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Dynamical Capture of the Moon by the Earth

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ABSTRACT

Numerical integrations of dynamical equations relating to the Sun, Earth, and Moon have been carried out to determine conditions under which the Moon could be temporarily captured from an independent planetary orbit to become an Earth satellite. The period of time $2T$ for which it would remain captured depends critically on the solar eccentricity, and on the major-axis and eccentricity of the lunar orbit. This orbit becomes unstable, in the sense that the Moon would eventually escape, if either a or e is increased beyond certain limits. The dependence of T on the initial values of a and e is shown in diagrams, as also are the paths of the Moon relative to the Sun and relative to the Earth in its motion of escape. Possible sources of dissipative action that could render such temporary dynamical capture permanent are briefly discussed.

I. INTRODUCTION

The extraction of the Moon from the Earth through some mechanism of rotational instability, and one that can also set it into orbital motion around the Earth, has come to be widely recognized as almost certainly dynamically impossible. Accordingly, ideas have turned toward the notion that the Moon first was formed as a separate planet and later was captured by the Earth. Although many authors have suggested this possibility, there does not seem to have been any systematic study showing exactly how such capture might occur. The object of the work of this report is to discover conditions under which this capture could take place. It can reasonably be conjectured that temporary capture is possible in a three-body system of the Sun, Earth, and Moon. Nevertheless, it is obviously of importance to establish as rigorously as possible that capture can take place. Moreover, it would be of value to obtain numerical indications of the initial dimensions that the lunar orbit might have on the basis of such an origin.

It seems certain that the essential features of any process of capture of a satellite by a planet must, in the final stages, be governed purely by inverse-square dynamical forces arising from the mutual gravitational attraction of the bodies concerned, and that these bodies can be regarded as point-masses for the purpose of investigating the motion. But dissipative action well may need to be operative within the system to bring the bodies, the Earth and the independent small planet (the Moon), into paths making capture possible, and again to be operative to render a capture permanent, once it has occurred.

Considerations of a phase-space or ergodic kind, for example, as are implied by Poincaré's recurrence theorem, imply that if the Moon were captured by purely dynamical forces it would eventually escape again. On the other hand, in order to arrive at actual conditions under which the Moon could be captured and the period of time it would remain a satellite, if otherwise unaffected, it is necessary to develop specific numerical instances by means of the equations of celestial mechanics.

If, as we shall suppose, the Moon is considered to be moving initially in a planetary orbit coplanar with that of the Earth and closely adjacent to it, there are still as many as four parameters associated with the Moon's motion (corresponding to the position and velocity at any selected time). Simultaneous adjustment of all of these parameters to assure that a capture eventually occurs would obviously be a matter of extreme difficulty, if not of impossibility. But this obstacle can be avoided quite simply in the following way, at the price of some restriction to the type of path.

Suppose that at the instant $t = 0$, the Earth E is at perihelion in an elliptic orbit round the Sun S and that the Moon M is collinear with S and E, and directly opposite to the Sun, so SEM is a straight line. Next, suppose M to be projected at right angles to SEM in the plane of the ecliptic and in the same sense as the rotation of E around S, and (in the first instance) with just such a speed as would put it into strictly circular motion around E in the absence of solar perturbations. With EM equal to the present lunar distance, it is highly probable that escape would never occur under purely dynamical forces, especially with a solar eccentricity e' remaining small, of the present order of 1/60. But if M is started at some sufficiently larger distance, it is to be expected that, sooner or later as the initial distance is increased in a series of trials, the Moon would circuit the Earth for a limited period of time and then escape into a separate planetary orbit. This in fact is borne out by detailed numerical calculations, described below.

An important feature of a motion started in this way is that it is accurately reversible about $t = 0$ to give a mirror-image of itself in the major-axis of the Earth–Sun orbit. Thus, if the calculated time to escape is T years, the whole motion can be regarded as consisting of a capture at $t = -T$ followed by a number of circuits of the Earth terminating in escape at $t = +T$. The procedure plainly involves a restriction to a limited class of all such orbits, beginning with capture and ending with escape, but it is clearly adequate to demonstrate the possibility of temporary dynamical capture. It has the added advantage of giving double value for the machine time, which has in fact to be fairly extensive in such calculations in order to make certain that the quantities emerging remain meaningful.

II. EQUATIONS OF MOTION

The mass of the Moon is regarded as negligible. This is not in any way a restriction, since the motion of the Moon is the same as that of a particle of negligible mass moving under two centers of force: one of mass $E^3/(E + M)^2$ situated at the center of mass of the Earth–Moon system, and the second the Sun itself. The motion of the center of mass of the Earth–Moon system about the Sun is known to be purely elliptic to an accuracy of about 10^{-7}. The relative motion of the Earth E and Sun S is therefore taken to be accurately elliptic, and the whole motion takes place in the plane of motion of E and S. An origin of coordinates is taken at the Earth with non-rotating axes. The negative *x-axis* is towards the Sun at the instant of perihelion, and the positive *y-axis* is perpendicular to this in the plane of the ecliptic in the sense ahead of Ox determined by the direction of motion of the Earth around the Sun. The motion of the Moon is regarded as limited to the Oxy plane. Suffixes are used to denote the coordinates of one object relative to another measured parallel to these axes, thus (x_{EM}, y_{EM}) are the coordinates of the Moon relative to the Earth. Also we write $r_{EM} = (x_{EM}^2 + y_{EM}^2)^{1/2}$. The equations of motion then take the form

$$\ddot{x}_{EM} = -E\frac{x_{EM}}{r_{EM}^3} - S\left(\frac{x_{SM}}{r_{SM}^3} - \frac{x_{SE}}{r_{SE}^3}\right)$$

with a similar equation for y_{EM}.

III. NUMERICAL RESULTS FOR INITIALLY CIRCULAR MOTION

A number of cases were investigated corresponding to different values of the solar eccentricity e'. It seemed beforehand that larger values of e' might be more conducive to escape of the Moon. The calculations showed this to be the case in the sense that, for stability, the initial starting distance must be smaller, the greater e' is made, but even when $e' = 0$ escape can still occur. In making the calculations, the instant of escape is defined to be that at which the osculating eccentricity of the lunar orbit first exceeds unity. But in every case it was also verified that the subsequent path of the Moon became planetary about the Sun. The initial stages of this path were also computed.

It was found that the time to escape, T yr, is extremely sensitive to the initial distance, EM, selected. Also, for each value of e' there exists a critical value of this initial distance such that for minutely smaller values escape never occurs, while for minutely larger values it occurs in a short time of the order of a few years. The results are exhibited diagrammatically in Fig. 1 which shows the time of escape T in years plotted (on a logarithmic scale) against the initial distance of the Moon (measured in 10^{-3} AU as unit) for values of the solar eccentricity e' of 0.2, 0.1, 0.05, 0.01675 (the present value), and 0. In each case the position of the crucial initial distance is indicated by the almost vertical parts of the curves (ended by heavy arrows). The steepness of these parts of the curves would be even greater than they appear if an ordinary linear scale were adopted. The critical values of the initial distance EM for the cases calculated are given below in Table 1.

To take a specific case: For $e' = 0.2$, with initial value $EM = 4.0 \times 10^{-3}$ AU, the value of T is in excess of 60 yr,

Table 1. Critical value of initial distance *EM* for the cases investigated

Solar eccentricity e'	Initial EM for escape, 10^{-3} AU
0.2	4.00
0.1	4.33
0.005	4.80
0.01675	4.90
0.0	4.92

and in the whole interval of $2T$ while the Moon is captured it makes several hundred revolutions around the Earth. (In this case, calculations were ceased at 60 yr before escape had actually taken place because of the large amount of machine time required.) Similar results emerged for other smaller values of e', as will be seen from Fig. 1. The feature that is common to all cases examined is the existence of this extremely steep *cliff* separating instances of escape from those of non-escape. As e' diminishes, the critical initial distance increases, with a limiting value, when $e' = 0$, of about 4.93×10^{-3} AU. In terms of the present mean lunar distance, the values of the initial distance for the cases considered range from about 1.56 (for $e' = 0.2$) to 1.90 (for $e' = 0$).

It is of some interest to follow the development of the orbit with time. For this purpose a case with shorter time of capture was selected, in fact, that corresponding to the fifth point from the left in Fig. 1 for $e' = 0.2$ for which the time to escape was 7.22 yr. In Fig. 2, the value of the osculating eccentricity is plotted. It is seen how for the first year e oscillates about a value near 0.1 before rising to values of about 0.3, though at the end of 2 years it has temporarily reached about 0.8, which it almost attains again at 2.8 yr and 4.0 yr. In the period 5 to 6 yr, the average value is again small, about 0.2, and gives little indication of the impending rapid increase to beyond unity. This general uncertainty of development is reflected otherwise in the curves of Fig. 1 where successive values of T, the time to escape, change discontinuously by considerable amounts when escape occurs within a matter of a few years.

In Fig. 3 is shown the actual path of the escaping Moon relative to the Sun. The inner curve (labelled from 6.00 yr to 6.96 yr) shows the path of the Earth relative to the Sun. This of course was the same for each preceding year and remains the same for subsequent years. The Earth–Moon distance is so small (initially 4.05×10^{-3} AU) compared with the Earth–Sun distance, that before escape becomes imminent the path of the Moon is indistinguishable in the diagram from that of the Earth, and no attempt is therefore made to represent it until $t > 7.00$ yr. The path of the Moon is then shown by the points labelled 7.03, 7.07, 7.11, 7.15, 7.17, and 7.22 yr (at which instant escape is completed). Thereafter, the heavier curve, labelled at times 7.22 yr, 7.26, 7.30, etc. shows the planetary path of the Moon around the Sun after escape.

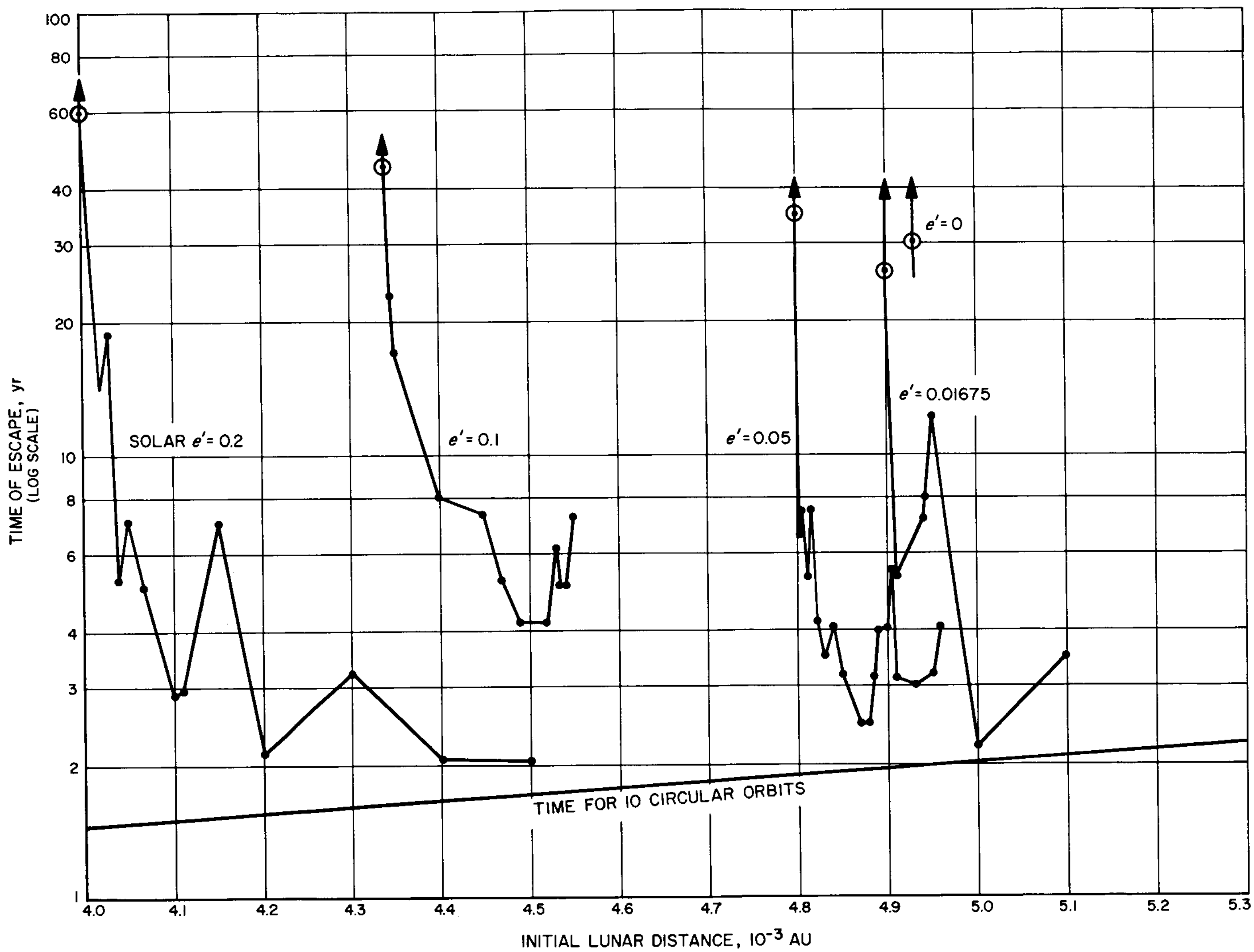

Fig. 1. Plot of *time to escape* against *initial lunar distance* for different values of solar eccentricity e′

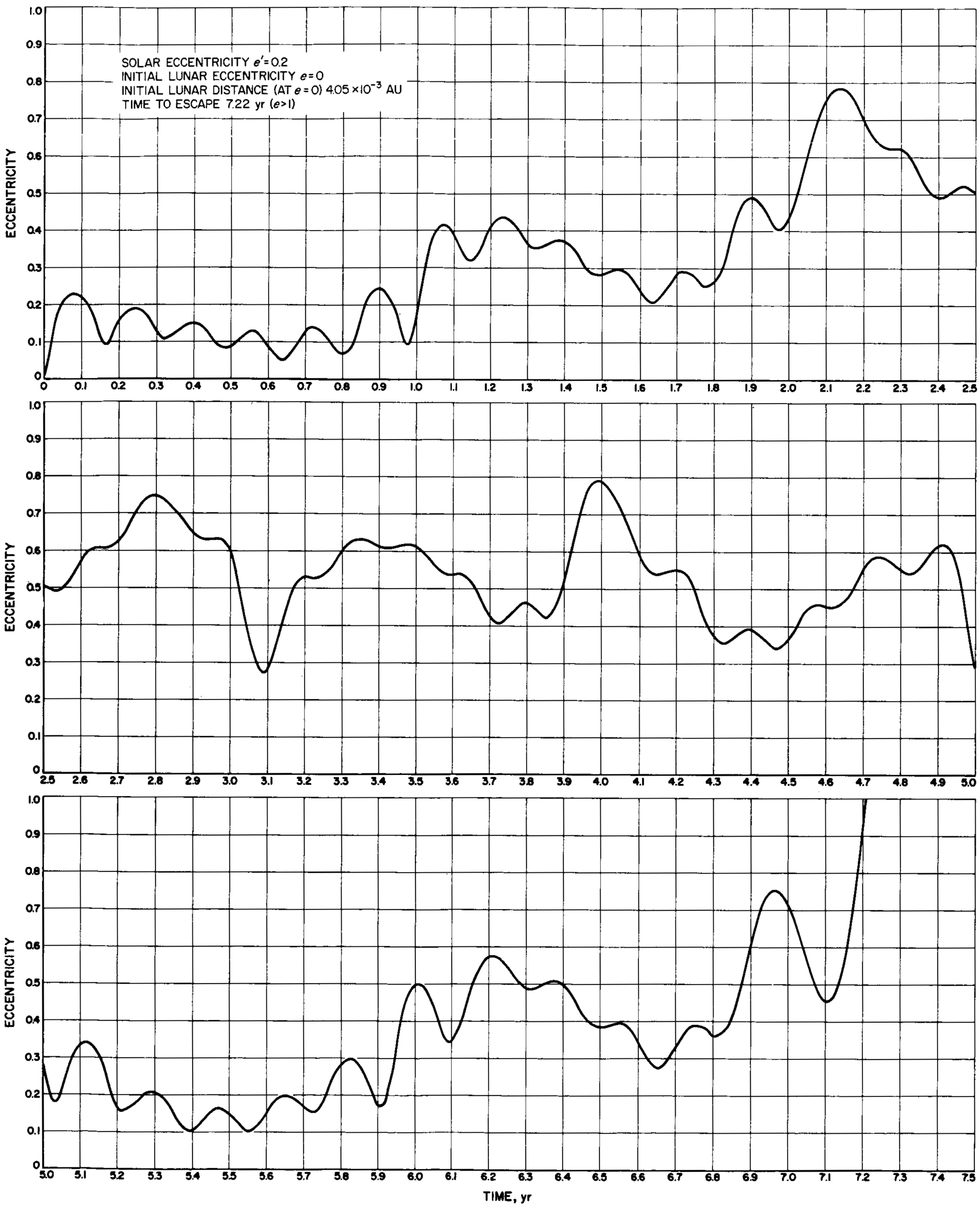

Fig. 2. Variation of osculating eccentricity of lunar orbit for the case $a = 4.05 \times 10^{-3}$ AU (initially), $T = 7.22$ yr, $e' = 0.2$

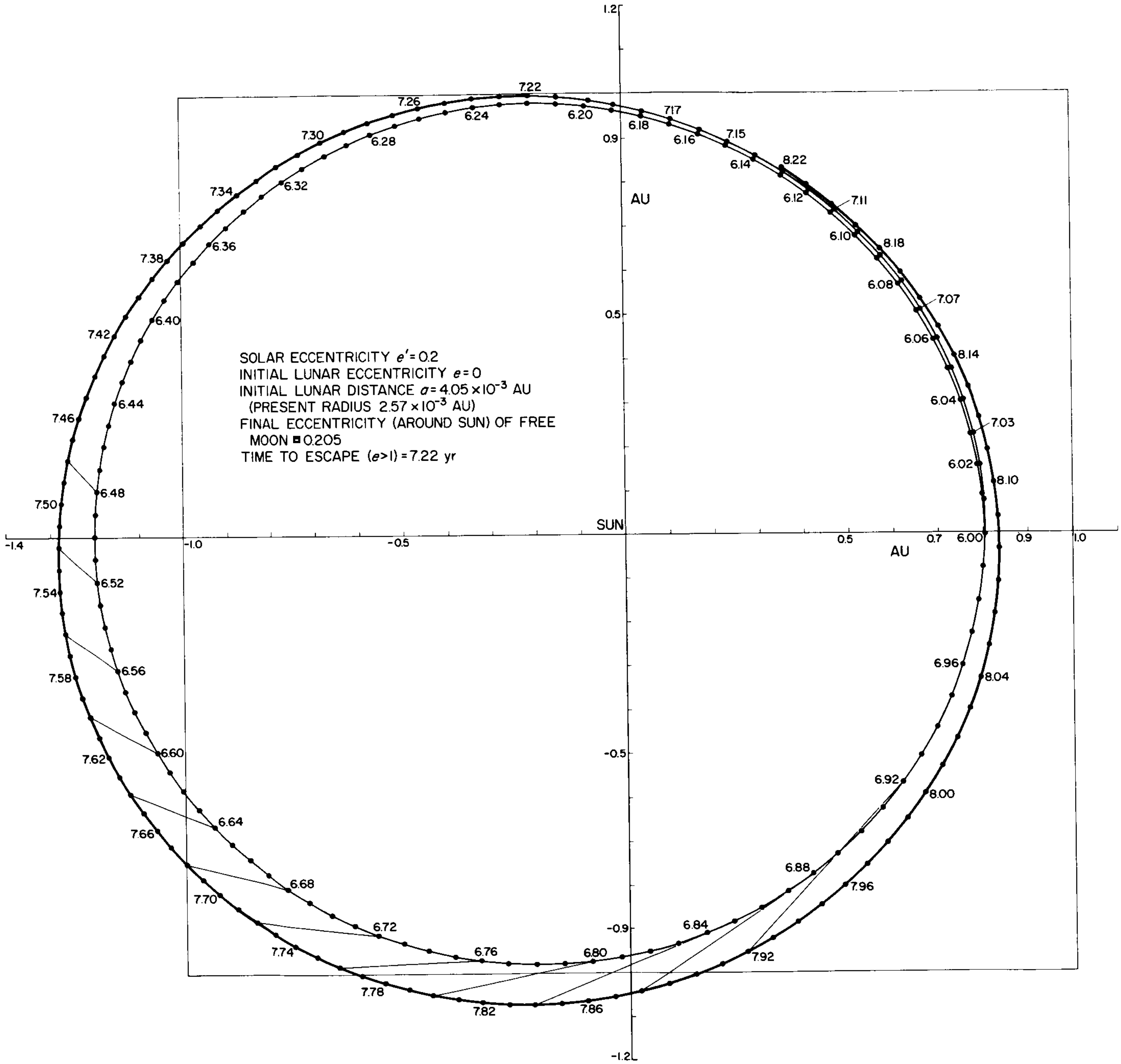

Fig. 3. Lunar escape orbit relative to Sun. ($a = 4.05 \times 10^{-3}$ AU, $T = 7.22$ yr, $e' = 0.2$)

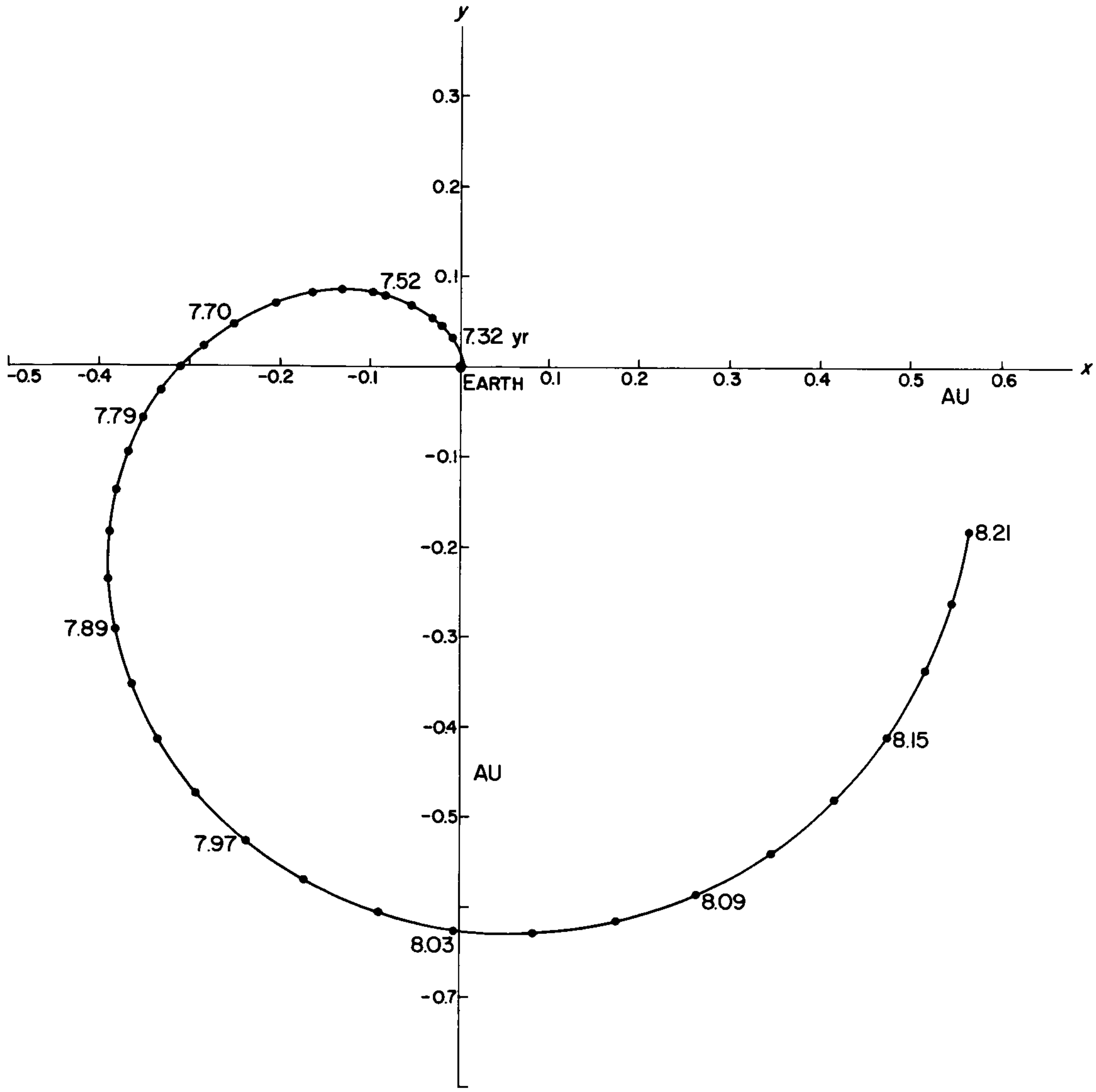

Fig. 4. Lunar escape orbit relative to Earth. (Same case as Figs. 2 and 3)

The thin straight lines, as for example, joining the point marked 6.48 for the Earth (which is also the position of the Earth at 7.48 yr) to 7.48 for the Moon, and so on, show the position of the Moon relative to the Earth. This relative path is shown on a comparable scale in Fig. 4 for the portion from about 7.3 yr to 8.21 yr, by which time the Moon has become an independent planet again. Because of the disparate sizes of the lunar orbit and that of the Earth around the Sun, the later stages of the path while the Moon is captured lie within the small black dot, labelled Earth, in Fig. 4. This has been drawn on about a tenfold scale in Fig. 5, and on about a hundredfold scale in Fig. 6, which shows the path just before escape from 6.57 to 7.12 yr.

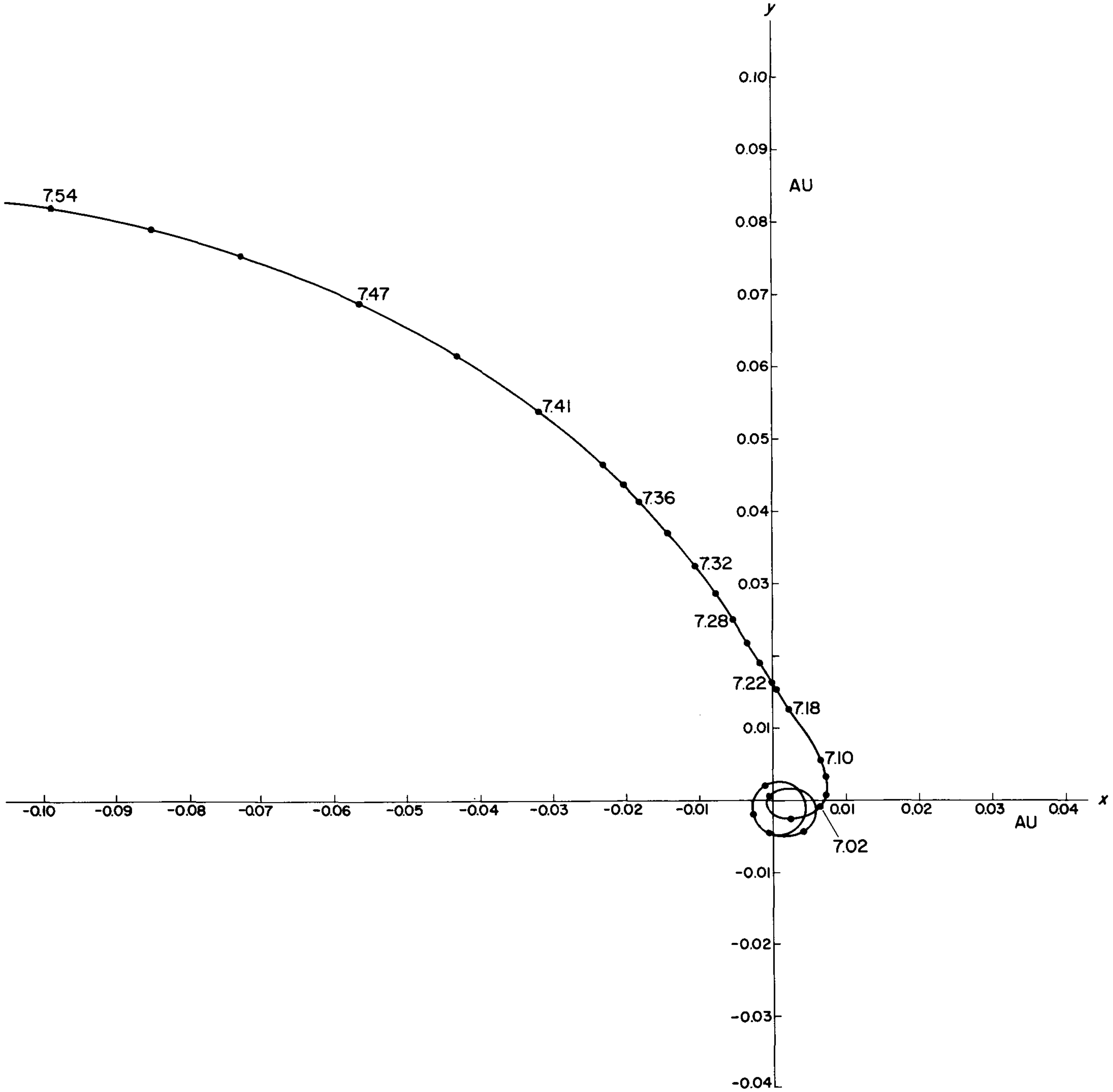

Fig. 5. Lunar escape orbit relative to Earth with scale increased tenfold

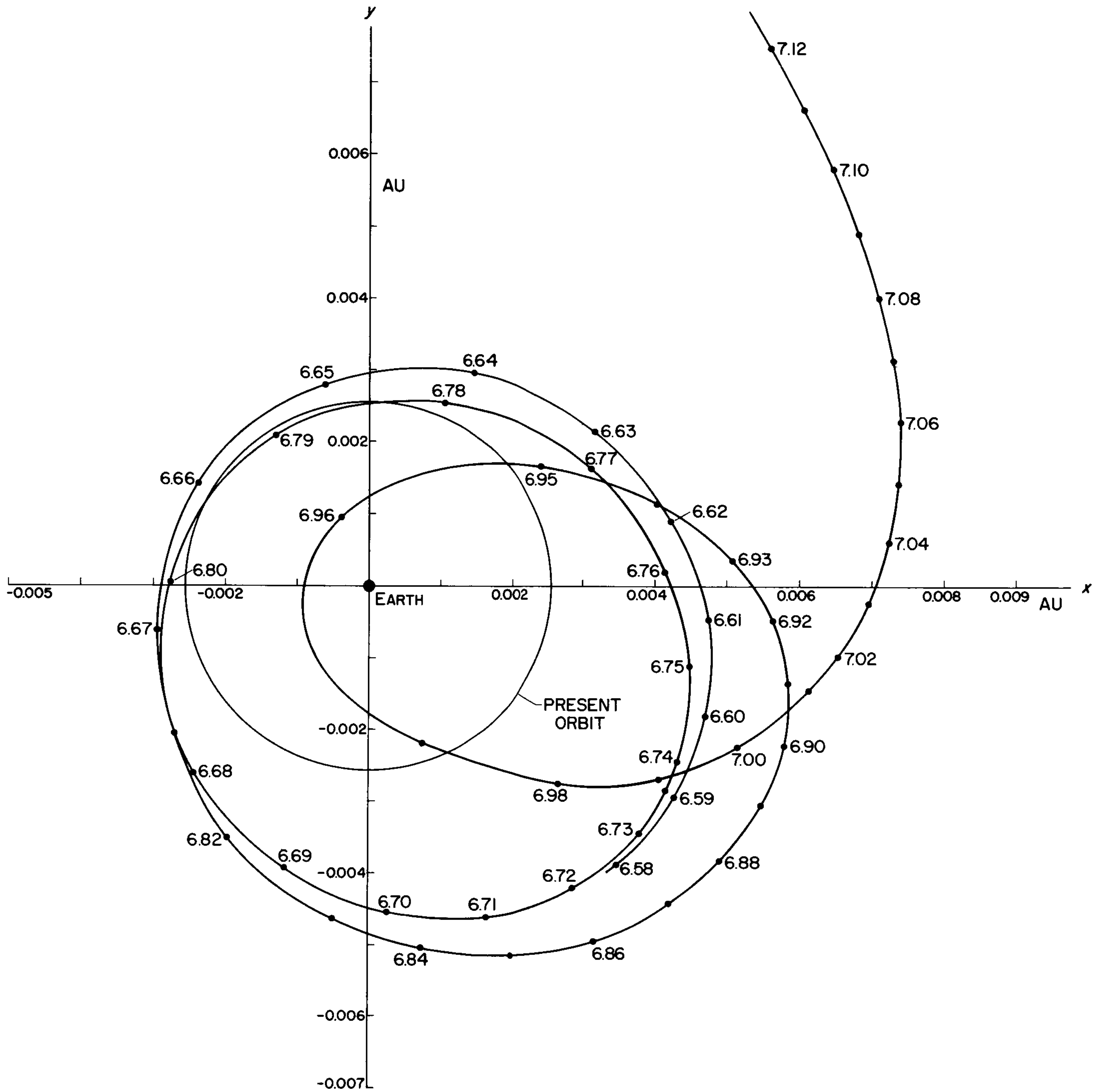

Fig. 6. Lunar escape orbit relative to Earth with scale increased hundredfold

IV. FACTORS TENDING TO STABILIZE AN ORBIT

Inspection of Fig. 1 shows that there are two effects that would be capable of changing an unstable orbit, with the Moon destined for escape, into a stable one. First, if the solar eccentricity e' were to undergo a decrease, then the corresponding *cliff* would move farther to the right in the diagram and the Moon would become trapped for good. Second, if the mean distance of the Moon were reduced during the time it was captured, then the representative point would move to the left (e' remaining fixed) with the same result. Clearly a combination of the two changes would have the effect of rendering the capture permanent.

A possible cause of such changes is to be found in the meteoritic bombardment of the Moon and Earth. If initially the Earth and Moon (as an independent planet) were moving in a region containing meteoritic material, it seems likely that their orbits would be rendered increasingly adjacent until circumstances appropriate to temporary capture occurred. If meteoritic impacts during a few hundred years of capture were sufficiently intense, this could have the effect of reducing the lunar distance and bringing about permanent capture. Interaction of the Earth with meteorites, and possibly with other material in the solar system, whether directly impacting it or not, would almost certainly have the effect of rounding up the orbit, that is, to reduce e' and render capture permanent. If a temporary dynamical capture were not thereby transformed into a permanent one, then clearly the Moon would be returned to an adjacent planetary orbit to await a further possibility of temporary capture, and so on.

It is doubtful whether decreases in e' resulting from the dynamical action of other planets could alone effect the capture of the Moon. This dynamical action requires periods of time of the order of 10^4 years for any important change to occur, and, although not periodic, is oscillatory and confined between fixed limits over periods of time of the order of 10^6 to 10^7 yr (Ref. 1). Accordingly it seems certain that dynamical action must be aided by dissipation effects in order to make a capture permanent.

V. EFFECT OF INITIAL ECCENTRICITY ON STABILITY

Since the present lunar distance is only 2.57×10^{-3} AU, subsequent reduction of the capture distance by about 40% would be needed to produce the present arrangement. Meteoritic impacts on the Moon could if sufficient bring this about, but it would require mass to the amount of about 1/5 that of the Moon. With the possibility of tidal friction operating in the reverse direction, the amount required might even be slightly greater. In view of the scanty knowledge of the origin and history of meteoritic material, and indeed of other forms of interplanetary material too, it is not possible to say whether this amount is excessive or not. It is to be remembered however in this connection that it is not necessary for such impacting material actually to be added permanently to the body of the Moon. There is growing evidence that some meteoritic impacts probably result in an overall loss of mass from the Moon because of the low escape speed. Such material ejected from the surface of the Moon would be available again for further impacts, except in so far as a portion of it might be permanently captured by the Earth and possibly by other planets. It is only incoming material that importantly affects the lunar distance, since, on an average, it has negligible angular momentum about the Earth, whereas outgoing material exploded off the lunar surface carries with it the existing angular momentum per unit mass and therefore has little effect on the Earth–Moon distance (Ref. 2).

On the other hand, it would seem that physically the rounding-up of an orbit without serious change in angular momentum could be more readily achieved than a large reduction in the major-axis. Almost all processes of dissipation (impacts, encounters with particles of all sizes down to individual molecules, and other actions) probably operate in the direction of rounding-up an orbit. If at any stages of the solar system the atmosphere of the Earth were more extensive than at present, for example, the dipping into it of the Moon when near perigee would result in the rounding-up of the orbit. Further calculations have therefore been carried out to ascertain how stability of the lunar orbit is affected by increasing the eccentricity without changing the present angular momentum.

For this purpose, suppose that $a(1 - e^2)$ is held fixed at its present value of 2.5618×10^{-3} AU, and the initial value of e is gradually increased in a series of trial cases. As before, S, E, and M can be taken as initially collinear with E at perihelion. To obtain a reversible motion, M can be started either at apogee or perigee. Both possibilities have been investigated, and calculations show that if the time to escape T is plotted against e, a *cliff* again exists for any adopted value of e' (the solar eccentricity) such that if e is slightly less than a critical value, defined by the *cliff*, then escape never occurs; while if e is only slightly greater, then escape occurs within a few years. The forms of the curves for four cases computed in detail are shown in Fig. 7. The critical values of e for the various cases calculated are given below in Table 2. Although these values of e are all substantially greater than the present lunar eccentricity (0.0549), curiously enough they do not imply very large departure from geometrical circularity. For $e = 0.6$ the ratio of the axes is about 5:4, and even for $e = 0.7$ the ratio does not differ much from unity at 10:7.

Table 2. Critical value of e for the various cases indicated

Solar eccentricity		Critical e
0.05	Apogee case	0.71
	Perigee case	0.59
0.01675	Apogee case	0.73
	Perigee case	0.61

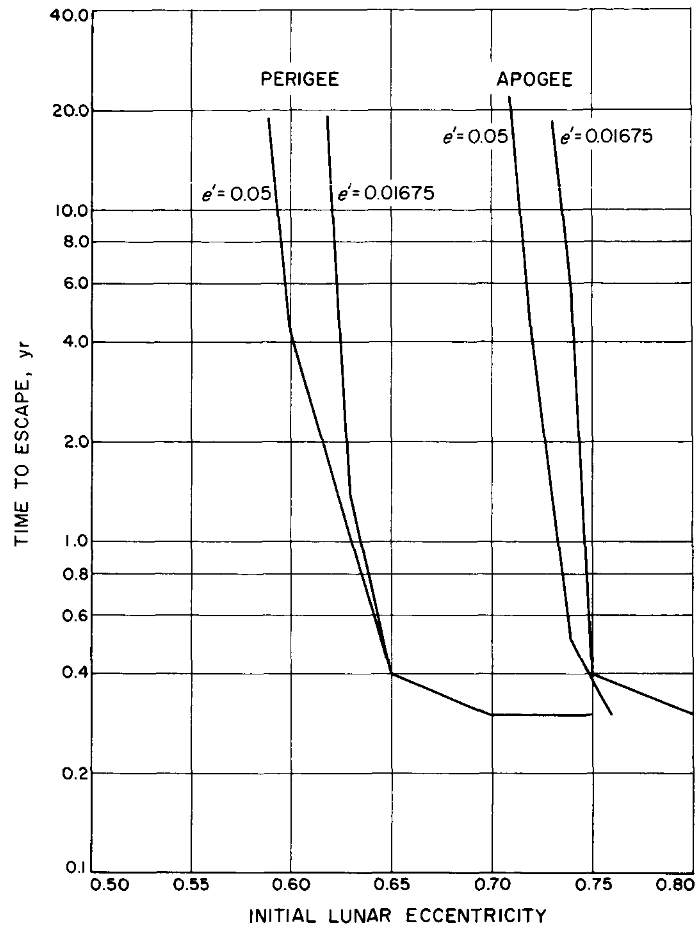

Fig. 7. Plot of *time to escape* against *initial lunar eccentricity* for two values of solar eccentricity e'

Figure 8 gives an example of how the osculating eccentricity varies with time up to the instant of escape. Figure 9 shows the escape path relative to the Sun for this same case, while Figs. 10, 11, and 12 show the lunar escape orbit relative to the Earth.

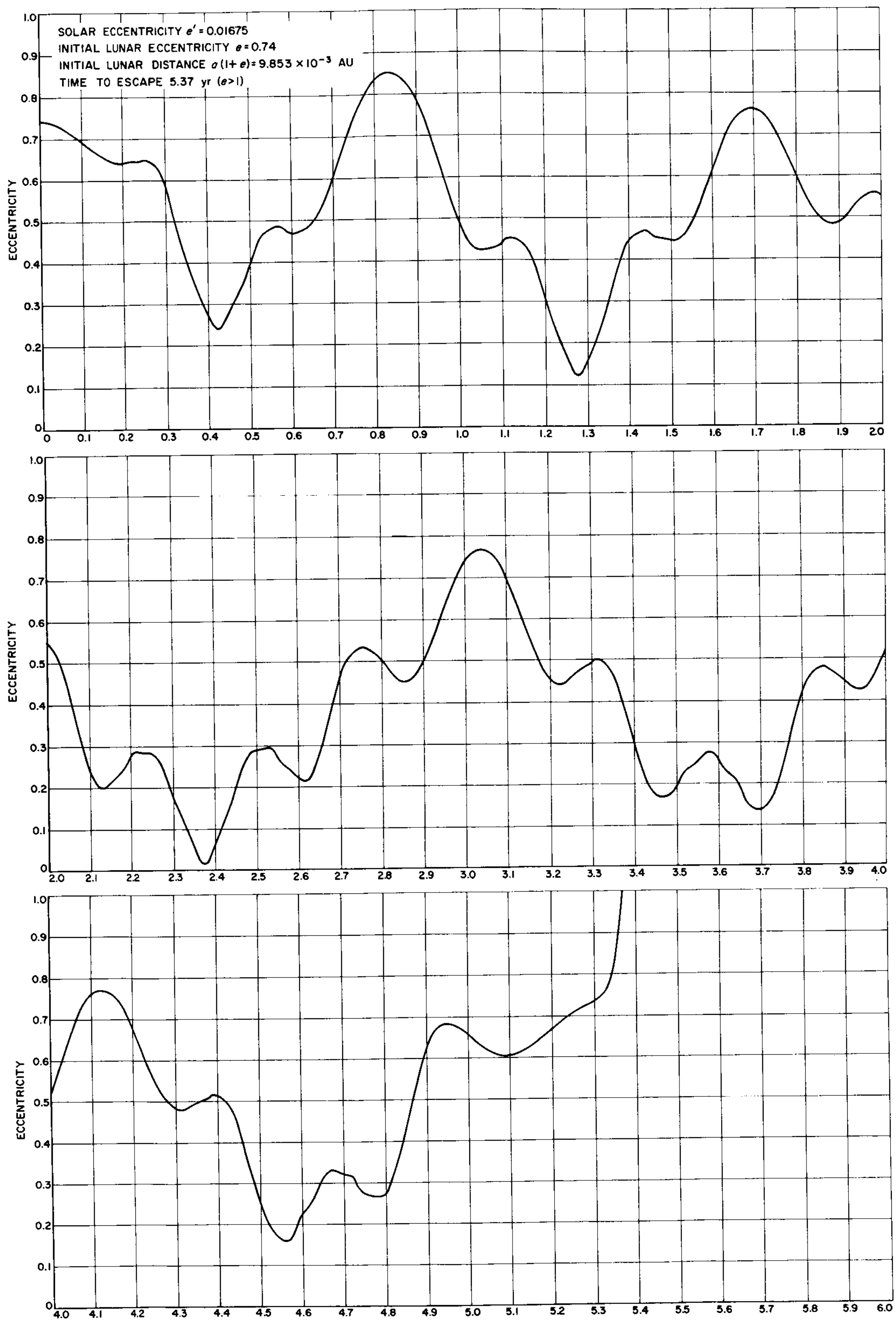

Fig. 8. Variation of osculating eccentricity of lunar orbit for the case $e = 0.74$ (initially), $T = 5.37$ yr, $a = 9.853 \times 10^{-3}$ AU, $e' = 0.01675$

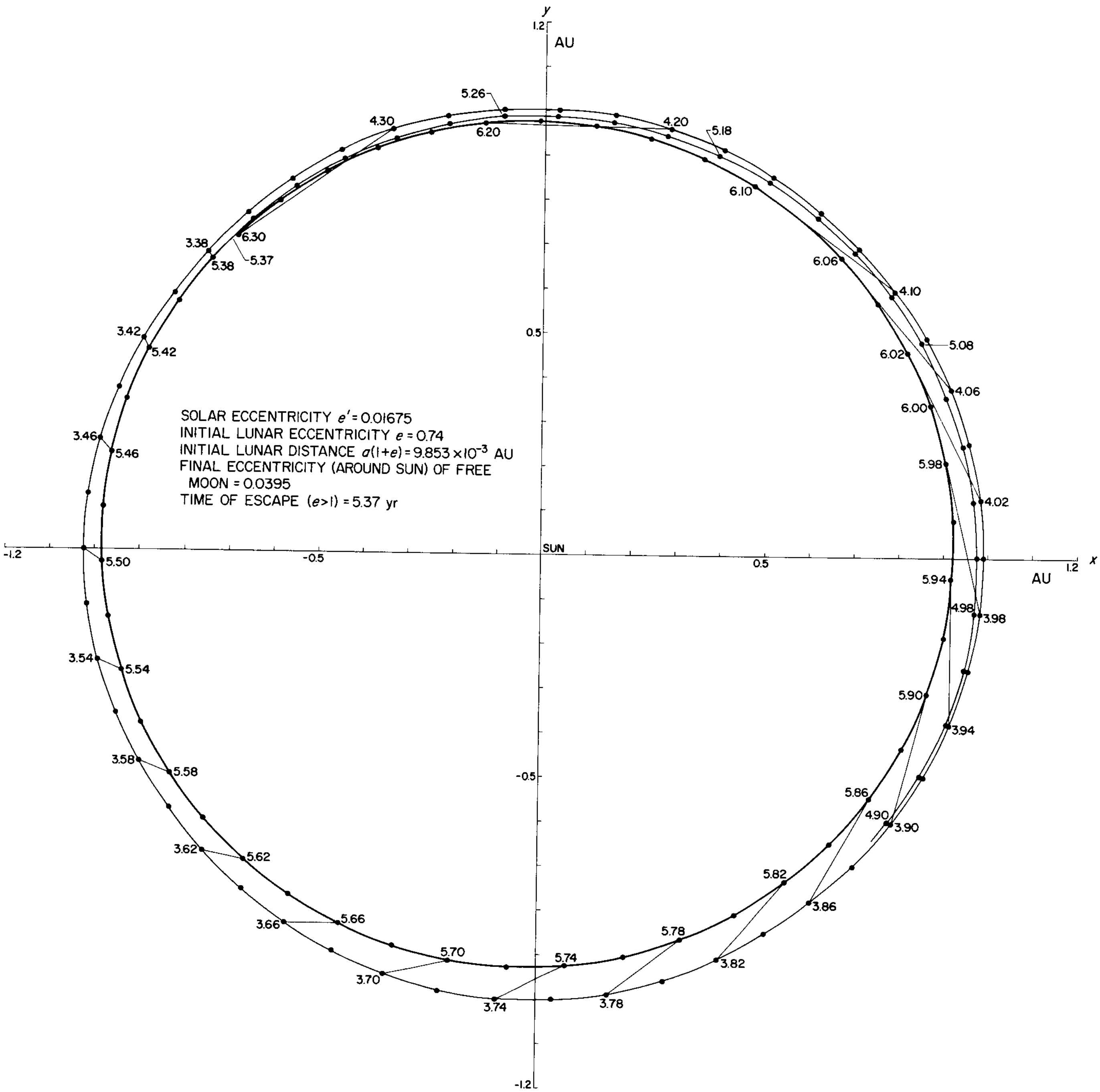

Fig. 9. Lunar escape orbit relative to Sun (Same case as Fig. 8)

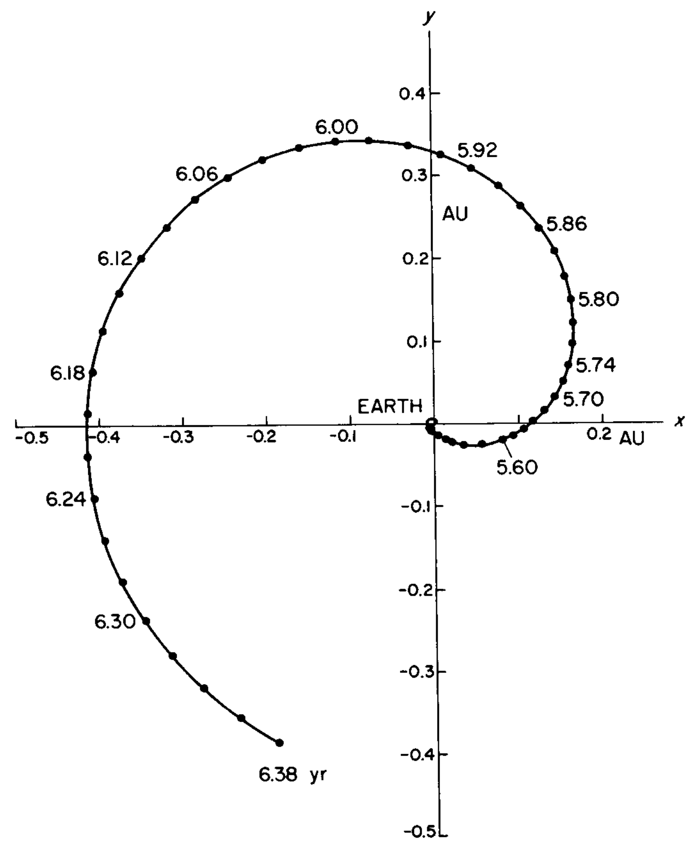

Fig. 10. Lunar escape orbit relative to Earth (Same case as Figs. 8 and 9)

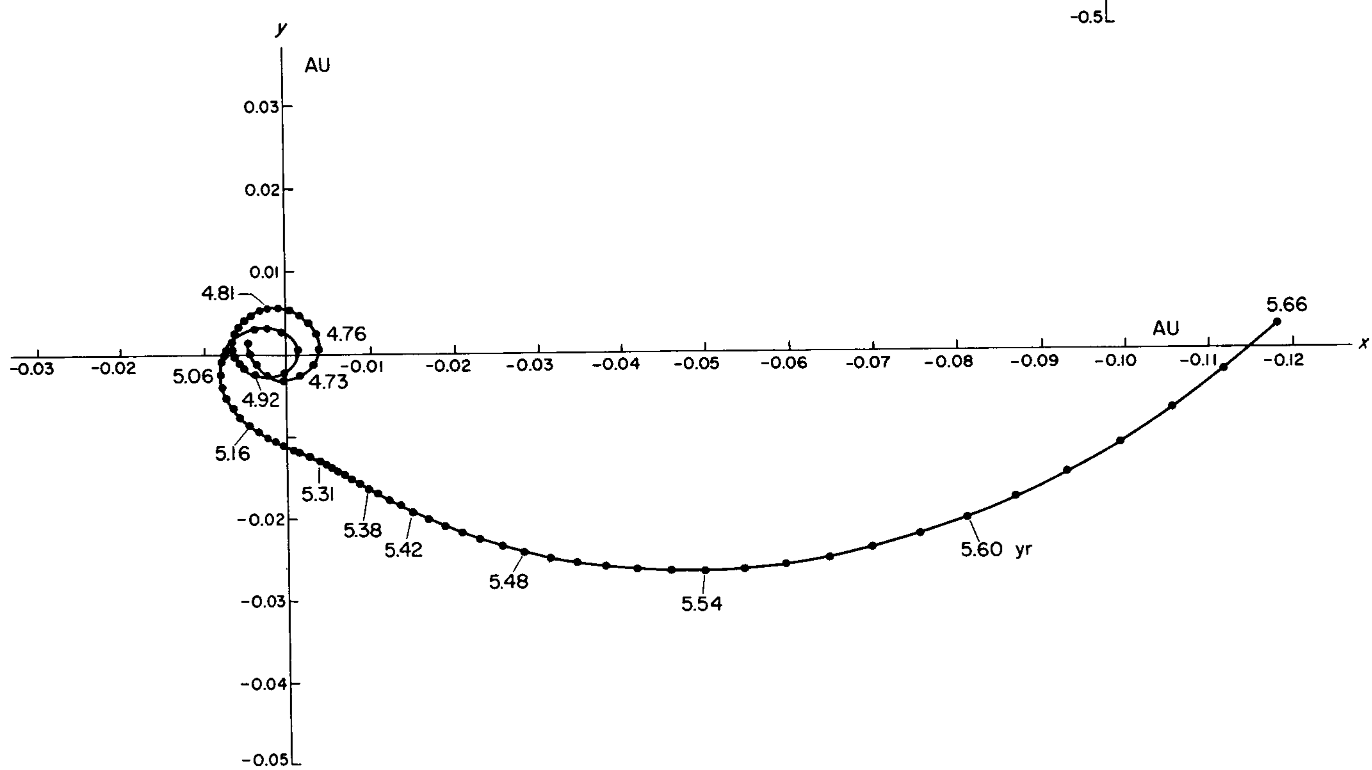

Fig. 11. Lunar escape orbit relative to Earth with scale increased tenfold

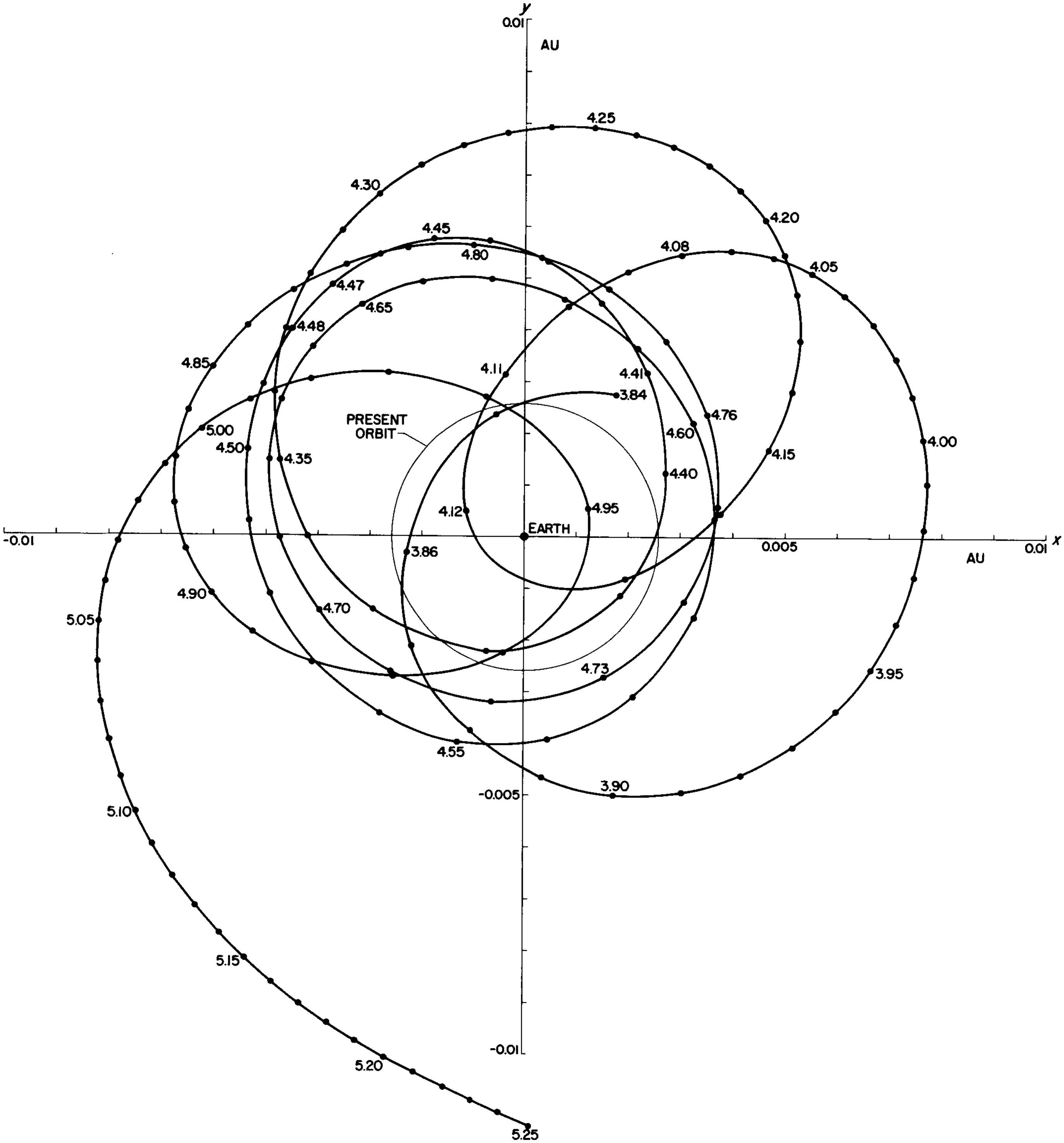

Fig. 12. Lunar escape orbit relative to Earth with scale increased hundredfold

VI. CONCLUSION

It may be concluded that instances can readily be found by the method here described of initial conditions leading to temporary capture of the Moon from an independent planetary orbit. Since tidal friction appears to be increasing the angular momentum of the Moon at the present time, there is some interest in initial conditions with less angular momentum than at present. Such cases remain to be calculated, but it may be conjectured with some degree of safety that the only requirement for instability would be correspondingly larger initial values of e than those shown in Table 2 above.

There remains also to be investigated the more numerous cases of unsymmetrical orbits. But, as explained above, this would involve extensive exploratory calculations to hit upon appropriate initial conditions, and this more difficult form of the problem has not been attempted.

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2. Lyttleton, R. A., "Effect on the Lunar Orbit of Meteoritic Accretion," *Icarus*, vol. 5, p. 162, 1966.

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