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"Thermal Conduction and Ion Temperatures in the Ionosphere"

by

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ABSTRACT

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An analysis is made of the ion energy balance equation for the ionosphere including the effects of  $O^+$ ,  $He^+$ , and  $H^+$  ions, thermal conduction, and the cooling by atomic hydrogen and helium. It is shown that thermal conduction strongly affects the ion temperature profile for conditions of moderate and low electron densities and that the high altitude ion temperature can be significantly less than the electron temperature. In addition,  $H^+$  in concentrations as low as 10% of the total ion density is found to be a major recipient of heat from the ionospheric electron gas and has an important influence upon the total ion energy budget.

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## I.- INTRODUCTION

Through numerous experiments it has been determined that the electron temperature above 150 km is consistently higher than the temperature of the neutral atmosphere. To explain this state of thermal nonequilibrium, a number of theoretical studies have been made of the electron heat budget equation using photoionization as the principal electron gas energy source [1,2,3,4,5]. The most recent of these analyses [5] gives results in general agreement with current measurements but, because the electron temperature is very sensitive to minor changes in the solar EUV flux, the electron density, and the magnitude of the assumed mechanisms of electron energy loss, a high degree of correspondence cannot be expected at the present time.

The primary source of electron energy loss above 300 km altitude occurs by means of elastic collisions between electrons and the ambient ions. The ions, in turn, transfer this excess thermal energy to the neutral gases of the atmosphere. Hanson [3] was the first to point out that, because the density of the neutral atmosphere decreases much more rapidly than that of the electron gas, the ion temperature should have a pronounced altitude dependence in response to the electron heating, increasing gradually from the neutral gas temperature near 300 km to the electron temperature above 800 km.

Experimental support for the transition behavior of the ion temperature can be inferred from satellite measurements [6] and incoherent radio backscatter data [7,8]. It is necessary to point out, however, that although the results of Hanson [3] indicate that the electron and ion temperatures should be approximately equal at high altitudes, in fact, in the available experimental data [6,7,8,9] there appear to be significant deviations of the ion temperature towards values which are consistently lower than the electron temperature, even in the atmospheric regions where the ion-neutral energy losses are not effective in cooling the ion gas. Further, in the regions 400-600 km it is difficult to match the calculated values of ion

temperature with those actually measured under conditions of low electron density.

To reconcile the differences between theory and experiment, a reanalysis has been made of the ion energy balance equations to include the previously neglected effects of ion thermal conduction, the presence of  $\text{He}^+$  and  $\text{H}^+$  ions, and the cooling by atomic hydrogen and helium. The results, presented in this paper, indicate that ion heat conduction can play an important part in determining the profiles of ion temperature, leading to values which are considerably below the electron temperature. It is also shown that  $\text{H}^+$  ions are important in the over-all ion energy balance, even in concentrations less than 10% of the total ion density.

## II.- ION ENERGY BALANCE

The time dependent energy balance equation for the ion gases of the ionosphere is

$$\frac{\partial U_i}{\partial t} = P_i - L_i - \nabla \cdot \bar{q}_i \quad (1)$$

where  $U_i = 3/2 n_i k T_i$  is the ion thermal energy per unit volume,  $n_i = n(\text{O}^+) + n(\text{He}^+) + n(\text{H}^+)$  is the total ion number density,  $k$  is Boltzmann's constant,  $T_i$  is the ion temperature (for a discussion of ion temperature coupling, see [10]),  $P_i$  is the rate of ion gas energy production,  $L_i$  is the rate of energy loss to the neutral atmosphere, and  $\bar{q}_i$  is the ion heat flux arising from conduction and diffusion effects.

Under the influence of thermal conduction and the non-local heating effects of photoelectrons, it is found [5,7] that the electron temperature above 300 km is significantly greater than the temperature of the neutral atmosphere, thus providing a heat source for the ion gases. By means of electron-ion collisions, the electron gas of temperature  $T_e$  and

density  $n_e$  transfers energy to the ions at the rates,

$$P_i(O^+) = 4.8 \times 10^{-7} n_e n(O^+) [T_e - T_i] T_e^{-3/2} \quad (2a)$$

$$P_i(He^+) = 1.9 \times 10^{-6} n_e n(He^+) [T_e - T_i] T_e^{-3/2} \quad (2b)$$

$$P_i(H^+) = 7.7 \times 10^{-6} n_e n(H^+) [T_e - T_i] T_e^{-3/2} \text{ ev cm}^{-3} \text{ sec}^{-1}, \quad (2c)$$

which are in the ratios 1:4:16. Thus, the heating rates for  $He^+$  and  $H^+$  are equal to the  $O^+$  rate when  $n(He^+)/n(O^+) = 0.25$  and  $n(H^+)/n(O^+) = 0.063$ , respectively, implying the electron energy transfer to the  $He^+$  and  $H^+$  ions plays an important part in the overall ion energy balance.

The rates of ion energy loss to neutral gases by means of elastic collisions have been discussed recently [10,12] and these results are adopted for this study. Briefly, for ions in parent gases the process of resonance charge exchange is dominant, while for ions in unlike gases the polarization interaction alone is assumed to be acting. For  $H^+$  and  $O^+$  in atomic oxygen and hydrogen, the effects of accidentally resonant charge exchange have been included.

The heat flux,  $\bar{q}_i$ , can be evaluated for a magnetic field-free medium as, neglecting thermal diffusion, [13]

$$\bar{q}_i = -K_i \nabla T_i + 5/2 n_i k T_i \bar{C}_i \quad (3)$$

where  $K_i$  is the ion thermal conductivity and  $\bar{C}_i$  is the ion diffusion velocity relative to the mass velocity of the combined ion gas and the neutral atmosphere. The evaluation of the two terms in (3) shows that heat conduction is generally dominant, but that there are some circumstances, especially for low temperatures and large ion densities, when the diffusion contribution resulting from a downward flux of ions cannot be neglected. However, because the calculation of  $\bar{C}_i$  requires the solution of the coupled ion temperature and density equations for conditions of chemical and diffusive equilibrium, it is difficult to adopt reliable values. Hence, in this analysis we take  $\bar{C}_i = 0$  and deal only with the heat flux arising from ion

temperature gradients in an equilibrium distribution of ion gases.

The calculation of  $K_i$  for a single ion gas follows from the work of Chapman<sup>[14]</sup> as,

$$K_i = 4.6 \times 10^4 T_i^{5/2} / A_i^{1/2} \text{ ev cm}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{K}^{-1} \quad (4)$$

where  $A_i$  is the ion mass in atomic units. Unlike the electron thermal conductivity, it is not necessary to include a thermoelectric reduction factor or, for altitudes above 300 km, to consider the effects of ion-neutral collisions<sup>[16]</sup>.

For the ion gas of the ionosphere, an extension of (4) must be made to include the effects of differing conductivities for each ion species. An exact expression is difficult to obtain and for the present analysis an approximate, density weighted, conductivity has been used in the form

$$K_i = 1.15 \times 10^4 T_i^{5/2} [n(O^+) + 2n(He^+) + 4n(H^+)] / n_e \text{ ev cm}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{K}^{-1}. \quad (5)$$

When the effect of the earth's magnetic field is considered it is found<sup>[13]</sup> that there is heat flow only along the lines of geomagnetic force. To include the influence of the inclination of the field lines upon vertical profiles of ion temperature, we introduce the magnetic dip angle,  $I$ , and rewrite (1) as

$$\frac{\partial U_i}{\partial t} = P_i - L_i + \sin^2 I \frac{d}{dz} \left[ K_i \frac{dT_i}{dz} \right] \quad (6)$$

where  $z$  is a vertical coordinate.

The solution of (6) for the ion temperature profile requires a knowledge of the electron and ion densities in the upper ionosphere. Following recent work<sup>[10,17]</sup>, we adopt a state of  $O^+$  diffusive equilibrium above 340 km while taking  $H^+$  and  $He^+$  to be in chemical equilibrium with  $N_2$ ,  $O_2$ ,

and 0 to an altitude of 500 km, above which a condition of ternary ion diffusive equilibrium is applied<sup>[17]</sup>.

The numerical solution of (6) requires a choice of ion temperature boundary conditions which relate to the influences of ion energy production outside the region of analysis. For the lower boundary condition it is assumed that conduction is ineffective at 300 km and the ion temperature is taken as the solution to (6) with  $K_1 = 0$ . At the upper limit ( $z = 1500$  km) it is convenient to take  $dT_1/dz = 0$  and neglect the heat conducted downwards from higher regions.

### III.- RESULTS

The ion energy balance has been solved numerically for steady-state conditions ( $\partial U_1/\partial t = 0$ ) using three atmospheric models of Nicolet<sup>[18]</sup> to represent the effects of changing aeronomic conditions. Throughout all calculations the electron temperature has been treated as a parameter which has the same value for all altitudes.

Several points of interest have been found. First, it appears that the ion thermal conduction can play an important part in determining the shape of the ion temperature profile at high altitudes, yielding, for a constant  $T_e$ , isothermal values which are significantly less than the electron temperature. This effect is shown in curve 1 of Figure 1 where a 1000°K neutral atmosphere has been used with only  $O^+$  ions present. The electron temperature was taken as 2500°K,  $I = 90^\circ$ , and the electron density determined from the boundary condition  $n_e = 2 \times 10^5 \text{ cm}^{-3}$  at 340 km. For curve 1 the cooling effects of atomic hydrogen and helium have not been included. To show the importance of thermal conduction, curve 2 (broken line) has been calculated with  $K_1 = 0$ . Since this choice for  $K_1$  is equivalent to the conditions found at the geomagnetic equator, it is seen that for identical aeronomic parameters the ion and electron temperature separation would have a range of



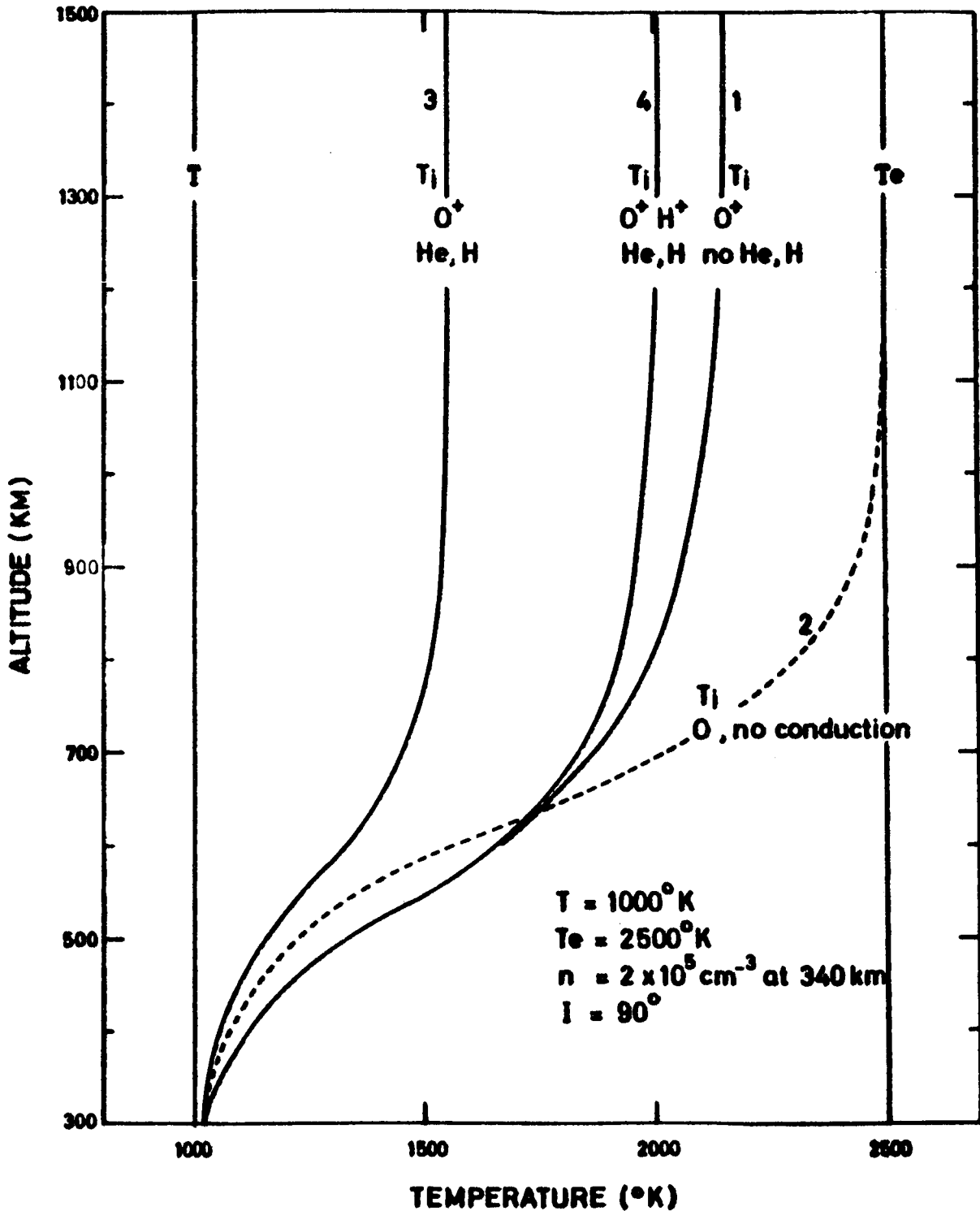


Fig. 1.- Ion temperatures with an electron temperature of 2500°K and neutral gas temperature of 1000°K, for a magnetic dip angle of 90° with an electron density of  $2 \times 10^5 \text{ cm}^{-3}$  at 340 km. Curve 1 includes  $\text{O}^+$  ions and thermal conduction but neglects cooling by He and H. In curve 2 the same conditions are used with no thermal conduction. Curve 3 includes  $\text{O}^+$  ions, thermal conduction, He and H. Curve 4 represents the final result with  $\text{O}^+$ ,  $\text{H}^+$ , He, H and thermal conduction.

350°K between the geomagnetic equator and poles.

A second effect occurs when atomic hydrogen and helium are included in the  $O^+$  energy loss. Curve 3 of Figure 1 shows that, for the model chosen here, the high altitude cooling, along with thermal conduction, reduces the isothermal value of the ion temperature to 1550°K. The introduction of  $H^+$  as part of the ionic composition reverses this situation, however, by providing an important source of ion heating at high altitudes. As shown by curve 4 of Figure 1, this results in an increase of 450°K in the isothermal ion temperature. In fact, the profile obtained for the ion temperature omitting the effects of  $H^+$  is very similar to the profile found when  $H^+$ , He, and H are included. The basis for this near equality lies in the counterbalancing of the superior heating rate of  $H^+$  by the large  $H^+$  charge exchange energy loss rate to atomic hydrogen and an improved ion thermal conductivity. The important point to note is that for the adopted model, the ion temperature is calculated to be about 500°K less than the electron temperature. The effects of  $He^+$ , which are not shown here, do not appear to be large and can be neglected in a first analysis.

The value of the electron density is important in determining the ion temperature profile. At high altitudes, where ion cooling by the neutral gases is ineffective, conduction must carry away the heat obtained from the electron gas. The changes brought about by varying the electron density boundary condition at 340 km are shown in Figure 2 for  $n_e = 1 \times 10^5$ ,  $2 \times 10^5$ ,  $5 \times 10^5$ , and  $1 \times 10^6 \text{ cm}^{-3}$  with  $T = 1000^\circ\text{K}$ ,  $T_e = 2500^\circ\text{K}$ , and both  $O^+$  and  $H^+$  ions being included. It is seen that ion heat conduction is generally important during conditions of low electron density or, more exactly, when the high altitude energy production rate is small. For enhanced electron densities at high altitudes, the ion conductivity is not large enough to transport downwards the energy produced in the ion gas following moderate separations between the ion and electron temperatures.

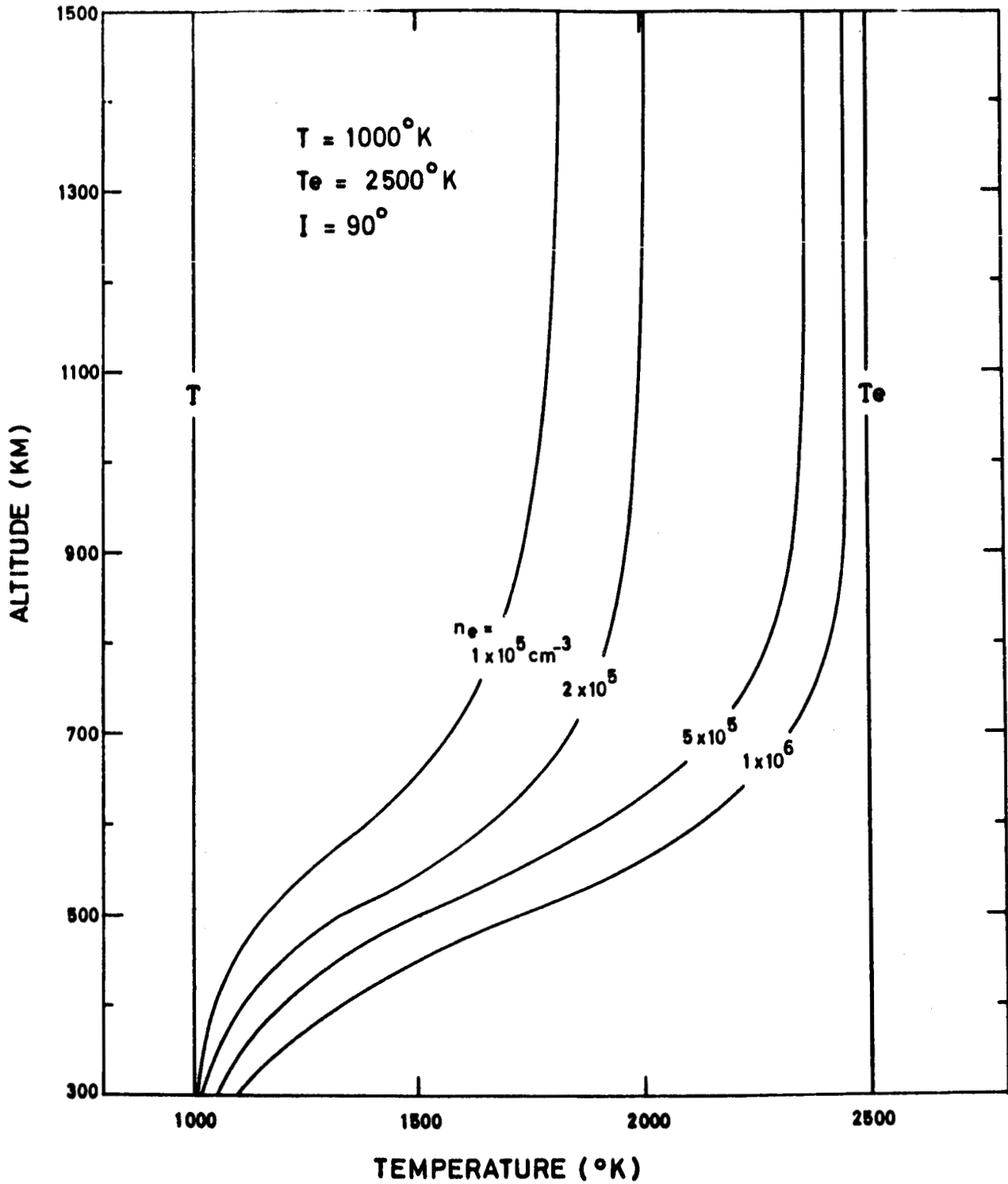


Fig. 2.- Effects of changing electron density boundary conditions upon ion temperatures with the electron density varying from  $10^5$  to  $10^6 \text{ cm}^{-3}$ .

Changes in the neutral atmosphere affect the ion temperature profiles by shifting the region of ion-neutral temperature decoupling with respect to the electron density profile. This effect is shown in Figure 3 where model atmospheres corresponding to neutral thermospheric temperatures of 800°, 1000°, and 1394°K have been used with  $T_e = 2500^\circ\text{K}$  and  $n_e = 2 \times 10^5 \text{ cm}^{-3}$ . The result, which is due to a combination of thermal conduction and change in the ion heating rate, is that for given values of electron temperature and density, the high altitude ion temperature decreases slightly with increasing neutral gas temperatures.

As a final point, a comparison has been made between the theoretical calculations based upon (6) with  $\text{O}^+$  and  $\text{H}^+$  ions, and the radar observations of Evans [7]. By choosing  $n_e = 3.4 \times 10^5 \text{ cm}^{-3}$  at 300 km,  $T_e = 2600^\circ\text{K}$ , and  $T = 1000^\circ\text{K}$  to match the observed parameters for 1200 EST, September, 1963, the results shown in Figure 4 have been obtained. The squares represent the experimental data, while the solid curve represents the theoretical result.

To evaluate the influence of diffusion (see discussion following (3)), which has not been included here, the divergence of the second term in (4) has been computed using the final results of the ion temperature and electron density profiles. For the diffusion effect to have an importance equal to that of thermal conduction, it is found that the ion diffusion velocity,  $\bar{C}_1$ , would have to be about  $7 \times 10^3 \text{ cm sec}^{-1}$  which, for the conditions indicated by Evan's data, appears to be too large to represent the true physical situation.

As a test for the existence of large ion diffusion velocities which could alter the ion temperature profile, it is noted that a net motion of  $7 \times 10^3 \text{ cm sec}^{-1}$  would yield a doppler shift of nearly 200 cycles/sec at a radar frequency of 430 Mc/sec. Hence, along with an asymmetry in the characteristic electron temperature peaks there should occur a detectable shift in the radar spectra half power points.

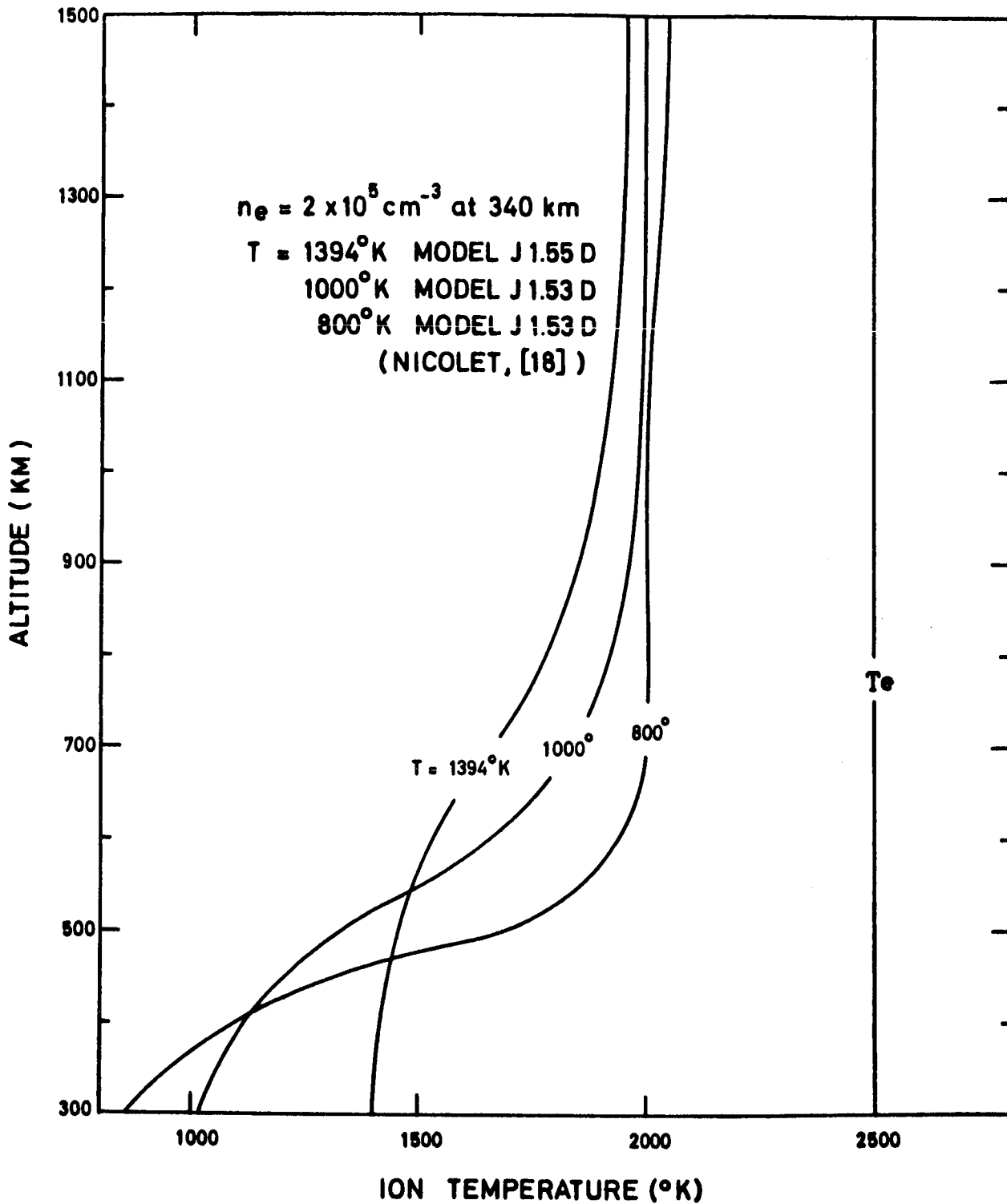


Fig. 3.- Effects of different models of the neutral atmosphere upon ion temperatures. The chosen models have thermospheric temperatures of 800°, 1000°, and 1394°K.

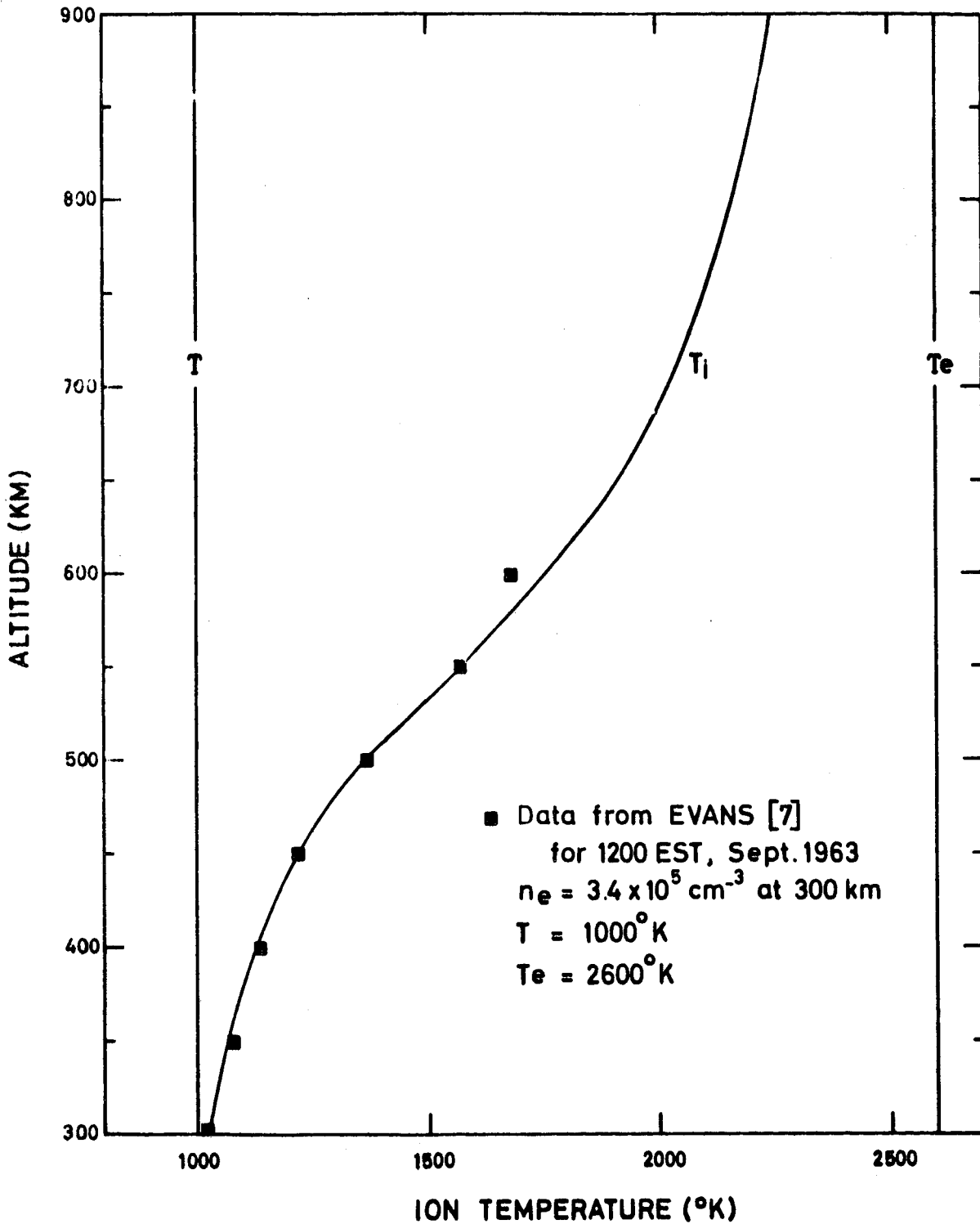


Fig. 4.- Comparison of calculated and measured ion temperatures. The solid curve was calculated using a 1000°K model neutral atmosphere and the experimental parameters of Evans [7]. The squares indicate the measured values.

#### IV.- CONCLUSIONS

From the present analysis it is concluded that thermal conduction within the ion gases of the ionosphere can be important in maintaining the high altitude ion temperature at values less than the electron temperature. Further, an ion energy budget which does not include both  $H^+$  ( and perhaps  $He^+$ ) and the presence of atomic hydrogen and helium cannot be expected to yield temperature profiles which agree with experimental data. The effect of ion cooling at high altitudes by means of downwards diffusion, although not included here, may also be important under particular conditions in further increasing the separation between the ion and electron temperatures.

Changes in the parameters of the charged and neutral atmospheres affect the importance of thermal conduction in the ion energy balance. Increases in the atmospheric temperature lead, for identical electron temperatures and charged particle densities, to a decrease in the high altitude ion temperature. Likewise, increases in the over-all electron density can be expected to increase the ion temperature and decrease the ion and electron temperature separation.

Since the effectiveness of the ion heat conduction term depends upon the dip angle of the earth's magnetic field, the separation of temperatures at high altitudes should be larger for higher geomagnetic latitudes. Thus, when the data of Farley<sup>[8]</sup>, taken at the geomagnetic equator, is compared with the results of Evans<sup>[7]</sup>, and Doupnik and Nisbet<sup>[9]</sup>, there is some indication that there are increases in the temperature separations for the higher latitude sites.

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