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SOLUTION OF NONLINEAR ALGEBRAIC EQUATIONS CHARACTERISTIC OF FILTER CIRCUITS

SUMMARY TECHNICAL REPORT

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GEORGE C. MARSHALL SPACE FLIGHT CENTER
Aero-Astrodynamics Laboratory

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NORTHROP SPACE LABORATORIES
6025 TECHNOLOGY DRIVE, HUNTSVILLE, ALABAMA 35805
TELEPHONE 837-0580
P. O. BOX 1484

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Research & Analysis Section Tech Memo No. 196

by

Frank B. Tatom
Theodore J. Thomas
Robert G. Schroeder

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AERO-ASTRODYNAMICS LABORATORY

Dr. Steve S. Hu
Director
Research & Analysis Section

HUNTSVILLE DEPARTMENT
NORTHROP SPACE LABORATORIES
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FOREWORD

The research effort described in this report was performed by Northrop Space Laboratories, Huntsville Department, for the Aero-Astrodynamics Laboratory of George C. Marshall Space Flight Center under Contract NAS8-20183. Mr. Mario Rheinfurth, Chief of Control Theory Branch, Dynamics and Flight Mechanics Division, acted as the NASA Contracting Officer's Representative for the study.

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Northrop Space Laboratories
Huntsville, Alabama

ABSTRACT

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This report presents the culmination of a research effort by the Huntsville Department of Northrop Space Laboratories concerned with the development of a digital computer program for use in filter circuit analysis problems. The program is designed for use in obtaining roots to sets of nonlinear algebraic equations which are characteristic of filter circuits. The program utilizes a combination of Kizner's method and the Freudenstein-Roth technique in solving for the roots to the equations. After obtaining the roots, the program selects standard circuit components whose values approximately match the actual roots, determines the transfer function characteristic of the circuit elements selected, and finally generates frequency response curves for this transfer function. Results of computer runs involving sets of equations in six, thirteen, and fifteen unknowns are discussed.

The report indicates that the program developed is especially suitable to filter circuit analysis problem for which the corresponding set of algebraic equations is not overly ill-conditioned. If the set of equations involved is ill-conditioned, there is difficulty in obtaining a solution and the program may fail to converge.

Certain possibilities concerning the extension of the program to algebraic equations in general are discussed. A brief description of several engineering problems involving simultaneous nonlinear differential equations is also presented, based on the idea that efficient numerical processes for simultaneous solving nonlinear algebraic equations may be useful in the numerical solution of sets of nonlinear differential equations.

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NOMENCLATURE

English

Symbol	Definition
A_j	The coefficient of a specified term in the j^{th} equation which is systematically reduced to unity during the Freudenstein-Roth technique.
$A_j^{(m)}$	The m^{th} value of the coefficient in the j^{th} equation.
C_n	The n^{th} capacitance, expressed in farads.
d_j	The degree of the j^{th} equation.
D_q	The q^{th} coefficient in series in denominator of transfer function.
F_j	The constant term of the j^{th} equation.
$F_j^{(m)}$	The m^{th} value of the constant term of the j^{th} equation in the Freudenstein-Roth technique.
G	Number of terms in the numerator of the transfer function.
H	Number of terms in the denominator of the transfer function.
$j\omega$	A complex quantity corresponding to s, the Laplacian variable; an imaginary representation of the angular frequency ω.
k_n	A constant relating the inductive resistance to the induction of the n^{th} inductance.
$k_1^{(m)}$	The first change in the variable x in the m^{th} application of the single variable Runge-Kutta integration.
$k_2^{(m)}$	The second change in x in the m^{th} application of the single variable Runge-Kutta integration.

NOMENCLATURE (Continued)

Symbol	Definition
$k_3^{(m)}$	The third change in x in the mth application of the single variable Runge-Kutta integration
$k_4^{(m)}$	The fourth change in x in the mth application of the single variable Runge-Kutta integration.
$k_{n1}^{(m)}$	The first change in the variable X_n in the mth application of the multi-variable Runge-Kutta integration.
$k_{n2}^{(m)}$	The second change in X_n in the mth application of Runge-Kutta integration.
$k_{n3}^{(m)}$	The third change in X_n in the mth application of Runge-Kutta integration.
$k_{n4}^{(m)}$	The fourth change in X_n in the mth application of Runge-Kutta integration.
L_n	The nth inductance, expressed in henries.
$n(j,i,k)$	The subscript for the kth factor in the ith term of the jth equation.
N_q	The qth coefficient in series in numerator of transfer function.
p	The number of unknowns.
Q_j	The number of terms in the jth equations.
$Q_{j(max)}$	The number of terms in the longest equation.
Q_{limit}	The number of applications of the coefficient method minus 1.

NOMENCLATURE (Continued)

Symbol	Definition
R_n	The n^{th} resistance, expressed in ohms.
$R_n^{(b)}$	Natural resistance for the n^{th} inductance $(m=1,2,\ldots v)$ expressed in ohms.
$R_n^{(s)}$	Surplus resistance in the n^{th} resistance R_n $(m=1,2,\ldots v)$ expressed in ohms.
s	Laplace transform variable.
T	Transfer function.
t_{ji}	The i^{th} term of the j^{th} equation.
u	The number of resistances in the circuit.
v	Number of inductances in the circuit.
V	The selected number of iterative steps in the Freudenstein-Roth technique.
V_{limit}	Maximum number of steps in the Freudenstein-Roth technique.
w	Number of capacitances in the circuit.
x	The independent variable of the single variable application of Kizner's method.
X_n	The n^{th} unknown, defined by equation (2-13).
$X_n^{(m)}$	The m^{th} estimate of X_n.
$\left[X_n\right]_{m\ \text{step}}$	The root X_n at the m^{th} step of the Freudenstein-Roth process.

NOMENCLATURE (Concluded)

Symbol	Definition
Y_n	The n^{th} circuit element (resistance, inductance, or reciprocal of capacitance) of unknown magnitude.
$Y_n^{(b)}$	The natural resistance of the inductor.
$Y_n^{(s)}$	Surplus resistance in series with inductor.
	Greek
γ	A non-trivial equation involving the functions ϕ_j. Equal to zero if the equations are dependent.
$\Delta X_n^{(m)}$	$(X_n^{(m+1)} - X_n^{(m)})$
$\varepsilon_j^{(m)}$	The m^{th} value of the j^{th} residual.
$\varepsilon_\ell^{(m)}$	The reference residual at the m^{th} step.
ζ_n	The derivative of an independent variable with respect to a function, as shown in equation (2-46).
$\xi(x)$	In a one variable function, the inverse of the derivative of the function with respect to its variable, as in equation (2-32).
ϕ_j	The j^{th} function of the form of equation (2-14).
$\phi_j^{(m)}$	$\phi_j(X_1^{(m)}, X_2^{(m)} \ldots X_p^{(m)})$
ϕ_j'	The dependent portion of the term ϕ_j in ill-conditioned systems.
ϕ_j''	The independent portion of the term ϕ_j in ill-conditioned systems.
ψ_j	The j^{th} function of the form of equation (2-2).

SUMMARY

A research effort by Northrop Space Laboratories/Huntsville Department has been carried out to develop a general digital computer program which is capable of solving, by numerical techniques, sets of simultaneous nonlinear algebraic equations which arise in problems involving filter circuit analysis, and presenting the solution in a form useful to filter circuit designers.

The Freudenstein-Roth technique modified to incorporate Kizner's method was found to be the most promising numerical technique. A technique was developed whereby the exact roots to the equation could be approximately matched by standard circuit components. The frequency response curves for the transfer function resulting from the approximate matching could then be plotted.

The processes described were incorporated into a digital computer program which was tested on sets of equations in six, thirteen, and fifteen unknowns. The program successfully solved the equations in six and thirteen unknowns including the selection of components to match roots, and the generation of frequency response curves. Only limited success was achieved in solving the set of equations in fifteen unknowns. However, all available evidence strongly supports the hypothesis that the latter set of equations is quite ill-conditioned.

The conclusion was reached that the program, utilizing the numerical techniques previously mentioned, is a useful tool in problems of filter circuit analysis so long as the algebraic equations involved are not overly ill-conditioned. The numerical techniques developed, along with all other available numerical techniques, encounter serious difficulties with ill-conditioned sets of equations.

Although the program is specifically designed to handle equations associated with filter circuit analysis, only minor modifications would enable it to be applied to other classes of simultaneous nonlinear equations.

SECTION I

INTRODUCTION

In filter circuit analysis, problems arise which involve the simultaneous solution of nonlinear algebraic equations. Solution of such sets of equations by hand can be extremely laborious, and, if large number of equations are involved, hand calculations become impractical. The use of digital computers, coupled with appropriate numerical techniques, is a logical approach to such problems. In developing the necessary digital computer program, consideration must be given to the fact that many different filter circuits exist, and the set of equations which correspond to one filter circuit will not generally correspond to other filter circuits. Therefore the most desirable program is one which is sufficiently general to solve a large number of different sets of filter circuit equations. In addition, it is highly desirable to present the solutions in a form that is most useful to filter circuit designers. For this reason, the program should incorporate routines to calculate attenuation and phase shift vs frequency plots on the basis of the solutions obtained.

The Huntsville Department of Northrop Space Laboratories has been engaged in the development of a digital computer program capable of solving sets of nonlinear algebraic equations associated with filter circuit analysis and presenting the results in a form useful to filter circuit designers. Initial research efforts under this contract were reported in reference 1.

Section II of this report provides a detailed technical discussion of the problem involved, the numerical techniques used, and digital computer considerations. A discussion of the computer program is presented in Section III. A discussion of the results obtained is provided in Section IV. Conclusions and recommendations are presented in Section V. Several appendixes are provided to augment the main body of the report. Throughout the report, the nomenclature used is generally the same as that employed in reference 1.

SECTION II

TECHNICAL DISCUSSION

2.1 STATEMENT OF PROBLEM

A digital computer program was developed under Contract NAS8-20183 which, by numerical procedure, is capable of solving sets of nonlinear algebraic equations for positive roots within a prescribed range of values. The unknowns in the equations are the values of resistances, inductances, and reciprocals of capacitances which occur in a filter circuit. Each equation consists of a sum of terms with each term consisting of the product of several unknowns and with the coefficient of each term equal to unity.

The research effort has been extended with the objective of allowing several refinements and additions to the existing computer program. The refinements under consideration should both improve convergence of the numerical techniques and shorten running time.

The need for additions to the program already developed results from the fact that the roots obtained in solving the equations are generally not equal to standard values of off-the-shelf electrical components, ordinarily used in actual filter circuits. Thus an actual filter circuit composed of standard off-the-shelf components, which most nearly match the values indicated by the equation's roots, would only approximate the theoretical circuit. The determination of the effect of such an approximation is important to circuit designers.

2.2 BACKGROUND

This section reviews portions of the technical sections of the previous report (ref. 1). Its purpose is to provide completeness and continuity to the present report.

Transfer functions associated with electronic filter circuits, such as that shown in Figure 2-1, have the general form

$$T = \sum_{q=1}^{G} N_{q-1} \quad s^{q-1} \; / \sum_{q=1}^{H} D_{q-1} \quad s^{q-1} \qquad (2\text{-}1)$$

where

T = transfer function

G = number of terms in the numerator

N_q = q^{th} coefficient of the series in the numerator

s = Laplace transform variable = complex representation of angular velocity ($j\omega$)

H = number of terms in the denominator

D_q = q^{th} coefficient of the series in the denominator.

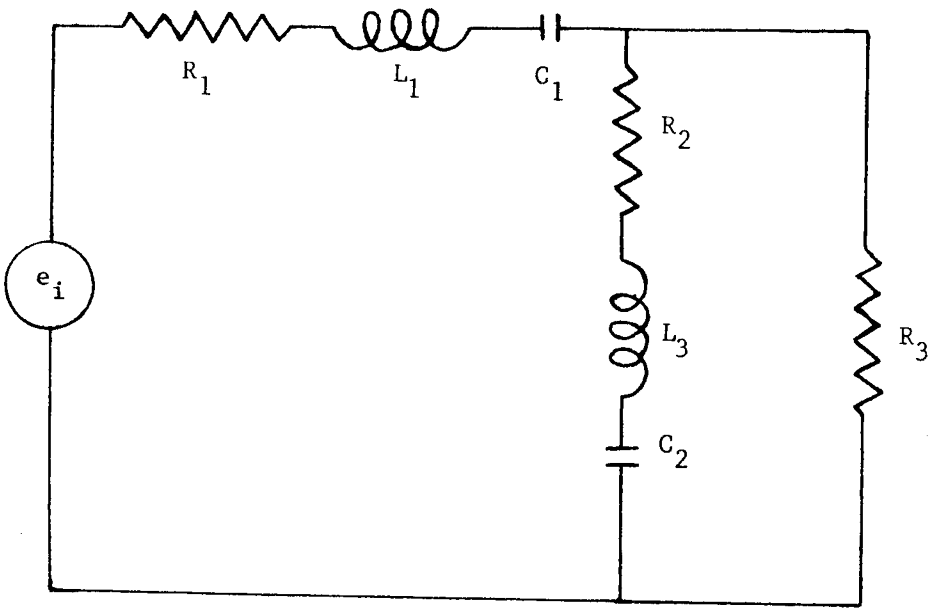


Figure 2-1. TYPICAL ELECTRONIC FILTER CIRCUIT

Generally, the numeric values of the coefficients, N_q and D_q are obtained by curve fitting. Based on circuit analysis, a set of algebraic equations containing the unknown circuit elements can be derived by means of a flow graph (ref. 2) or topology (ref. 3).

The number of these equations may be less than, equal to, or greater than the number of unknowns. Although not significant from the standpoint of filter circuit theory, this situation can present difficulties to the solution of such equations.

If there are less equations than unknowns, additional equations must be generated until there are as many equations as unknowns to form a solvable set. These additional equations may be generated by assigning values to the required number of unknowns. The only physical restriction is that the resulting equations should possess a set of real, positive roots.

If there are as many equations as unknowns, the equations possess a solution, if they are independent. If they constitute a dependent set of equations, discrete sets of roots do not exist. While it is true that a dependent set of equations may possess solutions, such solutions are not obtainable by general mathematical means.

If there are more equations than unknowns, a serious uncertainty exists. There is no a priori reason to believe that any set with as many equations as unknowns, taken from the available equations, will form an independent, hence uniquely solvable, set of equations. If such a case arises in connection with physical problems, some auxiliary means is necessary to generate a set of independent equations. The mathematical difficulties associated with dependent and nearly dependent, or ill-conditioned, sets of equations is discussed more fully in subsection 2.2.3.

The equations resulting from circuit analysis can be written as

$$\psi_j(Y_1, Y_2, \ldots, Y_p) = F_j \quad (j = 1, 2, \ldots, p) \tag{2-2}$$

where

p = the number of unknowns

and

$$F_j = \begin{cases} N_{j-1}(j = 1, 2, \ldots G) \\ D_{j-G-1}\ (j = G+1, \ldots G+H) \end{cases}$$

Y_n – circuit elements (resistances, inductances, and reciprocals of capacitances) of unknown magnitude

If the number of unknowns, p, is not equal to the number of coefficients in the transfer function, G+H, then steps must be taken, as already outlined, to generate or delete equations. Thus for each coefficient N_q or D_q there is an equation in which the coefficient appears as a constant, F_j. The reciprocal of capacitance is used because the resulting form of ψ_j is easier to work with.

These functions ψ_j consist of a sum of terms of the form

$$\psi_j = \sum_{i=1}^{Q_j} t_{ji} \tag{2-3}$$

where the term t_{ji} has the form

$$t_{ji} = \prod_{k=1}^{d_j} Y_{n(j,i,k)} \tag{2-4}$$

The expression n(j,i,k) denotes a subscripted subscript and specifies the subscript of an unknown corresponding to a given j (equation), i (term), and k (factor). For any equation, all terms of the equation are of the same degree, d_j, but d_j is not necessarily the same from equation to equation.

In order to establish an orderly relationship between Y_n and the resistances, capacitances, and conductances, it is convenient to use the following arrangement:

$$Y_n = R_n \quad (n = 1, 2, \ldots, u)$$

$$Y_n = L_{n+1-u} \quad (n = u + 1, \ldots, u+v) \tag{2-5}$$

$$Y_n = \frac{1}{C_{n+1-u-v}} \quad (n = u+v+1, \ldots, u+v+w)$$

where

R_n = the n^{th} resistance

L_n = the n^{th} inductance

C_n = the n^{th} capacitance

u = number of resistances in the circuit

v = number of inductances in the circuit

w = number of capacitances in the circuit.

Because the circuit element values are positive real numbers, the desired roots must also be in this category. For practical purposes there exist maximum and minimum values for the roots, as indicated in Table 2-1.

Table 2-1.

RANGE OF VALUES FOR FILTER CIRCUIT COMPONENTS

COMPONENT	MINIMUM	MAXIMUM
Resistor (ohms)	2.4×10^{-1}	2.2×10^{7}
Inductor (henrys)	5.0×10^{-5}	2.0×10^{3}
Capacitor (farads)	1.0×10^{-11}	1.5×10^{-1}

Because each inductance in a circuit also has a "built-in" or natural resistance associated with it in series, consideration must be given to the functional relationship between each inductance and its natural resistance. In formulating equation (2-2), these natural resistances are treated as portions of unknown resistances, but actually they are each dependent on a particular inductance. Thus, in the circuit these exists v resistances each of which contains a natural resistance. For ease in relating these resistances to the appropriate inductances it is convenient when numbering the circuit components to use the same numerical subscript for an inductance and the corresponding resistance. Thus R_1 contains the natural resistance for L_1, R_2 the natural resistance for L_2, etc. In general, based on the relationships provided in equations (2-5), the natural resistance for L_n, where

$$L_n = Y_{u+n} \ (n = 1, 2, \ldots, v) \tag{2-6}$$

would be found in R_n, where

$$R_n = Y_n \ (n = 1, 2, \ldots, v) \tag{2-7}$$

With the numbering arrangement outlined, all resistances with subscripts equal to or less than v are composed of two parts. One part is the natural resistance, $R_n^{(b)}$, for an inductance and the second part is a "surplus" resistance, $R_n^{(s)}$. Thus,

$$R_n = R_n^{(b)} + R_n^{(s)} \ (n = 1, 2, \ldots, v) \tag{2-8}$$

or

$$Y_n = Y_n^{(b)} + Y_n^{(s)} \tag{2-9}$$

The functional relationship between an inductance and its natural resistance is dependent on the electrical characteristics and physical dimensions of the wire which makes up the inductance. For practical purposes, however, a linear relationship between inductance and natural resistance appears satisfactory.

Thus,

$$R_n^{(b)} = K_n L_n \quad (n = 1, 2, \ldots, v) \tag{2-10}$$

or

$$Y_n^{(b)} = K_n Y_{u+n} \quad (n = 1, 2, \ldots, v) \tag{2-11}$$

where K_n = a constant (normally taken as unity).

Thus, by substitution,

$$Y_n = Y_n^{(s)} + K_n Y_{u+n} \quad (n = 1, 2, \ldots, v) \tag{2-12}$$

From equation (2-12) it can be seen that for n = 1, 2, ..., v, $Y_n^{(s)}$ are the true independent variables instead of Y_n. To avoid unnecessary use of superscripts, while at the same time positively identifying the true independent unknowns, a change of variable is convenient. Thus by definition,

$$X_n = \begin{cases} Y_n^{(s)} & (n = 1, 2, \ldots, v) \\ Y_n & (n = v+1, v+2, \ldots, p) \end{cases} \tag{2-13}$$

All previously mentioned physical constraints for Y_n apply also to X_n. In terms of the new variables, X_n, equations (2-2) may be written

$$\phi_j(X_1, X_2, \ldots X_n, \ldots X_p) = F_j \quad (j = 1, 2, \ldots p) \tag{2-14}$$

An examination of equations (2-14) reveals that while the form of functions has changed from ψ_j to ϕ_j the problem remains essentially the same.

As part of the original investigation, the Freudenstein-Roth technique (ref. 4) combined with the Newton-Raphson method was incorporated into a digital computer program designed to solve sets of equations of the type given by equation (2-14). In the subsections which follow, a description of these two numerical techniques is provided, along with a discussion of the difficulties generated by nonlinear dependent sets or ill-conditioned sets of equations.

2.2.1 Newton-Raphson Method

Probably the most widely used method for solving simultaneous nonlinear algebraic equations, as well as transcendental equations, is the Newton-Raphson method. The method is described in various numerical analysis texts (refs. 5 through 8) and only a brief description need be given here.

The Newton-Raphson method is a successive approximation technique. Based on an initial estimate of the unknowns, $X_n^{(o)}$, the values of $\phi_j^{(o)}$ are calculated and compared with the values F_j. The difference is the residual $\varepsilon_j^{(o)}$. Thus

$$\varepsilon_j^{(o)} = \phi_j^{(o)} - F_j \qquad (2\text{-}15)$$

where

$$\phi_j^{(o)} = \phi_j\,(X_1^{(o)}, X_2^{(o)}, \ldots, X_p^{(o)})$$

or, in general,

$$\varepsilon_j^{(m)} = \phi_j^{(m)} - F_j \qquad (2\text{-}16)$$

where

$$\phi_j^{(m)} = \phi_j\,(X_1^{(m)}, X_2^{(m)}, \ldots, X_p^{(m)})$$

and

$$X_n^{(m)} = m^{th} \text{ estimate of } X_n$$

Obviously, when the residuals are all simultaneously zero, a solution has been achieved. A first-order Taylor's series expansion is used to approximate the functions. Thus

$$\phi_j \cong \phi_j^{(o)} + \sum_{n=1}^{p} \frac{\partial \phi_j}{\partial X_n}^{(o)} \left[X_n^{(1)} - X_n^{(o)} \right] \tag{2-17}$$

By definition

$$\Delta X_n^{(m)} = X_n^{(m+1)} - X_n^{(m)} \tag{2-18}$$

By equation (2-14)

$$F_j \cong \phi_j^{(m)} + \sum_{n=1}^{p} \frac{\partial \phi_j^{(m)}}{\partial X_n} \Delta X_n^{(m)} \tag{2-19}$$

Based on the definition of the residual,

$$\epsilon_j^{(o)} \cong - \sum_{n=1}^{p} \frac{\partial \phi_j}{\partial X_n}^{(o)} \Delta X_n^{(o)} \tag{2-20}$$

or, in general,

$$\epsilon_j^{(m)} \cong - \sum_{n=1}^{p} \frac{\partial \phi_j}{\partial X_n}^{(m)} \Delta X_n^{(m)} \tag{2-21}$$

Equation (2-21) represents a set of p linear equations, with the $\Delta X_n^{(m)}$ as the unknowns. This system of equations can be solved by the Gaussian method of pivotal condensation (ref. 9).

In actual practice, the repeated approximation of $X_n^{(m)}$ by solution of equation (2-21) for $\Delta X_n^{(m)}$ will result in a systematic reduction of the residuals toward zero, if convergence occurs. Normally, a solution is considered to have been obtained when all residuals have been reduced to some prescribed level.

2.2.2 Freudenstein-Roth Technique

In applying the Newton-Raphson method, convergence is not likely to occur unless the initial estimates of the roots are in the neighborhood of the actual values. Obviously, in many cases, the locations of such neighborhoods are unknown. Application of the Freudenstein-Roth technique (ref. 4) enables convergence even though the estimates are much further out than the Newton-Raphson technique alone would allow.

The first step in the Freudenstein-Roth technique involves assuming a set of initial values $X_n^{(o)}$ for the roots. These initial values will in general not satisfy the original equations. However, one coefficient in each equation may be altered by increasing or decreasing its value so that the altered set of equations is satisfied by the original estimates of the roots. If the altered coefficients of the equations are changed slightly in the direction of their original values a new set of equations is generated which may be solvable by the Newton-Raphson method using the roots to the previous set of equations as initial estimates. The altered coefficients are then changed slightly further toward their original values and the resulting set of equations is again solved by the Newton-Raphson method, using the roots of the previous initial step as estimates. This stepwise process is repeated until the original equations are reproduced and solved. The solution of each intermediate set of equations completes what is termed, for convenience, a "Freudenstein-Roth step" or "step".

Two different methods of altering one coefficient in each equation have been used. For convenience, they are referred to as the "coefficient approach" and the "constant approach".

For the coefficient approach, one coefficient of a nonconstant term in each equation is multiplied by a constant, $A_j^{(o)}$, which is chosen so that the equation is satisifed by the original estimates. The altered equation satisfied by the original estimates can be written

$$F_j = \sum_{i=1}^{Q_j} t_{ji}^{(o)} + (A_j^{(o)} - 1)\, t_{jL} \qquad (2\text{-}22)$$

in which L is any integer from 1 to Q_j, thus specifying a specific term in the equation. The value of L can change from equation to equation. A recursion relation is used to vary the constant A_j for each Freudenstein-Roth step.

The relation is

$$A_j^{(m)} = \left[A_j^{(o)} \right]^{\left(\frac{V-m}{V}\right)} \qquad (m = 0,\ 1,\ 2,\ \ldots,\ V) \qquad (2\text{-}23)$$

The value of m is increased by one prior to starting each step. Obviously, when m is equal to V, the original equations are reproduced. The solution of this set of equations is the desired solution.

The constant approach method alters the constant term F_j. The initial value of the altered constant, $F_j^{(o)}$, is calculated by the equation

$$F_j^{(o)} = \phi_j^{(o)} \qquad (2\text{-}24)$$

The Fj's are modified for each Freudenstein-Roth step by the recursion relation

$$F_j^{(m)} = F_j \left[\frac{F_j^{(o)}}{F_j} \right]^{\left(\frac{V-m}{V}\right)} \qquad (2\text{-}25)$$

so that at the end of V steps

$$F_j^{(V)} = F_j \qquad (2\text{-}26)$$

The solution obtained at this step is the desired solution.

The convergence criteria for the Freudenstein-Roth technique are discussed in reference 4. The proper use of this method ensures that the initial estimates for the set of roots at each step are close to the true roots for that step. Obviously, if the step size is too large, reflecting a small value of V, the Newton-Raphson method may fail for some individual step. This may be corrected by increasing the value of V, but a point may be reached beyond which further increases of V are not practical. In such a case, the problem should be started over using a new set of estimates.

2.2.3 Nonlinear Dependent or Ill-Conditioned Systems

The Newton-Raphson method, in common with other numerical techniques, is incapable of solving a functionally dependent system of equations and encounters great difficulties solving ill-conditioned systems of equations. These two cases are not unrelated, for ill-conditioned system border on being functionally dependent. They differ in that functionally dependent systems of equations do not possess any discrete solutions whereas ill-conditioned systems possess discrete solutions but great practical difficulties are encountered in obtaining such solutions.

If a set of p equations of the form

$$\phi_j = F_j$$

are functionally dependent, based on reference 10, there exists a non-trivial equation involving the functions ϕ_j of the form

$$\gamma(\phi_1, \phi_2, \dots, \phi_p) = 0 \qquad (2\text{-}27)$$

This equation, which may be taken to be a definition of functional dependence, holds for all values of the independent variables. Therefore, it is impossible to vary the ϕ_j independently.

The general method of determining whether a set of equations is dependent is to determine whether their Jacobian matrix

$$\left[\frac{\partial \phi_j}{\partial X_n}\right]$$

is identically singular. Unfortunately, this method is not feasible when even a moderately large number of independent variables are involved, for it involves the direct expansion of the determinant of a high-order matrix, each term of which is an algebraic expression. Therefore, it is generally impractical to attempt to establish conclusively whether or not simultaneous equations having a large number of independent variables are dependent.

It appears more practical to detect the dependence of a set of equations by numerical means. This approach calls for the determinant corresponding to the Jacobian of a set of equations to be evaluated using several different sets of values of the X_n. If the determinant is zero or nearly so for each set of values, there is strong indication of a singular matrix. Unfortunately, if the magnitude of the unknowns within a set varies significantly, accurate numerical evaluation of the determinant is difficult even on a digital computer. This is primarily due to truncation error.

The term "ill-conditioned" as applied to a set of simultaneous equations is not clearly defined. The term is of a qualitative rather than a quantitative nature. Its practical value is that the term ill-conditioned singles out those sets of simultaneous equations which are exceedingly difficult to solve by numerical methods and which require great accuracy when exact methods are applicable.

To be more definitive, an ill-conditioned system may be considered to be a simultaneous set of equations between functions that can be transformed into a functionally dependent set by minor modification of one or more of the functions. That is, ill-conditioned systems border on being functionally dependent. The concept of "bordering on functional dependence" for p functions ϕ_j can be expressed by the relation

$$\gamma(\phi_1, \phi_2, \ldots, \phi_p) \tilde{=} 0 \tag{2-28}$$

For this case each function ϕ_j can be considered to consist of two parts

$$\phi_j = \phi_j' + \phi_j'' \tag{2-29}$$

in such a manner that

$$\gamma(\phi_1', \phi_2', \ldots, \phi_p') = 0 \tag{2-30}$$

and

$$|\phi_j| \tilde{=} |\phi_j'| >> |\phi_j''| \tag{2-31}$$

A truly <u>independent</u> variation of any ϕ_j can only be accomplished by a variation of ϕ_j'', but due to its small size, variation of ϕ_j'' can only result in small changes in ϕ_j. If ϕ_j' is varied in any equation then ϕ_j' and thus ϕ_j of the other equations are strongly affected. In actual cases the ϕ_j' and ϕ_j'' of most equations cannot be identified and separated. Thus any variation of ϕ_j for one equation in an ill-conditioned system is likely to have a strong influence in the ϕ_j of the other equations. When cast in this light, insight is gained into the difficulties of obtaining numerical solutions of ill-conditioned systems of simultaneous equations.

The numerical methods already described for obtaining solutions of simultaneous equations (Newton-Raphson and Freudenstein-Roth) involve approximations which are valid only for small changes in the independent variables X_n. These approximations yield a set of linear simultaneous equations for the changes in the independent variables. The solution of this set of linear equations gives a refinement to the original estimates of the roots. This process is repeated using the refined values of the roots as new estimates until sufficient accuracy is obtained.

In the case of ill-conditioned simultaneous equations, their near functional dependency generates situations in which the elimination of a relatively small residual, $\varepsilon_j^{(m)}$, in at least one equation calls for large changes in the values of the unknowns. These large changes often invalidate the approximations based on small changes of the independent variables X_n. This is the dilemma ill-conditioned systems present to numerical solution techniques.

2.3 IMPROVED NUMERICAL METHODS OF SOLUTION

The previous discussion presents ideas which resulted from the work accomplished under the original research effort. The discussion which follows presents the refinements to the original numerical approach which have been considered during the contract extension.

2.3.1 Kizner's Method

The Freudenstein-Roth technique removes the major limitation of the Newton-Raphson method in that the initial estimates of the roots of the simultaneous equations do not need to be close to the actual roots of the equations to ensure convergence. However, as originally presented, each step, or set of intermediate equations, of the Freudenstein-Roth technique is solved by the Newton-Raphson method.

For a given step, the roots of the previous step serve as initial estimates. These must be close to the roots of the given step for the Newton-Raphson method to converge. This requirement often results in an undesirably large number of steps being necessary to obtain a solution. Consequently, a method more strongly convergent than Newton-Raphson's is desirable for these steps.

Such a method is presented by Kizner in reference 11. Kizner showed that, by considering the independent variables X_n as functions of the dependent variables ϕ_j, a system of simultaneous algebraic equations can be treated as a simultaneous system of ordinary first-order differential equations. These differential equations can be approximately solved by a one-step Runge-Kutta numerical method, using the estimates of the roots and the functions evaluated at these estimates as initial values. Since these differential equations interchange the role of independent and dependent variables with respect to the original equations, the roots of the original equation are obtained by evaluating the solutions of the differential equations at zero. This process can be repeated, using the new approximations of the roots as initial estimates, until the desired accuracy is attained. A more detailed discussion of Kizner's method follows.

For simplicity, one equation in one unknown will be considered first. The equation is assumed to be of the form

$$f(x) = f = 0.$$

The initial estimate of the root is $x^{(o)}$, and

$$f^{(o)} = f(x^{(o)}).$$

The function ξ is defined by the differential equation

$$\xi(x) = \frac{dx}{df} = 1/\frac{df}{dx} \qquad (2\text{-}32)$$

It should be noted that the left-hand member of this equation is a function of the variable, x, only. The root of the original equation, x, can be written as

$$x = \int_{f(x^{(0)})}^{o} \frac{dx}{df} \; df + x^{(o)} = \int_{f(x^{(0)})}^{o} \xi (x) \; df + x^{(o)} \qquad (2\text{-}33)$$

Kizner's method approximates the required integral by a one step Runge-Kutta numerical process, which evaluates the integrand at four points and approximates it with a cubic expression. The resulting expression yields an approximation $x^{(1)}$ of the root x and can be written as follows:

$$x^{(1)} = x^{(o)} + \frac{1}{6} (k_1^{(o)} + 2k_2^{(o)} + 2k_3^{(o)} + k_4^{(o)}) \qquad (2\text{-}34)$$

where

$$k_1^{(o)} = -f^{(o)} \xi \; (x^{(o)})$$

$$k_2^{(o)} = -f^{(o)} \xi \; (x^{(o)} + k_1^{(o)}/2)$$

$$k_3^{(o)} = -f^{(o)} \xi \; (x^{(o)} + k_2^{(o)}/2)$$

$$k_4^{(o)} = -f^{(o)} \xi \; (x^{(o)} + k_3^{(o)})$$

In a more general form equation (2-34) can be written

$$x^{(m+1)} = x^{(m)} + \frac{1}{6} (k_1^{(m)} + k_2^{(m)} + k_3^{(m)} + k_4^{(m)}) \qquad (2\text{-}35)$$

where,

$$k_1^{(m)} = -f^{(m)} \; \xi \; (x^{(m)})$$

$$k_2^{(m)} = -f^{(m)} \; \xi \; (x^{(m)} + k_1^{(m)}/2)$$

$$k_3^{(m)} = -f^{(m)} \; \xi \; (x^{(m)} + k_2^{(m)}/_2)$$

$$k_4^{(m)} = -f^{(m)} \; \xi \; (x^{(m)} + k_3^{(m)}).$$

The method can be readily extended to systems of several equations in several unknowns. The original equations, ϕ_j, can be written in the residual form

$$\varepsilon_j = \phi_j - F_j \tag{2-36}$$

or

$$\varepsilon_j = \varepsilon_j\ (X_1, X_2, X_3, \ldots, X_p) = 0 \tag{2-37}$$

With the initial estimates $X_n^{(o)}$

$$\varepsilon_j^{(o)} = \varepsilon_j\ (X_1^{(o)}, X_2^{(o)}, X_3^{(o)}, \ldots, X_p^{(o)}) \tag{2-38}$$

If the independent variables X_n are considered to be functions of the dependent variables, ε_j, and if one of the ε_j's, designated ε_ℓ, is treated as the only independent variable, the total derivative of X_n with respect to the one variable ε_ℓ can be written

$$\frac{dX_n}{d\varepsilon_\ell} = \sum_j \frac{\partial X_n}{\partial \varepsilon_j} \quad \frac{d\varepsilon_j}{d\varepsilon_\ell} \tag{2-39}$$

In a manner analogous to the solution of one equation for one variable,

$$X_n = \int_{\varepsilon_\ell(o)}^{o} \sum \frac{\partial X_n}{\partial \varepsilon_j} \quad \frac{d\varepsilon_j}{d\varepsilon_\ell} \quad d\varepsilon_\ell + X_n^{(o)} \tag{2-40}$$

The total derivative $d\varepsilon_j/d\varepsilon_\ell$ can be established by assuming a linear relation between ε_j and ε_ℓ as follows:

$$\varepsilon_j = \alpha_j\ \varepsilon_\ell \tag{2-41}$$

Then

$$\frac{d\varepsilon_j}{d\varepsilon_\ell} = \alpha_j = \frac{\varepsilon_j}{\varepsilon_\ell} \tag{2-42}$$

In actual numerical calculations, the assumption of a linear relationship between ε_j and ε_ℓ does not exactly hold. For any iterative step, m, however,

$$\frac{d\varepsilon_j}{d\varepsilon_\ell} = \frac{\varepsilon_j^{(m)}}{\varepsilon_\ell^{(m)}} \tag{2-43}$$

A combination of equations (2-40) and (2-43) yields

$$X_n = \int_{\varepsilon_\ell^{(m)}}^{o} \sum_j \frac{\partial X_n}{\partial \varepsilon_j} \frac{\varepsilon_j^{(m)}}{\varepsilon_\ell^{(m)}} d\varepsilon_\ell + X_n^{(m)} \tag{2-44}$$

The partial derivatives $\partial X_n / \partial \varepsilon_j$ can be formally obtained through the well-known Jacobian matrix equation

$$\left[\frac{\partial \varepsilon_j}{\partial X_n} \right] \left[\frac{\partial X_n}{\partial \varepsilon_j} \right] = I \tag{2-45}$$

where I = the unit matrix.

In a manner analogous to that used for the case of one unknown, a function ζ_n can be defined by the differential equation

$$\zeta_n (X_1, X_2, \ldots, X_p) = \frac{dX_n}{d\varepsilon_\ell} = \sum_j \frac{\partial X_n}{\partial \varepsilon_j} \frac{d\varepsilon_j}{d\varepsilon_\ell} \tag{2-46}$$

Application of a one-step Runge-Kutta method to equation (2-44) then yields

$$X_n^{(m+1)} = X_n^{(m)} + \frac{1}{6} (k_{n1}^{(m)} + 2k_{n2}^{(m)} + 2k_{n3}^{(m)} + k_{n4}^{(m)}) \tag{2-47}$$

where

$$k_{n1}^{(m)} = - \varepsilon_\ell^{(m)} \zeta_n (X_1^{(m)}, X_2^{(m)} \ldots, X_p^{(m)}) \tag{2-48}$$

$$k_{n2}^{(m)} = - \varepsilon_\ell^{(m)} \zeta_n (X_1^{(m)} + \frac{k_{11}^{(m)}}{2}, \ldots, X_p^{(m)} + \frac{k_{p1}^{(m)}}{2}) \tag{2-49}$$

$$k_{n3}^{(m)} = - \varepsilon_{\ell}^{(m)} \quad \zeta_n \left(X_1^{(m)} + \frac{k_{12}^{(m)}}{2}, \ldots, X_p^{(m)} + \frac{k_{p2}^{(m)}}{2}\right) \tag{2-50}$$

$$k_{n4}^{(m)} = - \varepsilon_{\ell}^{(m)} \quad \zeta_n \left(X_1^{(m)} + k_{13}, \ldots, X_p^{(m)} + k_{p3}\right) \tag{2-51}$$

The quantities $k_{n1}^{(m)}$, $k_{n2}^{(m)}$, $k_{n3}^{(m)}$, and $k_{n4}^{(m)}$ can also be expressed as

$$k_{n1}^{(m)} = - \sum_j \frac{\partial X_n}{\partial \varepsilon_j} \varepsilon_j^{(m)} \qquad \left(X_1 = X_1^{(m)}, \ldots, X_p = X_p^{(m)}\right) \tag{2-52}$$

$$k_{n2}^{(m)} = - \sum_j \frac{\partial X_n}{\partial \varepsilon_j} \varepsilon_j^{(m)} \qquad \left(X_1 = X_1^{(m)} + \frac{k_{11}^{(m)}}{2}, \ldots, X_p = X_p^{(m)} + \frac{k_{p1}^{(m)}}{2}\right) \tag{2-53}$$

$$k_{n3}^{(m)} = - \sum_j \frac{\partial X_n}{\partial \varepsilon_j} \varepsilon_j^{(m)} \qquad \left(X_1 = X_1^{(m)} + \frac{k_{12}^{(m)}}{2}, \ldots, X_p = X_p^{(m)} + \frac{k_{p2}^{(m)}}{2}\right) \tag{2-54}$$

$$k_{n4}^{(m)} = - \sum_j \frac{\partial X_n}{\partial \varepsilon_j} \varepsilon_j^{(m)} \qquad \left(X_1 = X_1^{(m)} + K_{13}^{(m)}, \ldots, X_p = X_p^{(m)} + k_{p3}^{(m)}\right) \tag{2-55}$$

The evaluation of $k_{n1}^{(m)}$, $k_{n2}^{(m)}$, $k_{n3}^{(m)}$, and $k_{n4}^{(m)}$ can be accomplished by observing that

$$\frac{d\varepsilon_j}{d\varepsilon_{\ell}} = \sum_n \frac{\partial \varepsilon_j}{\partial X_n} \frac{dX_n}{d\varepsilon_{\ell}} \tag{2-56}$$

Then based on equations (2-43) and (2-46),

$$\frac{\varepsilon_j^{(m)}}{\varepsilon_{\ell}^{(m)}} = \sum_n \frac{\partial \varepsilon_j}{\partial X_n} \left(\sum_j \frac{\partial X_n}{\partial \varepsilon_j} \frac{\varepsilon_j^{(m)}}{\varepsilon_{\ell}^{(m)}} \right) \tag{2-57}$$

or

$$\varepsilon_j^{(m)} = \sum_n \frac{\partial \varepsilon_j}{\partial X_n} \left(\sum_j \frac{\partial X_n}{\partial \varepsilon_j} \varepsilon_j^{(m)} \right) \tag{2-58}$$

By means of equations (2-52 through (2-55)

$$\varepsilon_j^{(m)} = -\sum_n \frac{\partial \varepsilon_j}{\partial X_n} k_{n1}^{(m)} \qquad (X_1 = X_1^{(m)}, \ldots, X_p = X_p^{(m)}) \tag{2-59}$$

$$\varepsilon_j^{(m)} = -\sum_n \frac{\partial \varepsilon_j}{\partial X_n} k_{n2}^{(m)} \qquad (X_1 = X_1^{(m)} + \frac{k_{11}^{(m)}}{2}, \ldots, X_p = X_p^{(m)} + \frac{k_{p1}^{(m)}}{2}) \tag{2-60}$$

$$\varepsilon_j^{(m)} = -\sum_n \frac{\partial \varepsilon_j}{\partial X_n} k_{n3}^{(m)} \qquad (X_1 = X_1^{(m)} + \frac{k_{12}^{(m)}}{2}, \ldots, X_p = X_p^{(m)} + \frac{k_{p2}^{(m)}}{2}) \tag{2-61}$$

$$\varepsilon_j^{(m)} = -\sum_n \frac{\partial \varepsilon_j}{\partial X_n} k_{n4}^{(m)} \qquad (X_1 = X_1^{(m)} + k_{13}^{(m)}, \ldots, X_p = X_p^{(m)} + K_{p3}^{(m)}) \tag{2-62}$$

In each of the last four equations the partial derivatives $\partial \varepsilon_j/\partial X_n$ are the known elements of the Jacobian and thus serve as coefficients for the unknown k's. Likewise the $\varepsilon_j^{(m)}$'s are known and act as constants. Clearly then, equations (2-59) through (2-63) each represent a set of linear algebraic equations which can be solved by standard numerical means such as the Gaussian method of pivotal condensation. Furthermore, each of these four equations is identical in form with equation (2-21) which results from the Newton-Raphson method. Thus it can be seen that each step of Kizner's method involves calculations equivalent to four Newton-Raphson steps.

Based on an examination of Kizner's method, the question arises as to the possibility of treating the solution of nonlinear simultaneous equations entirely as the solution of their associated simultaneous ordinary differential equations by the Runge-Kutta method. This can be done by subdividing the required integration interval into several Runge-Kutta steps. This procedure would require a large number of Runge-Kutta steps to prevent the introduction of serious cumulative errors unless the initial estimates of the roots were quite close to the actual roots. To check for cumulative errors, one would have to verify the solution by substituting the results in the original equations. If the original equations were not sufficiently staisfied, the Runge-Kutta process would have to be repeated, using the previous results as new initial estimates.

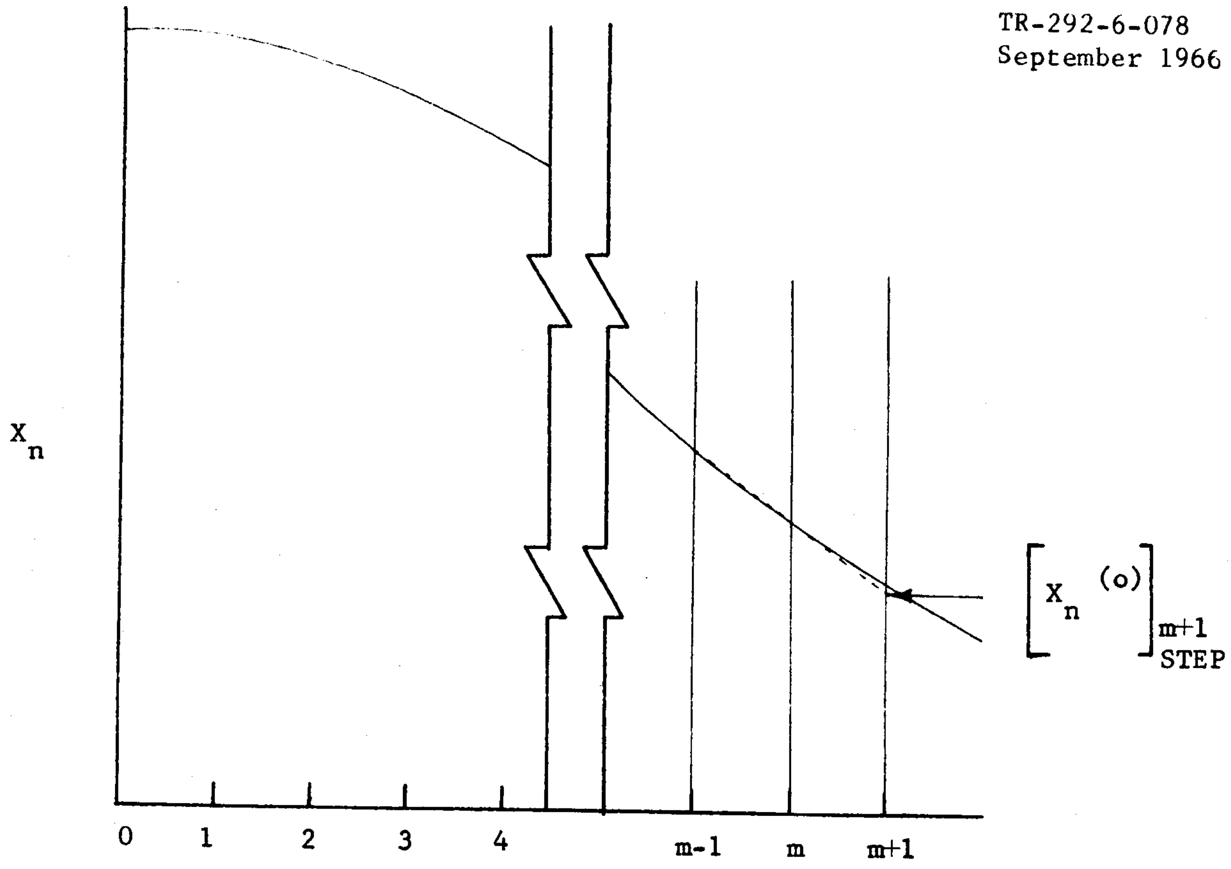
The Freudenstein-Roth technique modified to incorporate Kizner's method in conjunction with the root prediction technique presented in the next section, both eliminates any cumulative errors and lessens the number of Runge-Kutta steps required to obtain a satisfactory solution. Cumulative errors can not occur because the equations must be satisfied at each Freudenstein-Roth step.

2.3.2 Root Prediction

The equations for component values of filter circuits that are derived from transfer functions are regular. That is, the functions that form these equations are well behaved. Therefore, it appears likely that all of the roots of the intermediate equations corresponding to each Freudenstein-Roth step vary in a predictable manner from step to step.

An examination of the output of computer runs generated during the original research effort has indicated that for any three consecutive steps each root is an approximately linear function of the Freudenstein-Roth step number as shown in Figure 2-2. Then the accuracy of the initial estimate of the root X_n for any Freudenstein-Roth step (m + 1) can be greatly increased by means of the relation

$$\left[X_n^{(o)}\right]_{\substack{m+1\\ \text{STEP}}} = \left[X_n\right]_{\substack{m\\ \text{STEP}}} + \left[(X_n)_{\substack{m\\ \text{STEP}}} - (X_n)_{\substack{m-1\\ \text{STEP}}}\right]$$



FREUDENSTEIN-ROTH STEP

Figure 2-2. VARIATION OF ROOTS X_n WITH FREUDENSTEIN-ROTH STEP

2.4 COMPONENT SELECTION

The roots obtained by the numerical techniques previously described correspond to the values of circuit components necessary to build the circuit with the desired transfer function. However, it is usually impossible to obtain standard circuit components with the values which exactly match the roots found by the numerical techniques. A circuit built with components which only approximate the exact roots will only approximate the transfer function. To evaluate the change in the transfer function, it is first necessary to establish certain guidelines concerning the actual values obtainable in standard circuit components.

From an engineering standpoint the approximate components should be built up from standard components which are readily available. Parts A and B of Table 2-2 present standard decade tables for resistors and capacitors and their

TABLE 2-2

COMPONENT SELECTION VALUES

A. RESISTOR DECADE TABLES (Ω)

1.0

1.1

1.21

1.33

1.47

1.62

1.78

1.96

2.15

2.37

2.61

2.87

3.16

3.48

3.83

4.22

4.64

5.11

5.62

6.19

6.81

7.50

8.25

9.09

B. CAPACITOR DECADE TABLES

(10 - 2500 μμf)

1.0

1.2

1.5

1.8

2.0

2.2

2.5

2.7

3.0

3.3

3.6

3.9

4.7

5.0

5.1

5.6

6.8

7.5

8.2

Over 2500 μμf

1.0

1.2

1.5

1.8

2.2

2.7

3.3

3.9

4.7

5.6

6.8

8.2

C. INDUCTOR TABLE

(Less than 50 h)

Inductors of less than 50 henrys are matched to two significant figures by variable inductors.

(Greater than 50 h)

50	200	800	2000
100	400	1400	

D. INDUCTIVE RESISTANCE TABLE

(Variable Inductors - less than 50 h)

The resistance of the variable inductors is a multiple (K_m) of the inductance.

(Fixed Inductors greater than 50 h)

0.5 KΩ @ 50h	4.0 KΩ @ 400h	8.0 KΩ @ 2000h
1.0 KΩ @ 100h	8.0 KΩ @ 800h	
2.0 KΩ @ 200h	4.0 KΩ @ 1400h	

E. TOLERANCE TABLE

COMPONENT	TOLERANCE
Resistors	± 1%
Capacitors	± 5%
Inductors (> 50 h)	2 significant figures
Inductors (< 50 h)	± 10%
Inductive Resistance	Same as corresponding inductor

available tolerances. These decade tables are based on references 12 and 13. Inductors can be handled by assuming variable inductors under 50 henrys (ref. 14) and fixed values over 50 henrys (ref. 15). This procedure is also shown in Table 2-2, Part C. Values for inductive resistance, based on reference 15, are presented in Part D of Table 2-2. Tolerances for all components are found in Part E and are based on a survey of references 12 through 15.

The selection process for resistors and capacitors involves selecting the largest value from the decade table that is below the desired value and then adding smaller values until the component is within tolerance limits or until more than a specified number of values are used to form the component. For inductors over 50 henrys the selection scheme first matches the inductors to the largest fixed inductor value smaller than the desired value. Smaller increments are added with variable inductors. The selection of two values appears to be all that is needed for an approximate component to be within tolerance range of the desired component.

Application of the described scheme to each component yields a circuit with approximate component values that are easily obtainable.

2.5 FREQUENCY RESPONSE

As the components available for the circuit are only approximate, it is desirable to evaluate the effect of these approximations on the frequency response of the circuit.

The approximate transfer function may be found by evaluating the equations using the approximations to the components. The evaluation process results in values of F_j which in turn can be converted into values of the coefficients N_q and D_q in the numerator and denominator of the transfer function.

Evaluation of the complex quantity $N(j\omega)/D(j\omega)$, where N and D are the numerator and denominator of the approximate transfer function, for the desired values of frequency will yield the steady-state frequency-response curves for attenuation and phase shift as functions of frequency as discussed in reference 16. These steady-state frequency-response curves are the yardstick to use in the comparison of an approximate circuit with an exact circuit.

2.6 DIGITAL COMPUTER CONSIDERATIONS

Because of the overall numerical complexity of the problem the use of a digital computer is mandatory. The improved numerical techniques described in subsection 2.4 represent refinements to the original digital computer program described in reference 1. The component selection scheme is readily adaptable to a digital computer. The frequency response calculation discussed in subsection 2.5 has been previously programmed by Northrop as described in reference 16. Thus the most logical approach to the problem involves development of a master computer program capable of solving the equations, approximately matching the roots with standard circuit components, and calculating the resulting frequency response.

2.7 APPLICATION OF NUMERICAL TECHNIQUES TO NONLINEAR DIFFERENTIAL EQUATIONS

Because of their complexity, nonlinear differential equations are usually solved numerically. As a result, algebraic equations are generated. If a set of nonlinear differential equations is involved, then a set of nonlinear algebraic or transcental equations will generally result. Typical examples include:

- The equations of motion of a rocket flight (neglecting air resistance)
- The equations for supersonic flow around an axially symmetric body (assuming compressible inviscid flow).

The possibility exists that the sets of nonlinear algebraic equations generated in solving nonlinear differential equations may be efficiently solved by some combination of the techniques described in subsections 2.2 and 2.3. The primary considerations in establishing whether or not such a combination would offer any advantage over techniques already in use are the complexity and number of the nonlinear equations, and the accuracy to which the unknown can be estimated in any numerical step.

SECTION III

PROGRAM DESCRIPTION

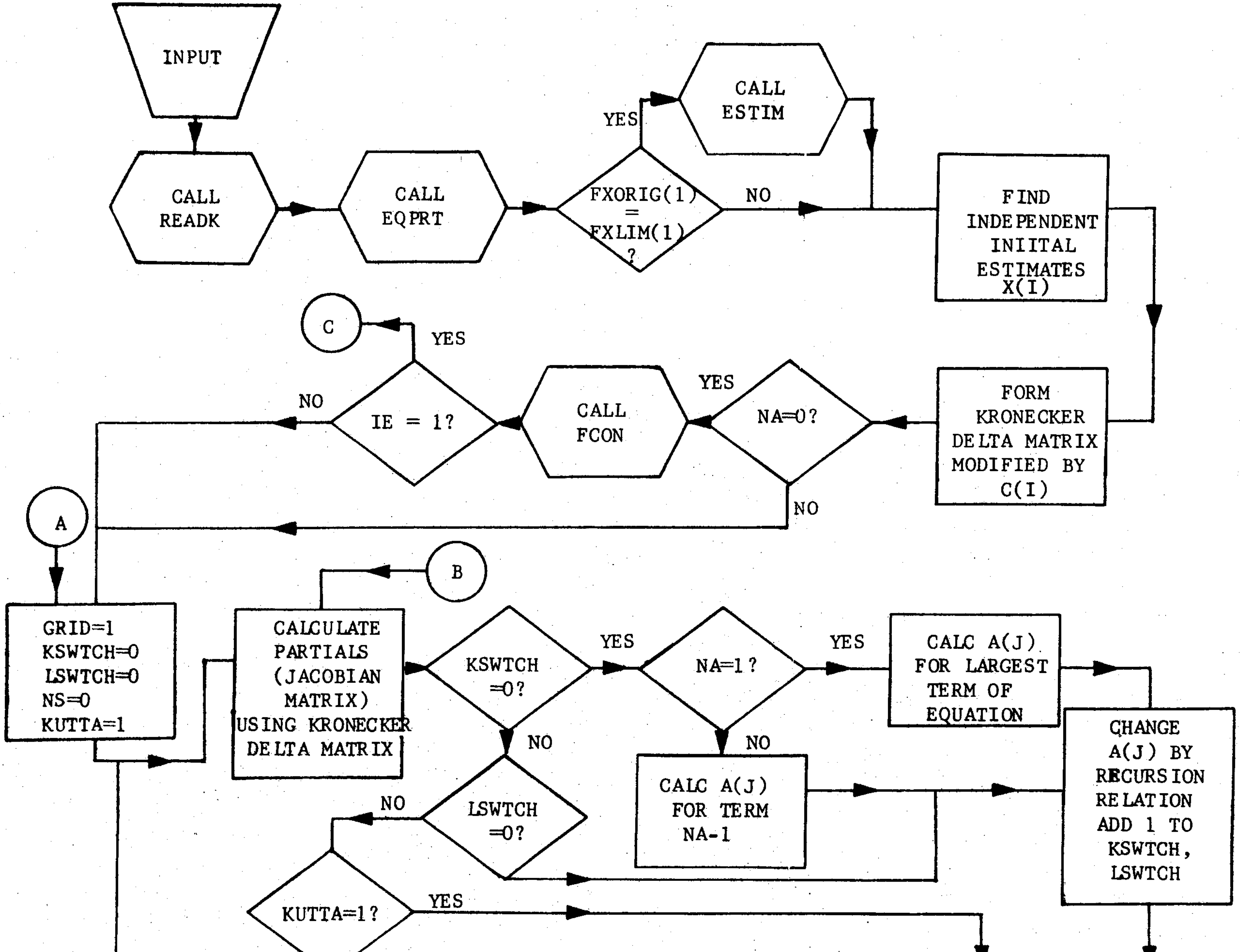
Based on the analytical development presented in subsections 2.2, 2.3, 2.4, and 2.5, a master digital computer program has been written. This program is designed to obtain the roots to the nonlinear algebraic equations, select standard circuit components which approximate the values of the roots obtained, and establish the frequency response of the circuit made up of the selected components.

The subsections which follow present a description of the various operations of the program throughout the running of a typical case, a description and necessary definitions of the input and output, and the flow charts of the program.

3.1 BASIC FEATURES

The program in its present form is designed to solve sets of nonlinear algebraic equations of the type indicated by equation (2-2). A general program flow chart is provided in Figure 3-1. A copy of the source program written in FORTRAN IV is included in Appendix A. A description of the program's subroutines is included in Appendix B. The overlay feature of the program is described in Appendix C. This program has been checked out for use on the IBM 7094 digital computer.

The program utilizes the Freudenstein-Roth technique in conjunction with Kizner's method. All partial derivatives needed for Kizner's method are calculated by analytical differentiation in contradistinction to finite-difference methods. The Gaussian pivotal technique is used to obtain the solutions of the linear algebraic equations that are necessary for the application of Kizner's method.


INPUT
CALL READK
CALL EQPRT
FXORIG(1) = FXLIM(1) ?
YES
NO
CALL ESTIM
FIND INDEPENDENT INIITAL ESTIMATES X(I)
FORM KRONECKER DELTA MATRIX MODIFIED BY C(I)
NA=0?
YES
NO
CALL FCON
IE = 1?
YES
NO
C
A
GRID=1
KSWTCH=0
LSWTCH=0
NS=0
KUTTA=1
B
CALCULATE PARTIALS (JACOBIAN MATRIX) USING KRONECKER DELTA MATRIX
KSWTCH =0?
YES
NO
NA=1 ?
YES
NO
CALC A(J) FOR LARGEST TERM OF EQUATION
CALC A(J) FOR TERM NA-1
LSWTCH =0?
NO
KUTTA=1?
YES
CHANGE A(J) BY RECURSION RELATION ADD 1 TO KSWTCH, LSWTCH

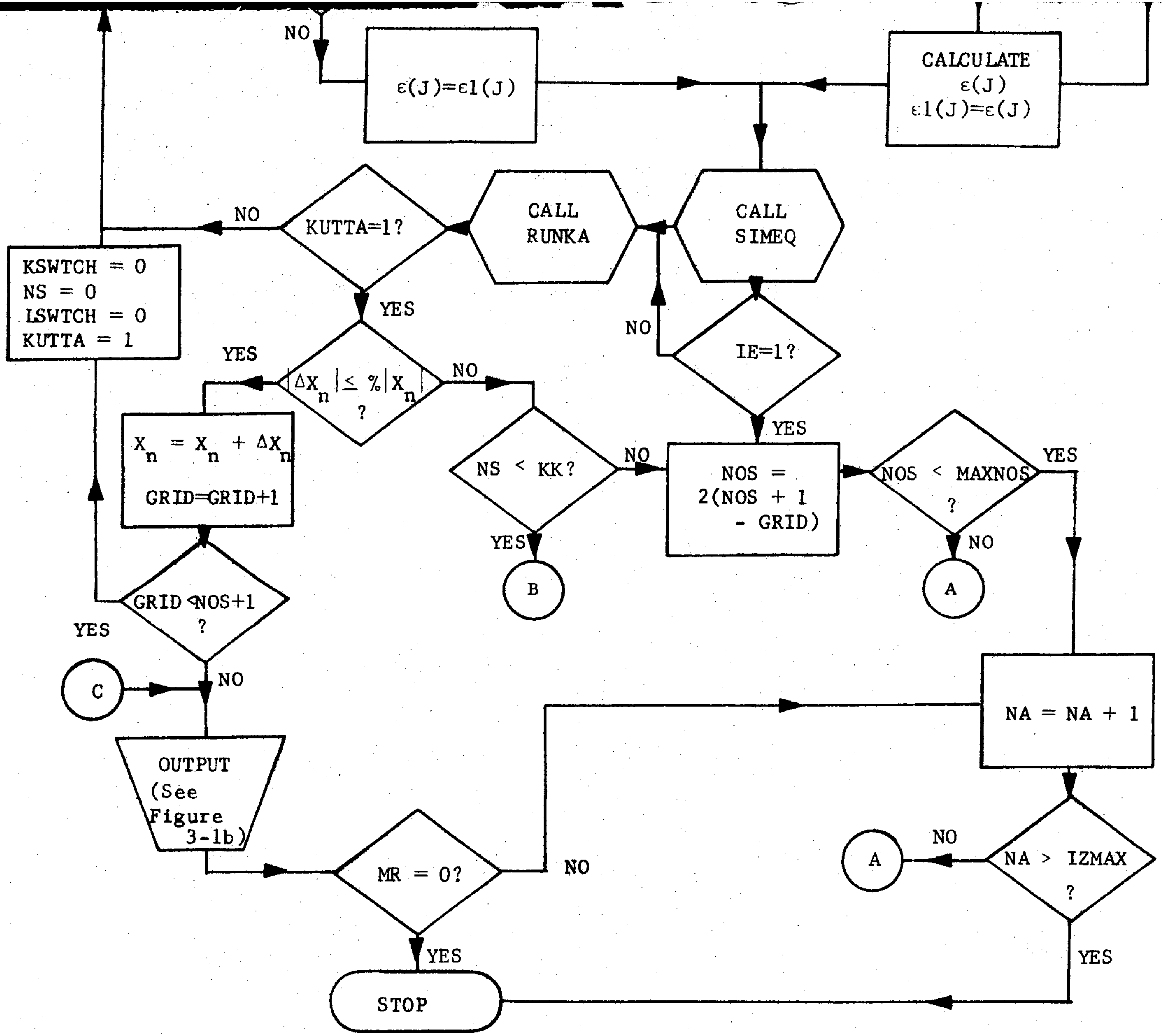


Figure 3-1a. MAIN PROGRAM FOR SOLUTION OF NONLINEAR ALGEBRAIC EQUATIONS

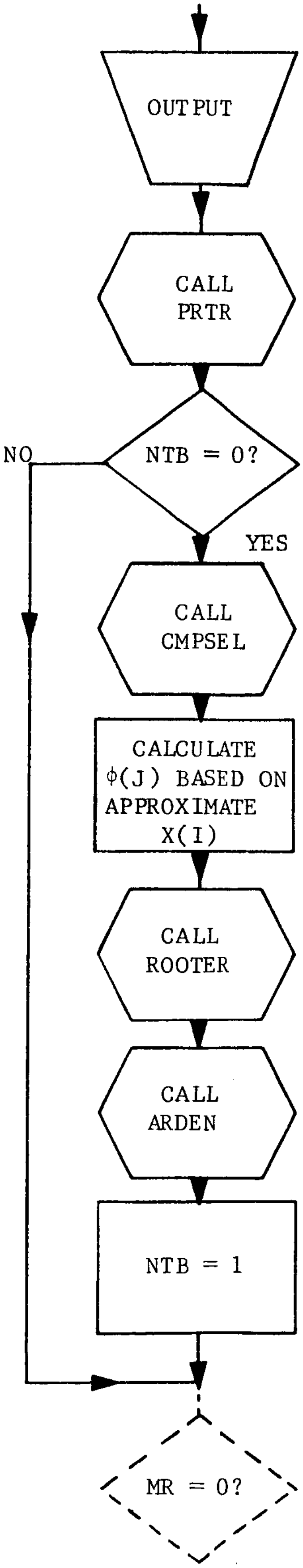


Figure 3-1b. MAIN PROGRAM (CONTINUED)

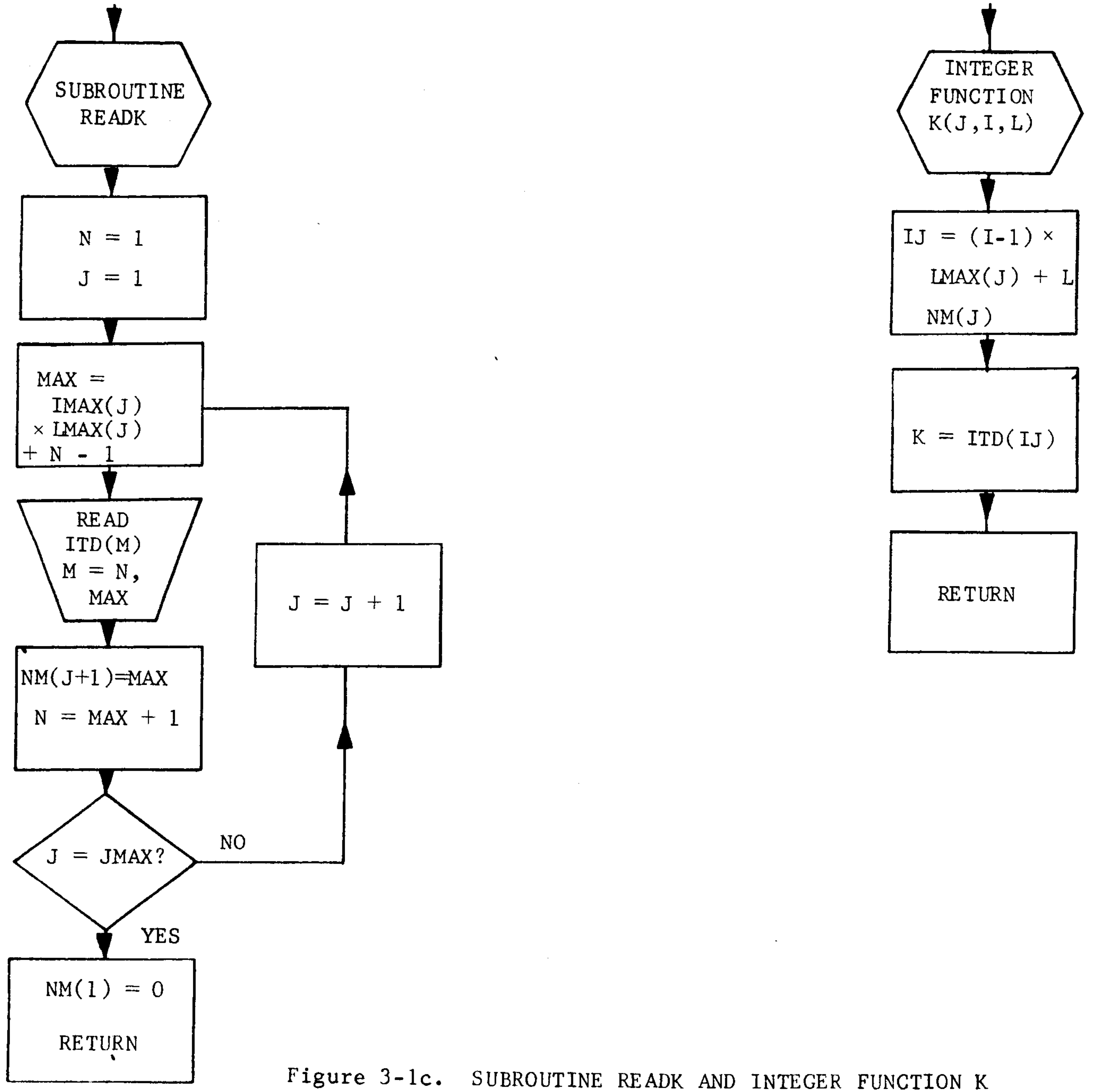


Figure 3-1c. SUBROUTINE READK AND INTEGER FUNCTION K

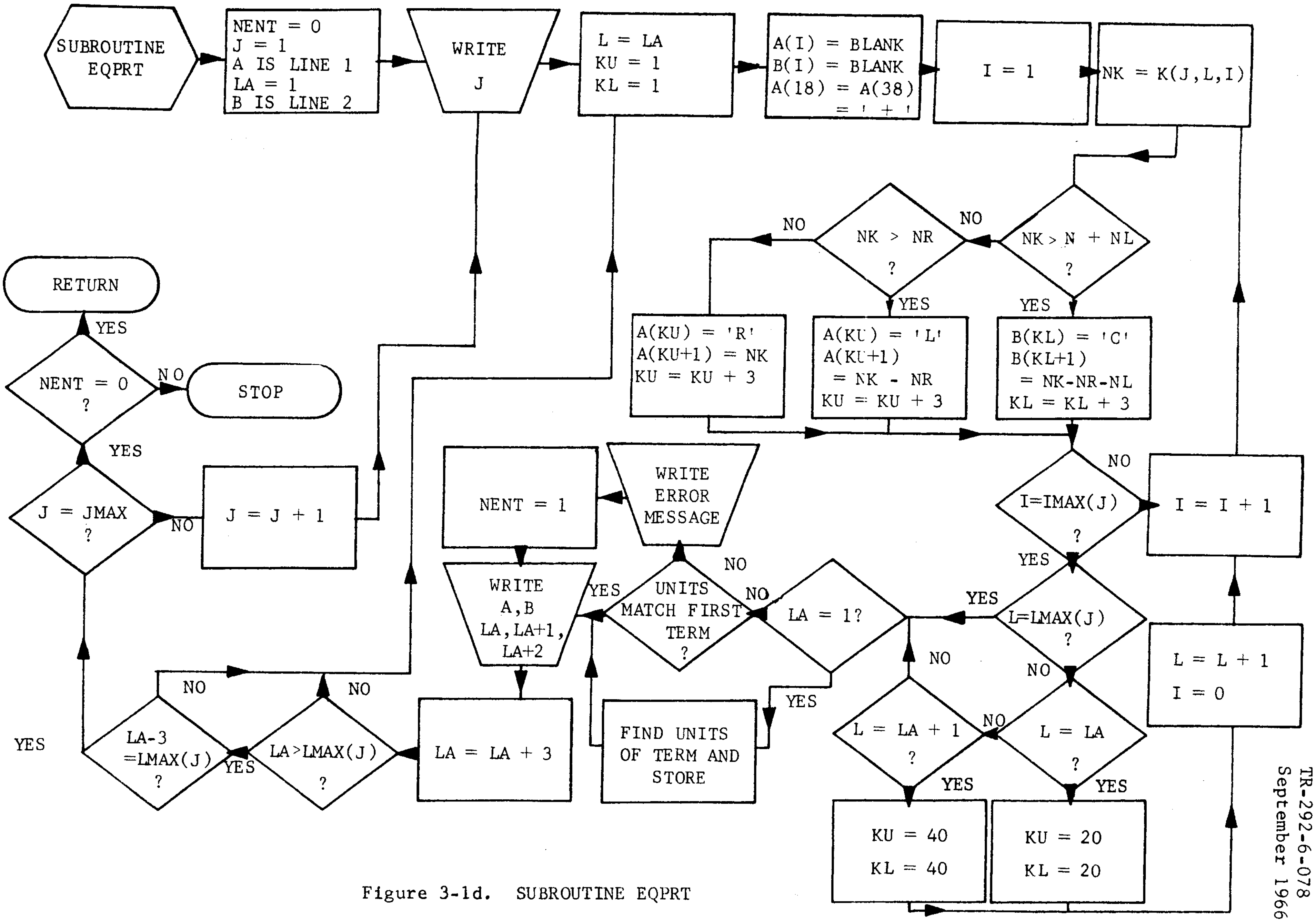


Figure 3-1d. SUBROUTINE EQPRT

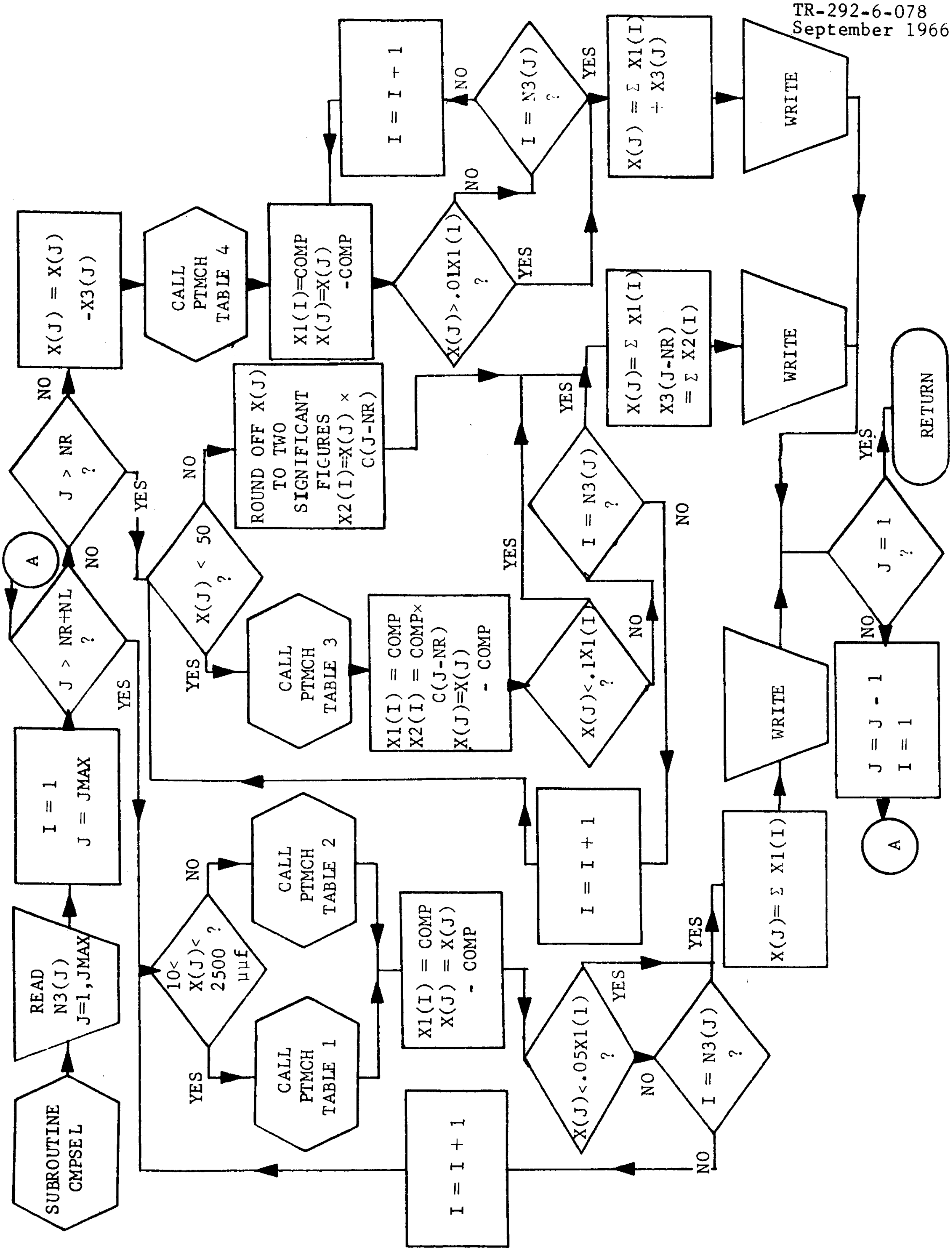
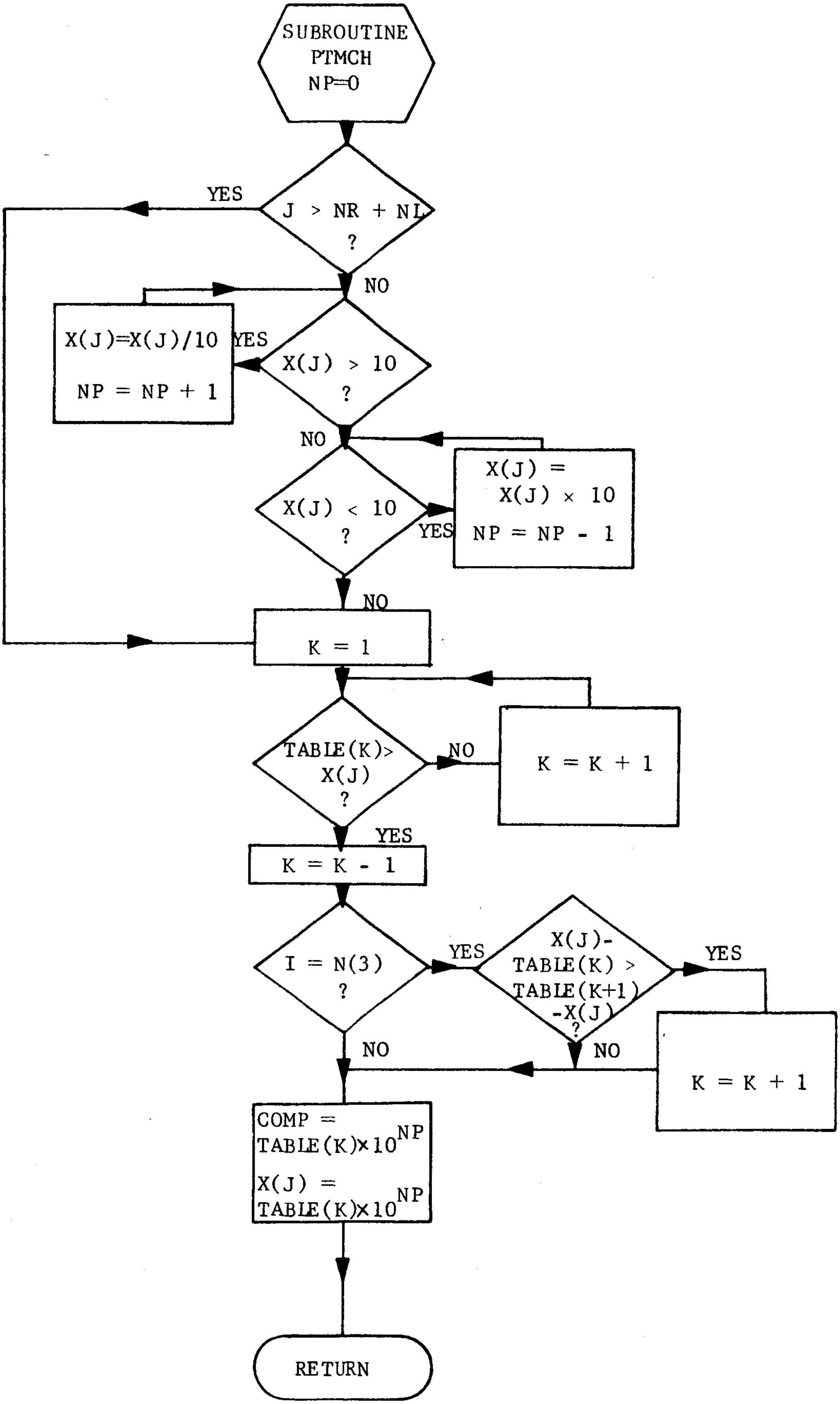


Figure 3-1e. SUBROUTINE CMPSEL



3-1f. SUBROUTINE PTMCH

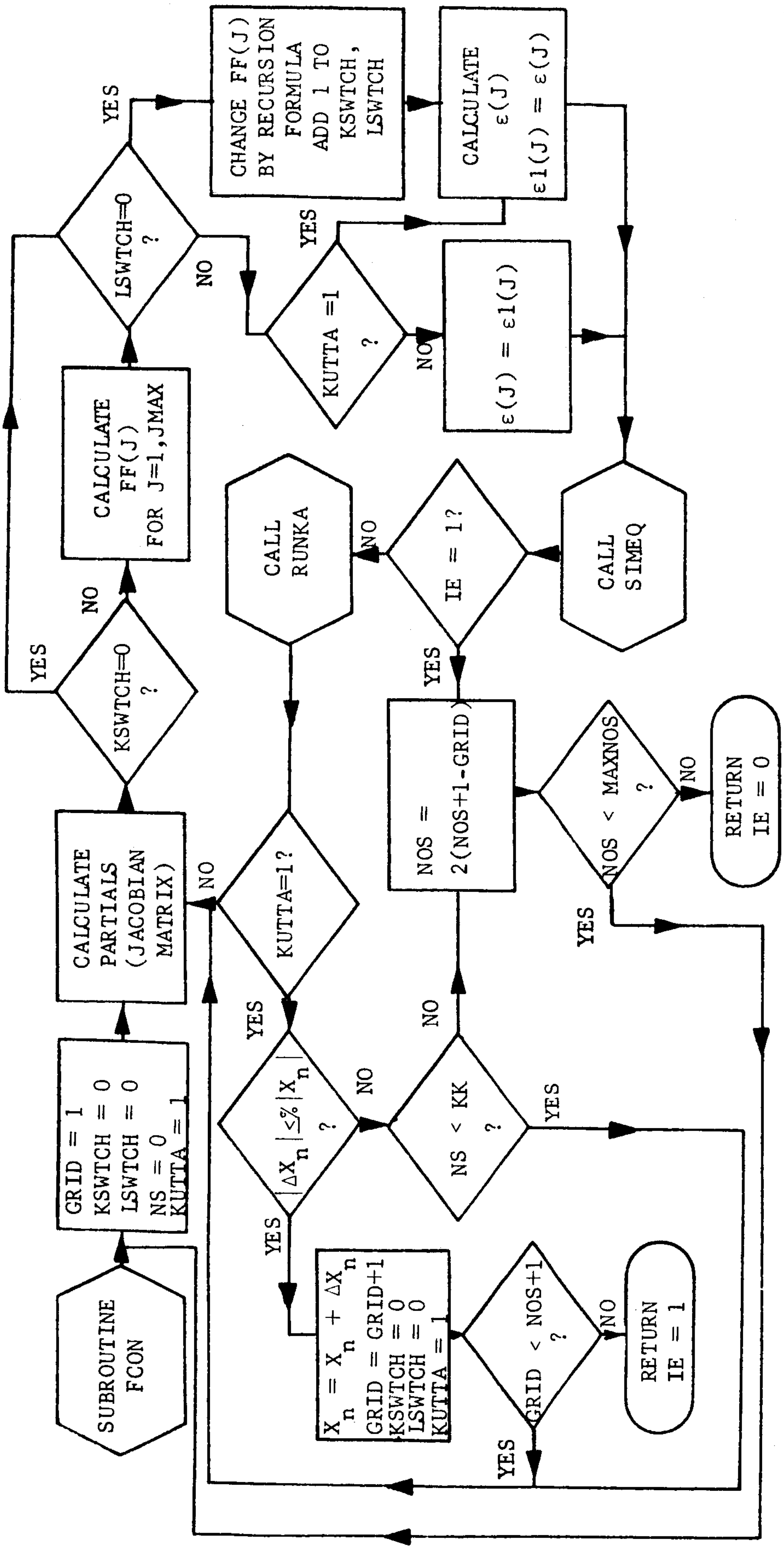


Figure 3-1g. SUBROUTINE FCON

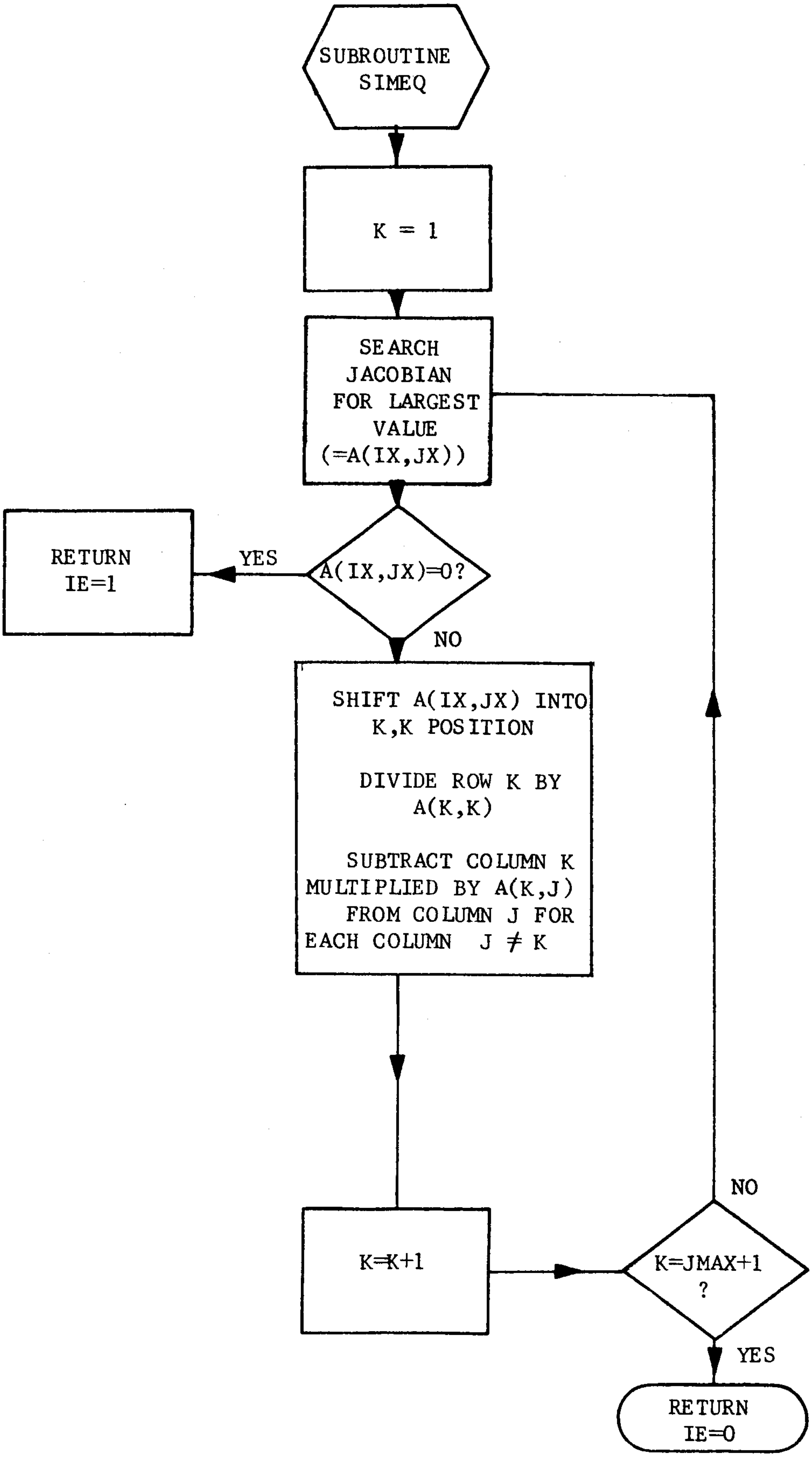

SUBROUTINE SIMEQ
K = 1
SEARCH JACOBIAN FOR LARGEST VALUE (=A(IX,JX))
A(IX,JX)=0?
YES
RETURN IE=1
NO
SHIFT A(IX,JX) INTO K,K POSITION
DIVIDE ROW K BY A(K,K)
SUBTRACT COLUMN K MULTIPLIED BY A(K,J) FROM COLUMN J FOR EACH COLUMN J ≠ K
K=K+1
K=JMAX+1 ?
NO
YES
RETURN IE=0

Figure 3-1h. SUBROUTINE SIMEQ

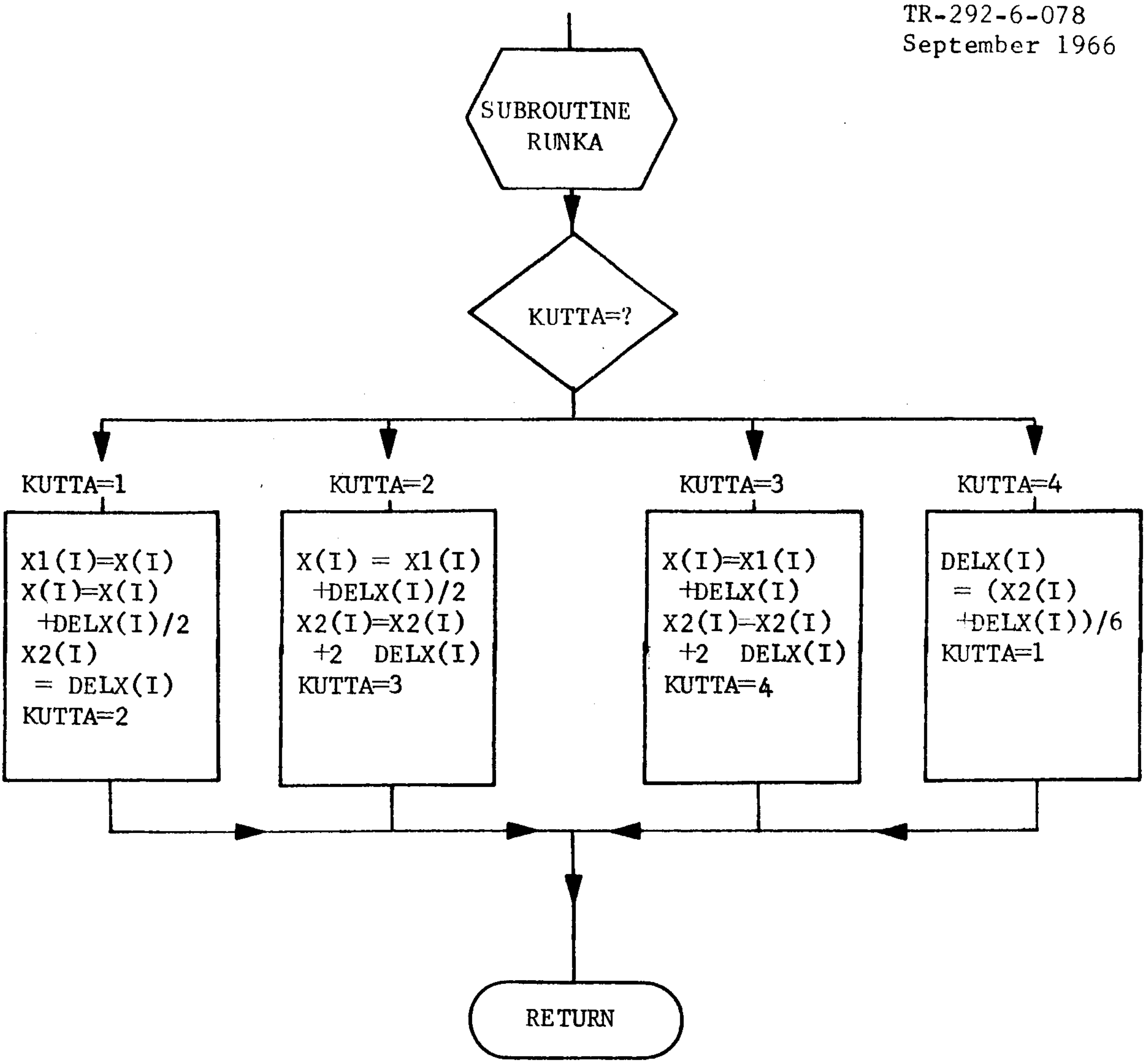


Figure 3-1i. SUBROUTINE RUNKA

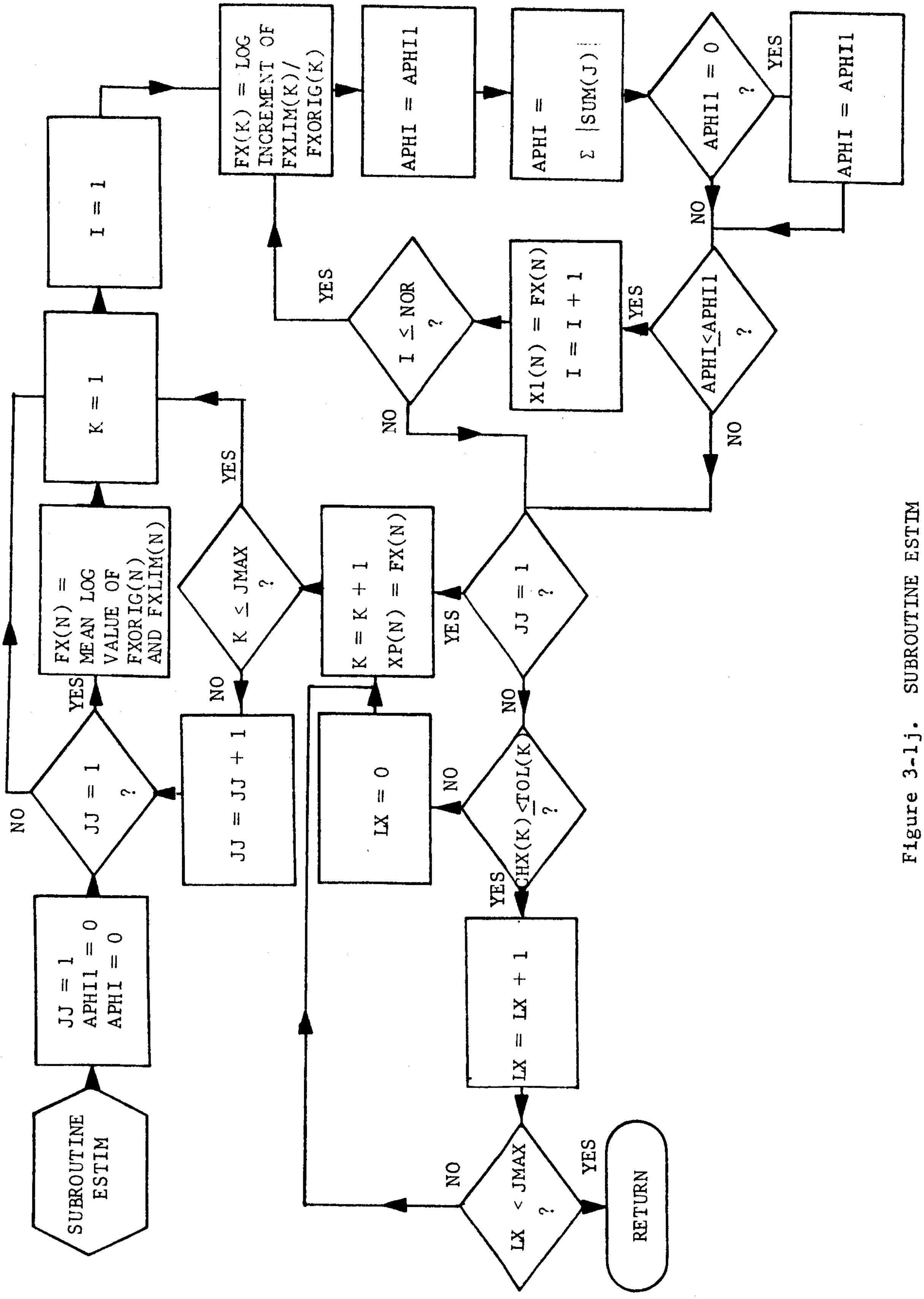


Figure 3-1j. SUBROUTINE ESTIM

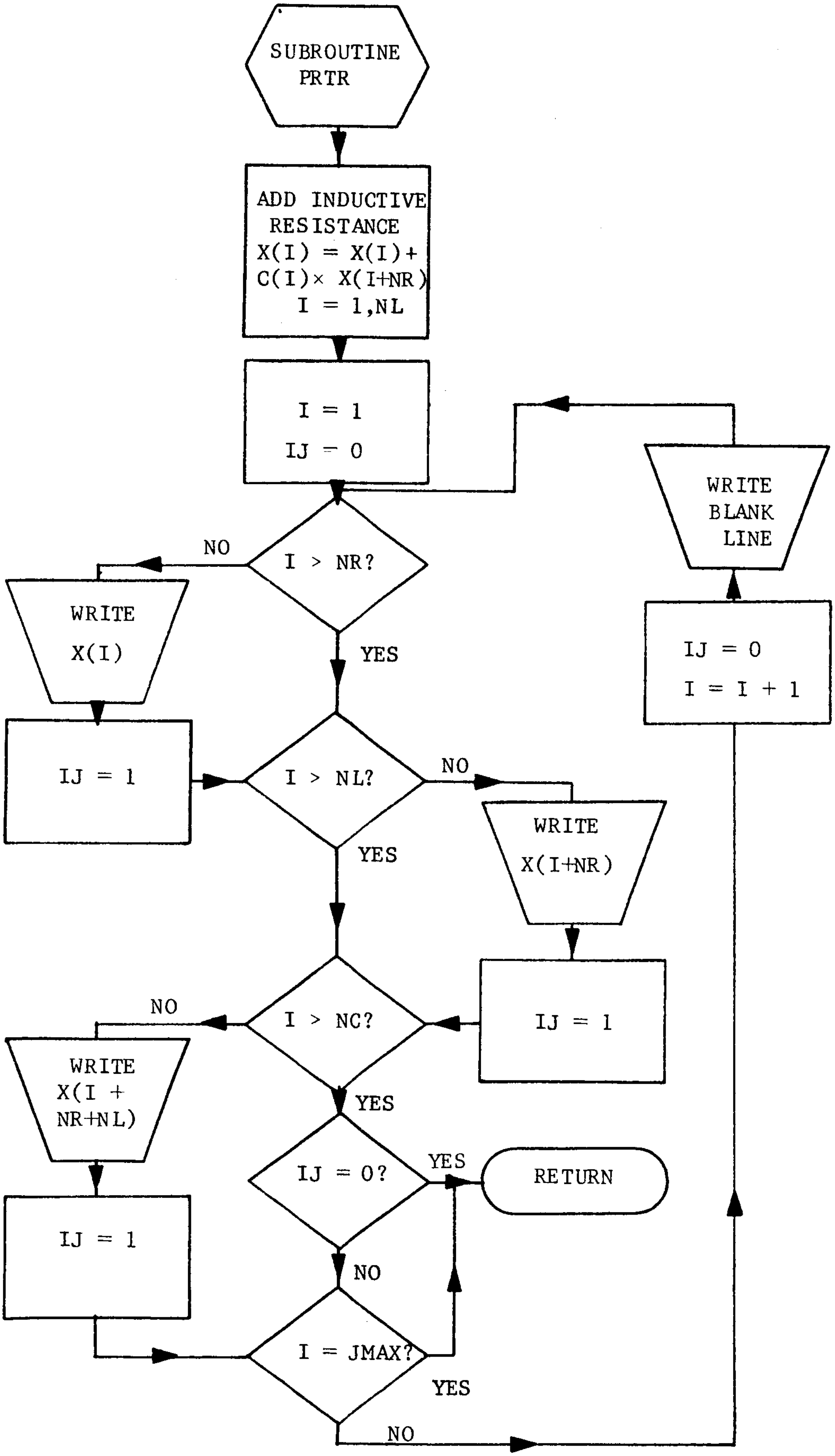


Figure 3-1k. SUBROUTINE PRTR

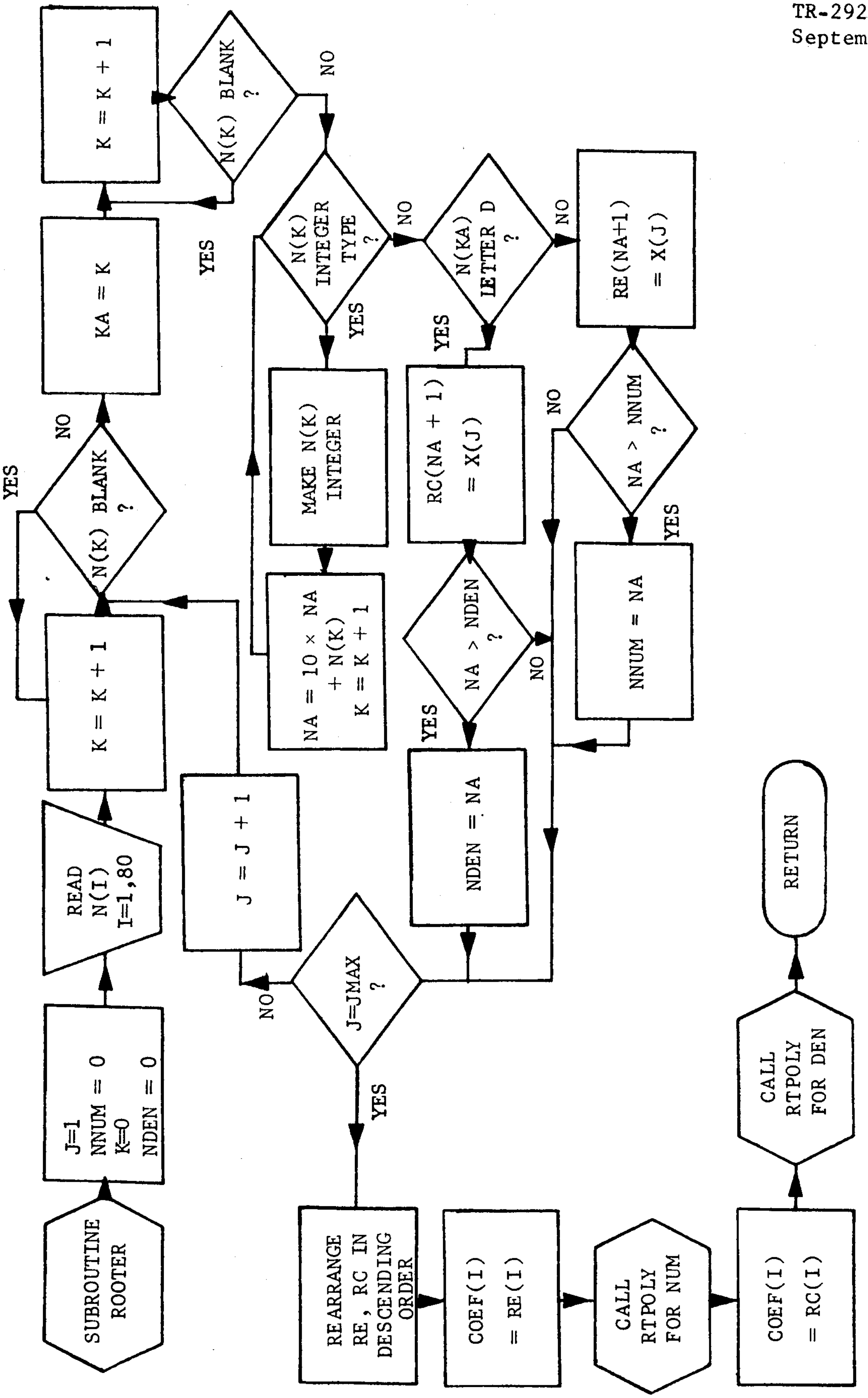


Figure 3-1l. SUBROUTINE ROOTER

After all the terms of the equations and the upper and lower limit for each unknown have been read into the computer, values for the initial estimate of each unknown are determined by the ESTIM subroutine.

The first attempt at solution (unless otherwise specified by the input value of NA) is the constant approach. In this method, the initial estimates are used to calculate constants that satisfy the equations. These constants are then varied stepwise, according to equation (2-25), toward their true values and the roots found at each intermediate step. When roots have been found for the case where the varying constants are the true (input) constants, a solution has been found. If at some step a singular matrix results or the attempt to find intermediate roots is unsuccessful, the number of iterative steps, V, is doubled to reduce the size of the incremental change in the constants and a solution is again attempted. This process continues until a solution is found or until the value of V exceeds some established limit, V_{limit}.

After successfully obtaining a set of satisfactory roots, the program can (by an input option) select approximate components and plot the frequency response of the resulting transfer function. If a set of roots is outside the physical limits specified in Table 2-1, or if no roots are found, the program starts over, using the coefficient approach. The coefficient approach involves finding a set of coefficients, A_j, applied to the largest terms in each equation, that will cause the equations to be satisfied. These coefficients are then varied stepwise toward unity in accordance with equation (2-23). When unity is reached, a solution has been found. If the coefficient approach using the largest terms fails, the process is repeated with the coefficients applied to the first term in each equation as originally read into the computer. If necessary the process can be performed repeatedly, applying the coefficients successively to the second term, third term, etc. in each equation. In any case, the method of approaching a solution is the same as the constant approach. The total number of such attempts, excluding the constant approach, is equal to some value, Q_{limit}, which is equal to or less than the number of terms in the longest equation plus one $(Q_{j(max)} + 1)$. In those equations where $Q_j > Q_{j(max)}$, and the coefficient approach specified application of A_j to a term number which is larger than Q_j, the coefficient A_j is applied to the last or $Q_j{}^{th}$ term of the equation.

For the case where satisfactory roots are obtained, the component selection subroutine takes one root at a time, starting with capacitors and ending with resistors, and matches components with the root in the same way a human might. It matches the root with values from a decade table of parts, picking the component that most nearly matches the root but is less than the root. This value is subtracted from the root, leaving a residual to be matched. This process continues until either the residual is less than the tolerance range of the first component selected for the root, or until a specified number of components for the root has been picked. In the latter case the last component is picked to match most nearly the residual. If the root is an inductor, its inductive resistance is calculated. If it is a resistor associated with an inductor, the natural or inductive resistance is subtracted from the total resistance prior to component matching. The natural resistance is added later to the sum of the components selected. The latter sum represents the "surplus" resistance as discussed in subsection 2.2. For inductance values of less than 50 henrys, the desired component is a variable inductor. The program assumes that the inductance in this case can be matched to two significant figures.

The program then forms the constant terms associated with the transfer function from either the actual roots or the approximations described above. From the specifications given on an input card, it matches the constant terms with the correct powers of s in the numerator N(s) and denominator D(s) of the transfer function. The program calculates the complex roots of N(s) and D(s) and then computes the magnitude and phase angle of the complex quantity $N(j\omega)/D(j\omega)$ for the desired values of frequency. The results are printed out and plotted on the SC-4020 plotter.

After the entire computational process has been successfully completed, the program may, based on input option, start over in search of additional sets of roots.

3.2 COMPUTER INPUTS AND OUTPUTS

All inputs are made through the familiar FORTRAN commands. The following is a listing, in alphabetical order, of the input items and their definitions, and a list of the format necessary for input of the items. The symbols in brackets are the corresponding symbols from the technical discussion. Example inputs and output for six equations with six unknowns are presented in Appendix D. Similar examples are provided in Appendix E for thirteen equations with thirteen unknowns.

3.2.1 Input Symbols

AMPMIN, AMPMAX } The minimum and maximum ordinate values for the amplitude versus frequency plot. If both are blank, the limits are taken as .001 and 100, respectively.

C(M) The constant term associating resistor (M) with inductor (M). $[K_m]$

DBMIN
DBMAX } The minimum and maximum ordinate values for the amplitude in decibels versus frequency plot. If both are blank, the limits are taken as -60 and +40, respectively.

F(J) The constant term associated with equation J. $\left[F_j\right]$

FRQMIN
FRQMAX } The minimum and maximum limits of frequency, respectively, to be plotted. If both are blank, the limits are taken as .001 cps to 25 cps.

FXORIG(J) The lower limits for the desired range on the variables X(J), where X(J) corresponds to X_n in Section II.

FXLIM(J) The upper limits for the desired range on the variables X(J).

ICPS An indicator. If it is not zero, the plots are made versus frequency in cps. If it is, the plots are made versus radians per second.

IMAX(J) The number of terms in equation J. $\left[Q_j\right]$

IZMAX The maximum number NA is allowed to attain.

JMAX The number of equations. $\left[p\right]$

KK The number of Runge-Kutta integrations allowed per Freudenstein-Roth step.

K(J,I,L) Subscript for each factor of each term of each equation. $\left[n(j,i,k)\right]$ L is varied most rapidly, J least rapidly. The subscripts for each equation begin on a new card.

LMAX(J) The number of factors per term for equation J. $\left[d_j\right]$

MAXNOS The maximum number of steps allowed in the Freudenstein-Roth technique. $\left[V_{Limit}\right]$

MR An indicator. If MR is zero, the program stops after obtaining one set of roots.

NA A column counter. If NA is zero, the constant approach is used. If NA is unity, the coefficient A (in the Freudenstein-Roth technique) is applied to the largest term in each equation. If NA is greater than unity, the coefficient A is applied to term NA-1. After each attempt at solution is fully exhausted, NA is increased by one. When NA equals IZMAX, the program stops.

NC The number of capacitors. [w]

NL The number of inductors. [v]

NMAX The number of derived equations in a circuit. Because in some cases there are more unknowns than there are derived equations, supplementary equations are made by assignation of values to components. These supplementary equations must follow the derived equations on input, and the number of derived equations must be specified (even if the number of derived equations is equal to the number of unknowns.

NOR The number of increments between FXORIG and FXLIM for ESTIM, the initial estimate subroutine.

NOS The initial number of steps for the Freudenstein-Roth technique. [V]

NTB An indicator. If NTB is zero, the program will plot the resulting transfer function from the first set of roots obtained.

NTC An indicator. If NTC is not zero the values of the roots are used to form the transfer function for the frequency-response subroutine. If it is zero approximate values found by CMPSEL are used.

NR The number of resistors. [u]

NSTPS — Number of points to be computed per decade of frequency in the frequency response program.

N3(J) — Specifies the maximum number of components to use in approximating X(J).

PTOL(J) — The desired tolerance for root X(J).

SPEC(J) — This specifies to the program to which power of s in N(s) or D(s) of the transfer function that F(J) belongs. The input is an 'N' or 'D' (specifying numerator or denominator) followed by a number (specifying a power). Thus D2 NO specifies that F(1) is the coefficient D_2 of s^2, and that F(2) is the coefficient N_0 of s^0. The input is free form, with blanks allowed anywhere except as part of a number (N 10 is allowed, but N 1 0 is not).

TX — The desired fractional tolerance for the initial estimates from ESTIM. When the estimates X(J) do not change more than TX x X(J) in an attempt to further modify the estimates, then the set X(J) is returned from ESTIM as the set of initial estimates.

XCMAX }
XCMIN } — The maximum and minimum practical values that are obtainable for capacitors.

XLMAX }
XLMIN } — The maximum and minimum practical values that are obtainalbe for inductors.

XRMAX }
XRMIN } — The maximum and minimum practical values that are obtainable for resistors.

3.2.2 Input Units

XRMIN	Resistance (ohms)
XLMIN	Inductance (henrys)
XCMIN	Capacitance (farads)
XRMAX	Resistance (ohms)
XLMAX	Inductance (henrys)
XCMAX	Capacitance (farads)

FXORIG }
FXLIM }

For J = 1,NR Resistance (Ohms)

For J = NR+L,NL Inductance (henrys)

For J = NR+NL+1, JMAX Capacitance (farads)

3.2.3 Input List and Format

The list which follows gives, in sequential order, all of the data that must be input into the computer for a run. The FORTRAN symbols defined in the previous section are used for the data. The word "CARD" in the left margin is used to designate that the Fortran symbols, corresponding to the input items, to the right of the word "CARD" must begin sequentially on a new card.

```
CARD        MAXNOS, NOX, KK, JMAX, IZMAX, NR, NL, NC, NOR, MR, NA, NTB
            FORMAT 20I4
CARD        IMAX(J)  J=1, JMAX
            FORMAT 20I4
CARD        LMAX(J)  J=1, JMAX
            FORMAT 20I4
CARD        F(J)  J=1, JMAX
            FORMAT 6E12.5
CARD        PTOL(J)  J=1, JMAX
            FORMAT 8E10.0
CARD        XRMIN, XLMIN, XCMIN, XRMAX, XLMAX, XCMAX
            FORMAT 8E10.0
CARD        FXORIG(J)  J=1, JMAX, FXLIM(J)  J=1, JMAX
            FORMAT  8E10.0
CARD        C(M)  M=1, NR
            FORMAT 6E12.5
```

CARD TX

FORMAT E10.0

CARD K(J,I,L) L=1, LMAX(J), I=1, IMAX(J)

FORMAT 20I4

Repeat above for J equals 1 to JMAX

CARD NTC,N3(J) J=1, JMAX

FORMAT I1, 4X, 15I1

CARD SPEC(J) J=1, JMAX

FORMAT 80A1

CARD NMAX

FORMAT I2

CARD ICPS, NSTPS, FRQMIN, FRQMAX, DBMIN, DBMAX, AMPMIN, AMPMAX

FORMAT I1, 4X, I5, 6F10.5

3.2.4 Output Nomenclature

The printout consists of a listing of the equations, the initial data, intermediate results, and, if roots are obtained, the roots and the results from the component selection and frequency-response subroutines.

The equations are listed three terms per line, with a term number for each term. The factors include a letter denoting resistance, capacitance, or inductance, and the corresponding component subscript. The lines indicating the division between the numerator and denominator terms are not printed.

The next portion of printout consists of certain input data. The "Maximum No. of Steps" referred to is MAXNOS; the "Number of Steps" is NOS; and the "Times through Runge-Kutta" is KK. The "Constants Terms" are F(J) arranged in order of subscripts reading in order from left to right. Following these terms, the range of interest for each variable is established by means of FXORIG(J) and FXLIM(J) which are arranged in the same order as F(J). The rest of the initial data printout describes the number of equations and unknowns, the number of resistances, capacitances, and inductances involved, and the maximum and minimum allowable components for such components.

After the printout of input data, the program is designed to indicate to the user the steps taken to obtain a solution. The terminology used is the same as that already provided for input with the following additions:

GRID The iterative step number in the Freudenstein-Roth technique ($1 \leq$ GRID $\leq$ NOS)

LX The counter used in the process of selecting initial estimates. When LX=JMAX the selection is complete.

NA The counter used to determine the method of solution. If NA is zero, the constant approach is tried. If NA is one, the coefficient approach is applied to the largest term of each equation. And if NA is greater than one, the coefficient approach is applied to term NA-1 of each equation ($0 \leq$ NA $\leq$ IZMAX).

The final output depends upon conditions arising within the program. Should a satisfactory set of roots be obtained (a set in which all elements are within the specified physical limits), a statement indicating this fact is printed out together with the roots appropriately denoted as resistances, capacitances, and inductances. In the case where roots are found but are not acceptable, a statement indicates this fact. A listing of the values of the unacceptable roots follows. As already noted, the computer contains an option that, in case a set of satisfactory roots is found, the process either stops or continues searching until NA=IZMAX. If a singular matrix is encountered in SIMEQ, the words "Singular Matrix" are printed out, and the computer proceeds as indicated in Figure 3-1.

Should a set of roots be found, the computer prints them out and then tests an indicator (ITB). If ITB is not zero the program searches for another set of roots. If ITB is zero the indicator ITC is tested. If this is non-zero the program skips CMPSEL and goes directly to the frequency-response subroutine. Otherwise, CMPSEL is used to approximate the roots by component selection.

The subroutine CMPSEL prints out, for each unknown, the various values of components selected and their summation. It also calculates the inductive resistances and prints them out.

Finally, the frequency-response subroutine is used. The printout from this subroutine consists of the transfer function, its roots and poles, and the calculated values of amplitude and phase shift over the specified frequency range. These points are plotted automatically on the SC-4020 plotter.

SECTION IV

DISCUSSION OF RESULTS

The goal of the present research effort has been to refine the computer program developed in the initial study for solving nonlinear sets of simultaneous algebraic equations,which occur in filter circuit analysis, and to extend the applications of the program and the numerical techniques upon which it is based.

4.1 PROGRAM REFINEMENTS ACHIEVED

The computational refinements achieved were the incorporation of Kizner's method for the solution of intermediate Freudenstein-Roth steps and the addition of a root prediction subroutine to provide better estimates of the roots of the Freudenstein-Roth steps. These refinements both shorten computational time and improve convergence of the computer program. In addition, certain subroutines were added to make the program more useful to filter circuit designers. These subroutines are designed to:

- Select standard, off-the-shelf components whose values most nearly match the theoretical values determined by the roots of the equations.
- Obtain the attenuation and phase shift vs frequency plots for the resulting filter circuit whose component values approximate a theoretical circuit.

4.2 APPLICATION TO ACTUAL PROBLEMS

The refined digital computer program was successfully used to solve sets of equations in six unknowns and thirteen unknowns. The equations represent filter circuits as described in reference 17 . In addition, attenuation and phase shift vs frequency plots were obtained for filter circuits composed of standard value components which approximate the above theoretical circuits. A solution was attempted for a set of equations in fifteen unknowns which represent the filter circuit described in reference 18. Although only limited success was achieved in obtaining a solution to this set of equations in fifteen unknowns, evidence was gathered which strongly supports the hypothesis that this set of equations is ill-conditioned.

4.2.1 Equations in Six Unknowns

The transfer function on page B-42 of reference 17 yielded six simultaneous equations in the six unknown component values. The occurrence of exactly six equations for six unknowns is not trivial, for transfer functions of other filter circuits often yield either a lesser or a greater number of equations than unknowns. These cases are discussed in subsequent sections.

The equations and the filter circuit associated with the equations are included in Appendix D. These equations were solved by the refined computer program. In addition, the computer program selected the standard value components which most nearly matched the values indicated by the roots of the equations and plotted attenuation and phase shift vs frequency curves for the resulting approximate circuit. The two sets of roots obtained, along with the upper and lower limits of each root used for the ESTIM subroutine, are presented in Appendix D. Figures D-1, D-2, and D-3 of the appendix present, respectively, the amplitude, phase shift and gain vs frequency plots for one of the circuits obtained.

4.2.2 Equations in Thirteen Unknowns

The transfer function of the filter circuit on page B-93 of reference 17 yielded twelve equations in thirteen unknowns. To obtain a solvable set of equations, one of the unknowns (i.e., component values) was assigned a fixed value. This value was chosen so that the resulting set of thirteen equations in thirteen unknowns had a set of roots that were real, positive numbers. This choise was made to insure that the component values of the filter were physically realizable.

The resulting set of thirteen simultaneous equations is listed in Appendix E. They were solved by the refined computer program. The set of roots obtained, as well as the upper and lower values of the roots used in the ESTIM subroutine, is included in Appendix E. This appendix also presents the standard component values selected by the computer program to most nearly match those indicated by the set of roots. Figures E-1, E-2, and E-2, respectively, present the amplitude, phase shift and gain vs frequency plots of the resulting approximate filter circuit.

4.2.3 Equations in Fifteen Unknowns

The transfer function of the filter circuit given on page 9 of reference 18 yielded the sixteen equations in fifteen unknowns shown in Appendix F. The task of generating the equations from circuit analysis proved quite laborious. This work involved expanding two determinants of eighth-order matrices, the elements of which were algebraic expressions. The two resulting algebraic polynomials contained over 800 terms which were grouped according to the exponent of the variable s. The sixteen algebraic expressions developed by this grouping represented the functions ψ_j discussed in subsection 2.2.

After deriving the expressions ψ_j, the next step was establishing the values for F_j. The original version of transfer functions given in reference 18 had already been normalized by dividing the numerator and denominator by N_0 and D_0, respectively. The gain factor for this original transfer function was also omitted. Northrop performed the necessary analysis to obtain the non-normalized transfer functions. The N_q and D_q of this transfer function were then matched with the corresponding algebraic expressions to form the sixteen equations of the form of equation (2-2).

A preliminary examination of equations indicated that they would have to be scaled to prevent computer overflow. For this reason, the circuit was scaled by multiplying all resistor and conductors by 10^{-6} and capacitors by 10^{6}. The constant terms, F_j, were correspondingly scaled by multiplying by 10^{-42}.

The circuit upon which the transfer function and the sixteen equations were based, contained only fifteen elements. Thus the set of sixteen equations contained only fifteen unknowns. As discussed in subsection 2.2, the existence of more equations than unknowns immediately raised the question as to which, if any, combination of the equations would form an independent set.

Various methods were used in an attempt to establish the independence or dependence of any of the sixteen sets of fifteen equations taken from the sixteen equations. Algebraic expansion of the determinant of the Jacobian matrix was not practical because a fifteenth-order matrix was involved. Numerical evaluation of this determinant for specific values of the unknowns proved inconclusive. For some values of the unknowns the matrix was numerically singular. For other values this was not the case. All numerical work of this nature was hampered by computer truncation error coupled with the significant differences in order of magnitude of the unknowns.

Numerous runs were made with several different sets of fifteen equations. In many cases the computer indicated a singular matrix had been encountered. In others rapid divergence occurred. These experiences indicated that the sets of equations selected were either dependent or extremely ill-conditioned.

One of the last computer runs carried out involved running the 16 different sets of 15 equations one after another, with the initial estimate of 14 of the 15 unknowns set equal to values of known roots taken from reference 18. The one unknown, which was not set equal to a root, was given a value 12 percent greater than the value of the corresponding root. For four of the sixteen cases, convergence did occur rapidly. The sets of equations used in these four cases can be most readily identified by specifying the coefficient N_q or D_q corresponding to the equation omitted. These four coefficients were N_0, N_1, D_2, and D_3. A singular matrix was not encountered in any of the remaining cases, and for some of these cases there was indication that convergence was occurring although not as rapidly as for the four cases already mentioned. Based on this last computer run it would appear that all of the sets of fifteen equations are independent but all are also ill-conditioned, some more so than others.

In carrying out this last computer run, the constant approach of the Freudenstein-Roth technique was used exclusively. This action was taken because of the fact that with the coefficient approach the Jacobian matrix changes algebraically with each step in the Freudenstein-Roth process. Thus a singular matrix might occur at some intermediate step in the process even though the true set of equations was independent. In the constant approach the Jacobian matrix remains constant algebraically through all steps. Thus the dependence or independence of a set of equations is more clearly indicated by means of the latter approach.

The ill-conditioned feature of the equations appears to be the result of the considerable differences in order of magnitude of the unknowns. An indication of the ill-conditioned characteristic is that the determinants of the Jacobian matrices corresponding to the 16 different sets of equations appear, in general, to have relatively small numerical values in that region within which the roots to the equations are most likely to occur. When computer truncation error is considered in conjunction with this characteristic of the Jacobian, it can be seen that accurate numerical calculations using either the Newton-Raphson method or Kizner's method are difficult if not impossible under such conditions.

4.3 APPLICATION TO NONLINEAR DIFFERENTIAL EQUATIONS

As noted in subsection 2.7, there exist a number of engineering problems in which sets of nonlinear differential equations are encountered. These problems are inherently complex and the techniques which have been developed to solve such problems tend to be somewhat specialized. The differential equations associated with the two specific problems listed in subsection 2.7 have been examined along with the appropriate boundary conditions. Because of time limitations, no attempt was made to apply the numerical techniques developed to the actual differential equations. It would appear that for situations in which boundary conditions or initial conditions are not well defined, the technique would prove useful for simultaneously satisfying finite-difference versions of the differential equations.

SECTION V

CONCLUSIONS AND RECOMMENDATIONS

Based on the experience gained in the research effort, the Freudenstein-Roth technique combined with Kizner's method appears to be a powerful tool in the simultaneous solution of nonlinear algebraic equations. The digital computer program, which contains this numerical technique combined with a circuit component selection scheme and a frequency-response curve plotter, is capable of analyzing complex filter circuits and represents a useful engineering tool.

The most significant feature of the program is its flexibility in handling any set of algebraic equations of the general type encountered in filter circuit analysis. The primary limitation of the program occurs when it is applied to circuits for which the corresponding algebraic equations are ill-conditioned.

With very minor modification the program could be extended to handle any set of algebraic equations. Extension of the program to sets of transcendental equations could also be accomplished with relatively small effort. The possibility also exists that the basic numerical techniques employed may be useful in the solution of sets of nonlinear differential equations and their associated boundary conditions.

The recommendation is made that an investigation be conducted concerning the extensions in application of the program and the associated numerical techniques discussed in the preceding paragraph. In addition, consideration should be given to the use of a digital computer to generate the algebraic equations characteristic of a filter circuit. Northrop is presently developing computer techniques capable of mathematical operations involving high-order polynomials with literal coefficients. The techniques developed in the latter research effort would be useful in writing a computer program capable of generating the desired equations.

There is a very evident need for an investigation into the problems of identifying independence/dependence in sets of nonlinear equations as well as identifying ill-conditioned sets of equations. The possibility of transforming an ill-conditioned set into a well-behaved set, by some numerical process, is also worthy of study. Such a transformation appears to offer the most promising approach to the solution of ill-conditioned sets of nonlinear algebraic equations.

SECTION VI

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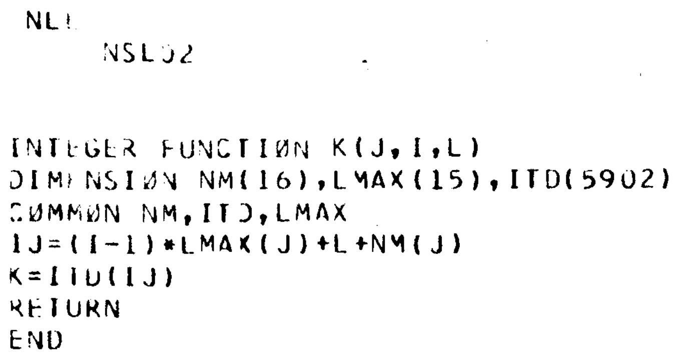
APPENDIX A

SOURCE LISTING OF COMPUTER PROGRAM

A source listing of the complete program is included in this appendix. Individual segments of the program are located on the pages indicated below:

```
      DIMENSIØN IMAX(15),F(15),X(15),C(15),FXØRIG(15),X1(15),DELX(15),
     DFX(15),DFX(15,15),SUM(15),PSUM(15,15),T(215),P(215,15),A(15,215),
     IR(15),AØRIG(15),PHI(15),PTØL(15),FL(15),FC(15),FXI(15),FXLIM(15),
     MXGUES(15),LMAX(15),          NM(16),ITD(5902),X2(15)
      DIMENSIØN PHIP(15),DX(15)
      CØMMØN NM,ITD,LMAX,IMAX,JMAX
      CØMMØN /PLØTER/RE(16),RC(16),NNUM,NDEN,RØØT(80),RØØT1(80)
100   FØRMAT (//4H NA=,I4/)
110   FØRMAT(/16H SINGULAR MATRIX/)
120   FØRMAT (20I4)
130   FØRMAT (2X10HINPUT DATA//21H MAXIMUM NO. ØF STEPS,3X,I4/16H NUMBER
     1 ØF STEPS,8X,I4/26H TIMES THRØUGH RUNGE KUTTA,4XI4/15H CØNSTANT TE
     2RMS/)
140   FØRMAT (/32H CØMMENCING CØEFFICIENT APPRØACH)
150   FØRMAT (/3X5HFXLIM/(6(3XE16.8)))
180   FØRMAT (6H GRID=,I4,3X,4HNØS=,I4)
210   FØRMAT (6(4XE16.8))
230   FØRMAT (8E10.0)
240   FØRMAT (6E12.5)
320   FØRMAT (/10H VARIABLES/)
330   FØRMAT (//72H ALL RØØTS IN THE FØLLØWING SET LIE WITHIN THE PHYSIC
     1AL LIMITS SPECIFIED//)
340   FØRMAT (49H USING THIS SET ØF ESTIMATES, NØ RØØTS WERE FØUND//)
350   FØRMAT (/75H THE FØLLØWING SET ØF RØØTS DØ NØT LIE WITHIN THE PHYS
     FICAL LIMITS SPECIFIED/)
360   FØRMAT (/20H RANGE FØR VARIABLES/3X6HFXØRIG/(6(3X,E16.8)))
370   FØRMAT (/11H THERE ARE ,I2,15H EQUATIØNS AND ,I2,24H UNKNØWNS,CØNS
     FISTING ØF ,I2,16H RESISTANCE(S), ,I2,19H INDUCTANCE(S),AND ,I2,16H
     D CAPACITANCE(S).)
380   FØRMAT (85H THE LØWER BØUNDARIES FØR THE RESISTANCES, THE INDUCTAN
     FCES, AND THE CAPACITANCES ARE ,2(E16.8,2H, )/5H AND ,E16.8,1H, 48H
     D RESPECTIVELY, WHILE THEIR UPPER BØUNDARIES ARE ,2(E16.8,2H, ),4HA
     RND /1XE16.8,14H RESPECTIVELY.)
      EQUIVALENCE (JMAX,NMAX)
      READ (5,120) MAXNØS,NØS,KK,JMAX,IZMAX,NR,NL,NC,NØR,MR,NA,NTB
      READ (5,120) (IMAX(J),J=1,JMAX)
      READ (5,120) (LMAX(J),J=1,JMAX)
      READ (5,240) (F(J),J=1,JMAX)
      READ (5,230) (PTØL(N),N=1,NMAX)
      READ (5,230) XRMIN,XLMIN,XCMIN,XRMAX,XLMAX,XCMAX
      READ (5,230) (FXØRIG(N),N=1,NMAX),(FXLIM(N),N=1,NMAX)
      READ(5,240) (C(M),M=1,NR)
      READ (5,230) TX
      CALL READK
      NCC=NMAX-NC
      CALL EQPRT(JMAX,IMAX,LMAX,  NR,NL,NC)
      NNØS=NØS
      WRITE (6,130) MAXNØS,NØS,KK
      WRITE (6,210) (F(J),J=1,JMAX)
      WRITE(6,360) (FXØRIG(I),I=1,JMAX)
      WRITE(6,150) (FXLIM(I),I=1,JMAX)
      WRITE (6,370) JMAX,NMAX,NR,NL,NC
      WRITE(6,380)XRMIN,XLMIN,XCMIN,XRMAX,XLMAX,XCMAX
      CALL ESTIM (NMAX,JMAX,NR,NL,NØR,TX,IMAX,LMAX,F,C,FXØRIG,FXLIM,FX)
      WRITE (6,320)
      NLE
          NSL01
      WRITE (6,210) (FX(N),N=1,NMAX)
      DØ 205 M=1,NMAX
      IF(M-NR) 206,206,207
  207 C(M)=0.
  206 NRM=NR+M
      DØ 7 N=1,NMAX
      DFX(M,N)=0.
      IF(M-N)9,8,9
    8 DFX(M,N)=1.
      GØ TØ 7
    9 IF(N-(NR+M))7,10,7
   10 DFX(M,N)=C(M)
    7 CØNTINUE
  205 X(M)=FX(M)-C(M)*FX(NRM)
      DØ 48 I=1,NMAX
   48 XGUES(I)=X(I)
      IF(NA.NE.0)GØ TØ 51
      CALL FCØN (MAXNØS,NØS,KK,JMAX,NMAX,NR,LMAX,IMAX,F,PTØL,X,C,XGUES,
     F           FX,IERR,FXLIM,DFX,X2,PHIP,DX)
      GØ TØ (112,52),IERR
   51 WRITE (6,100) NA
      NØS=NNØS
      DØ 50 I=1,NMAX
      X1(I)=XGUES(I)
   50 X(I)=XGUES(I)
      IGRID=1
   47 LL=0
      ANØS=NØS
      KSWTCH=0
      LSWTCH=0
      NS=0
   54 WRITE (6,180) IGRID,NØS
      KUTTA=1
   60 DØ 3 I=1,NMAX
    3 DELX(I)=0.
C     CALCULATE PARTIALS
      DØ 4 M=1,NMAX
      IF(M-NR)5,5,6
    6 C(M)=0.
    5 NRM=NR+M
      FX(M)=C(M)*X(NRM)+X(M)
    4 CØNTINUE
      DØ 11 J=1,JMAX
      SUM(J)=-F(J)
      DØ 12 N=1,NMAX
   12 PSUM(J,N)=0.
      IJMAX=IMAX(J)
      DØ 13 I=1,IJMAX
      T(I)=1.
      LJMAX=LMAX(J)
      DØ 14 L=1,LJMAX
      NK=K(J,I,L)
   14 T(I)=T(I)*FX(NK)
      DØ 15 N=1,NMAX
      P(I,N)=0.
      DØ 16 L=1,LJMAX
      NK=K(J,I,L)
   16 P(I,N)=P(I,N)+T(I)*DFX(NK,N)/FX(NK)
C     CALCULATE TØTAL PARTIALS
   15 PSUM(J,N)=PSUM(J,N)+P(I,N)
   13 SUM(J)=SUM(J)+T(I)
C     DETERMINE LARGEST CØEFFICIENT ØF EACH EQUATIØN
      IF(NA-1)17,17,18
   17 TX=0.
      IJMAX=IMAX(J)
      DØ 19 I=1,IJMAX
      IF(T(I)-TX)19,19,20
   20 TX=T(I)
      NX=I
   19 CØNTINUE
      GØ TØ 21
   18 IF(IMAX(J)+1-NA)22,23,23
   22 NX=IMAX(J)
      GØ TØ 21
   23 NX=NA-1
   21 IF(KSWTCH-1)24,25,25
C     CALCULATE CØEFFICIENTS
   24 AØRIG(J)=((-SUM(J))/T(NX))+1.
   25 IF(LSWTCH-1)30,29,29
   30 GRID=IGRID
      IF (AØRIG(J)) 1,125,125
    1 A(J,NA)=-(ABS(AØRIG(J)-2.)**(1.-GRID/ANØS))+2.
      GØ TØ 29
  125 A(J,NA)=AØRIG(J)**(1.-GRID/ANØS)
C     CALCULATE TØTAL PARTIALS (CØRRECTED)
   29 DØ 28 N=1,NMAX
   28 PSUM(J,N)=PSUM(J,N)+(A(J,NA)-1.0)*P(NX,N)
      IF(KUTTA-1) 11,281,11
  281 PHIP(J)=-(SUM(J)+(A(J,NA)-1.)*T(NX))
   11 PHI(J)=PHIP(J)
      KSWICH=1
      LSWICH=1
      CALL SIMEQ (PSUM,DELX,PHI,JMAX,IE)
      IF(IE.EQ.1) GØ TØ 32
      CALL RUNKA(X,DELX,FXLIM,PTØL,X2,      KUTTA,NMAX)
      GØ TØ (31,60,60,60),KUTTA
   31 CØNTINUE
      NS=NS+1
      DØ 33 I=1,NMAX
      IF (ABS(DELX(I))-PTØL(I)*ABS(X(I)))33,33,40
   33 CØNTINUE
      DØ 35 I=1,NMAX
   35 X(I)=X(I)+DELX(I)
   39 DØ 61 I=1,NMAX
      NRI=NR+I
   61 FX(I)=C(I)*X(NRI)+X(I)
      NS=0
      LSWICH=0
      IGRID=IGRID+1
      IF(IGRID-NØS-1)36,52,52
   36 DØ 26 N=1,NMAX
      DX(N)=X(N)-X1(N)
      X1(N)=X(N)
   25 X(N)=X(N)+DX(N)
      GØ TØ 54
   40 LSWICH=LSWTCH+1
      IF (NS-KK)37,43,43
   37 DØ 55 I=1,NMAX
   55 X(I)=X(I)+DELX(I)
      GØ TØ 50
   32 WRITE (6,110)
   43 NØS=2*(NØS+1-IGRID)
      IF(NØS-MAXNØS)44,38,38
   44 DØ 45 I=1,NMAX
      DX(I)=DX(I)*.5
   45 X(I)=X1(I)+DX(I)
      IGRID=1
      GØ TØ 47
   38 NØS=NNØS
      WRITE (6,340)
  211 NA=NA+1
      WRITE (6,140)
      IF(NA-1-IZMAX)51,49,49
   49 STØP
   52 DØ 76 I=1,NR
      IF (X(I)-XRMIN)121,76,76
   76 CØNTINUE
      DØ 77 I=1,NR
      IF (X(I)-XRMAX) 77,77,121
   77 CØNTINUE
      NRP1=NR+1
      NRPNL=NR+NL
      DØ 102 I=NRP1,NRPNL
      IF (X(I)-XLMIN) 121,102,102
  102 CØNTINUE
      DØ 104 I=NRP1,NRPNL
      IF(X(I)-XLMAX) 104,104,121
  104 CØNTINUE
      NCC=NR+NL+1
      DØ 106 I=NCC,NMAX
      X(I)=1./X(I)
      IF (X(I)-XCMIN) 121,106,106
  106 CØNTINUE
      DØ 108 I=NCC,NMAX
      IF (X(I)-XCMAX) 108,108,121
  108 CØNTINUE
      WRITE (6,330)
      GØ TØ 113
  112 WRITE (6,340)
      GØ TØ 211
  121 WRITE (6,350)
  113 CALL PRTR(X,C,NR,NL,JMAX)
   67 IF(NTB)68,68,69
   68 CØNTINUE
      CALL CMPSEL(JMAX,X,X1,X2,NR,NL,C,NTB)
  505 DØ 500 J=NCC,JMAX
  500 X(J)=1./X(J)
      DØ 520 J=1,JMAX
```

```
       NLL
           NSL01
      SUM(J)=0.
      IJMAX=IMAX(J)
      LJMAX=LMAX(J)
      DØ 520 I=1,IJMAX
      T(I)=1.
      DØ 510 L=1,LJMAX
      NK=K(J,I,L)
  510 T(I)=T(I)*X(NK)
  520 SUM(J)=SUM(J)+T(I)
      CALL RØØTER (SUM,JMAX)
      CALL ARDEN
      NTB=1
   69 IF(MR)211,212,211
  212 STØP
      END
```



```
 NL1
      NSL02

INTEGER FUNCTIØN K(J,I,L)
DIMENSIØN NM(16),LMAX(15),ITD(5902)
CØMMØN NM,ITD,LMAX
IJ=(I-1)*LMAX(J)+L+NM(J)
K=ITD(IJ)
RETURN
END
```

```
      SUBRØUTINE EQPRT(JMAX,LMAX,IMAX,  NR,NL,NC
      INTEGER E
      DIMENSIØN A(60),B(60),C(3),D(10),IMAX(16),LMAX(16),E(3)
  900 FØRMAT(1H1,50X9HEQUATIØN I2)
  901 FØRMAT(///)
  902 FØRMAT(36X60A1)
  903 FØRMAT(15X35H**********  ERRØR DETECTED IN TERM
     F,I3,13H ØF EQUATIØN ,I3,12H  **********)
  904 FØRMAT (36XI3,17XI3,17XI3)
  905 FØRMAT(1H1)
  906 FØRMAT(1H116X63HTHE FØLLØWING IS THE LIST ØF EQUATIØNS SPECIFIED T
     FØ THE PRØGRAM)
  907 FØRMAT(17X34HTHE FØRMAT IS....  EQUATIØN NUMBER)
  911 FØRMAT(36X19HNUMBER ØF EACH TERM)
  908 FØRMAT(36X35HTERMS ØF EQUATIØNS (THREE PER LINE))
  909 FØRMAT(1H016X62HA CHECK IS MADE ØF THE UNITS ØF EACH TERM. IF THE
     XUNITS DIFFER)
  910 FØRMAT(17X40HIN AN EQUATIØN, AN ERRØR MESSAGE RESULTS)
      DATA BLANK,C,D/1H ,1HR,1HL,1HC,1H1,1H2,1H3,1H4,1H5,1H6,
     D1H7,1H8,1H9,1H0/
      DATA PLUS/1H+/
      WRITE(6,905)
      WRITE(6,906)
      WRITE(6,907)
      WRITE(6,908)
      WRITE(6,911)
      WRITE(6,909)
      WRITE(6,910)
      NENT=0
      DØ 200 J=1,JMAX
      WRITE(6,900) J
      NECNT=0
      ICAP=0
      LA=1
    5 KU=1
      KL=1
      L=LA
      DØ 10 I=1,60
      A(I)=BLANK
   10 B(I)=BLANK
      A(18)=PLUS
      A(38)=PLUS
      LB=L+2
      IF(L .GT. LMAX(J)) GØ TØ 200
      IF((L+2) .LE. LMAX(J)) GØ TØ 20
      A(38)=BLANK
      LB=L+1
      IF((L+1) .EQ. LMAX(J)) GØ TØ 20
      A(18)=BLANK
      LB=L
   20 CØNTINUE
      IJ=IMAX(J)
      DØ 150 L=LA,LB
      IND=0
      DØ 100 I=1,IJ
      NLE
          NSL03
     NCØ-K(J,L,I)
     IF(NCØ .GT. (NR+NL)) GØ TØ 50
     IC=1
     IF(NCØ.LE.NR) GØ TØ 410
     IC=2
     IND-IND+1
     NCØ=NCØ-NR
 410 CØNTINUE
     A(KU)=C(IC)
     IF(NCØ .GT. 9) GØ TØ 30
     A(KU+1)=D(NCØ)
     KU=KU+3
     GØ TØ 100
  30 A(KU+1)=D(1)
     NCØ=NCØ-10
     A(KU+2)=D(10)
     IF(NCØ .NE. 0) A(KU+2)=D(NCØ)
     KU=KU+4
     GØ TØ 100
  50 B(KL)=C(3)
     IND-IND-1
     NCØ=NCØ-NR-NL
     IF(NCØ.GT.9) GØ TØ 70
     B(KL+1)=D(NCØ)
     KL=KL+3
     GØ TØ 100
  70 B(KL)=D(1)
     NCØ=NCØ-10
     B(KL+2)=D(10)
     IF(NCØ .NE. 0) B(KL+1)=D(NCØ)
     KL=KL+4
 100 CØNTINUE
     IF(ICAP .EQ. 0) ICAP=IND
     IF(IND .EQ. ICAP) GØ TØ 400
     WRITE (6,901)
     WRITE(6,903) L,J
     NECNT=NECNT+1
     NENT=1
 400 CØNTINUE
     IF(LA+1-L) 130,140,130
 130 KU=20
     KL=20
     GØ TØ 150
 140 KU=40
     KL=40
 150 CØNTINUE
     LB=LB-LA+1
     DØ 160 L=1,3
 160 E(L)=L+LA-1
     WRITE(6,901)
     WRITE(6,902) A
     WRITE(6,902) B
     WRITE(6,904)(E(L),L=1,LB)
     LA=LA+3
     IF(NECNT .LT. 5) GØ TØ 5
 200 CØNTINUE
```

```
  NLL
      NSL03

WRITE(5,905)
IF(NENT.NE.0) CALL EXIT
RETURN
END
```

```
    NLL
        NSL04

      SUBRØUTINE READK
      DIMENSIØN IMAX(15),LMAX(15),NM(16),ITD(5902)
      CØMMØN NM,ITD,LMAX,IMAX,JMAX
      N=1
      DØ 10 J=1,JMAX
      MAX=IMAX(J)*LMAX(J)+N-1
      READ(5,120) (ITD(M),M=N,MAX)
      NM(J+1)=MAX
   10 N=MAX+1
      NM(1)=0
      RETURN
  120 FØRMAT(20I4)
  972 FØRMAT(12A6)
      END
```

```
NLL
     NSL05

      SUBRØUTINE CMPSEL(JMAX,X,X1,X2,NR,NL,C,NTB)
      DIMENSIØN X(15),X1(15),X2(15),N3(15),X3(15),C(15)
      CØMMØN /DATA/ TABLE(69)
      READ(5,920)NTB,N3
      IF(NTB) 510,2,510
    2 WRITE(6,900)
      I=NR+NL+1
      DØ 5 J=I,JMAX
    5 X(J)=X(J)*1.0E+12
      DØ 10 J=1,JMAX
   10 X3(J)=0.
      J=JMAX
   20 IPMX=N3(J)
      IF(IPMX)23,23,25
   23 IPMX=1
   25 IF(J-NR-NL)200,200,30
   30 TØL=.05
      DØ 80 I=1,IPMX
      IF(X(J)-2500.)40,40,60
   40 IF(X(J)-10.)50,50,50
C     TABLE 2
   50 NB=13
      NT=31
      GØ TØ 70
C     TABLE 1
   60 NB=1
      NT=12
   70 NTB=NT-NB+1
      U=X(J)
      CALL PTMCH(U,I,K,NB,NT,IPMX,CØMP)
      X1(I)=CØMP
      X(J)=U-CØMP
      IF(X(J)-TØL*X1(1))85,85,80
   80 CØNTINUE
      GØ TØ 90
   85 IPMX=I
   90 X(J)=0.
      DØ 100 I=1,IPMX
  100 X(J)=X1(I)+X(J)
      I=J-NR-NL
      U=X(J)
      WRITE(5,901) I,IPMX
      WRITE(5,902) (X1(K),K=1,IPMX)
      WRITE(5,903) I,U
      GØ TØ 500
  200 NPC=0
      IF(J-NR) 400,400,210
  210 TØL=.1
      KKK=1
      DØ 240 I=1,IPMX
      IF(X(J)-50.) 220,230,230
  220 IF(X(J)-10.)221,222,222
  221 IF(X(J)-1.)223,224,224
  222 X(J)=X(J)/10.
      NPC=NPC+1
      NLE
          NSL05
      GØ TØ 220
  223 NPC=NPC-1
      X(J)=X(J)*10.
      GØ TØ 221
  224 X(J)=X(J)*10.
      NTB=X(J)+.5
      X1(I)=NTB
      NPC=NPC-1
      X(J)=X(J)*(10.**NPC)
      X1(I)=X1(I)*(10.**NPC)
      JNR=J-NR
      IF(C(JNR)) 226,226,225
  225 X2(KKK)=X1(I)*C(JNR)
      X(JNR)=X(JNR)-X2(KKK)
      KKK=KKK+1
  226 CØNTINUE
      GØ TØ 250
C     TABLES 3 AND 4
  230 NB=32
      NT=-38
      U=X(J)
      CALL PTMCH(U,I,K,NB,NT,IPMX,CØMP)
      NTB=NT-NB+1
      NTB=NTB+K
      X2(KKK)=TABLE(NTB)
      X1(I)=CØMP
      X(JNR)=X(JNR)-X2(KKK)
      X(J)=X(J)-CØMP
      KKK=KKK+1
      IF(X(J)-TØL*X1(1))250,250,240
  240 CØNTINUE
      GØ TØ 255
  250 IPMX=I
  255 X(J)=0.
      DØ 260 I=1,IPMX
  260 X(J)=X(J)+X1(I)
      KKK=KKK-1
      X3(JNR)=0
      DØ 270 K=1,KKK
  270 X3(JNR)=X3(JNR)+X2(K)
      WRITE(5,904) JNR,IPMX
      WRITE(5,902) (X1(K),K=1,IPMX)
      U=X(J)
      WRITE(5,905) JNR,U
      U=X3(JNR)
      WRITE(5,906) JNR,U
      GØ TØ 500
  400 TØL=.01
      DØ 410 I=1,IPMX
      NB=46
      NT=69
      U=X(J)
      CALL PTMCH(U,I,K,NB,NT,IPMX,CØMP)
      X1(I)=CØMP
      U=X(J)-CØMP
      X(J)=U
  NLF
        NSL05

     IF(U-TØL*X1(1))420,420,410
 410 CØNTINUE
     GØ TØ 430
 420 IPMX=I
 430 X(J)=X3(J)
     DØ 440 I=1,IPMX
 440 X(J)=X(J)+X1(I)
     WRITE(5,907) J,IPMX
     WRITE(5,902) (X1(K),K=1,IPMX)
     WRITE(5,908)
     IF(J-NL)450,450,460
 450 U=X3(J)
     IF(U) 460,460,455
 455 WRITE(5,909) U
     WRITE(5,908)
 460 U=X(J)
     WRITE(5,910) J,U
 500 J=J-1
     IF(J)505,505,20
 505 I=NR+NL+1
     DØ 506 J=I,JMAX
 506 X(J)=X(J)*1.E-12
 510 WRITE(5,900)
 900 FØRMAT(1H1)
 901 FØRMAT(///24X15HFØR CAPACITØR C,I2,5H THE ,I2,
    F17H CØMPØNENT(S) ARE/)
 902 FØRMAT(39X E16.8)
 903 FØRMAT(/24X1HC,I2,9H IS THUS ,E16.8,
    F17H MICRØMICRØFARADS)
 904 FØRMAT(///24X14HFØR INDUCTØR L,I2,5H THE ,I2,
    F17H CØMPØNENT(S) ARE/)
 905 FØRMAT(/24X1HL,I2,9H IS THUS ,E16.8,13H HENRIES, AND)
 906 FØRMAT(24X23HTHE INDUCTIVE PART ØF R,I2,4H IS ,E15.8,5H ØHMS)
 907 FØRMAT(///24X14HFØR RESISTØR R,I2,5H THE ,I2,
    F17H CØMPØNENT(S) ARE /)
 908 FØRMAT(1H )
 909 FØRMAT(24X31HWITH AN INDUCTIVE RESISTANCE ØF,E16.8,
    F5H ØHMS)
 910 FØRMAT(24X1HR,I2,9H IS THUS ,E16.8,5H ØHMS)
 920 FØRMAT(I1,4X15I1)
     RETURN
     END
```

```
      SUBROUTINE PTMCH(U,I,K,NB,NT,IPMX,CØMP)
      CØMMØN /DATA/ TABLE(69)
      NP=0
      IF(NT)300,300,100
 100  IF(U-10.)110,110,200
 110  IF(U-1.)250,305,305
 200  U=U/10.
      NP=NP+1
      GØ TØ 100
 250  U=U*10.
      NP=NP-1
      GØ TØ 110
 300  NT=-NT
 305  DØ 310 K=NB,NT
      IF(TABLE(K)-U)310,310,320
 310  CØNTINUE
      K=NT
 320  IF(K-NB)360,360,330
 330  IF(I-IPMX)350,340,340
 340  IF(TABLE(K)+TABLE(K-1)-2.*U)360,360,350
 350  K=K-1
 360  CØMP=TABLE(K)*(10.**NP)
      U=U*(10.**NP)
      RETURN
      END
```

```
NLF
    NSL07

  BLØCK DATA
  CØMMØN/DATA/TABLE(69)
  DATA  TABLE/  1.0, 1.2, 1.5, 1.8, 2.2, 2.7, 3.3, 3.9,  4.7, 5.6,
D    6.8, 8.2, 1.0, 1.2, 1.5, 1.8, 2.0, 2.2, 2.5, 2.7,  3.0, 3.3,
A    3.6, 3.9, 4.7, 5.0, 5.1, 5.6, 6.8, 7.5, 8.2, 50.,  100.,200.,
T    400., 800., 1400., 2000., 500., 1000., 2000., 4000., 8000.,
A    4000., 8000., 1.0, 1.1, 1.21, 1.33, 1.47, 1.62, 1.78, 1.96,
D    2.15, 2.37, 2.61, 2.87, 3.16, 3.48, 3.83, 4.22, 4.64, 5.11,
A    5.62, 6.19, 6.81, 7.50, 8.25, 9.09/
  END
```

NLE
NSL08

```
      SUBRØUTINE FCØN(MAXNØS,NØS,KK,JMAX,NMAX,NR,LMAX,IMAX,F,PTØL,X,
     $                    C,XGUES,FX,IERR,FXLIM,DFX,X2,PHIP,DX)
      DIMENSIØN IMAX(15),F(15),FØRG(15),X(15),DELX(15),C(15),SUM(15),
     DX1(15),FX(15),DFX(15,15),PSUM(15,15),P(215,15),T(215),PHI(15),
     1PTØL(15),FF(15),XGJES(15),LMAX(15),X2(15),FXLIM(15)
      DIMENSIØN PHIP(15),DX(15)
  110 FØRMAT(/16H SINGULAR MATRIX/)
  180 FØRMAT (5H GRID=,I4,3X,4HNØS=,I4)
  320 FØRMAT (//29H CØMMENCING CØNSTANT APPRØACH//)
      WRITE (6,320)
      IERR=1
      DØ 1 I=1,NMAX
      X(I)=XGUES(I)
    1 X1(I)=X(I)
      IGRID=1
   33 KSWICH=0
      LSWTCH=0
      ANØS=NØS
      NS=0
   22 WRITE (6,180) IGRID,NØS
      KUTTA=1
   43 DØ 2 I=1,NMAX
    2 DELX(I)=0.
C     CALCULATE PARTIALS
      DØ 3 M=1,NMAX
      IF(M-NR)4,4,5
    5 C(M)=0.
    4 NRM=NR+M
      FX(M)=C(M)*X(NRM)+X(M)
    3 CØNTINUE
      DØ 10 J=1,JMAX
      SUM(J)=0.
      DØ 11 N=1,NMAX
   11 PSUM(J,N)=0.
      IJMAX=IMAX(J)
      DØ 12 I=1,IJMAX
      T(I)=1.
      LJMAX=LMAX(J)
      DØ 13 L=1,LJMAX
      NK=K(J,I,L)
   13 T(I)=T(I)*FX(NK)
      DØ 14 N=1,NMAX
      P(I,N)=0.
      DØ 15 L=1,LJMAX
      NK=K(J,I,L)
   15 P(I,N)=P(I,N)+T(I)*DFX(NK,N)/FX(NK)
C     CALCULATE TØTAL PARTIALS
   14 PSUM(J,N)=PSUM(J,N)+P(I,N)
   12 SUM(J)=SUM(J)+T(I)
      IF(KSWTCH-1)28,29,29
C     CALCULATE CØNSTANT TERM
   28 FØRG (J)=SUM(J)
   29 IF(LSWTCH-1)40,41,41
   40 GRID=IGRID
      IF (FØRG (J)) 50,51,51
    NLE
       NSL08

 50 FF(J)=F(J)**(GRID/ANØS)*(-(ABS(FØRG(J))+2.*F(J))**(1.-GRID/ANØS))
   1+2.*F(J)
    GØ TØ 41
 51 FF(J)=F(J)**(GRID/ANØS)*FØRG(J)**(1.-GRID/ANØS)
 41 IF(KUTTA-1) 10,411,10
411 PHIP(J)=-SUM(J)+FF(J)
 10 PHI(J)=PHIP(J)
    KSWTCH=1
    LSWICH=1
    CALL SIMEQ (PSUM,DELX,PHI,JMAX,IE)
    IF(IE .EQ. 1) GØ TØ 17
    CALL RUNKA(X,DELX,FXLIM,PTØL,X2,KUTTA,NMAX)
    GØ TØ(200,43,43,43),KUTTA
200 NS=NS+1
 16 DØ 18 I=1,NMAX
    IF(ABS(DELX(I))-PTØL(I)*ABS(X(I)))18,18,19
 18 CØNTINUE
    DØ 20 I=1,NMAX
 20 X(I)=X(I)+DELX(I)
 21 DØ 34 I=1,NMAX
    NRI=NR+I
 34 FX(I)=C(I)*X(NRI)+X(I)
    NS=0
    LSWTCH=0
    IGRID=IGRID+1
    IF (IGRID-NØS-1) 42,99,99
 99 IERR=IERR+1
    RETURN
 42 DØ 30 I=1,NMAX
    DX(I)=X(I)-X1(I)
    X1(I)=X(I)
 30 X(I)=X(I)+DX(I)
    GØ TØ 22
 19 LSWTCH=LSWTCH+1
    IF(NS-KK)24,25,25
 24 DØ 26 I=1,NMAX
 26 X(I)=X(I)+DELX(I)
    GØ TØ 43
 17 WRITE (6,110)
 25 NØS=2*(NØS+1-IGRID)
    IF(NØS-MAXNØS)31,23,23
 31 DØ 32 I=1,NMAX
    DX(I)=DX(I)*.5
 32 X(I)=X1(I)+DX(I)
    IGRID=1
    GØ TØ 33
 23 DØ 35 N=1,NMAX
    NRM=NR+N
 35 FX(N)=C(N)*X(NRM)+X(N)
 36 RETURN
    END
 NLF
     NSL09

      SUBRØUTINE SIMEQ (A,X,B,N,IERR)
      SØLUTIØN ØF SIMULTANEØUS LINEAR EQUATIØNS
      DIMENSIØN A(15,15),X(15),B(15),IND(15)
      DØ 1 I=1,N
    1 IND(I)=I
      DØ 15 K=1,N
      SEARCH ARRAY FØR LARGEST VALUE
      IX=K
      JX=K
      DØ 3 I=K,N
      DØ 3 J=K,N
      IF(ABS(A(I,J))-ABS(A(IX,JX))) 3,3,2
    2 IX=I
      JX=J
    3 CØNTINUE
      IF (A(IX,JX)) 5,4,5
    4 IERR=1
      RETURN
    5 IF (IX-K) 8,8,6
C     EXCHANGE RØWS
    6 DØ 7 J=K,N
      TEMP=A(IX,J)
      A(IX,J)=A(K,J)
    7 A(K,J)=TEMP
      TEMP=B(IX)
      B(IX)=B(K)
      B(K)=TEMP
    8 IF (JX-K) 11,11,9
C     EXCHANGE CØLUMNS
    9 DØ 10 I=  1,N
      TEMP=A(I,JX)
      A(I,JX)=A(I,K)
   10 A(I,K)=TEMP
      INDEX=IND(JX)
      IND(JX)=IND(K)
      IND(K)=INDEX
   11 PIVØT=A(K,K)
      DØ 12 J=K,N
   12 A(K,J)=A(K,J)/PIVØT
      B(K)=B(K)/PIVØT
      DØ 15 I=1,N
      IF (I-K) 13,15,13
   13 TEMP=A(I,K)
      DØ 14 J=K,N
   14 A(I,J)=A(I,J)-A(K,J)*TEMP
      B(I)=B(I)-B(K)*TEMP
   15 CØNTINUE
      DØ 16 I=1,N
      INDEX=IND(I)
   16 X(INDEX)=B(I)
      IERR=0
      RETURN
      END
```

```
NLE
    NSL10

      SUBRØUTINE RUNKA(X,DELX,X1,PTØL,X2,       KUTTA,NMAX)
      DIMENSIØN X(15),DELX(15),X1(15),PTØL(15),X2(15)
  400 GØ TØ (500,520,540,560),KUTTA
  500 DØ 505 I=1,NMAX
      X1(I)=X(I)
      X(I)=X1(I)+DELX(I)/2.
  505 X2(I)=DELX(I)
      KUTTA=2
      GØ TØ 43
  520 DØ 525 I=1,NMAX
      X(I)=X1(I)+DELX(I)/2.
  525 X2(I)=X2(I)+2.*DELX(I)
      KUTTA=3
      GØ TØ 43
  540 DØ 545 I=1,NMAX
      X(I)=X1(I)+DELX(I)
  545 X2(I)=X2(I)+2.*DELX(I)
      KUTTA=4
      GØ TØ 43
  560 DØ 565 I=1,NMAX
      DELX(I)=(X2(I)+DELX(I))/6.
  565 X(I)=X1(I)
      KUTTA=1
   43 RETURN
      END
```

NLF
NSL11

```
      SUBRØUTINE ESTIM (NMAX,JMAX,NR,NL,NØR,TX,IMAX,LMAX,F,C,FXØRIG,
     S                 FXLIM,FX)
      DIMENSIØN FXØRIG(15),FXLIM(15),X1(15),XP(15),C(15),FX(15),SUM(15),
     DF(15),IMAX(15),LMAX(15),T(215),PHI(15),CHX(15),TØL(15)
  110 FØRMAT (/3X3HLX=,I4)
      NCC=NR+NL+1
      DØ 29 L=NCC,NMAX
      FXLIM(L)=1./FXLIM(L)
      FXØRIG(L)=1./FXØRIG(L)
      TEMP=FXØRIG(L)
      FXØRIG(L)=FXLIM(L)
   29 FXLIM(L)=TEMP
      IF (ABS(FXØRIG(1)-FXLIM(1)).LT.FXØRIG(1)/1000.) GØ TØ 200
      JJ=1
      LX=0
   20 IF (JJ-1) 16,15,16
   15 DØ 3 J=2,NMAX
    3 FX(J)=EXP((ALØG(FXLIM(J)*FXØRIG(J)))/2.0)
   16 DØ 1 JK=1,NMAX
      DØ 2 I=1,NØR
      AP=I-1
      XNØS=NØR
      FX(JK)=FXØRIG(JK)*EXP(AP*ALØG(FXLIM(JK)/FXØRIG(JK))/(XNØS-1.0))
      DØ 8 M=1,JMAX
      SUM(M)=-F(M)
      IMMAX=IMAX(M)
      DØ 9 J=1,IMMAX
      T(J)=1.
      LMMAX=LMAX(M)
      DØ 10 N=1,LMMAX
      NK=K(M,J,N)
   10 T(J)=T(J)*FX(NK)
    9 SUM(M)=SUM(M)+T(J)
    8 PHI(M)=-SUM(M)
      APHI=0.
      DØ 11 N=1,JMAX
   11 APHI=APHI+ABS(PHI(N))
      IF (I-1) 22,12,22
   22 IF (APHI-APHI1) 12,12,13
   12 APHI1=APHI
      DØ 19 N=1,NMAX
   19 X1(N)=FX(N)
    2 CØNTINUE
      GØ TØ 26
   13 DØ 28 N=1,NMAX
   28 FX(N)=X1(N)
   25 IF (JJ-1) 18,17,18
   18 CHX(JK)=ABS(ALØG(XP(JK)/X1(JK)))
      TØL(JK)=TX*ALØG(FXLIM(JK)/FXØRIG(JK))/(XNØS-1.)
      IF(CHX(JK)-TØL(JK)) 21,21,23
   21 LX=LX+1
      IF (LX-NMAX) 30,24,24
   23 LX=0
   30 WRITE (6,110) LX
   17 DØ 14 N=1,NMAX
```

```
      NLL
          NSL11

  14 XP(N)=FX(N)
   1 CØNTINUE
     JJ=JJ+1
     GØ TØ 20
  24 WRITE (6,110) LX
     DØ 25 I=1,NMAX
  25 FX(I)=XP(I)
     RETURN
 200 DØ 201 I=1,NMAX
 201 FX(I)=FXØRIG(I)
     RETURN
     END
```

```
NLL
     NSL12

      SUBROUTINE PRTR(X,C,NR,NL,JMAX)
      DIMENSIØN X(15),C(15)
      NC=JMAX-NR-NL
      DØ 10 I=1,NR
      NRM=NR+I
   10 X(I)=X(I)+C(I)*X(NRM)
      DØ 90 I=1,JMAX
      IJ=0
      WRITE(6,904)
      IF(I-NR) 30,30,40
   30 WRITE(6,901) I,X(I)
      IJ=1
   40 IF(I-NL) 50,50,60
   50 NRM=NR+I
      WRITE(6,902) I,X(NRM)
      IJ=1
   60 IF(I-NC) 70,70,80
   70 NRM=NR+NL+1
      WRITE(6,903) I,X(NRM)
      IJ=1
   80 IF(IJ) 90,100,90
   90 CØNTINUE
  100 RETURN
  901 FØRMAT(1H+3X2HR(,I2,2H)=,E16.8,2X4HØHMS)
  902 FØRMAT(1H+37X2HL(,I2,2H)=,E16.8,2X7HHENRIES)
  903 FØRMAT(1H+73X2HC(,I2,2H)=,E16.8,2X6HFARADS)
  904 FØRMAT(1H )
      END
      SUBRØUTINE RØØTER (X,JMAX)
      DIMENSIØN X(15),N(80),RE(16),RC(16),CØEF(41)
      DIMENSIØN CØNV(20)
      DØUBLE PRECISIØN CØEF,RTR(40),RTI(40),A(21),B(21),C(21),D(21),E(21
     1)
      CØMMØN/PLØTER/RE,RC,NNUM,NDEN,RØØT(80),RØØTI(80)
      INTEGER TWENTY
      INTEGER BLANK
      INTEGER DED
      DATA TWENTY,DED/2000000000000,1HD/
      DATA BLANK/1H /
      READ(5,900) N
      READ(5,901) NMAX
      NDEN=0
      NNUM=0
      DØ 10 J=1,16
      RE(J)=0.
   10 RC(J)=0.
      K=0
      DØ 60 J=1,NMAX
   20 K=K+1
      IF(K.EQ.80) STØP
      IF(N(K).EQ.BLANK) GØ TØ 20
      KA=K
      NA=0
   30 K=K+1
      IF(N(K).EQ.BLANK) GØ TØ 30
   40 N(K)=N(K)/1073741824
      NA=10*NA+N(K)
      K=K+1
      IF(N(K).LT.0) GØ TØ 45
      IF(N(K).LT.TWENTY) GØ TØ 40
   45 K=K-1
      IF(N(KA).EQ.DED) GØ TØ 50
      RE(NA+1)=X(J)
      IF(NA.GT.NNUM) NNUM=NA
      GØ TØ 60
   50 RC(NA+1)=X(J)
      IF(NA.GT.NDEN) NDEN=NA
   60 CØNTINUE
      ACC=1.E-12
      DØ 70 I=1,16
      CØEF(I)=RE(I)
   70  RE(I)=0.
      I1=NNUM+1
      DØ 75 I=1,I1
      I2=I1-I
   75 RE(I2+1)=CØEF(I)
      DØ 77 I=1,16
   77 CØEF(I)=RE(I)
      DØ 80 I=1,80
      RØØT(I)=0.
   80  RØØTI(I)=0.
      CALL RTPØLY(NNUM,CØEF,40,ACC,RTR,RTI,CØNV,A,B,C,D,E)
      DØ 90 I=1,NNUM
```

NLE
NSL13

```
    IK=I*2
    RØØT(IK-1)=RTR(I)
 90 RØØT(IK)=RTI(I)
    DØ 95 I=1,16
    CØEF(I)=RC(I)
 95 RC(I)=0.
    I1=NDEN+1
    DØ 97 I=1,I1
    I2=I1-I
 97 RC(I2+1)=CØEF(I)
    DØ 98 I=1,16
 98 CØEF(I)=RC(I)
    CALL RTPØLY(NDEN,CØEF,40,ACC,RTR,RTI,CØNV,A,B,C,D,E)
    DØ 100 I=1,NDEN
    IK=I*2
    RØØTI(IK-1)=RTR(I)
100 RØØTI(IK)=RTI(I)
    RETURN
900 FØRMAT(8 A1)
901 FØRMAT(I2)
    END
```

```
   NLF
       NSL14

      SUBRØUTINE ARDEN
C     FREQUENCY RESPØNSE PRØGRAM
      DIMENSIØN X(150),Y1(150),Y2(150),Y3(150),XLAB(12)
      DIMENSIØN FS(40,3),GS(40,3),FØ(40,2),GØ(40,2),
     1RØØT(80),           FM(40),FP(40),GM(40),GP(40),RØØT1(80),
     2BCDFRQ(12),BCDAMP(12),BCDPHZ(12),XFREQ(150),YAMP(150),YPHZ(150)
      DIMENSIØN BCDMAG(12),YMAG(150),Y1MAP(150),KRØSS(150),KRØS(150)
      DIMENSIØN XCPS(150),BCDCPS(12), F360(40), G360(40)
      DIMENSIØN SATF(40),SATG(40)
      DIMENSIØN RE(16),RC(16)
      CØMMØN/PLØTER/RE,RC,NN,ND,RØØT,RØØT1
      REAL MSQ
      DATA BCDFRQ(1)/72H                              FREQUENCY IN RADIANS/SECØN
     XD                     /
      DATA BCDAMP(1)/72H                                AMPLITUDE IN DECIBELS
     X                      /
      DATA BCDPHZ(1)/72H                               PHASE ANGLE IN DEGREES
     X                      /
      DATA BCDMAG(1)/72H                                  AMPLITUDE- GAIN
     X                      /
      DATA BCDCPS(1)/72H                              FREQUENCY IN CYCLES/SECØN
     XD                     /
      IF(NN*ND .EQ. 0) RETURN
      READ(5,1) ICPS,NSTPS,FRQMIN,FRQMAX,DBMIN,
     RDBMAX,AMPMIN,AMPMAX
    1 FØRMAT(I1,4X,I5,6F10.5)
      IF((FRQMAX-FRQMIN).GT..0001)GØ TØ 101
      FRQMAX=25.
      FRQMIN=.001
  101 WI=FRQMIN*6.2832
      WF=FRQMAX*6.2832
      IF((DBMAX-DBMIN).GT..0001)GØ TØ 201
      DBMIN=-60.
      DBMAX=40.
  201 IF((AMPMAX-AMPMIN).GT..0001)GØ TØ 301
      AMPMIN=.001
      AMPMAX=100.
  301 ICAM=0
      IF(NSTPS.EQ.0)NSTPS=25.
      NSTEPS=ALØG10(FRQMAX/FRQMIN)
      NSTEPS=NSTEPS*NSTPS
      KDUM=0
      KØLD1=0
      KØLD2=0
  100 LINES=50
      KTR=40
      DØ 200 I=1,40
      FS(I,1)=0.
      GS(I,1)=0.
      FS(I,2)=0.
      GS(I,2)=0.
      FS(I,3)=1.
  200 GS(I,3)=1.
      IPØINT=0
      WRITE(6,1540)
    NLF
         NSL14

      KPLØT=1
      IF(ICAM) 6,3,6
    3 ICAM=ICAM+KPLØT
      IF(ICAM) 5,6,5
    5 CALL CAMRAV(935)
    6 CØNTINUE
  260 N=NN
      WRITE(5,270) NN
  270 FØRMAT(/33X,26HTHE NUMERATØR IS ØF ØRDER ,I2,
     142H. THE PØLYNØMIAL IN DESCENDING ØRDER BELØW//)
      FACTF=RE(1)
      L=NN+1
      WRITE(6,280) (RE(I),I=1,L)
  280 FØRMAT(34X,4E16.8)
      WRITE(5,310)
  310 FØRMAT(/33X,14HTHE RØØTS ARE-)
      WRITE(6,320)
  320 FØRMAT(34X,9HREAL PART,8X,10HIMAG. PART,10X,9HREAL PART,8X,
     110HIMAG. PART)
      N=NN*2
      WRITE(5,340)(RØØT(I),I=1,N)
  340 FØRMAT(33X,E12.5,5X,E12.5,8X,E12.5,5X,E12.5)
  370 I=1
  375 J=I+I
      F360(I)=0.
      K=J+J-3
      IF(N-J+1) 900,400,380
  380 IF(RØØT(K+1)) 382,381,382
  381 FS(I,3)=RØØT(K)*RØØT(K+2)
      FS(I,2)=-RØØT(K)-RØØT(K+2)
  30  GØ TØ 383
  382 FS(I,3)=RØØT(K)*RØØT(K)+RØØT(K+1)*RØØT(K+1)
      FS(I,2)=-2.*RØØT(K)
  383 FS(I,1)=1.
      I=I+1
      GØ TØ 375
  400 FS(I,1)=0.
      FS(I,2)=1.
      FS(I,3)=-RØØT(K)
  900 CØNTINUE
  910 N=ND
      WRITE(6,920) ND
  920 FØRMAT(/33X,28HTHE DENØMINATØR IS ØF ØRDER ,I2,
     142H. THE PØLYNØMIAL IN DESCENDING ØRDER BELØW//)
      FACTG=RC(1)
      L=ND+1
      WRITE(6,280) (RC(I),I=1,L)
      WRITE(6,310)
      WRITE(6,320)
      N=ND*2
      DØ 935 I=1,N
  935 RØØT(I)=RØØT1(I)
      WRITE(6,340)(RØØT(I),I=1,N)
  970 I=1
  980 J=I+I
      G360(I)=0.
```

```
       K=J+J-3
       IF(N-J+1) 1200,1000,990
  990  IF(RØØT(K+1)) 992,991,992
  991  GS(I,3)=RØØT(K)*RØØT(K+2)
       GS(I,2)=-RØØT(K)-RØØT(K+2)
  180  GØ TØ 993
  992  GS(I,3)=RØØT(K)*RØØT(K)+RØØT(K+1)*RØØT(K+1)
       GS(I,2)=-2.*RØØT(K)
  993  GS(I,1)=1.
       I=I+1
       GØ TØ 980
 1000  GS(I,1)=0.
       GS(I,2)=1.
       GS(I,3)=-RØØT(K)
 1200  WRITE(6,1201)
 1201  FØRMAT(1X)
C      PHASE CHECKER LØØP 3200 THRU 3234
       DØ 3234 I=1,KTR
       IF (FS(I,1)) 3202,3205,3202
 3202  IF (FS(I,3)) 3210,3203,3208
 3203  FS(I,3)=ABS(FS(I,3))
       IF(FS(I,2)) 3213,3204,3207
 3204  FS(I,2)=ABS(FS(I,2))
       GØ TØ 3207
 3205  IF(FS(I,3)) 3207,3206,3207
 3206  FS(I,3)=ABS(FS(I,3))
 3207  SATF(I)=+1.0
       F360(I)=0.0
       GØ TØ 3214
 3208  IF(FS(I,2)) 3213,3209,3207
 3209  FS(I,2)=ABS(FS(I,2))
       SATF(I)=-1.0
       F360(I)=+1.0
       GØ TØ 3214
 3210  IF (FS(I,2)) 3213,3204,3207
 3213  SATF(I)=+1.0
       F360(I)=+1.0
 3214  CØNTINUE
       IF(GS(I,1))3222,3225,3222
 3222  IF(GS(I,3))3230,3223,3228
 3223  GS(I,3)=ABS(GS(I,3))
       IF(GS(I,2))3233,3224,3227
 3224  GS(I,2)=ABS(GS(I,2))
       GØ TØ 3227
 3225  IF(GS(I,3)) 3227,3226,3227
 3226  GS(I,3)=ABS(GS(I,3))
 3227  SATG(I)=+1.0
       G360(I)=0.0
       GØ TØ 3234
 3228  IF(GS(I,2))3233,3229,3227
 3229  GS(I,2)=ABS(GS(I,2))
       SATG(I)=-1.0
       G360(I)=+1.0
       GØ TØ 3234
 3230  IF(GS(I,2))3233,3224,3227
 3233  SATG(I)=+1.0
```

```
      G360(I)=+1.0
 3234 CØNTINUE
      STEPS=NSTEPS
      IF(NSTEPS) 1230,2000,1270
 1230 WRITE(6,1240)
 1240 FØRMAT(///30X,27HNUMBER ØF STEPS IS NEGATIVE)
 1245 CALL GETØUT(ICAM)
 1250 IF(NSTEPS.NE.1) GØ TØ 1300
 1260 W=WI
      ASSIGN 1200 TØ IFFY
      GØ TØ 1500
 1270 IF(WI) 1280,1280,1250
 1280 WRITE(6,1290)
 1290 FØRMAT(///30X,44HINITIAL ØMEGA IS EQUAL TØ ØR LESS THAN ZERØ.)
      GØ TØ 1245
 1300 IF(WF-WI) 1310,1310,1321
 1310 WRITE(6,1320)
 1320 FØRMAT(///30X,42HFINAL ØMEGA EQUAL TØ ØR LESS THAN INITIAL.)
      GØ TØ 1245
 1321 NUMPTS=STEPS+1.
 1330 XX=ALØG(WI)
      YY=ALØG(WF)
      ZZ=(YY-XX)/STEPS
      W=WI
 1335 ASSIGN 1340 TØ IFFY
      GØ TØ 1500
 1340 STEPS=STEPS-1.
      IF(STEPS) 1230,1360,1350
 1350 XX=XX+ZZ
      W=EXP(XX)
      GØ TØ 1335
 1360 W=WF
      ASSIGN 2000 TØ IFFY
 1500 IF(LINES-50) 1560,1520,1520
 1520 ASSIGN 1560 TØ JIFFY
 1530 WRITE(6,1540)
 1540 FØRMAT(1H1)
      WRITE(6,1550)
 1550 FØRMAT(/32X,41HØMEGA-RAD/SEC F-CYCLES/SEC   AMPLITUDE     ,
     124H 20LØG AMP      PHASE-DEG//)
      LINES=0
      GØ TØ JIFFY,(1560,1720)
 1560 WSQ=W*W
      ANSMAG=FACTF/FACTG
      ANSPHZ=0.
      DØ 1650 I=1,KTR
      FØ(I,1)=FS(I,3)-FS(I,1)*WSQ
      GØ(I,1)=GS(I,3)-GS(I,1)*WSQ
      FØ(I,2)=FS(I,2)*W
 1570 GØ(I,2)=GS(I,2)*W
      MSQ=FØ(I,1)*FØ(I,1)+FØ(I,2)*FØ(I,2)
      IF(MSQ-1.) 1580,1590,1580
 1580 FM(I)=SQRT(MSQ)
      GØ TØ 1600
 1590 FM(I)=MSQ
 1600 MSQ=GØ(I,1)*GØ(I,1)+GØ(I,2)*GØ(I,2)
```

```
      IF(MSQ-1.) 1610,1620,1610
 1610 GM(I)=SQRT(MSQ)
      GØ TØ 1630
 1620 GM(I)=MSQ
 1630 FP(I)=SATF(I)*ATAN2(FØ(I,2),FØ(I,1))+F360(I)*6.2831853
      GP(I)=SATG(I)*ATAN2(GØ(I,2),GØ(I,1))+G360(I)*6.2831853
      ANSMAG=ANSMAG*FM(I)/GM(I)
 1650 ANSPHZ=ANSPHZ+FP(I)-GP(I)
      FCPS=W/6.2831853
      IF(ANSMAG) 1660,1670,1670
 1660 ABSANS=-ANSMAG
      GØ TØ 1680
 1670 ABSANS=ANSMAG
 1680 EXPMAG=20.*ALØG10(ABSANS)
      ANSPHZ=57.2957795*ANSPHZ
      WRITE(5,1700) W,FCPS,ANSMAG,EXPMAG,ANSPHZ
 1700 FØRMAT(27X,5F14.5)
      IPØINT=IPØINT+1
      KRØS(IPØINT)=0
      KNEW1=0
      KNEW2=0
   10 IF(ANSPHZ.LT.0.) GØ TØ 20
      ANSPHZ=ANSPHZ-360.
      KNEW1=KNEW1+1
      GØ TØ 10
   20 IF(ANSPHZ.GT.-360.) GØ TØ 30
      ANSPHZ=ANSPHZ+360.
      KNEW2=KNEW2+1
      GØ TØ 20
   30 IF(KNEW1.NE.KØLD1.ØR.KNEW2.NE.KØLD2)KRØS(IPØINT)=1
      KØLD1=KNEW1
      KØLD2=KNEW2
      YPHZ(IPØINT)=ANSPHZ
      XFREQ(IPØINT)=W
      XCPS(IPØINT)=FCPS
      YAMP(IPØINT)=EXPMAG
      YMAG(IPØINT)=ABSANS
      LINES=LINES+1
      IF(LINES-50) 1720,1710,1710
 1710 ASSIGN 1720 TØ JIFFY
      GØ TØ 1530
 1720 GØ TØ IFFY,(1200,1340,2000)
 2000 IF(KPLØT.EQ.0) GØ TØ 100
      NPØINT=IPØINT
 2005 CØNTINUE
 2018 IF(ICPS.NE.0) GØ TØ 2100
      FRQMIN=WI
      FRQMAX=WF
      I=0
      J=0
 2020 I=I+1
      IF(I.GT.NPØINT) GØ TØ 2200
      IF(XFREQ(I).LT.FRQMIN) GØ TØ 2020
      IF(XFREQ(I).GT.FRQMAX) GØ TØ 2200
      J=J+1
      X(J)=XFREQ(I)
      Y1(J)=YPHZ(I)
      Y1MAP(J)-45.+.25*Y1(J)
      KRØSS(J)=KRØS(I)
      Y2(J)=YAMP(I)
      IF(Y2(J).LT.DBMIN) Y2(J)=DBMIN
      IF(Y2(J).GT.DBMAX) Y2(J)=DBMAX
      Y3(J)=YMAG(I)
      IF(Y3(J).LT.AMPMIN) Y3(J)=AMPMIN
      IF(Y3(J).GT.AMPMAX) Y3(J)=AMPMAX
      GØ TØ 2020
 2100 I=0
      J=0
 2120 I=I+1
      IF(I.GT.NPØINT) GØ TØ 2200
      IF(XCPS(I).LT.FRQMIN) GØ TØ 2120
      IF(XCPS(I).GT.FRQMAX) GØ TØ 2200
      J=J+1
      X(J)=XCPS(I)
      Y1(J)=YPHZ(I)
      Y1MAP(J)=45.+.25*Y1(J)
      KRØSS(J)=KRØS(I)
      Y2(J)=YAMP(I)
      IF(Y2(J).LT.DBMIN) Y2(J)=DBMIN
      IF(Y2(J).GT.DBMAX) Y2(J)=DBMAX
      Y3(J)=YMAG(I)
      IF(Y3(J).LT.AMPMIN) Y3(J)=AMPMIN
      IF(Y3(J).GT.AMPMAX) Y3(J)=AMPMAX
      GØ TØ 2120
 2200 NPNTS=J
      IF(ICPS.NE.0) GØ TØ 2220
      DØ 2210 I=1,12
 2210 XLAB(I)=BCDFRQ(I)
      GØ TØ 2240
 2220 DØ 2230 I=1,12
 2230 XLAB(I)=BCDCPS(I)
 2240 CALL QJKLG1(-1,FRQMIN,FRQMAX,-360.,0.,42,XLAB,BCDPHZ,NPNTS,X,Y1,
     X KRØSS,1,1,0,1.,10.)
      CALL QJKLG1(-1,FRQMIN,FRQMAX,DBMIN,DBMAX,42,XLAB,BCDAMP,NPNTS,X,Y2
     X ,KRØSS,0,1,0,1.,10.)
      CALL QJKLG1(-1,FRQMIN,FRQMAX,AMPMIN,AMPMAX,42,XLAB,BCDMAG,NPNTS,X,
     X Y3,KRØSS,0,1,1,1.,1.)
 3999 CALL CLEAN
      RETURN
      END
```

```
NLE
    NSL15

SUBRØUTINE GETØUT(ICAM)
IF(ICAM.NE.0) CALL CLEAN
CALL EXIT
RETURN
END
```

NLL
    NSL16

```
      SUBRØUTINE QJKLG1(L,XL,XR ,YB,YT,ISYM,BCDX,BCDY,NP,X,Y,NØLINE,
     X IBREAK,MX,MY,DX,DY)
C     PLØT LØG-LØG ØR SEMI-LØG
      DIMENSIØN X(500),Y(500),BCDX(12),BCDY(12),NØLINE(500)
      IF(L)20,200,100
 20   L1=1
      GØ TØ 110
 100  L1=2
  110 NCX=72
      NCY=72
      DCX=10.
      DCY=10.
      INCRY=-14
  140 CALL MARGIN(L,ICY)
      IX=524-4*NCX
      IY=ICY+7*NCY
      GØ TØ (142,144,146),L
 142  IY1=0
      GØ TØ 150
 144  IY1=ICY-253
      GØ TØ 150
 146  IY1=ICY-169
  150 NX=9
      NY=9
      CALL SMXYV(MX,MY)
      DC=10.
      NYY=4
      IF(MY)11,10,11
   10 CALL DXDYV(2,YB,YT,DY,M,J,NYY,DC,IERR)
   11 IF (MX) 12,13,12
   13 CALL DXDYV(1,XL,XR,DX,N,I,NXX,DC,IERR)
   12 CALL GRID1V(L1,XL,XR,YB,YT,DX,DY,N,M,I,J,NX,NY)
      CALL PRINTV(NCX,BCDX, IX,IY1)
      CALL  APRNTV(0,INCRY,NCY,BCDY,0, IY)
  200 DØ 270 K=1,NP
      NX1=NXV(X(K))
      NY1-NYV(Y(K))
      IF(K.EQ.1)GØ TØ 220
      IF(IBREAK.EQ.0)GØ TØ 210
      IF(NØLINE(K).NE.0)GØ TØ 215
  210 CALL LINEV(NX0,NY0,NX1,NY1)
  215 CALL PLØTV(NX1,NY1,ISYM)
 220  CØNTINUE
      NX0=NX1
      NY0=NY1
  270 CØNTINUE
      RETURN
      END
```

APPENDIX B

SUBROUTINES

A description of the operation of the MAIN program is provided in subsection 3.1. The discussion which follows in part B-1 provides a brief description of each of the subroutines used in conjunction with the MAIN program. For convenience, these descriptions are arranged in alphabetical order as opposed to sequential order of use. In part B-2, a discussion of internal routines is provided.

B-1. DESCRIPTION OF SUBROUTINES

ARDEN

This subroutine uses the complex roots obtained by ROOTER to compute the magnitude and phase angle of the complex quantity $N(j\omega)/D(j\omega)$ for the values of the frequency specified to it.

BLOCK DATA

The block data routine contains the necessary decade tables for CMPSEL and PTMCH.

CMPSEL

This subroutine utilizes the technique presented in subsection 2.4 to select the approximate components corresponding to each root. The relationship between inductance resistance and inductance is taken into consideration. External input to the routine consists of a control digit, specifying whether or not to select components and plot the transfer function, and a maximum number of components allowed for each variable.

EQPRT (Equation Printer)

This subroutine prints out the set of equations specified to the program in terms of resistance, inductance, and capacitance. It compares the units of each term in a particular equation to the units of the first term of the equation, and gives an error message if the units do not agree. If more than five error messages occur, the subroutine prints the following equations, and then stops execution. A term number is printed out under each term for easy reference.

ESTIM (Selection of Initial Estimates)

This subroutine is a technique for obtaining a set of initial estimates for the variables. The range of interest and the number of increments to be taken for each variable are inputs to the subroutine. The variables are first given the value of the logarithmic mean of their respective ranges. Each variable is then varied in turn over its range, according to its number of increments. The variable is then given the value which causes the equations to be most nearly satisfied, and the next variable processed. The process is repeated until an increment or decrement in any variable will cause the equations to be less nearly satisfied. The set of variables is then returned as the initial set of estimates.

FCON (Constant Approach)

This subroutine applies the Freudenstein-Roth Method in conjunction with Kizner's method to the set of equations and unknowns. It differs from the main program in that it increments (or decrements) the constant term associated with each equation, rather than a coefficient of one of the terms. Experience has shown this method to be superior to the coefficient approach.

GETOUT

If a severe error results in ARDEN, this subroutine is called to turn off the cameras (turned on for plotting in ARDEN) and to stop execution of the remainder of the program.

INTEGER FUNCTION K

The method used of storing equations involves storing the subscripts of the unknowns in positions that are a function of equation number, term number, and factor number. The rather standard method of storing the unknown's subscripts is by storing them in a variable with three dimensions. However, unless the equations all have the same number of terms and factors per term, this practice can lead to much unused (and needed) storage. A method was found to store these subscripts sequentially, using the previously used dimension variables to define a single subscript in the sequential storage. The function K is used to determine this subscript, and thus the desired unknown. In the case of the 15 equations and unknowns presented in this report, it reduced required storage for the equations from 24,000 to 6,000 words.

PRTR

This subroutine prints the roots obtained by the main program and FCON. This print routine was made into a subroutine that could be "overlaid" for additional storage.

PTMCH

This subroutine does the actual component matching for CMPSEL.

QUKLQ1

This subroutine plots the results from ARDEN using the SC4020 plotter on both microfilm and paper. The various options available allow the specification of the frequency range and the upper and lower limits of the amplitude plots. Upon exit from this program, control returns to the main program for further attempts at obtaining solutions to the set of equations and unknowns.

READK

This subroutine reads in the subscripts of the unknowns for each term of each equation.

ROOTER

As the equations may be input in any order, a method is necessary to specify to which powers of s in N(s) or D(s) (the numerator and denominator polynomials of the transfer function) the various constant terms belong, in order that the root plotting subroutine may have the correct transfer function. ROOTER does this, reading in the specifications off one card. ROOTER also obtains the complex roots of N(s) and D(s) necessary for the root plotting subroutine.

RUNKA

The Runge-Kutta integration necessary for Kizner's method is performed in RUNKA. The subroutine is called four times for each integration.

SIMEQ (Simultaneous Equation Solver)

This routine employs the Gauss-Jordan technique of reducing a matrix by the pivotal method. The matrix is the Jacobian matrix of the set of equations to be solved. The values of the unknowns used correspond to the current estimates. The largest element of the matrix is sought and, should this largest element be trivial, an error message is returned and printed out.

B-2 INTERNAL ROUTINES

The program makes use of several subroutines available on the 7094 library tape. These decks include POLRT, LOGB2, and the SC 4020 plot routines. These subroutines are included in the overlay structure of the program.

APPENDIX C

OVERLAY FEATURE

The complete deck, dimensioned to be able to handle a set of fifteen equations and fifteen unknowns, uses approximately 41,000 words. The IBM 7094 at the MSFC facility can store only 33,000 words. This obstacle was overcome by use of the overlay system, which stores the subroutines on a systems tape. The subroutines are then loaded into memory only when needed, and thus several subroutines can share the same storage locations. The major restriction to this system is that one subroutine cannot call another subroutine that would cause the first to be overlaid. The system is used by specifying with a $ORIGIN card the mnemonic or absolute storage location that the first command of the following subroutine is to take. All following subroutines and internal storage areas, such as input/output buffers, are loaded sequentially until the next $ORIGIN card. A schematic of the overlay system used for this deck is shown on Figure C-1. The two mnemonics used are ALPHA and BETA.

It is suggested that the user make no attempt to rearrange the sequence of the deck, to avoid the accidental overlaying of a portion of some subroutine.

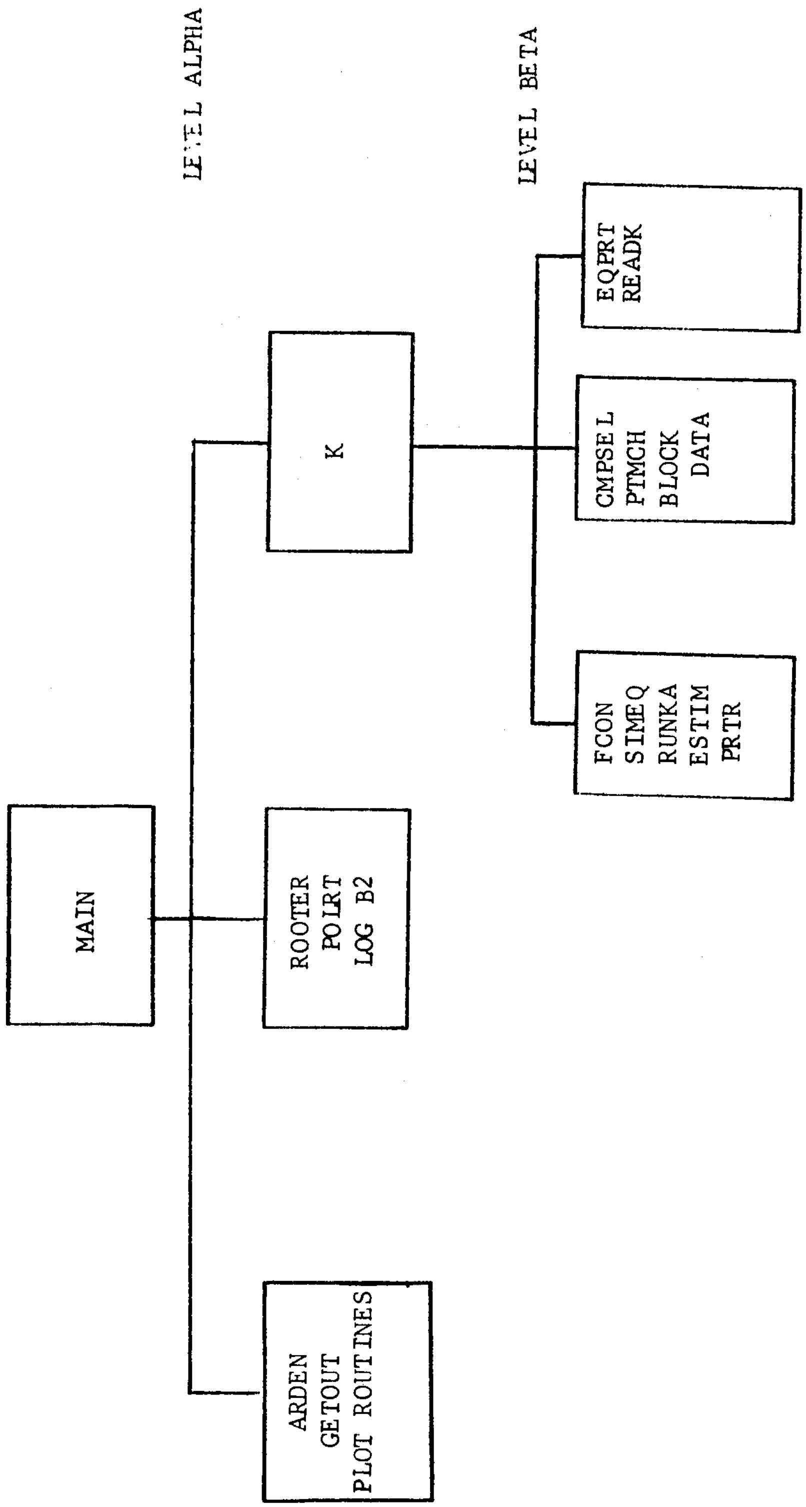
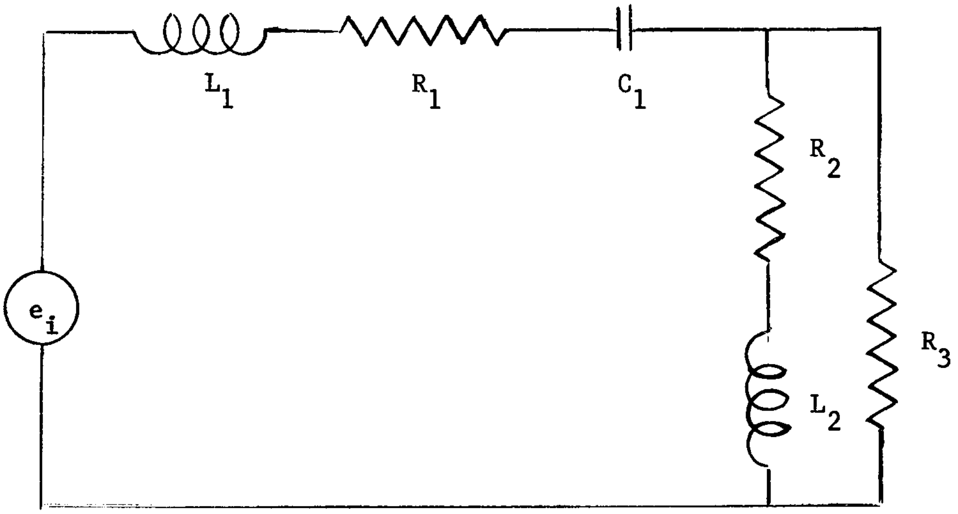


Figure C-1. OVERLAY STRUCTURE OF PROGRAM

APPENDIX D

FILTER CIRCUIT WITH SIX UNKNOWNS

D-1 Circuit Diagram



D-2 Identity of Unknowns

$Y_1 = R_1$ $\quad$ $Y_4 = L_1$ $\quad$ $Y_6 = 1/C_1$

$Y_2 = R_2$ $\quad$ $Y_5 = L_2$

$Y_3 = R_3$

D-3 Transfer Function

$$T = \frac{1.2 \times 10^6 s + 1.6 \times 10^4 s^2}{3.4 \times 10^7 + 8.4 \times 10^6 s + 1.64 \times 10^5 s^2 + 8.0 \times 10^2 s^3}$$

D-4 Example Inputs and Outputs

Two input samples and the outputs which resulted from them for the set of 6 equations in 6 unknowns are presented on the pages which follow. The plots from the frequency-response subroutine are included only with the first case. The range of interest of the unknowns in Case #1 is identical to that presented on page 35 in reference 1.

Case #2 presents an identical run, except that the range of interest of the unknowns was set equal to the maximum and minimum allowable values for components, as presented in Table 2-1. This was done to demonstrate the strength of convergence of the program. For brevity, the input items are listed without FORTRAN symbols, and the plots resulting from the roots obtained have been omitted.

EXAMPLE INPUT AND OUTPUT

FOR SIX EQUATIONS AND SIX UNKNOWNS

Case #1

SIMULTANEOUS SOLUTION OF SETS OF NONLINEAR ALGEBRAIC EQUATIONS

AC CODE DIV	SALES ORDER	DASH

INPUT DATA FOR SIX EQUATIONS AND SIX UNKNOWNS CASE # 1

MAXNOS	NOS	KK	JMAX	IZMAX	NR	NL	NC	NOR	MR	NA	NTB
100	25	20	6	5	3	2	1	4	1	0	0

IMAX(1)	IMAX(2)	IMAX(3)	IMAX(4)	IMAX(5)	IMAX(6)
1	4	4	2	1	1

LMAX(1)	LMAX(2)	LMAX(3)	LMAX(4)	LMAX(5)	LMAX(6)
2	2	2	2	2	2

F(1)	F(2)	F(3)	F(4)	F(5)	F(6)
8.0E+02	16.4E+04	84.0E+05	3.4E+07	16.0E+03	12.0E+05

PTOL(1)	PTOL(2)	PTOL(3)	PTOL(4)	PTOL(5)	PTOL(6)
.01	.01	.01	.01	.01	.01

XRMIN	XLMIN	XCMIN	XRMAX	XLMAX	XCMAX
.24	.00005	1.0E-11	22.0E+06	350.	1.5E-01

FXORIG(1)	FXORIG(2)	FXORIG(3)	FXORIG(4)	FXORIG(5)	FXORIG(6)	FXLIM(1)	FXLIM(2)
100.	100.	100.	10.	10.	1.0E-05	1.0E+05	1.0E+05

FXLIM(3)	FXLIM(4)	FXLIM(5)	FXLIM(6)
1.0E+05	1.0E+04	1.0E+04	1.0E-02

A.C CODE DIV.	SALES ORDER	DASH	EWA NO.		ENGINEER	2	3
					ANALYST		

INPUT DATA FOR SIX EQUATIONS AND SIX UNKNOWNS CASE #1 (Continued)

C(1)	C(2)	C(3)
1.0	1.0	1.0

TX
1.0

K(111)	K(112)
4	5

K(211)	K(212)	K(221)	K(222)	K(231)	K(232)	K(241)	K(242)
2	4	3	4	1	5	3	5

K(311)	K(312)	K(321)	K(322)	K(331)	K(332)	K(341)	K(342)
5	6	1	2	1	3	2	3

K(411)	K(412)	K(421)	K(422)
3	6	2	6

K(511)	K(512)
3	5

K(611)	K(612)
2	3

JOB TITLE				ENGINEER	3
ACCODE DIV	SALES ORDER	DASH	EWO NO.	FOR CHGNO. NO.	ANALYST

INPUT DATA FOR SIX EQUATIONS AND SIX UNKNOWNS CASE #1 (Concluded)

```
NTC    N3(J),J=1,JMAX
0      3 3 3 3 3 3

  SPEC(J),J=1,JMAX
 D3  D2  D1  D0  N2  N1
NMAX
  6
ICPS NSTPS              FRQMIN      FRQMAX
1              25         1.0         10.
```

```
THE FØLLØWING IS THE LIST ØF EQUATIØNS SPECIFIED TØ THE PRØGRAM
THE FØRMAT IS....  EQUATIØN NUMBER
                   TERMS ØF EQUATIØNS (THREE PER LINE)
                   NUMBER ØF EACH TERM

A CHECK IS MADE ØF THE UNITS ØF EACH TERM. IF THE UNITS DIFFER
IN AN EQUATIØN, AN ERRØR MESSAGE RESULTS



                              EQUATIØN  1


                L1 L2

                  1



                              EQUATIØN  2


                R2 L1             + R3 L1               + R1 L2

                  1                   2                     3


                R3 L2

                  4



                              EQUATIØN  3


                L2                + R1 R2               + R1 R3
                C1
                  1                   2                     3


                R2 R3

                  4
```

```
                         EQUATIØN   4


R3                        + R2
C1                          C1
  1                            2



                         EQUATIØN   5


R3 L2

  1



                         EQUATIØN   6


R2 R3

  1
```

```
    INPUT DATA

    MAXIMUM NO. ØF STEPS      100
    NUMBER ØF STEPS            25
    TIMES THRØUGH RUNGE KUTTA        20
    CØNSTANT TERMS
          0.80000000E 03       0.16400000E 06        0.84000000E 07

    RANGE FØR VARIABLES
      FXØRIG
         0.10000000E 03      0.10000000E 03       0.10000000E 03      0.1
      FXLIM
         0.10000000E 06      0.10000000E 06       0.10000000E 06      0.1

    THERE ARE  6 EQUATIØNS AND  6 UNKNØWNS,CØNSISTING ØF  3 RESISTAN
    THE LØWER BØUNDARIES FØR THE RESISTANCES, THE INDUCTANCES, AND T
    AND    0.10000000E-10, RESPECTIVELY, WHILE THEIR UPPER BØUNDARIES
      0.15000000E [illegible] RESPECTIVELY.

      LX=   1

      LX=   2

      LX=   3

      LX=   4

      LX=   5

      LX=   6

    VARIABLES
          0.10000000E 03       0.10000000E 03        0.99999994E 04


    CØMMENCING CØNSTANT APPRØACH

    GRID=    1    NØS=   25
    GRID=    2    NØS=   25
    GRID=    3    NØS=   25
    GRID=    4    NØS=   25
    GRID=    5    NØS=   25
    GRID=    6    NØS=   25
    GRID=    7    NØS=   25
    GRID=    8    NØS=   25
    GRID=    9    NØS=   25
    GRID=   10    NØS=   25
    GRID=   11    NØS=   25
    GRID=   12    NØS=   25
    GRID=   13    NØS=   25
    GRID=   14    NØS=   25
    GRID=   15    NØS=   25
    GRID=   16    NØS=   25
    GRID=   17    NØS=   25
    GRID=   18    NØS=   25
    GRID=   19    NØS=   25
    GRID=   20    NØS=   25
    GRID=   21    NØS=   25
    GRID=   22    NØS=   25
    GRID=   23    NØS=   25
```

D-9-1

```
0.34000000E 08        .16000000E 05        0.12000000E 07


0000000E 02      0.10000000E 02      0.10000000E-04

0000000E 05      0.10000000E 05      0.12000000E-01
E(S),   2 INDUCTANCE(S),AND   1 CAPACITANCE(S).
HE CAPACITANCES ARE    0.24000000E 00,    0.50000000E-04,
 ARE    0.22000000E 08,    0.35000000E 03, AND








0.10000000E 02       0.10000000E 02       0.99999998E 03
```

```
GRID=  24   NØS=  25
GRID=  25   NØS=  25

ALL RØØTS IN THE FØLLØWING SET LIE WITHIN THE PHYSICAL LIMITS SP

    R( 1)=  0.25356202E 04  ØHMS       L( 1)=  0.28086632E 02  HEN
    R( 2)=  0.21362475E 04  ØHMS       L( 2)=  0.28483302E 02  HEN
    R( 3)=  0.56173262E 03  ØHMS
```

```
C( 1)=  0.79352362E-04  FARADS
```

```
FØR CAPACITØR C 1 THE  2 CØMPØNENT(S) ARE
                         0.68000000E 08
                         0.10000000E 08
C 1 IS THUS    0.78000000E 08 MICRØMICRØFARADS


FØR INDUCTØR L 2 THE  1 CØMPØNENT(S) ARE
                         0.28000000E 02
L 2 IS THUS     .28000000E 02 HENRIES, AND
THE INDUCTIVE PART ØF R 2 IS    0.28000000E 02 ØHMS


FØR INDUCTØR L 1 THE  1 CØMPØNENT(S) ARE
                         0.28000000E 02
L 1 IS THUS     .28000000E 02 HENRIES, AND
THE INDUCTIVE PART ØF R 1 IS     .28000000E 02 ØHMS


FØR RESISTØR R 3 THE  2 CØMPØNENT(S) ARE
                         0.51100000E 02
                         0.46400000E 02
R 3 IS THUS     .55739999E 02 ØHMS


FØR RESISTØR R 2 THE  2 CØMPØNENT(S) ARE
                         0.19610000E 04
                         0.14700000E 03
WITH AN INDUCTIVE RESISTANCE ØF  0.28000000E 02 ØHMS
R 2 IS THUS     .21360000E 04 ØHMS


FØR RESISTØR R 1 THE  2 CØMPØNENT(S) ARE
                         0.23700000E 04
                         0.13300000E 03
WITH AN INDUCTIVE RESISTANCE ØF  0.28000000E 02 ØHMS
R 1 IS THUS    0.25310000E 04 ØHMS
```

```
THE NUMERATØR IS ØF ØRDER  2. THE PØLYNØMIAL IN DESCENDING ØRDER BELØW

    0.15607200E 05   0.11900490E 07   0.00000000E-38

THE RØØTS ARE-
 REAL PART             IMAG. PART              REAL PART             IMAG. PART
-0.76250E 02           0.00000E-38             0.00000E-38           0.00000E-38

THE DENØMINATØR IS ØF ØRDER  3. THE PØLYNØMIAL IN DESCENDING ØRDER BELØW

    0.78400000E 03   0.16186240E 06   0.83634874E 07   0.34517948E 08

THE RØØTS ARE-
 REAL PART             IMAG. PART              REAL PART             IMAG. PART
-0.80026E 02           0.00000E-38            -0.12192E 03           0.00000E-38
-0.45127E 01           0.00000E-38
```

```
ØMEGA-RAD/SEC F-CYCLES/SEC  AMPLITUDE      20LØG AMP      PHASE-DEG

   6.28320        1.00000     0.02009      -33.93979      -57.04230
   6.88939        1.09648     0.01887      -34.48554      -59.76622
   7.55407        1.20227     0.01766      -35.06258      -62.42667
   8.28297        1.31826     0.01647      -35.66861      -65.01367
   9.08199        1.44544     0.01531      -36.30117      -67.52055
   9.95820        1.58490     0.01419      -36.95777      -69.94389
  10.91895        1.73780     0.01313      -37.63595      -72.28334
  11.97239        1.90546     0.01212      -38.33342      -74.54123
  13.12746        2.08930     0.01116      -39.04875      -76.72227
  14.39398        2.29087     0.01026      -39.77794      -78.83314
  15.78268        2.51189     0.00942      -40.52144      -80.88212
  17.30527        2.75423     0.00863      -41.27715      -82.87880
  18.97496        3.01996     0.00790      -42.04394      -84.83374
  20.80563        3.31132     0.00723      -42.82096      -86.75828
  22.81292        3.63079     0.00660      -43.60762      -88.66432
  25.01396        3.98108     0.00602      -44.40358      -90.56405
  27.42716        4.36517     0.00549      -45.20880      -92.46998
  30.07228        4.78631     0.00500      -46.02350      -94.39458
  32.97469        5.24809     0.00455      -46.84818      -96.35023
  36.15612        5.75441     0.00413      -47.68363      -98.34897
  39.64430        6.30959     0.00375      -48.53094     -100.40225
  43.46911        6.91832     0.00339      -49.39151     -102.52067
  47.66293        7.58579     0.00307      -50.26706     -104.71360
  52.26137        8.31765     0.00277      -51.15958     -106.98874
  57.30345        9.12013     0.00249      -52.07138     -109.35166
  62.83207       10.00002     0.00224      -53.00496     -111.80532

CØMMENCING CØEFFICIENT APPRØACH


NA=      1
GRID=     1     NØS=   25
GRID=     2     NØS=   25
GRID=     3     NØS=   25
GRID=     4     NØS=   25
GRID=     5     NØS=   25
GRID=     6     NØS=   25
GRID=     7     NØS=   25
GRID=     8     NØS=   25
GRID=     9     NØS=   25
GRID=    10     NØS=   25
GRID=    11     NØS=   25
GRID=    12     NØS=   25
GRID=    13     NØS=   25
GRID=    14     NØS=   25
GRID=    15     NØS=   25
GRID=    16     NØS=   25
GRID=    17     NØS=   25
GRID=    18     NØS=   25
GRID=    19     NØS=   25
GRID=    20     NØS=   25
GRID=    21     NØS=   25
GRID=    22     NØS=   25
GRID=    23     NØS=   25
GRID=    24     NØS=   25
GRID=    25     NØS=   25
```

```
ALL RØØTS IN THE FØLLØWING SET LIE WITHIN THE PHYSICAL LIMITS SPE

     R( 1)=  0.25356148E 04   ØHMS         L( 1)=  0.28086535E 02   HENR
     R( 2)=  0.21362539E 04   ØHMS         L( 2)=  0.28483399E 02   HENR
     R( 3)=  0.56173078E 03   ØHMS

CØMMENCING CØEFFICIENT APPRØACH

NA=    2
GRID=    1    NØS=   25
GRID=    2    NØS=   25
GRID=    3    NØS=   25
GRID=    4    NØS=   25
GRID=    5    NØS=   25
GRID=    6    NØS=   25
GRID=    7    NØS=   25
GRID=    8    NØS=   25
GRID=    9    NØS=   25
GRID=   10    NØS=   25
GRID=   11    NØS=   25
GRID=   12    NØS=   25
GRID=   13    NØS=   25
GRID=   14    NØS=   25
GRID=   15    NØS=   25
GRID=   16    NØS=   25
GRID=   17    NØS=   25
GRID=   18    NØS=   25
GRID=   19    NØS=   25
GRID=   20    NØS=   25
GRID=   21    NØS=   25
GRID=   22    NØS=   25
GRID=   23    NØS=   25
GRID=   24    NØS=   25
GRID=   25    NØS=   25

ALL RØØTS IN THE FØLLØWING SET LIE WITHIN THE PHYSICAL LIMITS SPE

     R( 1)=  0.25356151E 04   ØHMS         L( 1)=  0.28086541E 02   HENR
     R( 2)=  0.21362545E 04   ØHMS         L( 2)=  0.28483394E 02   HENR
     R( 3)=  0.56173081E 03   ØHMS

CØMMENCING CØEFFICIENT APPRØACH

NA=    2
GRID=    1    NØS=   25
GRID=    2    NØS=   25
GRID=    3    NØS=   25
GRID=    4    NØS=   25
GRID=    5    NØS=   25
```

```
C( 1)=  0.79352520E-C4  FARAUS
```

```
C( 1)=  0.79352511E-C4  FARADS
```

```
     GRID=    6     NØS=   25
     GRID=    7     NØS=   25
     GRID=    8     NØS=   25
     GRID=    9     NØS=   25
     GRID=   10     NØS=   25
     GRID=   11     NØS=   25
     GRID=   12     NØS=   25
     GRID=   13     NØS=   25
     GRID=   14     NØS=   25
     GRID=   15     NØS=   25
     GRID=   16     NØS=   25
     GRID=   17     NØS=   25
     GRID=   18     NØS=   25
     GRID=   19     NØS=   25
     GRID=   20     NØS=   25
     GRID=   21     NØS=   25
     GRID=   22     NØS=   25
     GRID=   23     NØS=   25
     GRID=   24     NØS=   25
     GRID=   25     NØS=   25


     ALL RØØTS IN THE FØLLØWING SET LIE WITHIN THE PHYSICAL LIMITS SP


          R( 1)=  0.25356161E 04  ØHMS        L( 1)=  0.28086559E 02  HEN
          R( 2)=  0.21362532E 04  ØHMS        L( 2)=  0.28483376E 02  HEN
          R( 3)=  0.56173117E 03  ØHMS

     CØMMENCING CØEFFICIENT APPRØACH


     NA=    4
     GRID=    1     NØS=   25
     GRID=    2     NØS=   25
     GRID=    3     NØS=   25
     GRID=    4     NØS=   25
     GRID=    5     NØS=   25
     GRID=    6     NØS=   25
     GRID=    7     NØS=   25
     GRID=    8     NØS=   25
     GRID=    9     NØS=   25
     GRID=   10     NØS=   25
     GRID=   11     NØS=   25
     GRID=   12     NØS=   25
     GRID=   13     NØS=   25
     GRID=   14     NØS=   25
     GRID=   15     NØS=   25
     GRID=   16     NØS=   25
     GRID=   17     NØS=   25
     GRID=   18     NØS=   25
     GRID=   19     NØS=   25
     GRID=   20     NØS=   25
     GRID=   21     NØS=   25
     GRID=   22     NØS=   25
     GRID=   23     NØS=   25
     GRID=   24     NØS=   25
     GRID=   25     NØS=   25
```

```
C( 1)=  0.79352481E-C4  FARADS
```

```
ALL RØØTS IN THE FØLLØWING SET LIE WITHIN THE PHYSICAL LIMITS SP

     R( 1)=  0.25356154E 04   ØHMS        L( 1)=  0.28086548E 02  HEN
     R( 2)=  0.21362540E 04   ØHMS        L( 2)=  0.28483387E 02  HEN
     R( 3)=  0.56173095E 03   ØHMS

COMMENCING CØEFFICIENT APPRØACH


NA=     5
GRID=    1    NØS=   25
GRID=    2    NØS=   25
GRID=    3    NØS=   25
GRID=    4    NØS=   25
GRID=    5    NØS=   25
GRID=    6    NØS=   25
GRID=    7    NØS=   25
GRID=    8    NØS=   25
GRID=    9    NØS=   25
GRID=   10    NØS=   25
GRID=   11    NØS=   25
GRID=   12    NØS=   25
GRID=   13    NØS=   25
GRID=   14    NØS=   25
GRID=   15    NØS=   25
GRID=   16    NØS=   25
GRID=   17    NØS=   25
GRID=   18    NØS=   25
GRID=   19    NØS=   25
GRID=   20    NØS=   25
GRID=   21    NØS=   25
GRID=   22    NØS=   25
GRID=   23    NØS=   25
GRID=   24    NØS=   25
GRID=   25    NØS=   25


ALL RØØTS IN THE FØLLØWING SET LIE WITHIN THE PHYSICAL LIMITS SP

     R( 1)=  0.25356154E 04   ØHMS        L( 1)=  0.28086548E 02  HEN
     R( 2)=  0.21362540E 04   ØHMS        L( 2)=  0.28483387E 02  HEN
     R( 3)=  0.56173095E 03   ØHMS

COMMENCING CØEFFICIENT APPRØACH
```

```
C( 1)=   .79352499E-04   FARADS
```

```
C( 1)=  0.79352499E-04   FARADS
```

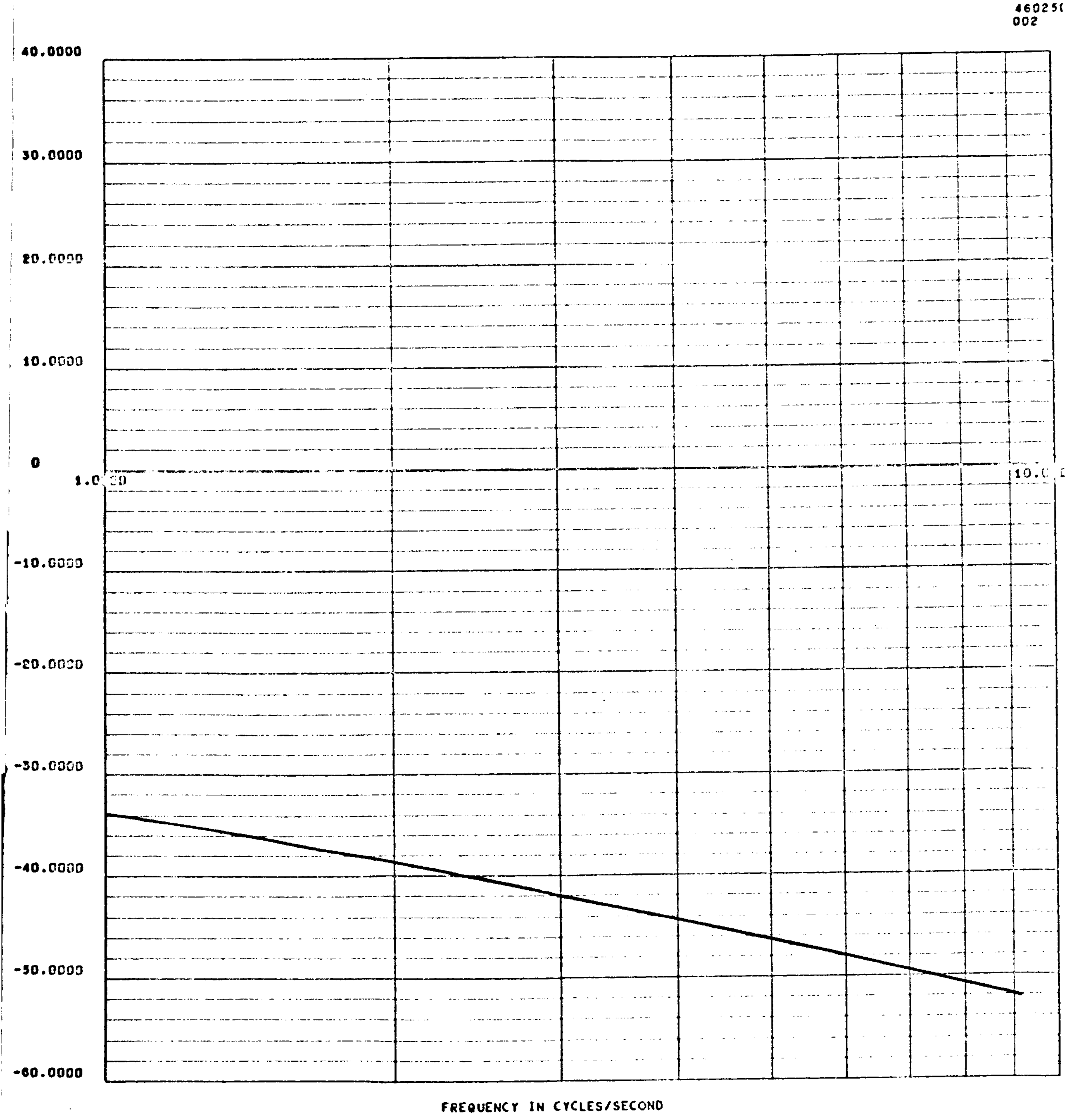


Figure D-1. AMPLITUDE VERSUS FREQUENCY
SIX EQUATIONS, SIX UNKNOWNS, CASE #1

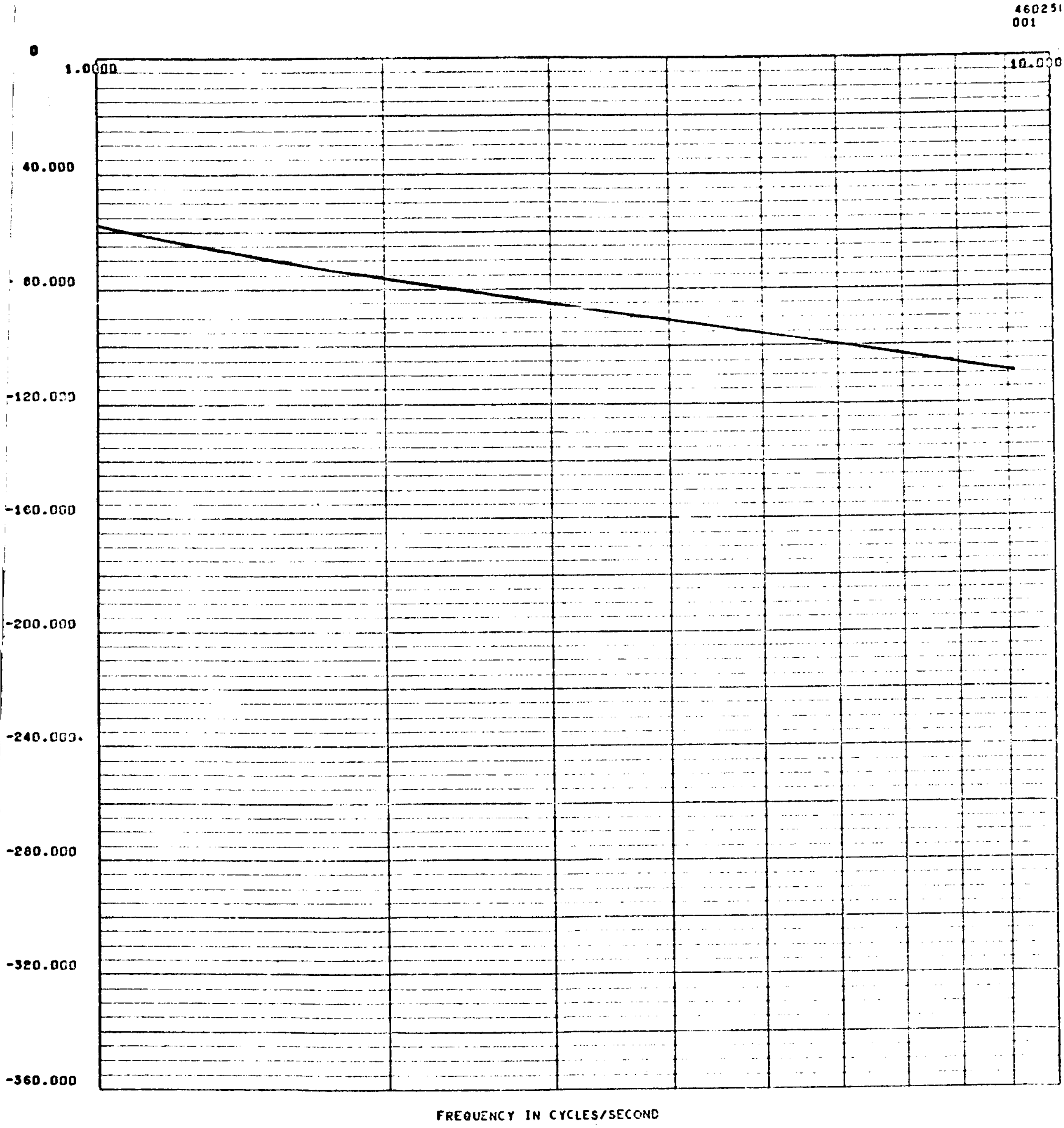


Figure D-2. PHASE SHIFT VERSUS FREQUENCY
SIX EQUATIONS, SIX UNKNOWNS, CASE #1

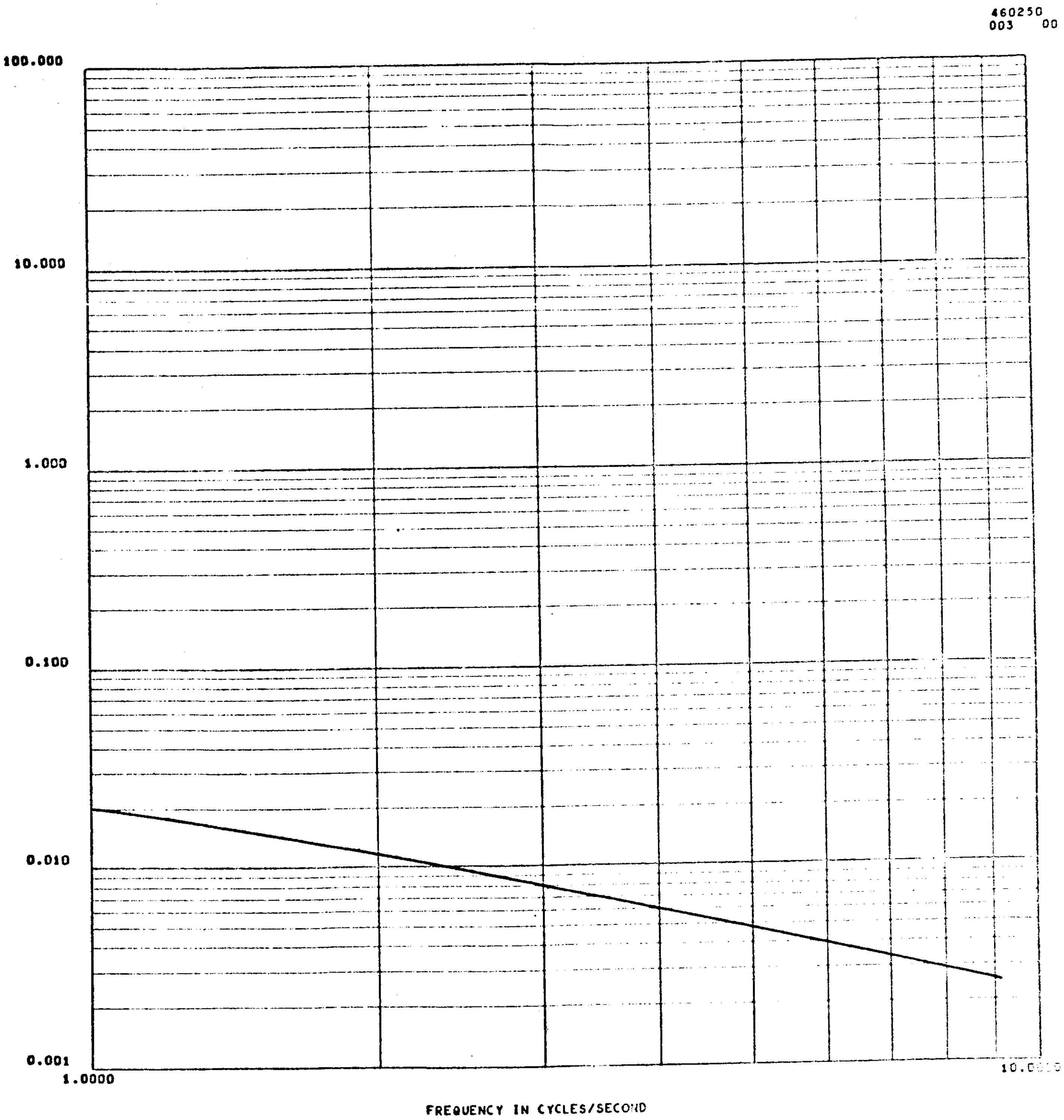


Figure D-3. GAIN VERSUS FREQUENCY
SIX EQUATIONS, SIX UNKNOWNS, CASE #1

EXAMPLE INPUT AND OUTPUT

FOR SIX EQUATIONS AND SIX UNKNOWNS

Case #2

```
                INPUT DATA FOR SIX EQUATIONS AND SIX UNKNOWNS

                                                   COLUMN NUMBER
000000000111111111122222222223333333333444444444455555555
123456789012345678901234567890123456789012345678901234567

 100  25  20   6   5   3   2   1   4   1
   1   4   4   2   1   1
   2   2   2   2   2   2
     8.0E+02    16.4E+04    84.0E+05     3.4E+07    16.0
       .01       .01       .01       .01       .01
       .24    .00005   1.0E-11  22.0E+06      350.   1.5
       .24       .24       .24    .00005    .00005   1.0
  22.0E+06      350.      350.   1.5E-01
         1.0         1.0         0.0
       1.0
   4   5
   2   4   3   4   1   5   3   5
   5   6   1   2   1   3   2   3
   3   6   2   6
   3   5
   2   3
     333333
 D3 D2 D1 D0 N2  N1
15
1               1.        1.
```

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```
2

6666677777777778
5678901234567890

12.0E+05

2.0E+06  22.0E+06
```

```
THE FØLLØWING IS THE LIST ØF EQUATIØNS SPECIFIED TØ THE PRØGRAM
THE FØRMAT IS....  EQUATIØN NUMBER
                   TERMS ØF EQUATIØNS (THREE PER LINE)
                   NUMBER ØF EACH TERM

A CHECK IS MADE ØF THE UNITS ØF EACH TERM. IF THE UNITS DIFFER
IN AN EQUATIØN, AN ERRØR MESSAGE RESULTS
------------------------------------------------------------------------



                EQUATIØN  1
------------------------------------------------------------------------

 L1 L2

   1
------------------------------------------------------------------------



                EQUATIØN  2
------------------------------------------------------------------------

 R2 L1               + R3 L1               + R1 L2

   1                     2                     3
------------------------------------------------------------------------

 R3 L2

   4
------------------------------------------------------------------------



                EQUATIØN  3
------------------------------------------------------------------------

 L2                  + R1 R2               + R1 R3
 C1
   1                     2                     3
------------------------------------------------------------------------

 R2 R3

   4
```

```
                    EQUATIØN  4

R3                   + R2
C1                     C1
  1                       2


                    EQUATIØN  5

R3 L2

  1


                    EQUATIØN  6

R2 R3

  1
  INPUT DATA

 MAXIMUM NO. ØF STEPS      100
 NUMBER ØF STEPS             25
 TIMES THRØUGH RUNGE KUTTA          20
 CØNSTANT TERMS
        0.80000000E 03        0.16400000E 06        0.84000000E 07

 RANGE FØR VARIABLES
   FXØRIG
      0.24000000E 00       0.24000000E 00       0.24000000E 00       0.50

   FXLIM
      0.22000000E 08       0.22000000E 08       0.22000000E 08       0.35

 THERE ARE  6 EQUATIØNS AND  6 UNKNØWNS,CØNSISTING ØF  3 RESISTANC
 THE LØWER BØUNDARIES FØR THE RESISTANCES, THE INDUCTANCES, AND TH
 AND   0.10000000E-10, RESPECTIVELY, WHILE THEIR UPPER BØUNDARIES
   0.15000000E 00 RESPECTIVELY.

   LX=    1

   LX=    2

   LX=    3

   LX=    4

   LX=    5

   LX=    6

 VARIABLES
      0.48792400E 05        0.10821357E 03        0.10821357E 03
```

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```
0000E 08        0.16000000E 05        0.12000000E 07


E-04      0.50000000E-04      0.10000000E-10

E 03      0.35000000E 03      0.15000000E 00
 2 INDUCTANCE(S),AND  1 CAPACITANCE(S).
CITANCES ARE    0.24000000E 00,    0.50000000E-04,
0.22000000E 08,    0.35000000E 03, AND





000E 03        0.18296528E 01        0.16441414E 05
```

```
CØMMENCING CØNSTANT APPRØACH

GRID=   1    NØS=  25
GRID=   2    NØS=  25
GRID=   3    NØS=  25
GRID=   4    NØS=  25
GRID=   5    NØS=  25
GRID=   6    NØS=  25
GRID=   7    NØS=  25
GRID=   8    NØS=  25
GRID=   9    NØS=  25
GRID=  10    NØS=  25
GRID=  11    NØS=  25
GRID=  12    NØS=  25
GRID=  13    NØS=  25
GRID=  14    NØS=  25
GRID=  15    NØS=  25
GRID=  16    NØS=  25
GRID=  17    NØS=  25
GRID=  18    NØS=  25
GRID=  19    NØS=  25
GRID=  20    NØS=  25
GRID=  21    NØS=  25
GRID=  22    NØS=  25
GRID=   1    NØS=   8
GRID=   2    NØS=   8
GRID=   3    NØS=   8
GRID=   4    NØS=   8
GRID=   5    NØS=   8
GRID=   6    NØS=   8
GRID=   7    NØS=   8
GRID=   8    NØS=   8

ALL RØØTS IN THE FØLLØWING SET LIE WITHIN THE PHYSICAL LIMITS SPE

   R( 1)=  0.19999919E 04  ØHMS        L( 1)=  0.19999894E 02  HENR
   R( 2)=  0.30000157E 04  ØHMS        L( 2)=  0.40000212E 02  HENR
   R( 3)=  0.39999788E 03  ØHMS
```

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```
C( 1)=  0.10000041E-03  FARADS
```

```
FØR CAPACITØR C 1 THE  1 CØMPØNENT(S) ARE
                   0.10000000E 09

C 1 IS THUS   0.10000000E 09 MICRØMICRØFARADS


FØR INDUCTØR L 2 THE  1 CØMPØNENT(S) ARE
                   0.40000000E 02

L 2 IS THUS   0.40000000E 02 HENRIES, AND
THE INDUCTIVE PART ØF R 2 IS   0.40000000E 02 ØHMS


FØR INDUCTØR L 1 THE  1 CØMPØNENT(S) ARE
                   0.19000000E 02

L 1 IS THUS   0.19000000E 02 HENRIES, AND
THE INDUCTIVE PART ØF R 1 IS   0.19000000E 02 ØHMS


FØR RESISTØR R 3 THE  2 CØMPØNENT(S) ARE
                   0.38300000E 03
                   0.16200000E 02

R 3 IS THUS   0.39920000E 03 ØHMS


FØR RESISTØR R 2 THE  2 CØMPØNENT(S) ARE
                   0.28700000E 04
                   0.82500000E 02

WITH AN INDUCTIVE RESISTANCE ØF   0.40000000E 02 ØHMS

R 2 IS THUS   0.29925000E 04 ØHMS


FØR RESISTØR R 1 THE  2 CØMPØNENT(S) ARE
                   0.19600000E 04
                   0.19600000E 02

WITH AN INDUCTIVE RESISTANCE ØF   0.19000000E 02 ØHMS

R 1 IS THUS   0.19986000E 04 ØHMS
```

```
THE NUMERATØR IS ØF ØRDER  2. THE PØLYNØMIAL IN DESCENDING ØRDER BELØW

   0.15968000E 05  0.11946060E 07  0.00000000E-38

THE RØØTS ARE-
 REAL PART            IMAG. PART            REAL PART            IMAG. PART
-0.74812E 02          0.00000E-38           0.00000E-38          0.00000E-38

THE DENØMINATØR IS ØF ØRDER  3. THE PØLYNØMIAL IN DESCENDING ØRDER BELØW

   0.76000000E 03  0.16035430E 06  0.83732574E 07  0.33916999E 08

THE RØØTS ARE-
 REAL PART            IMAG. PART            REAL PART            IMAG. PART
-0.79554E 02          0.00000E-38          -0.12702E 03          0.00000E-38
-0.44163E 01          0.00000E-38
```

```
ØMEGA-RAD/SEC F-CYCLES/SEC   AMPLITUDE      20LØG AMP      PHASE-DEG

    6.28320        1.00000       0.02024      -33.87695      -57.44433
    6.88939        1.09648       0.01899      -34.42981      -60.13132
    7.55407        1.20227       0.01776      -35.01335      -62.75003
    8.28287        1.31826       0.01655      -35.62521      -65.29106
    9.08199        1.44544       0.01536      -36.26289      -67.74831
    9.95820        1.58490       0.01425      -36.92388      -70.11890
   10.91895        1.73780       0.01317      -37.60573      -72.40289
   11.97239        1.90546       0.01215      -38.30614      -74.60390
   13.12746        2.08930       0.01119      -39.02299      -76.72416
   14.39398        2.29087       0.01029      -39.75440      -78.77322
   15.78268        2.51189       0.00944      -40.49871      -80.75856
   17.30537        2.75423       0.00866      -41.25454      -82.68978
   18.97496        3.01996       0.00792      -42.02073      -84.57743
   20.80563        3.31132       0.00725      -42.79641      -86.43283
   22.81292        3.63079       0.00662      -43.58095      -88.26782
   25.01386        3.98108       0.00604      -44.37398      -90.09466
   27.42716        4.36517       0.00551      -45.17539      -91.92585
   30.07328        4.78631       0.00502      -45.98532      -93.77405
   32.97469        5.24809       0.00457      -46.80420      -95.65187
   36.15603        5.75441       0.00415      -47.63274      -97.57173
   39.64430        6.30959       0.00377      -48.47195      -99.54563
   43.46911        6.91832       0.00342      -49.32314     -101.58489
   47.66293        7.58579       0.00309      -50.18797     -103.69974
   52.26137        8.31765       0.00280      -51.06839     -105.89895
   57.30345        9.12013       0.00252      -51.96667     -108.18030
   62.83200       10.00002       0.00227      -52.88534     -110.57508

CØMMENCING CØEFFICIENT APPRØACH


NA =     1
GRID=    1     NØS=   25
GRID=    2     NØS=   25
GRID=    3     NØS=   25
GRID=    4     NØS=   25
GRID=    5     NØS=   25
GRID=    6     NØS=   25
GRID=    7     NØS=   25
GRID=    8     NØS=   25
GRID=    9     NØS=   25
GRID=    1     NØS=   34
GRID=    2     NØS=   34
GRID=    3     NØS=   34
GRID=    4     NØS=   34
GRID=    5     NØS=   34
GRID=    6     NØS=   34
GRID=    7     NØS=   34
GRID=    8     NØS=   34
GRID=    1     NØS=   54
USING THIS SET ØF ESTIMATES, NØ RØØTS WERE FØUND
```

```
COMMENCING COEFFICIENT APPRØACH


NA=    2
GRID=    1    NØS=   25
GRID=    2    NØS=   25
GRID=    3    NØS=   25
GRID=    4    NØS=   25
GRID=    5    NØS=   25
GRID=    6    NØS=   25
GRID=    7    NØS=   25
GRID=    8    NØS=   25
GRID=    9    NØS=   25
GRID=   10    NØS=   25
GRID=   11    NØS=   25
GRID=   12    NØS=   25
GRID=   13    NØS=   25
GRID=   14    NØS=   25
GRID=   15    NØS=   25
GRID=   16    NØS=   25
GRID=   17    NØS=   25
GRID=   18    NØS=   25
GRID=   19    NØS=   25
GRID=   20    NØS=   25
GRID=   21    NØS=   25
GRID=   22    NØS=   25
GRID=   23    NØS=   25
GRID=   24    NØS=   25
GRID=   25    NØS=   25


ALL RØØTS IN THE FØLLØWING SET LIE WITHIN THE PHYSICAL LIMITS S


   R( 1)=  0.20000004E 04   ØHMS        L( 1)=  0.20000005E 02   HE
   R( 2)=  0.29999992E 04   ØHMS        L( 2)=  0.39999989E 02   HE
   R( 3)=  0.40000011E 03   ØHMS

CØMMENCING CØEFFICIENT APPRØACH


NA=    3
GRID=    1    NØS=   25
GRID=    2    NØS=   25
GRID=    3    NØS=   25
GRID=    4    NØS=   25
GRID=    5    NØS=   25
GRID=    6    NØS=   25
GRID=    7    NØS=   25
GRID=    8    NØS=   25
GRID=    1    NØS=   36
GRID=    2    NØS=   36
GRID=    1    NØS=   70
GRID=    2    NØS=   70
USING THIS SET ØF ESTIMATES, NØ RØØTS WERE FØUND
```

```
C( 1)=  0.99999980E-04  FARADS
```

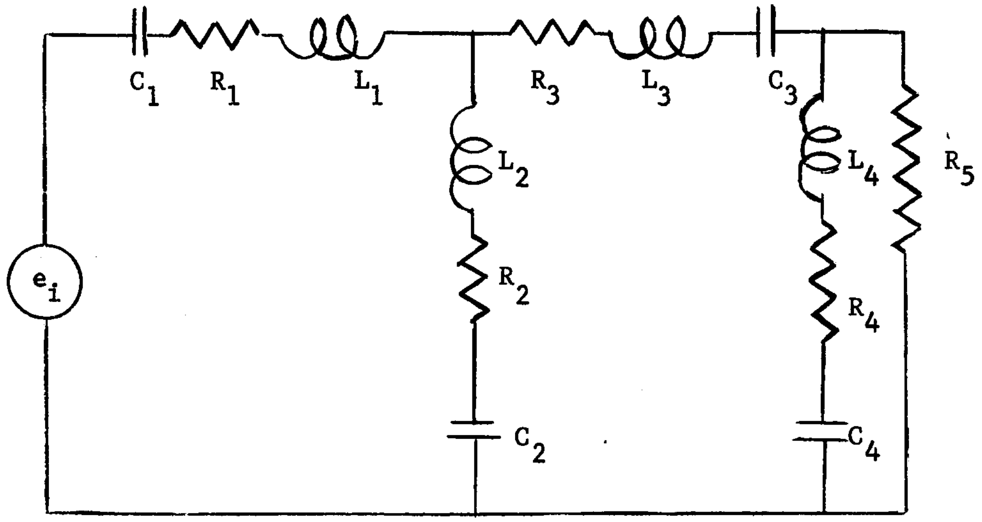
```
COMMENCING COEFFICIENT APPRØACH


 NA=    4
 GRID=    1    NØS=   25
 GRID=    2    NØS=   25
 GRID=    3    NØS=   25
 GRID=    4    NØS=   25
 GRID=    5    NØS=   25
 GRID=    6    NØS=   25
 GRID=    7    NØS=   25
 GRID=    8    NØS=   25
 GRID=    9    NØS=   25
 GRID=   10    NØS=   25
 GRID=   11    NØS=   25
 GRID=   12    NØS=   25
 GRID=   13    NØS=   25
 GRID=   14    NØS=   25
 GRID=   15    NØS=   25
 GRID=   16    NØS=   25
 GRID=   17    NØS=   25
 GRID=   18    NØS=   25
 GRID=   19    NØS=   25
 GRID=   20    NØS=   25
 GRID=   21    NØS=   25
 GRID=   22    NØS=   25
 GRID=   23    NØS=   25
 GRID=   24    NØS=   25
 GRID=   25    NØS=   25


ALL RØØTS IN THE FØLLØWING SET LIE WITHIN THE PHYSICAL LIMITS S


    R( 1)=  0.25356151E 04  ØHMS       L( 1)=  0.28086542E 02  HE
    R( 2)=  0.21362544E 04  ØHMS       L( 2)=  0.28483392E 02  HE
    R( 3)=  0.56173084E 03  ØHMS

COMMENCING COEFFICIENT APPRØACH


NA=    5
GRID=    1    NØS=   25
GRID=    2    NØS=   25
GRID=    3    NØS=   25
GRID=    4    NØS=   25
GRID=    1    NØS=   44
GRID=    2    NØS=   44
GRID=    3    NØS=   44
GRID=    1    NØS=   84
GRID=    2    NØS=   84
GRID=    3    NØS=   84
GRID=    4    NØS=   84
GRID=    5    NØS=   84
GRID=    6    NØS=   84
GRID=    7    NØS=   84
GRID=    8    NØS=   84
GRID=    9    NØS=   84
GRID=   10    NØS=   84
USING THIS SET ØF ESTIMATES, NØ RØØTS WERE FØUND


COMMENCING COEFFICIENT APPRØACH
```

```
C( 1)=  0.79352508E-04  FARADS
```

APPENDIX E

FILTER CIRCUIT WITH THIRTEEN UNKNOWNS

E-1 Circuit Diagram



E-2 Identity of Unknowns

$Y_1 = R_1$	$Y_6 = L_1$	$Y_{10} = 1/C_1$
$Y_2 = R_2$	$Y_7 = L_2$	$Y_{11} = 1/C_2$
$Y_3 = R_3$	$Y_8 = L_3$	$Y_{12} = 1/C_3$
$Y_4 = R_4$	$Y_9 = L_4$	$Y_{13} = 1/C_4$
$Y_5 = R_5$		

E-3 Transfer Function

$$\begin{aligned}T = (&1.2 \times 10^{11} s + 5.8 \times 10^{10} s^2 + 6.78 \times 10^9 s^3 \\ &+ 1.5 \times 10^8 s^4 + 9.0 \times 10^5 s^5)/(9.0 \times 10^{12} \\ &+ 7.225 \times 10^{12} s + 1.8186 \times 10^{12} s^2 \\ &+ 1.77245 \times 10^{11} s^3 + 5.5399 \times 10^9 s^4 \\ &+ 5.965 \times 10^7 s^5 + 2.22 \times 10^5 s)\end{aligned}$$

E-4 Example Input and Output

A sample input for the set of thirteen equations in thirteen unknowns and the output which resulted from it are presented in this portion of the appendix. For the sake of brevity, the input items are listed without FORTRAN symbols. The plots from the frequency-response subroutine for this sample input are included in this appendix as Figures E-1, E-2, and E-3. The range of interest of the unknowns is identical to that presented on page 37 of reference 1.

```
              INPUT DATA FOR THIRTEEN EQUATIONS AND THIRTEEN UNK

                                        COLUMN NUMBER
     000000000111111111122222222223333333333444444444455555
     12345678901234567890123456789012345678901234567890123456

     120   30   15   12   10    5    4    4    4
       3   14   28   34   23   15    3    1    2    3    2    1    1
       3    3    3    3    3    3    3    3    3    3    3    3    1
        2.22E+05     1.965E+07  5.5399E+09 1.77245E+11   1.8186E
         9.0E+12      9.0E+05    1.50E+08     6.78E+09     5.60E
         5.0E+02
             .01          .01          .01          .01          .01
             .01          .01          .01          .01          .01
             .24       .00005       .00001     22.5E+06         350.     15.E
       19.5E+02    39.4E+02     49.6E+02     29.7E+02     50.0E+01     50.0E
       10.0E+00    10.0E-05      0.1E-04      0.1E-04      0.1E-04     10.0E
       10.0E+03     0.5E+03     10.0E+01     10.0E+01     10.0E+01     10.0E
     0.667E-04 0.500E-04
                1.0              1.0             1.0             1.0
             1.0
       9    6    7    9    6    8    9    7    8
       4    6    7    4    6    8    4    7    8    5    6    7    5    6
       9    3    6    9    5    6    9    1    7    9    3    7    9    5
       8    9
       6    7   13    6    8   13    7    8   13    6    9   12    7    9
      11    7    9   10    8    9   10    2    4    6    2    5    6    2
       4    6    3    5    6    4    5    6    3    4    7    3    5    7
       1    5    7    1    4    3    1    5    8    1    2    9    1    3
       9    2    7    9
       2    6   13    3    6   13    5    6   13    1    7   13    3    7
      13    2    8   13    4    6   12    5    6   12    4    7   12    5
       9   12    4    6   11    5    6   11    4    8   11    5    8   11
       5    9   11    4    7   10    5    7   10    4    8   10    5    8
      10    3    9   10    1    2    4    1    3    4    2    3    4    1
       3    5
       6   12   13    7   12   13    6   11   13    7   10   13    8   10
      13    1    3   13    1    5   13    1    4   11    1    5   11    1
       3   13    2    5   13    2    4   10    2    5   10    2    4   12
       3    5   10    3    4   11    3    5   11
       1   12   13    1   11   13    2   12   13    2   10   13    3   11
      12    4   10   12    4   10   11    5   12   13    5   11   13    5
      10   12    5   10   11
      11   12   13   10   12   13   10   11   13
       5    7    9
       2    5    9    4    5    7
       5    9   11    5    7   13    2    4    5
       2    5   13    4    5   11
       5   11   13
       5
     0      2222222222222
     D6 D5 D4 D3 D2 D1 D0 D5 D4 D3  2  1
     12
```

E-3-1

```
66666677777777778
45678901234567890

7.225E+12
 1.20E+11

    .01         .01

0.0E+00   40.0E+00
0.0E+03   10.0E+03
0.0E-03 0.833E-04


5   7   8   2   6
9   1   8   9   4

6   9  11   8   9
8   2   5   8   3
5   7   1   4   7
1   5   9   2   3

5   7  13   1   8
2   1   9  12   2
9  11   2   9  11
2   9  10   3   9
5   1   3   5   2

8  11  13   1   2
2   1   5  12   2
5  12   3   4  10

8  10  13   4  11
2   5  10  13   5
```

```
HE FØLLØWING IS THE LIST ØF EQUATIØNS SPECIFIED TØ THE PRØGRAM
HE FØRMAT IS....  EQUATIØN NUMBER
                  TERMS ØF EQUATIØNS (THREE PER LINE)
                  NUMBER ØF EACH TERM

CHECK IS MADE ØF THE UNITS ØF EACH TERM. IF THE UNITS DIFFER
 AN EQUATIØN, AN ERRØR MESSAGE RESULTS



                 EQUATIØN  1


L4 L1 L2            + L4 L1 L3          + L4 L2 L3

  1                     2                   3



                 EQUATIØN  2


 4 L1 L2            + R4 L1 L3          + R4 L2 L3

  1                     2                   3

 5 L1 L2            + R5 L1 L3          + R5 L2 L3

 4                      5                   6

  L1 L4             + R3 L1 L4          + R5 L1 L4

 7                      8                   9

  L2 L4             + R3 L2 L4          + R5 L2 L4

                       11                  12

 L3 L4              + R4 L3 L4

                       14
                 ECUATIØN  3


L1 L2              + L1 L3              + L2 L3
C4                   C4                   C4
  1                     2                    3


L1 L4              + L2 L4              + L1 L4
C3                   C3                   C2
  4                     5                    6


L3 L4              + L2 L4              + L3 L4
C2                   C1                   C1
  7                     8                    9


R2 R4 L1           + R2 R5 L1           + R2 R4 L3

 10                    11                   12


R2 R5 L3           + R3 R4 L1           + R3 R5 L1

 13                    14                   15


R4 R5 L1           + R3 R4 L2           + R3 R5 L2

 16                    17                   18


R4 R5 L2           + R1 R4 L2           + R1 R5 L2

 19                    20                   21


R1 R4 L3           + R1 R5 L3           + R1 R2 L4

 22                    23                   24


R1 R3 L4           + R1 R5 L4           + R2 R3 L4

 25                    26                   27


R2 R5 L4

 28
```

```
                    ECUATIØN  4

R2  L1                  + R3  L1                 + R5  L1
C4                        C4                       C4
   1                           2                        3

R1  L2                  + R3  L2                 + R5  L2
C4                        C4                       C4
   4                           5                        6

R1  L3                  + R2  L3                 + R4  L1
C4                        C4                       C3
   7                           8                        9

R5  L1                  + R4  L2                 + R5  L2
C3                        C3                       C3
  10                          11                       12

R1  L4                  + R2  L4                 + R4  L1
C3                        C3                       C2
  13                          14                       15

R5  L1                  + R4  L3                 + R5  L3
C2                        C2                       C2
  16                          17                       18

R1  L4                  + R2  L4                 + R5  L4
C2                        C2                       C2
  19                          20                       21

R4  L2                  + R5  L2                 + R4  L3
C1                        C1                       C1
  22                          23                       24

R5  L3                  + R2  L4                 + R3  L4
C1                        C1                       C1
  25                          26                       27

R5  L4                  + R1  R2  R4             + R1  R3  R4
C1
  28                          29                       30
```

```
R2 R3 R4              + R1 R2 R5              + R1 R3 R5

 31                      32                      33


R2 R3 R5

 34
```

```
                      EQUATIØN  5

L1                    + L2                    + L1
C3 C4                   C3 C4                   C2 C4
  1                         2                       3


L2                    + L3                    + L3
C1 C4                   C1 C4                   C2 C4
  4                         5                       6


R1  R2                + R1  R3                + R1  R5
C4                      C4                      C4
  7                         8                       9


R1  R4                + R1  R5                + R1  R4
C2                      C2                      C3
 10                        11                      12


R1  R5                + R2  R3                + R2  R5
C3                      C4                      C4
 13                        14                      15


R2  R4                + R2  R5                + R2  R4
C1                      C1                      C3
 16                        17                      18


R2  R5                + R3  R4                + R3  R5
C3                      C1                      C1
 19                        20                      21


R3  R4                + R3  R5
C2                      C2
 22                        23
```

```
            EQUATIØN  6


R1                    + R1                    + R2
C3 C4                   C2 C4                   C3 C4
 1                        2                       3


R2                    + R3                    + R3
C1 C4                   C2 C4                   C1 C4
 4                        5                       6


R4                    + R4                    + R4
C2 C3                   C1 C3                   C1 C2
 7                        8                       9


R5                    + R5                    + R5
C3 C4                   C2 C4                   C2 C3
 10                      11                      12


R5                    + R5                    + R5
C1 C4                   C1 C3                   C1 C2
 13                      14                      15



                      EQUATIØN  7


                          +                       +
          C2 C3 C4            C1 C3 C4               C1 C2 C4
            1                   2                      3



                      EQUATIØN  8


          R5 L2 L4

            1
```

```
                    EQUATIØN  9

    R2 R5 L4            + R4 R5 L2

       1                     2


                    EQUATIØN 10

    R5 L4               + R5 L2                + R2 R4 R5
    C2                    C4
       1                     2                      3


                    EQUATIØN 11

    R2 R5               + R4 R5
    C4                    C2
       1                     2


                    EQUATIØN 12

    R5
    C2 C4
       1


                    EQUATIØN 13

    R5

       1
```

```
 INPUT DATA

MAXIMUM NO. ØF STEPS      120
NUMBER ØF STEPS            30
TIMES THRØUGH RUNGE KUTTA        15
CØNSTANT TERMS
      0.22700000E 06       0.5965CCCCE C8       C.55399CC0E 10
      0.90000000E 13       0.90C0CCC0E C6       C.15CCCCC0E C9
      C.5CCCC000E 03

RANGE FØR VARIABLES
  FXØRIG
    0.195C0000E 04      0.394000C0E 04      C.496C000CE 04      C.29
    0.600C0000E 02      C.40000000E 02      0.3C00000CE 02      C.10
    0.10CCCC00E-04

  FXLIM
    0.10000000E 05      0.100000C0E 05      C.1CCCC0CCE 05      C.10
    0.10000000E 03      0.10000CCCE 03      C.1CCCCCCCE 03      C.10
    0.500CCC00E-04

THERE ARE 13 EQUATIØNS AND 13 UNKNØWNS,CØNSISTING ØF  5 RESISTANC
THE LØWER BØUNDARIES FØR THE RESISTANCES, THE INDUCTANCES, AND TH
AND   0.1CC00000E-04, RESPECTIVELY, WHILE THEIR UPPER BØUNDARIES
  0.15000CCCE 06 RESPECTIVELY.

   LX=    1

   LX=    2

   LX=    3

   LX=    4

   LX=    5

   LX=    6

   LX=    7

   LX=    8

   LX=    9

   LX=    C

   LX=    1

   LX=    2

   LX=    3

   LX=    4

   LX=    5

   LX=    6
   LX=    7

   LX=    8

   LX=    9

   LX=   10

   LX=   11

   LX=   12

   LX=   13

 VARIABLES
       0.19500000E 04       0.39400000E 04       0.49600000E 04
       0.60000000E 02       0.40000000E 02       0.30000000E 02
       0.34199519E 05


 CØMMENCING CØNSTANT APPRØACH

 GRID=    1   NØS=  30
 GRID=    2   NØS=  30
 GRID=    3   NØS=  30
 GRID=    1   NØS=  56
 GRID=    1   NØS= 112
 GRID=    2   NØS= 112
 GRID=    3   NØS= 112
 USING THIS SET ØF ESTIMATES, NØ RØØTS WERE FØUND


 CØMMENCING CØEFFICIENT APPRØACH


 NA=    1
 GRID=    1   NØS=  30
 GRID=    2   NØS=  30
 GRID=    3   NØS=  30
 GRID=    4   NØS=  30
 GRID=    1   NØS=  54
 GRID=    2   NØS=  54
 GRID=    3   NØS=  54
 GRID=    1   NØS= 104
 GRID=    2   NØS= 104
 GRID=    3   NØS= 104
 GRID=    4   NØS= 104
 GRID=    5   NØS= 104
 GRID=    6   NØS= 104
 GRID=    7   NØS= 104
 GRID=    8   NØS= 104
 GRID=    9   NØS= 104
 GRID=   10   NØS= 104
```

E-11-1

```
E 12        0.12186COCE 13       C.7225000Cr 13
E 1C        0.58000CCCE 11       C.1200000CE 12


    0.50000000E 03      0.5000CCCOE 02
    0.10000000E-04      C.10CCCOCCE-04


    0.50000000E 03      0.10CCCCCCE 03
    0.83300000E-04      0.667CCCCOE-04

NDUCTANCE(S),AND  4 CAPACITANCE(S).
NCES ARE   0.24000000E 00,   0.5C000000E-04,
000000E 08,   0.350000COE C3, AND
```

```
CCOE 04        0.50000000E 03       0.5000000CE C2
346E 04        0.120048C2E C5       C.149925C4E C5
```

```
 D=    11    NØS= 104
 D=    12    NØS= 104
ID=    13    NØS= 104
ID=    14    NØS= 104
ID=    15    NØS= 104
ID=    16    NØS= 104
ID=    17    NØS= 104
ID=    18    NØS= 104
ID=    19    NØS= 104
ID=    20    NØS= 104
ID=    21    NØS= 104
ID=    22    NØS= 104
ID=    23    NØS= 104
ID=    24    NØS= 104
ID=    25    NØS= 104
ID=    26    NØS= 104
ID=    27    NØS= 104
ID=    28    NØS= 104
ID=    29    NØS= 104
ID=    30    NØS= 104
ID=    31    NØS= 104
ID=    32    NØS= 104
ID=    33    NØS= 104
ID=    34    NØS= 104
RID=   35    NØS= 104
RID=   36    NØS= 104
RID=   37    NØS= 104
RID=   38    NØS= 104
RID=   39    NØS= 104
RID=   40    NØS= 104
RID=   41    NØS= 104
RID=   42    NØS= 104
RID=   43    NØS= 104
RID=   44    NØS= 104
RID=   45    NØS= 104
RID=   46    NØS= 104
RID=   47    NØS= 104
RID=   48    NØS= 104
RID=   49    NØS= 104
RID=   50    NØS= 104
RID=   51    NØS= 104
RID=   52    NØS= 104
RID=   53    NØS= 104
RID=   54    NØS= 104
RID=   55    NØS= 104
RID=   56    NØS= 104
RID=   57    NØS= 104
RID=   58    NØS= 104
RID=   59    NØS= 104
RID=   60    NØS= 104
RID=   61    NØS= 104
RID=   62    NØS= 104
RID=   63    NØS= 104
RID=   64    NØS= 104
```

```
GRID=  65   NØS= 104
GRID=  66   NØS= 104
GRID=  67   NØS= 104
GRID=  68   NØS= 104
GRID=  69   NØS= 104
GRID=  70   NØS= 104
GRID=  71   NØS= 104
GRID=  72   NØS= 104
GRID=  73   NØS= 104
GRID=  74   NØS= 104
GRID=  75   NØS= 104
GRID=  76   NØS= 104
GRID=  77   NØS= 104
GRID=  78   NØS= 104
GRID=  79   NØS= 104
GRID=  80   NØS= 104
GRID=  81   NØS= 104
GRID=  82   NØS= 104
GRID=  83   NØS= 104
GRID=  84   NØS= 104
GRID=  85   NØS= 104
GRID=  86   NØS= 104
GRID=  87   NØS= 104
GRID=  88   NØS= 104
GRID=  89   NØS= 104
GRID=  90   NØS= 104
GRID=  91   NØS= 104
GRID=  92   NØS= 104
GRID=  93   NØS= 104
GRID=  94   NØS= 104
GRID=  95   NØS= 104
GRID=  96   NØS= 104
GRID=  97   NØS= 104
GRID=  98   NØS= 104
GRID=  99   NØS= 104
GRID= 100   NØS= 104
GRID= 101   NØS= 104
GRID= 102   NØS= 104
GRID= 103   NØS= 104
GRID= 104   NØS= 104


ALL RØØTS IN THE FØLLØWING SET LIE WITHIN THE PHYSICAL LIMITS SPECIFIED


    R( 1)=  0.19999993E 04  ØHMS        L( 1)=  0.49999994E 02  HENRIES
    R( 2)=  0.39999999E 04  ØHMS        L( 2)=  0.60000002E 02  HENRIES
    R( 3)=  0.50000002E 04  ØHMS        L( 3)=  0.40000006E 02  HENRIES
    R( 4)=  0.30000000E 04  ØHMS        L( 4)=  0.29999999E 02  HENRIES
    R( 5)=  0.50000000E 03  ØHMS
```

E-13-1

```
=   0.99999958E-04   FARACS
=   0.99999958E-04   FARACS
=   0.99999958E-04   FARACS
=   0.99999958E-04   FARACS
```

```
ØR CAPACITØR C 4 THE  1 CØMPØNENT(S) ARE
                     0.46999999E 08

 4 IS THUS   0.46999999E 08 MICRØMICRØFARADS



ØR CAPACITØR C 3 THE  2 CØMPØNENT(S) ARE
                     0.56000000E 08
                     0.10000000E 08

 3 IS THUS   0.66000000E 08 MICRØMICRØFARADS



FØR CAPACITØR C 2 THE  2 CØMPØNENT(S) ARE
                     0.68000000E 08
                     0.15000000E 08

C 2 IS THUS   0.83000000E 08 MICRØMICRØFARADS



FØR CAPACITØR C 1 THE  2 CØMPØNENT(S) ARE
                     0.68000000E 08
                     0.33000000E 08

C 1 IS THUS   0.10100000E 09 MICRØMICRØFARADS



FØR INDUCTØR L 4 THE  1 CØMPØNENT(S) ARE
                     0.30000000E 02

L 4 IS THUS   0.30000000E 02 HENRIES, AND
THE INDUCTIVE PART ØF R 4 IS   0.30000000E 02 ØHMS



FØR INDUCTØR L 3 THE  1 CØMPØNENT(S) ARE
                     0.40000000E 02

L 3 IS THUS   0.40000000E 02 HENRIES, AND
THE INDUCTIVE PART ØF R 3 IS   0.40000000E 02 ØHMS



FØR INDUCTØR L 2 THE  2 CØMPØNENT(S) ARE
                     0.50000000E 02
                     0.10000000E 02

L 2 IS THUS   0.60000000E 02 HENRIES, AND
THE INDUCTIVE PART ØF R 2 IS   0.51000000E 03 ØHMS
```

```
FØR INDUCTØR L 1 THE  1 CØMPØNENT(S) ARE
                  0.50000000E 02

L 1 IS THUS   0.50000000E 02 HENRIES, AND
THE INDUCTIVE PART ØF R 1 IS   0.50000000E 02 ØHMS



FØR RESISTØR R 5 THE  2 CØMPØNENT(S) ARE
                  0.46400000E 03
                  0.34800000E 02

R 5 IS THUS   0.49880000E 03 ØHMS



FØR RESISTØR R 4 THE  2 CØMPØNENT(S) ARE
                  0.28700000E 04
                  0.90900000E 02

WITH AN INDUCTIVE RESISTANCE ØF  0.30000000E 02 ØHMS

R 4 IS THUS   0.29909000E 04 ØHMS



FØR RESISTØR R 3 THE  2 CØMPØNENT(S) ARE
                  0.42200000E 04
                  0.23700000E 03

WITH AN INDUCTIVE RESISTANCE ØF  0.40000000E 02 ØHMS

R 3 IS THUS   0.44970000E 04 ØHMS



FØR RESISTØR R 2 THE  2 CØMPØNENT(S) ARE
                  0.38300000E 04
                  0.16200000E 03

WITH AN INDUCTIVE RESISTANCE ØF  0.51000000E 03 ØHMS

R 2 IS THUS   0.45020000E 04 ØHMS



FØR RESISTØR R 1 THE  2 CØMPØNENT(S) ARE
                  0.17800000E 04
                  0.16200000E 03

WITH AN INDUCTIVE RESISTANCE ØF  0.50000000E 02 ØHMS

R 1 IS THUS   0.19920000E 04 ØHMS
```

```
THE NUMERATØR IS ØF ØRDER  5. THE PØLYNØMIAL IN DESCENDING ØRCER BELØW

   0.89783998L C6  C.15687958E C9  C.75334129E 10  C.65752899E 11
   0.12786465E 12  C.C00C00C0E-38

THE RØØTS ARE-
 REAL PART           IMAG. PART            REAL PART          IMAG. PART
-C.72254E C2         C.CC00CL-38          -0.91987E 02        0.0000CE-38
-C.27791E C1         C.CC00CE-38          -0.771C0E C1        0.0000CE-38
 0.00C0CE-38         C.0C00CE-38

THE DENØMINATØR IS ØF ØRCER  6. THE PØLYNØMIAL IN DESCENDING ØRCER BELØW

   0.22200CCC0L C6  C.58627997E C8  C.551146C2E 10  0.17911432E 12
   0.18877981E 13  C.76232013E 13  C.96138811E 13

THE RØØTS ARE-
 REAL PART           IMAG. PART            REAL PART          IMAG. PART
-C.10691E C3        -C.45353E C2          -0.1C691E 03        0.45353E 02
-C.35567E C2         C.CC00CE-38          -0.61914E 01       -0.70123E CC
-C.61914E C1         C.70123E C0          -0.23254E 01        0.00000E-38
```

RAD/SEC	F-CYCLES/SEC	AMPLITUDE	20LOG AMP	PHASE-DEG
0628	0.00100	0.0500	-46.02858	-0.00367
0695	0.00111	0.0500	-46.02858	-0.00406
0769	0.00122	0.0500	-46.02858	-0.00449
0851	0.00135	0.0500	-46.02857	-0.00497
0942	0.00150	0.0500	-46.02857	-0.00550
1043	0.00166	0.0500	-46.02856	-0.00609
1154	0.00184	0.0500	-46.02856	-0.00674
1277	0.00203	0.0500	-46.02855	-0.00746
1413	0.00225	0.0500	-46.02854	-0.00825
1563	0.00249	0.0500	-46.02852	-0.00913
1730	0.00275	0.0500	-46.02851	-0.01010
1914	0.00305	0.0500	-46.02849	-0.01118
2118	0.00337	0.0500	-46.02846	-0.01237
2344	0.00373	0.0500	-46.02843	-0.01369
2594	0.00413	0.0500	-46.02839	-0.01515
2870	0.00457	0.0500	-46.02835	-0.01677
3176	0.00505	0.0500	-46.02829	-0.01856
3514	0.00559	0.0500	-46.02822	-0.02055
3889	0.00619	0.0500	-46.02814	-0.02274
4303	0.00685	0.0500	-46.02804	-0.02518
4762	0.00758	0.0500	-46.02791	-0.02787
5269	0.00839	0.0500	-46.02776	-0.03086
5831	0.00928	0.0500	-46.02757	-0.03417
6452	0.01027	0.0500	-46.02734	-0.03785
7140	0.01136	0.0500	-46.02706	-0.04193
7901	0.01257	0.0500	-46.02671	-0.04647
8743	0.01391	0.0500	-46.02629	-0.05150
9674	0.01540	0.0500	-46.02578	-0.05710
10705	0.01704	0.0500	-46.02515	-0.06335
11846	0.01885	0.0500	-46.02437	-0.07031
13109	0.02086	0.0500	-46.02342	-0.07809
14506	0.02309	0.0500	-46.02227	-0.08681
16052	0.02555	0.0500	-46.02085	-0.09659
17762	0.02827	0.0500	-46.01912	-0.10760
19655	0.03128	0.0500	-46.01700	-0.12004
21750	0.03462	0.0500	-46.01441	-0.13414
24068	0.03831	0.0501	-46.01124	-0.15021
26633	0.04239	0.0501	-46.00738	-0.16862
29471	0.04690	0.0501	-46.00266	-0.18984
32612	0.05190	0.0501	-45.99690	-0.21445
36088	0.05744	0.0502	-45.98989	-0.24322
39933	0.06356	0.0502	-45.98135	-0.27711
44189	0.07033	0.0503	-45.97096	-0.31737
48898	0.07782	0.0504	-45.95835	-0.36564
54110	0.08612	0.0504	-45.94308	-0.42400
59876	0.09530	0.0506	-45.92461	-0.49519
66257	0.10545	0.0507	-45.90235	-0.58271
73318	0.11669	0.0508	-45.87560	-0.69115
81132	0.12913	0.0510	-45.84360	-0.82638
89778	0.14289	0.0513	-45.80551	-0.99590

MEGA-RAD/SEC	F-CYCLES/SEC	AMPLITUDE	20LOG AMP	PHASE-DEG
0.99346	0.15811	0.00515	-45.76044	-1.20929
1.09934	0.17496	0.00518	-45.70750	-1.47854
1.21649	0.19361	0.00522	-45.64587	-1.81857
1.34614	0.21424	0.00526	-45.57488	-2.24757
1.48960	0.23708	0.00531	-45.49416	-2.78739
1.64835	0.26234	0.00537	-45.40376	-3.46361
1.82401	0.29030	0.00543	-45.30441	-4.30533
2.01840	0.32124	0.00550	-45.19769	-5.34467
2.23350	0.35547	0.00557	-45.08624	-6.61551
2.47153	0.39336	0.00564	-44.97396	-8.15186
2.73492	0.43528	0.00571	-44.86608	-9.98558
3.02639	0.48166	0.00577	-44.76923	-12.14369
3.34891	0.53300	0.00583	-44.69122	-14.64553
3.70581	0.58980	0.00586	-44.64079	-17.50012
4.10074	0.65265	0.00587	-44.62719	-20.70410
4.53777	0.72221	0.00585	-44.65959	-24.24063
5.02136	0.79917	0.00579	-44.74648	-28.07962
5.55649	0.88434	0.00569	-44.89512	-32.17928
6.14866	0.97859	0.00555	-45.11104	-36.48898
6.80393	1.08288	0.00537	-45.39773	-40.95298
7.52903	1.19828	0.00515	-45.75663	-45.51461
8.33141	1.32599	0.00491	-46.18717	-50.12015
9.21930	1.46730	0.00463	-46.68720	-54.72206
10.20181	1.62367	0.00434	-47.25335	-59.28102
11.28903	1.79671	0.00404	-47.88155	-63.76680
12.49212	1.98818	0.00373	-48.56748	-68.15797
13.82342	2.20007	0.00342	-49.30693	-72.44085
15.29660	2.43453	0.00313	-50.09602	-76.60782
16.92678	2.69398	0.00284	-50.93140	-80.65547
18.73068	2.98108	0.00257	-51.81019	-84.58277
20.72684	3.29878	0.00231	-52.72999	-88.38958
22.93572	3.65033	0.00207	-53.68869	-92.07549
25.38001	4.03935	0.00184	-54.68429	-95.63930
28.08479	4.46983	0.00164	-55.71472	-99.07911
1.07782	4.94619	0.00145	-56.77766	-102.39296
4.38983	5.47331	0.00128	-57.87044	-105.58017
8.05480	6.05661	0.00112	-58.99003	-108.64289
2.11034	6.70207	0.00098	-60.13314	-111.58784
6.59810	7.41632	0.00086	-61.29654	-114.42774
1.56412	8.20668	0.00075	-62.47734	-117.18196
7.05937	9.08128	0.00066	-63.67353	-119.87612
3.14026	10.04908	0.00057	-64.88435	-122.54027
9.86919	11.12003	0.00049	-66.11065	-125.20576
7.31524	12.30510	0.00043	-67.35501	-127.90099
5.55482	13.61647	0.00037	-68.62156	-130.64671
4.67251	15.06760	0.00032	-69.91551	-133.45203
4.76187	16.67337	0.00027	-71.24236	-136.31196
5.92647	18.45027	0.00023	-72.60707	-139.20741
8.28090	20.41654	0.00020	-74.01321	-142.10784
1.95196	22.59236	0.00017	-75.46248	-144.97586

```
GA-RAC/SEC F-CYCLES/SEC   AMPLITUDE        20LØG AMP      PHASE-CEG

7.08CCC        25.00C06          C.C0014      -76.95457    -147.7727C
```

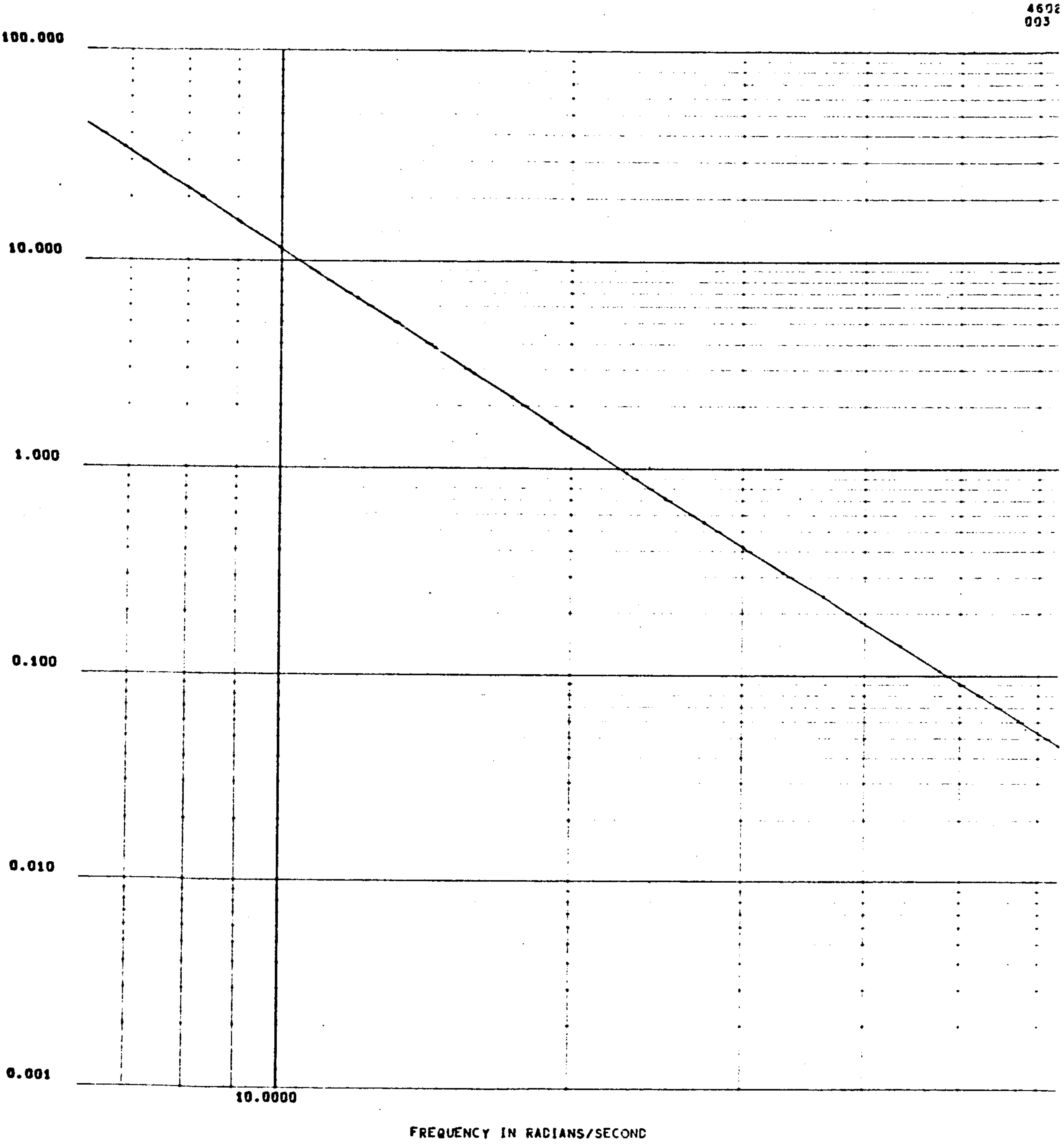


Figure E-3. GAIN VERSUS FREQUENCY
THIRTEEN EQUATIONS, THIRTEEN UNKNOWNS

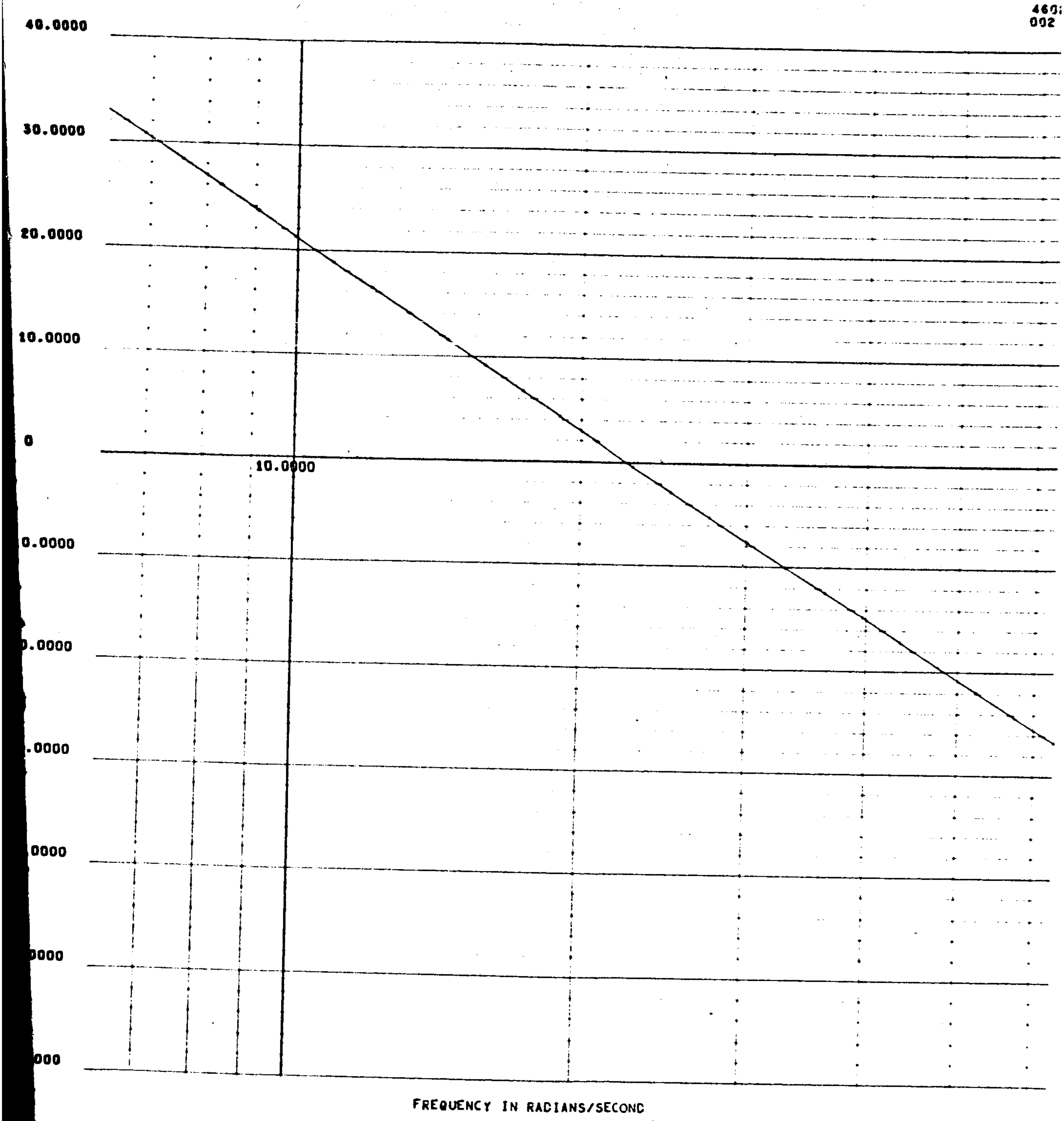


Figure E-1. AMPLITUDE VERSUS FREQUENCY
THIRTEEN EQUATIONS, THIRTEEN UNKNOWNS

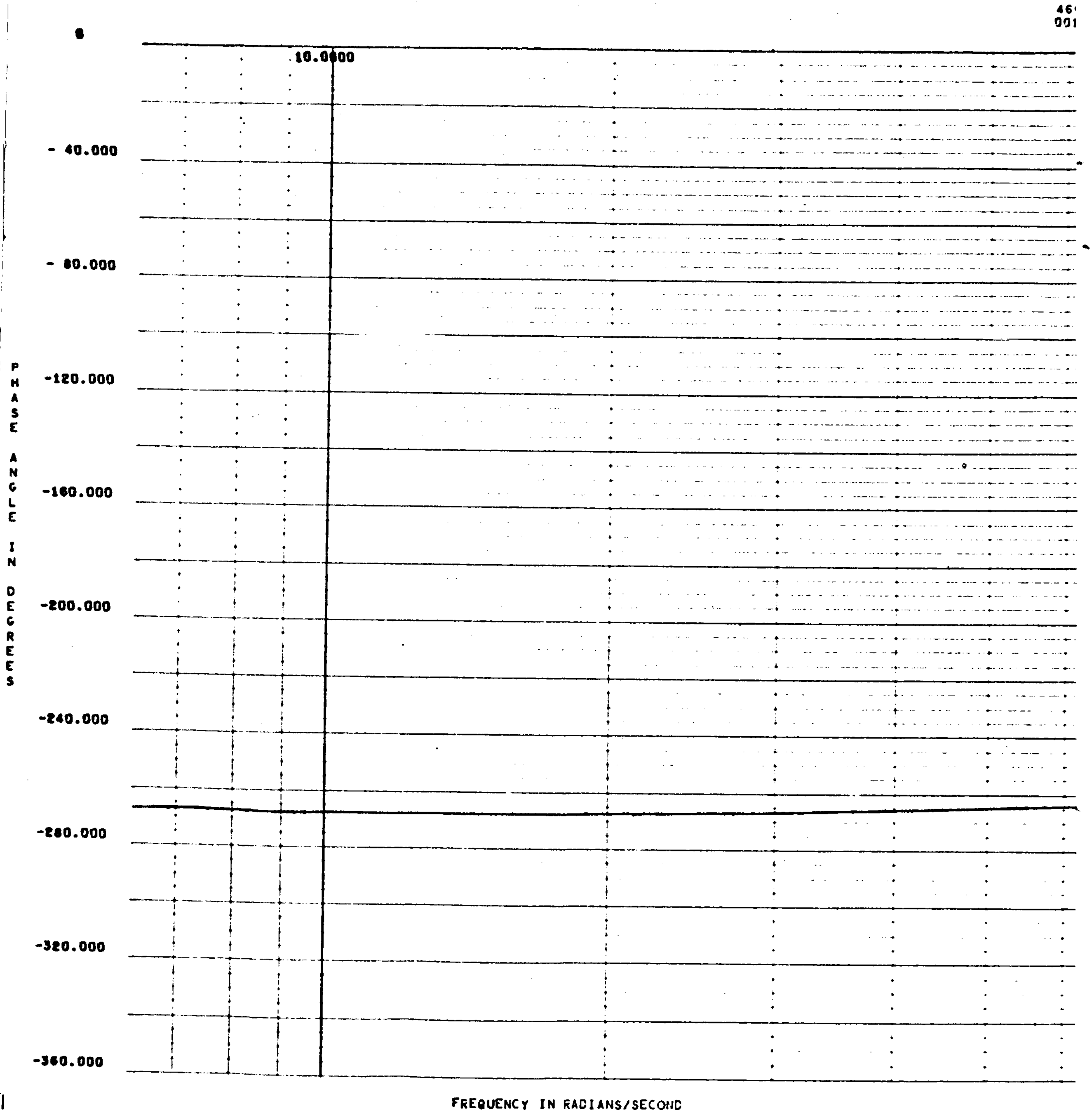


Figure E-2. PHASE SHIFT VERSUS FREQUENCY
THIRTEEN EQUATIONS, THIRTEEN UNKNOWNS

APPENDIX F

FILTER CIRCUIT WITH FIFTEEN UNKNOWNS

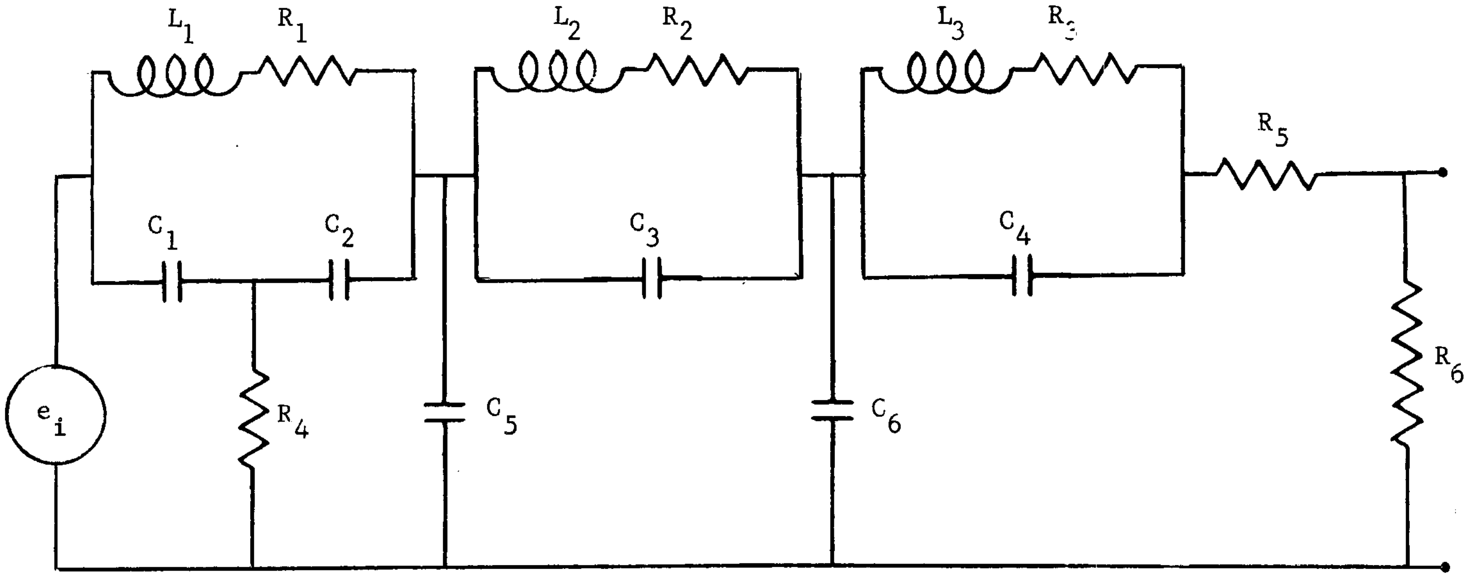
The circuit and its transfer function, from which the set of 16 equations was derived, is shown in Figure F-1. The transfer function is shown in a normalized form in equation (F-1). In order to obtain the true constants for the equations, it was necessary to find the true values of N_o and D_7. The remaining coefficients in the polynomial could then be found. After establishment of the coefficients, the circuit was scaled by a factor of 10^{-6}, changing the constant terms by a factor of 10^{-42}, in order to prevent overflow on the IBM 7094. The resulting transfer function is shown in equation (F-2).

The values of the circuit elements, as given in reference 18, are provided in Table F-1 below. The equations themselves, derived during the course of the study, are presented on the pages following Figure F-1.

Table F-1.

COMPONENT VALUES FOR THE FIFTEEN-ELEMENT CIRCUIT

Resistors	Inductors	Capacitors
R(1) 4580 Ω	L(1) 1400 h	C(1) 14 μf
R(2) 5700 Ω	L(2) 1200 h	C(2) 14 μf
R(3) 8610 Ω	L(3) 900 h	C(3) .6 μf
R(4) 12000 Ω		C(4) .7 μf
R(5) 220 K Ω		C(5) 10 μf
R(6) 2000 Ω		C(6) 10 μf



$$T = \frac{1 + .345s + .144\times10^{-1}s^2 + .385\times10^{-2}s^3 + .465\times10^{-4}s^4 + .473\times10^{-5}s^5 + .251\times10^{-7}s^6 + .149\times10^{-8}s^7}{1 + .565s + .137s^2 + .237\times10^{-1}s^3 + .163\times10^{-2}s^4 + .125\times10^{-3}s^5 + .17\times10^{-5}s^6 + .610\times10^{-7}s^7} \tag{F-1}$$

$$T = \frac{.243\times10^{-6} + .839\times10^{-7}s + .372\times10^{-8}s^2 + .938\times10^{-9}s^3 + .118\times10^{-10}s^4 + .115\times10^{-11}s^5 + .639\times10^{-14}s^6 + .363\times10^{-15}s^7}{.293\times10^{-4} + .166\times10^{-4}s + .395\times10^{-5}s^2 + .676\times10^{-6}s^3 + .455\times10^{-7}s^4 + .369\times10^{-8}s^5 + .511\times10^{-10}s^6 + .179\times10^{-11}s^7} \tag{F-2}$$

Figure F-1. CIRCUIT DIAGRAM

EQUATION 1 (N_0)

F_j (normalized) = 1.0

F_j (scaled) = $.243 \times 10^{-6}$

```
R6
C1 C2 C3 C4 C5 C6
```

EQUATION 2 (N_1)

F_j (normalized) = .345

F_j (scaled) = $.839 \times 10^{-7}$

```
R6 R3                + R2 R6                + R6 R4
C4 C5 C2 C6 C3         C5 C1 C6 C3 C4         C1 C6 C2 C4 C5

R6 R4
C2 C3 C5 C1 C6
```

EQUATION 3 (N_2)

F_j (normalized) = $.144 \times 10^{-1}$

F_j (scaled) = $.372 \times 10^{-8}$

```
R6 L2 R3             + L3 R6                + R6 R1
C2 C3 C5 C6            C1 C1 C4 C5 C6         C1 C2 C2 C5 C6

R6 R2 R4             + R6 R3 R4             + R6 R3 R4
C2 C4 C5 C6            C3 C5 C6 C3            C5 C6 C1 C4

R6 R2 R4             + R6 R2 R4
C6 C1 C4 C5            C2 C3 C5 C6
```

EQUATION 4 (N_3)

F_j (normalized) = $.385 \times 10^{-2}$

F_j (scaled) = $.938 \times 10^{-9}$

R6 R4 R2 C2 C5 C6 C2	+ R2 L2 R4 R4 C5 C6 C3	+ L3 R6 L3 C6 C2 C4 C5
R6 R2 C1 C4 C5 C1 C6	+ R4 R3 R6 C2 C5 C6 C3	+ L1 R4 R6 R4 C5 C6 C5
R6 R1 L2 C6 C3 C1 C6	+ R6 R2 R3 R6 C4 C5 C1	+ R3 R3 R4 R6 C5 C6 C4
L2 R4 R6 R4 C6 C3 C5	+ R6 R1 L3 C1 C2 C5 C6	

EQUATION 5 (N_4)

F_j (normalized) = $.465 \times 10^{-4}$

F_j (scaled) = $.118 \times 10^{-10}$

R6 L2 R4 R2 R4 C6 C5	+ L2 R6 L3 R4 C4 C6 C5	+ L3 R3 R6 L3 C5 C1 C6
R4 R3 R6 C1 C6 C5 C2	+ R6 L1 R4 R3 C2 C6 C5	+ R2 R6 L2 R4 C5 C3 C6
R4 R1 R6 L2 C6 C5 C2	+ R6 L1 R4 R1 C6 C5 C1	+ R1 R6 L3 R2 C4 C6 C5
R4 R2 R6 R3 C5 C3 C6		

EQUATION 6 (N_5)

F_j (normalized) = $.473 \times 10^{-5}$

F_j (scaled) = $.115 \times 10^{-11}$

R6 L3 R3 R1 C6 C5 C2	+ R4 R6 R4 R2 C6 C5 C3	+ L1 R2 R6 L2 R4 C6 C5
L2 R3 R4 R6 L3 C6 C5	+ R4 L2 R4 R6 C4 C6 C5	+ L1 L3 L1 R1 R6 C5 C6
R6 R4 L2 L3 C1 C5 C6		

EQUATION 7 (N_6)

F_j (normalized) = $.251 \times 10^{-7}$

F_j (scaled) = $.639 \times 10^{-14}$

$$\begin{array}{lll} R6\ L2\ R4\ R3\ L2 & +\ R1\ L3\ R6\ L1\ R4 & +\ R4\ R2\ L3\ R6\ L1 \\ C6\ C5 & \ \ C6\ C5 & \ \ C5\ C6 \end{array}$$

EQUATION 8 (N_7)

F_j(normalized) = $.149 \times 10^{-8}$

F_j (scaled) - $.363 \times 10^{-15}$

$$\begin{array}{l} R6\ R4\ L1\ L2\ L3 \\ C5\ C6 \end{array}$$

EQUATION 9 (D_7)

F_j (normalized) = $.610 \times 10^{-7}$

F_j (scaled) = $.179 \times 10^{-11}$

$$\begin{array}{lll} R4\ L1\ L3\ R6\ L2 & +\ R5\ L2\ R4\ L1\ L3 & +\ L1\ L3\ R6\ L2\ R4 \\ C3\ C3 & \ \ C1\ C5 & \ \ C6\ C2 \\ \\ L2\ R4\ L1\ L3\ R5 & +\ L3\ R6\ L2\ R4\ L1 & +\ R4\ L1\ L3\ R5\ L2 \\ C1\ C6 & \ \ C5\ C2 & \ \ C1\ C5 \\ \\ R6\ L2\ R4\ L1\ L3 & +\ R4\ L1\ L3\ R5\ L2 & +\ R6\ L2\ R4\ L1\ L3 \\ C3\ C2 & \ \ C3\ C5 & \ \ C1\ C6 \\ \\ L1\ L3\ R5\ L2\ R4 & +\ L2\ R4\ L1\ L3\ R6 & +\ L3\ R5\ L2\ R4\ L1 \\ C6\ C3 & \ \ C1\ C5 & \ \ C5\ C2 \\ \\ R4\ L1\ L3\ R6\ L2 & +\ R5\ L2\ R4\ L1\ L3 & +\ R4\ L1\ L3\ R6\ L2 \\ C1\ C6 & \ \ C3\ C2 & \ \ C5\ C5 \\ \\ R5\ L2\ R4\ L1\ L3 & & \\ C2\ C6 & & \end{array}$$

EQUATION 10 (D_6)

F_j (normalized) = $.17 \times 10^{-5}$

F_j (scaled) = $.511 \times 10^{-10}$

```
R5 L1 L2 L1 L3     + L1 L2 L3 L3 R3     + L2 L3 R1 R4
C2 C5                C1 C5                C3 C1 C6

L3 R2 R4 R5        + R3 R4 R5 L1        + R4 R4 R5 L2 L2
C1 C4 C5             C1 C4 C5             C2 C5

R4 L1 R5 L1 L3     + R6 L1 L2 L1 L3     + L1 L2 L3 L2 R3
C3 C3                C2 C5                C2 C5

L2 L3 R1 R4        + L3 R2 R4 R6        + R3 R4 R6 L1
C3 C1 C3             C1 C5 C5             C1 C4 C6

R4 R4 R6 L2 L2     + R4 L1 R6 L1 L3     + R5 L1 L2 L1 L3
C2 C6                C3 C3                C2 C5

L1 L2 L3 L2 R1     + L2 L3 R2 R4        + L3 R3 R4 R5
C2 C5                C4 C1 C3             C1 C5 C5

R1 R4 R5 L2        + R2 R4 R5 L1 L3     + R5 R4 R5 L1 L3
C1 C5 C6             C2 C6                C5 C3

R6 L1 R5 L2 L2     + L1 L2 L1 L3 R1     + L2 L3 L3 R2 R4
C2 C5                C2 C6                C1 C3

L3 R3 R4 R6        + R1 R4 R6 L2        + R2 R4 R6 L1 L3
C1 C1 C5             C1 C5 C3             C2 C6

R6 R4 R6 L1 L3     + R5 L1 R6 L2 L2     + L1 L2 L1 L3 R2
C5 C3                C2 C5                C2 C6

L2 L3 L3 R3 R4     + L3 R1 R4 R5        + R2 R4 R5 L1
C1 C3                C1 C1 C5             C1 C5 C3

R3 R4 R5 L1 L3     + R4 R4 R5 L2 L2     + R6 L1 R5 L1 L3
C2 C6                C6 C5                C2 C6

L1 L2 L1 L3 R2     + L2 L3 L2 R3 R4     + L3 R1 R4 R6
C2 C6                C1 C3                C2 C1 C6

R2 R4 R6 L1        + R3 R4 R6 L1 L3     + R4 R4 R6 L2 L2
C1 C3 C3             C2 C4                C6 C5

R4 L1 R6 L1 L3     + L1 L2 L1 L3 R3     + L2 L3 L2 R1 R4
C2 C6                C2 C6                C1 C3

L3 R2 R4 R5        + R3 R4 R5 L1        + R1 R4 R5 L2 L2
C2 C1 C6             C1 C3 C3             C3 C6

R4 R4 R5 L1 L3     + R5 L1 R5 L1 L3     + L1 L2 L2 L2 R3
C4 C5                C2 C6                C2 C6
```

EQUATION 10 (D_6) (Concluded)

```
L2 L3 L3 R1 R4      + L3 R2 R4 R6         + R3 R4 R6 L1
C1 C5                 C2 C1 C6              C1 C4 C3

R1 R4 R6 L2 L2      + R4 R4 R6 L1 L3      + R6 L1 R6 L1 L3
C3 C5                 C5 C5                 C2 C6

L1 L2 L2 L2 R1      + L2 L3 L3 R2 R4      + L3 R3 R4 R5
C2 C6                 C1 C5                 C2 C1 C6

R1 R4 R5 L2         + R2 R4 R5 L1 L3      + R4 R4 R5 L1 L3
C1 C4 C3              C3 C6                 C5 C5

R4 L1 R5 L2 L2      + L1 L2 L1 L3 R1      + L2 L3 L3 R2 R4
C3 C6                 C2 C5                 C1 C5

L3 R3 R4 R6         + R1 R4 R6 L2         + R2 R4 R6 L1 L3
C2 C1 C6              C1 C5 C5              C3 C6

R4 R4 R6 L1 L3      + R4 L1 R6 L2 L2
C6 C5                 C3 C6
```

EQUATION 11 (D_5)

F_j (normalized) = $.125 \times 10^{-3}$

F_j (scaled) = $.369 \times 10^{-8}$

```
R2 L2 R3 L2           + R4 R4 R1 L3           + L1 R4 R5 R2
C3 C5 C1                C5 C2 C2                C4 C3 C1

L3 R6 L2 R4 R3        + L2 R4 R6 R5           + L3 R6 L3 L1 R1
C5 C3                   C1 C1 C5                C3 C3

L1 R1 L2 R6           + R3 L2 R3 L2           + R4 R4 R1 L3
C6 C2 C2                C3 C5 C1                C5 C2 C2

L1 R3 R6 R3           + L2 R4 L2 R4 R3        + L1 R2 R5 R6
C4 C3 C1                C5 C3                   C1 C1 C5

L2 R4 L2 L1 R4        + L1 R2 L2 R5           + R4 L3 R3 L2
C3 C3                   C6 C1 C2                C3 C5 C1

R5 R4 R1 L3           + L1 R4 R5 R3           + L3 R5 L1 R4 R1
C4 C2 C2                C4 C3 C1                C5 C3

L1 R1 R6 R5           + L2 R4 L2 L2 R4        + L2 R2 L3 R6
C1 C1 C5                C3 C5                   C6 C1 C2

R4 L3 R3 L2           + R6 R4 R2 L3           + L1 R4 R6 R3
C3 C5 C1                C4 C2 C2                C4 C3 C1

L3 R6 L1 R4 R1        + L1 R3 R5 R6           + L2 R4 L1 L2 R2
C5 C3                   C1 C1 C5                C3 C5

L1 R1 L3 R4           + R1 L2 R2 L1           + R4 R4 R2 L3
C6 C1 C2                C3 C6 C1                C4 C2 C2

L2 R2 R5 R3           + L3 R4 L3 R4 R2        + L1 R4 R6 R5
C4 C3 C1                C6 C5                   C1 C1 C6

L3 R5 L1 L1 R2        + L1 R1 L3 R5           + R2 L2 R2 L1
C3 C5                   C6 C1 C2                C3 C6 C1

R4 R4 R1 L3           + L1 R1 R6 R2           + L3 R4 L3 R4 R2
C4 C2 C2                C4 C5 C1                C6 C5

L2 R4 R5 R6           + L3 R6 L3 L1 R2        + L1 R1 L3 R6
C2 C1 C6                C3 C5                   C6 C1 C2

R3 L2 R3 L1           + R4 R4 R1 L3           + L1 R3 R5 R2
C3 C6 C1                C4 C2 C2                C4 C5 C1

L2 R4 L2 R4 R3        + L1 R2 R6 R5           + L2 R4 L3 L1 R3
C6 C5                   C2 C1 C6                C3 C5

L1 R1 L2 R4           + R4 L3 R3 L1           + R5 R4 R1 L2
C6 C1 C2                C4 C6 C1                C5 C2 C2
```

```
L1 R4 R6 R3              + L3 R5 L2 R4 R3           + L1 R1 R5 R6
C4 C5 C1                   C6 C5                      C2 C1 C6

L2 R4 L2 L1 R3           + L2 R2 L2 R5              + R4 L3 R3 L1
C3 C5                      C6 C1 C2                   C4 C6 C1

R6 R4 R1 L2              + L1 R4 R5 R3              + L3 R6 L1 R4 R1
C5 C2 C2                   C4 C5 C1                   C6 C5

L1 R3 R6 R5              + L2 R4 L2 L2 R3           + L1 R2 L3 R6
C2 C1 C6                   C3 C6                      C6 C1 C2

R1 L2 R3 L1              + R4 R4 R2 L2              + L2 R2 R6 R3
C4 C6 C1                   C5 C3 C2                   C4 C5 C1

L3 R4 L1 R4 R1           + L1 R4 R5 R6              + L3 R5 L1 L2 R1
C5 C5                      C2 C1 C6                   C3 C6

L1 R1 L3 R5              + R2 L2 R2 L2              + R4 R4 R2 L3
C6 C1 C3                   C4 C5 C1                   C5 C3 C2

L1 R1 R5 R3              + L3 R4 L3 R4 R2           + L2 R4 R6 R5
C4 C5 C1                   C5 C5                      C3 C1 C6

L3 R6 L1 L1 R1           + L1 R1 L3 R6              + R3 L2 R2 L2
C5 C6                      C6 C1 C3                   C4 C5 C1

R4 R4 R1 L3              + L1 R3 R6 R2              + L2 R4 L3 R4 R2
C5 C3 C2                   C4 C6 C1                   C5 C5

L1 R2 R5 R6              + L2 R4 L3 L1 R4           + L1 R1 L3 R5
C3 C1 C6                   C5 C6                      C6 C1 C3

R4 L3 R3 L2              + R5 R4 R1 L3              + L1 R4 R5 R2
C4 C5 C1                   C6 C3 C2                   C4 C6 C1

L3 R5 L2 R4 R3           + L1 R1 R6 R5              + L2 R4 L3 L1 R4
C5 C5                      C3 C1 C6                   C5 C6

L2 R1 L2 R6              + R4 L3 R3 L2              + R6 R4 R1 L3
C6 C1 C3                   C4 C5 C1                   C6 C3 C2

L1 R4 R6 R3              + L3 R6 L2 R4 R3           + L1 R3 R5 R6
C4 C6 C1                   C5 C5                      C3 C1 C6

L2 R4 L2 L1 R1           + L1 R2 L2 R4              + R1 L2 R3 L2
C5 C6                      C6 C1 C3                   C4 C5 C1

R4 R4 R1 L3              + L2 R2 R5 R3              + L3 R4 L1 R4 R1
C6 C4 C2                   C5 C6 C1                   C6 C5

L1 R4 R6 R5              + L3 R5 L2 L2 L1           + L1 R2 L3 L2
C3 C2 C6                   C5 C3                      C6 C1 C3
```

EQUATION 11 (D_5) (Concluded)

```
R2 L2 R3 L3           + R4 R4 R2            + L1 R1 R6 R3
C4 C5 C1                C6 C4 C2 C1           C5 C6 C2

L3 R4 L1 R4 R1        + L2 R4 R5 R6         + L3 R6 L1 L2 L1
C6 C3                   C2 C2 C4              C5 C3

L1 R1 L3 L2           + R3 L2 R2 L3         + R4 R4 R2
C6 C1 C5                C4 C6 C1              C6 C4 C2 C1

L1 R3 R5 R3           + L2 R4 L3 R4 R4      + L1 R2 R6 R5
C5 C6 C2                C6 C3                 C2 C2 C6

L2 R4 L1 L1 L1        + L1 R1 L3 L2         + R1 L3 R2 L3
C5 C3                   C6 C2 C5              C3 C6 C1

R4 R4 R1              + L2 R4 R6 R2         + L3 R5 L3 R4 R4
C4 C4 C3 C1             C5 C5 C2              C6 C4

L1 R1 R5 R6           + L2 R4 L3 L1 L1      + L2 R1 L3 L2
C2 C2 C5                C5 C3                 C6 C2 C5

R4 L3 R3 L3           + R5 R4 R1            + L2 R4 R5 R2
C3 C6 C1                C4 C4 C3 C1           C5 C5 C2

L3 R6 L2 R4 R2        + L1 R2 R6 R5         + L2 R3 L3 L1 L1
C6 C4                   C2 C2 C6              C5 C3

R4 R1 L3 L2           + R4 R5 R3 L3         + R6 L1 R4 R1
C6 C2 C5                C3 C6 C1              C4 C3 C1

L2 R2 R6 R3           + L3 R4 L2 R4 R2      + L1 R2 R5 R6
C1 C5 C2                C3 C5                 C2 C2 C6

L3 R3 L2 L1 L1        + R4 R2 L3 L2         + R4 R6 R3 L3
C5 C3                   C6 C2 C5              C3 C6 C1

R5 L1 R4 R1           + L1 R1 R5 R3         + L3 R4 L1 R4 R3
C4 C3 C1                C1 C5 C3              C3 C4

L2 R1 R6 R5           + L3 R2 L2 L1 L1      + R4 R2 L2 L2
C2 C2 C5                C3 C5                 C5 C2 C5

R4 R5 R3 L3           + R6 L3 R4 R2         + L1 R3 R6 R5
C3 C6 C1                C4 C3 C1              C1 C5 C3

L3 R4 L1 L1 R3        + L1 R1 L3 R6         + L2 R2 L1 L1
C3 C5                   C2 C2 C6              C3 C5 C1

R4 R1 L2 L2           + R4 R6 R2 L3         + R5 L3 R4 R2
C5 C2 C2                C4 C3 C1              C5 C3 C1

L2 R4 R5 R6           + L3 R5 L3 L1 R1      + L1 R1 L3 R5
C1 C5 C4                C3 C5                 C2 C2 C6
```

EQUATION 12 (D_4)

F_j (normalized) = $.163 \times 10^{-2}$

F_j (scaled) = $.455 \times 10^{-7}$

```
R4 L2 R3 L3        + L1 R6 R2         + L3 R3 L2 R4
C5 C6 C2             C6 C2 C3 C1        C4 C4 C3

R4 R5 R3           + R5 R1 L3 L1      + L1 R4 L2 R2
C2 C5 C1 C5          C3 C2 C6           C4 C3 C1

R5 R2 L1           + R4 L2 R3 L3      + L2 R5 R2
C5 C3 C5 C1          C5 C6 C2           C6 C2 C4 C1

L3 R3 L1 R4        + R4 R6 R5         + R6 R1 L3 L1
C4 C5 C2             C2 C5 C1 C3        C3 C2 C6

L1 R4 L2 R5        + R6 R2 L1         + R4 L2 R3 L3
C4 C3 C1             C5 C3 C5 C1        C5 C6 C2

L1 R6 R1           + L3 R1 L1 R3      + R3 R5 R6
C6 C2 C4 C1          C4 C5 C2           C2 C5 C1 C3

R4 R2 L2 L1        + L2 R3 L2 R6      + R4 R1 L1
C3 C2 C6             C5 C3 C1           C6 C1 C5 C1

R4 L1 R2 L3        + L2 R5 R1         + L3 R1 L3 R3
C3 C6 C2             C4 C2 C4 C1        C4 C5 C2

R4 R6 R2           + R5 R2 L2 L1      + L2 R4 L3 R3
C2 C5 C1 C3          C3 C2 C6           C5 C5 C1

R5 R1 L1           + R4 L1 R2 L2      + L1 R6 R3
C6 C1 C5 C1          C3 C6 C2           C4 C2 C4 C1

L2 R1 L3 R4        + R4 R5 R1         + R6 R2 L2 L2
C4 C5 C2             C1 C5 C1 C3        C3 C2 C6

L2 R4 L3 R1        + R6 R1 L2         + R4 L1 R3 L3
C4 C5 C1             C6 C1 C5 C1        C3 C6 C2

L1 R5 R3           + L3 R2 L2 R4      + R3 R6 R3
C4 C2 C4 C1          C4 C5 C2           C1 C5 C1 C3

R4 R1 L2 L1        + L1 R2 L2 R5      + R4 R1 L1
C3 C2 C6             C4 C5 C1           C6 C1 C5 C1

R4 L3 R3 L3        + L1 R6 R1         + L2 R2 L2 R2
C3 C6 C2             C4 C2 C4 C1        C4 C6 C2

R4 R4 R5           + R5 R1 L3 L1      + L1 R3 L2 R5
C2 C5 C1 C3          C3 C2 C5           C4 C5 C1

R4 R2 L2           + R4 L2 R3 L3      + L1 R5 R1
C6 C1 C5 C1          C3 C6 C2           C4 C2 C4 C1
```

EQUATION 12 (D_4) (Continued)

```
L3 R2 L1 R3      + R4 R4 R6         + R6 R1 L2 L1
C4 C6 C2           C2 C6 C1 C3        C3 C2 C5

L1 R4 L2 R6      + R5 R2 L1         + R4 L2 R3 L3
C4 C5 C1           C6 C1 C5 C1        C3 C6 C2

L1 R6 R2         + L2 R1 L1 R3      + R2 R4 R2
C4 C2 C4 C1        C4 C6 C2           C3 C6 C1 C3

R4 R1 L1 L1      + L3 R4 L3 R6      + R6 R1 L2
C4 C2 C5           C5 C5 C1           C6 C1 C5 C1

R4 L2 R2 L3      + L1 R5 R2         + L3 R1 L3 R3
C3 C6 C2           C4 C2 C4 C1        C4 C6 C2

R3 R5 R1         + R4 R2 L1 L2      + L2 R3 L3 R1
C3 C6 C1 C3        C4 C2 C5           C5 C6 C1

R4 R1 L2         + R4 L1 R2 L3      + L1 R6 R2
C6 C1 C5 C1        C4 C6 C2           C5 C2 C4 C1

L2 R1 L3 R3      + R4 R6 R3         + R5 R2 L1 L1
C4 C6 C2           C2 C6 C1 C5        C4 C2 C6

L2 R4 L2 R5      + R5 R1 L2         + R4 L1 R3 L3
C5 C6 C1           C6 C1 C5 C1        C4 C6 C3

L2 R5 R1         + L3 R1 L2 R4      + R4 R5 R5
C5 C2 C4 C1        C4 C5 C2           C2 C6 C1 C5

R6 R2 L3 L1      + L2 R4 L2 R6      + R6 R1 L2
C4 C2 C6           C5 C6 C2           C6 C1 C3 C1

R1 L1 R3 L3      + R2 R6 R1         + R4 R2 L2 R4
C4 C5 C3           C5 C2 C4 C1        C4 C5 C2

L3 R3 R6 R6      + R4 R1 L3 L1      + L1 R2 L2 R2
C6 C1 C5           C1 C2 C6           C3 C6 C2

R4 R2 L1         + R1 L3 R3 L3      + R4 R5 R2
C6 C1 C3 C1        C4 C5 C3           C5 C2 C4 C1

R5 R2 L1 R4      + L3 R4 R5 R2      + R5 R1 L3 L1
C4 C5 C2           C6 C1 C5           C1 C2 C6

L1 R3 L3 R3      + R4 R2 L1         + R1 L2 R3 L2
C3 C6 C2           C6 C1 C3 C1        C4 C5 C3

R4 R6 R2         + R6 R2 L1 R4      + L3 R4 R6 R4
C5 C3 C4 C1        C4 C5 C2           C5 C1 C5

R6 R1 L3 L1      + L1 R4 L3 R1      + R5 R1 R2
C1 C2 C6           C3 C6 C2           C6 C1 C3 C1
```

EQUATION 12 (D_4) (Continued)

```
R1 L2 R2 R3             + R3 R5 R3 R4             + R4 R1 L3 R4 R5
C4 C5 C3                  C5 C3 C4                  C4 C5

L2 R2 R5 R1             + R4 R1 L1 L2             + L3 R4 L3 R1
C5 C1 C1                  C1 C3 C3                  C3 C5 C2

R6 R1 R2                + R2 L2 R3 R3             + R3 R5 R3 R4
C6 C1 C3 C1               C4 C5 C3                  C5 C3 C4

R4 R1 L2 R4 R6          + L1 R3 R6 R4             + R4 R2 L1 L2
C4 C5                     C5 C1 C1                  C1 C3 C3

L2 R3 L3 R1             + R4 R2 R2                + R3 L1 R4 R3
C3 C5 C2                  C6 C1 C3 C1               C4 C5 C3

R4 R5 R3 R4             + R5 R1 L3 R4 R5          + L1 R4 R5 R3
C6 C3 C4                  C4 C5                     C5 C1 C1

R5 R2 L2 L1             + L2 R4 L2 R1             + R5 R3 R2
C1 C3 C5                  C3 C5 C2                  C6 C1 C3 C1

R3 L1 R4 R3             + R4 R5 R3 R4             + R6 R1 L2 R4 R6
C4 C5 C4                  C6 C3 C5                  C4 C6

L1 R4 R6 R4             + R6 R2 L2 L1             + L2 R4 L2 R1
C5 C1 C1                  C1 C3 C5                  C3 C5 C2

R6 R1 R2                + R1 L1 R2 R3             + R2 R6 R1 R4
C6 C1 C3 C1               C4 C5 C4                  C6 C3 C5

R4 R2 L3 R2 R5          + L3 R3 R4 R5             + R4 R1 L3 L1
C4 C6                     C5 C1 C1                  C2 C3 C6

L1 R2 L2 R1             + R4 R1 R2                + R1 L3 R3 R3
C3 C5 C2                  C6 C1 C5 C1               C4 C6 C4

R4 R6 R1 R4             + R5 R2 L2 R3 R6          + L3 R4 R4 R6
C6 C3 C5                  C4 C6                     C5 C1 C1

R5 R1 L2 L1             + L1 R3 L2 R1             + R4 R2 R2
C2 C3 C6                  C3 C5 C2                  C6 C1 C5 C1

R1 L2 R4 R3             + R4 R6 R2 R4             + R6 R2 L3 R4 R5
C4 C6 C4                  C6 C4 C5                  C5 C6

L3 R4 R5 R2             + R6 R1 L3 L1             + L1 R4 L3 R1
C6 C1 C2                  C2 C3 C3                  C3 C5 C2

R5 R3 R2                + R1 L2 R4 R3             + R3 R6 R3 R4
C6 C1 C5 C1               C4 C6 C4                  C6 C4 C5

R4 R1 L2 R4 R6          + L2 R2 R5 R1             + R4 R1 L2 L2
C5 C6                     C6 C1 C2                  C2 C3 C3
```

EQUATION 12 (D_4) (Continued)

```
L3 R4 L3 R1           + R6 R2 R2             + R2 L2 R3 R3
C3 C5 C2                C6 C1 C5 C1            C4 C6 C4

R3 R5 R2 R4           + R4 R1 L1 R4 R5       + L1 R3 R6 R4
C6 C4 C5                C5 C6                  C6 C1 C2

R4 R2 L3 L2           + L2 R3 L3 R1          + R4 R3 R2
C2 C3 C5                C3 C5 C2               C6 C2 C5 C1

R3 L1 R4 R3           + R4 R5 R3 R4          + R5 R1 L1 R4 R6
C3 C6 C4                C4 C4 C5               C5 C6

L1 R4 R6 R2           + R5 R2 L3 L1          + L2 R4 L3 R1
C6 C1 C2                C2 C3 C5               C3 C5 C2

R5 R2 R2              + R3 L1 R3 R3          + R4 R6 R2 R4
C6 C2 C5 C1             C3 C6 C3               C4 C4 C5

R6 R1 L1 R3 R5        + L1 R4 R4 R4          + R6 R2 L1 L1
C5 C6                   C6 C1 C2               C2 C3 C6

L2 R4 L3 R1           + R6 R3 R2             + R1 L1 R4 R3
C3 C5 C2                C6 C2 C5 C1            C3 C6 C3

R2 R6 R3 R4           + R4 R2 L1 L1 R6       + L3 R3 L2 R5
C6 C4 C5                C5 C6                  C6 C1 C2

R4 R1 L1              + L1 R2 L3 R1          + R4 R1 R2
C3 C3 C1 C6             C5 C5 C2               C6 C2 C3 C1

R1 L3 R4 R3           + R4 R5 R5 R4          + R5 R2 L3 L1 R5
C3 C4 C3                C4 C4 C5               C5 C6

L3 R4 L2 R6           + R5 R1 L1             + L1 R2 L3 R1
C6 C1 C3                C3 C3 C1 C5            C5 C5 C2

R5 R1 R2              + R1 L3 R4 R3          + R4 R6 R6 R4
C6 C2 C3 C1             C3 C4 C3               C4 C1 C5

R6 R2 L3 L1 R6        + L3 R4 L2 R3          + R6 R1 L1
C2 C6                   C3 C1 C3               C3 C3 C1 C5

L1 R2 L2 R1           + R6 R1 R2             + R1 L3 R2 R3
C5 C5 C2                C6 C2 C3 C1            C3 C4 C3

R3 R5 R2 R4           + R4 R1 L3 L1 R5       + L2 R2 L3 R1
C4 C1 C5                C2 C6                  C3 C1 C5

R4 R1 L2              + L3 R3 L3 R1          + R5 R1 R2
C3 C5 C1 C6             C5 C6 C2               C6 C2 C3 C1

R2 L2 R2 R3           + R3 R6 R1 R4          + R4 R1 L3 L2 R6
C3 C4 C3                C4 C1 C5               C2 C6
```

EQUATION 12 (D_4) (Concluded)

```
L1 R3 L3 R4          + R4 R1 L2
C3 C1 C5               C3 C5 C1 C6
```

EQUATION 13 (D_3)

F_j (normalized) = $.237 \times 10^{-1}$

F_j (scaled) = $.676 \times 10^{-6}$

$$\frac{R1\ R4\ L3\ R6}{C5\ C5\ C4} + \frac{R2\ R2\ L1}{C2\ C1\ C6\ C5} + \frac{R3\ R3\ R2}{C3\ C2\ C1\ C6}$$

$$\frac{R4\ R4\ R5\ R1}{C6\ C3\ C3} + \frac{R1\ R5\ L1\ R4}{C1\ C6\ C4} + \frac{R2\ R3\ L3}{C3\ C2\ C1\ C5}$$

$$\frac{R4\ R6\ R3}{C4\ C3\ C4\ C2} + \frac{R1\ R5\ L1\ R4}{C5\ C5\ C4} + \frac{R2\ R2\ L1}{C2\ C1\ C6\ C5}$$

$$\frac{R4\ R3\ R4}{C3\ C2\ C1\ C6} + \frac{R5\ R4\ R5\ R2}{C6\ C3\ C3} + \frac{R1\ R6\ L2\ R4}{C1\ C6\ C4}$$

$$\frac{R2\ R1\ L3}{C3\ C2\ C1\ C5} + \frac{R4\ R3\ R4}{C4\ C3\ C4\ C2} + \frac{R1\ R6\ L2\ R5}{C5\ C5\ C4}$$

$$\frac{R2\ R1\ L1}{C2\ C1\ C6\ C5} + \frac{R4\ R2\ R1}{C3\ C2\ C1\ C6} + \frac{R6\ R3\ R6\ R1}{C6\ C3\ C3}$$

$$\frac{R1\ R4\ L2\ R5}{C1\ C6\ C4} + \frac{R2\ R1\ L3}{C3\ C2\ C1\ C5} + \frac{R3\ R3\ R4}{C4\ C5\ C4\ C1}$$

$$\frac{R1\ R4\ L2\ R6}{C6\ C5\ C2} + \frac{R2\ R2\ L1}{C3\ C1\ C6\ C3} + \frac{R3\ R3\ R2}{C5\ C2\ C1\ C5}$$

$$\frac{R4\ R4\ R6\ R3}{C6\ C3\ C3} + \frac{R1\ R5\ L1\ R5}{C1\ C4\ C4} + \frac{R2\ R2\ L1}{C4\ C2\ C1\ C5}$$

$$\frac{R4\ R3\ R3}{C5\ C5\ C4\ C1} + \frac{R1\ R5\ L1\ R4}{C6\ C5\ C2} + \frac{R2\ R2\ L2}{C3\ C1\ C6\ C3}$$

$$\frac{R4\ R3\ R4}{C5\ C2\ C1\ C5} + \frac{R5\ R4\ R6\ R2}{C6\ C3\ C3} + \frac{R1\ R6\ L2\ R5}{C1\ C4\ C4}$$

$$\frac{R2\ R1\ L3}{C4\ C2\ C1\ C5} + \frac{R4\ R5\ R4}{C5\ C5\ C4\ C1} + \frac{R1\ R6\ L2\ R5}{C6\ C5\ C2}$$

$$\frac{R2\ R2\ L2}{C3\ C1\ C6\ C3} + \frac{R4\ R4\ R1}{C5\ C2\ C1\ C5} + \frac{R6\ L1\ R2\ R3}{C6\ C3\ C3}$$

$$\frac{R1\ L3\ R5}{C1\ C1\ C4\ C4} + \frac{R2\ R2\ L2}{C4\ C3\ C1\ C5} + \frac{R3\ R5\ R4}{C5\ C4\ C4\ C1}$$

$$\frac{R1\ R4\ L1\ R6}{C6\ C5\ C2} + \frac{R2\ R3\ L2}{C1\ C1\ C6\ C3} + \frac{R3\ R4\ R2}{C2\ C2\ C1\ C5}$$

$$\frac{R4\ L1\ R4\ R1}{C3\ C3\ C3} + \frac{R1\ L1\ R6}{C1\ C1\ C4\ C4} + \frac{R2\ R1\ L3}{C4\ C3\ C1\ C5}$$

$$\frac{R4\ R6\ R1}{C6\ C4\ C4\ C1} + \frac{R1\ R5\ L2\ R2}{C6\ C5\ C2} + \frac{R2\ R4\ L3}{C1\ C1\ C6\ C3}$$

EQUATION 13 (D_3) (Continued)

```
R4 R5 R2        + R5 L1 R4 R3     + R1 L3 R6
C2 C2 C1 C5       C3 C3 C3          C1 C1 C4 C4

R2 R2 L1        + R4 R6 R1        + R1 R6 L1 R4
C4 C3 C1 C5       C6 C4 C4 C1       C6 C5 C2

R2 R4 L3        + R4 R6 R1        + R6 L1 R4 R2
C1 C1 C6 C3       C2 C2 C1 C5       C3 C3 C3

R1 L2 R6        + R2 R1 L3        + R3 R2 R2
C1 C1 C4 C4       C4 C3 C1 C5       C6 C4 C4 C1

R1 R4 L3 R4     + R2 R1 L3        + R3 R4 R1
C6 C5 C2          C1 C1 C6 C3       C2 C2 C1 C5

R4 L2 R2 R3     + R1 L3 R6        + R2 R1 L2
C5 C3 C3          C2 C1 C4 C4       C3 C3 C1 C5

R4 R3 R1        + R1 R5 L2 R4     + R2 R1 L3
C4 C4 C3 C1       C6 C5 C2          C1 C1 C6 C3

R4 R4 R1        + R5 L3 R5 R1     + R1 L3 R2
C2 C2 C2 C5       C5 C4 C3          C2 C1 C5 C5

R2 R2 L3        + R4 R3 R2        + R1 R6 L1 R4
C3 C3 C1 C6       C4 C4 C3 C1       C6 C5 C2

R2 R2 L3        + R4 R4 R1        + R6 L1 R6 R1
C1 C1 C6 C5       C2 C2 C2 C6       C5 C4 C3

R1 L3 R3        + R2 R1 L2        + R3 R5 R4
C2 C2 C5 C5       C3 C3 C1 C6       C4 C4 C3 C1

R1 R4 L2 R5     + R2 R3 L3        + R3 R4 R2
C6 C5 C2          C1 C1 C6 C5       C2 C2 C2 C6

R4 L1 R3 R2     + R1 L1 R5        + R2 R2 L3
C6 C4 C3          C2 C2 C5 C5       C4 C3 C1 C6

R4 R5 R4        + R1 R5 L1 R6     + R2 R4 L3
C5 C4 C3 C1       C6 C5 C2          C1 C1 C6 C5

R4 R5 R3        + R5 L1 R5 R3     + R1 L1 R5
C2 C2 C2 C6       C6 C4 C3          C2 C2 C5 C5

R2 R1 L2        + R4 R6 R3        + R1 R6 L2 R4
C4 C3 C1 C6       C5 C4 C3 C1       C6 C5 C2

R2 R4 L1        + R4 R6 R3        + R6 L1 R6 R2
C1 C1 C6 C5       C2 C2 C2 C6       C6 C4 C3

R1 L1 R6        + R2 R2 L3        + R3 R6 R3
C2 C2 C5 C5       C4 C3 C1 C6       C5 C4 C3 C1
```

EQUATION 13 (D_3) (Continued)

```
R1 R5 L1 R4       + R2 R1 L2          + R3 R4 R2
C6 C5 C2            C1 C1 C6 C5         C3 C2 C2 C6

R4 L2 R4 R3       + R1 L3 R6          + R2 R1 L2
C5 C4 C3            C2 C2 C5 C5         C4 C3 C1 C6

R3 R2 R3          + R1 R6 L3 R4       + R2 R1 L1
C6 C4 C3 C1         C6 C5 C2            C1 C1 C6 C5

R4 R4 R4          + R5 L3 R5 R2       + R2 L3 R3
C3 C2 C2 C6         C5 C4 C3            C2 C2 C5 C4

R3 R1 L1          + R4 R3 R3          + R1 R5 L2 R4
C4 C3 C1 C5         C6 C4 C3 C1         C6 C5 C2

R2 R2 L2          + R4 R4 R4          + R6 L1 R6 L1
C1 C1 C6 C5         C3 C2 C2 C6         C5 C4 C3

R2 L3 L3          + R3 R2             + R4 R3 R4
C2 C3 C6 C4         C4 C4 C1 C5 C1      C6 C5 C3 C2

R1 R6 L1 R5       + R2 R3 L1          + R3 R4 R1
C6 C5 C3            C1 C1 C6 C4         C3 C2 C2 C5

R4 L1 R3 L2       + R1 L2 L3          + R3 R1
C5 C4 C3            C3 C3 C6 C4         C4 C4 C1 C5 C1

R4 R5 R4          + R1 R5 L2 R5       + R2 R4 L2
C5 C5 C3 C2         C6 C5 C3            C1 C1 C6 C4

R4 R5 R3          + R5 L1 R4 L1       + R1 L2 L3
C3 C2 C2 C5         C5 C4 C3            C3 C3 C6 C4

R3 R2             + R4 R5 R4          + R1 R6 L1 R6
C4 C4 C1 C5 C1      C5 C5 C3 C2         C6 C5 C3

R2 R4 L1          + R4 R6 R1          + R6 L1 R4 L2
C1 C1 C6 C5         C3 C2 C2 C6         C5 C4 C3

R1 L3 L3          + R2 R1             + R3 R6 R4
C3 C3 C6 C4         C4 C4 C1 C5 C1      C5 C5 C3 C2

R1 R5 L2 R6       + R2 R1 L2          + R3 R4 R3
C6 C5 C3            C1 C1 C6 C5         C5 C2 C2 C6

R4 L2 R4 L1       + R2 L1 L2          + R3 R2
C6 C4 C3            C4 C3 C6 C4         C5 C4 C1 C5 C1

R4 R6 R1          + R1 R5 L1 R4       + R2 R1 L3
C6 C5 C3 C2         C6 C5 C4            C1 C1 C6 C5

R4 R4 R1          + R5 L3 R3 L2       + R1 L2 L3
C5 C2 C2 C6         C6 C4 C3            C4 C3 C6 C4
```

EQUATION 13 (D_3) (Concluded)

```
R2 R1                   + R3 R2 R2               + R1 R6 L3 R4
C5 C4 C1 C5 C1            C6 C5 C3 C2              C6 C4 C4

R2 R1 L3                + R4 R2 R1               + R6 L3 R5 L1
C1 C1 C5 C5               C5 C2 C2 C6              C6 C4 C3

R2 L2 L2                + R3 R1                  + R4 R3 R1
C4 C1 C6 C4               C5 C2 C1 C5 C1           C6 C3 C3 C2

R1 R6 L2 R4             + R2 R2 L2               + R3 R3 R1
C6 C4 C4                  C1 C1 C5 C5              C5 C4 C2 C6

R4 L1 R6 L1             + R1 L2 L3               + R2 R2
C6 C5 C4                  C1 C1 C6 C5              C3 C2 C1 C6 C1

R3 R3 R3                + R1 R4 L1 R4            + R2 R1 L2
C6 C3 C3 C3               C6 C4 C4                 C1 C1 C5 C5

R4 R5 R2                + R5 L3 R3 L1            + R1 L1 L2
C5 C4 C2 C6               C6 C5 C4                 C1 C1 C6 C5

R3 R3                   + R4 R4 R4               + R1 R5 L2 R5
C3 C2 C1 C6 C1            C6 C3 C3 C2              C6 C4 C3

R2 R3 L2                + R4 R5 R2               + R6 L1 R5 L1
C2 C1 C5 C4               C3 C4 C2 C6              C5 C5 C4

R1 L1 L3                + R3 R1                  + R4 R5 R4
C1 C1 C6 C5               C3 C2 C1 C6 C1           C6 C3 C3 C2

R1 R6 L2 R6             + R2 R1 L2               + R3 R6 R2
C6 C4 C3                  C2 C1 C5 C4              C3 C4 C2 C6
```

EQUATION 14 (D_2)

F_j (normalized) = .137

F_j (scaled) = $.395 \times 10^{-5}$

```
R1 R2 R6             + R2 R3 R1             + R4 R4 L3
C1 C3 C5 C5            C1 C3 C5 C6            C1 C3 C5 C6

R1 L1                + R3 R3                + R1 R4 R5
C1 C2 C4 C6 C1         C3 C4 C5 C1 C2         C4 C5 C3 C3

R1 R4 R6             + R1 R2 R6             + R3 R5 R2
C6 C1 C4 C5            C1 C2 C5 C6            C1 C3 C3 C6

R4 R5 L3             + R2 L1                + R1 R3
C1 C3 C5 C5            C1 C2 C4 C6 C1         C3 C4 C5 C1 C2

R2 R3 R5             + R1 R4 R4             + R1 R2 R6
C4 C5 C3 C3            C6 C1 C4 C5            C2 C2 C5 C6

R4 R6 R5             + R5 R6 L3             + R1 L1
C1 C3 C3 C6            C1 C3 C4 C5            C1 C2 C4 C5 C1

R1 R3                + R1 R4 R6             + R1 R2 R5
C3 C4 C5 C1 C2         C4 C5 C3 C3            C6 C1 C4 C5

R1 R2 R3             + R4 R3 R6             + R6 R1 L3
C2 C2 C5 C6            C1 C3 C3 C6            C1 C4 C4 C5

R2 L2                + R1 R3                + R1 R4 R6
C1 C2 C5 C5 C1         C3 C4 C6 C1 C2         C4 C6 C3 C3

R1 R2 R6             + R1 R2 R4             + R2 R5 R3
C6 C1 C4 C5            C2 C2 C5 C6            C1 C3 C3 C6

R4 R1 L1             + R1 L3                + R2 R2
C1 C4 C4 C5            C2 C2 C5 C5 C1         C3 C4 C6 C1 C2

R1 R3 R3             + R1 R2 R4             + R1 R2 R5
C4 C6 C3 C3            C6 C1 C4 C5            C2 C2 C5 C6

R3 R6 R3             + R4 R4 L2             + R2 L2
C1 C3 C5 C6            C1 C4 C4 C6            C2 C2 C5 C5 C1

R2 R3                + R1 R4 R5             + R1 R2 R5
C3 C4 C6 C1 C2         C4 C6 C3 C3            C6 C1 C4 C5

R1 R2 R6             + R4 R3 R1             + R5 R4 L2
C2 C2 C5 C6            C1 C3 C5 C6            C1 C4 C4 C6

R2 L3                + R2 R3                + R2 R4 R6
C2 C2 C5 C5 C1         C3 C3 C6 C1 C2         C4 C6 C3 C4

R1 R3 R6             + R1 R3 R4             + R4 R5 R3
C6 C1 C4 C5            C2 C2 C5 C6            C1 C3 C5 C6
```

EQUATION 14 (D_2) (Continued)

```
R6 R3 L2                + R2 L1                   + R1 L1
C1 C4 C4 C6               C2 C2 C5 C5 C1            C3 C3 C6 C1 C2

R2 R2                   + R1 R4 R4                + R1 R3 R5
C4 C6 C1 C2 C4            C6 C2 C3 C5               C2 C3 C4 C6

R2 R6 R5                + R4 R3 L2                + R3 L2
C1 C4 C4 C5               C1 C4 C5 C6               C3 C2 C5 C6 C1

R2 L1                   + R2 R3                   + R1 R4 R4
C4 C3 C6 C1 C2            C5 C6 C1 C2 C4            C6 C2 C3 C5

R1 R2 R6                + R3 R3 R6                + R4 R5 L2
C2 C3 C4 C6               C1 C4 C4 C5               C1 C4 C5 C6

R5 L1                   + R2 L1                   + R1 R4
C3 C3 C5 C6 C1            C4 C4 C6 C1 C2            C5 C5 C1 C2 C4

R1 R2 R5                + R1 R3 R3                + R4 R4 R2
C6 C2 C3 C5               C2 C3 C4 C6               C1 C4 C4 C5

R5 R5 L1                + R6 L2                   + R2 L1
C1 C3 C5 C6               C3 C3 C5 C6 C1            C4 C4 C6 C1 C2

R1 R4                   + R2 R3 R6                + R1 R3 R4
C5 C5 C1 C2 C4            C6 C2 C3 C5               C2 C3 C4 C6

R4 R4 R2                + R6 R6 L3                + R1 L1
C1 C4 C4 C5               C1 C3 C5 C6               C3 C3 C5 C6 C1

R1 L2                   + R1 R3                   + R1 R3 R4
C4 C4 C6 C1 C2            C5 C5 C1 C2 C4            C6 C2 C3 C5

R1 R2 R5                + R2 R5 R4                + R3 R6 L1
C2 C3 C4 C6               C1 C3 C4 C5               C1 C3 C5 C6

R1 L2                   + R2 L3                   + R1 R3
C1 C3 C5 C6 C2            C2 C4 C6 C1 C3            C3 C5 C1 C2 C4

R1 R3 R4                + R1 R4 R6                + R2 R5 R4
C4 C2 C3 C5               C2 C3 C4 C6               C1 C3 C4 C5

R5 R1 L2                + R2 L3                   + R3 L1
C1 C3 C5 C6               C1 C3 C5 C6 C2            C2 C4 C6 C1 C3

R2 R4                   + R2 R3 R5                + R1 R4 R4
C3 C5 C1 C2 C4            C4 C3 C3 C5               C2 C4 C4 C6

R2 R5 R4                + R6 R2 L3                + R3 L3
C1 C3 C5 C5               C1 C3 C5 C6               C1 C3 C5 C6 C2

R3 L1                   + R3 R4                   + R1 R4 R6
C2 C4 C6 C1 C3            C3 C5 C1 C2 C4            C4 C3 C3 C5
```

EQUATION 14 (D_2) (Concluded)

```
R1 R2 R5
C2 C4 C4 C6
```

EQUATION 15 (D_1)

F_j (normalized) = .565

F_j (scaled) = $.166 \times 10^{-4}$

```
R1 R4                 + R3 R1                  + R2 R2
C4 C2 C6 C4 C2          C5 C3 C1 C5 C3           C1 C5 C3 C6 C4

R6 R2                 + R1 R4                  + R2 L2
C2 C6 C4 C1 C5          C3 C1 C5 C3 C6           C4 C2 C6 C4 C2

R2                    + R2 R4                  + R3 R1
C5 C3 C1 C5 C3 C1       C4 C2 C6 C4 C2           C5 C4 C1 C5 C3

R1 R3                 + R3 R3                  + R2 R4
C1 C5 C3 C6 C4          C2 C6 C4 C1 C5           C3 C1 C5 C3 C6

R3 L3                 + R3                     + R1 R4
C4 C2 C6 C4 C2          C5 C3 C1 C5 C3 C1        C5 C2 C6 C4 C2

R5 R1                 + R2 R5                  + R3 R4
C6 C4 C1 C5 C3          C1 C5 C3 C6 C4           C2 C6 C4 C1 C5

R2 R5                 + R5 R1                  + R4 R2
C3 C1 C5 C3 C6          C4 C2 C6 C4 C2           C5 C3 C1 C5 C3

R2 R5                 + R5 R1                  + R3 R6
C5 C2 C6 C4 C1          C6 C4 C1 C5 C2           C1 C5 C3 C6 C3

R6 R4                 + R2 R6                  + R6 R1
C2 C6 C4 C1 C4          C3 C1 C5 C3 C6           C4 C2 C6 C4 C2

R4 R3                 + R1 R6                  + R6 R1
C5 C3 C1 C5 C3          C5 C2 C6 C4 C1           C6 C4 C1 C5 C2

R3 R4                 + R5 L1                  + R1
C1 C5 C3 C6 C3          C2 C6 C4 C2 C4           C3 C1 C5 C3 C1 C6

R1 R5                 + R1 R6
C1 C2 C3 C4 C6          C1 C2 C3 C4 C6
```

EQUATION 16 (D_0)

F_j (normalized) = 1.0

F_j (scaled) = $.293 \times 10^{-4}$

R1
C5 C3 C1 C6 C4 C2

+ R5
C1 C6 C4 C2 C5 C3

+ R2
C2 C5 C3 C1 C6 C4

R6
C3 C1 C6 C4 C2 C5

+ R3
C4 C2 C5 C3 C1 C6