THE DYNAMICS OF A SPINNING-SOLAR-PRESSURE-STABILIZED SATELLITE WITH PRECESSION DAMPING

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1. Introduction

A spinning, axially symmetric, solar orbiting satellite tends to be aligned with the solar radius vector when the centroid of the solar radiation pressure is displaced from the center of gravity of the satellite. If the only torque acting on the satellite is that furnished by the radially directed radiation pressure, the motion of the satellite will be a precessional motion about the radial direction with a cone angle determined by the initial conditions. To cause the spin axis to become co-incident with the solar radius vector, i.e. to reduce the cone angle, a torque orthogonal to the spin axes and to the radiation-applied torque must be exerted on the precessing vehicle. This may be accomplished either by active means involving the use of microthrusters or the reradiation of thermal energy in an orthogonal direction, or effectively by having a loss mechanism in the system sensitive to the component of angular velocity orthogonal to the spin axis, i.e. sensitive to the rates of rotation and precession of the spin axis.

Microthruster methods have been investigated by H. Marchetta[1] and the thermal reradiation possibilities by C. A. Peterson[2]. In an earlier study, J. L. Carroll and R. C. Limburg[3] the comprehensive dynamical equations for a solar-pressure-stabilized satellite on a computer and gave particular emphasis to the librational motion of a nonspinning satellite about the radius vector. G. Colombo[4] suggested that keeping the probe spinning, possibly at a constant rate, would help the stabilization problem. He analyzed the general behavior of the spinning satellite to orthogonal-erecting torques and proposed maintaining the spin of the satellite by the use of radial vanes acted upon by the solar radiation pressure.

It is the purpose of this present note to derive expressions for the dynamical behavior of both spinning and nonspinning radiation-torqued satellites when their motion is damped by a loss mechanism, sensitive either to the precessional or libration motion of the satellite. The equations of motion are formulated by using the Euler-Lagrange equations with the rotational and precessional losses represented by a Rayleigh dissipation term. Explicit solutions are found for the rotation angle and precession angle as functions of time. The coefficient of the Rayleigh dissipation term is further related to the specific measurable parameters of a vibrating viscous damper located on the periphery of the spinning satellite and driven by acceleration parallel to the spin axis. The slow rotation of the solar radius vector corresponding to the orbital angular velocity of the satellite prevents the spin axis from ever being completely aligned with the radius vector and, for a constant angular velocity, will cause it to precess at some equilibrium cone angle whose value is determined.

2. Basic Equations

We use the set of coordinates designated by the Euler angles:

θ (the rotation angle)

ϕ (the precession angle)

and ψ (the spin angle)

The satellite is assumed to be axially symmetric and to have the general shape shown in Figure 1 such that its moments of inertia are:

$$I_1 = I_2 = A; \quad I_3 = C \qquad [1]$$

The kinetic energy term may be written:

$$1/2\ I_1\ \omega_1^2 + 1/2\ I_2\ \omega_2^2 + 1/2\ I_3\ \omega_3^2 = T \qquad [2]$$

The angular velocity components about the body axes, written in terms of the Euler angles, are:

$$\omega_1 = \dot{\phi} \sin\theta \sin\psi + \dot{\theta} \cos\psi \qquad [3]$$

$$\omega_2 = \dot{\phi} \sin\theta \cos\psi - \dot{\theta} \sin\psi$$

$$\omega_3 = \dot{\psi} + \dot{\phi} \cos\theta = \Omega$$

The kinetic energy may then be expressed as:

$$T = 1/2\ A\ [(\dot{\theta})^2 + \dot{\phi}^2 \sin^2\theta] + 1/2\ C\ [\dot{\psi} + \dot{\phi}\cos\theta]^2 \qquad [4]$$

In general, the Rayleigh dissipation term is given by:

$$1/2\ F_1\omega_1^2 + 1/2\ F_2\omega_2^2 + 1/2\ F_3\omega_3^2 = f_D \qquad [5]$$

We shall assume negligible damping about the spin axis so that $F_3 = 0$, and we shall assume axial symmetry so that $F_1 = F_2$, in which case the effective Rayleigh damping term expressed in Euler angles is given by:

$$f_D = 1/2\ F\ [(\dot{\theta})^2 + \dot{\phi}^2 \sin^2 \theta] \qquad [6]$$

The potential energy term for the satellite is that due to the solar radiation pressure and is simply given by:

$$V = -\eta p S \ell \cos \theta \qquad [7]$$

where p is the incident pressure of radiation, S is the effective area of the vane and ℓ is the moment arm measured from the center of pressure of the vane to the center of gravity of the satellite and η is a coefficient having a value between 0 and 2, depending upon the manner in which the incident energy is reflected, absorbed, and reradiated. In our work we shall assume $\eta = 1$, corresponding to a nonspectrally reflecting vane surface with isotropic reradiation of the absorbed or scattered energy. By minimizing the energy given by [4], [6], and [7] through application of the Euler-Lagrange equations, we find the following three fundamental differential equations governing the dynamical behavior of the spinning satellite. The equations are similar to those for the spinning top in a gravity field.

$$I_1 \frac{d^2\theta}{dt^2} - I_1 \sin\theta\cos\theta(\frac{d}{dt})^2 + I_3\ [\dot{\psi} + \dot{\phi}\cos\theta]$$
$$\frac{d\phi}{dt} \sin\theta + F_1\frac{d\theta}{dt} = -(pS\ell)\sin\theta \qquad [8]$$

$$\frac{d}{dt}\ [I_1 \sin^2\theta\frac{d\phi}{dt} + I_3(\dot{\psi} + \dot{\phi}\cos\theta)\ \cos\theta]$$
$$+ F_1\ \sin^2\theta\frac{d\phi}{dt} = 0 \qquad [9]$$

$$\frac{d}{dt}\,[I_3\ (\dot{\psi} + \dot{\phi}\cos\,\theta)] = 0 \qquad [10]$$

Since we had earlier taken F_3 to be 0, we find that the momentum about the spin axis is conserved and that

$$\dot{\psi} + \dot{\phi}\ \cos\,\theta = \Omega = \text{constant} \qquad [11]$$

If we further restrict our analysis to the situation where the spin is high compared to the precessional or rotational velocities, i.e. $\Omega \approx \dot{\psi} >> \dot{\phi}$, then we can safely neglect the higher order derivatives in [8] and [9] and find the following two relationships which describe the desired motion.

$$C\Omega\sin\theta\ \frac{d\phi}{dt} + F\ \frac{d\theta}{dt} = -\ (pS\ell)\ \sin\,\theta \qquad [12]$$

$$\sin\theta\ \frac{d\phi}{dt} = \left(\frac{C\Omega}{F}\right)\ \frac{d\theta}{dt} \qquad [13]$$

It is clear from [12] that the rotational ($\dot{\theta}$) and precessional ($\dot{\phi}$) velocities are coupled through the loss mechanism and that [12] and [13] may be solved to obtain a differential equation in rotation only in which the effect of the precessional motion is simply to augment the pure rotational damping by an additional term, thus:

$$\left(\left(\frac{C\Omega}{F}\right)^2 + F\right)\ \ \frac{d\theta}{dt} = pS\ell\ \sin\,\theta \qquad [14]$$

When the satellite is not spinning, $\Omega = 0$, we find from [8] that:

$$I_1\ \frac{d^2\theta}{dt^2} + F\ \frac{d\theta}{dt} = -\ (pS\ell)\ \sin\,\theta \qquad [15]$$

This describes an oscillatory motion of the vehicle with a libration frequency when θ is small, given by:

$$\omega_\ell^2 = \frac{pS\ell}{I_1} = \frac{pS\ell}{A} \qquad [16]$$

The rotation rate [14] may then be written as:

$$\frac{d\theta}{dt} = \left(\frac{A\omega_\ell^2}{C\Omega}\right) \frac{\sin\theta}{\left(\frac{C\Omega}{F}\right) + \left(\frac{F}{C\Omega}\right)} \qquad [17]$$

and the precession rate [12] as:

$$\frac{d\phi}{dt} = \left(\frac{A\omega_\ell^2}{C\Omega}\right) \left(\frac{\frac{C\Omega}{F}}{\frac{C\Omega}{F} + \frac{F}{C\Omega}} \right) \qquad [18]$$

[17] may be integrated directly to obtain an exact solution for the cone angle decay as a function of time.

$$\log\left(\frac{\tan\frac{\theta}{2}}{\tan\frac{\theta_o}{2}} \right) = - \left(\frac{A\omega_\ell^2}{C\Omega}\right) \frac{t}{\left(\frac{C\Omega}{F} + \frac{F}{C\Omega}\right)} = - \beta t \qquad [19]$$

The decay is nearly exponential and even for large initial angles (Figure 2) is closely approximated by:

$$\theta \simeq \theta_o \varepsilon^{-\beta t} \qquad [20]$$

For $\theta < \frac{\pi}{4}$ the exponential approximation is quite accurate. Minimum relaxation time or alignment time is obtained when $\frac{C\Omega}{F} = 1$ (Figure 3). Under these conditions, the rotation and precession rates are approximately equal for large θ and the rate of precession is one-half of its low loss value.

A comparison of [18] and [19] will show that $\dot{\phi}$ and β are simply related by:

$$\dot{\phi} = \left(\frac{C\Omega}{F}\right) \beta \qquad [21]$$

so that

$$\tau_r = \left(\frac{C\Omega}{F}\right) \frac{1}{\dot{\phi}} \qquad [22]$$

For minimum relaxation time $(\frac{C\Omega}{F}) = 1$ in which case:

$$\tau_r = \frac{1}{\dot{\phi}_1} = \frac{\tau_p}{2\pi} \qquad [23]$$

where

$$\dot{\phi}_1 = \frac{1}{2} \frac{A}{C} \frac{\omega_\ell^2}{\Omega} \qquad [24]$$

If τ_r is to be small, the precession rate $\dot{\phi}$ must be large, and since this is proportional to ω_ℓ^2 a rather high libration rate is very desirable.

3. <u>Body Accelerations</u>

In the preceding section we described the dynamic behavior of the spinning vehicle in terms of an arbitrary damping factor F. The next step is then to relate the magnitude of F to some specific loss mechanism on the satellite so that a determination of its magnitude from explicit or measurable parameters of the damping mechanism may be made. Without appreciable loss of generality, we can assume that the damping will be furnished by the oscillations of a small mass elastically coupled to the vehicle at some point displaced from its center of gravity. If $\vec{r}$ is the position of the mass relative to the body axes (Figure 4), then the forces acting on the mass are:

$$m\ddot{\vec{r}} + f\,(\dot{\vec{r}} - \dot{\vec{r}}_p) + k\,(\vec{r} - \vec{r}_p) = 0 \qquad [25]$$

where r_p is the position on the body to which the mass is elastically attached. By rewriting [25] in the form

$$m\ddot{\delta} + f\dot{\delta} + k\delta = - m\ddot{\vec{r}}_p$$

where

$$\delta = (\vec{r} - \vec{r}_p) << r \qquad [26]$$

it is made more evident that the driving force for the dissipative relative motion between the mass and the body is the acceleration of the point to which the mass is attached measured relative to the body axes.

The components of angular velocity about the body axes were given earlier as equation [3]. The corresponding component of angular acceleration, when $\ddot{\theta}$ and $\ddot{\phi}$ are negligible, are given by:

$$\dot{\omega}_1 = \dot{\phi}\dot{\theta}\cos\theta\sin\psi + \dot{\phi}\dot{\psi}\sin\theta\cos\psi - \dot{\theta}\dot{\psi}\sin\psi$$
$$\dot{\omega}_2 = \dot{\phi}\dot{\theta}\cos\theta\cos\psi - \dot{\phi}\dot{\psi}\sin\theta\sin\psi - \dot{\theta}\dot{\psi}\cos\psi \qquad [27]$$
$$\dot{\omega}_3 = 0$$

These may also be written:

$$\dot{\omega}_1 = \omega_2\Omega + \Xi_1\ (\dot{\theta}^2, \dot{\phi}^2, \dot{\theta}\dot{\phi})$$
$$\omega_2 = -\omega_1\Omega + \Xi_2\ (\dot{\theta}^2, \dot{\phi}^2, \dot{\theta}\dot{\phi}) \qquad [28]$$

where Ξ signifies terms of the order of θ^2, etc. in magnitude. If we can neglect the orbital accelerations applied to the center of gravity of the spinning satellite, then the acceleration at a vector distance $\vec{r}$ from the center of gravity is given by

$$\vec{a} = \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \dot{\vec{\omega}} \times \vec{r} \qquad [29]$$

(Thompson, page 216)[5]. If we take $\vec{r} = r_1\vec{i} + r_3\vec{k}$, then by direct substitution of [3] and [28] in [29], we find that the acceleration vector is given by:

$$\vec{a} = \vec{i}\ [-r_1(\omega_2^2 + \Omega^2) + r_3\Xi_2(\dot{\theta}^2, \dot{\phi}^2, \dot{\phi}\dot{\theta})]$$
$$+ \vec{j}\ [r_1\omega_1\omega_2 - r_3\Xi_1(\dot{\theta}^2, \dot{\phi}^2, \dot{\phi}\dot{\theta})]$$
$$+ \vec{k}\ [r_1(2\omega_1\Omega + \Xi_2) - r_3(\omega_1^2 + \omega_2^2)] \qquad [30]$$

Recalling that

$$\Omega = \dot{\psi} + \dot{\phi} \cos \theta$$

then for the important case when

$$\dot{\psi} >> \dot{\phi}, \dot{\theta}$$

it follows that

$$\Omega \approx \dot{\psi}$$

and

$$\psi \approx \Omega t$$

The angular velocity components ω_1, ω_2 as given by [3] have magnitudes proportional to $\dot{\phi}$ and $\dot{\theta}$ and fluctuate at the spin rate Ω. Examination of [30] reveals that the dominant oscillatory term is:

$$\vec{a} \approx \vec{k}\ 2r_1\ \omega_1 \Omega$$

$$\vec{a} \approx \vec{k}\ [2r_1 \Omega (\dot{\phi} \sin\theta \sin\Omega t + \dot{\theta} \cos\Omega t)] \qquad [31$$

and is a single frequency term directed along the spin axis. If r_1 is taken equal to zero and r_3 is large (corresponding to the excitation of a damping mechanism at the end of the boom supporting the sail), the principal fluctuating acceleration components are in the i and j directions and are much smaller than [31] by the ratio of $\dot{\theta}$ or $\dot{\phi}$ to Ω.

4. Damper Dynamics

If we then return to the case of a small mass m constrained to move parallel to the spin (k) axis and located a distance r_1 from that axis, we find, using [31] as the value of $\ddot{r}_p$ in [26], that the displacement δ_k of the mass will be given by:

$$\delta_k = \frac{A_o \sin(\Omega t + \lambda)}{\sqrt{(\Omega_d^2 - \Omega^2)^2 + 4\zeta^2\Omega_d^2\Omega^2}} \quad [32]$$

where

$$\Omega_d^2 = \frac{k}{m}, \ \zeta = \frac{f}{2\sqrt{km}} \quad [33]$$

and

$$A_o = 2r_1\Omega\sqrt{(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta)} \quad [34]$$

The power dissipated by the damper is given by $f(\dot{\delta}_k)^2$. Since δ_k is oscillatory, it is also equal to $f\Omega^2(\delta_k)^2$. The expression for the average power loss in the damper is then:

$$P_D = \frac{1}{2} f \frac{4r_1^2\Omega^4}{[(\Omega_d^2 - \Omega^2)^2 + 4\zeta^2\Omega_d^2\Omega^2]}[\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta] \quad [35]$$

which, by comparison with [6], the Rayleigh dissipation term, shows F to be:

$$F = \frac{4fr_1^2\Omega^4}{[(\Omega_d^2 - \Omega^2)^2 + 4\zeta^2\Omega_d^2\Omega^2]} \quad [36]$$

or

$$F = \frac{8r_1^2\Omega^4\zeta\Omega_d m}{[(\Omega_d^2 - \Omega^2)^2 + 4\zeta^2\Omega_d^2\Omega^2]}$$

Hence we have found an expression for the precession and rotation damping factor in terms of the measurable parameters of the peripherally mounted linear vibrator.

While [36] was derived for the case of a single vibrator, if a number of identical vibrators are symmetrically spaced around the periphery as required for dynamic balance, then the same relationship holds with m in [33] being interpreted as the sum of the individual damper masses.

The spin-to-dissipation ratio $(\frac{C\Omega}{F})$ which appears in [19] is found to be:

$$\frac{C\Omega}{F} = \frac{1}{8}\left(\frac{I_3 + I_m}{I_m}\right)\left(\frac{\Omega}{\Omega_d}\right)\frac{1}{\zeta}\left[\left(\frac{\Omega_d^2}{\Omega^2} - 1\right)^2 + 4\zeta^2\frac{\Omega_d^2}{\Omega^2}\right] \qquad [37]$$

where

$$I_m = mr_1^2 \qquad\qquad I_3 + I_m = C$$

This expression is nondimensional and indicates that the effective damping of the rotational motion of the satellite depends on three ratios: the ratio of the moment of inertia of the satellite to that of the damper about the spin axis, the ratio of the spin angular velocity relative to the resonant frequency of the damper, and finally the damping ratio ζ for the damper itself.

The expression [37] for $(\frac{C\Omega}{F})$ may be simplified in three important situations for Ω at, well above, or well below, resonance.

I. <u>For $\Omega = \Omega_d$</u>

$$\frac{C\Omega}{F} = \frac{1}{2}\frac{C}{I_m}\zeta \qquad [38]$$

II. <u>For $\Omega >> \Omega_d$</u>

$$\frac{C\Omega}{F} \approx \frac{1}{8}\frac{C}{I_m}\left(\frac{\Omega}{\Omega_d}\right)\frac{1}{\;}\left[1 + \frac{4\zeta^2\Omega^2{}_d}{\Omega^2}\right] \qquad [39]$$

$$\approx \frac{1}{4}\frac{C}{I_m}\left[\Omega\tau_d + \frac{1}{\Omega\zeta_d}\right]$$

where $\frac{1}{2\Omega_d\zeta} = \frac{m}{f} = \tau_d$ is the damper relaxation time in the situation where $k = 0$.

III. <u>For $\Omega << \Omega_d$</u>

$$\frac{C\Omega}{F} \quad \frac{1}{8}\frac{C}{I_m}\left(\frac{\Omega_d}{\Omega}\right)^3\frac{1}{\zeta} \tag{40}$$

If we recall from the discussion on page 6 that the minimum rotation time will be obtained when $\frac{C\Omega}{F} = 1$, we observe that this is only achievable in cases I and II and not in III where the value of $\frac{C\Omega}{F}$ will be very much greater than unity. It is a practical possibility for the moment of inertia of the damper to be made to be one-fourth of the major moment of inertia C and for the linear vibrator to be underdamped such that $\zeta = 0.5$. At resonance $\frac{C\Omega}{F}$ will then be equal to unity and will produce the minimum rotational relaxation time.

Case II pertains to the important situation where the damper motion is highly viscous rather than resonant. Equation [39] implies that $\frac{C\Omega}{F}$ will be a minimum when $\Omega\zeta_d = 1$, i.e. when the relaxation time of the viscous damper is chosen to be $\frac{1}{2\pi}$ times the spin period. For ζ_d much larger than this, the large viscous drag does not allow sufficient motion of the small mass to produce much energy loss per cycle. Conversely, for $\zeta_d << \frac{1}{\Omega}$ the small mass simply oscillates at the spin frequency against a viscous drag too small to affect its motion significantly.

Thus for a viscous damper

$$\left(\frac{C\Omega}{F}\right)_{min} = \frac{1}{2}\frac{C}{I_m} \text{ when } \Omega\zeta_d = 1 \tag{41}$$

which is numerically identical to the case I value for $\zeta = 1$. The use of a resonant damper, particularly at $\zeta << 1$, will lead to a minimum weight damper but one effective over a narrow range of spin rates. The viscous damper on the other hand is effective over a broad spin interval.

5. Effect of An Orbital Angular Velocity

The earlier results indicate that with a sun line stationary in inertial space, the spinning satellite with $\dot{\theta}$ and $\dot{\phi}$ damping will come to equilibrium aligned with the sun line, i.e. with $\theta = \phi$. In fact, of course, the orbital angular velocity produces in effect a rotation of the sun line in inertial space and the consequences of this rotation on the alignment of the spacecraft with the moving sun line are of interest. The detailed analysis of the motion of a spinning vehicle relative to a moving coordinate system is at best a complicated process but in this special situation is susceptible to a simpler treatment.

If the satellite is precessing at constant angular velocity about the sun line, its instantaneous angular velocity in the direction perpendicular to the orbital plane at the point of maximum displacement from this plane is:

$$\omega_i = \dot{\phi} \tan \theta \qquad [42]$$

If this velocity is just equal to the orbital angular velocity, ω_o, an equilibrium condition results in which the spinning satellite will, in fact, continue to precess about the sun line but will appear to orbit the sun with its spin axis slightly inclined* by an angle θ_o to the orbital plane such that:

$$\omega_o = \dot{\phi} \tan \theta_o \qquad [43]$$

If the spin momentum vector is directed away from the sun, the inclination will be above the orbit plane and the reverse will be true for the opposite direction of spin. Since

$$\dot{\phi} \propto \omega_\ell^2 \propto \frac{1}{r^2} \qquad [44]$$

and from Kepler's second law

$$\omega_o r^2 = H = \text{constant}$$

then

$$\tan \theta_o = \frac{\omega_o}{\dot{\phi}} \qquad [45]$$

is independent of r and the inclination angle will be constant throughout an eccentric heliocentric orbit.

6. Application to Sunblazer Vehicle

For a short, axially symmetric body, the ratio of the moment about a diameter to that about the axis of symmetry is $\frac{1}{2}$. As the body is lengthened along the spin axis, the ratio increases toward unity. In moment-free precession stability considerations, i.e. the prevention of tumbling or spin conversion, require $I_3 >> I_1$. While this is not necessarily the case when solar

* This possibility was first suggested to me by Professor Leverett Davis.

pressure torques are present, it would still seem prudent to require the major moment of inertia to be about the spin axis so that a practical and acceptable value for the moment ratio is:

$$\frac{I_1}{I_3} = \frac{1}{2} \qquad [46]$$

The minimum relaxation time [23] may then be written:

$$\tau_{min} = 4\,\frac{\omega}{\omega_\ell^2} = 4\,\frac{\tau_\ell^2}{\tau_{spin}} \qquad [47]$$

The importance of a short libration period is again evident but the practicalities of achieving large vane areas without resorting to difficult unfolding structures which are light enough to avoid substantial increases in I_1 mitigate against τ_ℓ being much less than 10^3 sec. For example, the design of Figure 1, weighing 10 kg with an 0.2m radius of gyration, having a sail area of 1 m^2 and a moment arm of 1 m will have a libration frequency of:

$$\omega_\ell = \sqrt{\frac{pS\ell}{I}} = \frac{(0.5\cdot 10^{-5})(1)(1)}{\frac{10(0.2)^2}{2}} = 5.10^{-3} \text{ rad/sec}$$

or

$$\tau_\ell = \frac{2\pi}{\omega_\ell} = 1.2 \times 10^3 \text{ sec}$$

If the satellite spin rate is the 1 rps imparted during launch, then:

$$\tau_r = \frac{4(1.2)^2 10^6}{1} = 5.7 \cdot 10^6 \text{ sec} = 66 \text{ days}$$

which would be unacceptably long so that some degree of despinning is indicated.

In Table I the equilibrium inclination angle and the initial alignment time have been calculated for an achievable range of spin rates. It appears that despinning the vehicle to about 6 rpm even for minimal damping $\frac{C\Omega}{F} = 10$ (Table II) would allow it to align itself within 0.5° of the sun line and would permit alignment 24 days after injection. For more optimal damping, $\frac{C\Omega}{F} = 1$ (Table I), the inclination angle would still be less than one degree but equilibrium would be achieved 5 days after launch.

Recalling that for a uniformly loaded drum-shaped satellite of radius R and mass M, the axial moment of inertia C is:

$$C = M \frac{R^2}{2} + I_m$$

Further, since $I_m = mR^2$ for a peripherially mounted damper ($r_1 = R$), then:

$$\frac{C}{I_m} = \frac{(M + 2m)}{2m} = \frac{1}{2} (\frac{M}{m} + 2)$$

and at damper resonance [38] becomes:

$$\frac{C\Omega}{F} = \frac{1}{4} (\frac{M + 2}{m}) \zeta \qquad [48]$$

For this to be unity we require:

$$\frac{m}{M} = \frac{\zeta}{4} \cdot \frac{1}{1 - \frac{\zeta}{2}} = \frac{\frac{\zeta}{4}}{1 - \frac{\zeta}{2}} \qquad [49]$$

TABLE I. For $\frac{C\Omega}{F} = 1$

Spin Period t_s	Spin Rate	Precession Period[1] t_p	Relaxation Time[1] t_r	Spin Axis Inclination to Orbit Plane at Equilibrium[2] α_o	Alignment Time Following Injection[3] $\frac{t_a}{t_r}$	$\frac{t_a}{t_p}$	t_a
1	1 rps	66.7 days	10.6 days	8.91°	4.40	0.70	46.6 days
3.33	0.3	20	3.18	2.70	4.46	0.71	14.2
10	0.1	6.67	1.06	0.90	4.49	0.715	4.76
33.3	0.03	2.0	0.318	0.27	4.50	0.716	1.43
100	0.01	0.667	0.106	0.09	4.50	0.716	0.477

1. Based on a 1200 second libration period.
2. Based on the orbital angular rate $\omega_o = 0.85°$/day applying to Sunblazer after launch.
3. Defined as the time for θ to rotate from 90° to within 1° of the equilibrium value, i.e., $\frac{t_a}{t_r} = \ln\left(\frac{90 - \alpha_o}{1}\right)$.

TABLE II. For $\frac{C\Omega}{F} = 10$

Spin Period t_s	Spin Rate Ω	Precession Period[1] t_p	Relaxation Time[1] t_r	θ_{eq} Equilibrium Inclination Angle[2]	Alignment Time Following Injection[3] $\frac{t_a}{t_r}$	$\frac{t_a}{t_p}$	t_a
1	1 rps	33.7 days	53.5 days	4.55°	4.45	7.07	238 days
3.33	0.3	10.2	16.1	1.37	4.47	7.10	71.9
10	0.1	3.37	5.35	0.455	4.48	7.12	23.9
33.3	0.03	1.02	1.61	0.137	4.50	7.15	7.25
100	0.01	0.0337	0.535	0.0455	4.50	7.15	2.41

1. Based on a 1200 second libration period.
2. Based on the orbital angular rate ω_o = 0.85°/day applying to Sunblazer after launch.
3. Defined as the time for θ to rotate from 90° to within 1° of the equilibrium value, i.e., $\frac{t_a}{t_r} = \ln\left(\frac{90 - \alpha_o}{1}\right)$.

The relative weight of the damper may be smaller as its motion becomes more highly resonant (i.e. as $\zeta <<1$). Unfortunately, this also reduces the range of spins over which $\frac{C\Omega}{F} \approx 1$. Using $\zeta = 0.5$ implies initial damping which is not difficult to achieve; implies that the damper mass, if mounted at the satellite periphery, must be about $\frac{1}{6}$ of the satellite mass and will provide effective damping over a 10:1 range of spin rates (Figure 4). If the damper is mounted further from the spin axis or, even better, is incorporated in the antenna or vane assembly, a smaller mass is usable. For a nominal spin rate of 2 rpm (0.03 rps), the resonant period of the damper must be 30 sec and is achievable by simple mechanical means. For less than optimum damping, such as when $\frac{C\Omega}{F} = 10$ (Table II), the required damper mass is considerably reduced and is only $\frac{1}{80}$ of the satellite mass for the same ($\zeta = 0.5$) damper bandwidth.

The question of the constancy of the spin rate is an important one. It is difficult to envisage just what the dissipation mechanism in $\dot{\psi}$ could be that would produce appreciable spin decay. The eddy current losses experienced by spinning satellites in earth orbit are trivial beyond a few earth radii, and other body force mechanisms only produce $\dot{\theta}$ and $\dot{\phi}$ damping. As early experimental proof of this assertion, Pioneer VI's spin rate has been observed to decrease from 59 to about 58 rpm after two months in interplanetary space. Even this small decrease may be attributable to some suspected minor gas leakage in the attitude control system.

References

1. Marchetta, H.: S.M. Thesis on "Stabilization and Damping of a Small Satellite by Reaction Jet Control System," 1966.

2. Peterson, C. A.: "Use of Thermal Radiation Effects in Spacecraft Attitude Control," June 1966 (CSR R-66-3).

3. Carroll, J. R. and R. C. Limburg: "Dynamics of a Solar Pressure Stabilized Satellite," June 1965 (CSR T-65-1).

4. Colombo, G.: "Passive Stabilization of a Sunblazer Probe by Means of Radiation Pressure Torque," (CSR TR-66-5).

5. Thomson, W. T.: "Introduction to Space Dynamics," Wiley, 1961, p. 216.

6. I am very grateful to Professor G. Colombo for his helpful criticisms and suggestions on this manuscript.

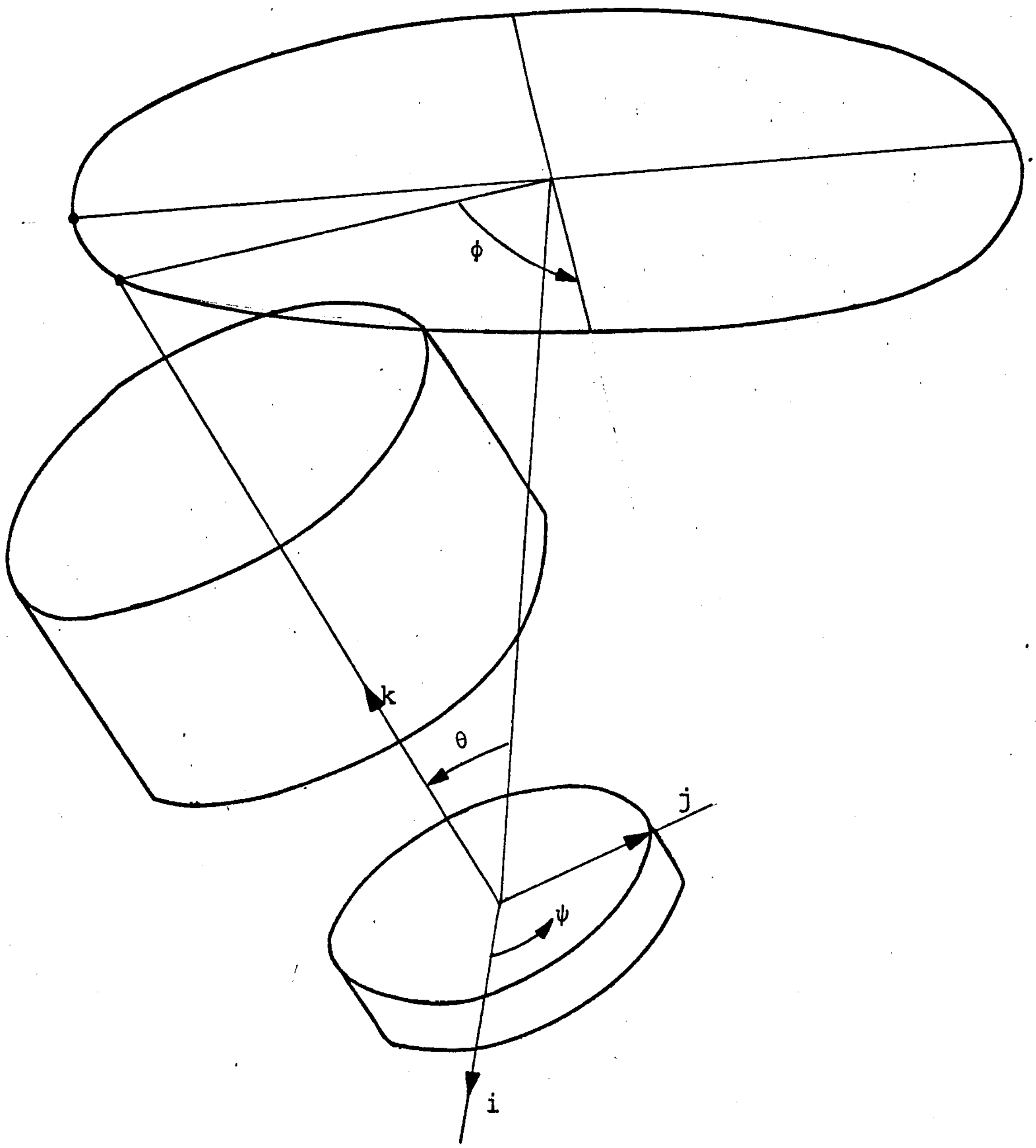

Figure 1

Coordinate Axes

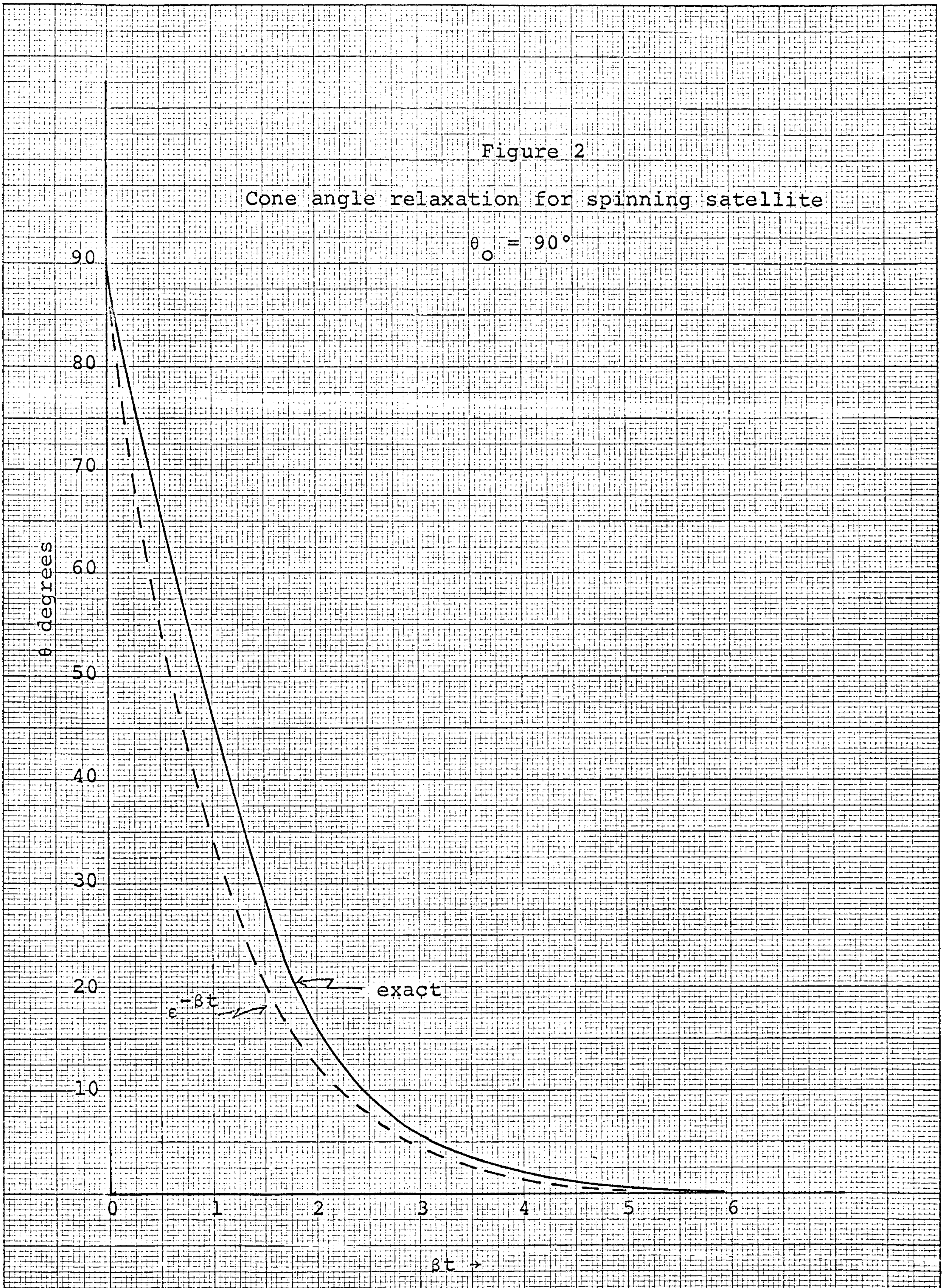

Figure 2

Cone angle relaxation for spinning satellite

$\theta_o = 90°$

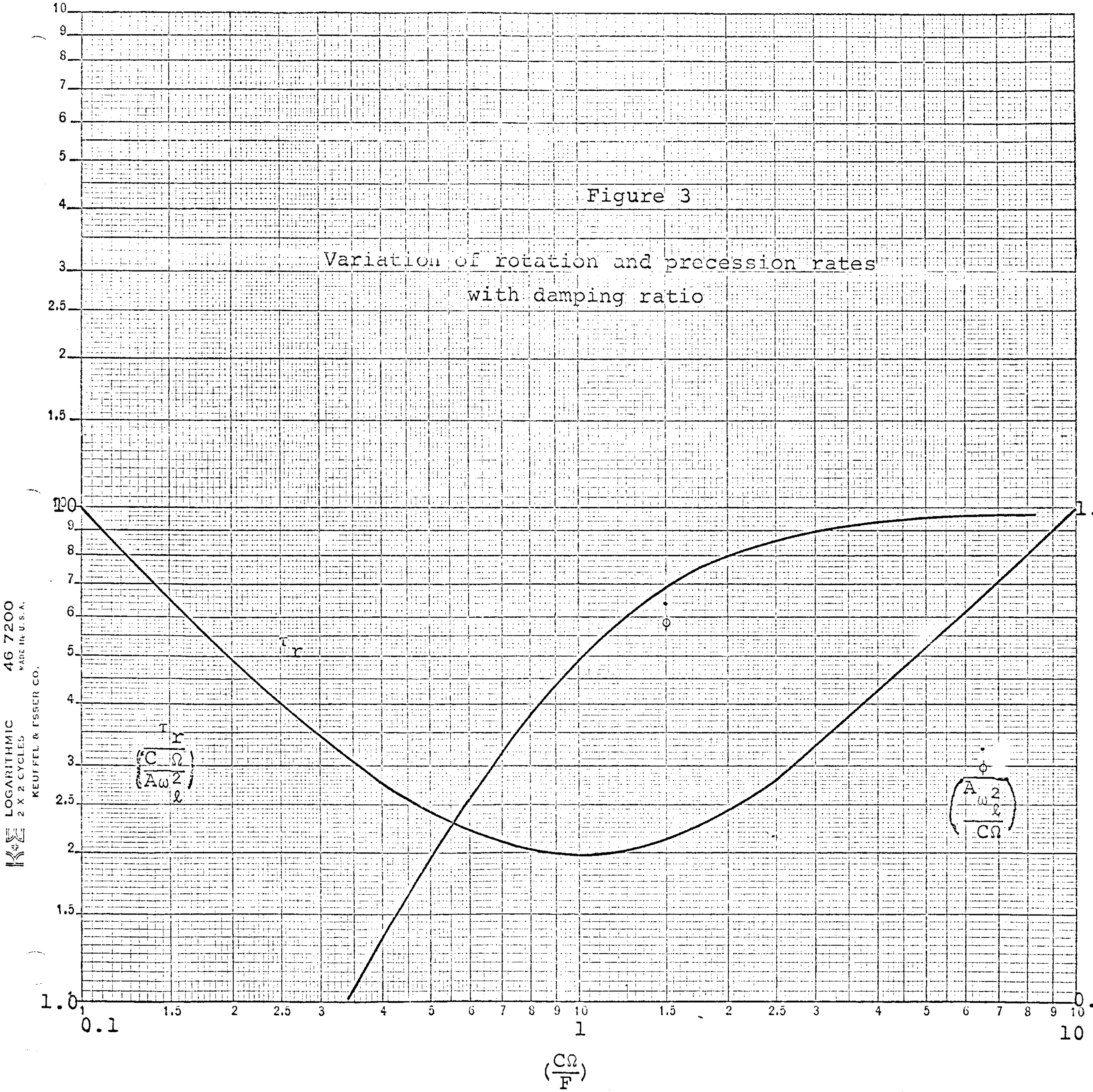

Figure 3

Variation of rotation and precession rates with damping ratio

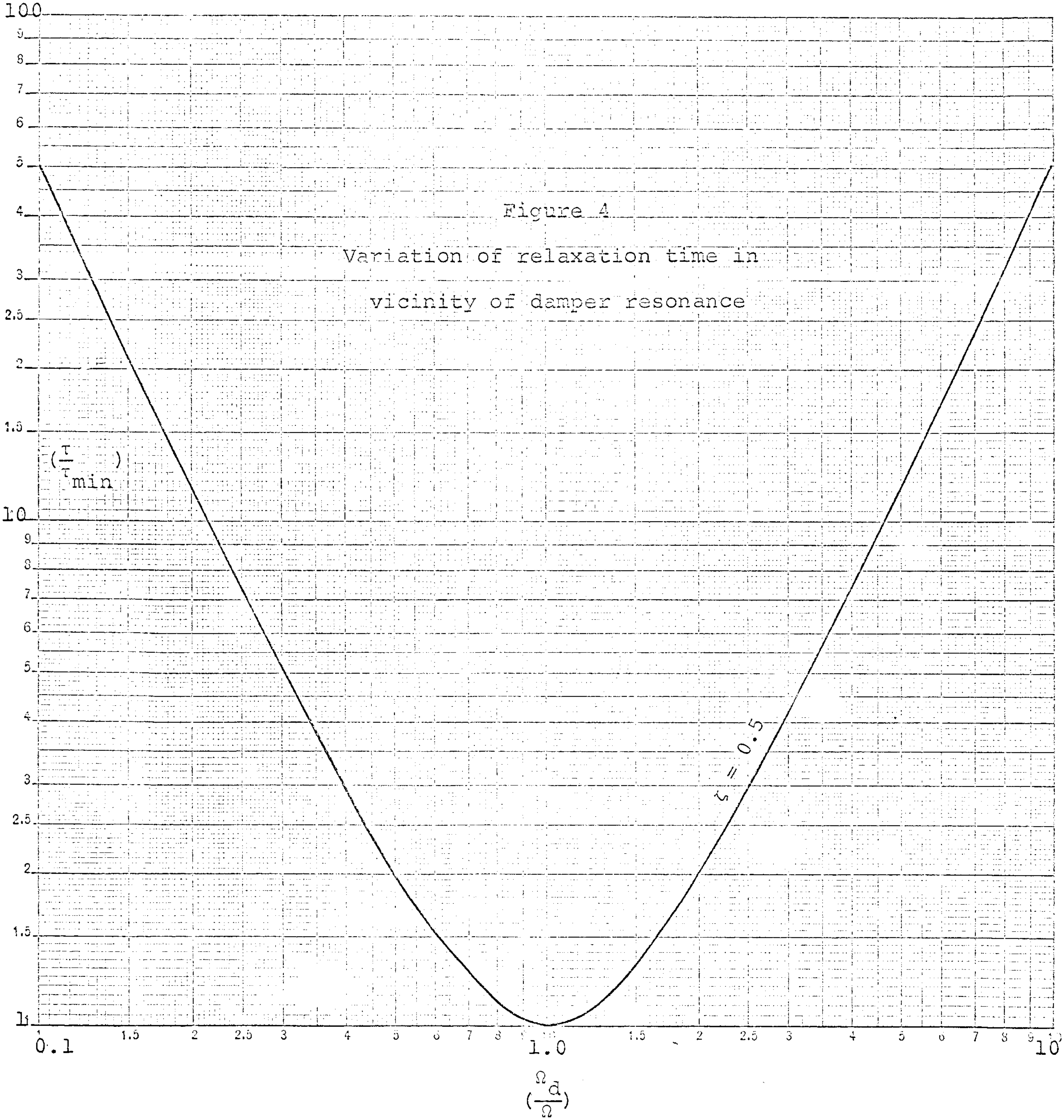

Figure 4

Variation of relaxation time in vicinity of damper resonance

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