54559	.49834	.49013	.51419	.51250	.37968	.49798	.45646	.50908
100	.57659	.52658	.51978	.50797	.51577	.59030	.50596	.51827	.54020
120	.60410	.55271	.54338	.50306	.51912	.56746	.55803	.56802	.57584
140	.62507	.57550	.55796	.50389	.52127	.61188	.62470	.62661	.61100
160	.63652	.59396	.56089	.51300	.52328	.62801	.65948	.66940	.63479
180	.63650	.60745	.55023	.52882	.52824	.62445	.65351	.66342	.63885
200	.62484	.61600	.52541	.54597	.53921	.60972	.62620	.63007	.62578
220	.60327	.61420	.48089	.55220	.55011	.58088	.58597	.58516	.60150

TABLE 4-123

LAMBDA 1 ROTS (4,4) STD. AA1C

		SUCCESSIVE SUBSTITUTIONS								
TIME	TRUE	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.90859	.93345	.93267	.91591	.92340	.98901	.91856	.90865	65.31	65.32
20	.92267	.94217	.94177	.92812	.93603	.99027	.93036	.92274	67.32	67.33
40	.93652	.95132	.95137	.94063	.94842	.99152	.94288	.93659	69.47	69.49
60	.94995	.96074	.96125	.95317	.96030	.99291	.95577	.95002	71.79	71.81
80	.96270	.97019	.97112	.96535	.97135	.99452	.96839	.96277	74.30	74.32
100	.97443	.97934	.98053	.97668	.98122	.99630	.97977	.97449	77.01	77.03
120	.98468	.98766	.98889	.98654	.98953	.99808	.98937	.98472	79.95	79.97
140	.99286	.99450	.99546	.99423	.99578	.99948	.99613	.99289	83.15	83.17
160	.99826	.99893	.99935	.99894	.99940	.99998	.99961	.99827	86.62	86.64
180	.99997	.99980	.99958	.99979	.99961	.99876	.99940	.99997	90.38	90.40
200	.99697	.99566	.99514	.99586	.99544	.99471	.99512	.99696	94.45	94.46
220	.98812	.98476	.98519	.98614	.98578	.98631	.98611	.98810	98.83	98.84
240	.97224	.96509	.96922	.96959	.96945	.97168	.97117	.97223	103.53	103.53
260	.94829	.93446	.94740	.94497	.94554	.94895	.94850	.94829	108.50	108.51
280	.91549	.89063	.93074	.91074	.91375	.91657	.91651	.91550	113.72	113.72
300	.87356	.83140	.89135	.86470	.87499	.87345	.87488	.87355	119.12	119.13
320	.82280	.75502	.86251	.80374	.83246	.81829	.82588	.82277	124.63	124.65
340	.76422	.65968	.83823	.72252	.79265	.74664	.77573	.78413	130.16	130.17
360	.69937	.50031	.81707	.56726	.76027	.59998	.73279	.67287	135.62	137.71

TABLE 4-124

LAMBDA 2 ROTS (4,4) STD. AA1C

				SUCCESSIVE SUBSTITUTIONS					
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.417679	.364825	.366960	.402042	.384351	.147811	.395324	.405007	.417555
40	.350603	.312018	.311858	.339702	.317206	.129882	.333132	.322251	.350408
80	.270545	.244131	.240315	.261004	.237677	.104504	.249440	.234209	.270301
120	.174357	.157042	.149046	.163500	.144321	.061861	.145411	.141243	.174096
160	.058916	.046165	.035886	.045997	.034474	-.005082	.027716	.037066	.058687
200	-.077688	-.093092	-.098523	-.090851	-.095289	-.102630	-.198665	-.089565	-.077830
240	-.333963	-.264208	-.248072	-.244759	-.245318	-.236283	-.238383	-.235466	-.233996
280	-.402325	-.467693	-.398121	-.413749	-.406984	-.399925	-.400067	-.399696	-.402312
320	-.568322	-.699965	-.524379	-.600079	-.557592	-.575676	-.564596	-.570225	-.568360
360	-.714753	-.999387	-.604855	-.855362	-.657758	-.810414	-.683459	-.788638	-.739759

TABLE 4-125

LAMBDA 3 ROTS (4,4) STD. AA1C

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.13846	.11335	.10542	.13788	.13460	.06680	.13302	.14323	.14280
40	.15426	.12852	.12091	.15598	.14881	.08565	.14930	.15374	.15743
80	.17193	.14594	.13930	.17477	.16412	.11059	.16439	.16551	.17319
120	.19152	.16595	.16078	.19309	.18116	.14075	.17806	.18114	.19034
160	.21280	.18897	.18512	.20968	.20020	.17564	.19348	.20097	.20928
200	.23485	.21532	.21107	.22412	.22006	.21385	.21641	.22415	.23030
240	.25582	.24523	.23567	.23813	.23802	.24928	.24842	.25105	.25254
280	.27282	.27887	.25341	.25553	.25206	.27330	.27543	.27578	,27193
320	.28275	.31603	.25529	.27846	.26185	.28386	.28304	.28420	.28275
360	.28383	COMPUTER TRUNCATION ERROR							

TABLE 4-126

LAMBDA 4 ROTS (4,4) STD. AA1C

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.25000	.17047	.14791	.25132	.24783	.09192	.23073	.27082	.26560
40	.26603	.19263	.17421	.27330	.25960	.12465	.25071	.27292	.27585
80	.28454	.21824	.20459	.29403	.27265	.16601	.26697	.27707	.28800
120	.30552	.24758	.23864	.31155	.28838	.21314	.27950	.28881	.30277
160	.32829	.28077	.27510	.32466	.30733	.26448	.29456	.30859	.32101
200	.35171	.31761	.31117	.33433	.32749	.31732	.32143	.33413	.34312
240	.37269	.35733	.34182	.34492	.34480	.36229	.36097	.36496	.36710
280	.38732	.39857	.35948	.36209	.35668	.38782	.39081	.39129	.38594
320	.39157	.43887	.35396	.38615	.36288	.39221	.39191	.39355	.39159
360	.38354	-2.30063	-2.47804	-2.40131	-2.63039	-2.39339	-2.51224	-2.40053	-2.36923

TABLE 4-127

LAMBDA 1 ROTS (4,4) STD. AA24C

TIME	TRUE	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
		SUCCESSIVE SUBSTITUTIONS								
0	.86585	.89618	.88910	.87313	.88974	.99965	.89298	.86591	59.98	59.99
20	.88190	.90583	.89942	.88780	.90513	.99952	.89931	.88200	61.87	61.88
40	.89828	.91654	.91110	.90362	.92075	.99926	.90767	.89840	63.93	63.95
60	.91487	.92829	.92406	.92030	.93621	.99893	.91882	.91502	66.18	66.21
80	.93149	.94092	.93808	.93739	.95106	.99869	.93301	.93164	68.66	68.69
100	.94782	.95416	.95274	.95424	.96488	.99874	.94948	.94797	71.40	71.44
120	.96340	.96751	.96734	.97000	.97720	.99914	.96645	.96354	74.45	74.48
140	.97755	.98018	.98087	.98363	.98755	.99970	.98174	.97760	77.83	77.87
160	.98931	.99099	.99192	.99390	.99532	.99999	.99329	.98937	81.61	81.65
180	.99736	.99834	.99875	.99943	.99958	.99938	.99939	.99739	85.83	85.87
200	.99995	.99963	.99935	.99877	.99889	.99690	.99877	.99995	90.53	90.56
220	.99497	.99218	.99183	.99035	.99126	.99076	.99108	.99494	95.74	95.76
240	.98002	.97227	.97480	.97253	.97432	.97784	.97641	.97999	101.47	101.48
260	.95278	.93595	.94822	.94328	.94596	.95381	.95291	.95278	107.67	107.68
280	.91153	.87937	.91401	.89993	.90544	.91453	.91535	.91157	114.28	114.28
300	.85576	.79945	.87638	.83848	.85500	.85721	.86092	.85580	121.15	121.15
320	.78663	.69454	.84151	.75301	.80197	.77882	.79508	.78660	128.12	128.13
340	.70694	.56444	.81625	.63445	.76060	.66879	.73713	.70681	135.01	135.02
360	.62061	.40855	.80664	.46701	.25236	.49146	.73027	.62006	141.63	141.68

TABLE 4-128

LAMBDA 2 ROTS (4,4) STD. AA24C

TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
				SUCCESSIVE SUBSTITUTIONS					
0	.500288	.455654	.470944	.489290	.457744	.026093	.450433	.521224	.500183
40	.439417	.408550	.421656	.429258	.390721	.038425	.419771	.423520	.439159
80	.363760	.343720	.351883	.348588	.309154	.051032	.359870	.301079	.363359
120	.268046	.254892	.255554	.243133	.212346	.041382	.256843	.181501	.267540
160	.145765	.134228	.127065	.110264	.096599	-.001040	.115581	.075187	.145235
200	-.009380	-.026932	-.035543	-.049575	-.046983	-.078605	-.049478	-.045259	-.009792
240	-.198888	-.235472	-.224482	-.232820	-.225186	-.209352	-.215887	-.206983	-.199026
280	-.411226	-.491176	-.414699	-.437044	-.425360	-.404573	-.402726	-.401259	-.411139
320	-.617421	-.781605	-.563009	-.666756	-.602565	-.628899	-.607788	-.619260	-.617455
360	-.784113	-.010876	-.621857	-.935254	-.667548	-.897259	-.686254	-.874626	-.784547

TABLE 4-129

LAMBDA 3 ROTS (4,4) STD. AA24C

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.15157	.13009	.11971	.15313	.14738	.02862	.13624	.16959	.16105
40	.17077	.14902	.13858	.17496	.16335	.04988	.16358	.18191	.17871
80	.19319	.17113	.16162	.19801	.18056	.08223	.19186	.18748	.19784
120	.21936	.19692	.18940	.22036	.20067	.12235	.21497	.19474	.21884
160	.24945	.22675	.22175	.23941	.22535	.16994	.23193	.21726	.24251
200	.28239	.26045	.25670	.25279	.25299	.32991	.24938	.25507	.27023
240	.31464	.29700	.28866	.26737	.27669	.29539	.29010	.29807	.30341
280	.33933	.33444	.30689	.28897	.29091	.33759	.34618	.34834	.33628
320	.34873	.37085	.29669	.31956	.30149	.34648	.35468	.35683	.34927
360	.34024	.38332	.22051	.32829	.29576	.32167	.29802	.31740	.33376

TABLE 4-130

LAMBDA 4 ROTS (4,4) STD. AA24C

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.27059	.19672	.16423	.27792	.27063	.00369	.22254	.32468	.30460
40	.29268	.22518	.19730	.30755	.28497	.04722	.26652	.33121	.31723
80	.31945	.25867	.23690	.33606	.30044	.10800	.31000	.32182	.33206
120	.35151	.29796	.28305	.35983	.32060	,17733	.34018	.31475	.35038
160	.38883	.34230	.33433	.37547	.34866	.25273	.35755	.33654	.37424
200	.42952	.39156	.38630	.38374	.38174	.34445	.37562	.38533	.40654
240	.46809	.44278	.42946	.39439	.40910	.43783	.42965	.44173	.44893
280	.49457	.49120	.44868	.42104	.42356	.49140	.50332	.50615	.48980
320	.49834	.53153	.42721	.45934	.43365	.49530	.50600	.50901	.49910
360	.47669	.53230	.32521	.46228	.42077	.45378	.42357	.44834	.46858

TABLE 4-131

LAMBDA 1 ROTS (5,4) STD. AA1

				SUCCESSIVE SUBSTITUTIONS						
TIME	TRUE	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.97371	.98173	.98312	.97395	.97189	.98157	.97752	.97374	76.84	76.84
10	.98231	.98844	.98968	.98289	.98167	.98722	.98540	.98232	79.21	79.21
20	.98952	.99388	.99483	.99023	.98960	.99238	.99175	.98953	81.69	81.71
30	.99507	.99776	.99833	.99571	.99545	.99654	.99640	.99508	84.31	84.32
40	.99867	.99978	.99992	.99904	.99899	.99922	.99922	.99867	87.04	87.05
50	.99999	.99961	.99936	.99997	.99997	.99997	.99997	.99999	89.90	89.91
60	.99874	.99690	.99644	.99824	.99821	.99836	.99838	.99874	92.86	92.87
70	.99463	.99132	.99099	.99363	.99351	.99404	.99414	.99463	95.93	95.94
80	.98740	.98254	.98290	.98595	.98571	.98672	.98693	.98740	99.10	99.10
90	.97686	.97027	.97216	.97503	.97472	.97617	.97650	.97685	102.34	102.35
100	.96288	.95422	.95882	.96077	.96049	.96225	.96264	.96287	105.65	105.66
110	.94542	.93420	.94304	.94312	.94307	.94490	.94527	.94541	109.01	109.02
120	.92456	.91005	.92508	.92287	.92258	.92416	.92446	.92454	112.39	112.40
130	.90043	.88168	.90529	.89763	.89926	.90010	.90037	.90041	115.78	115.79
140	.87331	.84909	.88412	.86988	.87346	.87292	.87333	.87328	119.15	119.16
150	.84352	.81222	.86201	.83879	.84556	.84277	.84371	.84348	122.48	122.49
160	.81147	.77086	.83936	.80413	.81590	.80966	.81182	.81135	125.76	125.77
170	.77758	-.12575	.55549	-.22701	.68964	-.64078	.89550	.19664	128.95	168.66

TABLE 4-132

LAMBDA 2 ROTS (5,4) STD. AA1

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.227749	.191126	.183691	.226774	.235470	.191085	.210807	.214134	.227659
20	.144370	.110587	.101614	.139388	.143798	.123184	.128171	.132088	.144266
40	.051507	.020652	.011871	.043667	.045028	.039289	.039455	.041652	.051413
60	-.050004	-.078678	-.084319	-.059148	-.059702	-.057192	-.056864	-.056300	-.050071
80	-.158201	-.186818	-.184888	-.167036	-.168426	-.182412	-.161092	-.161052	-.158241
100	-.269914	-.302563	-.286971	-.277417	-.278403	-.272149	-.270780	-.270721	-.269943
120	-.381036	-.424136	-.387078	-.387574	-.386335	-.382051	-.381315	-.381264	-.381073
140	-.487157	-.549379	-.481419	-.495180	-.488686	-.488120	-.487384	-.487520	-.487210
160	-.584391	-.676953	-.566813	-.599523	-.582568	-.587914	-.584881	-.582922	-.584552

TABLE 4-133

LAMBDA 3 ROTS (5,4) STD. AA1

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.178091	.161097	.155667	.185868	.191171	.141325	.171983	.168062	.173708
20	.189637	.171486	.166928	.190814	.193913	.166559	.178421	.179967	.184380
40	.200917	.182092	.178129	.194123	.196174	.186988	.189155	.191539	.195677
60	.211393	.192781	.188754	.196259	.198151	.202906	.202594	.203653	.207211
80	.220425	.203451	.198162	.198164	.200117	.214744	.215534	.215755	.217986
100	.227360	.214049	.205595	.201069	.202544	.222965	.225347	.225611	.226513
120	.231673	.224611	.210230	.206097	.206210	.228015	.230869	.231245	.231638
140	.233094	.235279	.211210	.213775	.212140	.230165	.232103	.232322	.233287
160	.231689	.239985	.201250	.217356	.214840	.222413	.222542	.222556	.232292

TABLE 4-134

LAMBDA 4 ROTS (5,4) STD. AA1

		SUCCESSIVE SUBSTITUTIONS								
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.45610	.40615	.39129	.48339	.50009	.33811	.43971	.42483	.44042
20	.47794	.42847	.41710	.48304	.49225	.40879	.44610	.44993	.46054
40	.49872	.45076	.44147	.48025	.48621	.45981	.46641	.47302	.48261
60	.51715	.47256	.46309	.47683	.48224	.49510	.49409	.49695	.50518
80	.53176	.49339	.48051	.47521	.48053	.51796	.51958	.52014	.52524
100	.54110	.51284	.49224	.47771	.48147	.53096	.53636	.53696	.53898
120	.54408	.53060	.49686	.48560	.48587	.53596	.54231	.54314	.54401
140	.54026	.54655	.49317	.49837	.49472	.53401	.53817	.53864	.54070
160	.52998	.54782	.46711	.50060	.49534	.51103	.51128	.51131	.53129

TABLE 4-135

LAMBDA 1 ROTS (5,4) AA24

		SUCCESSIVE SUBSTITUTIONS								
TIME	TRUE	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.89650	.91505	.90951	.88390	.87086	.99907	.99995	.89652	63.70	63.70
10	.91598	.93308	.92972	.90912	.90106	.99855	.96874	.91605	66.34	66.36
20	.93512	.95053	.94901	.93296	.92848	.99377	.95661	.93523	69.24	69.27
30	.95337	.96676	.96658	.95460	.95246	.99088	.96342	.95349	72.43	72.46
40	.96999	.98090	.98148	.97314	.97237	.99159	.97757	.97010	75.92	75.96
50	.98404	.99193	.99265	.98759	.98747	.99491	.99034	.98413	79.75	79.78
60	.99436	.99861	.99896	.99689	.99696	.99857	.99791	.99441	83.91	83.94
70	.99961	.99961	.99941	.99999	.99998	.99996	.99996	.99963	88.41	88.44
80	.99839	.99353	.99317	.99594	.99568	.99658	.99651	.99837	93.25	93.27
90	.98933	.97910	.97984	.98394	.98340	.98640	.98708	.98930	98.34	98.39
100	.97142	.95532	.95955	.96349	.96277	.96798	.96981	.97139	103.72	103.74
110	.94414	.92164	.93307	.93440	.93397	.94067	.94342	.94411	109.24	109.24
120	.90767	.87807	.90172	.89686	.89775	.90459	.90751	.90763	114.81	114.82
130	.86298	.82522	.86723	.85140	.85556	.86048	.86304	.86289	120.34	120.36
140	.81163	.76424	.83159	.79883	.80944	.80946	.81216	.81147	125.74	125.76
150	.75559	.69637	.79655	.73985	.76167	.75252	.75756	.75536	130.92	130.94
160	.69691	.62294	.76369	.67503	.71474	.69036	.70233	.69659	135.82	135.85
170	.63745	.54460	.73403	.60417	.67084	.62286	.64953	.63668	140.39	140.46

TABLE 4-136

LAMBDA 2 ROTS (5,4) STD. AA24

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.443039	.412174	.425398	.469124	.493416	-.042987	.009391	.247730	.442995
20	.354322	.314524	.219337	.360362	.371833	.111396	.291358	.312826	.354039
40	.243120	.195426	.192420	.230218	.233480	.129362	.210585	.184209	.242679
60	.106005	.052602	.045401	.078802	.077865	.053321	.064487	.066721	.105536
80	-.056708	-.113690	-.116875	-.090015	-.092771	-.082543	-.082151	-.081277	-.057042
100	-.237340	-.298913	-.284394	-.267812	-.270389	-.251027	-.243828	-.243019	-.237456
120	-.419666	-.493807	-.443345	-.443402	-.441564	-.426376	-.420117	-.418102	-.419750
140	-.584159	-.686666	-.580359	-.606963	-.591955	-.588187	-.584401	-.584318	-.584380
160	-.717156	-.868400	-.687464	-.754315	-.711619	-.728224	-.716061	-.721633	-.717469

TABLE 4-137

LAMBDA 3 ROTS (5,4) STD. AA24

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.24128	.23533	.23528	.28791	.31118	-.21797	-.11399	.10823	.23675
20	.26716	.25486	.24691	.29778	.30829	.00654	.24292	.23525	.25426
40	.29526	.27416	.26846	.30027	.30389	.15049	.28910	.23137	.27420
60	.32388	.29181	.28776	.29549	.29942	.25683	.28273	.28029	.28801
80	.34975	.30600	.30167	.28703	.29449	.31470	.31180	.31441	.32788
100	.36818	.31510	.30668	.28128	.28883	.34289	.35429	.35590	.36121
120	.37482	.31851	.30007	.28299	.28536	.24987	.37118	.37877	.37867
140	.36822	.31719	.28117	.29064	.28984	.34473	.36196	.36773	.37041
160	.35087	.30894	.24689	.29327	.30058	.32728	.33567	.33615	.34995

TABLE 4-138

LAMBDA 4 ROTS (5,4) STD. AA24

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.61036	.59011	.55770	.76651	.84344	-.85268	-.51228	.19563	.59415
20	.66867	.63523	.61216	.76674	.79920	-.11029	.60436	.57518	.62600
40	.73178	.67944	.66478	.75029	.76122	.35072	.72132	.55357	.66706
60	.79540	.71938	.71011	.72190	.73367	.61907	.68977	.68213	.72129
80	.85151	.75084	.74083	.69369	.71423	.76541	.75744	.76416	.79296
100	.88895	.76992	.74939	.68050	.69977	.83019	.85625	.85987	.87147
120	.89763	.77480	.73100	.68820	.69383	.84169	.88951	.90655	.90673
140	.87469	.76689	.68615	.70643	.70446	.82393	.86112	.87358	.87961
160	.82660	.74125	.61101	.70788	.72290	.77805	.79538	.79637	.82480

TABLE 4-139

LAMBDA 1 ROTS (5,4) STD. AA1A

		SUCCESSIVE SUBSTITUTIONS								
TIME	TRUE	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.97061	.97886	.98052	.97073	.96840	.97893	.97419	.97062	76.07	76.08
10	.97848	.98505	.98657	.97889	.97736	.98418	.98158	.97849	78.09	78.10
20	.98541	.99043	.99171	.98599	.98506	.98913	.98790	.98542	80.20	80.21
30	.99122	.99480	.99577	.99184	.99134	.99348	.99301	.99123	82.40	82.41
40	.99572	.99797	.99856	.99625	.99603	.99695	.99682	.99573	84.70	84.71
50	.99871	.99972	.99990	.99903	.99897	.99922	.99920	.99871	87.09	87.10
60	.99997	.99982	.99962	.99999	.99999	.99999	.99999	.99997	89.57	89.58
70	.99929	.99800	.99754	.99896	.99895	.99900	.99901	.99929	92.15	92.16
80	.99646	.99404	.99355	.99576	.99568	.99599	.99603	.99646	94.81	94.82
90	.99130	.98765	.98753	.99022	.99006	.99073	.99083	.99129	97.56	97.56
100	.98363	.97862	.97943	.98223	.98198	.98304	.98322	.98393	100.37	100.38
110	.97333	.96668	.96924	.97165	.97137	.97276	.97301	.97333	103.26	103.26
120	.96031	.95164	.95702	.95842	.95830	.95981	.96008	.96030	106.19	106.20
130	.94454	ERROR IN DATA								
140	.92604	.91153	.92703	.92372	.92433	.92570	.92593	.92603	112.17	112.18
150	.90491	.88616	.89968	.90218	.90383	.90457	.90478	.90489	115.18	115.19
160	.88128	.85712	.89117	.87780	.88124	.88080	.88117	.88125	118.20	118.21
170	.85537	.82437	.87181	.85047	.85677	.85443	.85526	.85532	121.19	121.20
180	.82743	.78757	.85204	.82012	.83084	.82557	.82742	.82733	124.16	124.17
190	.79773	.74462	.83110	.78458	.80234	.79243	.79633	.79693	127.08	127.16

TABLE 4-140

LAMBDA 2 ROTS (5,4) STD. AA1A

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.240655	.205618	.197366	.240218	.249463	.204190	.225717	.229748	.240597
20	.170187	.138342	.128693	.166777	.172186	.147012	.155087	.159638	.170111
40	.092349	.063576	.053536	.086480	.088998	.077937	.079575	.082928	.092272
60	.007357	-.018931	-.027550	-.000109	.000350	-.001515	-.001524	.000068	.007291
80	-.083985	-.109176	-.113535	-.091984	-.092786	-.089409	-.088992	-.088567	-.084033
100	-.180159	-.206784	-.202803	-.187688	-.188961	-.183369	-.182392	-.182345	-.180191
120	-.278911	-.310979	-.293174	-.285472	-.286189	-.280632	-.279710	-.279679	-.278937
140	-.377409	-.420639	-.382016	-.383575	-.382101	-.278279	-.377759	-.377731	-.377442
160	-.492582	-.634548	-.466519	-.480669	-.474211	-.473689	-.473010	-.473146	-.472634
180	-.561567	-.651801	-.573977	-.576343	-.560100	-.575058	-.562324	-.563263	-.561711

TABLE 4-141

LAMBDA 3 ROTS (5,4) STD. AA1A

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.16226	.14535	.13967	.16942	.17404	.12969	.15770	.15519	.15932
20	.17216	.15463	.14972	.17523	.17801	.14956	.16350	.16494	.16836
40	.18206	.16428	.16003	.17984	.18157	.16704	.17179	.17456	.17786
60	.19168	.17424	.17030	.18336	.18481	.18196	.18245	.18448	.18771
80	.20065	.18446	.18013	.18618	.18781	.19429	.19402	.19485	.19756
100	.20853	.19492	.18903	.18897	.19073	.20406	.20472	.20494	.20668
120	.21490	.20560	.19392	.19258	.19388	.21140	.21313	.21331	.21415
140	.21941	.21655	.20151	.19775	.19777	.21644	.21851	.21874	.21924
160	.22181	.22740	.20317	.20446	.20260	.21875	.22021	.22036	.22176
180	.22208	.23293	.19505	.20717	.20358	.21248	.21256	.21265	.22133

TABLE 4-142

LAMBDA 4 ROTS (5,4) STD. AA1A

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.39489	.34435	.32916	.41972	.43383	.29191	.38325	.37323	.38437
20	.41153	.36300	.35106	.42226	.43018	.34467	.38767	.39100	.39894
40	.42786	.38205	.37247	.42264	.42739	.38627	.40028	.40771	.41493
60	.44322	.40128	.39266	.42169	.42567	.41807	.41946	.42479	.43185
80	.45680	.42040	.41070	.42066	.42504	.44142	.44056	.44265	.44853
100	.46770	.43914	.42555	.42111	.42559	.45755	.45881	.45933	.46305
120	.47504	.45723	.43611	.42449	.42761	.46747	.47112	.47151	.47326
140	.47814	.47443	.44126	.43157	.43160	.47197	.47627	.47676	.47779
160	.47664	.48970	.43901	.44124	.43732	.47053	.47345	.47377	.47656
180	.47059	.49268	.41795	.44178	.43467	.45209	.45223	.45241	.46915

TABLE 4-143

LAMBDA 1 ROTS (5,4) STD. AA24A

		SUCCESSIVE SUBSTITUTIONS								
TIME	TRUE	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.90226	.91937	.91513	.89134	.88201	.99652	.96398	.90225	64.45	64.46
10	.91826	.93374	.93104	.91141	.90525	.99204	.94958	.91830	66.67	66.68
20	.93402	.94785	.94654	.93069	.92693	.98893	.94874	.93408	69.07	69.08
30	.94922	.96134	.96110	.94870	.94664	.98732	.956697	.94930	71.66	77.68
40	.96347	.97375	.97426	.96494	.96397	.98834	.96957	.96355	74.46	74.48
50	.97628	.98453	.98539	.97881	.97848	.99134	.98214	.97635	77.49	77.51
60	.98702	.99301	.99381	.98971	.98968	.99517	.99191	.98708	80.76	80.78
70	.99500	.99843	.99885	.99698	.99703	.99853	.99789	.99503	84.27	84.29
80	.99940	.99994	.99982	.99996	.99997	.99999	.99999	.99941	88.02	88.04
90	.99937	.99663	.99618	.99802	.99791	.99822	.99819	.99936	92.02	92.04
100	.99407	.98761	.98753	.99063	.99033	.99195	.99208	.99404	96.24	96.25
110	.98274	.97209	.97375	.97732	.97685	.98017	.98085	.98271	100.66	100.67
120	.96484	.94940	.95503	.95778	.95729	.96217	.96355	.96481	105.23	105.24
130	.94013	.91916	.93191	.93188	.93178	.93762	.93950	.94011	109.92	109.93
140	.90875	.88124	.90537	.89963	.90080	.90965	.90855	.90871	114.66	114.67
150	.87122	.83586	.87624	.86121	.86524	.86942	.87126	.87115	119.39	119.41
160	.82842	.78350	.84609	.81688	.82643	.82666	.82880	.82831	124.06	124.07
170	.78149	.72494	.81618	.76698	.78570	.77890	.78290	.78134	128.60	128.62
180	.73169	.66094	.78768	.71171	.74512	.72650	.73563	.73149	132.97	132.99
190	.68027	.59187	.76149	.65068	.70639	.66913	.68910	.67993	137.13	137.16
200	.62838	.50963	.73549	.57523	.66700	.59836	.64111	.62039	141.06	141.66

TABLE 4-144

LAMBDA 2 ROTS (5,4) STD. AA24A

		SUCESSIVE SUBSTITUTION							
TIME	TRUE	FIRST	SECOND	THIRD	FORTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.431192	.401567	.411972	.454563	.472748	.083268	.265960	.429432	.431189
20	.357209	.322928	.327050	.366326	.375698	.150989	.316065	.320946	.357050
40	.267784	.229085	.226874	.262543	.266070	.152259	.244716	.216870	.267501
60	.160551	.118152	.111184	.143040	.143264	.098117	.126907	.123354	.160208
80	.034446	-.010897	-.018473	.008868	.007478	.001217	.002931	.006530	.034127
100	-.108731	-.157368	-.157890	-.136556	-.138711	-.126579	-.125567	-.125190	-.108942
120	-.262823	-.318089	-.299896	-.287610	-.289246	-.272437	-.267514	-.267054	-.262915
140	-.417325	-.487348	-.435257	-.437676	-.435216	-.422099	-.417845	-.416767	-.417409
160	-.560093	-.658015	-.554799	-.581138	-.566998	-.563450	-.560258	-.560351	-.560259
180	-.681635	-.823470	-.651631	-.715008	-.676337	-.690397	-.680293	-.684768	-.681848
200	-.777901	.990321	-.727335	-.848494	-.762538	-.811758	-.774942	.797873	-.784292

TABLE 4-145

LAMBDA 3 ROTS (5,4) STD. AA24A

					SUCCESSIVE SUBSTITUTIONS				
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.21347	.20469	.19489	.24809	.26273	-.07859	.10684	.22921	.21142
20	.23313	.22082	.21260	.25904	.26584	.39049	.22428	.20701	.22557
40	.25440	.23740	.23100	.26575	.26764	.13380	.25583	.20646	.24114
60	.27660	.25384	.24912	.26758	.26890	.20578	.25372	.24034	.25874
80	.29837	.26929	.26539	.26531	.26957	.25818	.26292	.26738	.27928
100	.31756	.28278	.27775	.26161	.26906	.29343	.29184	.29323	.30317
120	.33148	.29344	.28387	.26037	.26747	.31318	.32137	.32250	.32657
140	.33777	.30092	.28176	.26429	.26692	.32033	.33485	.33904	.33921
160	.33544	.30543	.27016	.27259	.27106	.31880	.33155	.33509	.33673
180	.32543	.30670	.24780	.28023	.28133	.30985	.31681	.21744	.32480
200	.31003	.00998	-.08278	-.03791	-.06063	-.04848	-.05907	-.05305	.01092

TABLE 4-146

LAMBDA 4 ROTS (5,4) STD. AA24A

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS							NEW-RAPH
		FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	
0	.51448	.48425	.45332	.62860	.67672	-.40111	.18421	.56539	.50714
20	.55489	.51936	.49591	.63642	.65729	-.01634	.53437	.47940	.52989
40	.59855	.55514	.53880	.63142	.63975	.26475	.60796	.46687	.55779
60	.64379	.59019	.57949	.62258	.62672	.45944	.58764	.55063	.59259
80	.68742	.62257	.61432	.60611	.61795	.58922	.60141	.61233	.63626
100	.72456	.64998	.63879	.59228	.61129	.66923	.66479	.66817	.68844
120	.74930	.67034	.64833	.58897	.60588	.70942	.72694	.72935	.73772
140	.75655	.68243	.63966	.59885	.60468	.71989	.75044	.75926	.75979
160	.74453	.68615	.61165	.61609	.61274	.71075	.73662	.74382	.74729
180	.71569	.68063	.56362	.62772	.62970	.68542	.69896	.70019	.71452
200	.67542	.11894	-.05420	.02954	-.01286	.00890	-.00995	.00128	.12003

TABLE 4-147

LAMBDA 1 ROTS (5,4) STD. AA1B

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.96285	.97224	.97419	.96264	.95975	.97362	.96623	.96286	74.33	74.34
10	.97038	.97816	.98002	.97044	.96835	.97848	.97355	.97039	76.02	76.02
20	.97732	.98363	.98535	.97760	.97616	.98324	.98018	.97733	77.77	77.78
30	.98358	.98845	.99005	.98400	.98307	.98774	.98601	.98359	79.60	79.61
40	.98903	.99276	.99400	.98952	.98896	.99181	.99094	.98904	81.50	81.51
50	.99354	.99616	.99706	.99401	.99272	.99527	.99490	.99354	83.48	83.49
60	.99697	.99858	.99909	.99735	.99723	.99791	.99779	.99697	85.53	85.54
70	.99917	.99985	.99997	.99939	.99936	.99955	.99952	.99917	87.67	87.67
80	.99999	.99979	.99955	.99999	.99999	.99997	.99998	.99999	89.88	89.88
90	.99928	.99820	.99772	.99900	.99899	.99900	.99901	.99928	92.16	92.17
100	.99688	.99490	.99435	.99631	.99625	.99645	.99646	.99688	94.52	94.53
110	.99264	.98966	.98937	.99178	.99164	.99213	.99317	.99263	96.95	96.96
120	.98643	.98229	.98271	.98529	.98509	.98590	.98599	.98642	99.44	99.45
130	.97812	.97258	.97434	.97674	.97649	.97763	.97775	.97812	102.00	102.01
140	.96763	.96033	.96428	.96603	.96581	.96719	.96734	.96763	104.61	104.62
150	.95489	.94534	.95259	.95309	.95301	.95453	.95468	.95488	107.27	107.28
160	.93987	.92745	.93938	.93783	.93810	.93957	.93971	.93986	109.96	109.97
170	.92258	.90651	.92481	.92022	.92118	.92233	.92247	.92257	112.69	112.69
180	.90308	.88237	.90910	.90014	.90232	.90278	.90298	.90306	115.43	115.44
190	.88146	.85495	.89252	.87757	.88174	.88099	.88143	.88143	118.18	118.18
200	.85784	.82407	.87535	.85233	.85962	.85692	.85792	.85781	120.92	120.93
210	.83240	.78963	.85792	.82427	.83631	.83058	.83292	.83234	123.65	123.66

TABLE 4-148

LAMBDA 2 ROTS (5,4) STD. AA1B

					SUCCESSIVE SUBSTITUTIONS				
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.270017	.235592	.227191	.270854	.280957	.228151	.257675	.262336	.269993
20	.211729	.180917	.171146	.210473	.217049	.182288	.198067	.203433	.211684
40	.147697	.120262	.105940	.144372	.148153	.127648	.134270	.139319	.147639
60	.077774	.053226	.042465	.072664	.074352	.064530	.066346	.070027	.077712
80	.002083	-.020475	-.029695	-.004269	-.004060	-.006403	-.006256	-.004242	.002029
100	-.078883	-.100965	-.106207	-.085763	-.086505	-.084137	-.084025	-.083257	-.078925
120	-.164177	-.188157	-.185933	-.170883	-.172048	-.167279	-.166803	-.166627	-.164207
140	-.252353	-.281671	-.267257	-.258475	-.259324	-.254029	-.253449	-.253415	-.252378
160	-.341515	-.380914	-.348194	-.347392	-.346656	-.342369	-.341974	-.341944	-.341542
180	-.429457	-.485029	-.426376	-.436607	-.432022	-.430206	-.429776	-.429826	-.429494
200	-.513909	-.593167	-.499296	-.535996	-.513221	-.515844	-.514169	-.514703	-.513966
220	.592814	COMPUTER TRUNCATION ERROR							

TABLE 4-149

LAMBDA 3 ROTS (5,4) STD. AA1B

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.15020	.13268	.12648	.15718	.16129	.11755	.14779	.14580	.14867
20	.15898	.14109	.13558	.16390	.16633	.13385	.15309	.15410	.15660
40	.16796	.14995	.14513	.16981	.17109	.14937	.15950	.16248	.16488
60	.17698	.15924	.15499	.17485	.17560	.16377	.16766	.17098	.17353
80	.18586	.16895	.16496	.17906	.17987	.17677	.17737	.17982	.18247
100	.19434	.17908	.17472	.18269	.18390	.18819	.18784	.18909	.19148
120	.20212	.18961	.18389	.18617	.18772	.19787	.19795	.19837	.20014
140	.20888	.20058	.19199	.19008	.19148	.20575	.20667	.20682	.20783
160	.21433	.21199	.19841	.19500	.19542	.21179	.21316	.21340	.21394
180	.21822	.22386	.20244	.20134	.19991	.21596	.21734	.21747	.21812
200	.22040	.23564	.20261	.20854	.20472	.21751	.21806	.21818	.22028
220	.22085	COMPUTER TRUNCATION ERROR							

TABLE 4-150

LAMBDA 4 ROTS (5,4) STD. AA1B

TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
				SUCCESSIVE SUBSTITUTIONS					
0	.34150	.28777	.27145	.36510	.37723	.24034	.33637	.32825	.33606
20	.35437	.30345	.39021	.37029	.37687	.28136	.33897	.34079	.34648
40	.36741	.31974	.30914	.37369	.37686	.31670	.34545	.35308	.35794
60	.38029	.33656	.32703	.37551	.37733	.34638	.35614	.36535	.37039
80	.39262	.35378	.34575	.37626	.37832	.37068	.37226	.37824	.38353
100	.40388	.37124	.36226	.37674	.37978	.38992	.38899	.39191	.39670
120	.41349	.38875	.37655	.37795	38166	.40441	.40438	.40523	.40882
140	.42088	.40615	.38782	.38094	.38408	.41452	.41627	.51657	.41853
160	.42549	.42322	.39509	.38641	.38728	.42055	.42339	.42365	.42468
180	.42694	.42973	.43737	.39447	.39161	.42272	.42529	.42555	.42679
200	.42504	.45435	.39248	.40336	.39618	.41984	.42081	.42103	.42482
220	.41985	COMPUTER TRUNCATION ERROR							

TABLE 4-151

LAMBDA 1 ROTS (5,4) STD. AA24B

TIME	TRUE	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
		SUCCESSIVE SUBSTITUTIONS								
0	.90245	.91904	.91556	.89185	.88501	.98622	.93754	.90241	64.48	64.48
10	.91548	.93028	.92785	.90773	.90300	.98366	.93522	.91548	66.27	66.27
20	.92840	.94159	.94008	.92328	.92023	.98224	.93864	.92842	68.18	68.19
30	.94105	.95256	.95203	.93827	.93647	.98221	.94662	.94109	70.22	70.24
40	.95324	.96321	.96342	.95242	.95148	.98360	.95742	.95329	72.41	72.42
50	.96474	.97321	.97394	.96540	.96541	.98621	.96908	.96479	74.74	74.75
60	.97525	.98222	.98322	.97688	.97680	.98966	.97991	.97530	77.22	77.24
70	.98444	.98986	.99087	.98648	.98653	.99342	.98880	.98449	79.88	79.90
80	.99191	.99570	.99645	.99380	.99388	.99685	.99521	.99194	82.70	82.72
90	.99720	.99921	.99952	.99844	.99850	.99928	.99896	.99722	85.71	85.73
100	.99981	.99985	.99968	.99999	.99999	.99996	.99997	.99981	88.89	88.91
110	.99922	.99702	.99653	.99805	.99797	.99815	.99812	.99921	92.25	92.27
120	.99490	.99010	.98981	.99225	.99208	.99315	.99312	.99488	95.78	95.80
130	.98636	.97851	.97938	.98226	.98199	.98431	.98448	.98633	99.47	99.48
140	.97319	.96169	.96524	.96781	.96750	.97110	.97165	.97316	103.29	103.30
150	.95511	.93921	.94763	.94868	.94853	.95317	.95410	.95509	107.23	107.24
160	.93203	.91075	.92699	.92474	.92522	.93034	.93151	.93200	111.24	111.25
170	.90403	.87614	.90392	.89590	.89788	.90262	.90386	.90399	115.30	115.31
180	.87143	.83539	.87921	.86212	.86712	.87016	.87151	.87137	119.37	119.38
190	.83472	.78872	.85380	.82340	.83383	.83321	.83521	.83463	123.41	123.42
200	.79458	.73646	.82862	.77971	.79915	.79197	.79607	.79447	127.38	127.39
210	.75177	.67888	.80454	.73080	.76435	.74634	.75544	.75162	131.25	131.27
220	.70710	.61665	.78253	.67661	.73116	.69622	.71540	.70691	135.00	135.02
230	.66137	.54921	.76302	.61574	.7009;	.63995	.67777	.66083	138.59	138.64

TABLE 4-152

LAMBDA 2 ROTS (5,4) STD. AA24B

TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
				SUCCESSIVE SUBSTITUTIONS					
0	.430788	.402371	.410926	.453545	.466995	.165408	.347875	.440181	.430861
20	.371568	.342014	.346163	.384644	.391965	.187597	.344896	.347754	.371524
40	.302192	.271221	.270475	.304955	.307870	.180326	.288675	.262223	.302048
60	.221068	.188541	.183164	.213796	.214169	.143400	.199404	.186093	.220853
80	.126904	.092710	.084217	.111109	.110404	.079184	.097686	.098867	.126656
100	.019236	-.017063	-.025281	-.002239	-.003328	-.008932	-.007185	-.003299	.019003
120	-.100846	-.140684	-.142700	-.124224	-.125572	-.116799	-.117093	-.115934	-.101020
140	-.229989	-.276781	-.263639	-.251742	-.252903	-.238637	-.236415	-.236363	-.230090
160	-.362368	-.422482	-.382119	-.381088	-.379932	-.366713	-.363735	-.363230	-.362438
180	-.490519	-.573762	-.491488	-.508898	-.500110	-.493046	-.490653	-.490416	-.490622
200	-.607154	-.725996	-.585455	-.632835	-.606516	-.611912	-.606474	-.608381	-.607300
220	-.707108	-.875608	-.659493	-.752685	-.692857	-.722314	-.702384	-.713195	-.707296

TABLE 4-153

LAMBDA 3 ROTS (5,4) STD. AA24B

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.18868	.17721	.16732	.21415	.22301	-.00585	.14726	.20265	.18938
20	.20371	.19052	.18174	.22560	.22944	.05442	.20123	.18965	.20092
40	.21996	.20456	.19717	.23499	.23504	.11101	.22608	.18841	.21327
60	.23722	.21915	.21328	.24161	.24020	..6131	.23094	.20747	.22667
80	.25504	.23400	.22944	.24514	.24504	.20447	.23276	.22987	.24150
100	.27261	.24868	.24471	.24602	.24926	.24026	.24405	.24891	.25817
120	.28872	.26269	.25777	.24573	.25230	.26810	.26586	.26831	.27673
140	.30187	.27553	.26703	.24654	.25396	.28731	.28989	.29024	.29540
160	.31054	.28687	.27087	.25054	.25513	.29811	.30652	.30817	.30943
180	.31365	.29658	.26780	.25828	.25787	.30189	.31184	.31422	.31451
200	.31099	.30489	.25684	.26822	.26465	.30036	.30768	.30899	.31136
220	.30331	.30886	.23397	.27414	.27304	.29005	.29292	.29230	.30220

TABLE 4-154

LAMBDA 4 ROTS (5,4) STD. AA24B

TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
		SUCCESSIVE SUBSTITUTIONS							
0	.42576	.38543	.35493	.50728	.53643	-.17120	.29971	.46882	.42825
20	.45294	.41221	.38756	.51981	.53171	.02249	.44855	.41354	.44369
40	.48239	.44027	.42150	.52676	.52710	.18656	.50257	.39777	.46181
60	.51362	.46915	.45576	.52764	.52412	.31921	.50074	.43842	.48337
80	.54559	.49815	.48882	.52311	.52332	.42338	.49370	.48577	.50928
100	.57659	.52639	.51856	.51555	.52317	.50285	.51191	.52309	.54030
120	.60410	.55238	.54230	.50902	.52465	.55977	.55444	.55992	.57577
140	.62507	.57517	.55708	.50808	.52461	.59539	.60020	.60091	.61067
160	.63652	.59367	.56023	.51550	.52510	.61221	.62858	.63180	.63421
180	.63650	.60724	.54981	.52999	.52908	.61424	.63309	.63761	.63824
200	.62494	.61589	.52520	.54634	.53949	.60542	.61878	.62119	.62538
220	.60327	.61417	.48083	.55225	.55014	.58012	.58512	.58405	.60137

TABLE 4-155

LAMBDA 1 ROTS (5,4) STD. AA1C

		SUCCESSIVE SUBSTITUTIONS								
TIME	TRUE	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.90859	.92894	.92952	.90284	.89901	.94285	.90668	.90851	65.31	65.30
20	.92267	.93860	.93933	.91707	.91535	.95148	.92109	.92261	67.32	67.31
40	.93652	.94856	.94955	.93147	.93134	.95953	.93572	.93648	69.47	67.47
60	.94995	.95869	.95995	.94577	.94668	.96821	.95016	.94993	71.79	71.79
80	.96270	.96874	.97025	.95957	.96099	.97625	.96385	.96269	74.30	74.30
100	.97443	.97837	.97999	.97239	.97387	.98385	.97618	.97443	77.01	77.02
120	.98468	.98708	.98859	.98361	.98481	.99061	.98652	.98468	79.95	79.96
140	.99286	.99420	.99533	.99250	.99327	.99601	.99430	.99287	83.15	83.16
160	.99826	.99883	.99932	.99826	.99856	.99936	.99895	.99826	86.62	86.63
180	.99997	.99983	.99959	.99995	.99990	.99972	.99982	.99997	90.38	90.39
200	.99697	.99596	.99516	.99663	.99635	.99605	.99617	.99696	94.45	94.46
220	.98812	.98488	.98520	.98726	.98688	.98708	.98703	.98811	98.83	98.84
240	.97224	.96519	.96921	.97078	.97046	.97145	.97126	.97223	103.53	103.53
260	.94829	.93454	.94737	.94600	.94630	.94787	.94764	.94827	108.50	108.51
280	.91549	.89067	.92073	.91146	.91423	.91520	.91524	.91547	113.72	113.73
300	.87356	.83142	.89133	.86509	.87524	.87238	.87391	.87353	119.12	119.13
320	.82280	.75503	.86250	.80387	.83255	.81777	.82549	.82276	124.63	124.64
340	.76422	.65698	.83823	.72254	.79266	.74654	.77568	.76413	130.16	130.17
360	.69937	.50031	.81707	.56723	.76027	.60007	.73276	.67289	135.62	135.71

TABLE 4-156

LAMBDA 2 ROTS (5,4) STD. AA1C

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.417679	.376971	.375428	.430914	.438959	.333214	.421894	.443598	.417849
40	.350603	.320721	.317638	.364199	.364550	.280225	.352754	.365078	.350700
80	.270545	.350018	.243920	.281564	.276663	.216614	.266424	.275730	.270578
120	.174357	.160731	.151006	.180316	.173593	.136707	.163599	.172966	.174338
160	.058916	.048246	.036728	.058961	.053470	.035924	.045809	.053720	.058865
200	-.077688	-.092083	-.098317	-.082011	-.085271	-.088707	-.087430	-.083313	-.077745
240	-.233963	-.263822	-.248119	-.240006	-.241307	-.237230	-.237993	-.236982	-.234009
280	-.402325	-.467597	-.398189	-.412137	-.405876	-.403056	-.402973	-.402933	-.402368
320	-.568322	-.699956	-.524397	-.599891	-.557441	-.576419	-.565175	-.570889	-.568386
360	-.714753	-.999687	-.604855	-.855386	-.657759	-.810347	-.683490	-.788559	-.739741

TABLE 4-157

LAMBDA 3 ROTS (5,4) STD. AA1C

TIME	TRUE	FIRST	SECOND	SUCCESSIVE SUBSTITUTIONS THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.13846	.11564	.10640	.14332	.14494	.09122	.14127	.14979	.14286
40	.15426	.13042	.12170	.16138	.15922	.10922	.15644	.16209	.15750
80	.17193	.14745	.13991	.18015	.17429	.13231	.17008	.17583	.17327
120	.19152	.16710	.16123	.19838	.19051	.15945	.18352	.19139	.19042
160	.21280	.18977	.19541	.31471	.120799	.18944	.20001	.20857	.20935
200	.23485	.21583	.21122	.22853	.22571	.22057	.22220	.22788	.23035
240	.25582	.24550	.23569	.24139	.24152	.24912	.24851	.25008	.25253
280	.27282	.27897	.25338	.25723	.25381	.27038	.27105	.27117	.27185
320	.28275	.31605	.25526	.27886	.26240	.28241	.28084	.28202	.28266
360	.28383	COMPUTER TRUNCATION ERROR							

TABLE 4-158

LAMBDA 4 ROTS (5,4) STD. AA1C

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.25000	.17416	.14875	.26404	.26589	.11303	.25509	.28118	.26573
40	.26603	.19566	.17507	.28224	.37732	.15046	.26963	.28554	.27599
80	.28454	.22063	.20537	.30372	.28948	.19354	.37933	.29344	.28814
120	.30552	.24935	.23928	.31987	.30341	.23859	.28863	.30537	.30290
160	.32839	.28198	.37553	.33232	.31948	.28428	.30425	.32059	.32112
200	.35171	.31835	.31139	.34084	.33602	.32703	.32988	.33977	.34318
240	.37268	.35771	.34186	.34957	.34986	.36208	.36108	.36358	.36708
280	.38732	.39871	.35944	.36443	.35911	.38383	.38473	.38491	.38583
320	.39157	.43889	.35393	.38667	.36360	.39129	.38898	.39066	.39147
360	.38354	COMPUTER TRUNCATION ERROR							

TABLE 4-159

LAMBDA 1 ROTS (5,4) STD. AA24C

TIME	TRUE	SUCCESSIVE SUBSTITUTIONS SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH	CHI_{nom}	CHI_{nr}
0	.86585	.88975	.88426	.84517	.84415	.94294	.86856	.86564	59.98	59.96
20	.88190	.90058	.89553	.86287	.86504	.94958	.97916	.88174	61.87	61.85
40	.89828	.91236	.90806	.88170	.88626	.95590	.89205	.89816	63.93	63.92
60	.91487	.92506	.92178	.90138	.90739	.96227	.90765	.91480	66.18	66.18
80	.93149	.93852	.93645	.92149	.92793	.96898	.92562	.92145	68.66	68.66
100	.94782	.95245	.95164	.94138	.94728	.97612	.94475	.94781	71.40	71.41
120	.96340	.96637	.96667	.96109	.96482	.98349	.96331	.96341	74.45	74.45
140	.97755	.97949	.98050	.97684	.97982	.99052	.97948	.97757	77.83	77.84
160	.98931	.99064	.99177	.98999	.99142	.99631	.99173	.98933	81.61	81.63
180	.99736	.99813	.99812	.99814	.99850	.99966	.99876	.99737	85.83	85.85
200	.99995	.99967	.99938	.99965	.99968	.99906	.99943	.99995	90.53	90.55
220	.99497	.99228	.99184	.99277	.99276	.99250	.99264	.99495	95.74	95.76
240	.98002	.97239	.97479	.97569	.97595	.97742	.97698	.97999	101.47	101.48
260	.95278	.93604	.94818	.94639	.94732	.95093	.95038	.95274	107.67	107.68
280	.91153	.87943	.91396	.90236	.90643	.91038	.91045	.91148	114.28	114.29
300	.85576	.79948	.87634	.83994	.85561	.85352	.85645	.85569	121.15	121.16
320	.78663	.69455	.84148	.75362	.80227	.77660	.79302	.78652	128.12	128.14
340	.70694	.56444	.81624	.63459	.76069	.66799	.73690	.70677	135.01	135.03
360	.62061	.40855	.80664	.46701	.75237	.49139	.73032	.62006	141.63	141.68

TABLE 4-160

LAMBDA 2 ROTS (5,4) STD. AA24C

					SUCCESSIVE SUBSTITUTIONS				
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.500288	.469561	.481106	.537399	.539059	.332966	.495881	.561884	.500653
40	.439417	.418649	.428797	.473335	.464563	.293675	.452078	.476128	.439651
80	.363760	.350655	.356499	.388956	.373211	.247118	.378494	.371067	.363860
120	.268046	.259313	.258183	.279428	.262976	.180925	.268387	.256616	.268020
160	.145765	.136775	.128272	.141113	.130696	.085775	.128324	.127543	.145636
200	-.009380	-.025665	-.035204	-.026418	-.028855	-.043318	-.033521	-.027455	-.009557
240	-.198888	-.234970	-.224529	-.219176	-.218012	-.211281	-.213303	-.210459	-.199036
280	-.411226	-.491042	-.414807	-.431942	-.423244	-.413843	-.413690	-.413355	-.411324
320	-.617421	-.781590	-.563051	-.666000	-.602143	-.631696	-.610532	-.622634	-.617559
360	-.784113	-1.087608	-.621858	-.935252	-.667540	-.892106	-.686204	-.874704	-.784550

TABLE 4-161

LAMBDA 3 ROTS (5,4) STD. AA24C

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.15157	.13264	.12071	.16175	.16155	.06069	.14680	.17903	.16116
40	.17077	.15115	.13939	.18419	.17821	.08056	.17680	.19186	.17884
80	.19319	.17284	.16227	.20801	.19574	.11098	.20387	.20065	.19798
120	.21936	.19823	.18988	.23115	.21522	.14922	.22240	.21419	.21899
160	.24945	.22767	.22208	.25066	.23788	.19319	.23620	.23630	.24265
200	.28397	.26104	.25687	.26453	.26230	.24233	.25555	.26373	.27033
240	.31464	.29732	.28870	.27589	.28265	.29425	.29169	.29605	.30340
280	.33933	.33456	.30684	.29369	.29430	.32971	.33301	.33348	.33602
320	.34873	.37087	.29664	.32082	.30294	.34219	.34679	.34783	.34896
360	.34024	.38332	.22050	.32820	.29584	.32137	.29794	.32711	.33370

TABLE 4-162

LAMBDA 4 ROTS (5,4) STD. AA24C

		SUCCESSIVE SUBSTITUTIONS							
TIME	TRUE	FIRST	SECOND	THIRD	FOURTH	FIFTH	SIXTH	SEVENTH	NEW-RAPH
0	.27059	.20069	.16469	.29145	.29402	.00919	.24740	.34586	.30485
40	.29268	.22849	.19793	.32210	.30918	.06286	.30127	.34647	.31749
80	.31945	.27130	.23756	.35164	.32466	.13183	.34016	.33807	.33232
120	.35151	.29966	.28365	.27629	.34329	.20689	.36652	.34215	.35062
160	.38883	.34366	.33478	.39220	.36773	.28335	.36406	.36494	.37445
200	.42952	.39241	.38655	.39926	.39552	.36255	.38374	.39796	.40668
240	.46809	.44322	.42952	.40633	.40756	.43608	.43193	.43888	.44892
280	.49457	.49137	.44863	.42744	.42815	.48074	.48537	.48604	.48948
320	.48934	.53156	.42714	.46088	.43554	.48973	.49567	.49720	.49870
360	.47669	.53230	.32520	.46230	.42087	.45342	.42347	.44798	.46851

Table 4-163 COMPARISON OF λ_1 SOLUTIONS FOR DIFFERENT ORDERS OF ROTS EQUATIONS WITH CONSTANT ΔR AND VARIABLE ΔT

ΔR = 19.78 KM

STD. NUM.	ROTS(3,3)	ROTS (4,3)	ROTS (5,3)	ROTS (4,4)	ROTS (5,4)	MAX ΔT	THRUST
AA24	-6.89%	1.30%	.06%	-.06%	.00%	178.70	200 K lb
AA24A	-5.96%	1.55%	.09%	-.04%	.00%	203.60	175 K lb
AA24B	-5.25%	2.07%	.08%	-.03%	.00%	237.98	150 K lb
AA24C	-3.85%	7.34%	.09%	-.01%	.02%	373.74	100 K lb

Table 4-164 EFFECT OF STEERING ANGLE ERRORS ON INJECTION ACCURACY

NOMINAL CASE	SUBSTITUTION STEP FOR X	Δx (m)	Δy (m)	Δu (m/sec)	Δv (m/sec)
AA1	THIRD	1.1	-26.0	-1.48	4.86
AA1	FOURTH	187.4	-116.2	3.26	2.45
AA1	FIFTH	230.0	-137.6	2.86	2.80
AA1	SIXTH	207.0	-126.9	3.18	2.74
AA1C	THIRD	-1391.6	1045.6	-12.29	16.16
AA1C	FOURTH	543.3	-447.7	4.45	-.14
AA1C	FIFTH	859.8	-693.5	-.65	3.44
AA1C	SIXTH	1174.5	-935.9	3.48	-.31
AA24	THIRD	-314.6	90.0	-7.99	8.83
AA24	FOURTH	396.6	-259.9	5.31	1.07
AA24	FIFTH	480.4	-305.6	4.49	1.91
AA24	SIXTH	347.0	-247.7	5.31	2.29
AA24C	THIRD	-2218.1	1699.4	-24.04	29.13
AA24C	FOURTH	1467.5	-1199.2	.38	-5.24
AA24C	FIFTH	1443.0	-1187.3	-5.30	8.06
AA24C	SIXTH	2557.4	-2062.9	7.99	-4.53

Steering angle computed from ROTS (4,3) equations.

SECTION V

SOLUTION FOR TIME-TO-CUTOFF FUNCTION

5.1 INTRODUCTION

In this section the results of an analysis of the time-to-cutoff function are discussed. The analysis is based on the use of true or nominal values for the multipliers. As previously stated, the guidance problem was divided into two parts, the solution for the multipliers based on true Δt and solutions for Δt based on true multiplier values. Each part was analyzed somewhat independently of the other.

The Δt studies are particularly critical. The entire guidance problem solution hinges on obtaining an accurate representation for Δt. The accuracy of the steering angle obtained from the ROTS equations, or some analagous system of equations, depends on Δt. Ultimately, the mass loss depends on Δt, and its accuracy determines the degree of optimality achievable. A more immediate and bothersome problem is the complexity of the resulting formulas after a Δt expression is substituted into the ROTS equations.

5.2 CHOICE OF SERIES FOR Δt

Recall from Section III that the three functions chosen for the circular orbit terminal condition were expanded in Taylor series about the unknown Δt interval. An explicit solution for Δt was then obtained by reverting the "Velocity" series, equation (22).

Either of the other two series, equations (21) or (23), could have been used to solve for Δt. However, as explained in Section IV, the systems of algebraic equations in the multipliers which result from these two choices, and which must be solved for the multipliers, do not generally have satisfactory solutions. Moreover, use of the "Radius" or "Orthogonality" series for computing time-to-cutoff is not always satisfactory. Reversion of the "Radius" and "Orthogonality" series yields a term analogous to "Z" in the velocity series.

For the "Radius" series:

$$\phi = \frac{R^2_{co} - (x^2 + y^2)_o}{2(xu + yv)_o}$$

For the "Orthogonality" series:

$$\Omega = \frac{-(xu + yv)_o}{2(u\dot{u} + v\dot{v})_o} = \frac{-(RV \cos \theta)_o}{2(u\dot{u} + v\dot{v})_o}$$

where θ is the flight path-angle. Both of these terms can cause problems. In the case of ϕ, if the initial radius exceeds the cutoff radius before other orbit conditions are met, ϕ becomes zero and so does Δt. In some cases θ may pass through 90 degrees before orbit is achieved. For an optimal flight velocity should be increasing until cutoff and no singularity problems encountered.

5.3 REVERSION OF THE VELOCITY SERIES

The reversion or inversion of the velocity series, as noted in Section III, is discussed here in some detail. Equations (22) in expanded form are:

$$(u^2 + v^2 - V^2_{co})_o + 2(u\dot{u} + v\dot{v})(\Delta t) + (u\ddot{u} + v\ddot{v} + \dot{u}^2 + \dot{v}^2)(\Delta t)^2 + \ldots = 0$$

or
$$\frac{V^2_{co} - (u^2 + v^2)}{2(u\dot{u} + v\dot{v})} = Z = \Delta t + \frac{(u\ddot{u} + v\ddot{v} + \dot{u}^2 + \dot{v}^2)}{2(u\dot{u} + v\dot{v})}(\Delta t)^2 + \ldots$$

The series to be reverted may be written as,

$$Z = A_1(\Delta t) + A_2(\Delta t)^2 + \ldots + A_i(\Delta t)^i + \ldots \tag{31}$$

and the reverted series written as,

$$\Delta t = B_1 Z + B_2 Z^2 + \ldots + B_j Z^j + \ldots \tag{32}$$

where the B_j are in terms of the A_i. The coefficients can be obtained by several methods and are tabulated in numerous mathematical tables and handbooks. A discussion of series reversion is found in reference 8. Table 5-1 lists the B_j in terms of the u and v time derivatives for the reversion of a sixth-order series to a sixth-order series. Series of lesser orders can be derived from these formulas.

One problem that arises when the velocity series is reverted is the term in the denominator of Z.

$$\begin{aligned} \delta &= 2(u\dot{u} + v\dot{v}) \\ &= 2u \frac{F}{m} \lambda_1 + 2v \frac{F}{m} \lambda_2 + 2uV_x + 2uV_y \\ &= K_o + K_1\lambda_1 + K_2\lambda_2 \end{aligned}$$

The problem is that two multipliers occur in the denominator. They can be put into the numerator by expanding δ^{-1} into a series, but for the values encountered for K_o, K_1, and K_2 the series diverges. Because of this problem the "Z" term in the time-to-cutoff expression was left as shown above. It was assumed that the quantities in the denominator could all be measured at any initial time. The λ_1 and λ_2 which occur there would be the current values and would not be directly involved in the solution for the optimal steering function.

The reversion (or inversion) of a series representation for a function is actually a method for expressing the inverse of that function. It is equivalent to finding a zero of the original function. If the original series expansion is about the origin, the inverse series converges to the smallest real root of the function. If the function has no real roots, the series will, of course, not converge to real values. (Complex zeros can be computed, but the algorithm is very cumbersome and of no use in the guidance problem anyway.) There are no convergence problems with multiple real roots, provided enough terms are taken in the inverse series.

Equation (31) is a finite series and can be regarded as a polynomial in the unknown variable, Δt. Inversion of the polynomial, which is its own series, should produce the desired value for Δt if the original series is sufficiently accurate. A study was made to determine what orders of series are necessary to ensure that an accurate root (corresponding to the actual Δt) would be implicitly defined by equation (31).

The A_i in equation (31) were evaluated with nominal data from a number of trajectories. The resulting polynomials, of orders three through six, were then solved for all of their roots by the Bairstowe method. In nearly all cases there was one small, real positive root; the others were either complex or very large. These smallest roots were usually close to nominal Δt values, and are referred to as "actual roots".

After the actual roots of the polynomials were obtained, the roots obtained from reversions of these polynomials were evaluated and compared to the actual roots. The orders of the reverted series used (order with respect to Z) ranged from the order of the original series up through the sixth order. The notation, "I to J" indicates a series of order I reverted to one of order J.

These calculations were done for all nominal cases described in Table 4-2. Representative results are shown in the remainder of this section.

5.4 ITERATED REVERSION FOR Δt

From Table 5-6, it can be seen that a polynomial or series of relatively low order can have an acceptable actual root. The third-order series in Δt, for example, has roots which are close to actual Δt values and not too much more accuracy is obtained by taking roots of higher order series. Yet, when these series are reverted to orders three, four, five, or six, the values obtained are unacceptable. Had more terms been taken, the value obtained for Δt would have been much more accurate. However, a large number of terms will lead to an undesirable amount of algebra and complicated formulas for the guidance function. Because of this problem an investigation was made to find a simpler way to derive the expression for Δt. A possibility considered was an iteration of the series reversion.

As noted in Section 5.3, the reversion of a finite series is equivalent to finding a root of that series. If an estimate of the root is available, the reverted series may be made to converge more rapidly. Suppose equation (31) is written as

$$Z = A_1(\tau + dt) + A_2(\tau + dt)^2 + \ldots \tag{33}$$

where τ is an estimate for Δt and dt is an unknown correction such that $\tau + dt = \Delta t$. Equation (33) can be rewritten as

$$Z = (A_1\tau + A_2\tau^2 + A_3\tau^3 + \ldots) + (A_1 + 2A_2\tau + 3A_3\tau^3 + \ldots)(dt)$$

$$+ (A_3 + 3A_2\tau + \ldots)(dt)^2 + \ldots$$

or $$Z' = A'_1(dt) + A'_2(dt)^2 + \ldots \tag{34}$$

Equation (34) may be reverted for dt which is added to τ to obtain Δt. In some cases this procedure may involve less complicated formulas than the usual reversion, especially if a large number of terms are required in the reverted series.

It was assumed that the estimate, τ, would somehow be available. During flight, if Δt is computed along with the steering angle, the estimate would be obtained from a previous value. The estimate might also be made analytically. A simple and obvious estimate would be $\tau = Z$. Whatever estimate is used, it should be sufficiently accurate to determine Δt after only one iteration, because the expression for Δt must be substituted into the ROTS equations to be solved for the steering function.

This approach was tested using Z for the estimate. For most of the cases tested it was not worth the effort because of the complexity of equation (34) and its reversion. However, for a case involving a flight time of 300 seconds and a large ΔR such as shown in Table 5-6, the approach did seem to have merit. There is probably less complication in using the "iterative" approach than extending the reverted series past the sixth order for a certain class of missions. Further investigation of this topic would be desirable.

5.5 ERROR ANALYSIS

Studies of the errors in the Δt expression were carried out to determine what orders of series should be used for substitution into the ROTS equations. Specific questions considered were: (1) Does the Δt accuracy depend on ΔR as the χ accuracy seems to? (2) What effect does a Δt error have on the accuracy of the guidance function? (3) How does the Δt error affect the accuracy of the guidance function with respect to accuracy at the target orbit?

A number of nominal cases were tested for comparison against analytically predicted results in much the same way as done with the steering angle studies. Figures 5-1 through 5-4 show time histories of the Δt errors for one nominal case, and are representative of the pattern found on other cases. For longer duration flights and larger ΔR values, the behavior is the same, but the magnitudes tend to increase. This is shown in Figure 5-5 where the percent errors of actual roots at the beginning of flight are plotted against ΔR. There is a tendency for all orders of series to have increasingly inaccurate roots for Δt. No explanation has been found for the apparent increase in accuracy for the third-order series. Additional cases were observed which were identical to those shown in Figure 5-5 except for lower thrust levels to obtain longer flight times. For these cases the behavior was the same and errors were only slightly changed. Based on these results, it appears that ΔR is an indication of the order of series needed for Δt for a given mission; rather than the expected Δt.

To determine the sensitivity (to Δt) of the steering angle computation, perturbed nominal values of Δt were used in the ROTS equations. The behavior of the Δt error was crudely simulated by perturbing the initial (ignition) Δt value 2 percent and 5 percent true values and then linearly decreasing the error to zero at cutoff (see Figure 5-1). The perturbed Δt values were then used in the ROTS equations, and values for multipliers were computed. The effect of the Δt error on the solution for the multipliers was then observed. Figure 5-6 shows the effect on λ_1 for one case and is representative of what was found. It should be noted that the deliverate error in Δt is far worse than actual error. Based on these results, it was decided that no more than a 5 percent error in Δt would be acceptable.

The next step in analyzing the sensitivity of the steering angle solution to errors in time-to-cutoff was to stimulate the flight of a vehicle with a simulated linear error in Δt. This was done to determine the effect at the target orbit of an initial error in Δt combined with the error in the steering angle solution. The solution of the ROTS (4,3) equations was used to simulate flights from ignition to orbit for two cases. first with true Δt and then with a perturbed true Δt of -5 percent at ignition to 0 percent at cutoff. Coordinates at cutoff were then compared with each other and with nominal values. Table 5-7 indicates the increase in error due to the simulated error in Δt.

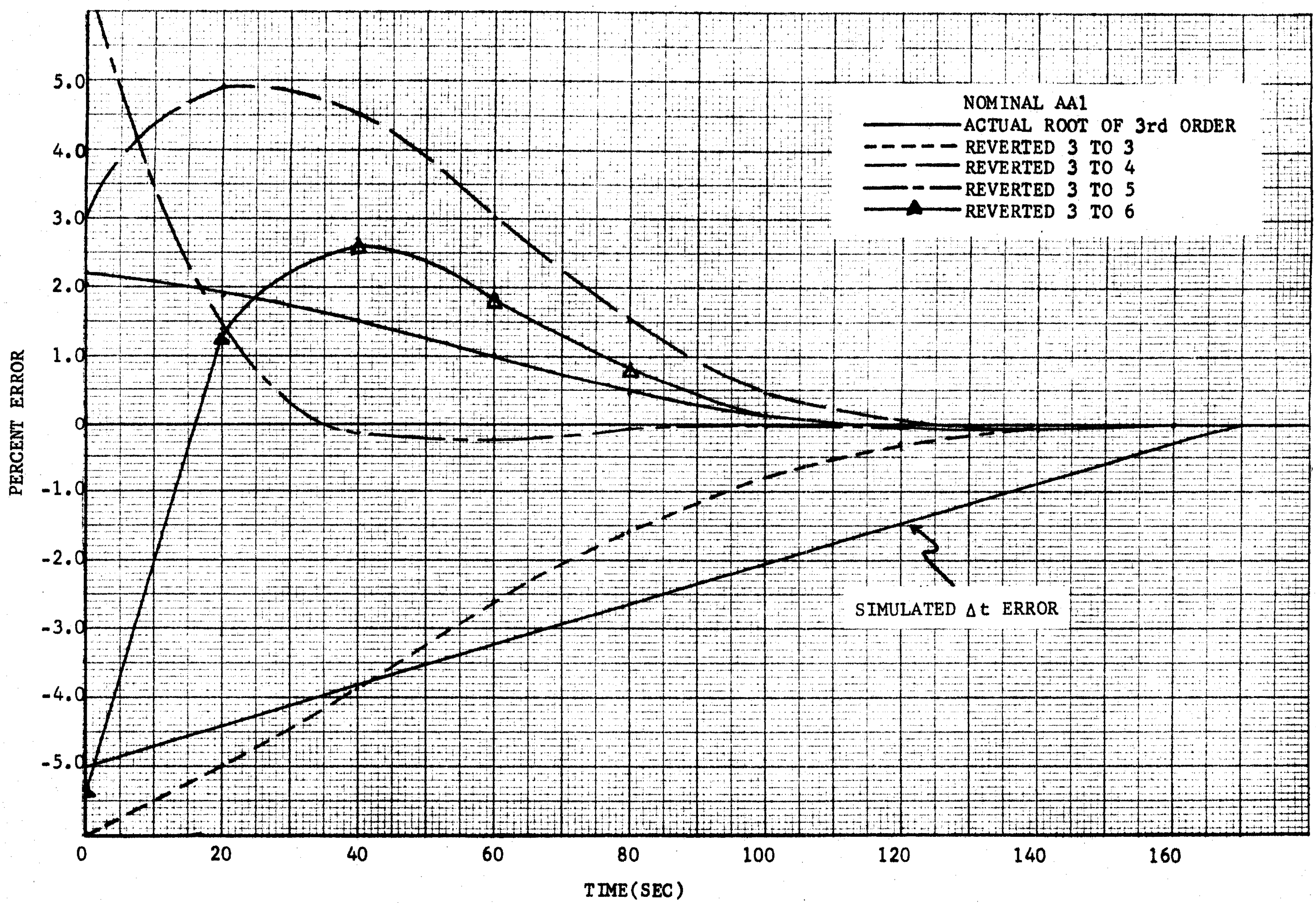

Figure 5-1. PERCENT ERROR OF ACTUAL ROOTS OF 3rd ORDER SERIES AND INDICATED REVERSION

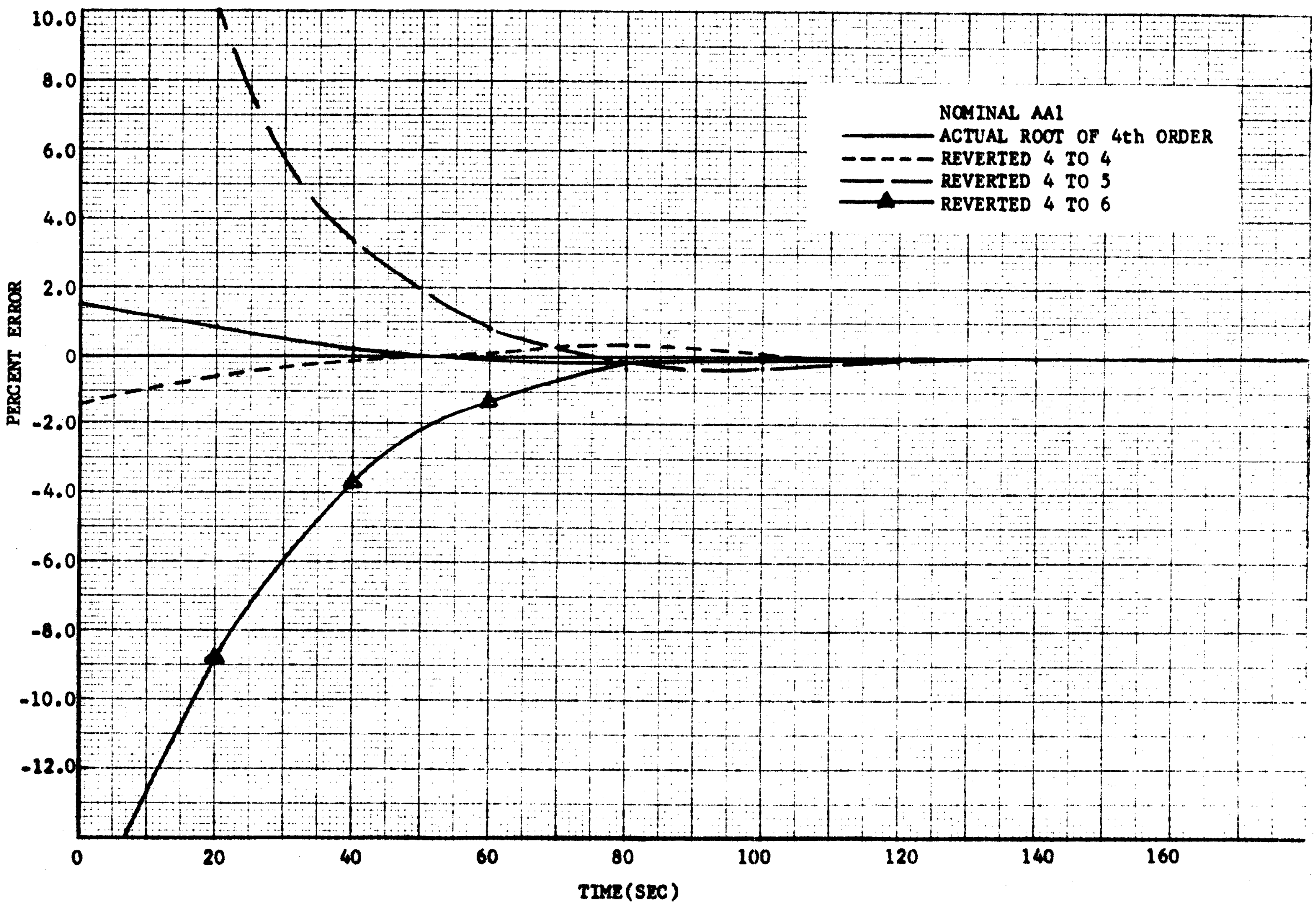

Figure 5-2. PERCENT ERROR OF ACTUAL ROOTS OF 4th ORDER SERIES AND INDICATED REVERSION

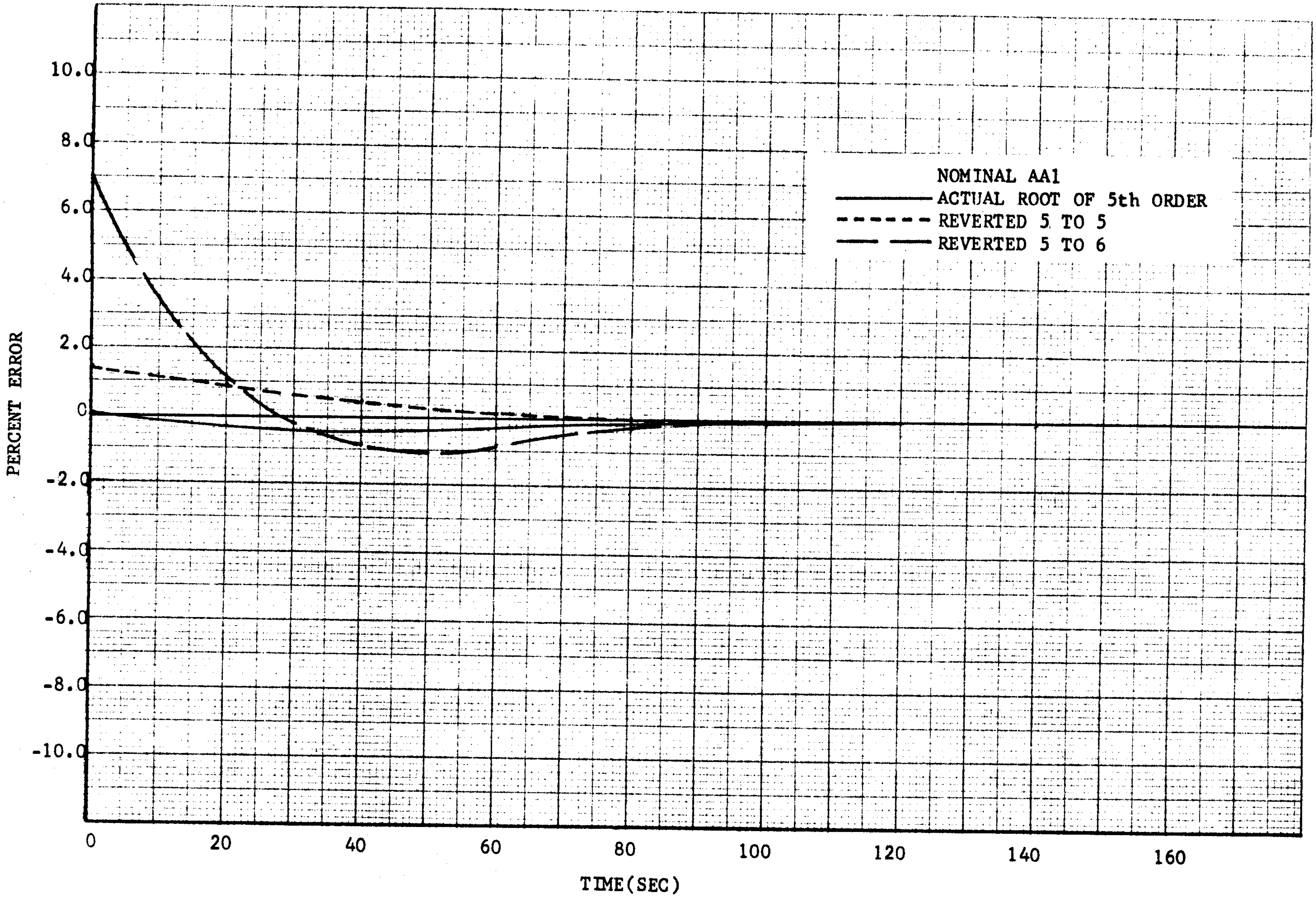

Figure 5-3. PERCENT ERROR OF ACTUAL ROOTS OF 5th ORDER SERIES AND INDICATED REVERSION

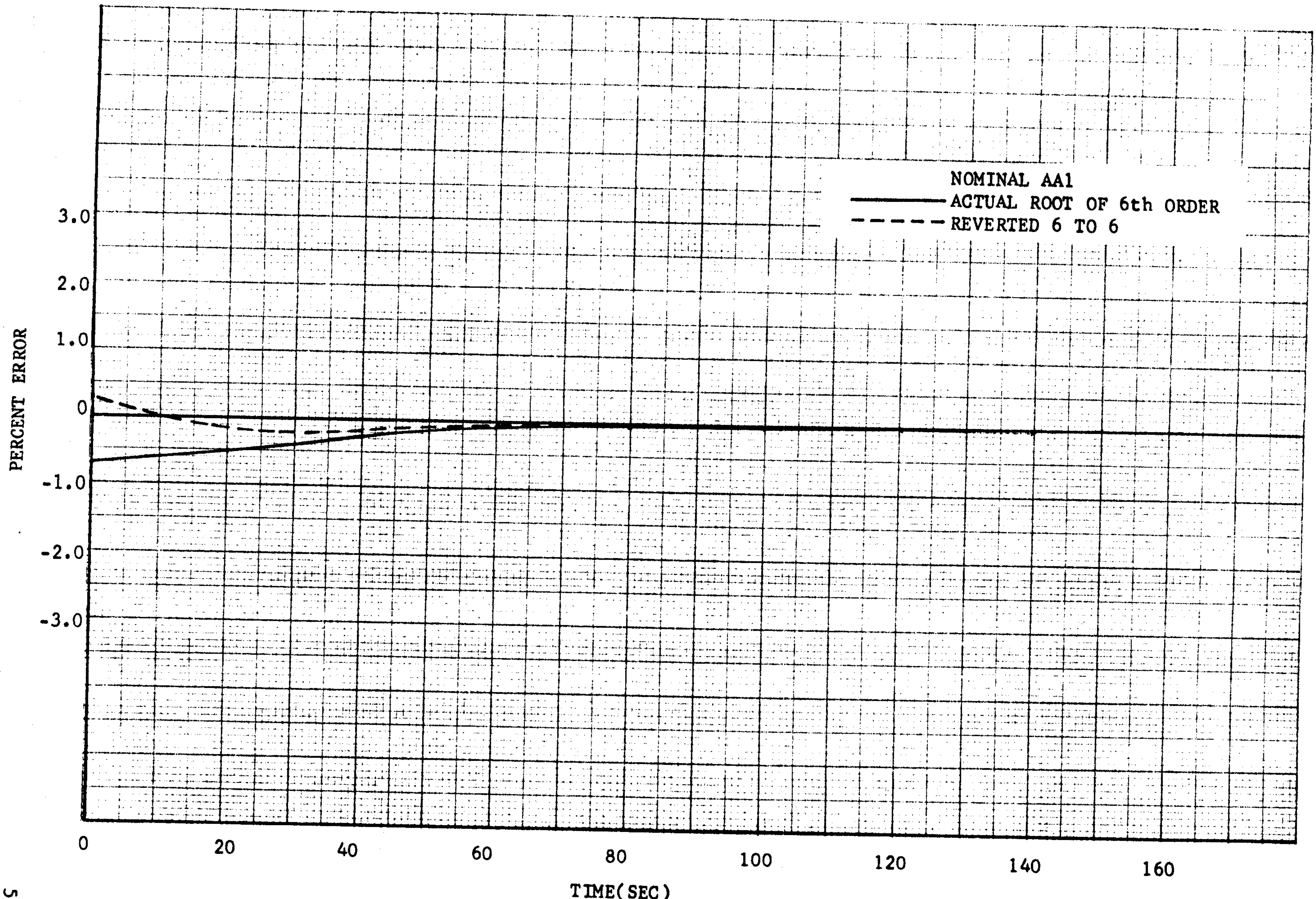

Figure 5-4. PERCENT ERROR OF ACTUAL ROOTS OF 6th ORDER SERIES AND INDICATED REVERSION

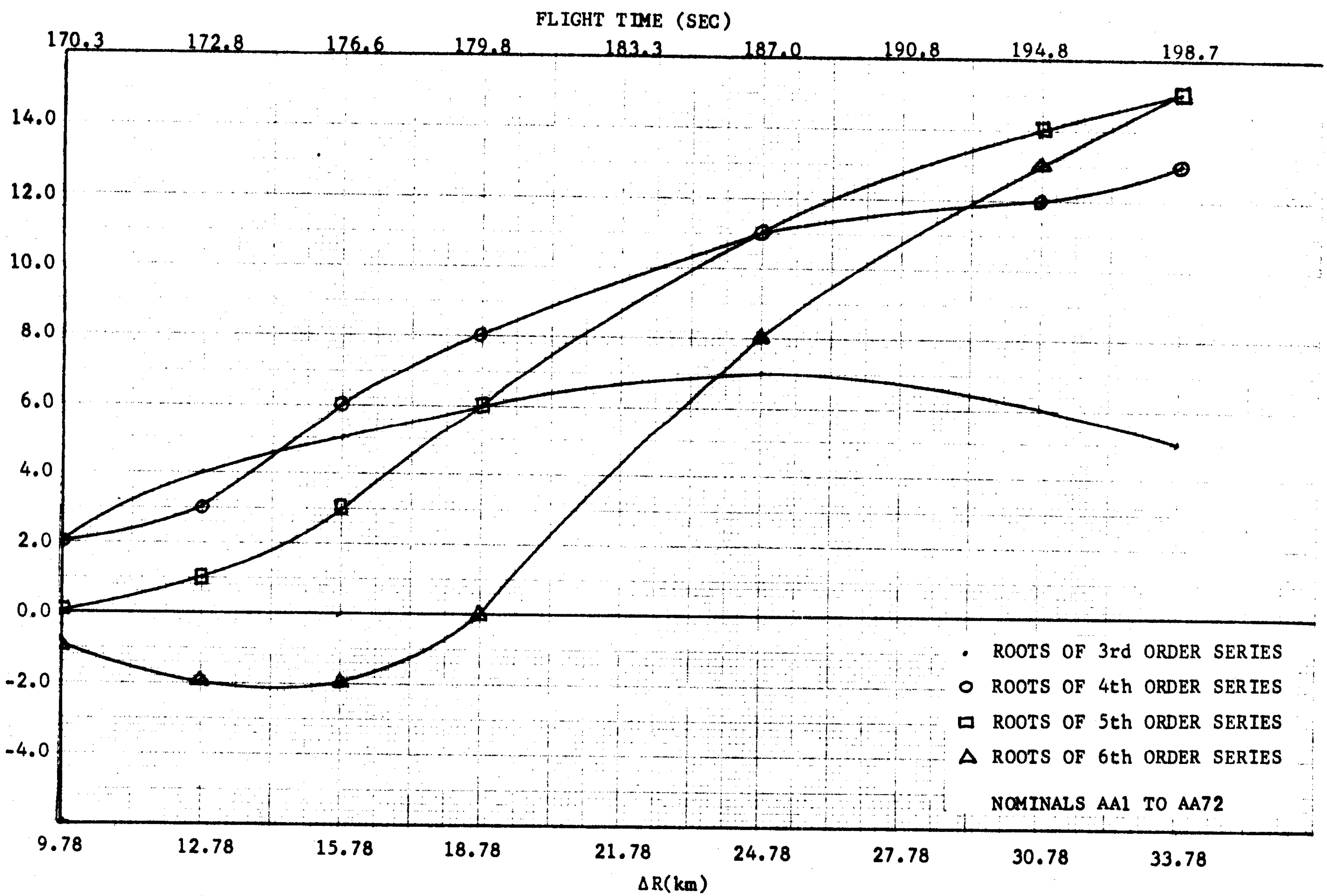

Figure 5-5. PERCENT ERROR OF "ACTUAL" Δt ROOTS vs. ΔR AND TRAJECTORY FLIGHT TIME. ERRORS ARE AT BEGINNING OF FLIGHT.

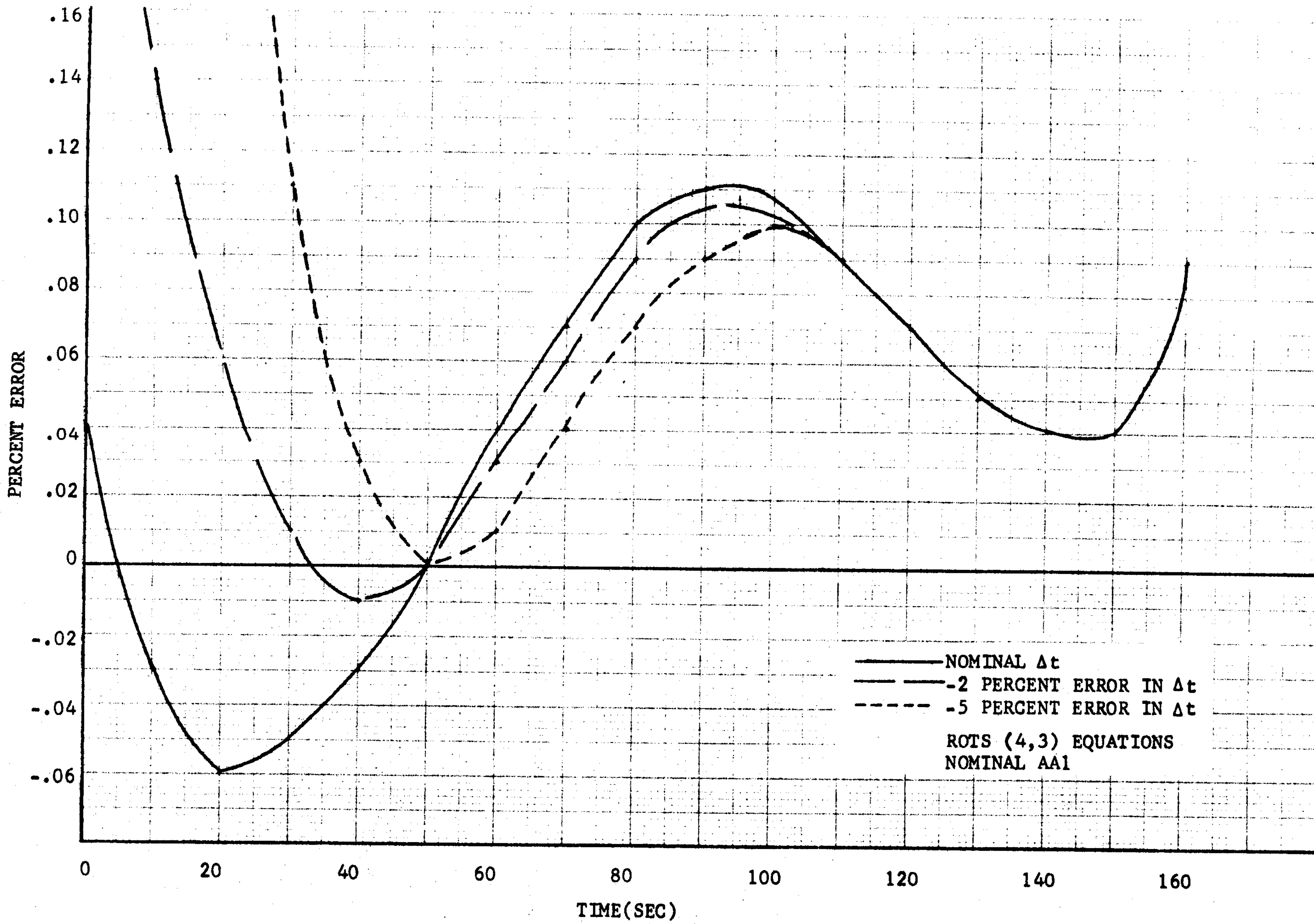

Figure 5-6. EFFECT OF PERTURBED Δt ON ACCURACY OF λ_1

Table 5-1. COEFFICIENTS OF REVERTED VELOCITY SERIES

$$\Delta t = \sum B_n (Z)^n$$

$$B_1 = 1$$

$$B_2 = -\frac{1}{\delta}\left(u\ddot{u} + v\ddot{v} + \dot{u}^2 + \dot{v}^2\right)$$

$$B_3 = \frac{2}{\delta^2}\left(u\ddot{u} + v\ddot{v} + \dot{u}^2 + \dot{v}^2\right)^2 - \frac{1}{\delta}\left(\frac{1}{3}u\dddot{u} + \frac{1}{3}v\dddot{v} + \dot{u}\ddot{u} + \dot{v}\ddot{v}\right)$$

$$B_4 = -\frac{5}{\delta^3}\left(u\ddot{u} + v\ddot{v} + \dot{u}^2 + \dot{v}^2\right)^3 + \frac{5}{\delta^2}\left(u\ddot{u} + v\ddot{v} + \dot{u}^2 + \dot{v}^2\right)\left(\frac{1}{3}u\dddot{u} + \frac{1}{3}v\dddot{v} + \dot{u}\ddot{u} + \dot{v}\ddot{v}\right) - \frac{1}{\delta}\left(\frac{1}{12}u\overset{IV}{u} + \frac{1}{12}v\overset{IV}{v} + \frac{1}{3}\dot{u}\dddot{u} + \frac{1}{3}\dot{v}\dddot{v} + \frac{1}{4}\ddot{u}^2 + \frac{1}{4}\ddot{v}^2\right)$$

$$B_5 = \frac{14}{\delta^4}\left(u\ddot{u} + v\ddot{v} + \dot{u}^2 + \dot{v}^2\right)^4 - \frac{21}{\delta^3}\left(u\ddot{u} + v\ddot{v} + \dot{u}^2 + \dot{v}^2\right)^2\left(\frac{1}{3}u\dddot{u} + \frac{1}{3}v\dddot{v} + \dot{u}\ddot{u} + \dot{v}\ddot{v}\right) + \frac{6}{\delta^2}\left(u\ddot{u} + v\ddot{v} + \dot{u}^2 + \dot{v}^2\right)\left(\frac{1}{12}u\overset{IV}{u} + \frac{1}{12}v\overset{IV}{v} + \frac{1}{3}\dot{u}\dddot{u} + \frac{1}{3}\dot{v}\dddot{v} + \frac{1}{4}\ddot{u}^2 + \frac{1}{4}\ddot{v}^2\right) + \frac{3}{\delta^2}\left(\frac{1}{3}u\dddot{u} + \frac{1}{3}v\dddot{v} + \dot{u}\ddot{u} + \dot{v}\ddot{v}\right)^2 - \frac{1}{\delta}\left(\frac{1}{60}u\overset{V}{u} + \frac{1}{60}v\overset{V}{v} + \frac{1}{12}\dot{u}\overset{IV}{u} + \frac{1}{12}\dot{v}\overset{IV}{v} + \frac{1}{6}\ddot{u}\dddot{u} + \frac{1}{6}\ddot{v}\dddot{v}\right)$$

where,

$$\delta = 2(u\dot{u} + v\dot{v})$$

$$Z = \frac{V_{co}^2 - (u^2 + v^2)}{\delta}$$

Table 5-2 COMPARISON OF ACTUAL ROOTS AND ROOTS OBTAINED BY REVERSION

TRUE Δt	ACTUAL 3rd*	3 to 3	3 to 4	3 to 5	3 to 6
170.34	166.57	180.90	165.33	158.65	179.56
150.34	147.32	158.03	142.85	148.16	148.46
130.34	128.30	135.38	124.40	130.54	127.00
110.34	109.24	113.24	107.02	110.58	108.39
90.34	89.24	91.75	89.03	90.39	89.66
70.34	70.26	70.89	70.02	70.36	70.22
50.34	50.36	50.50	50.32	50.37	50.35
30.34	30.36	30.37	30.36	30.36	30.36
10.34	10.34	10.35	10.35	10.35	10.35

*Actual "nth" is the smallest real root of the nth order velocity series. Data from Nominal AA1.

Table 5-2 (Concluded)

TRUE Δt	ACTUAL 6th*	ACTUAL 5th*	ACTUAL 4th*	6 to 6	5 to 5	5 to 6	4 to 4	4 to 5	4 to 6
170.34	171.55	170.33	167.74	169.90	167.95	158.51	172.72	147.71	196.84
150.34	151.15	150.89	149.08	150.57	148.94	148.79	151.24	138.84	163.56
130.34	130.71	130.91	130.00	130.55	129.75	131.49	130.50	126.09	135.21
110.34	110.45	110.68	110.44	110.44	110.17	111.26	110.27	109.44	111.68
90.34	90.37	90.48	90.51	89.92	90.33	90.68	90.27	90.39	90.67
70.34	70.36	70.39	70.44	70.37	70.36	70.42	70.32	70.45	70.44
50.34	50.36	50.36	50.38	50.36	50.36	50.36	50.35	50.38	50.38
30.34	30.35	30.35	30.36	30.35	30.35	30.35	30.35	30.36	30.36
10.34	10.35	10.35	10.35	10.35	10.35	10.35	10.35	10.35	10.35

*Actual "nth" is the smallest real root of the nth order velocity series. Data from Nominal AA1.

Table 5-3 COMPARISON OF ACTUAL ROOTS AND ROOTS OBTAINED BY REVERSION

TRUE Δt	ACTUAL 3rd*	3 to 3	3 to 4	3 to 5	3 to 6
195.26	191.72	206.22	193.55	178.89	209.67
175.26	172.12	183.50	169.52	169.73	176.81
155.26	152.82	161.10	149.27	154.00	152.80
135.26	133.64	139.08	130.87	135.12	132.82
115.26	114.38	117.49	112.81	115.22	113.89
95.26	94.90	96.39	94.24	95.21	94.74
75.26	75.17	75.73	74.98	75.25	75.15
55.26	55.27	55.41	55.23	55.28	55.26
35.26	35.28	35.30	35.27	35.28	35.28
15.26	15.27	15.27	15.27	15.27	15.27

*Actual "nth" is the smallest real root of the nth order velocity series. Data from Nominal AA1A.

Table 5-3 (Concluded)

TRUE Δt	ACTUAL 6th*	ACTUAL 5th*	ACTUAL 4th*	6 to 6	5 to 5	5 to 6	4 to 4	4 to 5	4 to 6
195.26	196.21	194.70	192.19	192.28	195.35	176.86	196.62	174.34	216.64
175.26	176.08	175.43	173.41	174.39	174.57	169.47	176.20	161.88	188.97
155.26	155.78	155.71	154.40	155.12	154.69	154.75	155.69	148.49	161.98
135.26	135.50	135.67	135.05	135.30	134.98	136.04	135.37	132.72	137.82
115.26	115.34	115.51	115.34	115.31	115.17	115.85	115.24	114.69	116.05
95.26	95.29	95.37	95.38	95.30	95.26	95.50	95.23	95.29	95.49
75.26	75.28	75.30	75.34	75.29	75.28	75.33	75.26	75.35	75.34
55.26	55.28	55.28	55.30	55.28	55.28	55.28	55.27	55.30	55.29
35.26	35.27	35.27	35.27	35.27	35.27	35.27	35.27	35.28	35.28
15.26	15.27	15.27	15.27	15.27	15.27	15.27	15.27	15.27	15.27

*Actual "nth" is the smallest real root of the nth order velocity series. Data from Nominal AA1A.

Table 5-4 COMPARISON OF ACTUAL ROOTS AND ROOTS OBTAINED BY REVERSION

TRUE Δt	ACTUAL 3rd*	3 to 3	3 to 4	3 to 5	3 to 6
229.41	226.25	242.09	233.08	203.96	254.72
209.41	206.17	218.83	207.27	197.10	218.11
189.41	186.46	196.38	184.60	184.29	190.12
169.41	167.02	174.43	164.31	167.58	167.36
149.41	147.71	152.84	145.39	148.78	147.22
129.41	128.37	131.57	126.87	129.12	127.97
109.41	108.89	110.63	108.14	109.25	108.72
89.41	89.23	90.00	88.94	89.34	89.18
69.41	69.38	69.65	69.30	69.40	60.37
49.41	49.42	49.49	49.41	49.43	49.42
29.41	29.42	29.43	29.42	29.42	29.42
9.41	9.41	9.41	9.41	9.41	9.41

*Actual "nth" is the smallest real root of the nth order velocity series. Data from Nominal AA1B.

Table 5-4 (Concluded)

TRUE Δt	ACTUAL 6th*	ACTUAL 5th*	ACTUAL 4th*	6 to 6	5 to 5	5 to 6	4 to 4	4 to 5	4 to 6
229.41	229.84	227.75	225.39	220.85	235.99	196.02	226.97	213.40	240.49
209.41	210.11	208.76	206.49	205.58	211.61	193.32	209.16	194.65	221.76
189.41	190.07	189.42	187.54	187.89	189.84	183.33	189.80	178.92	198.32
169.41	169.87	169.72	168.43	168.91	169.31	168.10	169.79	163.39	174.36
149.41	149.66	149.76	149.05	149.30	149.26	149.72	149.62	146.69	151.62
129.41	129.52	129.66	129.39	129.42	129.34	129.89	129.49	128.54	130.23
109.41	109.46	109.55	109.50	109.44	109.40	109.68	109.44	109.29	109.70
89.41	89.44	89.47	89.50	89.44	89.43	89.51	89.42	89.47	89.52
69.41	69.44	69.44	69.46	69.44	69.43	69.45	69.43	69.46	69.46
49.41	49.43	49.43	49.44	49.43	49.43	49.43	49.43	49.44	49.44
29.41	29.42	29.43	29.42	29.42	29.42	29.42	29.42	29.42	29.42
9.41	9.41	9.41	9.41	9.41	9.41	9.41	9.41	9.41	9.41

*Actual "nth" is the smallest real root of the nth order velocity series. Data from Nominal AA1B.

Table 5-5 COMPARISON OF ACTUAL ROOTS AND ROOTS OBTAINED BY REVERSION

TRUE Δt	ACTUAL 3rd*	3 to 3	3 to 4	3 to 5	3 to 6
362.05	369.85	406.78	395.52	87.11	486.58
322.05	323.84	335.31	358.70	260.27	387.58
282.05	279.95	283.85	303.55	245.12	308.54
242.05	238.19	241.72	247.98	224.65	248.66
202.05	198.28	202.46	199.55	195.60	200.58
162.05	159.59	162.99	158.42	159.97	159.50
122.05	121.10	122.74	120.41	121.40	120.95
82.05	81.91	82.29	81.79	81.95	81.90
42.05	42.06	42.08	42.06	42.06	42.06
2.05	2.05	2.05	2.05	2.05	2.05

*Actual "nth" is the smallest real root of the nth order velocity series. Data from Nominal AA1C.

Table 5-5 (Concluded)

TRUE Δt	ACTUAL 6th*	ACTUAL 5th*	ACTUAL 4th*	6 to 6	5 to 5	5 to 6	4 to 4	4 to 5	4 to 6
362.05	353.82	351.79	355.74	249.16	495.30	205.78	250.99	568.77	26.48
322.05	316.46	313.78	314.80	294.71	374.21	240.73	280.11	388.81	210.45
282.05	279.65	276.55	275.34	272.91	301.70	231.74	270.82	289.20	253.46
242.05	241.95	239.48	237.07	237.91	248.08	217.03	241.89	231.02	240.66
202.05	202.72	201.63	199.37	200.54	203.14	194.70	203.90	192.55	205.11
162.05	162.44	162.40	161.26	161.75	162.06	161.61	163.11	158.54	163.45
122.05	122.14	122.26	122.07	122.05	122.03	122.36	122.35	121.63	122.37
82.05	82.08	82.10	88.12	82.08	82.07	82.11	82.10	82.11	82.13
42.05	42.06	42.06	42.07	42.06	42.06	42.06	42.06	42.07	42.07
2.05	2.05	2.05	2.05	2.05	2.05	2.05	2.05	2.05	2.05

*Actual "nth" is the smallest real root of the nth order velocity series. Data from Nominal AA1C.

Table 5-6 COMPARISON OF ACTUAL ROOTS AND ROOTS OBTAINED BY REVERSION

TRUE Δt	ACTUAL 3rd*	3 to 3	3 to 4	3 to 5	3 to 6
301.53	391.61	-44.23	-221.6	1984.	3107.
261.53	290.17	-739.5	955.4	7064.	-12833.
221.53	204.96	-937.66	4835.	747.6	-68330.
181.53	131.04	-185.8	4978.	-18487.	6549.
141.53	72.57	219.29	266.9	-2013.	8416.
101.53	-104.1	130.8	196.8	363.1	680.0
61.53	62.27	65.66	41.15	44.56	80.58
21.53	BAD DATA				

*Actual "nth" is the smallest real root of the nth order velocity series. Data from Nominal AA151.

Table 5-6 (Concluded)

TRUE Δt	ACTUAL 6th*	ACTUAL 5th*	ACTUAL 4th*	6 to 6	5 to 5	5 to 6	4 to 4	4 to 5	4 to 6
301.53	260.59	278.23	310.80	-476.07	-5693.	10499.	-2368.	-857.99	17162.
261.53	205.56	218.15	240.51	23989.	735.48	47194.	-2600.	9734.	37811.
221.53	153.20	161.14	175.29	5935.	7821.	40359.	107.64	20593.	-24315.
181.53	105.75	108.92	115.84	-8350.	27.38	6563.	1589.	7791.	-64976.
141.53	COMPLEX	-107.5	71.64	856.2	-194.5	-5124.	200.3	-1536.	6322.
101.53	COMPLEX	48.90	61.16	116.94	139.6	-93.80	136.2	203.1	91.56
61.53	49.82	66.38	72.75	80.86	69.73	80.18	54.56	76.12	96.67
21.53	BAD DATA								

*Actual "nth" is the smallest real root of the nth order velocity series. Data from Nominal AA151.

Table 5-7 EFFECT OF LINEAR ERROR IN Δt ON INJECTION ACCURACY

NOMINAL COORDINATES	SIMULATION WITH NOMINAL Δt	SIMULATION WITH PERTURBED Δt	ERROR DUE TO PERTURBED Δt
x = 2914294.3 m	2914324.6	2914501.2	+ 176.6 m
y = 5883487.8 m	5883471.8	5883383.4	- 88.4 m
u = 6975.4069 m/sec	6980.6663	6981.3395	+ .6732 m/sec
v = -3454.5500 m/sec	-3457.4427	-3457.2857	- .1570 m/sec

Steering Angle computed from ROTS (4,3) equations 6th Substitution Step. True Δt perturbed -5% at ignition. Nominal AA1.

SECTION VI

TERM ELIMINATION AND SIMPLIFICATION

6.1 SIMPLIFICATION OF THE ROTS EQUATIONS.

Due to the number and complexity of terms in the ROTS (5,4), and lower order equations, some simplification or elimination of terms is essential. The need becomes particularly evident when the analytical expression for Δt is to be substituted into the ROTS equations. The speed of computation of the guidance function must also be considered.

Unfortunately, there is no obvious way to proceed in eliminating terms from the equations. To a large extent, this elimination has been determined empirically. A significant amount of algebraic simplification can be done, and this was exploited as far as possible. The formulas for the $R_{ijk\ell}$ and $Q_{ijk\ell}$ coefficients appear to some sort of power series in Δt. (See Table 4-1). Some time was spent in an effort to derive a general expression for these coefficients, but it was not successful.

Because each coefficient appears to be a series in Δt, one would expect the higher order terms in them to be small, and such is the case. In particular, the high order derivatives of the gravity potential function contribute little to the coefficients' values and can be dropped without adverse effects. Potential derivatives of order four and greater were set to zero and virtually no change was observed in the solutions of the ROTS equations. This did not cause any significant simplification, and third-order derivatives were set to zero. This caused approximately twice as much error in the solution of the ROTS equations, and so the third derivatives were retained.

The next approach to simplification was to eliminate entire R and Q coefficients. The approach was as follows: In the equation

$$\sum R_{ijk\ell}\lambda^{i}_{1}\lambda^{j}_{2}\lambda^{\ell}_{4} = S \approx 0$$

the terms are summed from the left by increasing powers of the λ's. When a term is encountered which does not significantly alter the previous value of S, it is eliminated. Solutions to the ROTS equations are computed and compared to those obtained before the elimination. If they are close, the term is dropped; otherwise, it is retained. The same is done for the Q coefficients. When all terms to be dropped have been identified, they are simultaneously eliminated and the ROTs equations' solution observed. From the above procedure, it was found that about one fourth of the terms in the ROTS (4,3) equations could generally be eliminated without varying the error of the solution by more than ± 1 percent. Some of these terms cause larger errors when eliminated alone; but when eliminated in combination with others, a smaller error results. Both the Newton-Raphson and Successive Substitution solutions were examined. Three terms were noted which could be eliminated when Newton-Raphson was used, but not when Successive Substitution was used. These terms were retained. The reason for this discrepency is due to the alteration of the equations' structure upon which Successive Substitution is dependent. This topic was discussed in Section 4.6. The coefficients of those terms which were finally eliminated from the ROTS (4,3) equations are as follows:

R_{0200}, R_{2100}, R_{1200},

R_{0210}, R_{0120}, R_{0300},

R_{3000}, R_{1220}, Q_{2100},

Q_{2100}, Q_{0120} and Q_{1220}.

Four nominal cases were used to test formulas for term elimination; AA1, AA24, AA1C, and AA24C. These cases included flight times of 170.34 sec to 373.74 sec and altitude changes of 9.8 km to 18.8 km. Tables 6-1 and 6-2 show comparisons of the Newtow-Raphson and Successive Substitutions solutions for λ_1 and χ before and after eliminating the terms listed above.

It was also noted that terms which could be eliminated by this approach did not appear to depend on the flight time or altitude change.

6.2 SIMPLIFICATION OF THE U AND V TIME DERIVATIVES

In Section 3.2 an explanation was given for the algebra used to obtain the ROTS equations. One of the intermediate steps was to develop polynomial-type expressions for the u and v time derivatives such that the four multipliers would appear explicitly and have coefficients involving the state variables and vehicle parameters.

These expressions have the form,

$$U^{(n)} = \sum L_n \lambda^i{}_1 \lambda^j{}_2 \lambda^k{}_3 \lambda^\ell{}_4$$

$$L_n = L(x, y, u, v, m, \dot{m}, F)$$

Two approaches to simplification were employed. The first attempt was to eliminate numerically insignificant terms and then differentiate the result; repeating the process until the desired derivatives were obtained. Such a scheme was mechanized, using the FORMAC computer language to obtain the derivatives in a symbolic form. Only a few terms were found which could be eliminated, and this approach was discarded.

The second approach was to evaluate the u and v derivatives term-by-term with nominal data and then sort them by magnitude from largest to smallest. After this, the cross terms in products of the derivatives could possibly be eliminated. This, too, was not feasible because the magnitudes of the terms changed too unpredictably and ordering them by magnitude was not generally possible. No notable success was achieved in the simplification of the u and v time derivatives.

6.3 SIMPLIFICATION OF THE Δt EXPRESSION

The expression for Δt can be considered as a polynomial-type expression in terms of the multipliers. (See Table 7-1.) The same approach to term elimination as used for the u and v derivatives was applied to its simplification.

Because Δt must be raised to at least the fourth power, the number of terms involved becomes enormous. The expression for Δt shown in Table 7-1 was evaluated, term-by-term, with nominal data and was squared, cubed, etc. Those cross products which were small compared to the known product were dropped. A large number of terms were eliminated. However, this work still requires further analysis because there are terms which change their relative magnitude quickly and by large amounts. Some of these terms may have been eliminated from the formulas when they should not have been.

6.4 SIMPLIFICATION OF THE SUCCESSIVE SUBSTITUTION SOLUTION

The result of applying the Successive Substitution method to the ROTS equation has not been carried analytically. Some preliminary work has been done, and it appears that a considerable amount of algebraic simplification of the result may be possible. It is planned to use the FORMAC computer language to carry out the necessary algebra and examine the resulting formula. In a practical sense, one might actually prefer not to have an explicit solution of the ROTS equations if the numerical equivalent (as is now being used) is as fast or faster.

Table 6-1 COMPARISON OF PERCENT ERRORS OF NEWTON-RAPHSON (N-R) AND SUCCESSIVE SUBSTITUTION SOLUTIONS OF THE ROTS (4,3) EQUATIONS BEFORE AND AFTER SIMPLIFICATION

TIME	N-R BEFORE	N-R AFTER	6th SUBSTITUTION BEFORE	6th SUBSTITUTION AFTER	X NOMINAL	X_{N-R} BEFORE	X_{N-R} AFTER
0	0.04	0.16	0.06	0.18	76.84	76.86	76.45
20.00	-0.06	0.02	-0.05	0.03	81.70	81.71	81.62
40.00	-0.03	-0.01	-0.03	-0.01	87.05	87.06	87.19
60.00	0.04	0.02	0.04	0.02	92.87	92.87	93.11
80.00	0.10	0.07	0.11	0.08	99.10	99.10	99.35
100.00	0.11	0.09	0.12	0.10	105.66	105.66	105.85
120.00	0.07	0.08	0.09	0.09	112.40	112.40	112.51
140.00	0.04	0.06	0.06	0.07	119.15	119.16	119.21
160.00	0.09	0.10	0.22	0.23	125.76	125.77	125.84

NOMINAL AA1

Table 6-2 COMPARISON OF PERCENT ERRORS OF NEWTON-RAPHSON (N-R) AND SUCCESSIVE SUBSTITUTION SOLUTIONS OF THE ROTS (4,3) EQUATIONS BEFORE AND AFTER SIMPLIFICATION

TIME	N-R BEFORE	N-R AFTER	6th SUBSTITUTION BEFORE	6th SUBSTITUTION AFTER	X NOMINAL	X_{N-R} BEFORE	X_{N-R} AFTER
0	1.30	1.03	1.25	0.93	63.70	63.70	62.53
20.00	-0.04	0.00	-0.11	-0.09	69.25	69.27	69.24
40.00	-0.51	-0.42	-0.54	-0.45	75.93	75.96	76.93
60.00	-0.29	-0.26	-0.29	-0.26	83.91	83.94	85.53
80.00	0.24	0.22	0.23	0.22	93.25	93.27	94.99
100.00	0.61	0.61	0.62	0.62	103.73	103.74	105.09
120.00	0.53	0.62	0.61	0.67	114.81	114.82	115.57
140.00	0.21	0.36	0.38	0.49	125.74	125.76	126.03
160.00	0.20	0.28	0.95	1.01	135.82	135.85	135.98

NOMINAL AA24

SECTION VII

COMBINATION OF STEERING ANGLE AND TIME-TO-CUTOFF EXPRESSIONS

It was noted in Section 4.1 that the expressions for the steering angle and time-to-cutoff would be analyzed independently of each other, and the results of each analysis would be combined. Based on the studies reported in previous sections, it was decided that a feasible undertaking would be the substitution of a third-order series for Δt into the ROTS (4,3) equations. Solution of the resulting equations would then yield an analytical solution which would be valid for certain classes of missions. In addition, term elimination and simplification would be used to reduce the complexity of the equations as far as possible. Such was done and the resulting formulas were coded, and numerical results were obtained for comparison with nominal cases.

The expression used for Δt is shown in Table 7-1. This series was substituted into the R and Q coefficients of the ROTS (4,3) equations. The resulting equations in the multipliers have coefficients similar to the R and Q coefficients shown in Table 4-1. Instead of the explicit appearance of Δt a series expression in terms of Z appears. Table 7-2 displays a sample of these coefficients.

Figures 7-1 through 7-4 are comparisons of nominal and analytical solutions for the steering angle. Figure 7-1 indicates a maximum error of about five degrees early in the flight and was the best result achieved. The other figures show cases where larger ΔR and Δt values are considered and indicate poorer results.

Time has not allowed the substitution of higher order series for Δt, nor a simulated flight according to these formulas. The results obtained were quite agreeable with what was predicted by the studies reported in Sections IV and V, and it appears that the development of rather accurate guidance functions for longer flight times than about 250 seconds can be accomplished without an unreasonable amount of labor. However, a significant span of time will be necessary because of the algebra required, and the coding, checking, and simplification of formulas. Use of a computer language such as FORMAC for routine algebra has been, and will continue to be, a major tool in this sort of endeavor.

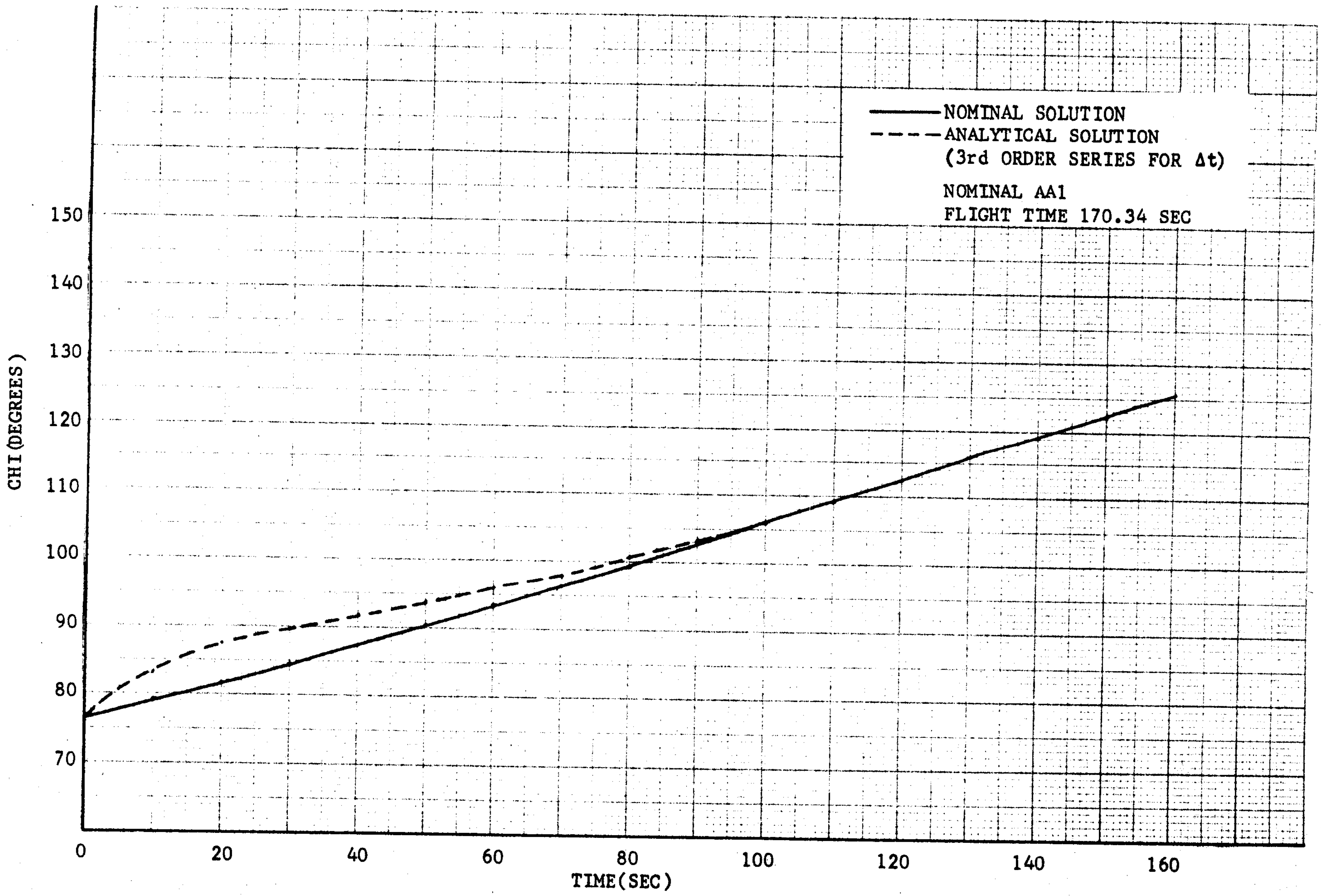

Figure 7-1. COMPARISON OF ANALYTICAL SOLUTION AND NOMINAL SOLUTION FOR STEERING ANGLE

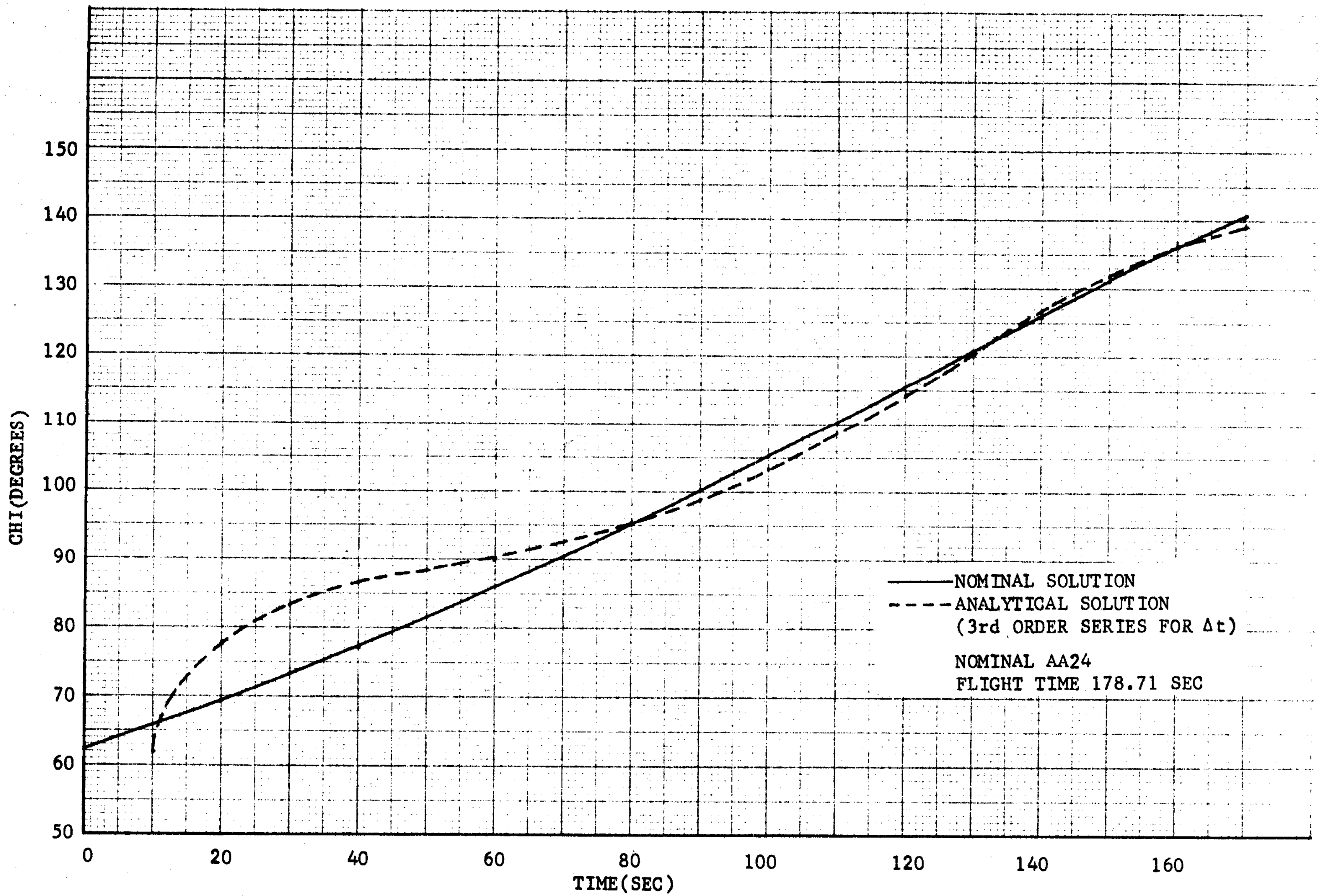

Figure 7-2. COMPARISON OF ANALYTICAL SOLUTION AND NOMINAL SOLUTION FOR STEERING ANGLE

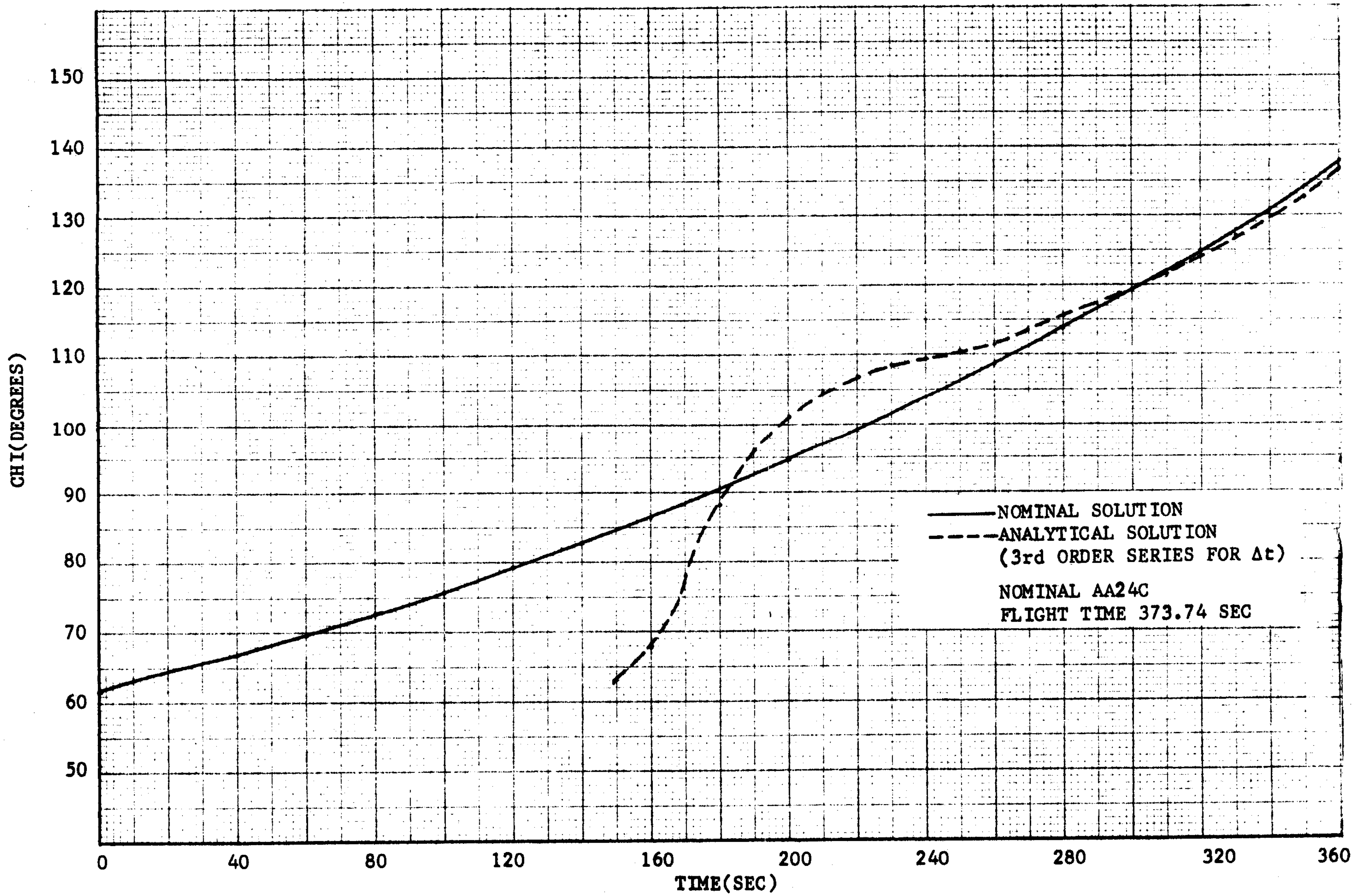

Figure 7-3. COMPARISON OF ANALYTICAL SOLUTION AND NOMINAL SOLUTION FOR STEERING ANGLE

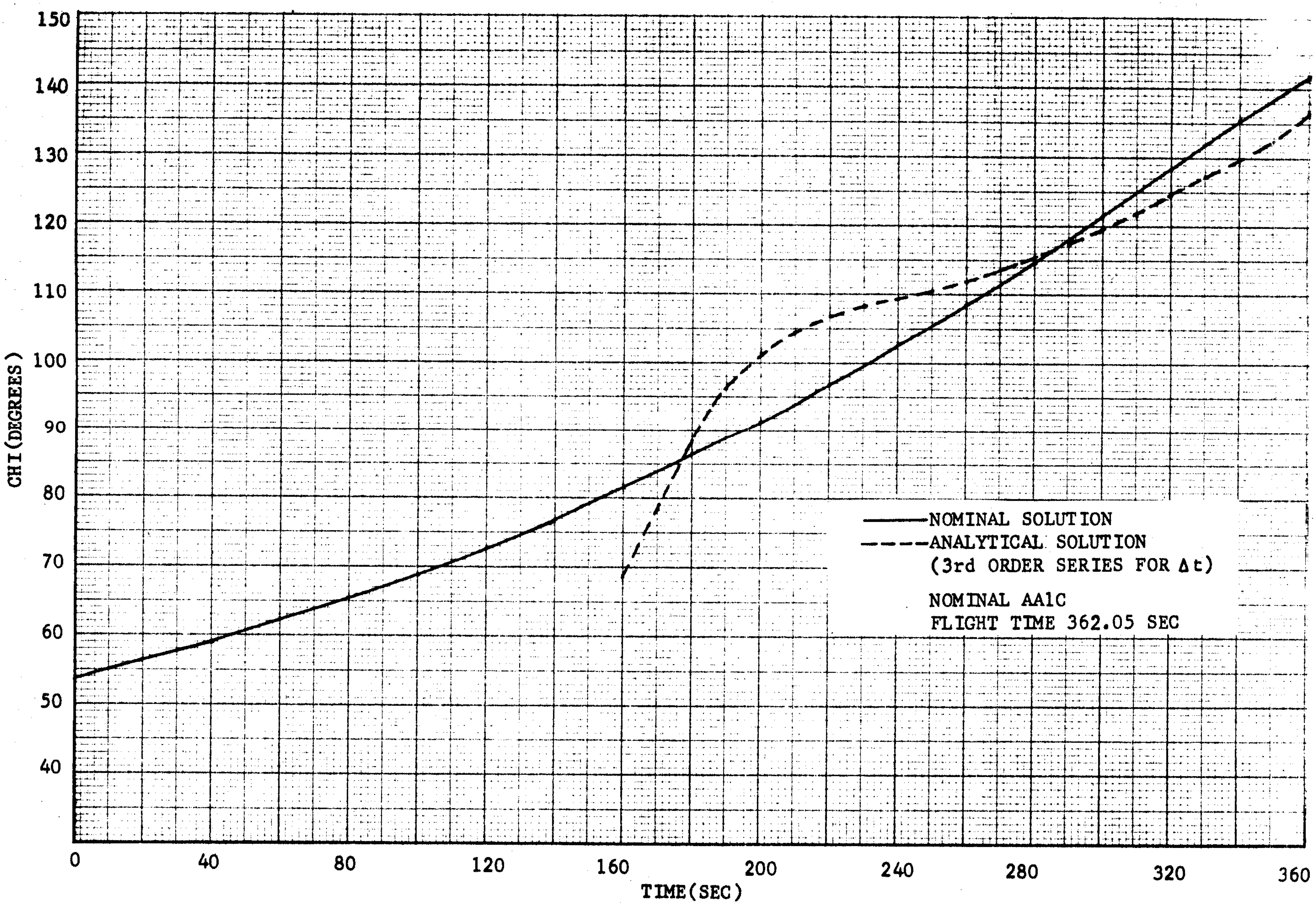

Figure 7-4. COMPARISON OF ANALYTICAL SOLUTION AND NOMINAL SOLUTION FOR STEERING ANGLE

Table 7-1. EXPRESSION FOR Δt USED TO DEVELOP ANALYTICAL SOLUTION FOR GUIDANCE FUNCTION, OBTAINED FROM THIRD-ORDER SERIES AND EXPRESSED IN TERMS OF MULTIPLIERS

$$
\begin{aligned}
\Delta t = {} & \gamma_{0000} + \gamma_{1000}\lambda_1 + \gamma_{0100}\lambda_2 + \gamma_{2000}\lambda_1^2 + \gamma_{0200}\lambda_2^2 \\
& + \gamma_{1100}\lambda_1\lambda_2 + \gamma_{0111}\lambda_2\lambda_3\lambda_4 + \gamma_{1011}\lambda_1\lambda_3\lambda_4 + \gamma_{1101}\lambda_1\lambda_2\lambda_4 \\
& + \gamma_{1110}\lambda_1\lambda_2\lambda_3 + \gamma_{1002}\lambda_1\lambda_4^2 + \gamma_{0120}\lambda_2\lambda_3^2 + \gamma_{0210}\lambda_2^2\lambda_3 \\
& + \gamma_{2001}\lambda_1^2\lambda_4 + \gamma_{1200}\lambda_1\lambda_2^2 + \gamma_{2100}\lambda_1^2\lambda_2 + \gamma_{3000}\lambda_1^3 \\
& + \gamma_{0300}\lambda_2^3 + \gamma_{2200}\lambda_1^2\lambda_2^2 + \gamma_{1201}\lambda_1\lambda_2^2\lambda_4 + \gamma_{2110}\lambda_1^2\lambda_2\lambda_3 \\
& + \gamma_{1210}\lambda_1\lambda_2^2\lambda_3 + \gamma_{2101}\lambda_1^2\lambda_2\lambda_4 + \gamma_{0310}\lambda_2^3\lambda_3 + \gamma_{3001}\lambda_1^3\lambda_4 \\
& + \gamma_{4000}\lambda_1^4 + \gamma_{0400}\lambda_2^4 + \gamma_{1220}\lambda_1\lambda_2^2\lambda_3^2 + \gamma_{1202}\lambda_1\lambda_2^2\lambda_4^2 \\
& + \gamma_{2120}\lambda_1^2\lambda_2\lambda_3^2 + \gamma_{2102}\lambda_1^2\lambda_2\lambda_4^2 + \gamma_{2210}\lambda_1^2\lambda_2^2\lambda_3 \\
& + \gamma_{2201}\lambda_1^2\lambda_2^2\lambda_4 + \gamma_{1211}\lambda_1\lambda_2^2\lambda_3\lambda_4 + \gamma_{2111}\lambda_1^2\lambda_2\lambda_3\lambda_4 \\
& + \gamma_{3101}\lambda_1^3\lambda_2\lambda_4 + \gamma_{1301}\lambda_1\lambda_2^3\lambda_4 + \gamma_{3110}\lambda_1^3\lambda_2\lambda_3 + \gamma_{1310}\lambda_1\lambda_2^2\lambda_3 \\
& + \gamma_{4001}\lambda_1^4\lambda_4 + \gamma_{0410}\lambda_2^4\lambda_3 + \gamma_{2202}\lambda_1^2\lambda_2^2\lambda_4^2 + \gamma_{2220}\lambda_1^2\lambda_2^2\lambda_3^2 \\
& + \gamma_{2211}\lambda_1^2\lambda_2^2\lambda_3\lambda_4 + \gamma_{1311}\lambda_1\lambda_2^3\lambda_3\lambda_4 + \gamma_{3111}\lambda_1^3\lambda_2\lambda_3\lambda_4 \\
& + \gamma_{3102}\lambda_1^3\lambda_2\lambda_4^2 + \gamma_{1320}\lambda_1\lambda_2^3\lambda_3^2 + \gamma_{4002}\lambda_1^4\lambda_4^2 \\
& + \gamma_{0420}\lambda_2^4\lambda_3^2
\end{aligned}
$$

Table 7-1. (Concluded)

$$\gamma_{0000} = Z - \frac{Z^2}{\delta} u L_5 - \frac{Z^2}{\delta} v L_6 - \frac{Z^2}{\delta} L_2^2 - \frac{Z^2}{\delta} L_3^2$$

$$+ \frac{2Z^3}{\delta^2} u^2 L_5^2 + \frac{2Z^3}{\delta^2} v^2 L_6^2 + 4 \frac{Z^3}{\delta^2} uv L_5 L_6$$

$$+ 4 \frac{Z^3}{\delta^2} u L_2^2 L_5 + 4 \frac{Z^3}{\delta^2} u L_3^2 L_5 + 4 \frac{Z^3}{\delta^2} v L_2^2 L_6$$

$$+ 4 \frac{Z^3}{\delta^2} v L_3^2 L_6 + 2 \frac{Z^3}{\delta^2} L_2^4 + 4 \frac{Z^3}{\delta^2} L_2^2 L_3^2 + 2 \frac{Z^3}{\delta^2} L_3^4$$

$$- \frac{Z^3}{3\delta} u L_{12} - \frac{Z^3}{3\delta} v L_{13} - \frac{Z^3}{\delta} L_2 L_5 - \frac{Z^3}{\delta} L_3 L_6$$

⋮

ETC.

where

$$\delta = 2 u\dot{u} + 2 v\dot{v}$$

$$Z = \frac{V_{co}^2 - u^2 - v^2}{\delta}$$

Table 7-2. SAMPLES OF Q' COEFFICIENTS AFTER SUBSTITUTION OF SERIES EXPRESSION FOR Δt

$$
\begin{aligned}
Q'_{0000} = {} & (xu + yv) + (x\,L_2 + y\,L_3 + u^2 + v^2)(Z - \frac{Z^2}{\delta} u\,L_5 \\
& - \frac{Z^2}{\delta} v\,L_6 - \frac{Z^2}{\delta} L_2^2 - \frac{Z^2}{\delta} L_3^2 + 2\frac{Z^3}{\delta^2} u^2 L_5^2 \\
& + 2\frac{Z^3}{\delta^2} v^2 L_6^2 + 4\frac{Z^3}{\delta^2} uv\,L_5 L_6 + 4\frac{Z^3}{\delta^2} u\,L_2^2 L_5 + 4\frac{Z^3}{\delta^2} u\,L_3^2 L_5 \\
& + 4\frac{Z^3}{\delta^2} v\,L_2^2 L_6 + 4\frac{Z^3}{\delta^2} v\,L_3^2 L_6 + 2\frac{Z^3}{\delta^2} L_2^4 + 4\frac{Z^3}{\delta^2} L_2^2 L_3^2 \\
& + 2\frac{Z^3}{\delta^2} L_3^4 - \frac{Z^3}{3\delta} u\,L_{12} - \frac{Z^3}{3\delta} v\,L_{13} - \frac{Z^3}{\delta} L_2 L_5 - \frac{Z^3}{\delta} L_3 L_6) \\
& + (\frac{1}{2} x\,L_5 + \frac{1}{2} y\,L_6 + \frac{3}{2} v\,L_3 + \frac{3}{2} u\,L_2)(Z - \frac{Z^2}{\delta} u\,L_5 \\
& - \frac{Z^2}{\delta} v\,L_6 - \frac{Z^2}{\delta} L_2^2 - \frac{Z^2}{\delta} L_3^2 + 2\frac{Z^3}{\delta^2} u^2 L_5^2 + 2\frac{Z^3}{\delta^2} v^2 L_6^2 \\
& + 4\frac{Z^3}{\delta^2} uv\,L_5 L_6 + 4\frac{Z^3}{\delta^2} u\,L_2^2 L_5 + 4\frac{Z^3}{\delta^2} u\,L_3^2 L_5 + 4\frac{Z^3}{\delta^2} v\,L_2^2 L_6 \\
& + 4\frac{Z^3}{\delta^2} v\,L_3^2 L_6 + 2\frac{Z^3}{\delta^2} L_2^4 + 4\frac{Z^3}{\delta^2} L_2^2 L_3^2 + 2\frac{Z^3}{\delta^2} L_3^4 \\
& - \frac{Z^3}{3\delta} u\,L_{12} - \frac{Z^3}{3\delta} v\,L_{13} - \frac{Z^3}{\delta} L_2 L_5 - \frac{Z^3}{\delta} L_3 L_6)^2 \\
& + (\frac{1}{6} x\,L_{12} + \frac{1}{6} y\,L_{13} + \frac{2}{3} u\,L_5 + \frac{2}{3} v\,L_6 + \frac{1}{2} L_1^2 + \frac{1}{2} L_2^2 \\
& + \frac{1}{2} L_3^2)(Z - \frac{Z^2}{\delta} u\,L_5 - \frac{Z^2}{\delta} v\,L_6 - \frac{Z^2}{\delta} L_2^2 - \frac{Z^2}{\delta} L_3^2 \\
& + 2\frac{Z^3}{\delta^2} u^2 L_5^2 + 2\frac{Z^3}{\delta^2} v^2 L_6^2 + 4\frac{Z^3}{\delta^2} uv\,L_5 L_6 + 4\frac{Z^3}{\delta^2} u\,L_2^2 L_5 \\
& + 4\frac{Z^3}{\delta^2} u\,L_3^2 L_5 + 4\frac{Z^3}{\delta^2} v\,L_2^2 L_6 + 4\frac{Z^3}{\delta^2} v\,L_3^2 L_6 + 2\frac{Z^3}{\delta^2} L_2^4 \\
& + 4\frac{Z^3}{\delta^2} L_2^2 L_3^2 + 2\frac{Z^3}{\delta^2} L_3^4 - \frac{Z^3}{3\delta} u\,L_{12} - \frac{Z^3}{3\delta} v\,L_{13} - \frac{Z^3}{\delta} L_2 L_5 \\
& - \frac{Z^3}{\delta} L_3 L_6)^3
\end{aligned}
$$

SECTION VIII

DESCRIPTION OF THE TOPSY COMPUTER PROGRAM

The numerical results shown in this report were generated by a computer program named "TOPSY". Because numerical tests of the formulas against nominal trajectories were necessary, the testing process was automated as far as possible in order to handle a larger number of cases. TOPSY is written almost entirely in FORTRAN IV for the IBM 7094 operating under IBSYS. The source program of about 3000 cards was put on tape, and the IBSYS's "alter" routines were used to modify the program when desired; e.g., for term elimination. A collection of nominal trajectories for purposes of comparison was obtained from the MSFC V-1 program and written on another tape. With this set-up a large variety of information could be conveniently obtained. Numerous options are available for timing certain computations, plotting results, etc. Approximately 28 hours of computer time were made available for this study through the MSFC Computation Laboratory under the Resource Sharing Program. Depending upon the options called for, average compilation and execution time for a single nominal time history is three to five minutes. Compilation time accounts for about 80 percent of the run time.

A skeleton flow chart of TOPSY is shown in Figure 8-1. A brief description of each routine is given below.

TOP1: MAIN SECTION

TOP1 serves as a driver for the rest of the program. Its main functions are to search the data tape for the correct nominal, and to call the subprograms in their proper sequence. Subprograms are called and their sequences are determined by the particular assignment and can be manipulated by alterations in this section of TOPSY.

TOP2: DATAIN

This subroutine reads vehicle constants, cut-off parameters, perturbations, and state variables at each time point.

TOP3: GRAPOT

This subroutine computes the gravitational potential derivatives through the sixth order.

TOP4: LCOEF

Using nominal data read by DATAIN and values from GRAPOT, this subroutine computes "L coefficients" which are the coefficients of the lambdas in the u and v derivatives.

TOP5: UVDER

Lambda values from the MAIN SECTION and L's" from LCOEF are used to compute u and v derivatives through the sixth order. These values are used in later subroutines to compute coefficients and remainder terms of the radius, velocity, and orthogonality Taylor series expansions.

TOP6: COEFDT

This routine computes coefficients of the Taylor series expansions for the Radius, Orthogonality, and Velocity series. Remainder terms are also calculated. Any order of these series up through sixth can be reverted to any order up through sixth. Actual roots of these series can also be calculated by the Bairstowe method.

TOP7: RPQ

This routine computes values for the R and Q coefficients for series of orders through the fifth for the radius equation and the fourth for the orthogonality equation. The values are used in the following two subroutines.

TOP8: REVSV

This subroutine solves the ROTS system of equations for lambdas using the successive substitution method. The matrix of the linear coefficients is solved by a Gauss-Jorday method, thus obtaining approximations for successive substitutions.

TOP9: ROTS

The same system of equations dealt with in REVSV is solved by the Newton-Raphson method using nominal lambdas for initial estimates and a Gaussian solution of the linear equations. The equations are solved simultaneously for the lambdas, and the results are converted to the steering angle values. The percent errors are printed for easy comparison with lambda and chi errors from REVSV.

TOP10: TRAJS

This subroutine integrates the equations of motion using a fourth-order Runge-Kutta integration scheme. Nominal or computed values are read at the initial time point and are used to obtain lambda values from REVSV. The calculated lambdas and initial nominal values are used in this subroutine to find new values for the state variables. These new values are then transferred to TOP1 where required calculations are performed before entering routines to compute new lambdas for the integration scheme in TRAJS.

TOP11: RUNKUT

This is a fourth-order Runge-Kutta integration scheme used in TRAJS to integrate the equations of motion.

TOP12: SIMEQ

This subroutine used in ROTS employs a Gaussian method for the solution of simultaneous linear equations.

TOP13: GAUJOR

This subroutine used in REVSV is a Gauss-Jorday solution of simultaneous linear equations similar to SIMEQ.

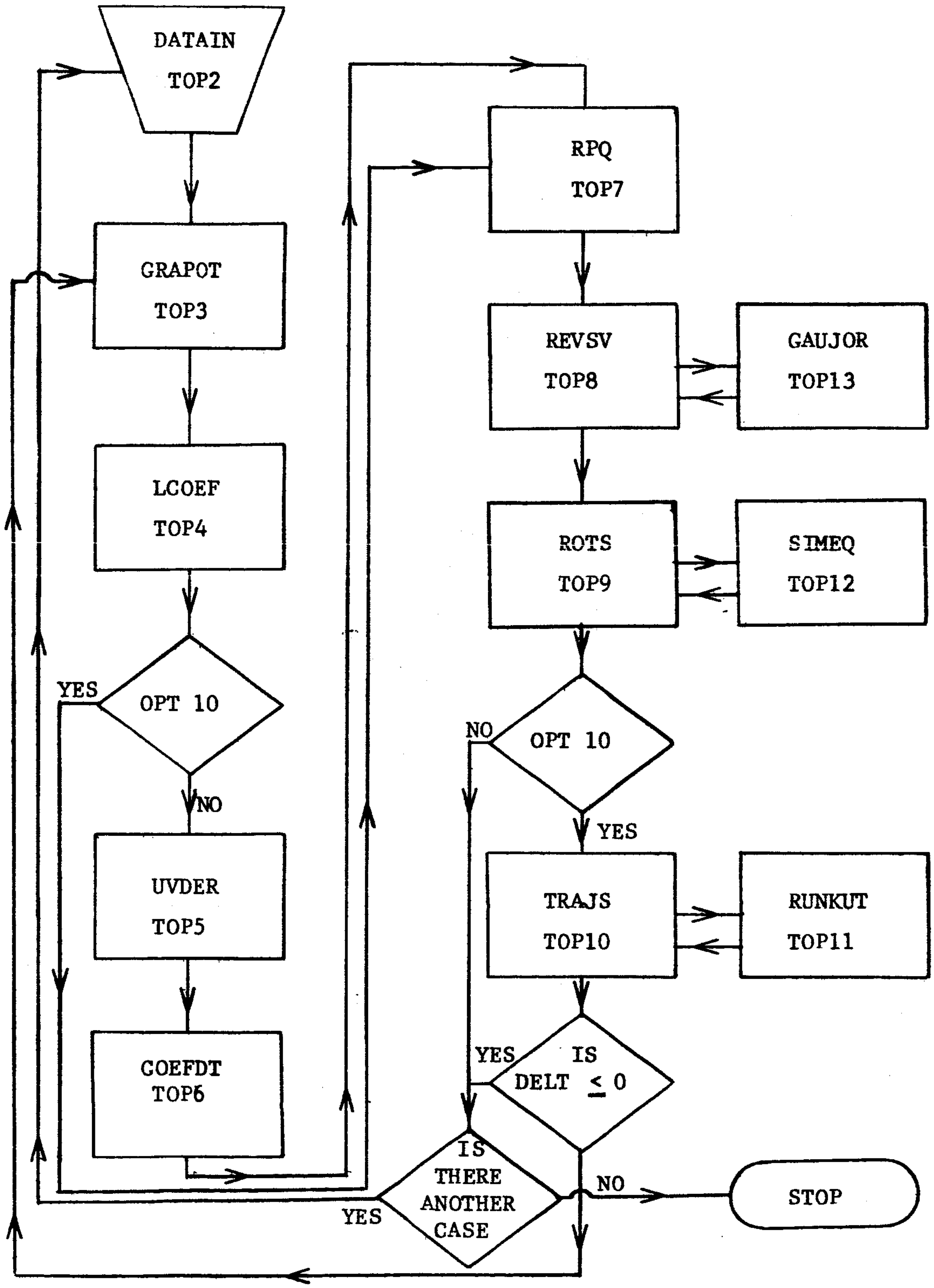

Figure 8-1. TOPSY FLOW CHART

SECTION IX

DISCUSSION OF THE FORMAC COMPUTER LANGUAGE FOR SYMBOLIC COMPUTATIONS

FORMAC, an experimental programming language developed by IBM, can manipulate mathematical expressions, and perform routine algebraic calculations and symbolic differentiation. It is an "extension" of FORTRAN IV, thus providing the capability of performing both numerical computations and symbol manipulations in the same program. Full FORTRAN IV capability and implementation under the IBM 7094 IBSYS Monitor system makes the transition to FORMAC extremely easy for an experienced FORTRAN programmer.

In this study, FORMAC was used to duplicate some of the symbolic differentiation and algebraic expansions that had previously been done by hand. The results were quite encouraging, especially in the area of differentiation. A major goal was to become familiar with the system and to fully realize the capabilities and limitations of the system.

The major problem encountered was maintaining an adequate amount of storage for free list, that is, the storage area used to develop and store expressions. Since the size of an expression and the number of computer words required to store it cannot be predicted, this situation caused the termination of numerous jobs because of non-availability of free list area.

The problem of insufficient core storage has two major aspects. First, the amount of core storage required for the FORTRAN library and FORMAC subroutines often leaves insufficient free list for a given program. Second, since the required size of the free list area cannot be accurately predicted, it is difficult to make efficient use of this area. Often trial and error is the only available procedure to debug a program.

Overlay can be used if it is anticipated that all available free list will be used. Object time routines can be swapped in and out of core storage when necessary. This allows the space they would occupy to be used as extra free list. However, an adequate free list area could not always be obtained by this technique because of necessary communication between some routines which required most of them to be in core storage simultaneously.

Elimination of expressions that are no longer needed releases the area that they occupied to become free list. This can be accomplished by the "erase" command. Also, expressions that may be required later but which are not needed at a given time may be erased and regenerated when needed again. This is, in effect, a trade of computing time for storage area. Also, intermediate results can be stored on tape and read from tape as required.

Some problems can be segmented. That is, one program may be broken into several smaller programs with the output of the first becoming input to the second, etc. Again, tape or cards may be used as a communication medium at the programmer's discretion.

Readability of output, especially where long complicated expressions were involved, was another area of concern. Output is in the regular FORTRAN expression format and hand translation is nearly always necessary before results become meaningful.

FORMAC promises to be a very powerful mathematical tool, both in capability and ease of program implementation. However, the size of the maximum expression which can be handled is dependent on the number of computer words which make up the free list. The size of the free list varies according to the size of the object program, the number of FORTRAN library routines used, and the number of FORMAC subroutines required. Overlay and other programming techniques can be used to increase the size of the free list. However, this still proves to be insufficient for any problem of considerable size.

SECTION X

CONCLUSIONS AND RECOMMENDATIONS

This formal, analytical approach to certain types of two-point boundary condition problems has been shown to be feasible. For problems of this type, which occur in optimal guidance, the approach has been shown to be effective. Results have been obtained which are in close agreement with numerical solutions obtained from the usual trial- and-error mehtods. Correlation between nominal trajectories and those predicted by the analytical solution has been demonstrated for a variety of circular orbit missions, and this indicates that favorable, special cases have not been inadvertently used.

One of the outstanding problems encountered was the series for time-to-cutoff. As the data in this report indicate, a rather high order series will be required for missions where "large" altitude changes and flight times are necessary. Problems will occur primarily in developing the powers of Δt. However, there will be a correspondingly greater number of terms that can be eliminated when these formulas are developed. Those terms which can be eliminated will be the most complicated expressions. Most of the labor expended in this area will be in eliminating nonessential terms and in simplification.

The "single iteration" method for obtaining Δt, using a formula for the estimate, should be investigated further. It may be worth consideration for certain classes of missions, and could conceivably reduce the complexity of the Δt expressions.

Even with the complicated expressions now being used to calculate the guidance functions, it requires only about one second of computer time to determine the optimal steering angle. This is for a FORTRAN-compiled program on the IBM 7094, computing the steering angle by the Successive Substitution method. A number of auxiliary computations are also performed in this program which are included in this time, but which are not essential to the steering angle evaluation.

Perhaps the most bothersome problem has been the elimination of terms and algebraic simplifications desired in order to reduce the guidance formulas to expressions that are manageable and feasible for implementation. This area of study still requires work. At best, it is a tedious and time-consuming activity. For the problem studied, much of the work was automated. Any future studies of this nature should be carefully planned, giving consideration to automation from the start. It would be very desirable to have expressions for "the general term" in many of the formulas used. This would not only be for the sake of elegance, but to have some sort of rational guide to the analytical developments and simplifications.

After consideration of the results achieved to date on the circular orbit problem, it is recommended that no further work be done on it unless an implementation study is anticipated. The result of using a higher order series for time-to-cutoff should be evident from the results obtained. Orders of series and numbers of terms required for given circular orbit missions can be estimated closely from the data presented in this report.

It is recommended that this same approach be applied to a more general problem in optimal guidance. Transfer from a specified point to a conic path or transfer between orbits appears to be a reasonable undertaking at this time. The availability of a computer language such as FORMAC to handle many of the routine differentiations and substitutions required is a tremendous asset in this sort of work. Preliminary analysis of some of these more general problems has been initiated. To date no refined numerical results are available, but it appears from initial calculations that these problems will not be much more difficult than the circular mission problem.

SECTION XI

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