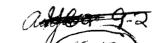
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ENERGY BALANCE OF COSMIC RAYS

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SUMMARY

The energy dissipation channels of the nuclear and electron components of cosmic rays in the Galaxy and Metagalaxy are analyzed. Definite forms of transition of energy, which is consumed on the ionization, are discussed. On the whole, the ionization energy is composed of slow-electron energy (~10 - 100 ev).

The basic fraction (\sim 75%) of slow electron energy is expended on the excitation of interstellar hydrogen (mainly for the (1s - 2p) transition, which is then exhausted).

The further destiny of the ultraviolet is discussed. A fraction of energy of slow secondary electrons is spent on the formation of bremmstrahlung photons. A comparatively large fraction of energy in this intermediate mechanism is consumed for the formation of X-rays.

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1. — It has been lately established that the energy density of cosmic rays in the Universe coincides in its order of magnitude with the energy density of the magnetic field and interstellar gas flow [1]. This circumstance is essential for cosmology and quite possible also for cosmogony, inasmuch as the question arises about the channels, along which this enormous energy emerges and vanishes. Numerous works are devoted to cosmic ray propagation (see first of all [1] however, to-date the question of energy dissipation ways of cosmic rays remains practically in infancy (to our knowledge, the only exception is the work, in which the heating of the Universe by cosmic rays is considered [2]).

In the present work we propose to estimate the contribution of various processes conditioning the energy losses of cosmic rays (c.r.).

^{*} ENERGETICHESKIY BALANS KOSMICHESKIKH LUCHEY.

2. — In the following we shall consider that cosmic rays consist of nucleons, and the interstellar gas — of hydrogen. Although heavier nuclei are also present in c.r., whereas the interstellar gas includes a certain fraction of ionized hydrogen ($\sim 10\%$) and atoms of heavier elements, we feel that the corrections, conditioned by the influence of these factors, are less than the uncertainty linked with the scarcity of data on the values of interaction cross sections of very low energy electrons (see section 7).

We shall assume the energy spectrum of cosmic rays in the form

$$P(E_0)dE_0 = kE_0^{-\gamma}dE_0, \quad E > 1000 \text{ MeV} = E_{0 \text{ min}}; \quad P(E_0) = 0, \quad E_0 < E_{0 \text{ min}}.$$
 (1)

Here k = const, $\forall = 2.6$ (see for example [1]).

In reality, there is at terrestrial atmosphere boundary a small "tail" of cosmic rays with energy $\sim 300 - 1000$ Mev. Under no condition the existence of these nucleons will influence the conclusions derived below.

Much more problematic is the influence of subcosmic rays with lesser energies, which do not penetrate the solar system on account of relatively powerful interplanetary magnetic fields. Assuming there are more hypothetical subcosmic rays at the atmosphere boundary than those observed, and this possibility is discussed in [3], this may substantially affect the final results.

For the subsequent quantitative estimates we shall postulate that the intensity of the galactic magnetic field is

$$H = 5 \cdot 10^{-6}$$
 oe

and the density of particles in the Galaxy is

$$n = 0.1 \text{ particle} \cdot \text{cm}^{-3}$$
.

The small departures from these figures are immaterial. More important is the assumption that the acceleration of c.r. takes place in the sources, and the acceleration mechanisms in the interstellar medium are ineffective [1]

- 3. Let us further bring up certain parameters of the nucleon-nucleon interactions (see for example the reviews [4,5]):
- a) the interaction cross section varies little or is totally independent from the energy of colliding particles, and is equal to about 30 mb.
- b) the inelasticity factor (that is, the fraction of energy passing to secondary mesons) is little dependent on (or totally independent from) the energy of colliding particles, and is equal to 0.5;
- c) amongst the secondary the fraction of K-mesons and hyperons is not great. In the following we shall postulate that all the secondary mesons are pions, among which one third are neutral and two-thirds are charged.

d) when computing the energy spectrum of pion generation it may be postulated [3] that there forms one pion with energy — $0.2\,E_0$ (E₀ being the energy of the primary particle).

4. — Let us compute the variation of c.r. energy at their passing the length x (measured in $g \cdot cm^{-2}$).

The equation describing the variation of energy has the form

$$-\frac{dE}{dx} = \frac{kE}{R} + c, \tag{2}$$

 $R = 60 \text{ g} \cdot \text{cm}^{-2}$ is the path of cosmic rays in the interstellar medium relative to nuclear interactions. $c = 3.65 \text{ Mev} \cdot \text{g} \cdot \text{cm}^{-2}$ (see for example [1]).

The equation (2) takes into account the nuclear collisions and the ionization losses, which may be assumed as constant through $E \sim 300 - 400 \text{ Mev}$ [7].

The solution of (2)

$$E = \frac{\left[\left(\frac{k}{R}E_0 + c\right)e^{-(k/R)x} - c\right]R}{k} \tag{3}$$

is valid up to energies E, for which the ionization losses are constant. At lesser energies the losses for the ionization rise rapidly; in this region it may be assumed that practically all energies are expended on the ionization.

At passing the path x, the ionization losses are

if E
$$>$$
 300 MeV, and $\epsilon_i = cx$, (4)

if E < 300 Mev.

$$\varepsilon_i = cx + (300 - E), \tag{4a}$$

The total losses for the ionization are

$$\varepsilon_{iT} = [cX + 300] \quad \text{KeV} \tag{5}$$

where X is determined by the equation

$$\frac{\left[\left(\frac{k}{R}E_0+c\right)e^{-(k/R)X}-c\right]R}{k}=300.$$
 (6)

The losses at nuclear interactions are:

$$e_N = E_0 - cx - E, \quad E > 300
e_N = E_0 - cx - 300, \quad E < 300$$
(7)

In the important particular case when $kx/R \ll 1$; $E_0 > 300$

$$\varepsilon_N = \frac{kE_0}{R} x. \tag{8}$$

5. - The relative rate of flow of cosmic rays (see section 4) depends on E and on their lifetime. Let us pause at the outset on the energy balance of c.r. in the Galaxy, assuming that their origin is galactic. The least lifetime of c.r. T_1 is apparently given by a Galaxy model without reflection from walls, provided we assume that the diffusive approximation is valid. In this case

$$T_1 \sim 10^8 - 10^9 \text{ years}[1] \ll T_n = 10^9 \text{ years},$$

where Tn is the lifetime of c.r. in the Galaxy relative to nuclear interaction.

In this case $k/R = T_1/T_N$ and for the estimate of life loss on the ionization and nuclear processes we may utilize (4) and (8). The energy, equal to $kE_0x/3R$, transfers into γ - quanta of high energies, which traverse the Galaxy practically without losses (1). A fraction $\sim kE_0x/2R$ changes to a neutrino, and 0.17 to electrons.

Another model (apparently quite little probable) corresponds to the assumption that $T_1 \geqslant T_n$ and this is why all the energy is expended in the Galaxy. It is appropriate, however, to consider this model, for it is, in a certain known sense, equivalent to the estimate of energy balance of c.r. in the Metagalaxy. Indeed, in this case the energy of c.r. as a whole is expended on the very same processes, and this is why the earlier conducted consideration is maintained (taking into account the variations of parameters H and n). In this case

$$X = \frac{R}{k} \ln \frac{E_0 k/R + c}{300 k/R + c} \tag{9}$$

and for the estimate of the relative role of the processes we must utilize (5) and (7) (assuming E = 0).

The fraction $oldsymbol{\delta}_i$ of energy loss on the ionization, averaged by the spectrum

(1), is

$$\delta_{i} = \frac{\int\limits_{E_{0 \text{ min}}}^{\infty} P(E_{0}) \, \varepsilon_{iT} \, dE_{0}}{\int\limits_{E_{0 \text{ min}}}^{\infty} P(E_{0}) \, E_{0} dE_{0}}.$$
(10)

Utilizing (5) and (9) and taking out the logarithmic multiplier of the integral sign, we obtain at $E_0=1000$ MeV,

$$\delta_{i} = \frac{\gamma - 2}{\gamma - 1} \frac{cX + 300}{E_{0 \min}}.$$
 (11)

Finally $\delta_i=0.25$. Consequently, the fraction δ_N of energy outgoing to the nuclear interactions is equal to 0.75.

It follows from (11) that δ_i is inversely proportional to $E_{0\,\mathrm{min}}$ and this is why, as the role of subcosmic rays increases, the relative losses on the ionization substantially increase. For our selection of $E_{0\,\mathrm{min}}$ the fraction of energy passing to photons is 0.25; $\delta_v = 0.37$ and $\delta_e = 0.13$ (δ_v and δ_e being respectively the fractions of energy changing to neutrino and electrons).

Neutrinos and photons (1) abandon the Galaxy practically without losses. More interesting is the destiny of electrons, whose energy is substantially (or even completely) expended within the bounds of the Galaxy. Making use of the assumptions formulated in sections 2 and 3, it is not difficult to compute the energy spectrum Pe (Ee0) of electrons emerging as a result of nuclear interactions and subsequent decays

$$P_{e}(E_{e0}) \sim 3 \cdot 10^{-3} P(E_{0}), \quad E_{e0} > 0.05 E_{0 \text{ min}};$$

$$P_{e}(E_{e0}) = 0, \quad E_{e0} < 0.05 E_{0 \text{ min}}, \quad (12)$$

Where E_{e0} is the initial energy of electrons.

The equation describing the behavior of electrons in time has the form

$$-\frac{dE_e}{dt} = c_1 + bE_e + aE_e^2, \tag{13}$$

 $-\frac{dE_e}{dt} = c_1 + bE_e + aE_e^2, \tag{13}$ where $c_1 = 2.5 \cdot 10^{-8}$ ev sec⁻¹; this term describes the losses on the ionization b = 5.5 · 10-17sec-1, bE is the term corresponding to losses in bremmstrahlung. In reality, in a certain region this term depends little (logarithmically) on the energy of electrons and photons. Its numerical value is chosen at $E_{e} = 50$ Mev (lower boundary of the spectrum) on the conditions of shielding absence (that is, at photon energy comparable with energy Ee. This region is more essential at computation of aggregate losses). For greater values of Ee and relatively low energies of photons, for example, the X-ray and luminous ones, shielding must be taken into account. In this case $b = 7.3 \cdot 10^{-17} \, \text{sec}^{-1}$. Thus, the losses according to formula (13) are about 1.3 times less than the true ones.

$$a = 10^{-25} [ev \cdot sec^{-1}] [1];$$

the last term describing the losses on bremmstrahlung and the inverse Compton effect.

When computing the coefficient a, it was assumed that in the Galaxy the density ω_{ϕ} of photons $\ll 1$ ev cm⁻³. The solution of the equation (13) gives the expression

$$E_{e} = \frac{\sqrt{\Delta}}{2a} \operatorname{tg} \left\{ \left[\frac{2}{\sqrt{\Delta}} \operatorname{arctg} \frac{b + 2aE_{e0}}{\sqrt{\Delta}} - t \right] \frac{\sqrt{\Delta}}{2} \right\} - \frac{b}{2a}, \qquad (14)$$

$$\Delta = 4ac_{1} - b^{2}.$$

Hence it is easy to determine the lifetime Te of electrons relative to aggregate losses

$$T_{e} = \frac{2}{\sqrt{\Delta}} \left\{ \arctan \frac{b + 2aE_{0}}{\sqrt{\Delta}} - \arctan \frac{b}{\sqrt{\Delta}} \right\}. \tag{15}$$

Let us write the energy losses on each of the processes separately:

$$\begin{aligned}
\varepsilon_{ei} &= c_1 T_e, \\
T_e & & \\
\varepsilon_{err} &= b \int_0^{\infty} E_e(t) dt, \quad \varepsilon_{emr} &= a \int_0^{T_e} [E_e(t)]^2 dt
\end{aligned} \tag{16}$$

where et, ee, eem are respectively the total losses on the ionization, bremmstrahlung and magnetic bremmstrahlung.

As an example, we shall bring forth the energy losses for two values of Ee0 taking into account the influence of the inverse Compton effect at $\omega_{\gamma}=0.3\,\mathrm{ev}$.

TABLE 1

TABLE ?

E _{e0} ×	e _{ei} ×	ε _{er} ×	E _{emr} ×	^e ek×
×10 ⁻¹ (se)	×10 [—] 1(эе)	×10 ⁻¹ (æ)	×10 ⁻⁷ (9e)	×10−* (эе)
50	28	13	6.4	2.2
200	45	52	60	20

Ę _{ei}	^Ę er	Eemr + Eek
0.7	0.15	0.15

The losses on the inverse Compton effect are compiled in the last column of Table 1. Utilizing formulas (16) and (12), it is possible to obtain the numerical value of the fractions ξ of energy expended by electrons after averaging by the spectrum $P(E_{e0})$. The results of computations are compiled in Table 2.

It follws from formula (15) that for the bulk of electrons with energy $E_{e0} \sim 100$ MeV, the lifetime $T_e \sim 10$ years. This is why one may assume ($T_e \lesssim T_1$) that the main fraction of energy is expended by electrons within the bounds of the Galaxy even if we assume the extreme "open" model. Assuming that the radioband is within the $10 - 10^3$ Mc[1] ($4 \cdot 10^{-8} + 4 \cdot 10^{-6}$ ev), the luminous band is in 0.5 - 5 ev, and the X-ray band in the 1.5 - 7 kev [7], we shall estimate the relative energy losses of electrons at bremmstrahlung and magnetic bremmstrahlung.

For the estimate of the role of bremmstrahlung we shall take advantage of the well known remark by Ginzburg and Syrovatskiy [1] that the energy spectrum of photons has a sharp maximum in this process. * This allows us to assume at tegration an unambiguous link between the energy Ee and the frequency of photons

$$E_e = 4.7 \cdot 10^2 \left(\frac{v}{H}\right)^{1/2} \quad \text{ev} \quad [v \text{ in cps}]. \tag{17}$$

This why there is a unilateral link for the magnetic bremm strahlung between the limits of the given band and the limit energy values of the electrons responsible for the radiation in this band. The results of estimates are compiled in Table 3.

TABLE 3

	Fraction of energy in the		
Process	radio band	luminous band	X-ray band
Bremmstrahlung Magnet. bremmst.	10 ⁻¹⁵	10 ⁻⁹ 2 · 10 ⁻⁶	10 ⁻⁶ 2 · 10 ⁻⁸

^{*} making use of the well known expression for the differential cross section $(d\sigma_r \sim dE_\gamma/E_e)$.

It may be seen from Table 3 that: 1) as already noted (see, for example, [1],) the magnetic bremmstrahlung is the prevailing source of nonthermal radioemission; 2) for the chosen parameters, however, the X-ray emission in the Galaxy is to a greater degree conditioned by bremmstrahlung, rather than by the magnetic bremmstrahlung. The inverse conclusion is formulated in ref. [7]. This discrepancy is conditioned by three causes: a) the influence of electrons with energy from 50 to 500 MeV the lower boundary of the spectrum in [7] is taken equal to the sensitivity threshold of the apparatus with which the photon intensity measurements were conducted; b) by taking into account in our work the simultaneous influence of all processes during the entire deceleration of electrons, while in [7] the comparison of the relative role of the various processes was conducted at the initial value of energy E_e ; c) it was assumed in our work that n=0, 1 while in [7] it was assumed n=0.01.

7. — We shall further attempt to refine the concrete forms of energy transfer, which was initially expended by c. r. on the ionization. To that effect it is necessary to know the effective collision cross sections for very low energies (\leq 100 ev). Unfortunately this question is insufficiently studied, on the one hand, on account of substantial experimental difficulties, and on the other hand, because the usual computation methods (mainly the Born approximation) in this region provide estimates valid only by order of magnitude. Whenever possible we shall rely on the following experimental data; in other cases we shall utilize the Born approximation.

We shall summarize the cross sections of elementary processes in the region of various energies and their approximations.

The ionization cross section of hydrogen atoms; the well known formula for the ionization over length equal to $g \cdot cm^{-2}$:

$$d\sigma_{\delta} = \frac{0.3m}{\beta^2} \frac{dE_{\delta}}{E_{\delta^2}} \left[1 - \beta^2 \frac{E_{\delta}}{E_m} \right], \tag{18}$$

Here β is the velocity of an incident particle, E_m is the maximum energy transferred to the electron and the speed of light =1. Formula (18) has been derived while neglecting the binding energy of the electron in the atom (that is, at energy $E_e > I$ (I being the ionization potential).

We shall utilize formula (18) without the second term through the energy $E_{\epsilon} = 35$ ev and at lower energies we shall postulate $d\sigma_{\delta} = 0$. Since an approximation has the following foundations: 1) in the interval of interest to us constitutes $10^{-3} - 10^{-5}$ of 1; 2) after integration of $d\sigma_{\delta}$ over E_{δ} we may obtain the total value of the ionization cross section G_{δ} and compare it with the experimental data of [3]. It was found that at $E_{\epsilon} = 200$ ev $G_{\epsilon} = 1.7$, whereas at $E_{\epsilon} = 40$ ev $G_{\epsilon} = 0.6$.

3) in accord with the estimates of [10] the total cross section at δ -electron formation in the region of energies 13-100 ev is $\sigma_{\delta} \sim 1/E_{\delta}$, which agrees also with formula (18); 4) the hydrogen atoms' excitation cross sections at collision of slow electrons.

Basically, the excitation is determined by the transition 1s-2p:
a) the cross section 1s-2p is by about one order greater than the transitions 1s-2s [11]; b) the excitation cross section on higher levels (3, 4...) is, in very rough approximation, $[\epsilon_3, \epsilon_{...}/\epsilon_2]^2$ times smaller than the cross section 1s-2p (E_k is the excitation energy from the ground to the k-th level [11]). The results of the experimental investigation of the excitation cross sections 1s-2s are brought out in the Fayt paper [12]. In the region 10-250 ev the cross section may be approximated by the following expression

$$\sigma_{12} = \frac{2\sigma_0(E_0 - e_2)}{E_e}, \quad 10 < E_e < 20 \text{ cv}$$

$$\frac{250\sigma_0}{E_e(ev)}, \quad 250 \text{ se} > E_e, \quad (19)$$

where $\sigma_0 = 0.5 \,\pi a_0^2$; a_0 is the radius of the first Bohr orbit; c) elastic collisions. At E_e ~ 12 ev, the cross section $\sigma_v \sim 6 \,\pi a_0^2$ and then it drops more rapidly than $1/E^2$ to energies ~ 50 ev [13]. As an average, in each elastic collision a fraction of energy, equal to Ee m/M is ~ 5 • 10 - 4 E_e, passes to hydrogen (m and M are respectively the mass of the electron and of hydrogen).

8. — Let us consider the processes, in the final count absorbing all the ionization energy. Inasmuch as at 100 kev > $\rm E_{\rm o}$ > 250 ev, the total ionization cross section and the excitation cross section are ~ 1 / $\rm E_{\rm e}$ and the cross section of elastic collisions drops significantly more rapidly, it is clear that there will take place a multiple ionization and excitation through energies $\rm E \gtrsim 250\,ev$, and the elastic processes will play an insignificant role. All the three processes referred to above will become substantial in the region $\rm E < 250\,ev$. (Here we neglect the recombination process of ionized hydrogen with the subsequent quantum emission in the light and ultraviolet bands). As the electrons decelerate, the excitation and the elastic collisions will play a greater and greater role. According to our estimates, based upon the cross sections and brought out in the section 7, it was found that about 25% of all ionization energy pass irreversibly to elastic collisions, and the remaining 75% pass to light, mainly into Layman series.

The question of further destiny of the L-series is at present not quite clear as yet. Apparently the basic absorption process of the Layman ultraviolet will be in the photoprocesses within interstellar dust [14]. For example, at some assumptions (number of dust particles being $5 \cdot 10^{-12} \, \mathrm{cm}^{-3}$, and their geometric cross section being equal to the effective ionization cross section), the path of ultraviolet is $\sim 1 \, \mathrm{ps}$. In this case, neglecting the possible ultraviolet decrease

within the limits of a parsec in other processes and at neglecting the secondary processes inside the dust particles, the ultraviolet energy will be transformed to heat nearly completely, without attaining either the solar system or the boundaries of the Galaxy. However, if the path of the γ —quantum is substantially greater than that of the parsec (and comparing with the dimensions of the Galaxy), or the regenration factor of the ultraviolet into visible light is near the unity, a significant part of the ionization energy will pass to light.

It is interesting to compare the total energy transfering into ultraviolet in the Galaxy, with the value of energy departing from the Galaxy in the form of visible light. Estimating only the order of both quantities, we shall make use of the following values: 1) the volume of the Galaxy $\sim 10^{69}~\rm cm^3$, 2) the area of the Galaxy $\sim 10^{46}~\rm cm^2$, $\omega_{\Phi} \sim 1$ ev cm $^{-3}$, 4) $c_1 = 10^{-8}~\rm sec^{-1}$, and assuming also that all the ionization energy goes to ultraviolet. It was found that in such a case the total energy passing from cosmic rays to the ultraviolet is $\sim 10^{51}~\rm ev~sec^{-1}$, whereas the energy outgoing from the Galaxy in the form of light is $\sim 10^{55}~\rm ev~sec^{-1}$.

9.— Let us estimate the energy, departing from δ —electrons in the bremmstrahlung process. Let us utilize the generation spectrum $\Gamma(E_{\delta})$ of the δ —electrons and the path $R(E_{\delta})$ of δ —electrons in hydrogen assumed in the work [3]. Inasmuch as the expression for $R(E_{\delta})$ was valid for $E_{\delta} \geqslant 1$ kev, we shall limit ourselves by the consideration of δ —electrons satisfying this condition. The fraction Δ of energy (from the total ionization energy), passing to radiation in the band $\omega_{\min} - \omega_{\max}$, is

$$\Delta = \frac{\int\limits_{E_{\text{omin}}}^{\infty} \Gamma(E_{\delta}) R(E_{\delta}) dE_{\delta} \int\limits_{\omega_{\text{min}}}^{\omega_{\text{max}}} \sigma(E_{\delta}, \omega) d\omega}{c}.$$
 (20)

Here

$$\sigma(E_{\delta}, \omega) d\omega = \frac{10^{-3}}{\omega} \frac{m}{E} \ln \frac{4E_{\delta}}{\omega} d\omega$$

is the bremmstrahlung cross section (nonrelativistic case) on 1 g·cm⁻². Computing (20) in the logarithmic approximation, we may obtain the following table:

TABLE 4

Fraction	bands				
of	radioband visible light X-ray				
energy	12	7			
Δ	10 -12	10-7	10 ⁻⁵		

The figures in Table 4, which provide only the lower limit of energy, passing to radiation, inasmuch as only two factors were taken here into account:

the emission of δ — electrons at $E_{\delta} < 1$ kev and the emission of δ —electrons of subsequent generations (δ —electrons fromed from δ —electrons of the first generation). But from Table 4 it follows already that δ — electrons may play an essential role in the formation of X-rays.

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**** THE END ****

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