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EFFECTS OF LIQUID SLOSH ON RENDEZVOUS DYNAMICS

RESEARCH REPORT

BROWN

ADVANCED STUDIES DEPARTMENT

ENGINEERING COMPANY, INC. • 300 SPARKMAN DRIVE, N.W. • HUNTSVILLE, ALABAMA/35805

TECHNICAL REPORT
EFFECTS OF LIQUID SLOSH ON
RENDEZVOUS DYNAMICS

by

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SUMMARY

This report presents a mathematical model for investigation of the effects of liquid slosh on rendezvous dynamics. The Euler-Lagrange formulation was used to describe the motions of the carrier and target vehicles. The slosh dynamics were simulated by two pendulums in the vehicle. The sophisticated nature of this mathematical model is ideal for control and guidance system studies, and permits constant monitoring of propellant usage.

This report also includes a study of low-gravity bubble dynamics. This study presents interesting possibilities for removal of liquid slosh during critical periods.

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DEFINITION OF SYMBOLS

Symbol	Definition
x, y, z	Inertial coordinate system origin, centered in the earth
$\bar{x}, \bar{y}, \bar{z}$	Body centered coordinate system, axes parallel to the inertial axes and centered in the vehicle
ξ, η, ζ	Body fixed coordinate axes, centered in and fixed with respect to the vehicle
$\hat{i}, \hat{j}, \hat{k}$	Unit vectors in the x, y, z directions respectively
$\hat{\xi}, \hat{\eta}, \hat{\zeta}$	Unit vectors in the ξ, η, ζ directions respectively
α, β, γ	Pitch, roll, and yaw angles, respectively required to transform $\bar{x}, \bar{y}, \bar{z}$ into ξ, η, ζ degrees
T_k	Kinetic energy
T_p	Gravitational potential energy
I	Moment of inertia
Q	Generalized force
g_0	Gravitational constant, 9.80665 m/s^2
r_e	Earth radius
v	Velocity
m, M	Mass
r	Distance from center of earth to the vehicle
c	Constant of integration
δ	Constant of integration
A_{ij}	Transformation matrix

DEFINITION OF SYMBOLS - cont'd

Symbol	Definition
k	Function defined by equation (25)
F	Total force on vehicle
q	Generalized coordinate
n	Gravitational force
Λ	Control, guidance, and thrust forces on vehicle
l	Pendulum length
b	Distance of pendulum pivot from center of mass of vehicle and the stationary fuel
h	One half of tank length
a	Radius of tank
ϵ_p	Zero of $J_1^1(\epsilon) = 0$ a bessel function
A_p	Fraction of fuel mass in p th slosh mode
B_p	Function defined by equation (54)
L	Distance from top of tank
V_c	Terminal velocity of a bubble
N	Number of bubbles
ℓ	Diameter of a bubble
σ	Bond number
λ	Proportionality constant
K	Velocity proportionality constant

DEFINITION OF SYMBOLS - cont'd

Symbol	Definition
t	Time
f	Fraction of bubbles escaping fluid
Subscript	Definition
t	Target vehicle
o	Center of mass of fuel and vehicle
1, 2	Relative to pendulums 1 and 2 respectively
i, j	Free and dummy subscripts
p	Mode of oscillation
P	Most probable
F	Fuel
x, y, z	Relative to the inertial axes
α, β, γ	Relative to the pitch, roll, and yaw angles
ξ, η, ζ	Relative to the ξ, η, ζ axes respectively
r	Relative to the center of mass of the stationary fuel

Time derivatives are indicated by the dot notation.

Vector quantities are indicated by the arrow (\rightarrow) notation.

SECTION I. INTRODUCTION

Future missions such as the Manned Mars Landing may require as many as twelve successive rendezvous and docking maneuvers, while missions of more immediate interest will require one or two such maneuvers. The rendezvous and docking maneuver places stringent requirements on the control and guidance systems and the trajectories. Within the past few years, considerable effort has been expended on docking dynamics and liquid slosh studies. However, little work has been completed on the effects of liquid slosh on rendezvous and docking dynamics.

The purpose of this analysis is to present a sophisticated, straightforward approach to the mathematical description of the effects of liquid slosh modes on rendezvous dynamics. The primary objective of the liquid slosh studies reviewed for this analysis has been to define the basic phenomena associated with liquid motion and to determine the effects of such motion on vehicle dynamics during the boost phase. Liquid slosh during the boost phase has led to control and stabilization problems which have caused mission failures.

Observations of early flights were sufficient to define the elemental sources of slosh instabilities and to suggest areas in which immediate basic research was required. However, little work was completed on low-gravity liquid slosh and its effects on vehicle dynamics.

Due to the complexity of liquid behavior, the boundary conditions imposed by the tank walls, and the presence of dissipative forces caused by external damping devices, the derivation of analytical expressions which describe the liquid motion presents a difficult problem. The most promising approach, at present, to the problem of incorporating liquid slosh dynamics in the vehicle equations of motion is to simulate the liquid motion by means of an analogous mechanical system.

The simulation of fluid dynamics forces and moments by analogous system is assured by the term-by-term comparison of linearized fluid dynamic equations with the equations of motion of the analogy.

In this analysis, liquid slosh dynamics are simulated by a pendulum analogy. Analytically the liquid motion is represented by an arbitrary number of pendulums and a fixed mass. However, since the dynamic effects of the n th pendulum varies as $1/n^4$ only one or two pendulums need be considered. The use of two pendulums permits the simulation of two liquid slosh modes in a given tank or the simulation of two tanks.

Other investigators have suggested using a spring mass system rather than the pendulum system as the mechanical analogy. However, the two analogies have been shown to be equivalent for small displacements and both provide excellent simulation of the stiffness and inertial terms for undamped liquid oscillations. The pendulum analogy is a better approximation in this application since the pendulum frequencies exhibit the same dependence on the longitudinal acceleration of the carrier vehicle as do the natural liquid frequencies.

SECTION II. PROBLEM FORMULATION

A. General

The mathematical formulation of the rendezvous problem was accomplished in two parts: (1) derivation of the target vehicle equations of motion, and (2) development of the carrier vehicle equations of motion including liquid slosh modes. These analyses are presented in detail in Section IV.

The Lagrangian formulation presents a sophisticated, straight forward approach to the rendezvous problem. The explicit nature of the thrust and control forces in the equations of motion is conducive to control system studies, and permits continuous observation of propellant usage.

The equations of motion are based on the following assumptions:

1. Spherical Earth
2. Inverse Gravity Law
3. Forces on the Carrier Vehicle Include:
Thrust, Fuel Slosh, and Gravity
4. Force on Target Vehicle: Gravity
5. Target and carrier vehicle are Initially
Close to Identical Orbits.

B. Vehicle Models

The equations of motion for each vehicle are derived in a general form to permit modification between computer runs. The parameters which determine vehicle dynamics and simulate different vehicle configurations can be readily changed.

The target is an arbitrary, passive vehicle constrained to move in a circular orbit, maintaining a constant attitude relative to the radius vector. The initial orbital position and velocity are inputs to the problem.

The carrier vehicle is an arbitrary vehicle with two spherical pendulums attached to the longitudinal axis, simulating liquid slosh. The length, and masses of the pendulums are determined by comparison with the linearized fluid dynamic equations. The comparisons are shown in Section V.

The masses, moments of inertia, orientation, and initial orbital position and velocity are inputs to the problem.

SECTION III. COORDINATE SYSTEMS

A. Inertial System

An inertial, Cartesian, coordinate system (xyz) is defined as being centered in the earth, with the z axis through the North Pole, and the x and y axes completing a right handed triad, as shown in Figure 1. The x axis passes through the vernal equinox.

B. Body Centered Coordinates

The body centered coordinate system ($\bar{x}\bar{y}\bar{z}$) is defined as having the origin at the center of mass of the vehicle, and each axis parallel to the corresponding axis of the inertial coordinate system, as shown in Figure 1. The system is denoted by

$$\bar{x}^i = \begin{matrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{matrix} . \quad (1)$$

The inertial and body centered systems are related through the linear translations

$$\bar{x}^i = x^i - x_0^i \quad i = 1, 2, 3, \quad (2)$$

where the x_0^i are the coordinates of the center-of-mass of the vehicle in the inertial coordinate set.

C. Body Fixed Coordinates

A body fixed, Cartesian, coordinate system (ξ, η, ζ) is defined as having the origin at the center of mass of the vehicle. The ξ axis is the longitudinal axis of the vehicle, and the η and ζ axes in the body completing the set. The ζ axis is always perpendicular to the radius vector.

The relationship between the body centered and body fixed coordinate systems is the three dimensional rotation

$$\xi^i = A_{ij} \bar{x}^j \quad i, j = 1, 2, 3. \quad (3)$$

The necessary rotation and transformation matrices are shown in Figure 2. The rotations are carried out in the order of pitch (α), roll (β), and

yaw (γ). The complete rotation matrix has the form

$$A_{ij} = \begin{vmatrix} \cos \alpha \cos \gamma & & \cos \alpha \sin \beta \sin \gamma \\ + \sin \alpha \sin \beta \sin \gamma & \cos \beta \sin \gamma & - \sin \alpha \cos \gamma \\ \sin \alpha \sin \beta \cos \gamma & & \cos \alpha \sin \beta \cos \gamma \\ - \cos \alpha \sin \gamma & \cos \beta \cos \gamma & + \sin \alpha \sin \gamma \\ \sin \alpha \cos \beta & - \sin \beta & \cos \alpha \cos \beta \end{vmatrix} \cdot (4)$$

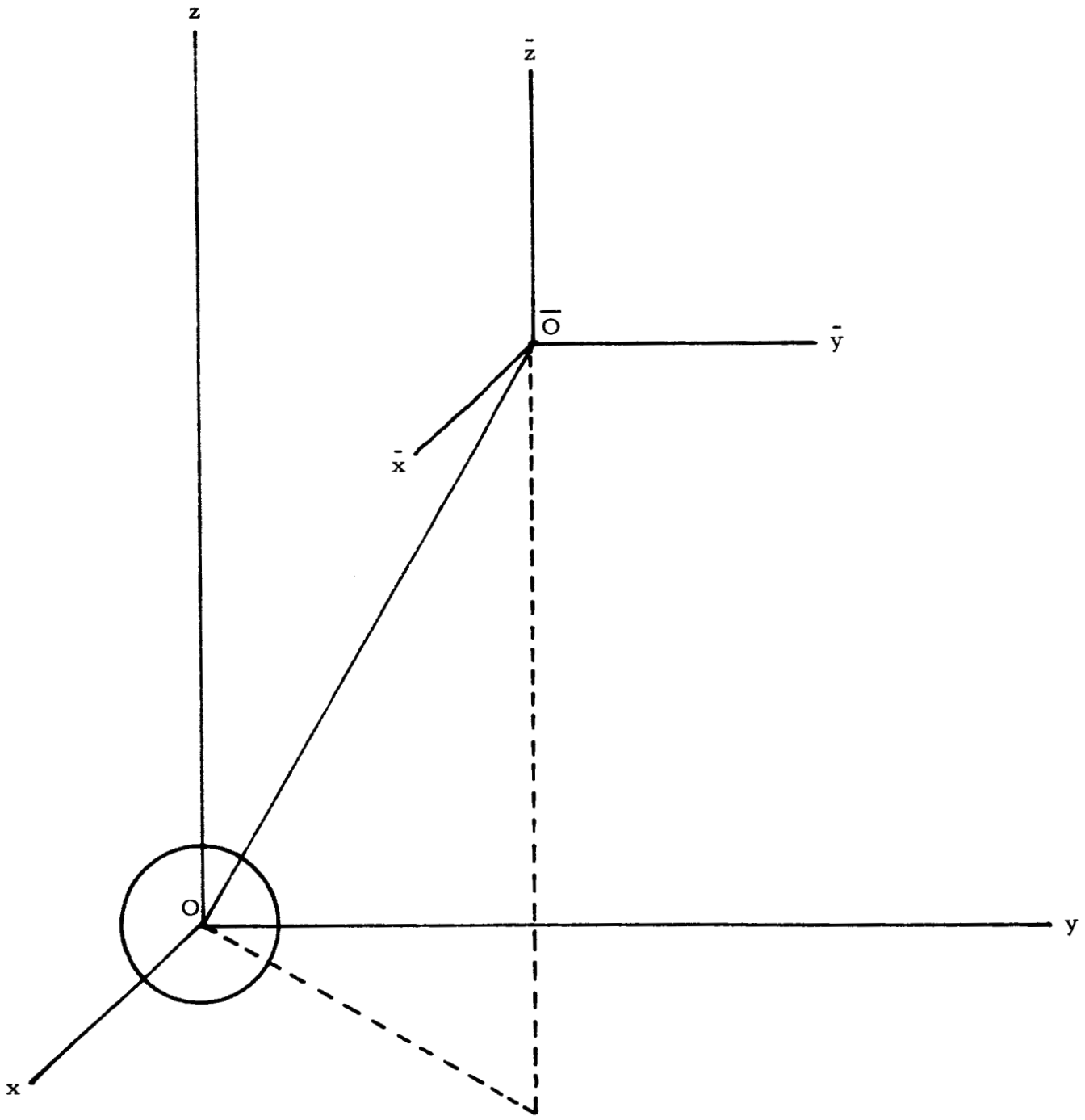
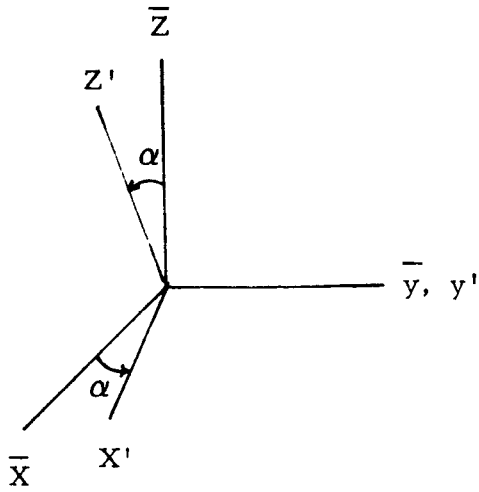
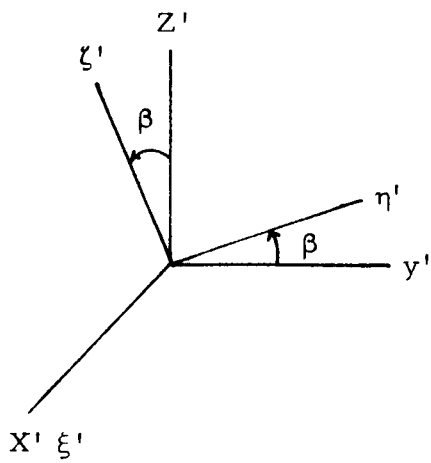


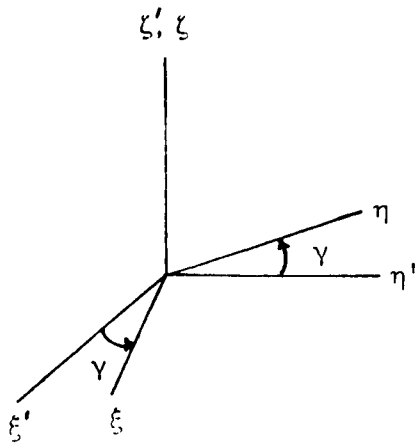
FIGURE 1. EARTH CENTERED AND BODY CENTERED COORDINATE SYSTEMS



$$X'^i = \begin{vmatrix} \cos \alpha & 0 & -\sin \alpha \\ 0 & 1 & 0 \\ \sin \alpha & 0 & \cos \alpha \end{vmatrix} \bar{X}^i$$



$$\xi'^i = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos \beta & \sin \beta \\ 0 & -\sin \beta & \cos \beta \end{vmatrix} X'^i$$



$$\xi^i = \begin{vmatrix} \cos \gamma & \sin \gamma & 0 \\ -\sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{vmatrix} \xi'^i$$

FIGURE 2. COORDINATE ROTATIONS

SECTION IV. EQUATIONS OF MOTION

A. Target Vehicle

The target vehicle is moving in a circular orbit in the earth's gravitational field and maintains a constant orientation with respect to the radius vector. The equations of motion of the target vehicle are derived in the inertial system to simplify the application of the termination conditions.

The total kinetic energy of the vehicle and the gravitational potential field of the spherical earth are written as

$$T_k = 1/2 [m_t (\dot{x}_t^2 + \dot{y}_t^2 + \dot{z}_t^2) + I_\xi \dot{\beta}^2 + I_\eta \dot{\alpha}^2 + I_\zeta \dot{\gamma}^2] \quad (5)$$

$$T_p = - \frac{m_t g_0 r_e^2}{(x_t^2 + y_t^2 + z_t^2)^{1/2}} \quad (6)$$

The form of the Euler-Lagrange equation used in the derivation is

$$\frac{d}{dt} \left(\frac{\partial T_k}{\partial \dot{q}_i} \right) - \frac{\partial T_k}{\partial q_i} = Q_i, \quad (7)$$

where the Q_i are the generalized forces on the vehicle, for this case-gravity. Substituting, the following six equations of motion are found

$$\ddot{x}_t + \frac{g_0 r_e^2 x_t}{r^3} = 0 \quad (8)$$

$$\ddot{y}_t + \frac{g_0 r_e^2 y_t}{r^3} = 0 \quad (9)$$

$$\ddot{z}_t + \frac{g_0 r_e^2 z_t}{r^3} = 0 \quad (10)$$

$$I_\xi \ddot{\beta} = 0 \quad (11)$$

$$I_\eta \ddot{\alpha} = 0 \quad (12)$$

$$I_\zeta \ddot{\gamma} = 0. \quad (13)$$

Since the moments of inertia are constant and not zero, it is possible to conclude that $\dot{\alpha}$, $\dot{\beta}$, and $\dot{\gamma}$ are constants.

The constraints on the target vehicle may be written as

$$\vec{v} \cdot \vec{r} = 0, \quad (14)$$

$$\vec{\xi} \cdot \vec{r} = 0, \quad (15)$$

and
$$\vec{\zeta} \cdot \vec{r} = 0. \quad (16)$$

The $\vec{\xi}$ and $\vec{\zeta}$ unit vectors can be written from the transformation matrix (Equation 4)

$$\begin{aligned} \hat{\xi} = & (\cos \gamma \cos \alpha + \sin \alpha \sin \beta \sin \gamma) \hat{i} + \sin \gamma \cos \beta \\ & \hat{j} + (\sin \gamma \sin \beta \cos \alpha - \sin \alpha \cos \gamma) \hat{k}, \end{aligned} \quad (17)$$

$$\hat{\zeta} = \sin \alpha \cos \beta \hat{i} - \sin \beta \hat{j} + \cos \beta \cos \alpha \hat{k} \quad (18)$$

thus, the solutions to the equations of motion may be written as

$$x_t = c_x \sin (kt + \delta_x) \quad (19)$$

$$y_t = c_y \sin (kt + \delta_y) \quad (20)$$

$$z_t = c_z \sin (kt + \delta_z) \quad (21)$$

$$\alpha = \alpha_0 + c_\alpha t \quad (22)$$

$$\beta = \beta_0 + c_\beta t \quad (23)$$

$$\gamma = \gamma_0 + c_\gamma t \quad (24)$$

where

$$k = \left(\frac{g_0 r_e^2}{r^3} \right)^{1/2} \quad (25)$$

and the boundary conditions in explicit form are

$$\begin{aligned} c_x^2 \sin^2 (kt + \delta_x) + c_y^2 \sin^2 (kt + \delta_y) + \\ c_z^2 \sin^2 (kt + \delta_z) = 0, \end{aligned} \quad (26)$$

$$\begin{aligned}
& c_x \sin (kt + \delta_x) [\cos \gamma \cos \alpha + \sin \alpha \sin \beta \sin \gamma] + \\
& c_y \sin (kt + \delta_y) \sin \gamma \cos \beta + \\
& c_z \sin (kt + \delta_z) [\sin \gamma \sin \beta \cos \alpha - \sin \alpha \cos \gamma] = 0, \quad (27)
\end{aligned}$$

$$\begin{aligned}
& c_x \sin (kt + \delta_x) \sin \alpha \cos \beta - c_y \sin (kt + \delta_y) \sin \beta \\
& + c_z \sin (kt + \delta_z) \cos \beta \cos \alpha = 0. \quad (28)
\end{aligned}$$

The boundary conditions eliminate three constants of integration, leaving nine to be evaluated for the complete solution of the problem. These represent the nine orbital elements necessary to specify an orbit, and are obtained from the initial position, velocity, and orbit plane.

B. Carrier Vehicle

The carrier vehicle is represented by three masses (M , m_1 , and m_2) in earth orbit. The point masses m_1 and m_2 are attached by rigid massless rods of length l_1 and l_2 respectively to the points b_1 and b_2 on the longitudinal (ξ) axis of the vehicle. M is a rigid body representing the vehicle and the motionless portion of the liquids (Figure 3).

A problem development similar to that in Section IV, Part A, is used to describe the motion of this system. In this use, however, the generalized forces include not only gravity but thrust and controls and have the form

$$Q_i = \sum_{j=0}^2 \vec{F}_j \cdot \frac{\partial \vec{r}_j}{\partial q_i} \quad (29)$$

where the \vec{F}_j are the forces acting on the j th mass, the \vec{r}_j is the radius vector from the origin of the inertial system to the center of mass of the j th mass, and the q_i are the state variables. The total force is written

$$\vec{F}_j = \vec{N}_j + \vec{\Lambda}_j \quad j = 0, 1, 2 \quad (30)$$

where the \vec{N}_j is the gravitational force on the j th mass and the $\vec{\Lambda}_j$ represent the unspecified forces on the j th mass. For $j=0$, the forces would be: control and guidance, thrust, drag, etc. For $j=1, 2$ only internal forces such as viscous damping can be represented, and are not considered here.

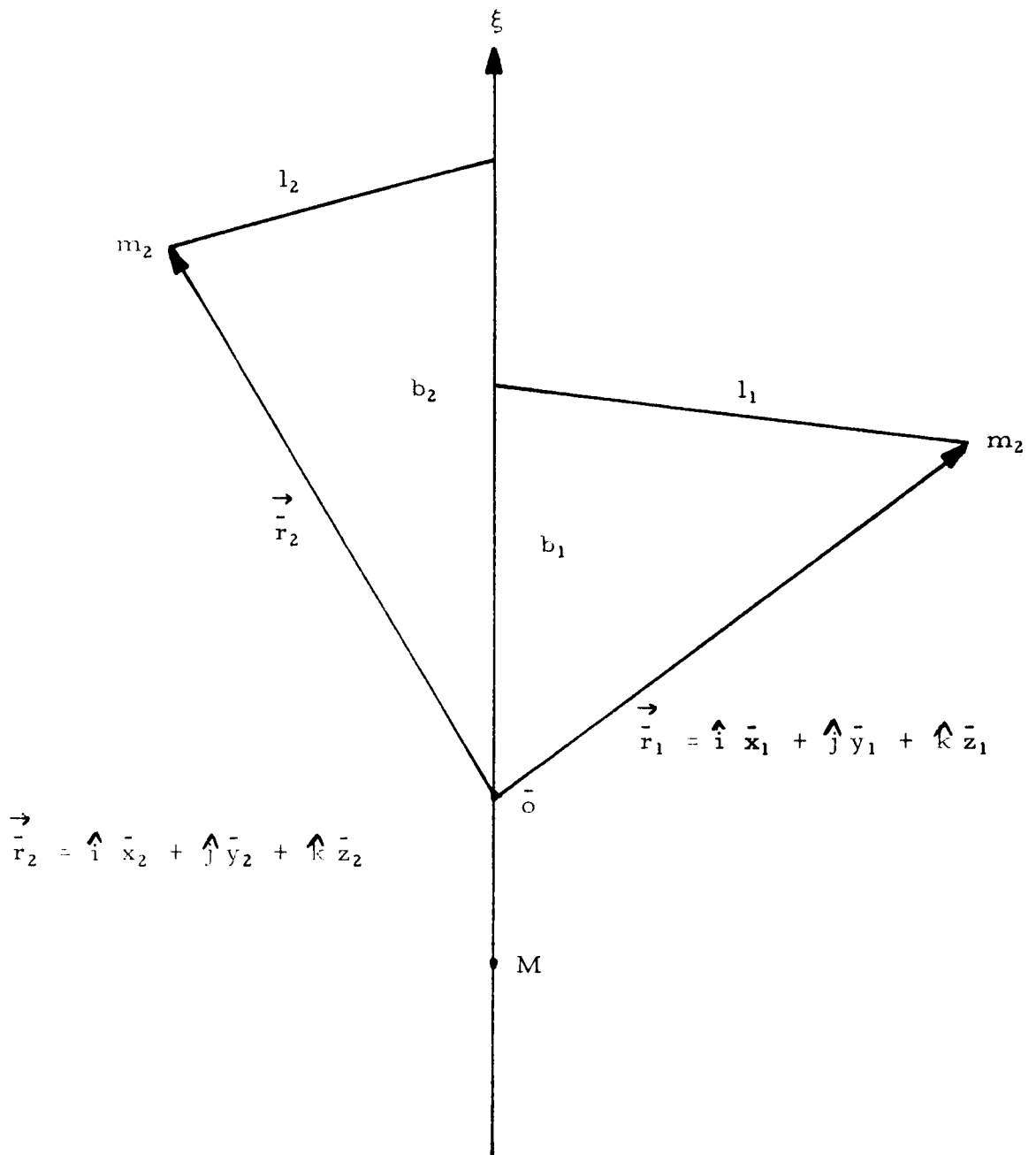


FIGURE 3. CARRIER VEHICLE MODEL

The two pendulums (m_1 and m_2) each have two degrees of freedom, with six degrees of freedom for M . A total of ten equations of motion are required to completely describe the system.

The constraint equations for m_1 and m_2 are, when solved for ζ_1 and ζ_2 ,

$$\zeta_1^2 = [l_1^2 - (\xi_1 - b_1)^2 - \eta_1^2], \quad (31)$$

$$\zeta_2^2 = [l_2^2 - (\xi_2 - b_2)^2 - \eta_2^2]. \quad (32)$$

The force on the three masses, because of the earth's gravitational field, are given by

$$\vec{N}_0 = - \frac{g_0 r_e^2 M \vec{r}_0}{r_0^3}, \quad (33)$$

$$\vec{N}_1 = - \frac{g_0 r_e^2 m_1 \vec{r}_1}{r_1^3}, \quad (34)$$

$$\vec{N}_2 = - \frac{g_0 r_e^2 m_2 \vec{r}_2}{r_2^3}. \quad (35)$$

Gravity gradient effects may be neglected by setting $\vec{r}_0 = \vec{r}_1 = \vec{r}_2$ for the \vec{N}_i . This has been done in the following equations.

The total kinetic energy of the systems is

$$T_k = 1/2 [M \dot{r}_0^2 + m_1 \dot{r}_1^2 + m_2 \dot{r}_2^2 + I_\xi \dot{\beta}^2 + I_\eta (\dot{\alpha}^2 + \dot{\gamma}^2)], \quad (36)$$

where

$$\vec{r}_0 = x_0 \hat{i} + y_0 \hat{j} + z_0 \hat{k} \quad (37)$$

and for

$$i = 1, 2,$$

$$\vec{r}_i = \{ X_0 + (\cos \alpha \cos \gamma + \sin \alpha \sin \beta \sin \gamma) \xi_i + (\sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma) \eta_i + \sin \alpha \cos \beta \zeta_i \} \hat{i} \quad (38)$$

$$\begin{aligned}
& + \{Y_0 + \cos \beta \sin \gamma \xi_i + \cos \beta \cos \gamma \eta_i - \sin \beta \zeta_i\} \hat{i} \\
& + \{Z_0 + (\cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma) \xi_i \\
& + (\sin \alpha \sin \gamma + \cos \alpha \sin \beta \cos \gamma) \eta_i + \cos \alpha \cos \beta \zeta_i\} \hat{K}.
\end{aligned}$$

Putting equations (33) through (38) into equation (7) and assuming gravity gradient to be negligible, the following equations of motion are obtained.

The equations of motion (39 through 46) are:

$$\begin{aligned}
 M\ddot{x}_0 + \sum_{n=1}^2 m_n [\ddot{x}_0 + (\ddot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\alpha}^2 \sin \alpha \sin \beta \sin \gamma + 2 \dot{\alpha} \dot{\beta} \cos \alpha \cos \beta \sin \gamma + 2 \dot{\alpha} \dot{\gamma} \cos \alpha \sin \beta \cos \gamma - \dot{\beta}^2 \sin \alpha \cos \beta \sin \gamma - \dot{\beta}^2 \sin \alpha \sin \beta \sin \gamma + 2 \dot{\beta} \dot{\gamma} \sin \alpha \cos \beta \cos \gamma + \dot{\gamma} \ddot{\alpha} \sin \alpha \sin \beta \cos \gamma - \dot{\gamma}^2 \sin \alpha \cos \beta \cos \gamma + \dot{\gamma} \ddot{\alpha} \sin \alpha \sin \beta \cos \gamma) \xi_n + (\cos \alpha \cos \gamma + \sin \alpha \sin \beta \sin \gamma) \ddot{\xi}_n + (\ddot{\alpha} \cos \alpha \sin \beta \cos \gamma - \dot{\alpha}^2 \sin \alpha \sin \beta \cos \gamma - 2 \dot{\alpha} \dot{\beta} \sin \alpha \cos \beta \sin \gamma - \dot{\gamma} \ddot{\alpha} \sin \alpha \sin \beta \sin \gamma - \dot{\gamma}^2 \sin \alpha \sin \beta \cos \gamma + \dot{\alpha} \ddot{\alpha} \sin \alpha \sin \gamma + \dot{\alpha}^2 \cos \alpha \sin \gamma + \dot{\gamma} \ddot{\alpha} \sin \alpha \sin \beta \sin \gamma + \dot{\alpha} \ddot{\alpha} \sin \alpha \sin \gamma - \dot{\gamma} \cos \alpha \cos \gamma) \eta_n + (\sin \alpha \sin \beta \cos \gamma - \cos \gamma \sin \gamma) \ddot{\eta}_n + (\ddot{\alpha} \cos \alpha \cos \beta \sin \gamma - \dot{\alpha}^2 \sin \alpha \sin \beta \sin \gamma) \zeta_n + (\sin \alpha \cos \beta) \ddot{\zeta}_n] = \vec{\Lambda} \cdot \hat{i} - g_0 r_e^2 (M + m_1 + m_2) x_0 (x_0^2 + y_0^2 + z_0^2)^{-3/2};
 \end{aligned}$$

$$\begin{aligned}
 M\ddot{y}_0 + \sum_{n=1}^2 m_n [\ddot{y}_0 + (\dot{\gamma} \cos \beta \cos \gamma - 2 \dot{\beta} \dot{\gamma} \sin \beta \cos \gamma - \dot{\gamma}^2 \cos \beta \sin \gamma - \dot{\beta}^2 \sin \beta \sin \gamma - \dot{\beta}^2 \cos \beta \sin \gamma - 2 \dot{\beta} \dot{\gamma} \sin \beta \sin \gamma + \dot{\gamma} \ddot{\alpha} \cos \beta \sin \gamma + \dot{\gamma}^2 \cos \beta \cos \gamma) \eta_n - 2 (\dot{\beta} \sin \beta \cos \gamma + \dot{\gamma} \cos \beta \sin \gamma) \dot{\eta}_n + (\cos \beta \cos \gamma - \cos \beta \sin \gamma) \ddot{\eta}_n - g_0 r_e^2 y_0 (M + m_1 + m_2) (x_0^2 + y_0^2 + z_0^2)^{-3/2};
 \end{aligned}$$

$$\begin{aligned}
 M\ddot{z}_0 + \sum_{n=1}^2 m_n [\ddot{z}_0 + (\dot{\beta} \cos \alpha \cos \beta \sin \gamma - 2 \dot{\alpha} \dot{\beta} \sin \alpha \cos \beta \sin \gamma - \dot{\beta}^2 \cos \alpha \sin \beta \sin \gamma + 2 \dot{\beta} \dot{\gamma} \cos \alpha \sin \beta \cos \gamma + \dot{\gamma} \ddot{\alpha} \cos \alpha \sin \beta \cos \gamma - \dot{\gamma}^2 \cos \alpha \sin \beta \sin \gamma - \dot{\alpha} \cos \alpha \cos \gamma + \dot{\alpha}^2 \sin \alpha \cos \gamma + 2 \dot{\alpha} \dot{\gamma} \cos \alpha \sin \gamma + \dot{\gamma} \ddot{\alpha} \cos \alpha \sin \gamma - \dot{\gamma} \cos \alpha \cos \gamma + \dot{\gamma} \sin \alpha \sin \gamma) \xi_n + (\cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma) \ddot{\xi}_n + (\dot{\alpha} \cos \alpha \sin \gamma - \dot{\alpha}^2 \sin \alpha \sin \gamma - 2 \dot{\alpha} \dot{\beta} \sin \alpha \cos \beta \cos \gamma + 2 \dot{\alpha} \dot{\gamma} \sin \alpha \sin \beta \sin \gamma + \dot{\beta} \cos \alpha \cos \beta \cos \gamma - \dot{\beta}^2 \cos \alpha \sin \beta \cos \gamma - 2 \dot{\beta} \dot{\gamma} \cos \alpha \sin \beta \cos \gamma - \dot{\alpha} \sin \alpha \sin \beta \cos \gamma + \dot{\beta} \cos \alpha \cos \beta \cos \gamma - \dot{\gamma} \cos \alpha \sin \beta \sin \gamma) \eta_n + (\sin \alpha \sin \gamma + \cos \alpha \sin \beta \cos \gamma) \ddot{\eta}_n - 2 (\dot{\alpha} \sin \alpha \cos \beta + \dot{\beta} \cos \alpha \sin \beta) \zeta_n + (\cos \alpha \cos \beta) \ddot{\zeta}_n] = \vec{\Lambda} \cdot \hat{k} - g_0 r_e^2 z_0 (x_0^2 + y_0^2 + z_0^2)^{-3/2};
 \end{aligned}$$

$$\begin{aligned}
 \sum_{n=1}^2 m_n \left\{ [\ddot{x}_0 + (\ddot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\alpha}^2 \sin \alpha \sin \beta \sin \gamma + 2 \dot{\alpha} \dot{\beta} \cos \alpha \cos \beta \sin \gamma + 2 \dot{\alpha} \dot{\gamma} \cos \alpha \sin \beta \cos \gamma - \dot{\beta}^2 \sin \alpha \cos \beta \sin \gamma - \dot{\beta}^2 \sin \alpha \sin \beta \sin \gamma + 2 \dot{\beta} \dot{\gamma} \sin \alpha \cos \beta \cos \gamma + \dot{\gamma} \ddot{\alpha} \sin \alpha \sin \beta \cos \gamma - \dot{\gamma}^2 \sin \alpha \cos \beta \cos \gamma + \dot{\gamma} \ddot{\alpha} \sin \alpha \sin \beta \cos \gamma) \xi_n + (\cos \alpha \cos \gamma + \sin \alpha \sin \beta \sin \gamma) \ddot{\xi}_n + (\ddot{\alpha} \cos \alpha \sin \beta \cos \gamma - \dot{\alpha}^2 \sin \alpha \sin \beta \cos \gamma - 2 \dot{\alpha} \dot{\beta} \sin \alpha \cos \beta \sin \gamma - \dot{\gamma} \ddot{\alpha} \sin \alpha \sin \beta \sin \gamma - \dot{\gamma}^2 \sin \alpha \sin \beta \cos \gamma + \dot{\alpha} \ddot{\alpha} \sin \alpha \sin \gamma + \dot{\alpha}^2 \cos \alpha \sin \gamma + \dot{\gamma} \ddot{\alpha} \sin \alpha \sin \beta \sin \gamma + \dot{\alpha} \ddot{\alpha} \sin \alpha \sin \gamma - \dot{\gamma} \cos \alpha \cos \gamma) \eta_n + (\sin \alpha \sin \beta \cos \gamma - \cos \gamma \sin \gamma) \ddot{\eta}_n + (\ddot{\alpha} \cos \alpha \cos \beta \sin \gamma - \dot{\alpha}^2 \sin \alpha \sin \beta \sin \gamma) \zeta_n + (\sin \alpha \cos \beta) \ddot{\zeta}_n] \right\} = \vec{\Lambda} \cdot \hat{i} - g_0 r_e^2 x_0 (x_0^2 + y_0^2 + z_0^2)^{-3/2};
 \end{aligned}$$

$$\begin{aligned}
& \cos \alpha \sin \beta \cos \gamma - \ddot{\alpha} \sin \alpha \cos \gamma - \dot{\alpha}^2 \cos \alpha \cos \gamma + 2 \ddot{\alpha} \dot{\gamma} \sin \alpha \sin \gamma - \ddot{\gamma} \cos \alpha \sin \gamma - \dot{\gamma}^2 \cos \alpha \cos \gamma \quad (39) \\
& \sin \beta \sin \gamma) \xi_n + 2 (\dot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\alpha} \sin \alpha \cos \gamma - \dot{\gamma} \cos \alpha \sin \gamma + \dot{\beta} \sin \alpha \cos \beta \sin \gamma \\
& \cos \gamma + 2 \ddot{\alpha} \dot{\beta} \cos \alpha \cos \beta \cos \gamma - 2 \ddot{\alpha} \dot{\gamma} \cos \alpha \sin \beta \sin \gamma + \ddot{\beta} \sin \alpha \cos \beta \cos \gamma - \dot{\beta}^2 \sin \alpha \sin \beta \cos \gamma \\
& 2 \ddot{\alpha} \dot{\gamma} \sin \alpha \cos \gamma - \ddot{\gamma} \cos \alpha \cos \gamma + \dot{\gamma}^2 \cos \alpha \sin \gamma) \eta_n + 2 (\dot{\alpha} \cos \alpha \sin \beta \cos \gamma + \dot{\beta} \sin \alpha \cos \beta \cos \gamma \\
& \alpha \cos \beta - \dot{\alpha}^2 \sin \alpha \cos \beta - 2 \dot{\alpha} \dot{\beta} \cos \alpha \sin \beta - \dot{\beta} \sin \alpha \sin \beta - \dot{\beta}^2 \sin \alpha \cos \beta) \zeta_n \\
& + z_0^2)^{-3/2}; \quad (39)
\end{aligned}$$

$$\dot{\gamma}) \xi_n + 2 (\dot{\gamma} \cos \beta \cos \gamma - \dot{\beta} \sin \beta \sin \gamma) \xi_n + (\cos \beta \sin \gamma) \ddot{\xi}_n - (\dot{\beta} \sin \beta \cos \gamma + \dot{\beta}^2 \cos \beta \cos \gamma \quad (40)$$

$$\dot{\gamma}) \eta_n - (\dot{\beta} \cos \beta - \dot{\beta}^2 \sin \beta) \zeta_n - 2 (\dot{\beta} \cos \beta) \dot{\zeta}_n - (\sin \beta) \ddot{\zeta}_n] = \vec{\Lambda} \cdot \vec{J} \quad (40)$$

$$\alpha \cos \beta \cos \gamma - \ddot{\alpha} \sin \alpha \sin \beta \sin \gamma - \dot{\alpha}^2 \cos \alpha \sin \beta \sin \gamma - 2 \dot{\alpha} \dot{\gamma} \sin \alpha \sin \beta \cos \gamma \quad (41)$$

$$\begin{aligned}
& \sin \alpha \sin \gamma + \dot{\gamma}^2 \sin \alpha \cos \gamma) \xi_n + 2 (\dot{\beta} \cos \alpha \cos \beta \sin \gamma - \dot{\alpha} \sin \alpha \sin \beta \sin \gamma + \dot{\gamma} \cos \alpha \sin \beta \cos \gamma \\
& \gamma + 2 \dot{\alpha} \dot{\gamma} \cos \alpha \cos \gamma + \ddot{\gamma} \sin \alpha \cos \gamma - \dot{\gamma}^2 \sin \alpha \sin \gamma - \ddot{\alpha} \sin \alpha \sin \beta \cos \gamma - \dot{\alpha}^2 \cos \alpha \sin \beta \cos \gamma \\
& \cos \alpha \cos \beta \sin \gamma - \ddot{\gamma} \cos \alpha \sin \beta \sin \gamma - \dot{\gamma}^2 \cos \alpha \sin \beta \cos \gamma) \eta_n + 2 (\dot{\alpha} \cos \alpha \sin \gamma + \dot{\gamma} \sin \alpha \cos \gamma \\
& - (\ddot{\alpha} \sin \alpha \cos \beta + \dot{\alpha}^2 \cos \alpha \cos \beta - 2 \dot{\alpha} \dot{\beta} \sin \alpha \sin \beta + \dot{\beta} \cos \alpha \sin \beta + \dot{\beta}^2 \cos \alpha \cos \beta) \zeta_n \quad (41)
\end{aligned}$$

$$\beta \cos \gamma - \ddot{\alpha} \sin \alpha \cos \gamma - \dot{\alpha}^2 \cos \alpha \cos \gamma + 2 \dot{\alpha} \dot{\gamma} \sin \alpha \sin \gamma - \ddot{\gamma} \cos \alpha \sin \gamma - \dot{\gamma}^2 \cos \alpha \cos \gamma \quad (42)$$

$$\alpha \sin \beta \sin \gamma) \xi_n + 2 (\dot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\alpha} \sin \alpha \cos \gamma - \dot{\gamma} \cos \alpha \sin \gamma$$

$$\begin{aligned}
& \beta \cos \gamma - \dot{\alpha}^2 \sin \alpha \sin \beta \cos \gamma + 2 \dot{\alpha} \dot{\beta} \cos \alpha \cos \beta \cos \gamma - 2 \dot{\alpha} \dot{\gamma} \cos \alpha \sin \beta \sin \gamma + \ddot{\beta} \sin \alpha \cos \beta \cos \gamma \quad (42 \\
& \hspace{15em} \text{cont'd)} \\
& \sin \gamma + \dot{\alpha}^2 \cos \alpha \sin \gamma + 2 \dot{\alpha} \dot{\gamma} \sin \alpha \cos \gamma - \ddot{\gamma} \cos \alpha \cos \gamma + \dot{\gamma}^2 \cos \alpha \sin \gamma \eta_n + 2 (\dot{\alpha} \cos \alpha \sin \beta \cos \gamma \\
& \alpha \sin \gamma) \ddot{\eta}_n + (\ddot{\alpha} \cos \alpha \cos \beta - \dot{\alpha}^2 \sin \alpha \cos \beta - 2 \dot{\alpha} \dot{\beta} \cos \alpha \sin \beta - \ddot{\beta} \sin \alpha \sin \beta - \dot{\beta}^2 \sin \alpha \cos \beta) \zeta_n \\
& \sin \beta \cos \gamma + \sin \alpha \sin \gamma) \eta_n + (\cos \alpha \cos \beta) \zeta_n] + [\dot{x}_0 + (\dot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\alpha} \sin \alpha \cos \gamma - \dot{\gamma} \cos \alpha \sin \gamma \\
& \cos \gamma + \dot{\beta} \sin \alpha \cos \beta \cos \gamma - \dot{\gamma} \sin \alpha \sin \beta \sin \gamma + \dot{\alpha} \sin \alpha \sin \gamma - \dot{\gamma} \cos \alpha \cos \gamma) \eta_n + (\sin \alpha \sin \beta \cos \gamma \\
& \sin \beta \sin \gamma + \dot{\gamma} \cos \alpha \sin \beta \cos \gamma - \dot{\alpha} \sin \alpha \cos \gamma + \dot{\gamma} \sin \alpha \sin \gamma) \xi_n + (\cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma) \xi_n \\
& (\cos \alpha \sin \beta \cos \gamma + \sin \alpha \sin \gamma) \eta_n - (\alpha \sin \alpha \cos \beta + \beta \cos \alpha \sin \beta) \zeta_n + (\cos \alpha \cos \beta) \zeta_n] + [z_0 \\
& \sin \alpha \sin \beta \sin \gamma - \dot{\alpha}^2 \cos \alpha \sin \beta \sin \gamma - 2 \dot{\alpha} \dot{\gamma} \sin \alpha \sin \beta \cos \gamma + \ddot{\gamma} \cos \alpha \sin \beta \cos \gamma - \dot{\gamma}^2 \cos \alpha \sin \gamma \\
& \cos \alpha \cos \beta \sin \gamma - \dot{\alpha} \sin \alpha \sin \beta \sin \gamma + \dot{\gamma} \cos \alpha \sin \beta \cos \gamma - \dot{\alpha} \cos \alpha \cos \gamma + \dot{\gamma} \sin \alpha \sin \gamma) \xi_n \\
& \alpha \cos \gamma - \dot{\gamma}^2 \sin \alpha \sin \gamma - \ddot{\alpha} \sin \alpha \sin \beta \cos \gamma - \dot{\alpha}^2 \cos \alpha \sin \beta \cos \gamma - 2 \dot{\alpha} \dot{\beta} \sin \alpha \cos \beta \cos \gamma \\
& \dot{\gamma} \cos \alpha \sin \beta \sin \gamma - \dot{\gamma}^2 \cos \alpha \sin \beta \cos \gamma) \eta_n + 2 (\dot{\alpha} \cos \alpha \sin \gamma + \dot{\gamma} \sin \alpha \cos \gamma - \dot{\alpha} \sin \alpha \sin \beta \cos \gamma \\
& \beta + \dot{\alpha}^2 \cos \alpha \cos \beta - 2 \dot{\alpha} \dot{\beta} \sin \alpha \sin \beta + \ddot{\beta} \cos \alpha \sin \beta + \dot{\beta}^2 \cos \alpha \cos \beta) \zeta_n - 2 (\dot{\alpha} \sin \alpha \cos \beta \\
& \sin \beta \cos \gamma) \eta_n - (\sin \alpha \cos \beta) \zeta_n] + [z_0 + (\beta \cos \alpha \cos \beta \sin \gamma - \dot{\alpha} \sin \alpha \sin \beta \sin \gamma + \dot{\gamma} \cos \alpha \sin \beta \cos \gamma \\
& \sin \gamma - \dot{\alpha} \sin \alpha \sin \beta \cos \gamma + \dot{\beta} \cos \alpha \cos \beta \cos \gamma - \dot{\gamma} \cos \alpha \sin \beta \sin \gamma) \eta_n + (\sin \alpha \sin \gamma \\
& \cos \alpha \sin \gamma - \dot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\beta} \sin \alpha \sin \beta \sin \gamma - \dot{\gamma} \sin \alpha \sin \beta \cos \gamma) \xi_n - (\sin \alpha \sin \beta \sin \gamma \\
& \alpha \sin \beta \sin \gamma) \eta_n + (\cos \alpha \sin \gamma - \sin \alpha \sin \beta \cos \gamma) \eta_n + (\beta \sin \alpha \sin \beta - \dot{\alpha} \cos \alpha \cos \beta) \zeta_n - (\cos \alpha \cos \beta) \zeta_n] \} \\
& \sin \gamma + \dot{\gamma} \sin \alpha \sin \beta \cos \gamma) \xi_n + (\cos \alpha \cos \gamma + \sin \alpha \sin \beta \sin \gamma) \xi_n + (\dot{\alpha} \cos \alpha \sin \beta \cos \gamma \\
& \cos \alpha \sin \gamma) \eta_n + (\dot{\alpha} \cos \alpha \cos \beta - \dot{\beta} \sin \alpha \sin \beta) \zeta_n + (\sin \alpha \cos \beta) \zeta_n] [(\dot{\gamma} \sin \alpha \sin \gamma - \dot{\alpha} \sin \alpha \sin \beta \sin \gamma \\
& + (\beta \cos \alpha \cos \beta \cos \gamma - \dot{\alpha} \sin \alpha \sin \beta \cos \gamma - \dot{\gamma} \cos \alpha \sin \beta \sin \gamma + \dot{\alpha} \cos \alpha \sin \gamma + \dot{\gamma} \sin \alpha \cos \gamma) \eta_n \\
& - (\beta \cos \alpha \cos \beta \sin \gamma - \dot{\alpha} \sin \alpha \sin \beta \sin \gamma + \dot{\gamma} \cos \alpha \sin \beta \cos \gamma - \dot{\alpha} \cos \alpha \cos \gamma + \dot{\gamma} \sin \alpha \sin \gamma) \xi_n
\end{aligned}$$

$$\begin{aligned}
& + (\cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma) \dot{\xi}_n + (\dot{\alpha} \cos \alpha \sin \gamma + \dot{\gamma} \sin \alpha \cos \gamma - \dot{\alpha} \sin \alpha \sin \beta \cos \gamma + \dot{\beta} \cos \alpha \cos \\
& + \dot{\beta} \cos \alpha \sin \beta) \dot{\zeta}_n + (\cos \alpha \cos \beta) \dot{\zeta}_n] [\dot{\alpha} \sin \alpha \cos \gamma - \dot{\beta} \sin \alpha \cos \beta \sin \gamma - \dot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\gamma} \sin \alpha \\
& - \dot{\alpha} \sin \alpha \sin \gamma - \dot{\alpha} \cos \alpha \sin \beta \cos \gamma - \dot{\beta} \sin \alpha \cos \beta \cos \gamma + \dot{\gamma} \sin \alpha \sin \beta \sin \gamma) \eta_n + (\cos \alpha \sin \gamma - \sin \\
& \sum_{n=1}^2 m_n \{ x_0 \cdot [(\cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma) \dot{\xi}_n + (\cos \alpha \sin \beta \cos \gamma + \sin \alpha \sin \gamma) \eta_n + (\cos \alpha \cos \beta) \\
& \sum_{n=1}^2 m_n \{ [\ddot{x}_0 + (\ddot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\alpha}^2 \sin \alpha \sin \beta \sin \gamma + 2 \dot{\alpha} \dot{\beta} \cos \alpha \cos \beta \sin \gamma + 2 \dot{\alpha} \dot{\gamma} \cos \alpha \sin \\
& + \dot{\beta} \sin \alpha \cos \beta \sin \gamma - \dot{\beta}^2 \sin \alpha \sin \beta \sin \gamma + 2 \dot{\beta} \dot{\gamma} \sin \alpha \cos \beta \cos \gamma + \dot{\gamma} \dot{\gamma} \sin \alpha \sin \beta \cos \gamma - \dot{\gamma}^2 \sin \alpha \sin \\
& + \dot{\gamma} \sin \alpha \sin \beta \cos \gamma) \dot{\xi}_n + (\cos \alpha \cos \gamma + \sin \alpha \sin \beta \sin \gamma) \ddot{\xi}_n + (\ddot{\alpha} \cos \alpha \sin \beta \cos \gamma - \dot{\alpha}^2 \sin \alpha \sin \beta \cos \\
& - 2 \dot{\beta} \dot{\gamma} \sin \alpha \cos \beta \sin \gamma - \dot{\gamma} \dot{\gamma} \sin \alpha \sin \beta \sin \gamma - \dot{\gamma}^2 \sin \alpha \sin \beta \cos \gamma + \ddot{\alpha} \sin \alpha \sin \gamma + \dot{\alpha}^2 \cos \alpha \sin \gamma \\
& - \dot{\gamma} \sin \alpha \sin \beta \sin \gamma + \dot{\alpha} \sin \alpha \sin \gamma - \dot{\gamma} \cos \alpha \cos \gamma) \eta_n + (\sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma) \ddot{\eta}_n + (\ddot{\alpha} \cos \alpha \cos \\
& + (\sin \alpha \cos \beta) \dot{\zeta}_n] [(\sin \alpha \cdot \cos \beta \sin \gamma) \dot{\xi}_n + (\sin \alpha \cos \beta \cos \gamma) \eta_n - (\sin \alpha \sin \beta) \dot{\zeta}_n] + [x_0 + (\dot{\alpha} \cos \alpha \sin \beta \\
& \sin \gamma) \dot{\xi}_n + (\dot{\alpha} \cos \alpha \sin \beta \cos \gamma + \dot{\beta} \sin \alpha \cos \beta \cos \gamma - \dot{\gamma} \sin \alpha \sin \beta \sin \gamma + \dot{\alpha} \sin \alpha \sin \gamma - \dot{\gamma} \cos \alpha \cos \gamma \\
& \sin \gamma - \dot{\beta} \sin \alpha \sin \beta \sin \gamma + \dot{\gamma} \sin \alpha \cos \beta \cos \gamma) \dot{\xi}_n + (\sin \alpha \cos \beta \sin \gamma) \dot{\xi}_n + (\dot{\alpha} \cos \alpha \cos \beta \cos \gamma - \dot{\beta} \sin \alpha \\
& + [\dot{y}_0 + (\dot{\gamma} \cos \beta \cos \gamma - 2 \dot{\beta} \dot{\gamma} \sin \beta \cos \gamma - \dot{\gamma}^2 \cos \beta \sin \gamma - \dot{\beta} \sin \beta \sin \gamma - \dot{\beta}^2 \cos \beta \sin \gamma) \dot{\xi}_n + 2 (\dot{\gamma} \cos \beta \cos \gamma \\
& \sin \gamma + \dot{\gamma}^2 \cos \beta \cos \gamma) \eta_n - 2 (\dot{\beta} \sin \beta \cos \gamma + \dot{\gamma} \cos \beta \sin \gamma) \eta_n + (\cos \beta \cos \gamma) \ddot{\eta}_n - (\dot{\beta} \cos \beta - \dot{\beta}^2 \sin \beta \\
& \dot{\beta} \sin \beta \sin \gamma) \dot{\xi}_n + (\cos \beta \sin \gamma) \dot{\xi}_n - (\dot{\beta} \sin \beta \cos \gamma + \dot{\gamma} \cos \beta \sin \gamma) \eta_n + (\cos \beta \cos \gamma) \dot{\eta}_n - (\dot{\beta} \cos \beta) \dot{\zeta}_n \\
& (\sin \beta \cos \gamma) \dot{\eta}_n + (\dot{\beta} \sin \beta) \dot{\zeta}_n - (\cos \beta) \dot{\zeta}_n] + [z_0 + (\dot{\beta} \cos \alpha \cos \beta \sin \gamma - 2 \dot{\alpha} \dot{\beta} \sin \alpha \cos \beta \sin \gamma - \dot{\beta} \\
& \beta \cos \gamma + \dot{\gamma} \cos \alpha \sin \beta \cos \gamma - \dot{\gamma}^2 \cos \alpha \sin \beta \sin \gamma - \dot{\alpha} \cos \alpha \cos \gamma + \dot{\alpha}^2 \sin \alpha \cos \gamma + 2 \dot{\alpha} \dot{\gamma} \cos \\
& \gamma - \dot{\alpha} \cos \alpha \cos \gamma + \dot{\gamma} \sin \alpha \sin \gamma) \dot{\xi}_n + (\cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma) \ddot{\xi}_n + (\ddot{\alpha} \cos \alpha \sin \gamma - \dot{\alpha}^2 \sin \alpha \\
& - 2 \dot{\alpha} \dot{\beta} \sin \alpha \cos \beta \cos \gamma + 2 \dot{\alpha} \dot{\gamma} \sin \alpha \sin \beta \sin \gamma + \dot{\beta} \cos \alpha \cos \beta \cos \gamma - \dot{\beta}^2 \cos \alpha \sin \beta \cos \gamma - 2
\end{aligned}$$

$$\beta \cos \gamma - \dot{\gamma} \cos \alpha \sin \beta \sin \gamma) \eta_n + (\sin \alpha \sin \gamma + \cos \alpha \sin \beta \cos \gamma) \dot{\eta}_n - (\dot{\alpha} \sin \alpha \cos \beta$$

$$(\sin \beta \cos \gamma + \dot{\gamma} \cos \alpha \sin \gamma) \dot{\xi}_n - (\sin \alpha \sin \beta \sin \gamma + \cos \alpha \cos \gamma) \dot{\xi}_n + (\dot{\gamma} \cos \alpha \cos \gamma$$

$$\alpha \sin \beta \cos \gamma) \dot{\eta}_n + (\beta \sin \alpha \sin \beta - \dot{\alpha} \cos \alpha \cos \beta) \dot{\zeta}_n - (\sin \alpha \cos \beta) \dot{\zeta}_n \} = -g_0 r e^2 (x_0^2 + y_0^2 + z_0^2)^{-3/2}$$

$$[\dot{\xi}_n] + z_0 [-(\sin \alpha \sin \beta \sin \gamma + \cos \alpha \cos \gamma) \dot{\xi}_n + (\cos \alpha \sin \gamma - \sin \alpha \sin \beta \cos \gamma) \dot{\eta}_n - (\sin \alpha \cos \beta) \dot{\zeta}_n] \} \quad (42)$$

$$\sin \beta \cos \gamma - \ddot{\alpha} \sin \alpha \cos \gamma - \dot{\alpha}^2 \cos \alpha \cos \gamma + 2 \dot{\alpha} \dot{\gamma} \sin \alpha \sin \gamma - \ddot{\gamma} \cos \alpha \sin \gamma - \dot{\gamma}^2 \cos \alpha \cos \gamma \quad (43)$$

$$+ \dot{\beta} \sin \gamma) \dot{\xi}_n + 2 (\dot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\alpha} \sin \alpha \cos \gamma - \dot{\gamma} \cos \alpha \sin \gamma + \dot{\beta} \sin \alpha \cos \beta \sin \gamma$$

$$+ \dot{\beta} \dot{\gamma} + 2 \dot{\alpha} \dot{\beta} \cos \alpha \cos \beta \cos \gamma - 2 \dot{\alpha} \dot{\gamma} \cos \alpha \sin \beta \sin \gamma + \ddot{\beta} \sin \alpha \cos \beta \cos \gamma - \dot{\beta}^2 \sin \alpha \sin \beta \cos \gamma$$

$$+ 2 \dot{\alpha} \dot{\gamma} \sin \alpha \cos \gamma - \ddot{\gamma} \cos \alpha \cos \gamma + \dot{\gamma}^2 \cos \alpha \sin \gamma) \dot{\eta}_n + 2 (\dot{\alpha} \cos \alpha \sin \beta \cos \gamma + \dot{\beta} \sin \alpha \cos \beta \cos \gamma$$

$$+ \dot{\beta} \dot{\gamma} - \dot{\alpha}^2 \sin \alpha \cos \beta - 2 \dot{\alpha} \dot{\beta} \cos \alpha \sin \beta - \ddot{\beta} \sin \alpha \sin \beta - \dot{\beta}^2 \sin \alpha \cos \beta) \dot{\zeta}_n + 2 (\dot{\alpha} \cos \alpha \cos \beta - \dot{\beta} \sin \alpha \sin \beta) \dot{\zeta}_n$$

$$\sin \gamma - \dot{\alpha} \sin \alpha \cos \gamma - \dot{\gamma} \cos \alpha \sin \gamma + \dot{\beta} \sin \alpha \cos \beta \sin \gamma + \dot{\gamma} \sin \alpha \sin \beta \cos \gamma) \dot{\xi}_n + (\cos \alpha \cos \gamma + \sin \alpha \sin \beta$$

$$) \dot{\eta}_n + (\sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma) \dot{\eta}_n + (\dot{\alpha} \cos \alpha \cos \beta - \dot{\beta} \sin \alpha \sin \beta) \dot{\zeta}_n + (\sin \alpha \cos \beta) \dot{\zeta}_n] [\dot{\alpha} \cos \alpha \cos \beta$$

$$\sin \beta \cos \gamma - \dot{\gamma} \sin \alpha \cos \beta \sin \gamma) \dot{\eta}_n + (\sin \alpha \cos \beta \cos \gamma) \dot{\eta}_n - (\dot{\alpha} \cos \alpha \sin \beta + \dot{\beta} \sin \alpha \cos \beta) \dot{\zeta}_n - (\sin \alpha \sin \beta) \dot{\zeta}_n]$$

$$- \dot{\beta} \sin \beta \sin \gamma) \dot{\xi}_n + (\cos \beta \sin \gamma) \dot{\xi}_n - (\ddot{\beta} \sin \beta \cos \gamma + \dot{\beta}^2 \cos \beta \cos \gamma - 2 \dot{\beta} \dot{\gamma} \sin \beta \sin \gamma + \ddot{\gamma} \cos \beta$$

$$) \dot{\zeta}_n - 2 (\dot{\beta} \cos \beta) \dot{\zeta}_n - (\sin \beta) \ddot{\zeta}_n] [- (\sin \beta \sin \gamma) \dot{\xi}_n - (\sin \beta \cos \gamma) \dot{\eta}_n - (\cos \beta) \dot{\zeta}_n] + [\dot{\gamma}_0 + (\dot{\gamma} \cos \beta \cos \gamma -$$

$$- (\sin \beta) \dot{\zeta}_n] \cdot [- (\dot{\beta} \cos \beta \sin \gamma + \dot{\gamma} \sin \beta \cos \gamma) \dot{\xi}_n - (\sin \beta \sin \gamma) \dot{\xi}_n + (\dot{\gamma} \sin \beta \sin \gamma - \dot{\beta} \cos \beta \cos \gamma) \dot{\eta}_n -$$

$$\dot{\beta} \cos \alpha \sin \beta \sin \gamma + 2 \dot{\beta} \dot{\gamma} \cos \alpha \cos \beta \cos \gamma - \ddot{\alpha} \sin \alpha \sin \beta \sin \gamma - \dot{\alpha}^2 \cos \alpha \sin \beta \sin \gamma - 2 \dot{\alpha} \dot{\gamma} \sin \alpha \sin$$

$$\alpha \sin \gamma + \ddot{\gamma} \sin \alpha \sin \gamma + \dot{\gamma}^2 \sin \alpha \cos \gamma) \dot{\xi}_n + 2 (\dot{\beta} \cos \alpha \cos \beta \sin \gamma - \dot{\alpha} \sin \alpha \sin \beta \sin \gamma + \dot{\gamma} \cos \alpha \sin \beta \cos$$

$$\sin \gamma + 2 \dot{\alpha} \dot{\gamma} \cos \alpha \cos \gamma + \ddot{\gamma} \sin \alpha \cos \gamma - \dot{\gamma}^2 \sin \alpha \sin \gamma - \ddot{\alpha} \sin \alpha \sin \beta \cos \gamma - \dot{\alpha}^2 \cos \alpha \sin \beta \cos \gamma$$

$$+ \dot{\beta} \dot{\gamma} \cos \alpha \cos \beta \sin \gamma - \ddot{\gamma} \cos \alpha \sin \beta \sin \gamma - \dot{\gamma}^2 \cos \alpha \sin \beta \cos \gamma) \dot{\eta}_n + 2 (\dot{\alpha} \cos \alpha \sin \gamma + \dot{\gamma} \sin \alpha$$

$$\begin{aligned}
& \cos \gamma - \dot{\alpha} \sin \alpha \sin \beta \cos \gamma + \dot{\beta} \cos \alpha \cos \beta \cos \gamma - \dot{\gamma} \cos \alpha \sin \beta \sin \gamma \dot{\eta}_n + (\sin \alpha \sin \gamma + \cos \alpha \sin \beta \cos \gamma) \dot{\xi}_n \\
& - 2 (\dot{\alpha} \sin \alpha \cos \beta + \dot{\beta} \cos \alpha \sin \beta) \dot{\zeta}_n + (\cos \alpha \cos \beta) \ddot{\zeta}_n \left[(\cos \alpha \cos \beta \sin \gamma) \dot{\xi}_n + (\cos \alpha \cos \beta \cos \gamma) \dot{\eta}_n \right. \\
& - \dot{\alpha} \cos \alpha \cos \gamma + \dot{\gamma} \sin \alpha \sin \gamma \dot{\xi}_n + (\cos \alpha \sin \beta \sin \gamma - \sin \alpha \sin \gamma) \dot{\xi}_n + (\dot{\alpha} \cos \alpha \sin \gamma + \dot{\gamma} \sin \alpha \cos \gamma) \dot{\eta}_n \\
& \left. - (\dot{\alpha} \sin \alpha \cos \beta + \dot{\beta} \cos \alpha \sin \beta) \dot{\zeta}_n + (\cos \alpha \cos \beta) \dot{\zeta}_n \right] \left[(\dot{\gamma} \cos \alpha \cos \beta \cos \gamma - \dot{\alpha} \sin \alpha \cos \beta \sin \gamma - \right. \\
& \left. \cos \beta \sin \gamma) \dot{\eta}_n + (\cos \alpha \cos \beta \cos \gamma) \dot{\eta}_n + (\dot{\alpha} \sin \alpha \sin \beta - \dot{\beta} \cos \alpha \cos \beta) \dot{\zeta}_n - (\cos \alpha \sin \beta) \dot{\zeta}_n \right] \left. + I \dot{\xi}_n \right. \\
& \left. + \dot{\gamma} \sin \alpha \sin \beta \cos \gamma \dot{\xi}_n + (\cos \alpha \cos \gamma + \sin \alpha \sin \beta \sin \gamma) \dot{\xi}_n + (\dot{\alpha} \cos \alpha \sin \beta \cos \gamma + \dot{\beta} \sin \alpha \cos \beta \cos \gamma) \dot{\eta}_n \right. \\
& \left. + (\dot{\alpha} \cos \alpha \cos \beta - \dot{\beta} \sin \alpha \sin \beta) \dot{\zeta}_n + (\sin \alpha \cos \beta) \dot{\zeta}_n \right] \left[(\dot{\alpha} \cos \alpha \cos \beta \sin \gamma - \dot{\beta} \sin \alpha \sin \beta \sin \gamma - \right. \\
& \left. \cos \beta \sin \gamma) \dot{\eta}_n + (\sin \alpha \cos \beta \cos \gamma) \dot{\eta}_n - (\dot{\alpha} \sin \alpha \cos \beta + \dot{\beta} \cos \alpha \sin \beta) \dot{\zeta}_n - (\sin \alpha \sin \beta) \dot{\zeta}_n \right] + [\dot{y}_0 + (\dot{\gamma} \cos \alpha \cos \beta \cos \gamma - \\
& - (\dot{\beta} \cos \beta) \dot{\zeta}_n - (\sin \beta) \dot{\zeta}_n] \cdot [- (\dot{\gamma} \sin \beta \cos \gamma + \dot{\beta} \cos \beta \sin \gamma) \dot{\xi}_n - (\sin \beta \sin \gamma) \dot{\xi}_n + (\dot{\gamma} \sin \beta \sin \gamma + \\
& \sin \beta \sin \gamma + \dot{\gamma} \cos \alpha \sin \beta \cos \gamma - \dot{\alpha} \cos \alpha \cos \gamma + \dot{\gamma} \sin \alpha \sin \gamma) \dot{\xi}_n + (\cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma) \dot{\eta}_n \\
& + (\sin \alpha \sin \gamma + \cos \alpha \sin \beta \cos \gamma) \dot{\eta}_n - (\dot{\alpha} \sin \alpha \cos \beta + \dot{\beta} \cos \alpha \sin \beta) \dot{\zeta}_n + (\cos \alpha \cos \beta) \dot{\zeta}_n] \left[\dot{\gamma} \cos \alpha \cos \beta \cos \gamma \right. \\
& \left. + \dot{\beta} \cos \alpha \sin \beta \cos \gamma + \dot{\gamma} \cos \alpha \cos \beta \sin \gamma \right] \dot{\eta}_n + (\cos \alpha \cos \beta \cos \gamma) \dot{\eta}_n + (\dot{\alpha} \sin \alpha \sin \beta - \dot{\beta} \cos \alpha \cos \beta) \dot{\zeta}_n \\
& \left. \cos \gamma \right] \dot{\eta}_n - (\sin \alpha \sin \beta) \dot{\zeta}_n \left. \right] - \gamma_0 \left[(\sin \beta \sin \gamma) \dot{\xi}_n + (\sin \beta \cos \gamma) \dot{\eta}_n + (\cos \beta) \dot{\zeta}_n \right] + z_0 \left[(\cos \alpha \cos \beta \sin \gamma) \right. \\
& \sum_{n=1}^2 m_n \left\{ [x_0 + (\dot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\alpha}^2 \sin \alpha \sin \beta \sin \gamma + 2 \dot{\alpha} \dot{\beta} \cos \alpha \cos \beta \sin \gamma + 2 \dot{\alpha} \dot{\gamma} \cos \alpha \sin \beta \cos \gamma) \right. \\
& \left. + \dot{\beta} \sin \alpha \cos \beta \sin \gamma - \dot{\beta}^2 \sin \alpha \sin \beta \sin \gamma + 2 \dot{\beta} \dot{\gamma} \sin \alpha \cos \beta \cos \gamma + \dot{\gamma} \sin \alpha \sin \beta \cos \gamma - \dot{\gamma}^2 \sin \alpha \sin \beta \cos \gamma \right. \\
& \left. + \dot{\gamma} \sin \alpha \sin \beta \cos \gamma \right] \dot{\xi}_n + (\cos \alpha \cos \gamma + \sin \alpha \sin \beta \sin \gamma) \ddot{\xi}_n + (\dot{\alpha} \cos \alpha \sin \beta \cos \gamma - \dot{\alpha}^2 \sin \alpha \sin \beta \cos \gamma) \dot{\xi}_n \\
& \left. - 2 \dot{\beta} \dot{\gamma} \sin \alpha \cos \beta \sin \gamma - \dot{\gamma} \sin \alpha \sin \beta \sin \gamma - \dot{\gamma}^2 \sin \alpha \sin \beta \cos \gamma + \dot{\alpha} \sin \alpha \sin \gamma + \dot{\alpha}^2 \cos \alpha \sin \gamma \right. \\
& \left. - \dot{\gamma} \sin \alpha \sin \beta \sin \gamma + \dot{\alpha} \sin \alpha \sin \gamma - \dot{\gamma} \cos \alpha \cos \gamma \right] \dot{\eta}_n + (\sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma) \ddot{\eta}_n + (\dot{\alpha} \cos \alpha \sin \beta \cos \gamma - \dot{\alpha}^2 \sin \alpha \sin \beta \cos \gamma) \dot{\eta}_n
\end{aligned}$$

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$$\begin{aligned}
& \cos \gamma) \ddot{\eta}_n - (\ddot{\alpha} \sin \alpha \cos \beta + \dot{\alpha}^2 \cos \alpha \cos \beta - 2 \dot{\alpha} \dot{\beta} \sin \alpha \sin \beta + \ddot{\beta} \cos \alpha \sin \beta + \dot{\beta}^2 \cos \alpha \cos \beta) \zeta_n \quad (43 \\
& \text{cont'd)} \\
& \sin \gamma) \eta_n - (\cos \alpha \sin \beta) \zeta_n] + [\dot{z}_0 + (\dot{\beta} \cos \alpha \cos \beta \sin \gamma - \dot{\alpha} \sin \alpha \sin \beta \sin \gamma + \dot{\gamma} \cos \alpha \sin \beta \cos \gamma \\
& \sin \gamma - \dot{\alpha} \sin \alpha \sin \beta \cos \gamma + \dot{\beta} \cos \alpha \cos \beta \cos \gamma - \dot{\gamma} \cos \alpha \sin \beta \sin \gamma) \eta_n + (\sin \alpha \sin \gamma + \cos \alpha \sin \beta \cos \gamma) \dot{\eta}_n \\
& \dot{\beta} \cos \alpha \sin \beta \sin \gamma) \xi_n + (\cos \alpha \cos \beta \sin \gamma) \dot{\xi}_n - (\dot{\alpha} \sin \alpha \cos \beta \cos \gamma + \dot{\beta} \cos \alpha \sin \beta \cos \gamma + \dot{\gamma} \cos \alpha \\
& \ddot{\beta} - \sum_{n=1}^2 m_n \{ [\dot{x}_0 + (\dot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\alpha} \sin \alpha \cos \gamma - \dot{\gamma} \cos \alpha \sin \gamma + \dot{\beta} \sin \alpha \cos \beta \sin \gamma \\
& \sin \beta \cos \gamma - \dot{\gamma} \sin \alpha \sin \beta \sin \gamma + \dot{\alpha} \sin \alpha \sin \gamma - \dot{\gamma} \cos \alpha \cos \gamma) \eta_n + (\sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma) \dot{\eta}_n \\
& + \dot{\gamma} \sin \alpha \cos \beta \cos \gamma) \xi_n + (\sin \alpha \cos \beta \sin \gamma) \dot{\xi}_n + (\dot{\alpha} \cos \alpha \cos \beta \cos \gamma - \dot{\beta} \sin \alpha \sin \beta \cos \gamma - \dot{\gamma} \sin \alpha \\
& \sin \beta \cos \gamma - \dot{\beta} \sin \beta \sin \gamma) \xi_n + (\cos \beta \sin \gamma) \dot{\xi}_n - (\beta \sin \beta \cos \gamma + \dot{\gamma} \cos \beta \sin \gamma) \eta_n + (\cos \beta \cos \gamma) \dot{\eta}_n \\
& \gamma - \dot{\beta} \cos \beta \cos \gamma) \eta_n - (\sin \beta \cos \gamma) \dot{\eta}_n + (\dot{\beta} \sin \beta) \zeta_n - (\cos \beta) \dot{\zeta}_n] + [\dot{Z}_0 + (\dot{\beta} \cos \alpha \cos \beta \sin \gamma - \dot{\alpha} \sin \alpha \\
& \sin \alpha + (\dot{\alpha} \cos \alpha \sin \gamma + \dot{\gamma} \sin \alpha \cos \gamma - \dot{\alpha} \sin \alpha \sin \beta \cos \gamma + \dot{\beta} \cos \alpha \cos \beta \cos \gamma - \dot{\gamma} \cos \alpha \sin \beta \sin \gamma) \eta_n \\
& \sin \beta \cos \gamma - \dot{\beta} \cos \alpha \sin \beta \sin \gamma - \dot{\alpha} \sin \alpha \cos \beta \sin \gamma) \xi_n + (\cos \alpha \cos \beta \sin \gamma) \dot{\xi}_n - (\dot{\alpha} \sin \alpha \cos \beta \cos \gamma \\
& \sin \beta - (\cos \alpha \sin \beta) \dot{\zeta}_n] \} = -g_0 r_e^2 (x_0^2 + y_0^2 + z_0^2)^{-3/2} \sum_{n=1}^2 m_n \{ x_0 \cdot [(\sin \alpha \cos \beta \sin \gamma) \xi_n + (\sin \alpha \cos \beta \\
& \sin \gamma) \xi_n + (\cos \alpha \cos \beta \cos \gamma) \eta_n - (\cos \alpha \sin \beta) \zeta_n] \}; \quad (43)
\end{aligned}$$

$$\begin{aligned}
& \sin \beta \cos \gamma - \dot{\alpha} \sin \alpha \cos \gamma - \dot{\alpha}^2 \cos \alpha \cos \gamma + 2 \dot{\alpha} \dot{\gamma} \sin \alpha \sin \gamma - \dot{\gamma}^2 \cos \alpha \sin \gamma - \dot{\gamma}^2 \cos \alpha \cos \gamma \quad (44) \\
& \sin \beta \sin \gamma) \xi_n + 2 (\dot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\alpha} \sin \alpha \cos \gamma - \dot{\gamma} \cos \alpha \sin \gamma + \dot{\beta} \sin \alpha \cos \beta \sin \gamma \\
& \sin \gamma + 2 \dot{\alpha} \dot{\beta} \cos \alpha \cos \beta \cos \gamma - 2 \dot{\alpha} \dot{\gamma} \cos \alpha \sin \beta \sin \gamma + \dot{\beta}^2 \sin \alpha \cos \beta \cos \gamma - \dot{\beta}^2 \sin \alpha \sin \beta \cos \gamma \\
& + 2 \dot{\alpha} \dot{\gamma} \sin \alpha \cos \gamma - \dot{\gamma}^2 \cos \alpha \cos \gamma + \dot{\gamma}^2 \cos \alpha \sin \gamma) \eta_n + 2 (\dot{\alpha} \cos \alpha \sin \beta \cos \gamma + \dot{\beta} \sin \alpha \cos \beta \cos \gamma \\
& \sin \beta - \dot{\alpha}^2 \sin \alpha \cos \beta - 2 \dot{\alpha} \dot{\beta} \cos \alpha \sin \beta - \dot{\beta}^2 \sin \alpha \sin \beta - \dot{\beta}^2 \sin \alpha \cos \beta) \zeta_n + 2 (\dot{\alpha} \cos \alpha \cos \beta
\end{aligned}$$

$$\begin{aligned}
& -\dot{\beta} \sin \alpha \sin \beta \dot{\zeta}_n + (\sin \alpha \cos \beta) \ddot{\zeta}_n] [(\sin \alpha \sin \beta \cos \gamma \cos \alpha \sin \gamma) \dot{\xi}_n - (\sin \alpha \sin \beta \sin \gamma + \cos \alpha \cos \gamma) \dot{\eta}_n \\
& \sin \beta \cos \gamma) \dot{\xi}_n + (\cos \alpha \cos \gamma + \sin \alpha \sin \beta \sin \gamma) \dot{\xi}_n + (\dot{\alpha} \cos \alpha \sin \beta \cos \gamma + \dot{\beta} \sin \alpha \cos \beta \cos \gamma - \dot{\gamma} \sin \alpha \sin \beta \sin \gamma + \\
& -\dot{\beta} \sin \alpha \sin \beta) \dot{\zeta}_n + (\sin \alpha \cos \beta) \ddot{\zeta}_n] [(\dot{\alpha} \cos \alpha \sin \beta \cos \gamma + \dot{\beta} \sin \alpha \cos \beta \cos \gamma - \dot{\gamma} \sin \alpha \sin \beta \sin \gamma + \\
& \cos \alpha \sin \beta \sin \gamma - \dot{\beta} \sin \alpha \cos \beta \sin \gamma - \dot{\gamma} \sin \alpha \sin \beta \cos \gamma) \dot{\eta}_n - (\sin \alpha \sin \beta \sin \gamma + \cos \alpha \cos \gamma) \dot{\eta}_n] + [\dot{\gamma} \cos \beta \cos \gamma - \dot{\beta} \sin \beta \sin \gamma + \\
& -\dot{\beta} \sin \beta \sin \gamma) \dot{\xi}_n + (\cos \beta \sin \gamma) \ddot{\xi}_n - (\dot{\beta} \sin \beta \cos \gamma + \dot{\beta}^2 \cos \beta \cos \gamma - 2 \dot{\beta} \dot{\gamma} \sin \beta \sin \gamma + \dot{\gamma} \cos \beta \sin \gamma) \dot{\xi}_n \\
& -2 (\dot{\beta} \cos \beta) \dot{\zeta}_n - (\sin \beta) \ddot{\zeta}_n] [(\cos \beta \cos \gamma) \dot{\xi}_n - (\cos \beta \sin \gamma) \dot{\eta}_n] + [\dot{\gamma} \cos \beta \cos \gamma - \dot{\beta} \sin \beta \sin \gamma) \dot{\xi}_n \\
& [- (\dot{\beta} \sin \beta \cos \gamma + \dot{\gamma} \cos \beta \sin \gamma) \dot{\xi}_n + (\cos \beta \cos \gamma) \dot{\xi}_n + (\dot{\beta} \sin \beta \sin \gamma - \dot{\gamma} \cos \beta \cos \gamma) \dot{\eta}_n - (\cos \beta \sin \gamma) \dot{\eta}_n \\
& \cos \beta \cos \gamma - \dot{\alpha} \sin \alpha \sin \beta \sin \gamma - \dot{\alpha}^2 \cos \alpha \sin \beta \sin \gamma - 2 \dot{\alpha} \dot{\gamma} \sin \alpha \sin \beta \cos \gamma + \dot{\gamma} \cos \alpha \sin \beta \cos \gamma \\
& \cos \gamma) \dot{\xi}_n + 2 (\dot{\beta} \cos \alpha \cos \beta \sin \gamma - \dot{\alpha} \sin \alpha \sin \beta \sin \gamma + \dot{\gamma} \cos \alpha \sin \beta \cos \gamma - \dot{\alpha} \cos \alpha \cos \gamma + \dot{\gamma} \sin \alpha \cos \gamma \\
& + \dot{\gamma} \sin \alpha \cos \gamma - \dot{\gamma}^2 \sin \alpha \sin \gamma - \dot{\alpha} \sin \alpha \sin \beta \cos \gamma - \dot{\alpha}^2 \cos \alpha \sin \beta \cos \gamma - 2 \dot{\alpha} \dot{\beta} \sin \alpha \cos \beta \cos \gamma + 2 \dot{\alpha} \dot{\gamma} \sin \alpha \sin \beta \cos \gamma \\
& \cos \alpha \sin \beta \cos \gamma) \dot{\eta}_n + 2 (\dot{\alpha} \cos \alpha \sin \gamma + \dot{\gamma} \sin \alpha \cos \gamma - \dot{\alpha} \sin \alpha \sin \beta \cos \gamma + \dot{\beta} \cos \alpha \cos \beta \cos \gamma - \dot{\gamma} \cos \alpha \sin \beta \cos \gamma \\
& + \dot{\beta}^2 \cos \alpha \cos \beta) \dot{\zeta}_n - 2 (\dot{\alpha} \sin \alpha \cos \beta + \dot{\beta} \cos \alpha \sin \beta) \dot{\zeta}_n + (\cos \alpha \cos \beta) \ddot{\zeta}_n] [(\cos \alpha \sin \beta \cos \gamma + \sin \alpha \sin \beta \sin \gamma) \dot{\xi}_n \\
& + \dot{\gamma} \cos \alpha \sin \beta \cos \gamma - \dot{\alpha} \cos \alpha \cos \gamma + \dot{\gamma} \sin \alpha \sin \gamma) \dot{\xi}_n + (\cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma) \dot{\xi}_n + (\dot{\alpha} \cos \alpha \cos \gamma - \dot{\gamma} \sin \alpha \sin \gamma) \dot{\eta}_n \\
& + \cos \alpha \sin \beta \cos \gamma) \dot{\eta}_n - (\dot{\alpha} \sin \alpha \cos \beta + \dot{\beta} \cos \alpha \sin \beta) \dot{\zeta}_n + (\cos \alpha \cos \beta) \ddot{\zeta}_n] [(\dot{\beta} \cos \alpha \cos \beta \cos \gamma - \dot{\alpha} \sin \alpha \sin \beta \cos \gamma \\
& + (\dot{\alpha} \cos \alpha \cos \gamma - \dot{\gamma} \sin \alpha \sin \gamma + \dot{\alpha} \sin \alpha \sin \beta \sin \gamma - \dot{\beta} \cos \alpha \cos \beta \sin \gamma - \dot{\gamma} \cos \alpha \sin \beta \cos \gamma) \dot{\eta}_n + (\sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma) \dot{\xi}_n \\
& - \dot{\gamma} \cos \alpha \sin \gamma + \dot{\beta} \sin \alpha \cos \beta \sin \gamma + \dot{\gamma} \sin \alpha \sin \beta \cos \gamma) \dot{\xi}_n + (\cos \alpha \cos \gamma + \sin \alpha \sin \beta \sin \gamma) \dot{\xi}_n + (\dot{\alpha} \cos \alpha \cos \gamma - \dot{\gamma} \sin \alpha \sin \gamma) \dot{\eta}_n \\
& + (\sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma) \dot{\eta}_n + (\dot{\alpha} \cos \alpha \cos \beta - \dot{\beta} \sin \alpha \sin \beta) \dot{\zeta}_n + (\sin \alpha \cos \beta) \ddot{\zeta}_n] [(\dot{\alpha} \cos \alpha \sin \beta \cos \gamma - \dot{\beta} \sin \alpha \sin \beta \sin \gamma) \dot{\xi}_n \\
& - \cos \alpha \sin \gamma) \dot{\xi}_n + (\dot{\alpha} \sin \alpha \cos \gamma - \dot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\beta} \sin \alpha \cos \beta \sin \gamma - \dot{\gamma} \sin \alpha \sin \beta \cos \gamma + \dot{\gamma} \cos \alpha \sin \beta \cos \gamma) \dot{\eta}_n \\
& - (\dot{\beta} \sin \beta \cos \gamma + \dot{\gamma} \cos \beta \sin \gamma) \dot{\eta}_n + (\cos \beta \cos \gamma) \dot{\eta}_n - (\dot{\beta} \cos \beta) \dot{\zeta}_n - (\sin \beta) \ddot{\zeta}_n] \cdot [-(\dot{\gamma} \cos \beta \sin \gamma + \dot{\beta} \sin \beta \cos \gamma) \dot{\xi}_n + (\cos \beta \cos \gamma) \dot{\xi}_n - (\cos \beta \sin \gamma) \dot{\eta}_n]
\end{aligned}$$

$$\begin{aligned}
 & \eta_n] + [\dot{x}_0 + (\dot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\alpha} \sin \alpha \cos \gamma - \dot{\gamma} \cos \alpha \sin \gamma + \dot{\beta} \sin \alpha \cos \beta \sin \gamma + \dot{\gamma} \sin \alpha \\
 & \alpha \sin \beta \sin \gamma + \dot{\alpha} \sin \alpha \sin \gamma - \dot{\gamma} \cos \alpha \cos \gamma) \eta_n + (\sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma) \dot{\eta}_n + (\dot{\alpha} \cos \alpha \cos \beta \\
 & \dot{\alpha} \sin \alpha \sin \gamma - \dot{\gamma} \cos \alpha \sin \gamma) \xi_n + (\sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma) \dot{\xi}_n + (\dot{\alpha} \sin \alpha \cos \gamma + \dot{\gamma} \cos \alpha \sin \gamma - \dot{\alpha} \\
 & + (\dot{\gamma} \cos \beta \cos \gamma - 2 \dot{\beta} \dot{\gamma} \sin \beta \cos \gamma - \dot{\gamma}^2 \cos \beta \sin \gamma - \dot{\beta} \sin \beta \sin \gamma - \dot{\beta}^2 \cos \beta \sin \gamma) \xi_n + 2 (\dot{\gamma} \cos \beta \cos \gamma \\
 & + \dot{\gamma}^2 \cos \beta \cos \gamma) \eta_n - 2 (\dot{\beta} \sin \beta \cos \gamma + \dot{\gamma} \cos \beta \sin \gamma) \dot{\eta}_n + (\cos \beta \cos \gamma) \ddot{\eta}_n - (\dot{\beta} \cos \beta - \dot{\beta}^2 \sin \beta) \dot{\xi}_n \\
 & + (\cos \beta \sin \gamma) \dot{\xi}_n - (\dot{\beta} \sin \beta \cos \gamma + \dot{\gamma} \cos \beta \sin \gamma) \eta_n + (\cos \beta \cos \gamma) \dot{\eta}_n - (\dot{\beta} \cos \beta) \dot{\xi}_n - (\sin \beta) \dot{\xi}_n] \\
 &] + [\ddot{z}_0 + (\ddot{\beta} \cos \alpha \cos \beta \sin \gamma - 2 \dot{\alpha} \dot{\beta} \sin \alpha \cos \beta \sin \gamma - \dot{\beta}^2 \cos \alpha \sin \beta \sin \gamma + 2 \dot{\beta} \dot{\gamma} \cos \alpha \\
 & - \dot{\gamma}^2 \cos \alpha \sin \beta \sin \gamma - \ddot{\alpha} \cos \alpha \cos \gamma + \dot{\alpha}^2 \sin \alpha \cos \gamma + 2 \dot{\alpha} \dot{\gamma} \cos \alpha \sin \gamma + \ddot{\gamma} \sin \alpha \sin \gamma + \dot{\gamma}^2 \sin \alpha \\
 & \sin \gamma) \xi_n + (\cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma) \ddot{\xi}_n + (\dot{\alpha} \cos \alpha \sin \gamma - \dot{\alpha}^2 \sin \alpha \sin \gamma + 2 \dot{\alpha} \dot{\gamma} \cos \alpha \cos \gamma \\
 & \sin \alpha \sin \beta \sin \gamma + \dot{\beta} \cos \alpha \cos \beta \cos \gamma - \dot{\beta}^2 \cos \alpha \sin \beta \cos \gamma - 2 \dot{\beta} \dot{\gamma} \cos \alpha \cos \beta \sin \gamma - \dot{\gamma} \cos \alpha \sin \beta \sin \gamma - \dot{\gamma}^2 \\
 & \beta \sin \gamma) \eta_n + (\sin \alpha \sin \gamma + \cos \alpha \sin \beta \cos \gamma) \ddot{\eta}_n - (\dot{\alpha} \sin \alpha \cos \beta + \dot{\alpha}^2 \cos \alpha \cos \beta - 2 \dot{\alpha} \dot{\beta} \sin \alpha \sin \beta + \dot{\beta} \cos \alpha \sin \beta \\
 & \alpha \sin \gamma) \xi_n + (\sin \alpha \cos \gamma - \cos \alpha \sin \beta \sin \gamma) \eta_n] + [\dot{z}_0 + (\dot{\beta} \cos \alpha \cos \beta \sin \gamma - \dot{\alpha} \sin \alpha \sin \beta \sin \gamma \\
 & \sin \gamma + \dot{\gamma} \sin \alpha \cos \gamma - \dot{\alpha} \sin \alpha \sin \beta \cos \gamma + \dot{\beta} \cos \alpha \cos \beta \cos \gamma - \dot{\gamma} \cos \alpha \sin \beta \sin \gamma) \eta_n + (\sin \alpha \sin \gamma \\
 & \sin \alpha \sin \beta \cos \gamma - \dot{\gamma} \cos \alpha \sin \beta \sin \gamma + \dot{\alpha} \cos \alpha \sin \gamma + \dot{\gamma} \sin \alpha \cos \gamma) \xi_n + (\cos \alpha \sin \beta \cos \gamma + \sin \alpha \sin \gamma) \dot{\xi}_n \\
 & \alpha \cos \gamma - \cos \alpha \sin \beta \sin \gamma) \dot{\eta}_n] \} + I_{\eta} \ddot{\gamma} - \sum_{n=1}^2 m_n \{ [\dot{x}_0 + (\dot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\alpha} \sin \alpha \cos \gamma \\
 & \cos \alpha \sin \beta \cos \gamma + \dot{\beta} \sin \alpha \cos \beta \cos \gamma - \dot{\gamma} \sin \alpha \sin \beta \sin \gamma + \dot{\alpha} \sin \alpha \sin \gamma - \dot{\gamma} \cos \alpha \cos \gamma) \eta_n \\
 & \beta \cos \gamma + \dot{\alpha} \sin \alpha \sin \gamma - \dot{\gamma} \cos \alpha \cos \gamma + \dot{\beta} \sin \alpha \cos \beta \cos \gamma - \dot{\gamma} \sin \alpha \sin \beta \sin \gamma) \xi_n + (\sin \alpha \sin \beta \cos \gamma \\
 & \alpha \sin \gamma) \eta_n - (\sin \alpha \sin \beta \sin \gamma + \cos \alpha \cos \gamma) \dot{\eta}_n] + [\dot{y}_0 + (\dot{\gamma} \cos \beta \cos \gamma - \dot{\beta} \sin \beta \sin \gamma) \xi_n + (\cos \beta \sin \gamma) \dot{\xi}_n \\
 & \gamma) \xi_n + (\cos \beta \cos \gamma) \dot{\xi}_n + (\dot{\beta} \sin \beta \sin \gamma - \dot{\gamma} \cos \beta \cos \gamma) \eta_n - (\cos \beta \sin \gamma) \dot{\eta}_n] + [\dot{z}_0 + (\dot{\beta} \cos \alpha \cos \beta \sin \gamma
 \end{aligned}$$

$$\begin{aligned}
& - \dot{\alpha} \sin \alpha \sin \beta \sin \gamma + \dot{\gamma} \cos \alpha \sin \beta \cos \gamma - \dot{\alpha} \cos \alpha \cos \gamma + \dot{\gamma} \sin \alpha \sin \gamma \xi_n + (\cos \alpha \sin \beta \sin \gamma - \sin \\
& - \dot{\gamma} \cos \alpha \sin \beta \sin \gamma) \eta_n + (\sin \alpha \sin \gamma + \cos \alpha \sin \beta \cos \gamma) \dot{\eta}_n - (\dot{\alpha} \sin \alpha \cos \beta + \dot{\beta} \cos \alpha \sin \beta) \zeta_n + (\cos \alpha \\
& + (\cos \alpha \sin \beta \cos \gamma + \sin \alpha \sin \gamma) \dot{\xi}_n + (\dot{\alpha} \cos \alpha \cos \gamma - \dot{\gamma} \sin \alpha \sin \gamma + \dot{\alpha} \sin \alpha \sin \beta \sin \gamma - \dot{\beta} \cos \alpha \cos \beta \\
& \sum_{n=1}^2 m_n \{ x_0 \cdot [(\sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma) \xi_n - (\sin \alpha \sin \beta \sin \gamma + \cos \alpha \cos \gamma) \eta_n] + y_0 [(\cos \beta \\
& \sin \gamma) \eta_n] \};
\end{aligned}$$

$$\begin{aligned}
& \sum_{n=1}^2 m_n \{ \ddot{x}_0 + (\dot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\alpha}^2 \sin \alpha \sin \beta \sin \gamma + 2 \dot{\alpha} \dot{\beta} \cos \alpha \cos \beta \sin \gamma + 2 \dot{\alpha} \dot{\gamma} \cos \alpha \sin \beta \cos \\
& - \dot{\beta}^2 \sin \alpha \sin \beta \sin \gamma + 2 \dot{\beta} \dot{\gamma} \sin \alpha \cos \beta \cos \gamma + \dot{\gamma} \ddot{\alpha} \sin \alpha \sin \beta \cos \gamma - \dot{\gamma}^2 \sin \alpha \sin \beta \sin \gamma) \xi_n + 2 (\dot{\alpha} \cos \\
& + \sin \alpha \sin \beta \sin \gamma) \ddot{\xi}_n + (\dot{\alpha} \cos \alpha \sin \beta \cos \gamma - \dot{\alpha}^2 \sin \alpha \sin \beta \cos \gamma + 2 \dot{\alpha} \dot{\beta} \cos \alpha \cos \beta \cos \gamma - 2 \dot{\alpha} \dot{\gamma} \cos \\
& - \dot{\gamma}^2 \sin \alpha \sin \beta \cos \gamma + \dot{\alpha} \ddot{\alpha} \sin \alpha \sin \gamma + \dot{\alpha}^2 \cos \alpha \sin \gamma + 2 \dot{\alpha} \dot{\gamma} \sin \alpha \cos \gamma - \dot{\gamma} \ddot{\alpha} \cos \alpha \cos \gamma + \dot{\gamma}^2 \cos \alpha \sin \\
& + (\sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma) \ddot{\eta}_n + (\dot{\alpha} \cos \alpha \cos \beta - \dot{\alpha} \sin \alpha \cos \beta - 2 \dot{\alpha} \dot{\beta} \cos \alpha \sin \beta - \dot{\beta} \ddot{\alpha} \sin \alpha \sin \\
& \sin \gamma) + (\sin \alpha \cos \beta) \frac{\partial \dot{\zeta}_n}{\partial \dot{\xi}_j} \} + [x_0 + (\dot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\alpha} \sin \alpha \cos \gamma - \dot{\gamma} \cos \alpha \sin \gamma + \dot{\beta} \sin \alpha \cos \\
& \cos \beta \cos \gamma - \dot{\gamma} \sin \alpha \sin \beta \sin \gamma + \dot{\alpha} \sin \alpha \sin \gamma - \dot{\gamma} \cos \alpha \cos \gamma) \eta_n + (\sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma) \dot{\eta}_n \\
& + \dot{\beta} \sin \alpha \cos \beta \sin \gamma + \dot{\gamma} \sin \alpha \sin \beta \cos \gamma) + (\dot{\alpha} \cos \alpha \cos \beta - \dot{\beta} \sin \alpha \sin \beta) \frac{\partial \dot{\zeta}_n}{\partial \dot{\xi}_j} + (\sin \alpha \cos \beta) \frac{d}{dt} \left(\frac{\partial \zeta_n}{\partial \dot{\xi}_j} \right) \\
& + 2 (\dot{\gamma} \cos \beta \cos \gamma - \dot{\beta} \sin \beta \sin \gamma) \dot{\xi}_n + (\cos \beta \sin \gamma) \ddot{\xi}_n - (\dot{\beta} \sin \beta \cos \gamma + \dot{\beta}^2 \cos \beta \cos \gamma - 2 \dot{\beta} \dot{\gamma} \sin \beta \\
& - (\dot{\beta} \cos \beta - \dot{\beta}^2 \sin \beta) \zeta_n - 2 (\dot{\beta} \cos \beta) \dot{\zeta}_n - (\sin \beta) \ddot{\zeta}_n] [(\cos \beta \sin \gamma) - (\sin \beta) \frac{\partial \dot{\zeta}_n}{\partial \dot{\xi}_j}] + [y_0 + (\dot{\gamma} \cos \beta \cos \\
& - (\sin \beta) \dot{\zeta}_n] [(\dot{\gamma} \cos \beta \cos \gamma - \dot{\beta} \sin \beta \sin \gamma) - (\dot{\beta} \cos \beta) \frac{\partial \dot{\zeta}_n}{\partial \dot{\xi}_j} - (\sin \beta) \frac{d}{dt} \left(\frac{\partial \zeta_n}{\partial \dot{\xi}_j} \right)] + [z_0 + (\dot{\beta} \cos \alpha \cos \\
& - \dot{\alpha}^2 \cos \alpha \sin \beta \sin \gamma - 2 \dot{\alpha} \dot{\gamma} \sin \alpha \sin \beta \cos \gamma + \dot{\gamma} \ddot{\alpha} \cos \alpha \sin \beta \cos \gamma - \dot{\gamma}^2 \cos \alpha \sin \beta \sin \gamma - \dot{\alpha} \ddot{\alpha} \cos \alpha \cos
\end{aligned}$$

$$x \cos \gamma) \dot{\xi}_n + (\dot{\alpha} \cos \alpha \sin \gamma + \dot{\gamma} \sin \alpha \cos \gamma - \dot{\alpha} \sin \alpha \sin \beta \cos \gamma + \dot{\beta} \cos \alpha \cos \beta \cos \gamma \quad (44)$$

cont'

$$x \cos \beta) \dot{\xi}_n] [(\dot{\beta} \cos \alpha \cos \beta \cos \gamma - \dot{\alpha} \sin \alpha \sin \beta \cos \gamma - \dot{\gamma} \cos \alpha \sin \beta \sin \gamma + \dot{\alpha} \cos \alpha \sin \gamma + \dot{\gamma} \sin \alpha \cos \gamma) \dot{\xi}_n$$

$$\sin \gamma - \dot{\gamma} \cos \alpha \sin \beta \cos \gamma) \eta_n + (\sin \alpha \cos \gamma - \cos \alpha \sin \beta \sin \gamma) \dot{\eta}_n] \} = -g_0 r_e^2 (x_0^2 + y_0^2 + z_0^2)^{-3/2}$$

$$\cos \gamma) \dot{\xi}_n - (\cos \beta \sin \gamma) \dot{\eta}_n] + z_0 [(\cos \alpha \sin \beta \cos \gamma + \sin \alpha \sin \gamma) \dot{\xi}_n + (\sin \alpha \cos \gamma - \cos \alpha \sin \beta$$

(44)

$$\ddot{\gamma} - \ddot{\alpha} \sin \alpha \cos \gamma - \ddot{\alpha}^2 \cos \alpha \cos \gamma + 2 \dot{\alpha} \dot{\gamma} \sin \alpha \sin \gamma - \ddot{\gamma} \cos \alpha \sin \gamma - \dot{\gamma}^2 \cos \alpha \cos \gamma + \ddot{\beta} \sin \alpha \cos \beta \sin \gamma \quad (45)$$

$$\sin \alpha \sin \beta \sin \gamma - \dot{\alpha} \sin \alpha \cos \gamma - \dot{\gamma} \cos \alpha \sin \gamma + \dot{\beta} \sin \alpha \cos \beta \sin \gamma + \dot{\gamma} \sin \alpha \sin \beta \cos \gamma) \dot{\xi}_n + (\cos \alpha \cos \gamma$$

$$\sin \alpha \sin \beta \sin \gamma + \ddot{\beta} \sin \alpha \cos \beta \cos \gamma - \dot{\beta}^2 \sin \alpha \sin \beta \cos \gamma - 2 \dot{\beta} \dot{\gamma} \sin \alpha \cos \beta \sin \gamma - \ddot{\gamma} \sin \alpha \sin \beta \sin \gamma$$

$$+ \dot{\gamma}) \dot{\eta}_n + 2 (\dot{\alpha} \cos \alpha \sin \beta \cos \gamma + \dot{\beta} \sin \alpha \cos \beta \cos \gamma - \dot{\gamma} \sin \alpha \sin \beta \sin \gamma + \dot{\alpha} \sin \alpha \sin \gamma - \dot{\gamma} \cos \alpha \cos \gamma) \dot{\eta}_n$$

$$- \dot{\beta}^2 \sin \alpha \cos \beta) \dot{\xi}_n + 2 (\dot{\alpha} \cos \alpha \cos \beta - \dot{\beta} \sin \alpha \sin \beta) \dot{\xi}_n + (\sin \alpha \cos \beta) \ddot{\xi}_n] [(\cos \alpha \cos \gamma + \sin \alpha \sin \beta$$

$$\sin \gamma + \dot{\gamma} \sin \alpha \sin \beta \cos \gamma) \dot{\xi}_n + (\cos \alpha \cos \gamma + \sin \alpha \sin \beta \sin \gamma) \dot{\xi}_n + (\dot{\alpha} \cos \alpha \sin \beta \cos \gamma + \dot{\beta} \sin \alpha$$

$$(\dot{\alpha} \cos \alpha \cos \beta - \dot{\beta} \sin \alpha \sin \beta) \dot{\xi}_n + (\sin \alpha \cos \beta) \dot{\xi}_n] [(\dot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\alpha} \sin \alpha \cos \gamma - \dot{\gamma} \cos \alpha \sin \gamma$$

$$\dot{\xi}_n) + [y_0 + (\dot{\gamma} \cos \beta \cos \gamma - 2 \dot{\beta} \dot{\gamma} \sin \beta \cos \gamma - \dot{\gamma}^2 \cos \beta \sin \gamma - \dot{\beta} \sin \beta \sin \gamma - \dot{\beta}^2 \cos \beta \sin \gamma) \dot{\xi}_n$$

$$\sin \gamma + \dot{\gamma} \cos \beta \sin \gamma + \dot{\gamma}^2 \cos \beta \cos \gamma) \dot{\eta}_n - 2 (\dot{\beta} \sin \beta \cos \gamma + \dot{\gamma} \cos \beta \sin \gamma) \dot{\eta}_n + (\cos \beta \cos \gamma) \ddot{\eta}_n$$

$$\cos \gamma - \dot{\beta} \sin \beta \sin \gamma) \dot{\xi}_n + (\cos \beta \sin \gamma) \dot{\xi}_n - (\dot{\beta} \sin \beta \cos \gamma + \dot{\gamma} \cos \beta \sin \gamma) \dot{\eta}_n + (\cos \beta \cos \gamma) \dot{\eta}_n - (\dot{\beta} \cos \beta) \dot{\xi}_n$$

$$\sin \gamma - 2 \dot{\alpha} \dot{\beta} \sin \alpha \cos \beta \sin \gamma - \dot{\beta}^2 \cos \alpha \sin \beta \sin \gamma + 2 \dot{\beta} \dot{\gamma} \cos \alpha \cos \beta \cos \gamma - \dot{\alpha} \sin \alpha \sin \beta \sin \gamma$$

$$\cos \gamma + \dot{\alpha}^2 \sin \alpha \cos \gamma + 2 \dot{\alpha} \dot{\gamma} \cos \alpha \sin \gamma + \dot{\gamma} \sin \alpha \sin \gamma + \dot{\gamma}^2 \sin \alpha \cos \gamma) \dot{\xi}_n + 2 (\dot{\beta} \cos \alpha \cos \beta \sin \gamma$$

$$\begin{aligned}
& - \dot{\alpha} \sin \alpha \sin \beta \sin \gamma + \dot{\gamma} \cos \alpha \sin \beta \cos \gamma - \dot{\alpha} \cos \alpha \cos \gamma + \dot{\gamma} \sin \alpha \sin \gamma \xi_n + (\cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma) \dot{\xi}_n \\
& - \ddot{\alpha} \sin \alpha \sin \beta \cos \gamma - \ddot{\alpha}^2 \cos \alpha \sin \beta \cos \gamma - 2 \dot{\alpha} \dot{\beta} \sin \alpha \cos \beta \cos \gamma + 2 \dot{\alpha} \dot{\gamma} \sin \alpha \sin \beta \sin \gamma + \ddot{\beta} \cos \alpha \cos \beta \cos \gamma \\
& + 2 (\dot{\alpha} \cos \alpha \sin \gamma + \dot{\gamma} \sin \alpha \cos \gamma - \dot{\alpha} \sin \alpha \sin \beta \cos \gamma + \dot{\beta} \cos \alpha \cos \beta \cos \gamma - \dot{\gamma} \cos \alpha \sin \beta \sin \gamma) \eta_n + (\sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma) \dot{\eta}_n \\
& + \dot{\beta}^2 \cos \alpha \cos \beta \xi_n - 2 (\dot{\alpha} \sin \alpha \cos \beta + \dot{\beta} \cos \alpha \sin \beta) \dot{\xi}_n + (\cos \alpha \cos \beta) \ddot{\xi}_n \left[(\cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma) \dot{\xi}_n \right. \\
& \left. - \dot{\alpha} \cos \alpha \cos \gamma + \dot{\gamma} \sin \alpha \sin \gamma \right] \xi_n + (\cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma) \dot{\xi}_n + (\dot{\alpha} \cos \alpha \sin \gamma + \dot{\gamma} \sin \alpha \cos \gamma - \dot{\alpha} \sin \alpha \cos \beta \\
& - (\dot{\alpha} \sin \alpha \cos \beta + \dot{\beta} \cos \alpha \sin \beta) \xi_n + (\cos \alpha \cos \beta) \dot{\xi}_n \left[(\dot{\beta} \cos \alpha \cos \beta \sin \gamma - \dot{\alpha} \sin \alpha \sin \beta \sin \gamma + \dot{\gamma} \cos \alpha \cos \beta \sin \gamma \right. \\
& \left. + \dot{\gamma} \sin \alpha \sin \beta \cos \gamma) \right] - \sum_{n=1}^2 m_n \left\{ [x_0 + (\dot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\alpha} \sin \alpha \cos \gamma - \dot{\gamma} \cos \alpha \sin \gamma + \dot{\beta} \sin \alpha \cos \beta \sin \gamma + \dot{\gamma} \sin \alpha \cos \beta \sin \gamma \right. \\
& \left. + \dot{\alpha} \sin \alpha \sin \gamma - \dot{\gamma} \cos \alpha \cos \gamma) \eta_n + (\sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma) \eta_n + (\dot{\alpha} \cos \alpha \cos \beta - \dot{\beta} \sin \alpha \sin \beta) \frac{\partial \xi_n}{\partial \xi_j} \right. \\
& \left. + (\dot{\gamma} \cos \beta \cos \gamma - \dot{\beta} \sin \beta \sin \gamma) + (\dot{\alpha} \cos \alpha \cos \beta - \dot{\beta} \sin \alpha \sin \beta) \frac{\partial \xi_n}{\partial \xi_j} + (\sin \alpha \cos \beta) \frac{\partial \xi_n}{\partial \xi_j} \right] + [y_0 + (\dot{\gamma} \cos \beta \cos \gamma - \dot{\beta} \sin \beta \sin \gamma) \\
& \left. - (\dot{\beta} \cos \beta) \frac{\partial \xi_n}{\partial \xi_j} - (\sin \beta) \frac{\partial \xi_n}{\partial \xi_j} \right] + [z_0 + (\dot{\beta} \cos \alpha \cos \beta \sin \gamma - \dot{\alpha} \sin \alpha \sin \beta \sin \gamma + \dot{\gamma} \sin \alpha \cos \beta \sin \gamma - \dot{\alpha} \sin \alpha \sin \beta \cos \gamma \\
& + \dot{\beta} \cos \alpha \cos \beta \cos \gamma - \dot{\gamma} \cos \alpha \sin \beta \sin \gamma) \eta_n + (\sin \alpha \sin \gamma + \cos \alpha \sin \beta \sin \gamma + \dot{\gamma} \cos \alpha \sin \beta \cos \gamma - \dot{\alpha} \cos \alpha \cos \gamma + \dot{\gamma} \sin \alpha \sin \gamma) \\
& - (\dot{\alpha} \sin \alpha \cos \beta + \dot{\beta} \cos \alpha \sin \beta) \frac{\partial \xi_n}{\partial \xi_j} + (\cos \alpha \cos \beta) \frac{\partial \xi_n}{\partial \xi_j} \left. \right\}, \text{ where}
\end{aligned}$$

$$\begin{aligned}
& \sum_{n=1}^2 m_n \left\{ [x_0 + (\ddot{\alpha} \cos \alpha \sin \beta \sin \gamma - \ddot{\alpha}^2 \sin \alpha \sin \beta \sin \gamma + 2 \dot{\alpha} \dot{\beta} \cos \alpha \cos \beta \sin \gamma + 2 \dot{\alpha} \dot{\gamma} \cos \alpha \sin \beta \cos \gamma \right. \\
& \left. + \dot{\beta} \sin \alpha \cos \beta \sin \gamma - \dot{\beta}^2 \sin \alpha \sin \beta \sin \gamma + 2 \dot{\beta} \dot{\gamma} \sin \alpha \cos \beta \cos \gamma + \dot{\gamma} \sin \alpha \sin \beta \cos \gamma - \dot{\gamma}^2 \sin \alpha \sin \beta \cos \gamma) \xi_n \right. \\
& \left. + (\cos \alpha \cos \gamma + \sin \alpha \sin \beta \sin \gamma) \dot{\xi}_n + (\ddot{\alpha} \cos \alpha \sin \beta \cos \gamma - \ddot{\alpha}^2 \sin \alpha \sin \beta \cos \gamma + 2 \dot{\alpha} \dot{\beta} \cos \alpha \cos \beta \cos \gamma \right. \\
& \left. - \dot{\gamma} \sin \alpha \sin \beta \sin \gamma - \dot{\gamma}^2 \sin \alpha \sin \beta \cos \gamma + \ddot{\alpha} \sin \alpha \sin \gamma + \ddot{\alpha}^2 \cos \alpha \sin \gamma + 2 \dot{\alpha} \dot{\gamma} \sin \alpha \cos \gamma - \dot{\gamma} \cos \alpha \cos \gamma) \eta_n \right. \\
& \left. + (\sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma) \dot{\eta}_n + (\dot{\alpha} \cos \alpha \cos \beta - \dot{\alpha}^2 \sin \alpha \cos \beta - 2 \dot{\alpha} \dot{\beta} \cos \alpha \sin \beta \sin \gamma + \dot{\beta} \sin \alpha \sin \beta \cos \gamma \right. \\
& \left. + \dot{\gamma} \sin \alpha \cos \beta \sin \gamma - \dot{\alpha} \sin \alpha \cos \beta) \frac{\partial \xi_n}{\partial \xi_j} + (\cos \alpha \cos \beta) \frac{\partial \xi_n}{\partial \xi_j} \right\}, \text{ where}
\end{aligned}$$

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$$\begin{aligned}
& \ddot{\alpha} \sin \alpha \cos \gamma - \dot{\alpha}^2 \cos \alpha \cos \gamma + 2 \dot{\alpha} \dot{\gamma} \sin \alpha \sin \gamma - \dot{\gamma}^2 \cos \alpha \cos \gamma \\
& \cos \beta \cos \gamma - \dot{\beta}^2 \cos \alpha \sin \beta \cos \gamma - 2 \dot{\beta} \dot{\gamma} \cos \alpha \cos \beta \sin \gamma - \dot{\gamma}^2 \cos \alpha \sin \beta \cos \gamma \\
& \dot{\alpha} \sin \gamma + \cos \alpha \sin \beta \cos \gamma \ddot{\eta}_n - (\dot{\alpha} \sin \alpha \cos \beta + \dot{\alpha}^2 \cos \alpha \cos \beta - 2 \dot{\alpha} \dot{\beta} \sin \alpha \sin \beta + \dot{\beta}^2 \cos \alpha \sin \beta \\
& \sin \gamma) + (\cos \alpha \cos \beta) \frac{\partial \dot{\zeta}_n}{\partial \dot{\xi}_j} \Big] + \left[\dot{z}_0 + (\dot{\beta} \cos \alpha \cos \beta \sin \gamma - \dot{\alpha} \sin \alpha \sin \beta \sin \gamma + \dot{\gamma} \cos \alpha \sin \beta \cos \gamma \right. \\
& \sin \alpha \sin \beta \cos \gamma + \dot{\beta} \cos \alpha \cos \beta \cos \gamma - \dot{\gamma} \cos \alpha \sin \beta \sin \gamma) \eta_n + (\sin \alpha \sin \gamma + \cos \alpha \sin \beta \cos \gamma) \dot{\eta}_n \\
& \sin \beta \cos \gamma - \dot{\alpha} \cos \alpha \cos \gamma + \dot{\gamma} \sin \alpha \sin \gamma - (\dot{\alpha} \sin \alpha \cos \beta + \dot{\beta} \cos \alpha \sin \beta) \frac{\partial \dot{\zeta}_n}{\partial \dot{\xi}_j} + (\cos \alpha \cos \beta) \\
& \sin \alpha \sin \beta \cos \gamma \Big] \xi_n + (\cos \alpha \cos \gamma + \sin \alpha \sin \beta \sin \gamma) \dot{\xi}_n + (\dot{\alpha} \cos \alpha \sin \beta \cos \gamma + \dot{\beta} \sin \alpha \cos \beta \cos \gamma - \dot{\gamma} \sin \alpha \\
& \sin \beta \cos \gamma) \zeta_n + (\sin \alpha \cos \beta) \dot{\zeta}_n \Big] [(\dot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\alpha} \sin \alpha \cos \gamma - \dot{\gamma} \cos \alpha \sin \gamma + \dot{\beta} \sin \alpha \cos \beta \sin \gamma \\
& \sin \gamma) \xi_n + (\cos \beta \sin \gamma) \dot{\xi}_n - (\dot{\beta} \sin \beta \cos \gamma + \dot{\gamma} \cos \beta \sin \gamma) \eta_n + (\cos \beta \cos \gamma) \dot{\eta}_n - (\dot{\beta} \cos \beta) \zeta_n - (\sin \beta) \dot{\zeta}_n \Big] \\
& \sin \gamma + \dot{\gamma} \cos \alpha \sin \beta \cos \gamma - \dot{\alpha} \cos \alpha \cos \gamma + \dot{\gamma} \sin \alpha \sin \gamma) \xi_n + (\cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma) \dot{\xi}_n + (\dot{\alpha} \cos \alpha \\
& \sin \beta \cos \gamma) \eta_n - (\dot{\alpha} \sin \alpha \cos \beta + \dot{\beta} \cos \alpha \sin \beta) \zeta_n + (\cos \alpha \cos \beta) \dot{\zeta}_n \Big] [(\dot{\beta} \cos \alpha \cos \beta \sin \gamma - \dot{\alpha} \sin \alpha \\
& \cos \alpha \cos \beta) \frac{\partial \dot{\zeta}_n}{\partial \dot{\xi}_j} \Big] \Big\} = -g_0 r_e^2 (x_0^2 + y_0^2 + z_0^2)^{-3/2} \sum_{n=1}^2 m_n \{x_0 \cdot [(\cos \alpha \cos \gamma + \sin \alpha \sin \beta \sin \gamma) + (\sin \alpha \cos \beta) \\
& j = 1, 2; \tag{45}
\end{aligned}$$

$$\begin{aligned}
& \ddot{\alpha} \sin \alpha \cos \gamma - \dot{\alpha}^2 \cos \alpha \cos \gamma + 2 \dot{\alpha} \dot{\gamma} \sin \alpha \sin \gamma - \dot{\gamma}^2 \cos \alpha \cos \gamma \\
& \sin \gamma) \xi_n + 2 (\dot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\alpha} \sin \alpha \cos \gamma - \dot{\gamma} \cos \alpha \sin \gamma + \dot{\beta} \sin \alpha \cos \beta \sin \gamma + \dot{\gamma} \sin \alpha \sin \beta \\
& \cos \beta \cos \gamma - 2 \dot{\alpha} \dot{\gamma} \cos \alpha \sin \beta \sin \gamma + \dot{\beta}^2 \sin \alpha \cos \beta \cos \gamma - \dot{\beta}^2 \sin \alpha \sin \beta \cos \gamma - 2 \dot{\beta} \dot{\gamma} \sin \alpha \cos \beta \sin \gamma \\
& \cos \gamma + \dot{\gamma}^2 \cos \alpha \sin \gamma) \eta_n + 2 (\dot{\alpha} \cos \alpha \sin \beta \cos \gamma + \dot{\beta} \sin \alpha \cos \beta \cos \gamma - \dot{\gamma} \sin \alpha \sin \beta \sin \gamma + \dot{\alpha} \sin \alpha \sin \gamma \\
& \sin \beta - \dot{\beta} \sin \alpha \sin \beta - \dot{\beta}^2 \sin \alpha \cos \beta) \zeta_n + 2 (\dot{\alpha} \cos \alpha \cos \beta - \dot{\beta} \sin \alpha \sin \beta) \dot{\zeta}_n + (\sin \alpha \cos \beta) \ddot{\zeta}_n \Big] \\
& \tag{46}
\end{aligned}$$

$$\begin{aligned}
& [(\sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma) + (\sin \alpha \cos \beta) \frac{\partial \dot{\zeta}_n}{\partial \dot{\eta}_j}] + [\dot{x}_0 + (\dot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\alpha} \sin \alpha \cos \gamma - \dot{\gamma} \cos \alpha \cos \gamma) \eta_n + (\dot{\alpha} \cos \alpha \sin \beta \cos \gamma + \dot{\beta} \sin \alpha \cos \beta \cos \gamma - \dot{\gamma} \sin \alpha \sin \beta \sin \gamma + \dot{\alpha} \sin \alpha \sin \gamma - \dot{\gamma} \cos \alpha \cos \gamma) \eta_n + (\dot{\beta} \sin \alpha \cos \beta \cos \gamma - \dot{\gamma} \sin \alpha \sin \beta \sin \gamma + \dot{\alpha} \sin \alpha \sin \gamma - \dot{\gamma} \cos \alpha \cos \gamma) + (\dot{\alpha} \cos \alpha \cos \beta - \dot{\beta} \sin \alpha \sin \beta \sin \gamma - \dot{\beta}^2 \sin \beta \sin \gamma - \dot{\beta}^2 \cos \beta \sin \gamma) \xi_n + 2 (\dot{\gamma} \cos \beta \cos \gamma - \dot{\beta} \sin \beta \sin \gamma) \dot{\xi}_n + (\cos \beta \sin \gamma) \ddot{\xi}_n - (\dot{\beta} \sin \beta \cos \gamma - \dot{\beta} \sin \beta \sin \gamma) \dot{\eta}_n + (\cos \beta \cos \gamma) \ddot{\eta}_n - (\dot{\beta} \cos \beta - \dot{\beta}^2 \sin \beta) \zeta_n - 2 (\dot{\beta} \cos \beta) \dot{\zeta}_n - (\sin \beta) \ddot{\zeta}_n] \cdot [(\cos \beta \cos \gamma) - (\sin \beta \sin \gamma) \eta_n + (\cos \beta \cos \gamma) \dot{\eta}_n - (\dot{\beta} \cos \beta) \zeta_n - (\sin \beta) \dot{\zeta}_n] [- (\dot{\beta} \sin \beta \cos \gamma + \dot{\gamma} \cos \beta \sin \gamma) - (\dot{\beta} \cos \beta) \frac{\partial \dot{\zeta}_n}{\partial \dot{\eta}_j} - (\sin \beta) \ddot{\zeta}_n] \\
& + 2 \dot{\beta} \dot{\gamma} \cos \alpha \cos \beta \cos \gamma - \dot{\alpha}^2 \sin \alpha \sin \beta \sin \gamma - \dot{\alpha}^2 \cos \alpha \sin \beta \sin \gamma - 2 \dot{\alpha} \dot{\gamma} \sin \alpha \sin \beta \cos \gamma + \dot{\gamma}^2 \cos \alpha \cos \beta \cos \gamma + \dot{\gamma}^2 \sin \alpha \cos \gamma \xi_n + 2 (\dot{\beta} \cos \alpha \cos \beta \sin \gamma - \dot{\alpha} \sin \alpha \sin \beta \sin \gamma + \dot{\gamma} \cos \alpha \sin \beta \cos \gamma - \dot{\alpha} \cos \alpha \cos \gamma + \dot{\gamma} \sin \alpha \cos \gamma - \dot{\gamma}^2 \sin \alpha \sin \gamma - \dot{\alpha}^2 \sin \alpha \sin \beta \cos \gamma - \dot{\alpha}^2 \cos \alpha \sin \beta \cos \gamma - 2 \dot{\alpha} \dot{\beta} \sin \alpha \cos \beta \cos \gamma + \dot{\gamma} \cos \alpha \sin \beta \sin \gamma - \dot{\gamma}^2 \cos \alpha \sin \beta \cos \gamma) \eta_n + 2 (\dot{\alpha} \cos \alpha \sin \gamma + \dot{\gamma} \sin \alpha \cos \gamma - \dot{\alpha} \sin \alpha \sin \beta \cos \gamma + \dot{\alpha}^2 \cos \alpha \cos \beta - 2 \dot{\alpha} \dot{\beta} \sin \alpha \sin \beta + \dot{\beta} \cos \alpha \sin \beta + \dot{\beta}^2 \cos \alpha \cos \beta) \zeta_n - 2 (\dot{\alpha} \sin \alpha \cos \beta + \dot{\beta} \cos \alpha \sin \beta - \dot{\alpha} \sin \alpha \sin \beta \sin \gamma + \dot{\gamma} \cos \alpha \sin \beta \cos \gamma - \dot{\alpha} \cos \alpha \cos \gamma + \dot{\gamma} \sin \alpha \sin \gamma) \xi_n + (\cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma) \eta_n + (\sin \alpha \sin \gamma + \cos \alpha \sin \beta \cos \gamma) \dot{\eta}_n - (\dot{\alpha} \sin \alpha \cos \beta + \dot{\beta} \cos \alpha \sin \beta) \zeta_n + (\cos \alpha \cos \beta) \dot{\zeta}_n] [(\dot{\alpha} \cos \alpha \sin \beta \cos \gamma + \dot{\beta} \cos \alpha \sin \beta) \frac{\partial \dot{\zeta}_n}{\partial \dot{\eta}_j} + (\cos \alpha \cos \beta) \frac{d}{dt} \left(\frac{\partial \dot{\zeta}_n}{\partial \dot{\eta}_j} \right)] \} - \sum_{n=1}^2 m_n \{ [\dot{x}_0 + (\dot{\alpha} \cos \alpha \sin \beta \sin \gamma - \dot{\alpha} \sin \alpha \cos \gamma) \eta_n + (\dot{\alpha} \cos \alpha \sin \beta \cos \gamma + \dot{\beta} \sin \alpha \cos \beta \cos \gamma - \dot{\gamma} \sin \alpha \sin \beta \sin \gamma + \dot{\alpha} \sin \alpha \sin \gamma - \dot{\gamma} \cos \alpha \cos \gamma) \eta_n + (\dot{\beta} \sin \alpha \cos \beta \cos \gamma - \dot{\gamma} \sin \alpha \sin \beta \sin \gamma + \dot{\alpha} \sin \alpha \sin \gamma - \dot{\gamma} \cos \alpha \cos \gamma) + (\dot{\alpha} \cos \alpha \cos \beta - \dot{\beta} \sin \alpha \sin \beta \sin \gamma - \dot{\beta}^2 \sin \beta \sin \gamma - \dot{\beta}^2 \cos \beta \sin \gamma) \xi_n + 2 (\dot{\gamma} \cos \beta \cos \gamma - \dot{\beta} \sin \beta \sin \gamma) \dot{\xi}_n + (\cos \beta \sin \gamma) \ddot{\xi}_n - (\dot{\beta} \sin \beta \cos \gamma - \dot{\beta} \sin \beta \sin \gamma) \dot{\eta}_n + (\cos \beta \cos \gamma) \ddot{\eta}_n - (\dot{\beta} \cos \beta) \zeta_n - (\sin \beta) \dot{\zeta}_n] [- (\dot{\beta} \sin \beta \cos \gamma + \dot{\gamma} \cos \beta \sin \gamma) - (\dot{\beta} \cos \beta) \frac{\partial \dot{\zeta}_n}{\partial \dot{\eta}_j} - (\sin \beta) \ddot{\zeta}_n] \}
\end{aligned}$$

$$\alpha \sin \gamma + \beta \sin \alpha \cos \beta \sin \gamma + \dot{\gamma} \sin \alpha \sin \beta \cos \gamma) \dot{\xi}_n + (\cos \alpha \cos \gamma + \sin \alpha \sin \beta \sin \gamma) \dot{\xi}_n$$

$$\alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma) \dot{\eta}_n + (\dot{\alpha} \cos \alpha \cos \beta - \dot{\beta} \sin \alpha \sin \beta) \dot{\xi}_n + (\sin \alpha \cos \beta) \dot{\xi}_n] [(\dot{\alpha} \cos \alpha \sin \beta \cos \gamma$$

$$\beta) \frac{\partial \dot{\xi}_n}{\partial \dot{\eta}_j} + (\sin \alpha \cos \beta) \frac{d}{dt} \left(\frac{\partial \dot{\xi}_n}{\partial \dot{\eta}_j} \right)] + [\dot{\gamma}_0 + (\dot{\gamma} \cos \beta \cos \gamma - 2 \dot{\beta} \dot{\gamma} \sin \beta \cos \gamma - \dot{\gamma}^2 \cos \beta \sin \gamma$$

$$\dot{\gamma} + \dot{\beta}^2 \cos \beta \cos \gamma - 2 \dot{\beta} \dot{\gamma} \sin \beta \sin \gamma + \dot{\gamma} \cos \beta \sin \gamma + \dot{\gamma}^2 \cos \beta \cos \gamma) \dot{\eta}_n - 2 (\dot{\beta} \sin \beta \cos \gamma + \dot{\gamma} \cos \beta$$

$$\beta) \frac{\partial \dot{\xi}_n}{\partial \dot{\eta}_j}] + [\dot{\gamma}_0 + (\dot{\gamma} \cos \beta \cos \gamma - \dot{\beta} \sin \beta \sin \gamma) \dot{\xi}_n + (\cos \beta \sin \gamma) \dot{\xi}_n - (\dot{\beta} \sin \beta \cos \gamma + \dot{\gamma} \cos \beta \sin \gamma) \dot{\eta}_n$$

$$\frac{d}{dt} \left(\frac{\partial \dot{\xi}_n}{\partial \dot{\eta}_j} \right)] + [\dot{z}_0 + (\dot{\beta} \cos \alpha \cos \beta \sin \gamma - 2 \dot{\alpha} \dot{\beta} \sin \alpha \cos \beta \sin \gamma - \dot{\beta}^2 \cos \alpha \sin \beta \sin \gamma$$

$$\alpha \sin \beta \cos \gamma - \dot{\gamma}^2 \cos \alpha \sin \beta \sin \gamma - \dot{\alpha} \cos \alpha \cos \gamma + \dot{\alpha}^2 \sin \alpha \cos \gamma + 2 \dot{\alpha} \dot{\gamma} \cos \alpha \sin \gamma + \dot{\gamma} \sin \alpha \sin \gamma$$

$$\dot{\gamma} \sin \alpha \sin \gamma) \dot{\xi}_n + (\cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma) \dot{\xi}_n + (\dot{\alpha} \cos \alpha \sin \gamma - \dot{\alpha}^2 \sin \alpha \sin \gamma + 2 \dot{\alpha} \dot{\gamma} \cos \alpha \cos \gamma$$

$$2 \dot{\alpha} \dot{\gamma} \sin \alpha \sin \beta \sin \gamma + \dot{\beta} \cos \alpha \cos \beta \cos \gamma - \dot{\beta}^2 \cos \alpha \sin \beta \cos \gamma - 2 \dot{\beta} \dot{\gamma} \cos \alpha \cos \beta \sin \gamma$$

$$\dot{\beta} \cos \alpha \cos \beta \cos \gamma - \dot{\gamma} \cos \alpha \sin \beta \sin \gamma) \dot{\eta}_n + (\sin \alpha \sin \gamma + \cos \alpha \sin \beta \cos \gamma) \dot{\eta}_n - (\dot{\alpha} \sin \alpha \cos \beta$$

$$\beta) \dot{\xi}_n + (\cos \alpha \cos \beta) \dot{\xi}_n] [(\sin \alpha \sin \gamma + \cos \alpha \sin \beta \cos \gamma) + (\cos \alpha \cos \beta) \frac{\partial \dot{\xi}_n}{\partial \dot{\eta}_j}] + [\dot{z}_0 + (\dot{\beta} \cos \alpha \cos \beta \sin \gamma$$

$$\cos \gamma) \dot{\xi}_n + (\dot{\alpha} \cos \alpha \sin \gamma + \dot{\gamma} \sin \alpha \cos \gamma - \dot{\alpha} \sin \alpha \sin \beta \cos \gamma + \dot{\beta} \cos \alpha \cos \beta \cos \gamma - \dot{\gamma} \cos \alpha \sin \beta \sin \gamma) \dot{\eta}_n$$

$$\dot{\gamma} + \dot{\gamma} \sin \alpha \cos \gamma - \dot{\alpha} \sin \alpha \sin \beta \cos \gamma + \dot{\beta} \cos \alpha \cos \beta \cos \gamma - \dot{\gamma} \cos \alpha \sin \beta \sin \gamma) - (\dot{\alpha} \sin \alpha \cos \beta$$

$$\dot{\gamma} - \dot{\gamma} \cos \alpha \sin \gamma + \dot{\beta} \sin \alpha \cos \beta \sin \gamma + \dot{\gamma} \sin \alpha \sin \beta \cos \gamma) \dot{\xi}_n + (\cos \alpha \cos \gamma + \sin \alpha \sin \beta \sin \gamma) \dot{\xi}_n$$

$$\alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma) \dot{\eta}_n + (\dot{\alpha} \cos \alpha \cos \beta - \dot{\beta} \sin \alpha \sin \beta) \dot{\xi}_n + (\sin \alpha \cos \beta) \dot{\xi}_n] \cdot [(\dot{\alpha} \cos \alpha \sin \beta \cos \gamma$$

$$\beta) \frac{\partial \dot{\xi}_n}{\partial \dot{\eta}_j} + (\sin \alpha \cos \beta) \frac{\partial \dot{\xi}_n}{\partial \dot{\eta}_j}] + [\dot{\gamma}_0 + (\dot{\gamma} \cos \beta \cos \gamma - \dot{\beta} \sin \beta \sin \gamma) \dot{\xi}_n + (\cos \beta \sin \gamma) \dot{\xi}_n - (\dot{\beta} \sin \beta \cos \gamma$$

$$\beta) \frac{\partial \dot{\xi}_n}{\partial \dot{\eta}_j} - (\sin \beta) \frac{\partial \dot{\xi}_n}{\partial \dot{\eta}_j}] + [\dot{z}_0 + (\dot{\beta} \cos \alpha \cos \beta \sin \gamma - \dot{\alpha} \sin \alpha \sin \beta \sin \gamma + \dot{\gamma} \cos \alpha \sin \beta \cos \gamma$$

$$\begin{aligned}
& - \dot{\alpha} \cos \alpha \cos \gamma + \dot{\gamma} \sin \alpha \sin \gamma) \dot{\xi}_n + (\cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma) \dot{\xi}_n + (\dot{\alpha} \cos \alpha \sin \gamma + \dot{\gamma} \sin \alpha \cos \gamma - \\
& - (\dot{\alpha} \sin \alpha \cos \beta + \dot{\beta} \cos \alpha \sin \beta) \dot{\zeta}_n + (\cos \alpha \cos \beta) \dot{\zeta}_n] [(\dot{\alpha} \cos \alpha \sin \gamma + \dot{\gamma} \sin \alpha \cos \gamma - \dot{\alpha} \sin \alpha \sin \beta \cos \\
& \frac{\partial \dot{\zeta}_n}{\partial \eta_j}] \} = -g_0 r_e^2 (x_0^2 + y_0^2 + z_0^2) \sum_{n=1}^2 m_n \{ x_0 \cdot [(\sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma) + (\sin \alpha \cos \beta) \frac{\partial \dot{\zeta}_n}{\partial \eta_j}]
\end{aligned}$$

where $j = 1, 2$.

$$\begin{aligned}
& \alpha \sin \alpha \sin \beta \cos \gamma + \beta \cos \alpha \cos \beta \cos \gamma - \gamma \cos \alpha \sin \beta \sin \gamma) \eta_n + (\sin \alpha \sin \gamma + \cos \alpha \sin \beta \cos \gamma) \dot{\eta}_n \quad (46) \\
& \text{cont'd)} \\
& \gamma + \beta \cos \alpha \cos \beta \cos \gamma - \gamma \cos \alpha \sin \beta \sin \gamma) - (\alpha \sin \alpha \cos \beta + \beta \cos \alpha \sin \beta) \frac{\partial \zeta_n}{\partial \eta_j} + (\cos \alpha \cos \beta) \\
& + y_0 \left[(\cos \beta \cos \gamma) - (\sin \beta) \frac{\partial \zeta_n}{\partial \eta_j} \right] + z_0 \left[(\sin \alpha \sin \gamma + \cos \alpha \sin \beta \cos \gamma) + (\cos \alpha \cos \beta) \frac{\partial \zeta_n}{\partial \eta_j} \right] \},
\end{aligned}$$

(46)

SECTION V. MECHANICAL ANALOG

As a result of this study, it has been determined that a simple mechanical analogy in the form of a pendulum will simulate the forces and moments produced by fluid motion. The pendulum parameters (length, pivot points) must be carefully chosen if the analogy is to be valid. However, the pendulum problem has been analyzed in the literature surveyed for this study and after careful examination the results are presented here.

For a flat "bottomed" cylindrical tank of height $2h$ and radius "a" the following relationships hold for the pendulum length (l_p), pendulum mass (m_p), distance from the top of the tank (L_p) of the pendulum pivot points and the position (L_v), and mass (m_r) of the fuel at rest:

$$l_p = \frac{a}{\epsilon_p} \coth \frac{\epsilon_p h}{a} \quad (47)$$

$$m_p = M_F A_p \quad (48)$$

$$L_p = \left\{ \sum_{p=1}^n \left[A_p - \frac{a^2}{4h} - \left(L + \frac{h}{2} \right) \right] \right\} / \left[1 - \sum_{p=1}^n A_p \right] \quad (49)$$

$$L_p = -B_p / A_p \quad (50)$$

$$m_r = M_F \left[1 - \sum_{p=1}^n A_p \right] \quad (51)$$

where the ϵ_p are the zeros of $J_1'(\epsilon) = 0$

$$\epsilon_1 = 1.84, \epsilon_2 = 5.335, \quad (52)$$

M_F is the total fuel mass, A_p is the fraction of the fuel mass in the p th pendulum given by

$$A_p = \left[2a \tanh \frac{\epsilon_p h}{a} \right] / \epsilon_p \left[\epsilon_p^2 - 1 \right] h \quad (53)$$

and B_p is given by

$$B_p = \left\{ 2a^2 \left[2 - \cosh \frac{\epsilon_p h}{a} - \frac{L \epsilon_p}{a} \sinh \frac{\epsilon_p h}{a} \right] \right\} / \left[\epsilon_p^2 h (\epsilon_p^2 - 1) \cosh \frac{\epsilon_p h}{a} \right] \quad (54)$$

where L is the distance of the free surface from the top of the tank.

SECTION VI. BUBBLE DYNAMICS

A knowledge of the behavior of bubbles under low-gravity conditions can be of value during the rendezvous phase of an orbital mission. If the ullage volume is broken up and distributed through the liquid, the system becomes incapable of a slosh mode. The results of several studies (boost cutoff, tankage vibration, etc.) indicate that it is extremely difficult to avoid such a mixture if the tank is over 70% full. The question that remains to be answered is just how long will such a mixture exist.

To obtain a conservative estimate of the settling time, assume that there is no combination, condensation, or regeneration of the bubbles. The distribution of bubble sizes, derived in a manner similar to that of the Maxwell-Boltzman distribution in thermodynamics, is found to be

$$\frac{N(\ell)}{N} = \frac{\ell}{\ell_p^2} e^{-1/2 \left(\frac{\ell}{\ell_p}\right)^2},$$

where $N(\ell)$ is the number of bubbles with diameter " ℓ ", N is the total number of bubbles, σ the bond number in dynes/cm, and ℓ_p the most probable diameter

$$\ell_p = \frac{1}{\sqrt{\lambda'\sigma}} = \frac{.231}{\sqrt{\sigma}},$$

λ' is a parameter determined by the system.

The terminal velocity of a bubble in a liquid is

$$V_t = k'\ell,$$

where $k' \propto g/\eta$ with g the gravitational acceleration and η the viscosity of the liquid. For water, with ℓ in cm, V_t in cm/sec, and $g = 980 \text{ cm/sec}^2$,

$$k' = 1, 2$$

thus, for any liquid

$$k' = 1, 2 \frac{g_\ell}{g_w} \frac{\eta_w}{\eta_\ell}.$$

The mean time for a bubble to escape to the surface is

$$\bar{t} = \frac{h}{V_t} = \frac{h}{k'\ell},$$

where h is one half of the mean depth of the fluid. From this, the rate at which the bubbles with diameters between ℓ and $\ell + d\ell$ escape the surface is

$$\frac{N(\ell)}{N_{\bar{t}}} d\ell = \left[\frac{k' \ell^2}{h \ell_p^2} e^{-1/2 \left(\frac{\ell}{\ell_p}\right)^2} \right] d\ell.$$

For a time interval Δt , a fraction f , of the bubbles will escape to the surface

$$f = \left[\int_0^{\infty} \frac{k' \ell^2}{h \ell_p^2} e^{-1/2 \left(\frac{\ell}{\ell_p}\right)^2} d\ell \right] \Delta t.$$

This can be integrated, and it becomes

$$f = \frac{k' \bar{\ell}_p}{h} \sqrt{\frac{\pi}{2}} \Delta t,$$

where $\bar{\ell}_p$ is the average, most probable diameter during the time interval.

Using the S-IVB stage as a carrier, and liquid hydrogen as the fluid, the time interval to collect 10% of the bubbles in a time averaged g-field of magnitude 10^{-4} g is found to be approximately 10^4 sec.

SECTION VII. CONCLUSIONS

The equations of motion for an orbital vehicle including two liquid slosh modes, and for a passive target vehicle maintaining a constant attitude relative to the radius vector, have been developed. A total of sixteen degrees of freedom were required, six for each vehicle and two each for the pendulums used to simulate liquid slosh. Guidance and control forces on the carrier vehicle are not specified, but appear as part of an arbitrary function in each equation. The equations represent the motions of any vehicle with cylindrical symmetry, arbitrary control and guidance system, and include the effects of two slosh modes.

An analysis of bubble dynamics was presented which indicated that, for critical periods of four or more hours, the problem of fuel slosh could be eliminated.

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