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in the Upper Atmosphere"

by

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ABSTRACT

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The thermal conductivity of the electron gas plays an important part in determining the profiles of electron and ion temperature in the upper atmosphere. Previous calculations have been based upon the thermal conductivity derived for a fully ionized gas. In aeronomic conditions however, the contribution made by neutral gas particles must be introduced. In this paper an analysis is made of the effect of electron-neutral particle collisions and an appropriate expression is developed for the electron thermal conductivity which is applicable to a weakly ionized plasma. It is found that a correction term needed to include the effect of neutral particles is of importance during both day and night at all altitudes below 220 km. In fact, at altitudes below 135 km it appears that the conduction term can be neglected in the electron energy balance equation. Using the derived expression for the effective electron thermal conductivity, an example of calculated temperatures characteristic of quiet solar conditions is presented.

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## I.- INTRODUCTION

There have been many recent measurements of electron and ion temperatures in the ionosphere which have involved satellite, rocket and ground based techniques. Without exception these investigations have shown that throughout the day and during much of the night there exist considerable differences between the temperatures of the charged and neutral gases. While specific experiments yield a range of temperature values, it is generally agreed that the gross features of the results above 150 km can be explained by assuming that photoionization is the principal source of heating in the daytime electron gas. A comprehensive review of current experimental techniques and results has been given by Evans (1965).

The problem of calculating theoretical values of electron and ion temperatures in the upper atmosphere is difficult.

The first study of the effect of photoelectrons in heating the ambient electrons of the ionosphere appears to have been made by Drukarev (1946) who derived an explicit expression for the difference between the charged and neutral particle temperatures in terms of the average energy of electrons created through photoionization.

A more comprehensive study of the electron gas energy balance was made by Hanson and Johnson (1961), demonstrating the importance of energy introduced into the electron gas by photoelectrons. It was shown that this process alone could create large differences between electron and neutral gas temperatures in the altitude range 150-350 km. Later papers by Hanson (1963), Dalgarno et al. (1963), and Geisler and Bowhill (1965a) successively developed the theoretical problem in greater detail by improving our knowledge of the various terms which enter into the energy balance equations for the electron and ion gases.

In this paper we continue the study of thermal nonequilibrium in the upper atmosphere, being primarily concerned with the influence of heat conduction through the electron gas, the way in which the neutral atmosphere acts to reduce the thermal flux, and the net effect this has upon calculations of electron and ion temperatures.

The energy balance equation for the electron gas of the ionosphere can be written as

$$\frac{\partial U_e}{\partial t} = P_e - L_e - \nabla \cdot \vec{q} \quad (1)$$

where  $U_e$  is the total electron gas thermal energy,  $P_e$  is the rate of energy production in the electron gas,  $L_e$  is the rate of energy loss from electron gas, and  $\vec{q}$  is the flux of thermal energy through the electron gas.

From Chapman and Cowling (1952) the general expression for  $\vec{q}$  is, for a gas mixture in which thermal diffusion is neglected,

$$\vec{q} = -K \nabla T_e + 5/2 n_e k T_e \vec{C}_e \quad (2)$$

where  $K$  is the electron gas thermal conductivity,  $T_e$  is the electron temperature,  $n_e$  is the electron number density,  $k$  is Boltzmann's constant, and  $\vec{C}_e$  is the average electron diffusion velocity.

For a complete analysis of the problem of electron and ion temperatures it is necessary to consider the coupled set of second order, nonlinear differential equations describing the heat balance and number density of the charged particle gases. In fact, at the present time it is not possible to separate cause and effect between changes in electron temperature and density. The solution to the complete problem is very complex and will require considerable effort. A first start can be made, however, by assuming that the electron and ion densities are known at each altitude and that we have a quasi-equilibrium density condition such that  $\vec{C}_e = 0$ . In this approximation equation (2) becomes

$$\vec{q} = -K \nabla T_e \quad (3)$$

and the only term in the heat flux equation arises from a gradient in the electron temperature.

The importance of the electron gas thermal conductivity in determining distributions of electron temperature was first discussed by Chapman (1956) in applications concerned with the extension of the solar corona into interplanetary space. Hanson and Johnson (1961) initially rejected the importance of the electron gas thermal conductivity on the basis of its small value relative to that of the neutral atmosphere. Hanson (1963), however, re-evaluated the problem and drew specific attention to the contribution which the thermal conductivity makes in keeping the electron temperature gradient small at high altitudes for a low altitude energy (photoionization) source. Later, Geisler and Bowhill (1965a) made extensive calculations of electron temperatures which included the effect of conduction.

A basic assumption in all of these studies has been that the electron gas thermal conductivity is correctly given by the expression,

$$K = 7.7 \times 10^5 T_e^{5/2} \text{ ev cm}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{K}^{-1}, \quad (4)$$

which was derived originally by Spitzer and Harm (1953) for a fully ionized gas. This form for the thermal conductivity has important consequences upon the boundary conditions applying to equation(1) since it accepts that the energy flux, given by equation (3), is independent of the neutral atmosphere density. This result, however, can not apply to a situation where the total gas mixture is only weakly ionized and electron energy transport is limited by electron collisions with the neutral gases rather than by charged particle collisions.

For the ionosphere the essential problem is to derive an expression for the total electron thermal conductivity which incorporates the effects of both charged and neutral particle collisions. In section II

this derivation is made on the basis of a mean free path method, yielding a general expression for the thermal conductivity which is valid for a plasma having an arbitrary degree of ionization. In section III the derived result is applied to the problem of calculating profiles of electron and ion temperatures and it is shown that the neutral atmospheric gases exert a significant influence at all times on the temperature profiles.

## II.- ELECTRON GAS THERMAL CONDUCTIVITY

The calculation of the electron gas thermal conductivity differs from the usual gas kinetic derivations in that we are not concerned with the total energy transported by the mixture of charged and neutral particles, but only with that portion which travels through the electron gas in response to gradients in electron temperature. The most convenient approach to this problem lies in the mean free path technique which permits a separation of the collision effects of each component within a weakly ionized gas. Using this method the total electron thermal conductivity can be derived in terms of component conductivities which arise from a consideration of charged and neutral particle collisions. The problem of finding the effective electron thermal conductivity is then reduced to the determination of appropriate expressions for the individual conductivities.

From equation (3) and the work of Chapman and Cowling (1952), the electron energy flux arising from a gradient in electron temperature is proportional to the electron thermal conductivity,  $K'$ , given by

$$K' = \frac{3}{4} n_e \bar{v}_e k \lambda \quad (5)$$

where  $n_e$  is the electron density,  $\bar{v}_e$  is the average electron velocity,  $k$  is Boltzmann's constant, and  $\lambda$  is the electron mean free path.

In a fully ionized gas the electron mean free path is determined by the effect of collisions with ions and, to a lesser extent, with other electrons. In a weakly ionized gas, however, it is necessary to take into account the presence of neutral particles since, for sufficiently large neutral particle concentrations, the free path will be determined by electron-neutral collisions alone. To introduce these effects we express the total electron free path as

$$\lambda = \frac{1}{\sum_j n_j Q_j} \quad (6)$$

where  $n_j$  is a particle number density and  $Q_j$  is the appropriate scattering cross section for the  $j$ -th species. Equation (6) may be further decomposed as

$$\lambda = \frac{\lambda_c}{1 + \lambda_c / \lambda_n} \quad (7a)$$

where  $\lambda_c$  is the electron-charged particle free path and

$$\lambda_n = 1 / \sum_n n_n Q_n \quad (7b)$$

is the electron-neutral free path which includes the effects of  $n$  different neutral gas species.

With equations (7) it is now possible to rearrange equation (5) and to write the electron thermal conductivity as

$$K' = \frac{K_i}{1 + K_i \left[ \sum_n 1/K_n \right]} \quad (8)$$

where  $K_i$  is the electron thermal conductivity of a fully ionized gas and  $K_n$  is the conductivity of electrons in a very dense neutral gas of the  $n$ -th species where charged particle collisions are unimportant. This result is identical with the more common thermodynamic problem of the heat conductivity of two combined slabs of insulating materials.



Equation (8) shows that at low neutral particle densities where  $\lambda_n$  and  $K_n$  become large,  $K' \simeq K_i$ , while for large neutral particle densities  $\lambda_n$  and  $K_n$  become small, leading to  $K' \simeq 1 / \sum_n (1/K_n)$ .

Before proceeding with the analysis needed to derive expressions for  $K_i$  and  $K_n$ , it is necessary to discuss two additional aspects of the thermal conductivity which are important for a plasma. The first revolves about the effect of a static magnetic field in constraining the motions of electrons into helical paths along the magnetic lines of force. This problem has been discussed by Chapman and Cowling (1952) where it is shown that under such conditions the thermal conductivity becomes anisotropic with different values parallel and perpendicular to the field lines. While the component of conductivity parallel to the field lines is the same as derived here for the field-free case, the perpendicular component is reduced by the factor  $1 + \omega^2 \tau^2$ , where  $\omega$  is the electron cyclotron frequency and  $\tau$  is essentially a mean time between collisions. Since  $\omega \sim 10^7 \text{ sec}^{-1}$  and  $\tau \sim 10^8 / n_n \text{ sec}$  for the earth's magnetic field and atmosphere, it is found that there will be essentially no energy conducted through the electron gas perpendicular to the magnetic field for altitudes above about 70 km. Thus, only the component of  $\nabla T_e$  parallel to the field will be effective in transporting energy through the electron gas. As a result, the thermal conductivity derived here will be considered as the component parallel to the direction of the magnetic field lines and the effect of the perpendicular component will be ignored.

The second effect which must be considered arises from the possibility of different thermoelectric effects in a plasma. As a consequence of the gradient of electron temperature there will be both a flow of energy and a flux of electrons. Spitzer and Härm (1953) have analyzed the conditions appropriate to this situation and conclude that in order for there to be no divergence of electric current it is necessary for a secondary electric field to be established within the plasma which reduces the electron

thermal conductivity by the factor  $\epsilon = 0.419$ . Although this factor has been specifically derived for electron-charged particle collisions, it is possible to extend the concept to include electron-neutral interactions as well. For the present purposes it is assumed that the factor  $\epsilon = 0.419$  can be applied directly to the case of a weakly ionized gas. Thus, we take the effective value,  $K$ , for the total thermal conductivity to be  $K = \epsilon K'$ .

The calculation of an accurate expression for the effective thermal conductivity of the electron gas requires a knowledge of both  $K_i$  and  $K_n$ . The derivation of  $K_i$  for a fully ionized gas composed of electrons and singly charged ions having Maxwellian velocity distributions has been made by Spitzer and Harm (1953). They find

$$K_i = 20 \left( \frac{2}{\pi} \right)^{3/2} k \frac{(kT_e)^{5/2} \delta_\tau}{m_e^{1/2} e^4 \ln \Lambda} \quad (9)$$

which, with  $\delta_\tau = 0.225$  and  $\ln \Lambda = 15$ , reduces to equation (4).

The derivation of the electron gas thermal conductivity which takes into account only the effects of electron-neutral particle collisions can be based upon the work of Chapman and Cowling (1952) for a Lorentzian gas. The general expression for the electron thermal conductivity is

$$K_n = 1/3 k n_e n_n \left[ \left\{ \bar{A}, \bar{A} \right\} - \left\{ \bar{A}, \bar{D} \right\}^2 / \left\{ \bar{D}, \bar{D} \right\} \right] \quad (10)$$

where

$$\left\{ \bar{A}, \bar{A} \right\} = \frac{1}{2\pi n_e n_n^2} \int f_e \left[ \frac{m_e v_e^2}{2kT_e} - \frac{5}{2} \right]^2 \frac{v_e^2}{\Phi_{en}^{(1)}(v_e)} d^3 v_e \quad (11a)$$

$$\left\{ \bar{A}, \bar{D} \right\} = \frac{-1}{2\pi n_e n_n^2} \int f_e \left[ \frac{m_e v_e^2}{2kT_e} - \frac{5}{2} \right] \frac{v_e^2}{\Phi_{en}^{(1)}(v_e)} d^3 v_e \quad (11b)$$

$$\left\{ \bar{D}, \bar{D} \right\} = \frac{1}{2\pi n_e^3 n_n^2} \int f_e \frac{v_e^2}{\Phi_{en}^{(1)}(v_e)} d^3\vec{v}_e, \quad (11c)$$

and

$$\Phi_{en}^{(1)}(v_e) = \int (1 - \cos \theta) v_e \sigma(\theta) \sin \theta d\theta. \quad (12)$$

In these equation  $f_e$  represents the electron velocity distribution function,  $v_e$  is the electron velocity,  $d^3\vec{v}_e$  is a velocity space volume element,  $\theta$  is the center of mass scattering angle, and  $\sigma(\theta)$  is the electron-neutral differential scattering cross section.

By means of the definition of the velocity dependent momentum transfer cross section,  $q_D$ , (Banks, 1966a) it is possible to reduce equation (12) to the form

$$\Phi_{en}^{(1)} = v_e q_D(v_e)/2\pi \quad (13)$$

which is similar in form to the velocity dependent momentum transfer collision frequency,  $\nu_{en}$ , given by

$$\nu_{en} = n_n v_e q_D(v_e). \quad (14)$$

Thus, by taking account of the neutral particle densities appearing in equations(11a-c) it is possible to show that equation (10) involves terms depending upon the different velocity-weighted averages of the inverse collision frequency or, conversely, the mean collision interval between electron-neutral collisions.

In order to reduce equations (11a-c) to obtain  $K_n$ , it is necessary to evaluate  $\Phi_{en}^{(1)}(v_e)$  for the problem of electron-neutral collisions. This, in general, is difficult since there are a number of effects which determine the actual energy dependence of the scattering cross sections.

Only for certain simple interaction models is it possible to obtain analytical solutions. To overcome this difficulty an alternative method of approach has been introduced which is based upon the thermal conductivity appropriate for elastic sphere collisions. From the work of Chapman and Cowling (1952) the thermal conductivity for elastic electron-neutral collisions where  $\sigma(\theta)$  is independent of the electron velocity and scattering angle is

$$K_n = \frac{2}{3} \left( \frac{n_e}{n_n} \right) k \left( \frac{8k T_e}{\pi m_e} \right)^{1/2} \frac{1}{Q}, \quad (15)$$

where  $Q$  is the electron-neutral scattering cross section. This approximation is accurate for neutral gases such as He and O but for others, such as  $O_2$  and  $N_2$ , it ignores the increases in cross section which occur at the higher electron velocities. To take these variations into account it has been found adequate to introduce the average momentum transfer cross section,  $\bar{Q}_D$ , into equation (15) in place of the elastic cross section  $Q$ . In terms of the velocity dependent momentum transfer cross section,  $q_D$ , we have

$$\bar{Q}_D = \left( \frac{m_e}{2 k T_e} \right)^3 \int v_e^5 q_D(v_e) \exp \left\{ -\frac{m_e v_e^2}{2 k T_e} \right\} dv_e \quad (16)$$

Hence, the approximation involved is essentially that of replacing equations(11a-c) by the terms

$$\left\{ \bar{A}, \bar{A} \right\} = \frac{1}{n_e n_n} \left( \frac{1}{\bar{Q}_D} \right) \int f_e \left[ \frac{m_e v_e^2}{2 k T_e} - \frac{5}{2} \right] v_e d^3 \vec{v}_e \quad (17a)$$

$$\left\{ \bar{A}, \bar{D} \right\} = \frac{-1}{n_e^2 n_n^2} \left( \frac{1}{\bar{Q}_D} \right) \int f_e \left[ \frac{m_e v_e^2}{2 k T_e} - \frac{5}{2} \right] v_e d^3 \vec{v}_e \quad (17b)$$

$$\left\{ \bar{D}, \bar{D} \right\} = \frac{1}{n_e^3 n_n^2} \left( \frac{1}{\bar{Q}_D} \right) \int f_e v_e d^3 \vec{v}_e, \quad (17c)$$

which can be solved immediately to give

$$K_n = \frac{2}{3} \left( \frac{n_e}{n_n} \right) k \left( \frac{8 kT_e}{\pi m_e} \right)^{1/2} \frac{1}{\bar{Q}_D} \quad (18)$$

The error involved in this process has been found to be less than 2% for simple power law dependences of the cross section. Since the actual values of  $q_D(v_e)$  are only known to within 20% for  $O_2$  and  $N_2$ , the approximation appears to be adequate. Expressions for  $\bar{Q}_D$  have been derived by Banks (1966a) in an analysis of electron collisions. For the atmospheric gases we adopt

$$\bar{Q}_D(N_2) = 2.82 (1 - 1.21 \times 10^{-4} T_e) T_e^{1/2} \times 10^{-17} \text{ cm}^2, \quad (19a)$$

$$\bar{Q}_D(O_2) = 2.2 (1 + 3.6 \times 10^{-2} T_e^{1/2}) \times 10^{-16} \text{ cm}^2, \quad (19b)$$

$$\bar{Q}_D(O) = 3.4 \times 10^{-16} \text{ cm}^2. \quad (19c)$$

In terms of  $K_i$  and  $K_n$  the effective electron thermal conductivity now becomes, including the thermoelectric factor,

$$K = \frac{7.7 \times 10^5 T_e^{5/2}}{1 + 3.22 \times 10^4 \frac{T_e^2}{n_e} \sum_n n_n \bar{Q}_D} \text{ ev cm}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{K} \quad (20)$$

where the summation in the denominator is taken over all neutral gas species present. For low neutral particle densities this expression reduces to equation (4) derived by Spitzer and Härm (1953) for a fully ionized gas. At the opposite extreme, however, the correction factor is large and the thermal conductivity becomes

$$K = \frac{23.9 n_e T_e^{1/2}}{\sum_n n_n \bar{Q}_D} \text{ ev cm}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{K}^{-1}, \quad (21)$$

which shows the relative insensitivity of the electron thermal conductivity to changes of temperature in a dense neutral atmosphere. The transition between the two limiting conditions is shown in Figure 1 where the thermal conductivity is plotted for several values of  $T_e$  as a function of the density reduced cross section  $\sum_n \frac{n}{n_e} \bar{Q}_D$ . Since  $K_1$  is independent of the number densities, the departure of the curves from the horizontal indicates the effect of the neutral gases.

As an example of the practical importance of the influence of the neutral atmosphere upon the electron thermal conductivity, we may investigate a typical daytime condition. From the Flight 606 data of Spencer, et al. (1965) it was found that at 160 km  $T_e = 1208$  °K,  $n(N_2) = 1.24 \times 10^{10}$  cm<sup>-3</sup>, and  $n_e = 2.1 \times 10^5$  cm<sup>-3</sup>. To supply the missing neutral particle densities we use a 1000° atmospheric model (J 1.5 5D) of Nicolet (1967) to obtain  $n(O) = 7.41 \times 10^9$  cm<sup>-3</sup> and  $n(O_2) = 2.95 \times 10^9$  cm<sup>-3</sup>. With these parameters the daytime atmospheric effect leads to a factor of 4.2 decrease in the electron thermal conductivity below that predicted for a fully ionized gas. For lower altitudes the reduction factor increases rapidly in proportion to the neutral atmosphere concentration. At 100 km, for example, it is found that there is essentially no thermal conduction in the electron gas.

The ionospheric situation during the nighttime is generally described by low values of  $T_e$  in comparison with the daytime. However, because the electron density may decrease by a factor of  $10^2$  in the nighttime E-region, it is found that the effective conductivity is substantially less than the fully ionized gas conductivity through at least 220 km. As a specific example, the radar data of Doupanik and Nisbet (1966) taken during early morning hours in July, 1965, give at 150 km :  $T_e = 900$ °K,  $n_e = 1.1 \times 10^3$  cm<sup>-3</sup>. Using an 800°K neutral atmosphere model (J 1.5 3D) from Nicolet (1967), the calculated reduction due to electron-neutral collisions is in excess of 1800. Since the conditions used here are typical

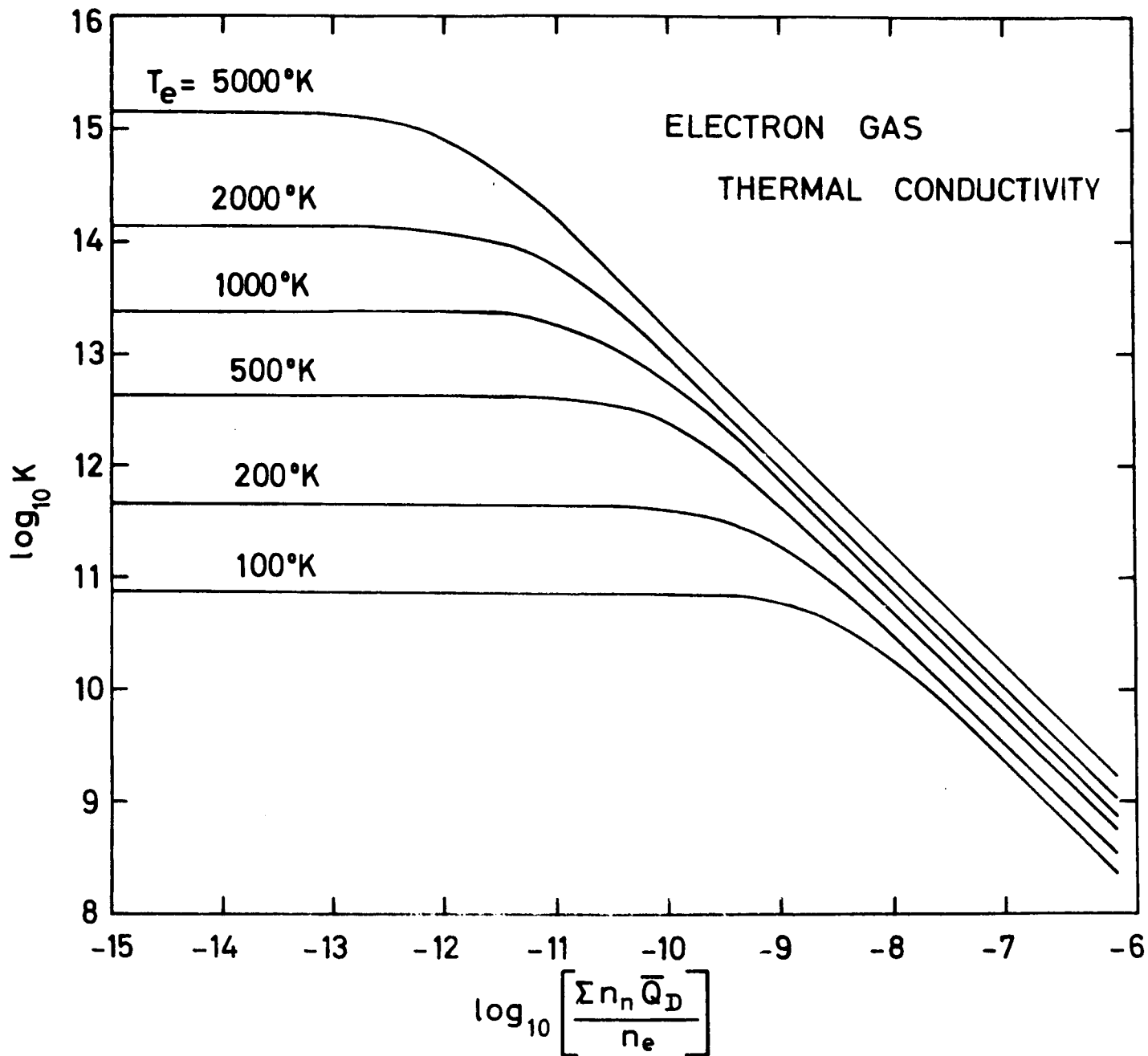


Fig. 1.- Electron Gas Thermal Conductivity. The effects of elastic collisions between electrons and neutral gas particles in reducing the thermal conductivity below the values for a fully ionized gas are seen in the bending of the curves from the horizontal.

of nighttime conditions at and below 150 km, it appears that there will essentially no thermal conduction acting in the nighttime electron gas within the ionospheric E-region.

These results indicate that there is a substantial decoupling of the high altitude electron gas from the important electron energy losses of the E-region. If electron-neutral collisions were not present, large amounts of electron thermal energy could be conducted rapidly to low altitudes. The net effect of this would be to lower the calculated values of  $T_e$  at higher altitudes,

There have been several laboratory measurements of the electron thermal conductivity (Goldstein and Sekiguchi, 1958 ; Sekiguchi and Herndon, 1958 ; Rostas, et al., 1963). In each of these experiments it was found that the electron thermal conductivity was independent of the neutral gas pressure. However, analysis of the experimental conditions using equation (20) has shown that in all cases the laboratory parameters of  $T_e$ ,  $n_e$  and  $n_n$  were not adequate to lead to any appreciable reduction in the conductivity below that predicted by equation (4).

### III.- APPLICATION

The reduction in the electron gas thermal conductivity brought about by the neutral atmosphere at altitudes below 200 km plays an important part in determining the calculated profiles of electron and ion temperature in the ionosphere. In this section the problem of equilibrium electron and ion temperature profiles is presented for steady state conditions between energy production, loss, and conduction. It is shown that the use of the uncorrected conductivity can lead to a considerable underestimate of charged particle temperatures at altitudes above 200 km.

For the following calculations, which are intended to emphasize the importance of the neutral atmosphere upon the electron gas thermal



conductivity, it is convenient to use models of the neutral and charged particle concentrations which are characteristic of quiet solar conditions. Further, it is assumed that the heating of the ionospheric electron gas is brought about only through the excess kinetic energy carried by electrons created through photoionization. The contribution of electron thermal energy stored at high altitudes within the exosphere, discussed by Geisler and Bowhill (1965b), is not included.

The rate of energy production within the electron gas arising from photoelectron heating for quiet solar conditions with solar zenith angle  $X = 0$  is listed in Table 1. These values have been derived in an analysis of electron and ion temperatures for the different solar conditions, the details of which will be presented in a subsequent paper.

The neutral atmospheric model used in this study has been taken from the work of Nicolet (1967) and corresponds to his model  $J_{1.5} 3D$  which, with a thermospheric temperature of  $800^\circ K$ , closely approximates the conditions found in the upper atmosphere during periods of minimum solar activity.

For a model of the electron and ion densities the rocket results of Bauer, et al. (1963) have been adopted for altitudes below 300 km with the assumption that only  $O^+$  ions are present, the effect of  $NO^+$  being neglected. Above 300 km the electron density,  $n_e$ , has been found through integration of the equation

$$n_e = n_{ea} \left( \frac{T_{ea} + T_{ia}}{T_e + T_i} \right) \exp \left\{ - \frac{m_i}{k} \int_a^z \frac{g dz}{(T_e + T_i)} \right\}, \quad (22)$$

which corresponds to a configuration of diffusive equilibrium when there is only a single ion species of mass  $m_i$  and temperature  $T_i$ . Since both  $T_e$  and  $T_i$  are altitude dependent in a complex fashion, the actual value of the electron density at an altitude  $z$  above the reference level  $a$  has been obtained numerically as an integral part of the calculations of the electron and ion temperature profiles. Table 1 lists the electron density values which

were found upon termination of the calculation of electron and ion temperatures.

**TABLE 1 : Model atmospheric parameters**

$z$ (km)	$P(\text{ev cm}^{-3} \text{ sec}^{-1})$	$n_e (10^5 \text{ cm}^{-3})$
116	$1.27 \times 10^3$	0.52
128	4.33	0.85
142	9.60	1.27
157	9.52	1.80
173	6.74	2.30
192	4.36	2.93
212	2.48	3.42
234	1.47	3.58
259	$8.84 \times 10^2$	3.41
286	5.20	2.88
316	3.03	2.57
349	1.62	2.24
386	$8.00 \times 10^1$	1.89
426	3.49	1.53
471	1.56	1.21
521	$6.08 \times 10^0$	0.94

The energy loss terms for the electron gas are composed of the elastic losses to O, N<sub>2</sub> and O<sup>+</sup> described in Banks (1966a) and inelastic rotational losses to N<sub>2</sub> (Mentzoni and Row, 1963) and O<sub>2</sub> (Mentzoni and Rao, 1965). Thus,

$$L_e = + 2.47 \times 10^{-18} n_e n(O) [T_e - T] T_e^{1/2} \\ + 1.41 \times 10^{-18} n_e n(N_2) [1 - 3.84 \times 10^{-2} T_e^{1/2}] T_e^{1/2} (T_e - T)$$

$$\begin{aligned}
 &+ 3.1 \times 10^{-14} n_e n(N_2) [T_e - T] T_e^{-1/2} \\
 &+ 1.0 \times 10^{-13} n_e n(O_2) [T_e - T] T_e^{-1/2} \\
 &+ 4.8 \times 10^{-7} n_e n(O^+) [T_e - T_i] T_e^{-3/2} \text{ ev cm}^{-3} \text{ sec}^{-1}, \quad (23)
 \end{aligned}$$

where  $T$  and  $T_i$  are the respective neutral and ion gas temperatures.

As shown by Hanson (1963) the ion gas temperature at high altitudes can be decoupled from the neutral gas temperature. Hence, to determine the effectiveness of ions in cooling the electron gas, it is necessary to consider the details of the ion energy balance equation which, in analogy with equation (1), can be written for a Maxwellian ion gas as,

$$\frac{\partial U_i}{\partial t} = P_i - L_i + \nabla \cdot (K_i \nabla T_i). \quad (24)$$

Here  $U_i = 3/2 n_i kT_i$  is the ion gas thermal energy,  $P_i$  is the rate at which energy is produced in the ion gas,  $L_i$  is the energy loss rate, and  $K_i$  is the ion thermal conductivity. For the present application it is found from the work of Chapman (1954) and Spitzer (1956) that the thermal conductivity of the ion component of a fully ionized gas of electrons and oxygen ions is smaller than the electron conductivity by a factor of approximately 172.

In fact, if an extension of the ion conductivity is made to include the effects of ion-neutral collisions a much larger reduction follows. Hence, for convenience in this problem we take  $K_i = 0$  and assume that the ion temperature is a function only of local effects. A more precise analysis of the ion temperature problem taking into account the important effects of ion thermal conduction will be presented in a subsequent paper.

The rate at which the oxygen ion gas gains energy from the electron gas as a result of electron-ion collisions is

$$P_i = 4.8 \times 10^{-7} n_e n(O^+) [T_e - T_i] T_e^{-3/2} \text{ ev cm}^{-3} \text{ sec}^{-1}. \quad (25)$$

For oxygen ions the energy loss is composed of terms corresponding to resonance charge exchange between  $O^+$  ions and O atoms and elastic collisional loss to  $N_2$  and  $O_2$  molecules. The appropriate expressions, taken from Banks (1966b) are,

$$\begin{aligned} L_i = & 2.1 \times 10^{-15} n(O^+) n(O) [T_i + T]^{1/2} [T_i - T] \\ & + 6.6 \times 10^{-14} n(O^+) n(N_2) [T_i - T] \\ & + 6.5 \times 10^{-14} n(O^+) n(O_2) [T_i - T] \text{ ev cm}^{-3} \text{ sec}^{-1}. \end{aligned} \quad (26)$$

Equations (25) and (26), when used in conjunction with equation (24) and a model of the neutral atmosphere, permit the solution of the coupled electron and ion time dependent energy balance equations.

A direct approach has been taken to solve the coupled time dependent and ion energy balance equations. Use has been made of the implicit integration method of Diaz (1958) for parabolic equations to obtain a set of linearized mesh equations which lead to the time development of the electron and ion temperature profiles. Because the electron temperature at low altitudes is strongly coupled to the neutral gas temperature, it is not possible to choose the time integration elements in an arbitrary manner without introducing the possibility of numerical oscillation and eventual divergence away from the correct solution. For the example given here it was found that at 122 km the maximum permissible time increment was  $9.8 \times 10^{-2}$  seconds. Hence, a large number of calculations were necessary to enhance the long term conduction effects.

The boundary and initial conditions necessary to solve the second order, non-linear electron energy balance equation are related to the assumed sources of thermal energy in the upper atmosphere. The upper boundary condition is connected to the flux of electron thermal energy entering into the ionosphere from the protonosphere. This problem has been discussed by Geisler

and Bowhill (1965b). For the purpose of illustrating the effect of the thermal conductivity upon the temperature profiles, it is not necessary to include this additional energy source and it is assumed that there is no gradient in electron temperature above 1000 km.

The lower boundary determines the flux of electron thermal energy to the lower atmosphere. Because the effective thermal conductivity is small at altitudes below 125 km, the electron temperature is determined by the local characteristics of electron energy production and loss. For the present problem, which considers only photoionization, it is adequate to assume that  $T_e = T$  at 110 km. In fact, there is very little effect upon the calculated temperature profiles if this condition is applied at higher (up to 135 km) or, of course, lower altitudes (below 100 km).

We consider first the temperature profiles which are characteristic of minimum solar activity under equilibrium conditions with a constant energy production. The results, shown in Figure 2, were obtained by relaxing the electron and ion temperatures from initial profiles which satisfied the equations  $P_e = L_e$  and  $P_i = L_i$ ; that is, the solutions to the two energy balance equations which ignore the effect of thermal conduction. Nearly 1000 seconds of elapsed time were required to reach the indicated steady-states, defined here to be the point when  $(dT_e/dt) < 2 \times 10^{-6} \text{ } ^\circ\text{K sec}^{-1}$  at all altitudes. Curve 1, indicated in the figure, is the initial electron temperature profile. Curve 2 is the steady state electron temperature which results from the use of an electron gas thermal conductivity which includes the effect of the neutral atmosphere. Likewise, curve 3 is the temperature profile which results if the conductivity is taken to be that of a fully ionized gas, independent of the neutral particle concentration. Curves 4 and 5 are, respectively, the profiles of ion and neutral particle temperature.

The effects of the effective thermal conductivity are clearly evident at both low and high altitudes. By permitting a large energy flux to

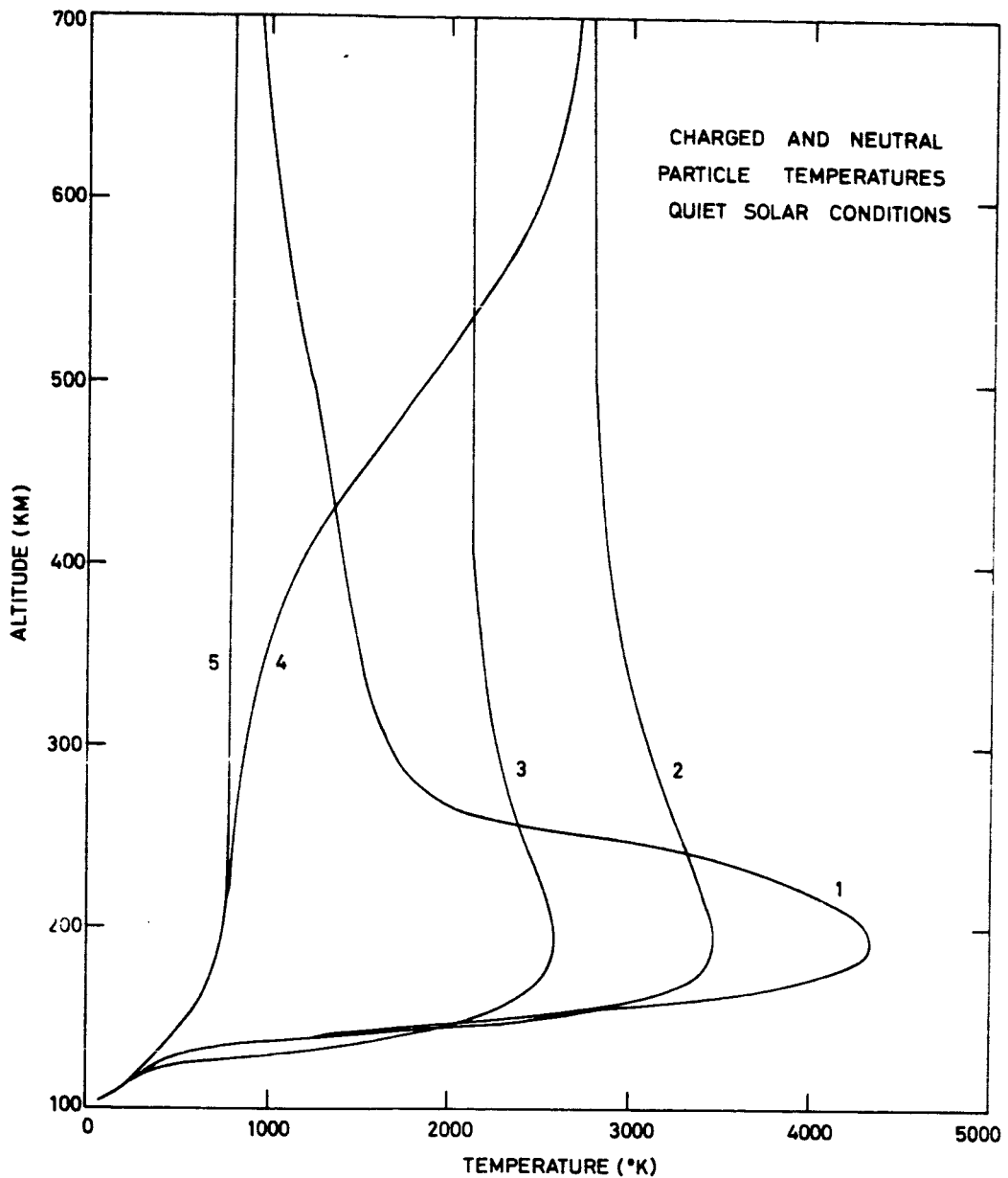


Fig. 2.- Charged and Neutral Gas Temperatures Typical of Quiet Solar Conditions. Curve 1 is the solution to the daytime electron energy balance equation which ignores heat conduction through the electron gas. When heat conduction is included the electron temperature profile is substantially changed, as shown by curves 2 and 3. Curve 2 results when an effective thermal conductivity including the effects of electron-neutral particle collisions is introduced. For curve 3 the thermal conductivity for a fully ionized gas was used. Curves 4 and 5 are the calculated ion and the assumed neutral atmosphere temperatures.

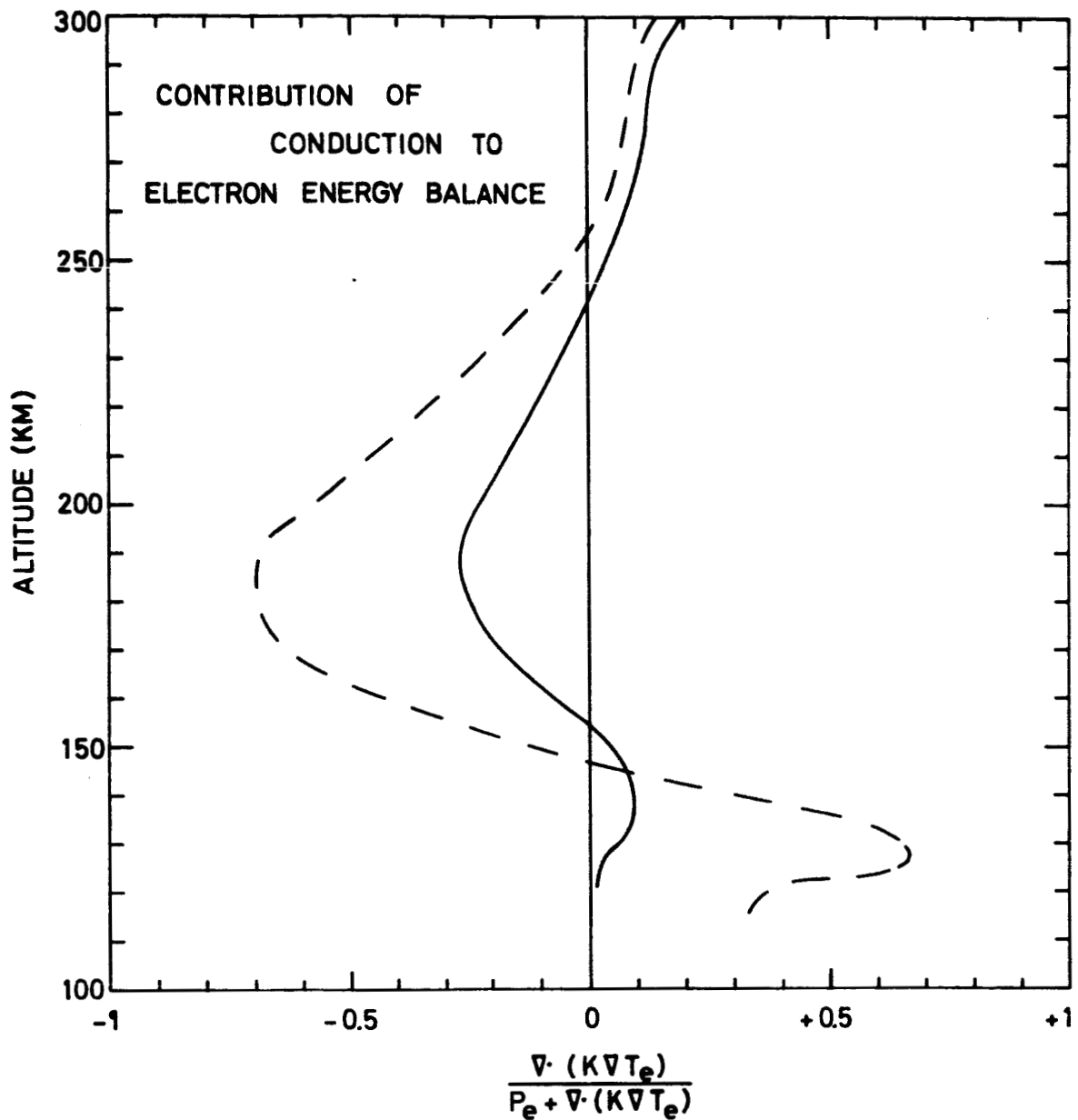
flow to the ionospheric E region, the conductivity for a fully ionized gas results in a substantial increase in  $T_e$  at altitudes below 145 km above those values found using the effective conductivity. This energy drain to low altitudes lowers the peak value of  $T_e$  from 3400 °K to 2600 °K at 190 km. This decrease is maintained at all higher altitudes through the action of thermal conduction and results in a substantial difference of 800°K for the high altitude isothermal regions.

The conduction term in the electron energy balance equation can act at any given altitude as an effective source or sink of thermal energy. With a coefficient of thermal conductivity which is reduced at low altitudes by the presence of the neutral atmosphere the net effect of this term can be substantially diminished. To show this the ratio

$$\frac{\nabla \cdot (K \nabla T_e)}{P_e + \nabla \cdot (K \nabla T_e)}$$

has been evaluated at each altitude for both the fully ionized and the density corrected models of the electron thermal conductivities. Since  $P_e$  is the same in both cases this ratio gives a direct indication of the importance of energy conduction in the overall energy balance. The results are presented in Figure 3 and show that the conduction term is substantially reduced in importance when neutral atmospheric effect are included. In fact, even for this daytime model the atmosphere corrected conduction term (solid curve) is seen to constitute less than 10% of the ratio for all altitudes below 160 km. In contrast, it is seen that the use of the fully ionized gas thermal conductivity (dotted curve) leads to a ratio of 0.66 even at the comparatively low altitude of 128 km.

The effect of the neutral atmosphere upon the electron thermal conductivity can also be shown by considering the reduction of the conductivity below that predicted by equation (4). Thus, we take



**Fig. 3.-** Contribution of Conduction to the Electron Energy Balance. The dashed curve indicates the relative importance of heat conduction in the electron energy balance at low altitudes when only the fully ionized conductivity is used. By including the effects of electron-neutral collisions the conduction term is greatly reduced, as shown by the solid curve.



$$K = \alpha T_e^{5/2} \quad (27)$$

where for a fully ionized gas  $\alpha = 7.7 \times 10^5 \text{ ev cm}^{-1} \text{ }^\circ\text{K}^{-1/2} \text{ sec}^{-1}$  but at lower altitudes smaller values prevail. Curve 1 in figure 4 shows a plot of the quantity  $\alpha$  as a function of altitude for the steady-state model of effective thermal conductivity discussed here. Curve 2 represents the density independent expression. A reduction by at least a factor of ten is seen at all altitudes below 180 km when electron-neutral collisions are considered. The local maximum centered around 128 km is caused by changes in the ratio  $T_e^2/n_e$  brought about by rapidly increasing densities and temperatures.

Due to the particular form of the electron-ion elastic energy exchange rate there is a maximum rate at which the ions can cool the electron gas which occurs at  $T_e = 3 T_i$ . For  $T_e < 3 T_i$ , the energy loss rate is an increasing function of  $T_e$ . When  $T_e > 3 T_i$  the electron-ion loss rate decreases with increasing  $T_e$  and the ion gas is unable to constrain the electron temperature. In this case a stable equilibrium must depend upon the combined effects of conduction and electron-neutral energy transfer. Since at high altitudes the losses to neutral particles are small, the introduction of sufficient energy into the electron gas to cause  $T_e$  to rise above the critical point into the "runaway" region must result in an increase in  $T_e$  to the point where conduction alone carries the excess thermal energy to other sources of energy loss.

Another factor in this process is the possibility of the ion temperature rising above the neutral gas temperature. As the electron temperature attempts to rise in response to the local energy production, it will raise the ion temperature in the manner of two coupled, non-linear springs. This will diminish the electron-ion energy transfer rate and further contribute to the rise in electron temperature. At the lower altitudes, where neutral losses become dominant, the point where  $T_e = 3 T_i$  loses its physical significance since only small increases in  $T_e$  are required

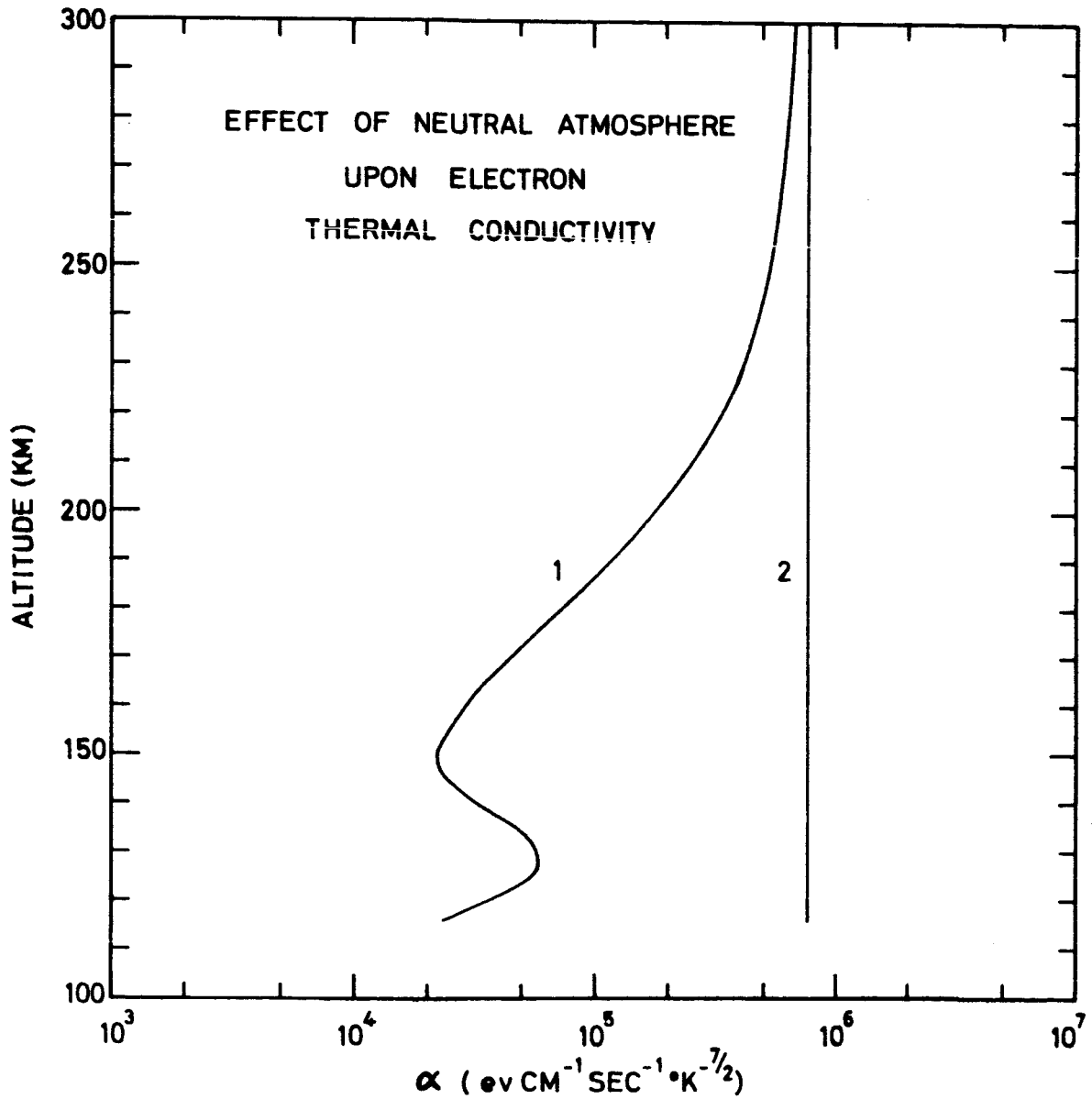


Fig. 4.- Effect of the Neutral Atmosphere Upon the Electron Thermal Conductivity. By taking  $K = \alpha T_e^{5/2}$  ev cm<sup>-1</sup> °K sec<sup>-1</sup>, it is possible to calculate  $\alpha$  as a function of altitude. If electron-neutral particle collisions are ignored,  $\alpha = 7.7 \times 10^5$ , as shown by curve 2, while when collisions are introduced,  $\alpha$  is significantly reduced, as indicated by curve 1. The local maximum in  $\alpha$  at 128 km is produced by changes in the ratio  $T_e^2/n_e$ .

to match the decrease in the electron-ion cooling rate.

The problem of electron runaway has been considered previously by Geisler and Bowhill (1965a) in their study of electron temperatures during a period of minimum solar activity. By equating local energy production and electron-ion energy loss according to the equation

$$P_e = 4.7 \times 10^{-7} n_e^2 \frac{(T_e - T_i)}{T_e^{3/2}} \text{ ev cm}^{-3} \text{ sec}^{-1}, \quad (28)$$

they found values of  $P_e/n_e^2$  which implied that the runaway condition was present over their entire altitude range above 200 km. It must be pointed out, however, that it is not possible to use equation (28) to evaluate the condition of runaway since at all times conduction is constantly acting to reduce the electron temperatures in the peak energy production regions. Thus, while  $P_e/n_e^2$  may indicate runaway for equation (28), when the correct equation

$$P_e = 4.7 \times 10^{-7} n_e^2 \frac{(T_e - T_i)}{T_e^{3/2}} - \nabla \cdot (K \nabla T_e) \text{ ev cm}^{-3} \text{ sec}^{-1}, \quad (29)$$

is investigated, there may be a substantial reduction in  $T_e$  below the value  $3 T_i$ . Thus, an inspection of the calculated profiles of Geisler and Bowhill (1965a) indicates that above 200 km  $T_e < 3 T_i$  and runaway had not been reached in their model. For the present calculations it is found that  $T_e < 3 T_i$  in the profiles for the corrected and uncorrected thermal conductivity expressions. In both cases the high altitude electron temperature profiles are isothermal in nature.

#### IV.- SUMMARY AND CONCLUSIONS.

In calculating profiles of electron temperature in the upper atmosphere the effect of electron thermal conduction must be included. To determine the coefficient of electron thermal conductivity appropriate

to the upper atmosphere, an analysis has been made of electron heat transport on the basis of a mean free path approach to evaluate the relative importance of charged and neutral particle collisions. An explicit expression for the thermal conductivity has been derived using the results of Spitzer and Härm (1952), Chapman and Cowling (1952), and Banks (1966a). At altitudes above 250 km it is found that the electron gas thermal conductivity is unchanged by the neutral atmosphere and that the expression derived by Spitzer and Härm (1952) for a fully ionized gas can be used.

At lower altitudes electron-neutral particle collisions become effective in reducing the electron mean free path and there occurs a significant decrease in the value of the thermal conductivity below that predicted for a fully ionized gas. In addition, the temperature dependence changes from  $T_e^{5/2}$  to a more moderate  $T_e^{1/2}$ . Since the effective thermal conductivity depends upon both the electron temperature and the weighted ratio of neutral to electron densities, it is found that at night, when the electron densities are greatly reduced, there will be essentially no electron thermal conduction below 150 km.

During the daytime the electron densities are large and the effect of electron-neutral collisions upon the thermal conductivity becomes less. However, because  $T_e$  is higher during the day, the ratio  $T_e^2/n_e$  must be evaluated in each case. From the calculation given in Section II for quiet solar conditions it is found that the effective conductivity was a factor of 10 less than the fully ionized conductivity up to an altitude of 180 km, while at 225 km there was still a factor of 5 involved.

To illustrate the effect of the neutral atmosphere on steady-state temperature profiles, a model characteristic of quiet solar conditions was presented. It was shown that by reducing the flow of electron

thermal energy to low altitudes there followed a significant rise in the high altitude electron temperature. Hence, previous calculations which have been made to derive a correspondance between measured values of electron temperature and rates of electron energy production probably over-estimate the actual energy given to the electron gas.

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